The influence of anisotropy on elastic full-waveform inversion

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Trondheim
April 28th 2015
Outline

Introduction

Theory

Model and survey setup

Results

Conclusions

Acknowledgments
• Recently implemented anisotropic (VTI) modeling and FWI.
• Test code on different assumptions used in FWI.
• For synthetic data that are both elastic and anisotropic, investigate quality of inverted $V_{P0}$ model for:
  • Acoustic vs. elastic
  • Isotropic vs. anisotropic
• Try to invert for Thomsen anisotropy parameters $\varepsilon$ and $\delta$. 
Theory

- In FWI we want to find a parameter model $m$ that can produce modeled data $u$ which is close to some measured data $d$.
- Apply a numerical wave operator that maps $m$ from the model domain into the data domain:
  \[
  \mathcal{L}(m) = u. \tag{1}
  \]

- Ideally, find an inverse operator to map $d$ from the data domain to the model domain:
  \[
  m = \mathcal{L}^{-1}(d). \tag{2}
  \]
Theory

- Define a misfit functional:
  \[
  F(m) = \frac{1}{2} \sum_{j=0}^{n_s} \sum_{i=0}^{n_r} ||\hat{u}_{i,j}(m) - \hat{d}_{i,j}||_2^2. \tag{3}
  \]

- The solution is an extreme point of \( F(m) \):
  \[
  m' = \arg \min_m F(m). \tag{4}
  \]
Theory

• Update the model iteratively:

\[ m_{k+1} = m_k - \alpha_k H_k^{-1} \delta m_k. \]  \hspace{1cm} (5)

• Hessian matrix contains second derivatives of the misfit functional
  • Approximated from previous gradients (L-BFGS)
  • Gradients are found via the adjoint method, Mora (1987).

\[ \delta \hat{m}(x) = \sum_{n_s} \int dt \sum_{n_r} \frac{\partial u_i(x_S, x_R, t)}{\partial m(x)} \delta u_i(x_S, x_R, t). \]  \hspace{1cm} (6)

\[ \delta u_i(x_S, x_R, t) = \int_V dV \frac{\partial u_i(x_S, x_R, t)}{\partial m(x)} \delta m(x). \]  \hspace{1cm} (7)
Gradients

\[ \delta \rho = - \sum_{n_s} \int dt \dot{u}_j \dot{\Psi}_j, \]

\[ \delta c_{11} = - \sum_{n_s} \int dt (u_{1,1} + u_{2,2})(\Psi_{1,1} + \Psi_{2,2}), \]

\[ \delta c_{33} = - \sum_{n_s} \int dt u_{3,3} \Psi_{3,3}, \]

\[ \delta c_{13} = - \sum_{n_s} \int dt \left[ \Psi_{3,3}(u_{1,1} + u_{2,2}) + (\Psi_{1,1} + \Psi_{2,2})u_{3,3} \right], \]

\[ \delta c_{44} = - \sum_{n_s} \int dt \left[ (\Psi_{3,1} + \Psi_{1,3})(u_{3,1} + u_{1,3}) + (\Psi_{3,2} + \Psi_{2,3})(u_{3,2} + u_{2,3}) \right], \]

\[ \delta c_{66} = - \sum_{n_s} \int dt \left[ (\Psi_{2,1} + \Psi_{1,2})(u_{2,1} + u_{1,2}) - 2(\Psi_{2,2}u_{1,1} + \Psi_{1,1}u_{2,2}) \right]. \]
Model

- Synthetic model representative of the Gullfaks field
- 10 km long, 3 km deep
- $1001 \times 300$ grid points
- Total of 101 shots and 1001 receivers
- Source: 5 Hz Ricker wavelet
- Receivers: Pressure
- Gradient muted in the water layer
Model
Starting model

![Graph showing the starting model with coordinates and values for x (m), z (m), and VP₀ (m/s).]
Starting model
Figure: Inverted model for $V_{P0}$ with exact $\varepsilon$ and $\delta$, elastic.
Figure: Inverted model for $V_{P0}$ with exact $\varepsilon$ and $\delta$, acoustic.
Elastic inversion

$x$ (m)

$z$ (m)

$V_{P0}$ (m/s)
Results

Figure: Inverted model for $V_{P0}$ with smooth $\varepsilon$ and $\delta$
Figure: Inverted model for $V_{P0}$ with $\varepsilon = \delta = 0$, elastic
Figure: Inverted model for $V_{P0}$ with $\epsilon = \delta = 0$, acoustic.
Results

Figure: Inverted model for $\varepsilon$. 
Starting model
**True model**

![Graph](image-url)
Inverted model
Results

Figure: Inverted model for $\delta$. 
Conclusions

- Four different inversion assumptions applied to an elastic, anisotropic dataset.
- Acoustic approximation holds, due to long offset data.
- Anisotropy cannot be completely neglected.
- A perfect anisotropy model is not needed, but some knowledge is necessary.
- Inverting for $\varepsilon$ and $\delta$ is in principle possible.
Acknowledgments

We thank the ROSE consortium and their sponsors for support.