An Improved Nonlinear Acoustic Inverse Algorithm

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Summary

A nonlinear full waveform inverse algorithm and examples of its application have been given by Tarantola (1984) and Lailly (1983). This scheme attempts to recover density and compressibillity as a function of spatial coordinates from seismic pressure recordings. The recorded (incoming) pressure wavefield is extrapolated backwards in time and used together with forward modelling to update estimates of density and compressibility in an iterative manner. The algorithm will not give correct results when using marine seismic data recorded along a horizontal array at a single depth. This is due to the fact that it is not possible to reconstruct the amplitude and direction of the incoming wavefield uniqely, since reflected waves comming from the free surface above the receiver array will give the same pressure as the reflections comming from the subsurface below the receivers.

Their inverse algorithm can however be modified by taking the direction of the incoming wavefield properly into account. One way of doing this is to use measurments of both pressure and the normal component of particle velocity, since these two measurments will define both amplitude and direction of the incoming wavefield. This work gives the details of a modified algorithm taking these measurments properly into account. It is shown by numerical examples that the modified inverse algorithm gives better results than Tarantola and Lailly's algorithm.

Introduction

A nonlinear full waveform inverse algorithm and examples of its application have been given by Tarantola (1984) and Lailly (1983). This scheme attempts to recover density and compressibillity as a function of spatial coordinates from seismic pressure recordings. The recorded (incoming) pressure wavefield is extrapolated backwards in time and used together with forward modelling to update estimates of density and compressibility in an iterative manner. It is important that the pressure data is measured in such a way that it is possible to reconstruct the incoming wavefield properly. Also, the backward extrapolation must be formulated such that the measurments are taken properly into account. If this is not the case, then the inverse algorithm will fail to give correct results. When using marine seismic data recorded along a horizontal array at a single depth, it is not possible to reconstruct the amplitude and direction of the incoming wavefield uniqely, since waves comming from the free surface above the receiver array will give the same pressure as the waves comming from the subsurface. Using this kind of recording geometry in conjunction with Tarantola and Lailly's inverse scheme, leads to incorrect results.

Their inverse algorithm can however be modified by taking the direction of the incoming wavefield properly into account. One way of doing this is to use measurments of both pressure and the normal component of particle velocity, since these two measurments will define both amplitude and direction of the incoming wavefield. In the next section it is shown how this can be done by changing the objective function of the inverse algorithm.

Inversion

The ultimate goal of acoustic inverse theory is to recover the compressibility $\kappa(\boldsymbol{x})$ and density $\rho(\boldsymbol{x})$ from the measured data. In practice one seeks densities and compressibilities (or densities and wave velocities) which minimizes the error between synthetic seismic data and observed data.

Assume that recordings of pressure are made on a surface (or part of a surface) enclosing the region to be imaged. The surface is denoted by S. Using an initial guess of $\kappa(\boldsymbol{x})$ and $\rho(\boldsymbol{x})$, synthetic data are generated by forward modelling. The synthetic pressure is denoted by p, while the difference between the observed pressure and the synthetic pressure at receiver position $\boldsymbol{x}_r$ is $\Delta p(\boldsymbol{x}_r, t)$. The problem one wants to solve is to find $\kappa(\boldsymbol{x})$ and $\rho(\boldsymbol{x})$ such that Δp is as small as possible. Tarantola (1984) and Lailly (1983) achieved this by minimizing the following objective function

$$J = \frac{1}{2}\int_0^T dt \int dS \,(\Delta p(\boldsymbol{x}_r, t))^2 . \tag{1}$$

One problem associated with the function given by equation (1) is that waves arriving at the receiver from above (in a conventional seismic experiment) give raise to the same pressure as those arriving from below the receiver. The strength and direction of the incoming wavefield can not be measured correctly, leading to degraded results of the inversion.

What is needed is an objective function taking into account both the direction and the amplitude of the incoming waves. This can be done by including both pressure and the normal component of the particle velocity,v_n in the object function, as follows

$$J = \frac{1}{2}\int_0^T dt \int dS \left[\kappa\rho(\Delta \dot{v}_n(\boldsymbol{x}_r, t))^2 + (\Delta \dot{p}(\boldsymbol{x}_r, t))^2\right] . \tag{2}$$

Here Δv_n is the difference between the synthetic normal component of the particle velocity and the observed normal component of particle velocity. A dot denotes time-differentiation. The time derivatives are introduced to allow for efficient computation of the gradient of the objective function with respect to density and compressibility. In a marine seismic experiment is it difficult to measure both particle velocity and pressure at the same depth. Instead pressure could be measured at two (or more) depth levels, and the particle velocity calculated.

The solution of the minimazation problem is found by an iterative procedure (e.g. steepest descent method)

$$\begin{aligned} \kappa(\boldsymbol{x})_{n+1} &= \kappa(\boldsymbol{x})_n + \alpha\nabla_\kappa J(\boldsymbol{x})_n, \\ \rho(\boldsymbol{x})_{n+1} &= \rho(\boldsymbol{x})_n + \alpha\nabla_\rho J(\boldsymbol{x})_n. \end{aligned} \tag{3}$$

α is a scaling factor chosen to obtain rapid convergence. The above expressions are iterated to give better estimates of the density and compressibility for each iteration.

The gradients $\nabla_\kappa J$ and $\nabla_\rho J$ of the objective function are are given by

$$\nabla_\kappa J = \frac{1}{\kappa^2}\int_0^T dt\, \ddot{p}(\boldsymbol{x},t)\phi(\boldsymbol{x},t), \quad (4)$$

$$\nabla_\rho J = \frac{1}{\rho^2}\int_0^T dt\, \nabla\phi(\boldsymbol{x},t)\cdot\nabla p(\boldsymbol{x},t). \quad (5)$$

The wavefield ϕ is the solution of the equation

$$\frac{1}{\kappa(\boldsymbol{x})}\frac{\partial^2\phi(\boldsymbol{x},t)}{\partial t^2} - \nabla\left(\frac{1}{\rho(\boldsymbol{x})}\nabla\phi(\boldsymbol{x},t)\right) = h(\boldsymbol{x},t), \quad (6)$$

Here $h(\boldsymbol{x},t)$ is a source term equal to

$$h(\boldsymbol{x},t) = -\int dS\,[\Delta\ddot{p}(\boldsymbol{x}_r,t)\delta(\boldsymbol{x}_r-\boldsymbol{x}) + \kappa\Delta\dot{v}_n n\cdot\nabla\delta(\boldsymbol{x}_r-\boldsymbol{x})] \quad (7)$$

The surface normal is denoted by $\boldsymbol{n}$. The source term consists of a distribution of point sources and dipole sources. The strength of the dipoles at any given time and position is given by the normal component of the residual particle velocity. The strength of the point sources is given by the residual pressure.

The objective function of Tarantola (1984) and Lailly (1983) leads to expressions for the gradients of J similar to equation (5), but with ϕ replaced by a new field ψ. This field is the solution of the equation

$$\frac{1}{\kappa(\boldsymbol{x})}\frac{\partial^2\psi(\boldsymbol{x},t)}{\partial t^2} - \nabla\left(\frac{1}{\rho(\boldsymbol{x})}\nabla\psi(\boldsymbol{x},t)\right) = \int dS\Delta p(\boldsymbol{x},t)\delta(\boldsymbol{x}_r-\boldsymbol{x}). \quad (8)$$

The source term consists here of a distribution of point sources only,with the residual pressure as the strength of the sources.

Numerical examples

In the following the simple problem of reconstructing a point diffractor in an otherwise homogenous two dimensional medium is considered. Figure 1 shows the "true" model, with the point diffractor in the middle of the model. At the top of the model was a free surface. A shot was fired at the position indicated by a star, and the resulting reflected wavefield was recorded with a horisontal receiver array, as indicated in the figure. The acoustic wave equations for p and ϕ were solved numerically with a fast finite difference technique described by Holberg (1987). This tecnique steps the wavefield forward in time with a source function as input to generate the synthetic pressure p, and synthetic particle velocity v_n. To compute ϕ the wave equation was solved backwards in time using the residual wavefields Δp and Δv_n as input data. Figures 2 and 3 show the reconstruction of the diffractor after one iteration of the inverse algorithm. For comparision is the corresponding result using the algorithm proposed by Tarantola (1984) and Lailly (1983) shown in figures 4 and 5. It is quite clear that this method does not perform well, showing large "migration smiles". The reason is simply that there is not enough information in the pressure field recorded at a receiver array at a single depth, to reconstruct the diffractor. In this particular example is the "smile" partly caused by the ghost reflection from the free surface. In order to reconstruct the diffractor, directional information contained in the normal component of the particle velocity is also needed.

Conclusions

A modification to Tarantola's (1984) and Lailly's (1983) nonlinear inverse algorithm has been given. The modified theory takes into account both amplitude and direction of the incoming (recorded) wavefield, by using both measurments of pressure and the normal component of the particle velocity. Simple numerical experiments show that inversion using both pressure and particle velocity data gives substancially better results than the method of Tarantola (1984) and Lailly (1983).

References

Holberg, O., 1987, Computational aspects of the choice of operator and sampling intervall for numerical differentiation in large-scale simulation of wave phenomena. *Geophysical prospecting* 35, 629–655.

Lailly, P., 1983, The seismic inverse problem as a sequence of before stack migrations. In *Conference on Inverse Scattering:Inverse scattering and applications*, SIAM 1983.

Tarantola, A., 1984, Inversion of seismic reflection data in the acoustic approximation. *Geophysics* 49, 1259–1266,

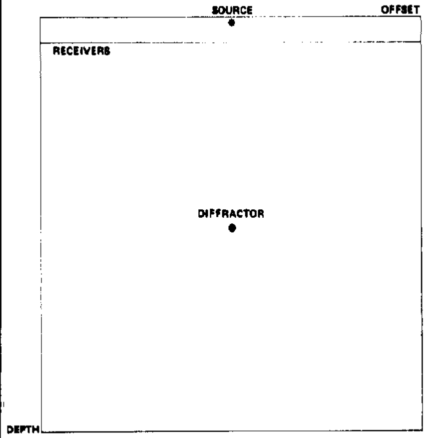

FIG. 1. True diffractor model.

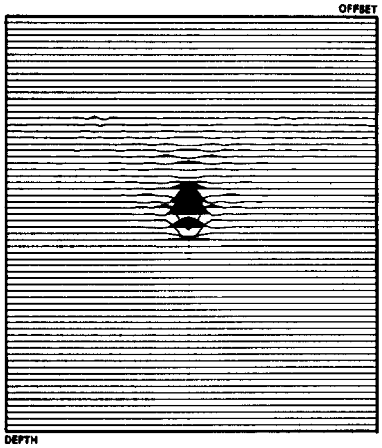

FIG. 2. Inversion result after one iteration.

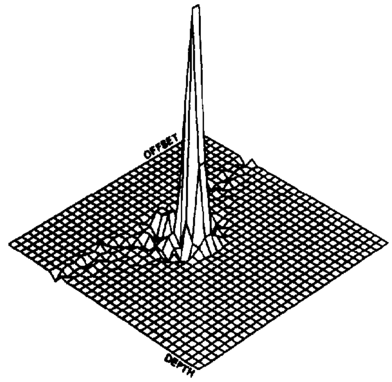

FIG. 3. Perspective view of result shown in Figure 2.

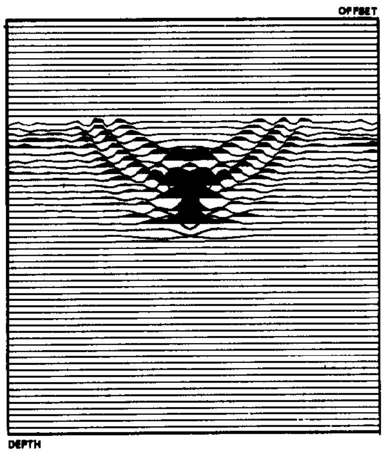

FIG. 4. Inversion result after one iteration using Tarantola (1984) and Lailly's (1983) methods.

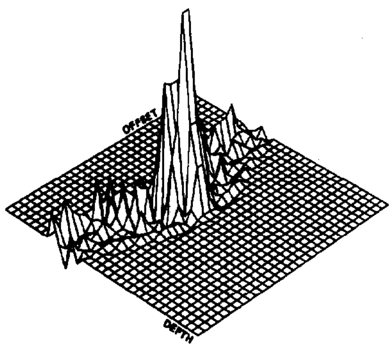

FIG. 5. Perspective view of the result shown in Figure 3.