P273

A New True-amplitude Imaging Condition for Shot-profile Migration

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SUMMARY

We derive a new, stable, true-amplitude cross-correlation type imaging condition for shot-profile migration by changing the source of the downgoing wavefield. Essentially this modification deconvolves the spatial response of the seismic point source and restores the amplitudes in the final image. Numerical examples demonstrate that the resulting common-angle gathers give the correct amplitude-versus-angle behaviour for a simple model.

Introduction

Claerbout's (1971) cross-correlation imaging condition for shot-profile migration has been extensively used due to its simplicity and stability. Designed for structural imaging it is expected and well known that stack amplitudes produced with this approach do not represent a true average of the angle dependent reflectivity. Claerbout (1971) also introduced the U/D imaging condition which gives correct stack amplitudes, but is difficult to implement due to the instability of spectral division. A wide variety of approaches to stabilize the U/D imaging have been investigated (see f.ex. Schleicher et al. 2007 and Guitton et al. 2007) but a simple and satisfactory solution remains to be found.

We derive a stable cross-correlation type imaging condition for shot-profile migration which produces common-angle gathers with correct amplitude-versus-angle relationship. The method is simple to implement and requires a modification of the initial wavefield in the downward propagation and decomposition into plane-waves in the midpoint-slowness domain (de Bruin et al, 1990). Zhang et al. (2007) introduced a similar cross-correlation type imaging condition, but based on ray-theoretical arguments. Here we show that the cross-correlation true-amplitude imaging condition arises naturally from wave theory through the solution of a simple forward problem.

Theory

We consider the simplified situation of a single reflecting interface at depth z with reflection coefficient R and where the wave velocity c and density ρ are constant above the reflector. In the frequency-wavenumber domain the relation between the upgoing wave U and downgoing wave D, can then be expressed as

$$U(\boldsymbol{k},z,\omega) = R(\boldsymbol{k},z,\omega)D(\boldsymbol{k},z,\omega), \tag{1}$$

where $\boldsymbol{k}$ is the horizontal wavenumber with components (k_x,k_y), $\boldsymbol{z}$ is the depth and ω is the frequency. The downgoing wave at depth $\boldsymbol{z}$ is related to the downgoing wave at the surface, D_0, by the relation

$$D(\boldsymbol{k},z,\omega) = \exp\left[-ik_z(\boldsymbol{k},\omega)z\right]D_0(\boldsymbol{k},\omega), \tag{2}$$

where k_z is the vertical wavenumber. For a point source the downgoing wave at the surface is given by

$$D_0(\boldsymbol{k},\omega) = \exp(-i\boldsymbol{k}q)\frac{S(\omega)}{2ik_z(\boldsymbol{k})}, \tag{3}$$

where $\boldsymbol{q}$ is the horizontal position of the source and $S(\omega)$ is the source pulse. Equation (1) can be solved as

$$R(\boldsymbol{k},z,\omega) = \frac{U(\boldsymbol{k},z,\omega)}{D(\boldsymbol{k},z,\omega)}, \tag{4}$$

which becomes by inserting equation (2) into equation (4)

$$R(\boldsymbol{k},z,\omega) = U(\boldsymbol{k},z,\omega)D'^*(\boldsymbol{k},z,\omega). \tag{5}$$

Here D' is a downgoing wavefield

$$D'(\boldsymbol{k},z,\omega) = \exp\left[-ik_z(\boldsymbol{k},\omega)z\right]D'_0(\boldsymbol{k},\omega), \tag{6}$$

with the initial wavefield D'_0 at the surface equal to

$$D'_0(\boldsymbol{k},\omega) = \exp(-i\boldsymbol{k}q)\frac{-2ik^*_z}{S^*(\omega)} \,. \qquad (7)$$

Equation (6) expresses the reflection coefficient as a product between the wavefields U and D'^* instead of a division as in equation (4). This has the advantage of avoiding numerical instabilities related with spectral division.

For a medium with vertical and lateral velocity changes, it can be shown that equation (6) must be modified to read

$$R(\boldsymbol{k_r},\boldsymbol{k}_s,z,\omega) = U(\boldsymbol{k_r},z,\omega)D'^*(\boldsymbol{k}_s,z,\omega)\,, \qquad (8)$$

where $R(\boldsymbol{k}_r,\boldsymbol{k}_s,\omega)$ is the reflectivity matrix, $\boldsymbol{k}_r$ is the horizontal wavenumber of the upgoing wave, and $\boldsymbol{k}_s$ is the horizontal wavenumber of the downgoing wave. The initial downgoing wavefield at the surface, $D'_0(\boldsymbol{k}_s,\omega)$, is given by equation (3), except that the source pulse $S(\omega)$ must be replaced by $S(\omega)(c_0{}^2/\omega^2)$ where c_0 is the velocity at the source position. In the spatial domain equation (8) becomes

$$R(\boldsymbol{x_m},\boldsymbol{h}\;,z,\omega) = U(\boldsymbol{x}_m-\boldsymbol{h},z,\omega)D'^*(\boldsymbol{x}_m+\boldsymbol{h},z,\omega)\,, \qquad (9)$$

where we have also introduced the midpoint and offset coordinates $\boldsymbol{x_m}$ and $\boldsymbol{h}$. Finally, inverse transforming equation (9) over the offset coordinate $\boldsymbol{h}$ and introducing the offset slowness $\boldsymbol{p_h}$ we get after integration over frequencies and summation over shots

$$R(\boldsymbol{x}_m,\boldsymbol{p}_h,z) = \sum_{shots}\int_{-\infty}^{+\infty} d\omega \int_{-\infty}^{+\infty} dh \exp(i\omega\boldsymbol{p}_h)R(\boldsymbol{x}_m,\boldsymbol{h},z,\omega)\,, \qquad (10)$$

which gives the reflectivity matrix as a function of midpoint and slowness.
Equation (9) is similar to Rickett and Sava's (2002) offset imaging condition, except that the downgoing field D is replaced by the modified downgoing field D'. Claerbout's (1971) classical cross-correlation shot-profile imaging condition is obtained as a special case for $h=0$ from equation (9).

Numerical Examples

Figure 1 shows a simple horizontally layered model with density contrasts between the layers. The wave velocity is constant equal to 2000 m/s. A line of synthetic data consisting of 200 shots where acquired across the model using a split-spread geometry with a maximum half-offset of 5km. A single shot is shown in the left hand part of figure 2. The middle part of figure 2 shows a common angle gather computed using equation (10) and then converting the slowness $\boldsymbol{p}_h$ into the corresponding angle using all 200 shots.

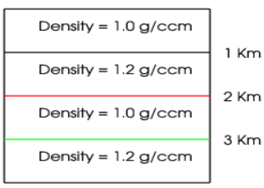

Figure 1 *Horizontally layered acoustic earth model with density contrasts only. The wave velocity is constant equal to 2000 m/s.*

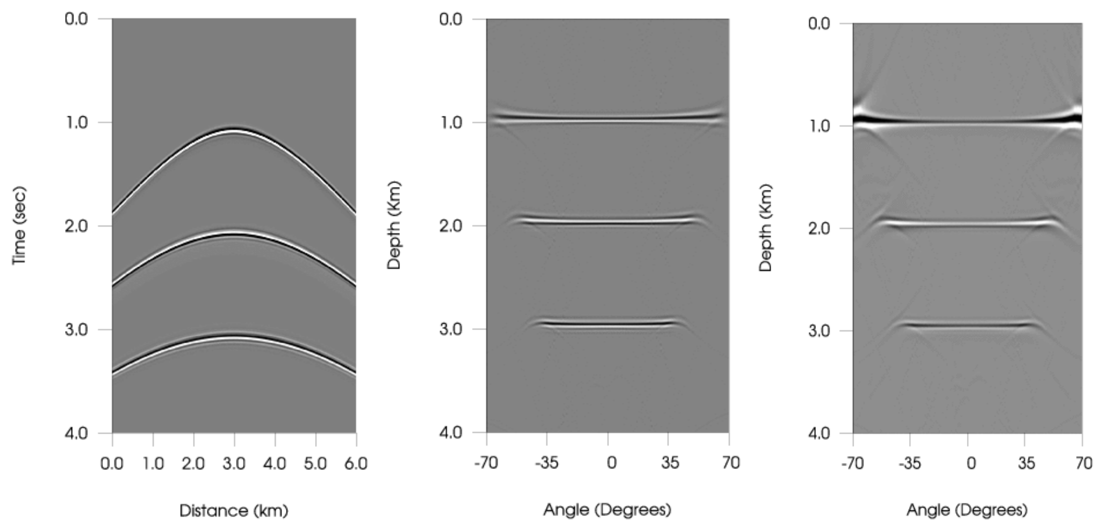

***Figure 2** Single shot (left) acquired over the earth model shown in figure 1. The acquisition geometry is of split-spread type with maximum half offset of 5 km. A common angle gather (middle) was computed using equation (10) and 200 shots. The section to the far right shows an angle gather computed using Rickett and Sava's(2002) offset imaging condition by employing equation (11) and then using equation (10).*

The left hand part of figure 3 shows rms amplitude picks of the angle gather shown in the middle part of figure 2. The rms amplitude values were computed in a 200 ms window around each of the three reflectors. A common scaling factor for all the three amplitude graphs were used, such that the relative amplitude relations between the three reflectors are preserved. We see that the correct amplitude-versus-angle behaviour is recovered, since a reflector with a pure density contrast has an angle independent reflection coefficient.

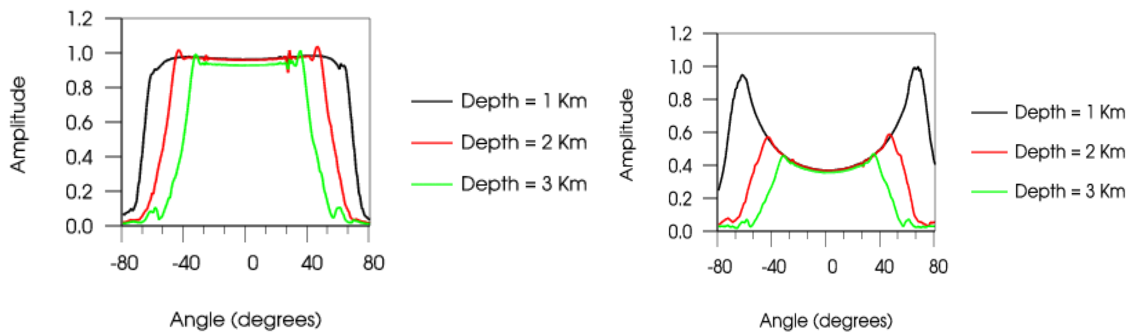

***Figure 3** Rms amplitude picks (left) of the common angle gather shown in the middle of fig 2. Rms amplitude picks (right) of the common angle gather shown in the far right of fig 2.*

The imaging condition given by Rickett and Sava (2002) is designed for structural imaging and not expected to yield correct amplitude-versus-angle behaviour, but it is still of interest to compare the amplitude response of this imaging condition with our condition given by equation (10). Since Rickett and Sava (2002) compute the reflectivity matrix in the midpoint-offset domain

$$R_{RS}(\boldsymbol{x}_m, \boldsymbol{h}, z, \omega) = U(\boldsymbol{x}_m - \boldsymbol{h}, z, \omega) D^*(\boldsymbol{x}_m + \boldsymbol{h}, z, \omega) \quad (11)$$

where D is now the downgoing wavefield due to a point source (see equation (3)), we use this expression to generate angle gathers by using equation (10) with R replaced by R_{Rs}. The resulting gather is shown in the right hand part of figure 2, and the corresponding amplitude picks are shown in the right hand part of figure 3. We see that the angle-versus-angle behaviour is incorrect, particularly for large angles. Claerbout's classical cross-correlation imaging condition corresponds to stacking the gather in the right hand part of figure 2 across all angles, and it is clear that the amplitudes of a stack section in this case do not represent an average of the true angle-dependent reflectivity but is instead biased. For the case of a plane reflector the large angles would contribute too much, and this would in particular tend to overestimate the reflection strength of shallow reflectors relative to deeper ones.

The imaging condition given in equation (10) can also be used for cases with complex velocity models, provided that the corresponding up- and downgoing wavefields are computed correctly. To that end any one-way extrapolation scheme can be used. For cases of non-flat reflectors, although the reflectivity matrix is correctly computed, the mapping from slowness to angle is non-trivial and will also involve corrections of the amplitude. These corrections can, however, be computed separately after the migration itself.

Conclusions

Claerbout's (1971) cross-correlation imaging condition can be modified to a true-amplitude cross-correlation type imaging condition by changing the source of the downgoing wavefield. Essentially this modification deconvolves the spatial response of the seismic point source and restores the amplitudes in the final image.

Acknowledgements

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