Wave equation versus Kirchhoff prestack depth migration of OBC data

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Summary

In areas with complex velocity models, wave equation depth migration has been shown to give significantly better results than the industry standard Kirchhoff depth migration. For very simple velocity models, on the other hand, Kirchhoff depth migration is expected to give similar results as wave equation algorithms. In this paper, we compare Kirchhoff and wave equation prestack depth migration using an ocean bottom cable (OBC) data set from a North Sea area with a velocity model of intermediate complexity. The wave equation depth migration gives better resolution than the Kirchhoff method, but the wave equation algorithm seems not to image the steeply dipping part of the reflectors at large depths as well as the Kirchhoff depth migration. However, this can be an artificial result due to the blurring introduced by the Kirchhoff method.

Introduction

Kirchhoff migration has for a long time been the workhorse for depth imaging of marine surface seismic data. Seismic data processing companies have gained extensive experience with Kirchhoff depth migration methods from application to surface data from all parts of the world. However, in areas with complex velocity structure several examples have shown that depth migration based on wave equation algorithms gives significantly better results than Kirchhoff depth migration methods. Targets below high velocity salt bodies appear in particular to be better resolved by wave equation algorithms (Albertin et al., 2001) than by Kirchhoff methods. In theory this is not surprising since Kirchhoff depth migration usually relies on the ray approximation, which is expected to break down for structures with high contrast velocity fields. On the other hand, the approximations involved in most wave equation migration algorithms seem to be better suited for handling complex velocity structures.

It is very difficult to establish exactly under which conditions the ray approximation break down. Also the range of validity of the approximations involved in different wave equations depth migration methods are not exactly known. Because of the theoretical difficulties involved in choosing between Kirchhoff and wave equation methods for prestack depth migration, we suspect that structures less extreme than sub salt sediments might be better imaged with wave equation depth migration algorithms than with Kirchhoff methods.

In this paper, we present a comparison between a Kirchhoff prestack depth migration method and a wave equation prestack depth migration algorithm using an OBC data set from the North Sea. The target zone is of intermediate complexity involving rotated fault blocks below a high-contrast velocity inversion. The data set contains 360 degree azimuths and gives superior images compared to conventional marine surface seismic data. We show that for this intermediate complexity case, the image computed with a wave equation finite-difference migration algorithm seems to give better resolution than the corresponding image made using a Kirchhoff migration method.

Prestack depth migration algorithms

Most prestack depth migration algorithms can be expressed as a wave field extrapolation step followed by an imaging condition.

The wave field extrapolation step can be derived from the Kirchhoff integral

$$p(\boldsymbol{x},\omega) = \int_S d\mathbf{S}\cdot\nabla g(\boldsymbol{x},\boldsymbol{x}_s,\omega)\; p(\boldsymbol{x}_s,\omega), \qquad (1)$$

where $\boldsymbol{x}$ and ω denote position and (angular) frequency, respectively, while $p(\boldsymbol{x},\omega)$ is the extrapolated wave field at depth. The integral extends over a surface S and $p(\boldsymbol{x}_s,\omega)$ is either the data or a source wave field, while $g(\boldsymbol{x},\boldsymbol{x}_s,\omega)$ is the Greens function or it's complex conjugate.

In most Kirchhoff depth migration schemes the Greens function is approximated with a ray approximation as

$$g(\boldsymbol{x},\boldsymbol{x}_s,\omega) = A(\boldsymbol{x},\boldsymbol{x}_s)\exp[-i\omega\times\tau(\boldsymbol{x},\boldsymbol{x}_s)], \qquad (2)$$

where $A(\boldsymbol{x},\boldsymbol{x}_s)$ is an amplitude and $\tau(\boldsymbol{x},\boldsymbol{x}_s)$ is a travel time function. In the Kirchhoff depth migration scheme utilized here, the travel time is computed using a finite difference technique, while the amplitude is computed with a simplified analytical formula. Downward extrapolation and imaging are then performed in one step by summation of data and source values contributing to the depth location at $\boldsymbol{x}$.

The approximation for the Greens function used in wave equation finite-difference prestack depth migration is

$$g(\boldsymbol{x},\boldsymbol{x}_s,\omega) = \exp[-ik\times r(\boldsymbol{x},\boldsymbol{x}_s)]/r(\boldsymbol{x},\boldsymbol{x}_s), \qquad (3)$$

where $r(\boldsymbol{x},\boldsymbol{x}_s)$ is the distance from point $\boldsymbol{x}$ to point $\boldsymbol{x}_s$, $k=\omega/c(\boldsymbol{x}_s)$ is the wavenumber, and $c(\boldsymbol{x}_s)$ is the velocity (Hale, 1991; Sollid and Arntsen, 1993). The Greens function in equation (3) is strictly speaking only valid for constant velocity, but by implementing the Kirchhoff integral in equation (1) recursively in depth and assuming that the velocity model is locally smooth, laterally inhomogeneous velocity fields can be handled. Our implementation uses a numerically optimized technique for the wave equation finite-difference extrapolation operators as suggested by Mittet (2001). Wave field extrapolation is done separately for the data and the source wave field and an image is obtained by cross correlation of the two extrapolated wave fields.

Imaging of the OBC data set

The OBC data set from the North Sea was acquired using three 6000 meter long multicomponent receiver cables with a receiver group separation of 25 meters. The cables were separated by a distance of 400 meters. Flip-flop shooting was used to generate a 50 by 50 meter hexagonal shooting grid, covering an area of 12000 by 2800 square meters. The data were preprocessed to remove multiples, sorted to common receiver gathers and then binned on a 12.5 meter by 12.5 meter regular surface grid. No interpolation was used, so that each common receiver gather contained a large number of zero traces. A zero-phase bandpass filter was applied to the data prior to the migration.

Figure 1 shows an inline prestack depth migrated image using a 2D version of the Kirchhoff algorithm described in the preceding section. Figure 2 shows the same inline as figure 1, but instead prestack depth migrated using a 2D version of the wave equation algorithm also described in the preceding section. Note that no additional interpolation was performed before application of the wave equation depth migration algorithm.

Overall the two sections in figure 1 and figure 2 are quite similar, but there are some notable differences. The wave equation depth migration seems to yield better resolution than the Kirchhoff method, this is particularly notable in the shallow part of the section and in the target zone below the velocity inversion at a depth of approximately 3000 meter. The Kirchhoff depth migration method appears to handle the steeply dipping parts of the reflectors in a better way than the wave equation algorithm. This is mainly notable in the target area around 3000-4000 meters (compare figure 3 and figure 4), but can also be an artificial result due to smearing.

Discussion

The noticeable difference between the Kirchhoff and the wave equation prestack depth migration methods in the shallow parts of the stack sections shown in figures 1 and 2 is somewhat surprising. The velocity field in this part of the sections is uncomplicated and roughly a function of depth only. However, similar differences have been observed in other studies (Albertin et al., 2001) and can possibly be attributed to the high frequency approximation inherent in the ray tracing.

The differences in the target zone below 3000 meter of the stack sections shown in figures 3 and 4 are easier to understand, since the velocity model shows a sharp inversion at this depth. Sudden velocity changes are poorly handled by the the ray approximation and can result in shadow zones or missing phases of the wave field.

The cross correlation imaging technique used by our wave equation depth migration is derived from inversion theory (Amundsen et al., 1993), and is an approximation to the gradient of the data with respect to the velocity field. However, this approximation neglects partially the geometrical spreading effect and can result in shadow zones in areas where the downward extrapolated source wave field is defocused due to a sharp velocity contrast. This effect may also explain some of the differences observed in the target zone below the velocity inversion at a depth of approximately 3000 meter.

Conclusion

The utilized Kirchhoff and wave equation prestack depth methods seem to give similar results for a typical data set from the North Sea with a velocity field of medium complexity. However, the wave equation approach results in images with higher resolution.

Acknowledgments

The utilized Kirchhoff prestack depth migration method is from the Seismic Unix package (Cohen and Stockwell, 2000). We thank Anders Sollid for modifying this method to accommodate OBC geometries, and we would like to thank Statoil for allowing us to publish this work.

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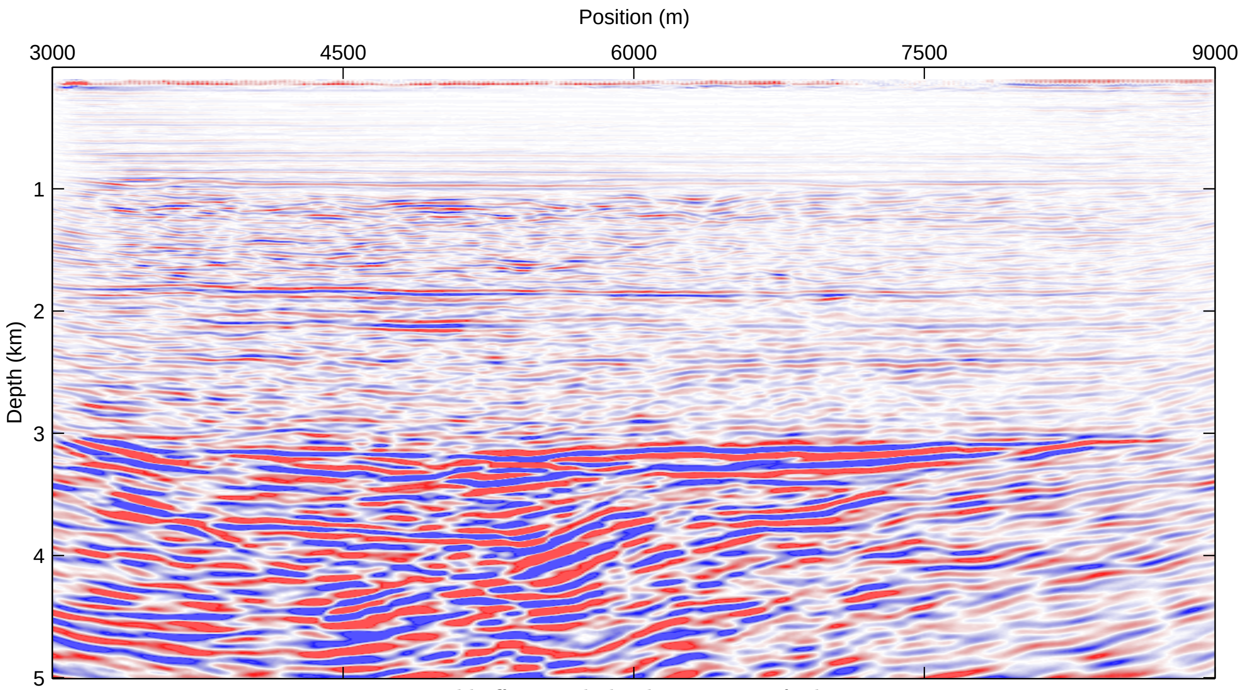

Fig. 1: Kirchhoff prestack depth migration of inline 1079.

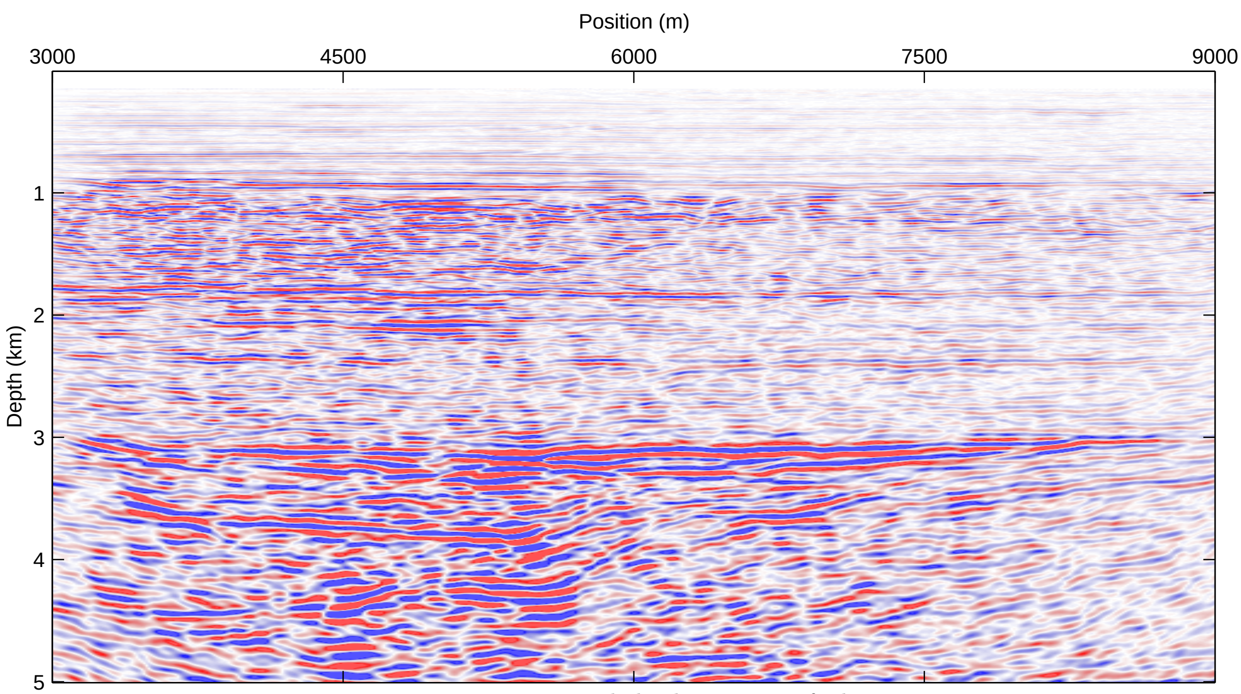

Fig. 2: Wave equation prestack depth migration of inline 1079.

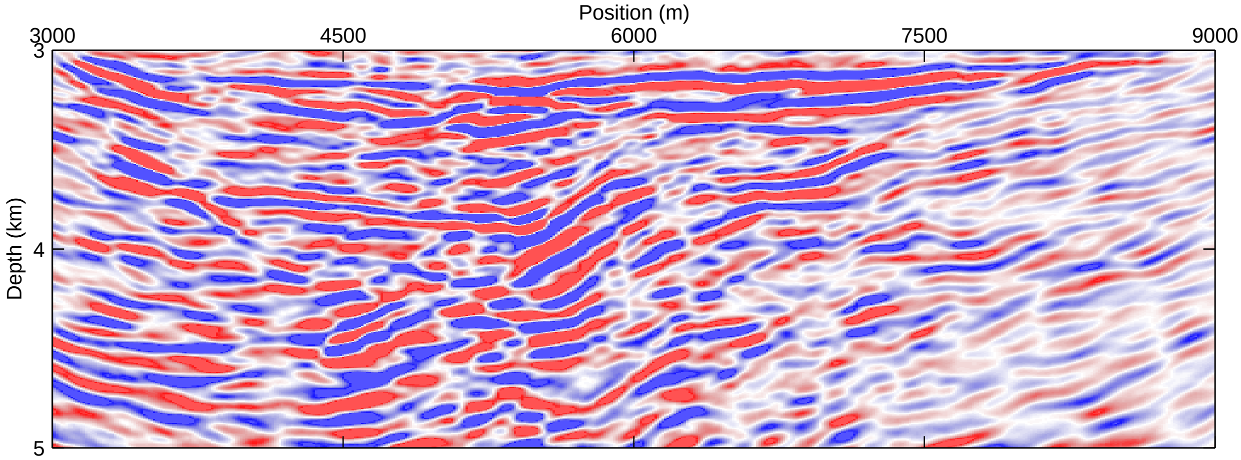

Fig. 3: Kirchhoff prestack depth migration around the target of inline 1079.

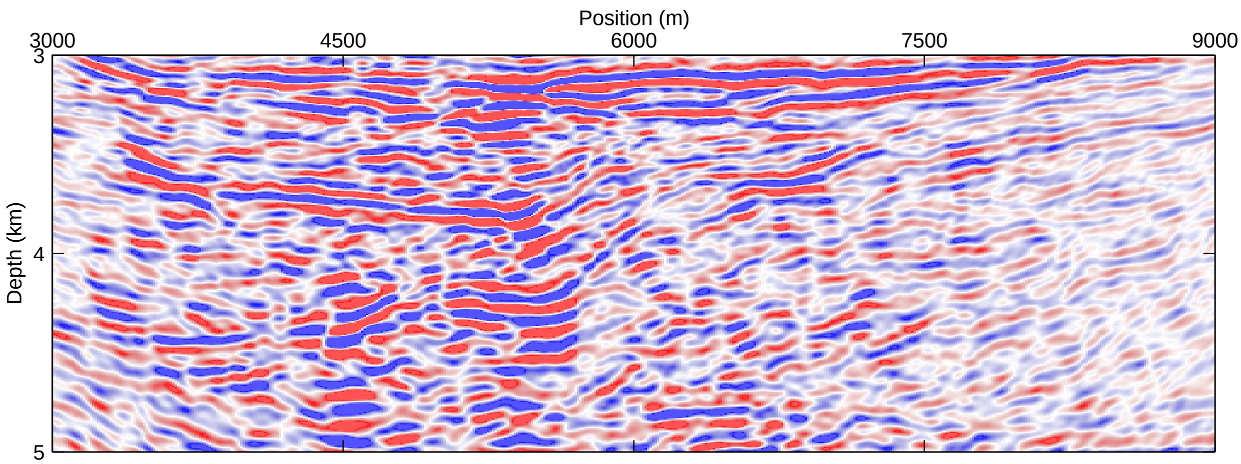

Fig. 4: Wave equation prestack depth migration around the target of inline 1079.