Velocity analysis with a non-hyperbolic two-term traveltime approximation

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Summary

Non-hyperbolic approximations of offset-traveltime curves usually have at least three terms. This complicates the practical calculation and interpretation of velocity spectra. We show here how conventional velocity spectra (i.e. computed using hyperbolae) can be improved using a non-hyperbolic traveltime approximation with only two terms.

Introduction

Standard velocity analysis uses hyperbolic offset-traveltime curves parameterized by two coefficients:

$$t^2(x) = a_0 + a_2 x^2 . \tag{1}$$

Optimal coefficients for each reflector are found by scanning the (a_0, a_2) space and computing the coherency of signals along the obtained traveltime curves.

In many situations seismic reflection traveltimes deviate significantly from a hyperbola. To take this into account, equation (1) is often replaced by a more complicated traveltime function. Different types of non-hyperbolic traveltime approximations have been presented in the literature. One approach consists of adding an extra term to equation (1) (Taner and Koehler, 1969; Hake et al., 1984):

$$t^2(x) = a_0 + a_2 x^2 + a_4 x^4 . \tag{2}$$

The third term of this equation can be modified as proposed by Tsvankin and Thomsen (1994):

$$t^2(x) = a_0 + a_2 x^2 + \frac{a_4 x^4}{1 + a_5 x^2} . \tag{3}$$

Another approach keeps a hyperbola, but allows a shift along the time axis (de Bazelaire, 1988; Castle, 1994)

$$(t(x) - \tau)^2 = b_0 + b_2 x^2 \tag{4}$$

All these equations are much more accurate than the hyperbolic approximation. They all involve one extra parameter (at least), and reduce to the hyperbolic equation (1) when this parameter is set to zero. With these equations, the parameter space to scan is now three-dimensional. For each value of vertical traveltime (i.e. of a_0), one should test not only N_2 values of a_2, but for instance $N_2 \times N_4$ values of (a_2, a_4) for equation (2). Because this would be too computer intensive, simpler strategies have to be used. For instance, a_2 may be estimated first, keeping only small offsets and assuming that a_4 is zero. a_4 can then be scanned for, using the previously determined value of a_2 and taking all offsets in the data (Gidlow and Fatti, 1990). Scanning a_2 and a_4 separately is not optimal because these parameters are not independent. In some situations the signal-to-noise ratio is very poor at small offsets, resulting in uncertainties on a_2 and in subsequent errors on a_4 caused by the trade-off between these coefficients.

For velocity analysis we propose to use a non-hyperbolic traveltime approximation that is much more accurate than the hyperbolic approximation, but which is parameterized by two coefficients only. With this equation, we may obtain more reliable velocity profiles without the practical problems associated to three-parameter equations.

Method

We usually have some a priori information about the velocity profiles we wish to estimate, e.g. from knowledge about the geology in the area of the survey, from well logs or from previous hyperbolic velocity analyses. Our idea is to use this a priori information to constrain the ensemble of curves tested during the scanning to only contain realistic traveltime curves. This allows to more efficiently extract the information on seismic velocities contained in reflection traveltimes.

We assume the available a priori information can be represented by a distribution of possible realistic depth-velocity profiles (called reference models hereafter). Introducing a general form of traveltime approximation

$$t^\alpha(x) = c_1 f_1(x) + c_2 f_2(x) + c_3 f_3(x) + \ldots , \tag{5}$$

where exponent α is e.g. equal to 1 or 2, Causse and Hokstad (2000) and Causse (2002) have explained how the functions $f_i(x)$ can be optimally chosen for describing reflection traveltimes in all the reference models. If these models are properly chosen, equation (5) is optimal for accurately describing the traveltime of actual reflections in our data. The $f_i(x)$ form an orthogonal basis of functions.

The coefficients c_i can be estimated as for other types of approximations. These coefficients can then be transformed into accurate estimates of the coefficients of other approximations, as explained by Causse (2002).

In this paper we take $\alpha = 2$ and use only two terms of (5):

$$t^2(x) = c_1 f_1(x) + c_2 f_2(x)\ . \qquad (6)$$

To explain and illustrate the method we take the velocity model given by the thick black line in Figure 1. The dotted lines show the reference models used to calculate functions $f_1(x)$ and $f_2(x)$. To have optimal approximations, $f_1(x)$ and $f_2(x)$ are allowed to vary with depth, i.e. we calculate a set of functions for each possible discrete value of zero-offset traveltime t_0. The functions are shown in Figure 2 for values of t_0 corresponding to reflectors in the exact model. Dark markers on the curves indicate larger depths. Function $f_1(x)$ represents the most important component of the approximation. It describes the trend of squared reflection traveltime curves in the reference models, and we see that its curvature decreases with increasing depth. Function $f_2(x)$ is the optimal function for describing the deviation of squared reflection traveltime curves from the trend.

Reflection traveltimes in the exact model were calculated and convolved with a Ricker wavelet to obtain simple synthetic data (Figure 3). We want to estimate the velocity profile from these data.

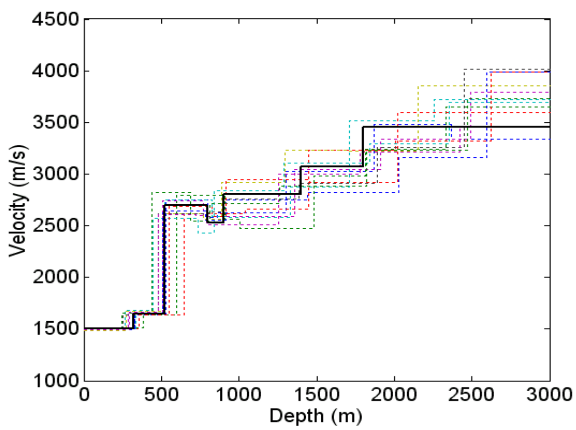

Figure 1: Exact velocity profile (thick line) and reference models (dotted lines)

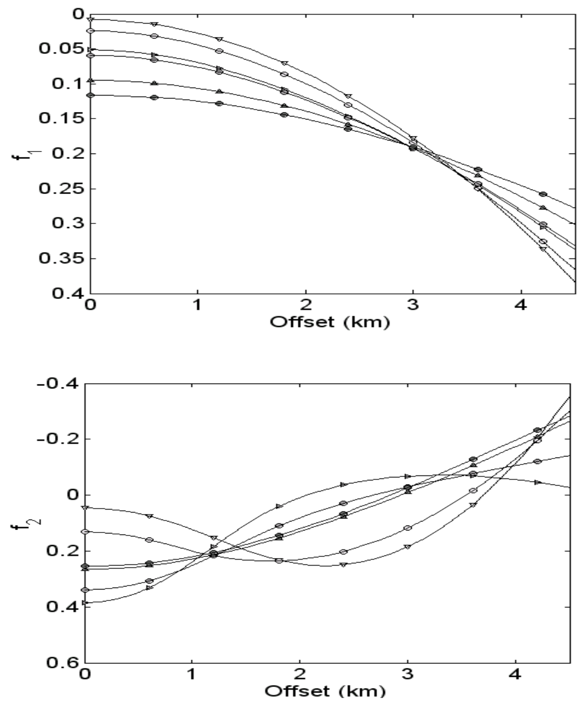

Figure 2: Basis functions of equation (6) at reflector depths.

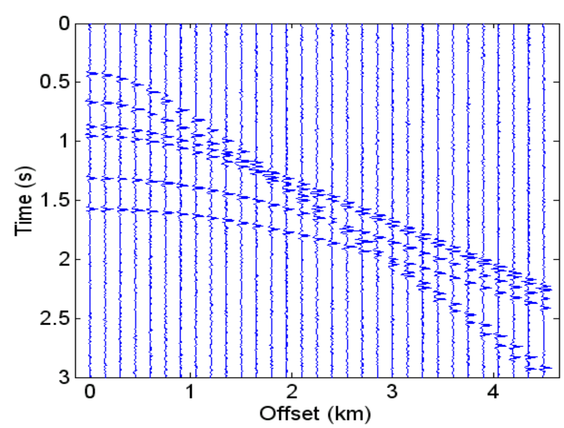

Figure 3: Synthetic data used to illustrate the method

The data are scanned in the following way: for each t_0 we first scan over c_2. For each value of c_2, c_1 is automatically chosen to ensure that t is equal to t_0 at $x = 0$ in equation (6)**.** Semblance is then calculated in a window along the obtained curved and stored. Each pair (c_1, c_2) is also transformed into an (a_0, a_2) pair and stored. This process is repeated for each value of t_0 .

The proposed procedure provides two semblance maps: one in the (t_0, c_2) plane, and the other in the (t_0, a_2) plane. The first map can be used for further processing with equation (6), like moveout correction, migration, etc. The second map represents an alternative to conventional velocity spectra obtained with the hyperbolic approximation (we could display V_{stack} instead of a_2).

With the hyperbolic approximation, we obtained the semblance map shown in Figure 4. The traveltime curves corresponding to the selected peaks are compared with the exact traveltimes in Figure 5. The water bottom reflection has a hyperbolic moveout and a high semblance peak. For deeper reflectors, the moveout is non-hyperbolic, especially below the strong velocity contrast at 520 m. Reflectors 3 and 4 have a semblance about 0.2 only. Their traveltime cannot be reproduced properly by a hyperbola.

Figures 6 and 7 show the semblance map and approximations of real traveltimes computed for the non-hyerbolic two-term equation (6). Much higher semblance values and more accurate traveltimes are obtained. In the upper part of the semblance map the peaks are narrow and cover a large range of c_2 values, but they are well defined, with a clear maximum (we applied a zoom on the peaks when picking). Since coefficient c_2 has no obvious physical meaning, it may be difficult to interpret semblance maps in the (t_0, c_2) domain. In Figure 7, a remapping of Figure 6 was performed by transforming c_2 values into a_2 values as explained above. The peaks have now moved to positions close to the ones in Figure 4, but they are much higher, except for the first hyperbolic reflection. The new method also positioned the peaks more properly, since the velocity model reconstructed from these peaks is more accurate than for the peaks of Figure 4 (see Figure 8).

Conclusions

Non-hyperbolic traveltime correction for velocity analysis can be done with a traveltime equation described by two parameters only.

This approach associates the simplicity of conventional velocity analysis (using hyperbolae) with the benefits of non-hyperbolic traveltime corrections. Higher semblance peaks are obtained. The traveltime curves associated with these peaks are accurate in the whole range of offsets (not only at small offsets). During the velocity analysis, the information on velocities contained at all available offsets in the data can be efficiently used in this way.

This method should be particularly beneficial in situations of non-hyperbolic traveltimes, or when the data have a poor signal-to-noise ratio at small offsets.

Acknowledgments

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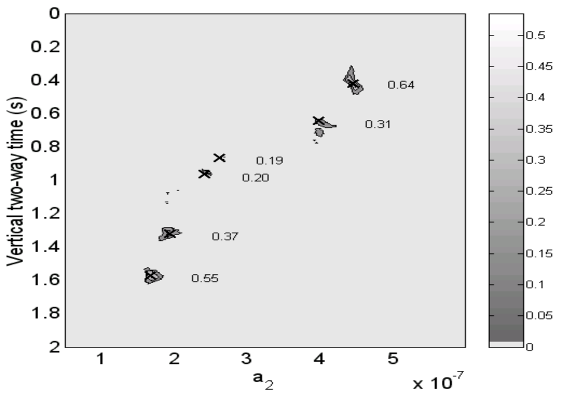

Figure 4: Semblance map obtained with hyperbolic traveltime correction. The crosses indicate the selected semblance peaks and the numbers indicate the peak values.

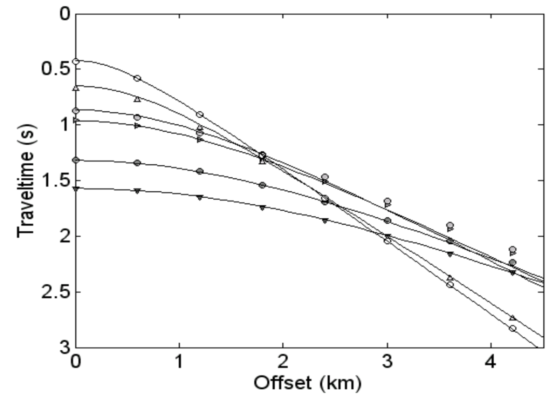

Figure 5: hyperbolic traveltime approximations corresponding to semblance peaks in Figure 4. The markers show the exact traveltimes for the different reflectors.

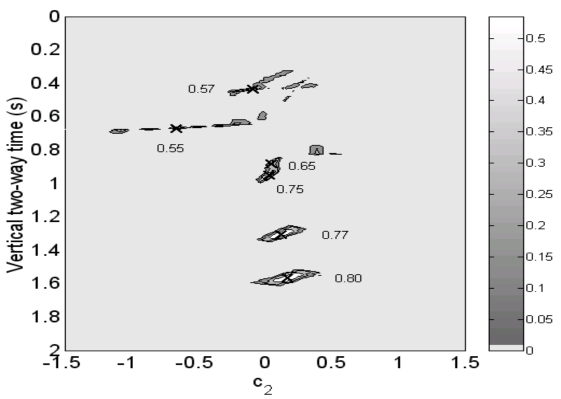

Figure 6: Semblance map obtained with the non-hyperbolic traveltime correction. (6).

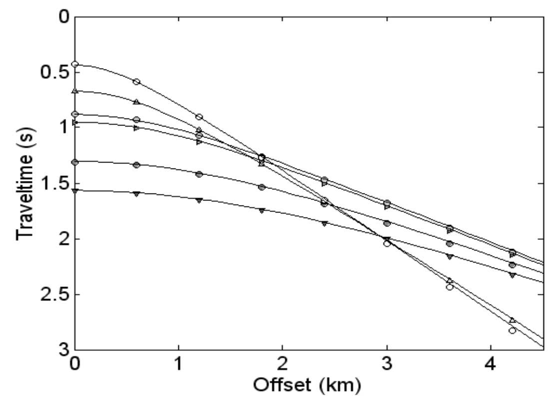

Figure 7: traveltime approximations (6) for the semblance peaks in Figure 6

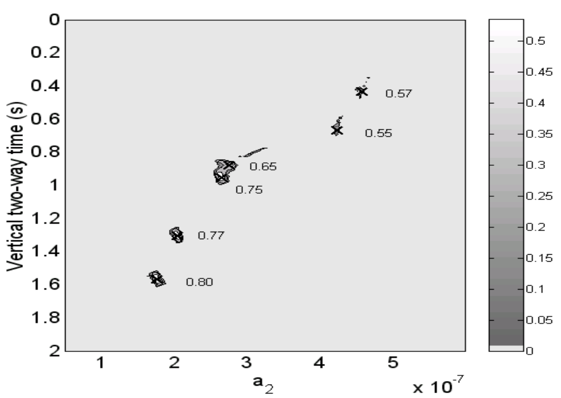

Figure 7: Semblance map obtained with the non-hyperbolic traveltime correction, remapped into the (t_0, a_2) domain.

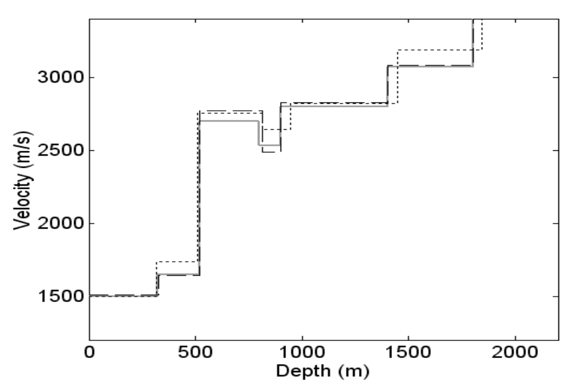

Figure 8: velocity profile reconstructed with the hyperbolic (dotted) and the non-hyperbolic approximation (dashed). The thick grey line shows the exact profile.