# P001 VELOCITY ANALYSIS WITH A TWO-TERM NON-HYPERBOLIC TRAVELTIME APPROXIMATION 

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#### Abstract

Non-hyperbolic approximations of time-offset curves usually have at least three terms. This complicates the practical calculation and interpretation of velocity spectra. We show here how velocity spectra can be improved by using a non-hyperbolic traveltime approximation with only two terms.


## Introduction

Standard velocity analysis uses hyperbolic traveltime curves parameterized by two coefficients:

$$
\begin{equation*}
t^{2}(x)=a_{0}+a_{2} x^{2} \tag{1}
\end{equation*}
$$

Optimal coefficients for each reflector are found by scanning the ( $a_{0}, a_{2}$ ) space and computing the coherency of signals along the obtained traveltime curves.

In many situations seismic reflection traveltimes deviate significantly from a hyperbola. To take this into account, equation (1) is often replaced by a more complicated traveltime function. Different types of non-hyperbolic traveltime approximations have been presented in the literature. One approach consists of adding an extra term to equation (1) (Taner and Koehler, 1969; Hake et al., 1984):

$$
\begin{equation*}
t^{2}(x)=a_{0}+a_{2} x^{2}+a_{4} x^{4} \tag{2}
\end{equation*}
$$

The third term of this equation can be modified as proposed by Tsvankin and Thomsen (1994):

$$
\begin{equation*}
t^{2}(x)=a_{0}+a_{2} x^{2}+\frac{a_{4} x^{4}}{1+a_{5} x^{2}} \tag{3}
\end{equation*}
$$

Another approach keeps a hyperbola, but allows a shift along the time axis (de Bazelaire, 1988; Castle, 1994)

$$
\begin{equation*}
(t(x)-\tau)^{2}=b_{0}+b_{2} x^{2} \tag{4}
\end{equation*}
$$

All these equations are much more accurate than the hyperbolic approximation (1). They all involve an extra parameter (at least), and reduce to the hyperbolic equation (1) when this parameter is set to zero. With these equations, the parameter space to scan is now three-dimensional. For each value of vertical traveltime (i.e. of $a_{0}$ ), one should test not only $N_{2}$ values of $a_{2}$, but for instance $N_{2} X N_{4}$ values of ( $a_{2}, a_{4}$ ) for equation (2). Because this would be too computer intensive, simpler strategies have to be used. For instance, $a_{2}$ may be estimated first, keeping only small offsets and assuming that $a_{4}$ is zero. $a_{4}$ can then be scanned for, using the previously determined value of $a_{2}$, and taking all offsets in the data (Gidlow and Fatti, 1990). Scanning $a_{2}$ and $a_{4}$ separately is not optimal because these parameters are not independent. In some situations the signal-to-noise ratio is very poor at small offsets, resulting in uncertainties on $a_{2}$ and in subsequent errors on $a_{4}$ caused by the trade-off between these coefficients.
For velocity analysis, we propose to use a non-hyperbolic traveltime approximation that is much more accurate than the hyperbolic approximation, but which is parameterized by two coefficients only. With this equation, we may obtain more reliable velocity profiles without the practical problems associated to three-parameter equations.

## Method

We usually have some a priori information about the velocity profiles we wish to estimate, e.g. from knowledge about the geology in the area of the survey, from well logs or from previous hyperbolic velocity analyses. Our idea is to use this a priori information to constrain the ensemble of curves tested during the scanning to only contain realistic traveltime curves. This allows to more efficiently extract the information on seismic velocities contained in reflection traveltimes.

We assume the available a priori information can be represented by a distribution of possible realistic depth-velocity profiles (called reference models hereafter). Introducing a general form of traveltime approximation

$$
\begin{equation*}
t^{\alpha}(x)=c_{1} f_{1}(x)+c_{2} f_{2}(x)+c_{3} f_{3}(x)+\ldots \tag{5}
\end{equation*}
$$

where exponent $\alpha$ is e.g. equal to 1 or 2, Causse and Hokstad (2000) and Causse (2002) have explained how the functions $f_{i}(x)$ can be optimally chosen for describing reflection traveltimes in all the reference models. If these models are properly chosen, equation (5) is optimal for accurately describing the traveltime of actual reflections in our data as well. The $f_{i}(x)$ form an orthogonal basis of functions. The coefficients $c_{i}$ can be estimated as for other types of approximations. These coefficients can then be transformed into accurate estimates of the coefficients of other approximations, as explained by Causse (2002). In this paper $\alpha=2$, and we use only two terms of (5):

$$
\begin{equation*}
t^{2}(x)=c_{1} f_{1}(x)+c_{2} f_{2}(x) \tag{6}
\end{equation*}
$$

To explain and illustrate the method we take the velocity model given by the dashed black line in Figure 1. Reflection traveltimes in the exact model were calculated and convolved with a Ricker wavelet to obtain the simple synthetic data shown on the right. We want to estimate the velocity profile from these data. The colored lines show the reference models used to calculate functions $f_{1}(x)$ and $f_{2}(x)$. To have optimal approximations, $f_{1}(x)$ and $f_{2}(x)$ are allowed to vary with depth, i.e. we calculate a set of functions for each possible discrete value of zero-offset traveltime $t_{0}$. The functions are shown in Figure 2 for values of $t_{0}$ corresponding to the reflectors in the exact model. Function $f_{l}(x)$ represents the most important component of the approximation. It describes the trend of squared reflection traveltime curves in the reference models, and we see that its curvature decreases with increasing depth. Function $f_{2}(x)$ is the optimal function for describing the deviation of squared reflection traveltime curves from the trend.

The data are scanned in the following way: for each $t_{0}$ we first scan over $c_{2}$. For each value of $c_{2}, c_{1}$ is automatically chosen to ensure that $t$ is equal to $t_{0}$. at $x=0$ in equation (6). Semblance is then calculated in a window along the obtained curve and stored. Each pair $\left(c_{1}, c_{2}\right)$ is also transformed into an ( $a_{0}, a_{2}$ ) pair and stored. This process is repeated for each value of $t_{0}$.
The proposed procedure provides two semblance maps: one in the $\left(t_{0}, c_{2}\right)$ plane, and the other in the $\left(t_{0}, a_{2}\right)$ or ( $\left.t_{0}, V_{s t a c k}\right)$ plane. The first map can be used for further processing with equation (6), like moveout correction, migration, etc. The second map represents an alternative to conventional velocity spectra obtained with the hyperbolic approximation.


Figure 1. Left: exact velocity model (dashed black line) and reference velocity models (solid lines). Right: synthetic data in the exact model. The colored dots show the exact traveltimes associated to each reflector.


Figure 2: basis functions of equation (6) at reflector depth: $f_{1}$ (left) and $f_{2}$ (right).

With the hyperbolic approximation, we obtained the semblance map shown in Figure 3. The traveltime curves corresponding to the selected peaks are compared with the exact traveltimes on the right. The water bottom reflection has a hyperbolic moveout and a high semblance peak. For deeper reflectors, the moveout is non-hyperbolic, especially below the strong velocity contrast at 520 m . Reflectors 3 and 4 have a very low semblance. Their traveltime cannot be reproduced properly by a hyperbola.
Figure 4 shows the semblance map and approximations of real traveltimes computed with the nonhyperbolic two-term equation (6). Much higher semblance values and more accurate traveltimes are obtained. In the upper part of the semblance map the peaks are narrow and cover a large range of $c_{2}$ values, but they are well defined, with a clear maximum (we applied a zoom on the peaks when picking). Since coefficient $c_{2}$ has no obvious physical meaning, it may be difficult to interpret semblance maps in the $\left(t_{0}, c_{2}\right)$ domain. In Figure 5, a remapping of Figure 4 was performed by transforming $c_{2}$ values into $a_{2}$ values (shown here as $V_{\text {stack }}$ ) as explained above. The peaks are much higher, except for the first hyperbolic reflection, and they are also much closer to the exact RMS values than the ones in Figure 3. Consequently, the new method allows a more accurate reconstruction of the velocity profile, as shown on the right of Figure 5.

## Conclusions

Non-hyperbolic traveltime correction for velocity analysis can be done with a traveltime equation described by two parameters only.
This approach associates the simplicity of conventional velocity analysis (approximations described by only two parameters) with the benefits of non-hyperbolic traveltime corrections. Higher semblance peaks are obtained. The traveltime curves associated with these peaks are accurate in the whole range of offsets (not only at small offsets). During the velocity analysis, the information on velocities contained at all available offsets in the data can be efficiently used in this way.
This method should be particularly beneficial in situations of non-hyperbolic traveltimes, or when the data have a poor signal-to-noise ratio at small offsets.

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Figure 3. Left: velocity spectrum obtained with classical NMO equation. The dashed line shows the exact RMS velocity. Right: hyperbolae for maxima of semblance (circles), compared to the exact traveltimes (solid).


Figure 4. Left: semblance map for coefficient $c_{2}$ of equation (6). Right: traveltime approximations for the maxima of semblance (circles), compared to the exact traveltimes (solid).


Figure 5: Left: semblance map of Figure 4 remapped into the ( $t_{0}, V_{\text {stack }}$ ) domain. The dashed line shows the exact RMS velocity. Right: exact velocity profile, and its reconstruction by the classical and the new methods.

