# Estimation of Reflection Coefficients from Zero-Offset Field Data

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## Summary

The classical one-dimensional inverse problem consists of estimating reflection coefficients from surface seismic data using the one-dimensional wave equation. Several authors have found stable solutions to this problem using least squares model fitting methods. However, the one-dimensional wave equation can only describe plane waves. Data generated by a point source can not be modelled by plane waves since the geometrical spreading is different for primary reflections and multiple reflections. We solve this difficulty by using a least-squares model fitting scheme based on a ray series expansion describing vertically travelling waves originating from a point source. The amplitudes of post-stack data are in some cases incorrect, so that pre-stack data must be used in an inversion scheme. Near-trace data from conventional seismic surveys are not zero-offset and can not be used with a method assuming zero-offset geometry. We use pre-stack data obtained from a purpose designed true zero-offset seismic experiment as input to estimate reflection coefficients. Stacking velocities from a conventional seismic survey were used to estimate the geometrical spreading. The resulting reflection coefficients are shown to correlate reasonably well with a well log.

## Introduction

The one-dimensional inverse problem has been treated by many authors. Reviews of these works are found in Burridge (1980) and Bube and Burridge (1983). Bamberger et al. (1982) gave a stable method based on least squares model fitting. This work also contained inversion results using real pre-stack surface seismic data. The one-dimensional wave equation describes plane waves propagating along one axis. Spherical waves originating from a point source can not be adequately described. In order to use methods based on the one-dimensional wave equation, some kind of scaling of the input data must be performed. As shown by Ursin and Berteussen (1986) and Ursin and Arntsen (1985), this leads to incorrect amplitudes of the multiple reflections. In some cases incorrect estimates of the reflection coefficients could be obtained. We avoid this difficulty by modelling zero-offset seismic data as vertically travelling spherical waves in a one-dimensional horizontally layered medium, taking geometrical spreading properly into account. A conventional marine seismic experiment is not a true zero-offset experiment so that a special zero-offset marine seismic experiment was performed in an area which is known to be approximately horizontally stratified.

The seismic inverse problem is solved by a least-squares model fitting scheme. The forward modelling is based on ray-theory and is able to simulate a true zero-offset experiment in a horizontally layered medium. The effects of three-dimensional geometrical spreading are included, as well as multiple reflections.

## Modelling

A medium consisting of a stack of L elastic plane layers is considered. The stack is bounded by a free surface at the top and a half-space at the bottom, which are numbered 0 and L+1, respectively. Layer $k$ is above interface $k$. Each layer is characterized by the thickness $D_k$, P-wave velocity $c_k$ and density $\rho_k$. A coordinate system is defined, with the positive $z$-axis downwards. The coordinate at the bottom of layer $k$ is $z_k$, with $z_0 = 0$. The receiver and source are located in the first layer.It is assumed that the pressure at the source position is given. As shown by Ursin and Arntsen (1985), the elastic wave-equation can be approximately solved by a ray-series expansion. The solution is valid for waves with the direction of propagation along the $z$-axis. The arclength along any ray is denoted by $s$, with $s = 0$ at the source position $z_s$.

The pressure $p$ at the receiver due to a single ray can be expressed in the following form:

$$p(s,t) = AF(s)g(t - \tau(s)). \tag{1}$$

$A$ is a factor containing products of reflection and transmission coefficients, while $F$ accounts for the geometrical spreading. $\tau$ is the traveltime. The source function $g$ is a sum of the source pulse and the source and receiver ghost reflections from the free surface.

The function $F(s)$ accounts for geometrical spreading and is written

$$F(s) = c_1/n(s), \tag{2}$$

where

$$n(s) = \sum_k h_k \Delta n_k. \tag{3}$$

The sum runs over all the layers traversed by the ray. The contribution $h_k \Delta n_k$ to the geometrical spreading from one layer is

$$h_k \Delta n_k = h_k \int_{z_{k-1}}^{z_k} c(\sigma) d\sigma = h_k c_k D_k = h_k c_k^2 \Delta\tau_k. \tag{4}$$

$\Delta\tau_k$ is the one-way traveltime of layer k. The total pressure at the receiver position consist of a sum of infinitely many rays. In any practical calculation a finite set of rays must be selected. A reasonable selection criterion is the number of times a given ray has been reflected (Arntsen 1988). The most important contribution to the total field will in many cases consist of primary reflections together with surface related multiples. The total pressure at the receiver position can then be written as the sum of primary reflections and surface multiples:

$$p(t) = p_1 + p_2. \tag{5}$$

The contribution from the primary reflections is

$$p_1(t) = \sum_{k=1}^{L} A_k c_1 n_k^{-1} g(t - \tau_k). \tag{6}$$

$c_1 n_k^{-1}$ is the geometrical spreading for the primary reflection from interface k, and $\tau_k$ is the corresponding traveltime. Note that $n_k = \tau_k V^2_{RMS,k}$ where $V_{RMS,k}$ is the RMS-velocity. The amplitude $A_k$ is the product of reflection and transmission coefficients for the primary reflection from interface k. For surface multiples, the contribution to the total pressure is given by:

$$p_2(t) = \sum_{l=1}^{L}\sum_{m=1}^{L} K A_l A_m r_0 c_1 (n_l + n_m)^{-1} g(t - \tau_l - \tau_m). \qquad (7)$$

$r_0$ is the reflection coefficient of the free surface. $K$ is a factor equal to 2 when $l \neq m$ and 1 when $l = m$. It is a simple ray count factor taking into account that two multiples with different raypath could have the same amplitude and traveltime (Arntsen 1988).

## Inversion of zero-offset field data

The pressure can in general be regarded as a (nonlinear) function of the parameters characterizing the layered medium. By writing the pressure $\boldsymbol{p}$ as a vector-function of a parameter vector $\boldsymbol{\theta}$ this is simply expressed:

$$\boldsymbol{p} = \boldsymbol{p}(\boldsymbol{\theta}) \qquad (8)$$

The pressure vector is defined by the measured samples $p_k$ at time $t = (k-1)\Delta t$

$$\boldsymbol{p} = (p_1, p_2, ..., p_{N_T}), \qquad (9)$$

where $N_T$ is the number of samples in the pressure vector. The parameter vector is in our case defined as:

$$\boldsymbol{\theta} = (r_1, ..., r_L), \qquad (10)$$

where $r_k$ is the reflection coefficient of layer $k$. All the layers have thickness equal to the sampling interval of the data measured in two-way traveltime. Equation (8) is solved with an iterative procedure where the time integrated squared difference between the measured pressure and the pressure from the forward model is minimized with respect to the reflection coefficients.

A zero-offset experiment was conducted to obtain proper field data. The source was a single airgun with a chamber volume of 9.5 liters, while the receiver was a short section (15 m long) of a conventional streamer. With this setup a line was shot in the North-Sea over an area which is known to be approximately horizontally stratified. A borehole was located very close to the line, and the sonic and density logs from this borehole were available. The data from a single hydrophone on the streamer section were bandpass-filtered with an upper cut off frequency at 62 Hz and resampled to a sampling interval of 8 ms. The resulting data is shown in Figure 1. A scale factor proportional with time was applied to the data before plotting. The shot interval was 50 m. The source wavelet was estimated from the direct arrival measured on the streamer section.

### First run

In the first run, only one trace at the bore-hole position was inverted. No scaling of the data before inversion was applied. The initial model was taken to be reflection coefficients equal to zero for all times. A conventional seismic survey had been performed along a line intersecting the bore-hole. It was then possible to use RMS velocities from a conventional velocity analysis to compute the geometrical spreading factor. The inverted reflection coefficients are shown in Figure 2. The norm of the error trace is about 0.05 times the norm of the data trace, and is also shown in Figure 2. The resulting reflection coefficients were obtained after five iterations. The last two iterations reduced the norm of the error trace only marginally.

Sonic and density logs were available, and these were resampled and then used to compute the reflection coefficients as function of two-way traveltime. After filtering with a zero-phase bandpass filter, the resulting reflection series was plotted in Figure 2. Comparing the reflection coefficients from the log with the estimated reflection coefficients, the major events correlate well.

### Second run

In the second run, eighty traces were inverted. The resulting section of reflection coefficients is shown in Figure 4. The reflection coefficients were only scaled by a constant factor before plotting. The error traces are shown in Figure 3. Comparing the estimated reflection coefficients with the input data, several features can be noted. The long tail of the source signature has been reduced, and the waterbottom multiple at 0.8s has been attenuated. The effects of geometrical spreading have of course been removed. The lateral correlation of the reflection coefficients are particulary strong below 1.4 seconds of two-way traveltime.

## Discussion and conclusions

We have given an inverse method for computing reflection coefficients from zero offset field data. Geometrical spreading is properly taken into account by using RMS velocities from a conventional seismic survey. This eliminates the need for any scaling of the input data.

A zero offset experiment has been conducted, and the inverse method was used to estimate reflection coefficients from a short marine seismic line. The estimates seem to correlate well with reflection coefficients obtained from well logs. It is however clear that a straightforward comparison is not without problems. After all, well log measurements are performed with sampling intervals of the order of 10 cm, which is much less than seismic wavelengths. Since the seismic waves "see" another medium than the well log measurments, one should expect some differences between the estimated reflection coefficients and the reflection coefficients obtained from the well log.

Difference between the estimates and the log not accounted for by differences of scale may in general be explained by inaccuracies in the forward model. In the present example we do not take into account absorption effects, which will lead to estimates of the reflection coefficients smaller than their actual value. Deviations from horizontal layering would lead to unpredictable errors in the estimates of the reflection coefficients. Errors of this kind should be expected to be present in the estimates of the reflection coefficients in the left part of Figure 4, since deviations from a horizontally stratified medium are evident. The ray series solution used in the forward model is valid only under certain conditions (Ben-Menahem and Beydoun 1985) which are not necessarily fulfilled in the examples we have shown.

## Acknowledgement

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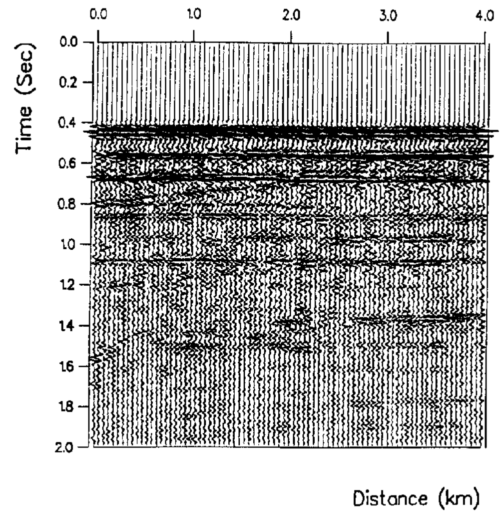


Figure 1: Zero-offset field data

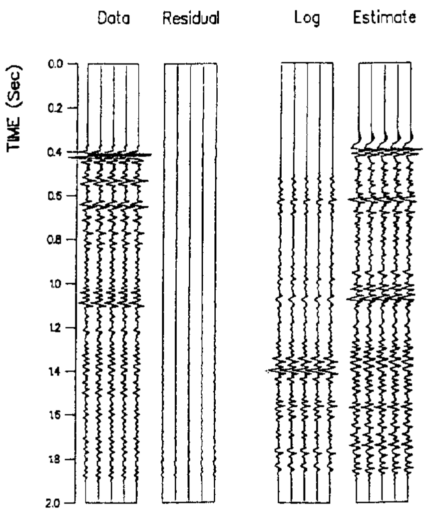


Figure 2: Inverted reflection coefficients, log, data and residuals at the bore-hole position. All traces are plotted five consequtive times.

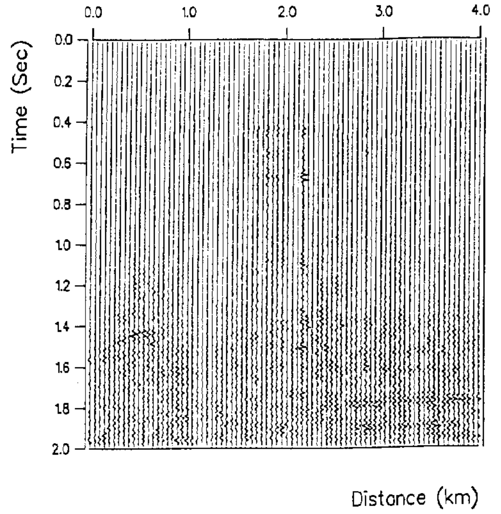


Figure 3: Zero-offset field data error section.

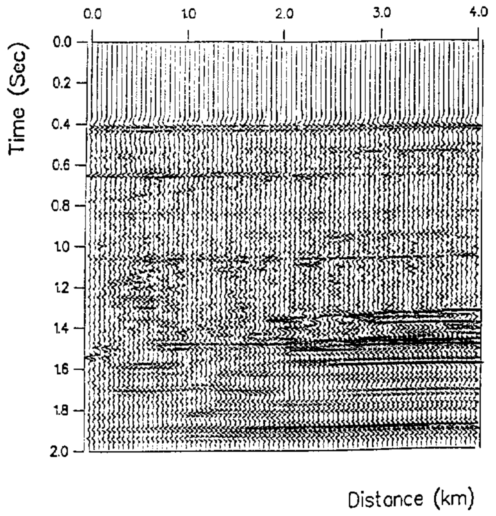


Figure 4: Estimated reflection coefficients.