Wave equation based prestack depth migration of converted wave data in TIV media

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Summary

This paper describes $\omega - x$ finite-difference prestack depth migration of converted-wave data in TIV media. The migration is accomplished by numerical wavefield extrapolation where the upgoing and downgoing wavefields are extrapolated in depth with space-variant filter operators. The filter operators are precomputed in a separate filter operator design program and accessed by the migration algorithm. The ratio between the temporal frequency and the local velocity, together with the anisotropic media type (defined by one specific set of Thomsen parameters) and wave mode (qP or qSV), are used to determine the correct filter operator at each grid point during the downward and upward continuation. Imaging is performed by crosscorrelating the source wavefield with the data wavefield divided by the source illumination at each depth level.

Introduction

Wave equation based prestack depth migration techniques might provide better solutions to the imaging of converted wave data than traditional Kirchhoff techniques. In areas with complex structures and strong lateral velocity variations several examples using compressional waves have shown that depth migration based on wave equation algorithms give significantly better results than Kirchhoff depth migration methods. We expect this to be true also for converted waves. Several authors have investigated methods for wave equation based prestack depth migration using compressional waves. Holberg (1988) presented a method for 2D numerical wave-field extrapolation in the space-frequency domain using space-variant symmetric convolutional operators. Sollid and Arntsen (1993) extended the work of Holberg to describe wave extrapolation in 3D. Another 3D solution was presented by Mittet (2001). In all of these papers the wave extrapolation algorithms were derived and tested with respect to extrapolation of compressional waves in isotropic media. These techniques can however, easily be modified to handle extrapolation of converted waves in TIV media.

The earth is anisotropic in nature and converted waves are more affected by this than compressional waves. A migration technique for converted waves must be designed to handle anisotropy. Thomsen (1986) considered wave propagation in elastic anisotropic media. Zhang et al. (2001) combined the work of Holberg and Thomsen and derived formulas for seismic wavefield depth extrapolation in anisotropic media using iterative application of spatial convolution.

In this paper, a method for anisotropic wave equation prestack depth migration of converted wave data is presented. The method is valid for TIV media with arbitrary strength of the anisotropy. The method is evaluated based on a study of migration impulse responses.

Prestack depth migration algorithm

Most prestack depth migration algorithms can be expressed as a wave field extrapolation step followed by an imaging condition.The wave field extrapolation step can be derived from the Kirchhoff integral

$$p(\boldsymbol{x}, \omega) = \int_S d\mathbf{S} \cdot \nabla g(\boldsymbol{x}, \boldsymbol{x}_s, \omega)\ p(\boldsymbol{x}_s, \omega), \tag{1}$$

where $\boldsymbol{x}$ and ω denote spatial position and (angular) frequency, respectively, while $p(\boldsymbol{x}, \omega)$ is the extrapolated wave field at depth. The integral extends over a surface S and $p(\boldsymbol{x}_s, \omega)$ is either the data or a source wave field at location $\boldsymbol{x}_s$, while $g(\boldsymbol{x}, \boldsymbol{x}_s, \omega)$ is the associated Greens function or it's complex conjugate.

The Greens function used in wave equation finite-difference prestack depth migration is

$$g(\boldsymbol{x}, \boldsymbol{x}_s, \omega) = \exp[-ikr(\boldsymbol{x}, \boldsymbol{x}_s)]/r(\boldsymbol{x}, \boldsymbol{x}_s), \tag{2}$$

where $r(\boldsymbol{x}, \boldsymbol{x}_s)$ is the distance from point $\boldsymbol{x}$ to point $\boldsymbol{x}_s$ and k is the wavenumber. The Greens function given in equation (2) is strictly speaking only valid for constant velocity. However, by implementing the Kirchhoff integral in equation (1) iteratively in depth, assuming that the velocity model is locally smooth and applying a laterally varying extrapolator, inhomogeneous velocity fields can be handled. The Greens function may be approximately represented by a finite length discrete filter. Downward wave field extrapolation can then be expressed as a space-variant convolution in the $\omega - x$ domain by

$$p(\omega, x, z + \Delta z) = \sum_{l=0}^{L} w_l [p(\omega, x + l\Delta x, z) + p(\omega, x - l\Delta x, z)], \tag{3}$$

where w_l are the numerically optimized discrete filter coefficients of the appropriate filter. The filter is then a approximation of the exact extrapolation operator. Here is Δz the extraploation step length in the vertical direction while Δx is the horizontal distance between data points. In TIV media the filter coefficients becomes functions of ω/V_{P0}, the Thomsen parameters ε, δ and the ratio V_{P0}/V_{S0}. The filter coefficients will also depend on wave mode (qP or qSV). V_{P0} and V_{S0} are the vertical qP- and qSV-wave velocities, respectively.

The wave field extrapolation is done separately for the data and the source wave fields and an image can be obtained by cross-correlation of the two extrapolated wave fields. The cross correlation imaging technique used is derived from inversion theory (Amundsen et al., 1993), and is an approximation to the gradient of the data with respect to the velocity field.

The exact extrapolation operator

The exact representation of the extrapolation operator may be expressed in the frequency, wavenumber domain as

$$G(k,\omega) = \exp[-ik_z\Delta z], \tag{4}$$

where k_z is the vertical wavenumber.

For TIV media the phase velocities for qP- and qSV-waves can be expressed exactly as (Thomsen, 1986)

$$\frac{V^2(\theta)}{V_{P0}^2} = 1 + \varepsilon\sin^2\theta - \frac{f}{2} \pm \frac{f}{2}\sqrt{(1+\frac{2\varepsilon\sin^2\theta}{f})^2 - \frac{2(\varepsilon-\delta)\sin^2 2\theta}{f}}, \tag{5}$$

where θ is the phase angle and $f = 1 - V_{S0}^2/V_{P0}^2$. The Thomsen parameters ε and δ describe the strength of the anisotropy.

By introducing plane waves equation (5) yields a dispersion equation for TIV media which becomes

$$k_z = \pm\sqrt{\frac{-a \pm \sqrt{a^2-4b}}{2}}, \tag{6}$$

where $\pm$ in front of the square-root is related to up- or down-going waves and the $\pm$ inside the square-root is related to qP- or qSV-waves, respectively. The coefficients a and b are given by

$$a = \frac{(-2(1+\varepsilon-f-f\delta)k_x^2 + (\frac{\omega}{V_{P0}})^2(2-f))}{(f-1)} \tag{7}$$

$$b = \frac{((2+2\varepsilon-f)(\frac{\omega}{V_{P0}})^2k_x^2 - (\frac{\omega}{V_{P0}})^4}{(f-1)} - \frac{(1-f)(1+2\varepsilon)k_x^4}{(f-1)}. \tag{8}$$

This dispersion equation will provide the exact forward phase-shift extrapolation operator for qP- and qSV-waves propagating in TIV media.

The approximated extrapolation operator

The approximation of the extrapolation operator can be formulated as the numerical optimization problem of finding the filter coefficients w_l of a finite-length filter with a Fourier transform that approximates the desired Fourier transform of the exact phase-shift extrapolator defined in equation (4), in a least-squares sense. The filter operators are pre-computed and made accessible in tables such that the ratio between the temporal frequency and the local velocity, the Thomsen parameters, and the wave mode (qP or qSV) are used to determine the correct operator at each spatial grid point during the downward continuation. To ensure stability of the migration scheme, the optimization of the operators is constrained such that the evanescent energy and waves propagating at higher angles than the maximum design angle are damped in the iterative downward continuation process.

Numerical results

The performance of the suggested migration scheme is evaluated through a study of migration impulse responses. The P and P-S impulse responses for a two layered TIV medium with $V_{P0} = 2000m/s$, $V_{S0} = 1000m/s$, $\varepsilon = 0.05$ and $\delta = 0.05$ in the first layer and $V_{P0} = 2500m/s$, $V_{S0} = 1250m/s$, $\varepsilon = 0.1$ and $\delta = 0.05$ in the second layer are shown in figures 1 and 2. The P-P and P-S impulse responses for the isotropic case are for comparison shown in figures 3 and 4.

Conclusion

A prestack migration scheme for converted wave data using space-variant filter (convolutional based) operators has been developed for TIV media. Impulse responses demonstrate good dip response and correct kinematic behavior.

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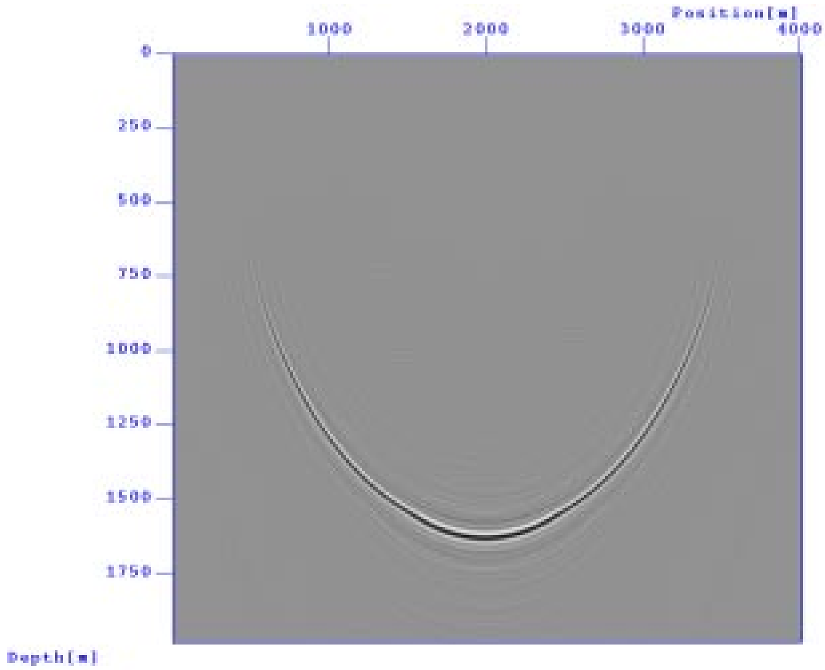

Fig. 1: P-P Impulse response in TIV media with two anisotropic layers.

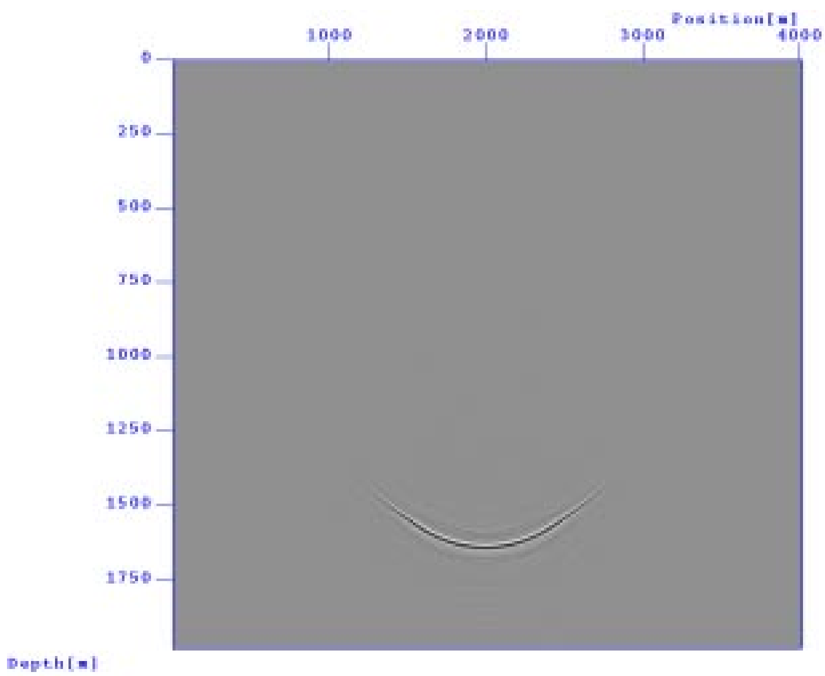

Fig. 2: P-S Impulse response in TIV media with two anisotropic layers.

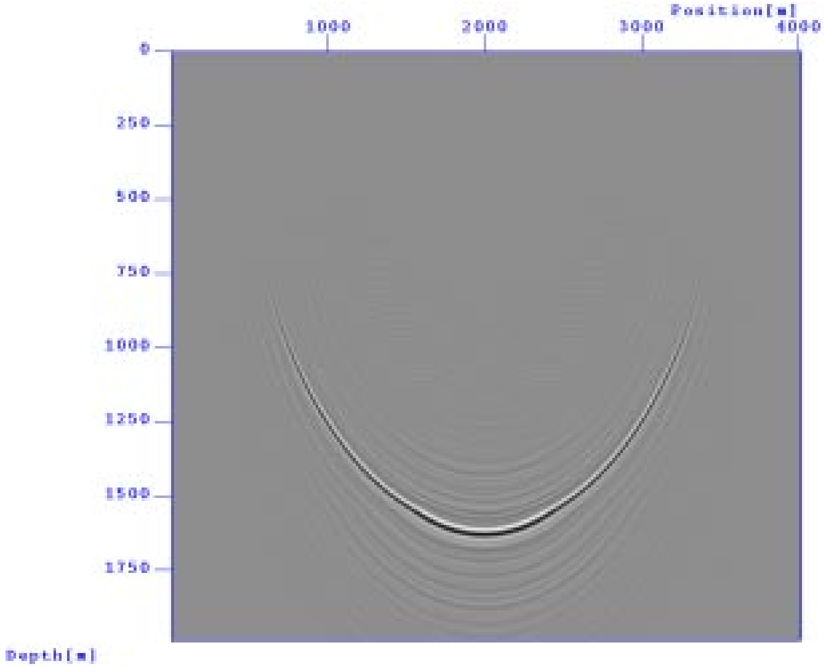

Fig. 3: P-P Impulse response in TIV media with two isotropic layers.

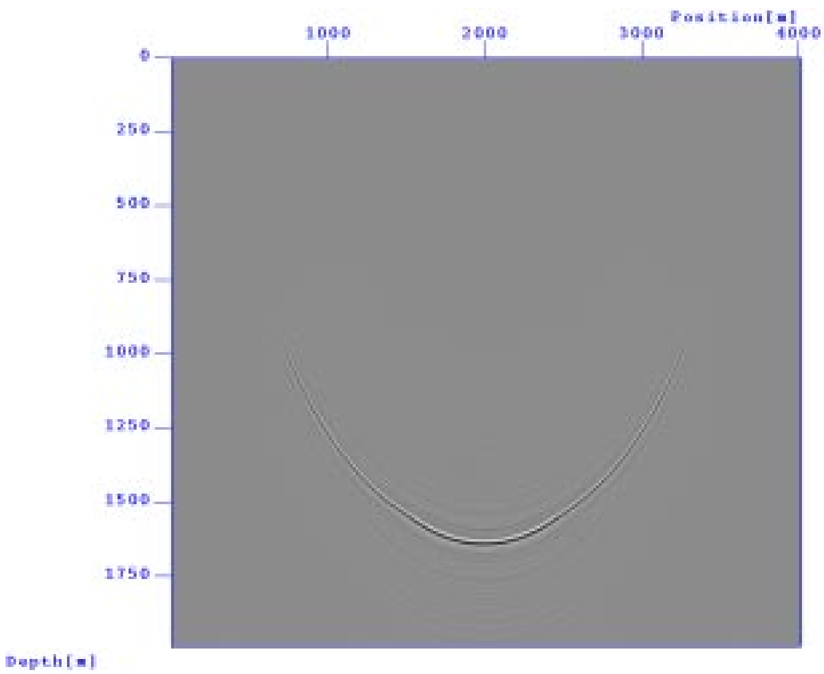

Fig. 4: P-S Impulse response in TIV media with two isotropic layers.