Inversion of seismic surface waves for shear wave velocities

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Summary

In this paper we present a method for determining the shear wave velocity in the uppermost parts (50-200m) of the sea bed. The method is based on analyzing dispersive characteristics of surface waves. The propagation velocity of surface waves is dominated by shear parameters within a distance from the sea floor, the particular distance being a function of wavelength. The phase velocity of the surface wave varies with frequency according to the variation of shear parameters with depth. To estimate the phase velocity, a frequency wavenumber representation of the wave field is used. Particular attention must be paid to resolution when computing the space wavenumber transform. Thus, a spectral line estimator, the Prony method, is used. Once the phase velocity characteristics are available, a simple inversion scheme is used to invert for shear parameters in the sea bed. The method is applied to data from the Tommeliten field and we provide estimates for shear wave velocities along a 10km seismic line.

Introduction

Obtaining information about the shear wave velocity in the uppermost 50-200 meters of the sea bed is of great importance in many situations. Shear parameters are needed as input in multiple reduction algorithms, imaging problems and for modeling. The problem of estimating shear parameters in the sea bed has for the last decade recieved attention within the underwater acoustics community. Losses due to conversions to surface waves is a major problem in sonar and underwater acoustical transmission applications (Hovem, 1993). As a result, a method was developed for mapping the sea bed for predicting transmission losses as a function of sediment properties. The method was recently converted for geotechnical applications (Stoll and Bautista, 1994). A similar method is well-known from monitoring the earth's crust by analyzing Rayleigh waves excited by earth quakes (Deiewonski et al, 1969). The most popular way of estimating dispersive characteristics is to apply a sliding window technique to a recorded time series in order to analyze the distribution of energy with frequency at different times. Estimating the time delay of the energy peak in each frequency band provides an estimate for the group velocity within the different frequency bands. As was demonstrated by McClellan (1986) , the sliding window approach is not capable of resolving two modes if they overlap in time and propagate at nearly the same velocity. We have chosen to adapt a scheme for dispersion analysis, based on array data, that was proposed by McClellan (1986) and later applied by Lang et al. (1987) to analyze dispersion properties of the pseudo-stoneley mode in a borehole. The twodimentional spectral analysis that we apply is capable to resolve several surface wave modes. Allnor et al. (1996) showed that inverting for two modes gives better estimates for the shear parameters. In this paper we first give a summary of the method that was used and then show the results from the analysis of data from the Tommeliten field.

The algorithm

The signal processing scheme is based on the well-known integral representation

$$s(x,t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} A(k_x,\omega)\exp j(\omega t - k_x x)\,dk_x\,d\omega \qquad (1)$$

of a wavefield comprising plane waves. Surface waves are included as poles in the $A(k_x,$ w) function. Thus, in principle, it suffices to compute the 2D Fourier transform of the signal and track the ridges in $(k_x,$ w) space that corresponds to the surface wave modes. Assuming a lateral homogenous formation, this can be written

$$A(k_x,\omega) = \mathcal{F}_x\left\{\mathcal{F}_\omega\left\{s(x,t)\right\}\right\} \qquad (2)$$

where $\mathcal{F}_x$ is the 1D Fourier transform with respect to parameter x. Thus, one can utilize eq. 2 by analyzing the output from an array of sensors simultaneously.

$$k_x(\omega) = \frac{\omega}{v_p(\omega)} \qquad (3)$$

The dispersion equation, eq. 3, relates the horizontal wavenumber k_x to the angular frequency w and phase velocity v_p of the mode. As v_p is lower bounded for most wave modes, k_x is upper bounded and proportional to w. Thus, for low frequencies, the dispersion lines for the wavefield tend to cluster in frequency wavenumber domain. Hence, sufficient resolution of spectral lines is of uttermost importance to resolve the field representation in (w, k_x) domain. It is well known that the resolution of the Fourier transform is constrained by the Heisenberg inequality (Papoulis, 1962). In the discrete case, resolution decreases with the length of the sequence to be transformed. For the inner transform in eq. 2, from time to frequency, resolution causes no problems as one usually have a few thousands time samples per trace. For the outer transform, however, the situation differs. One whishes to keep the number of sensors in the array as low as possible both to assure localization of the parameter estimates and to reduce the risk for encountering lateral variations of parameters within the array. Thus, the use of the FFT algorithm for the space wavenumber transform is discouraged. An alternative to using the FFT is to use a spectral line estimator. Several different estimators are available and we have chosen to follow McClellan (1986)and use Prony's method. Prony's method solves a least squares problem derived from the spatial autocorrelation matrix, with certain constraints on the equations to be solved (Kay and Marple, 1981). Prony's method solves for the spectral lines in two steps: First for. the wavenumbers, second, if desireable, for amplitudes. In this paper only wavenumbers are considered. The estimated dispersion curves go as input to the inversion algorithm. The inversion algorithm is basically a regularized multidimentional Newton-Raphson scheme (Caiti et al, 1994). The forward model is a fast Tompson-Haskell type model (Takeuchi and Saito, 1972). One advantage with the inversion scheme is that it provides both quality control parameters for determining how deep below the seafloor the result is accurate, and error bounds for the estimated parameters (Caiti et al, 1994).

Case study: Tommeliten field

Data has been available from a SUMIC survey at the Tommeliten field in the North Sea (Granli et al.). The SUMIC data aqcuisition scheme consist of placing a triaxial gepohone and a hydrophone at the sea floor and have a vessel towing an air-gun source along the survey line. In the survey to be considered, the source was fired every 28m. Surface Waves were most easily found in the vertical velocity, v_z, component of the data. Examples of surface waves to be found in the data is shown in figure 1. The left hand plot in figure 1 shows the data while some key structures of the data is marked in the right hand plot. From the plots, one can clearly see the dispersive characteristics of the surface waves as the wavelet stretches as the wave travels. One can also get an idea that there are more than one surface wave mode present in the wave train. The estimated dispersion curves for the surface wave modes are shown in figure 2. In this plot one clearly see the two modes, the first mode starting at 400m/s at 1.5Hz and ending at 250m/s at 5 Ha, and the second starting at 500m/s at 2Hz and ending at 350m/s at 6Hz. The crosses are the sampled estimates of phase velocity v_p, with error bounds, that go into the inversion procedure. The resulting profile of shear wave velocity vs depth is shown in figure 3. The error bounds on the velocities are estimated from the error bound on the dispersion and parameters internal to the inversion procedure (Caiti et al, 1994). The combined result from inverting for a number of profiles along the line is shown in figure 4. The dominating feature in figure 4 is the low-velocity layer in the uppermost 10 to 20m. The variation of shear wave velocity with depth dominates, relative to variation along the line.

Conclusions

In this paper we have demonstrated that the dispersion properties of seismic surface waves can be used to estimate shear parameters vs depth in the sea bed along a seismic line.

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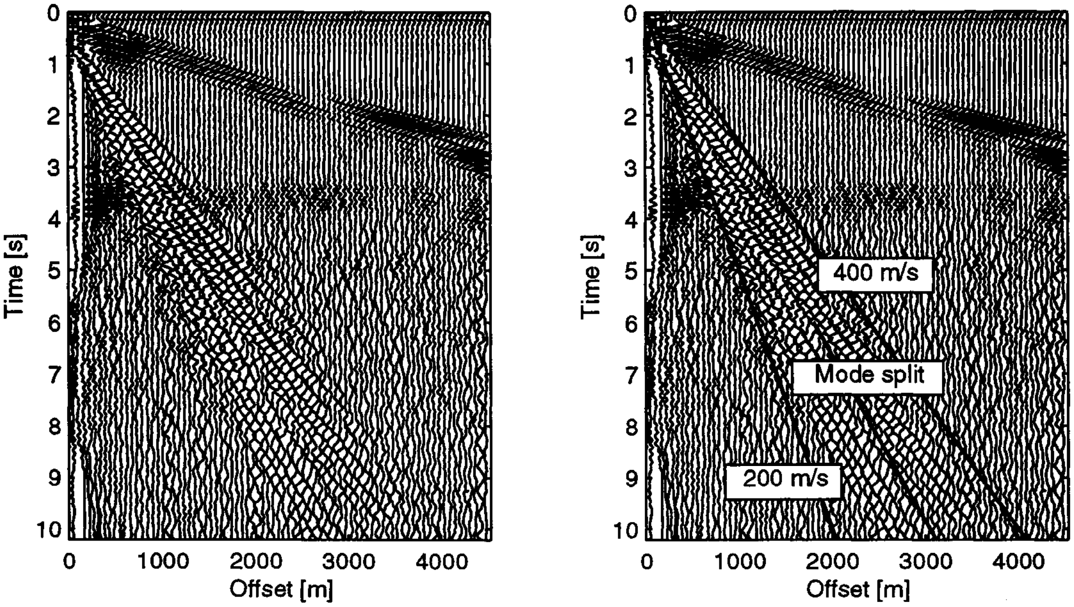

FIG. 1. Surface waves at Tommeliten. Data are enhanced to visualize surface waves.

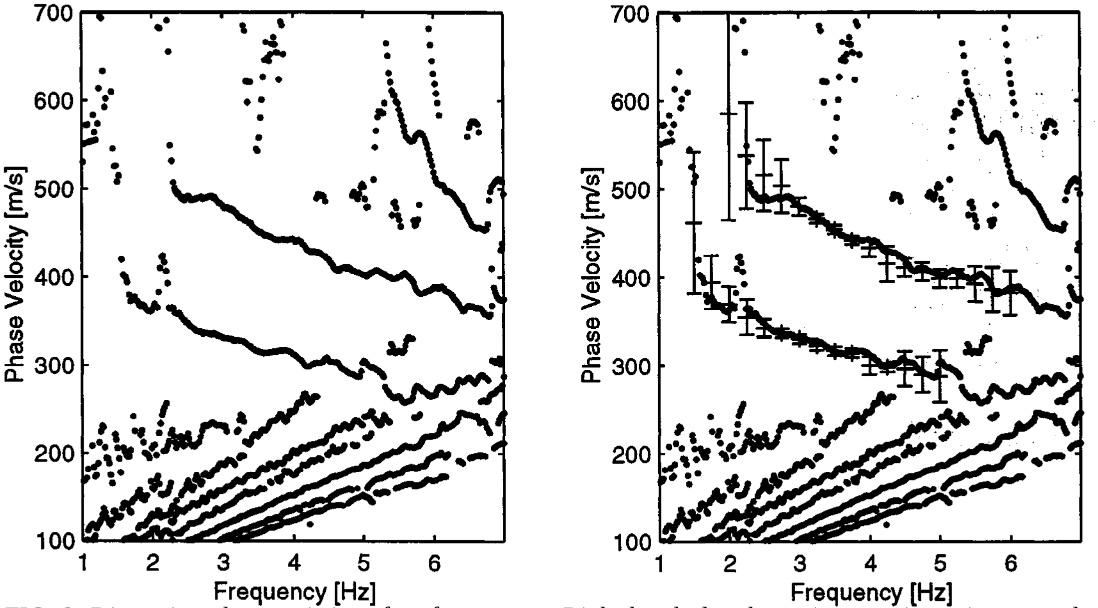

FIG. 2. Dispersion characteristics of surface waves. Right hand plot shows input to inversion procedure with error estimates.

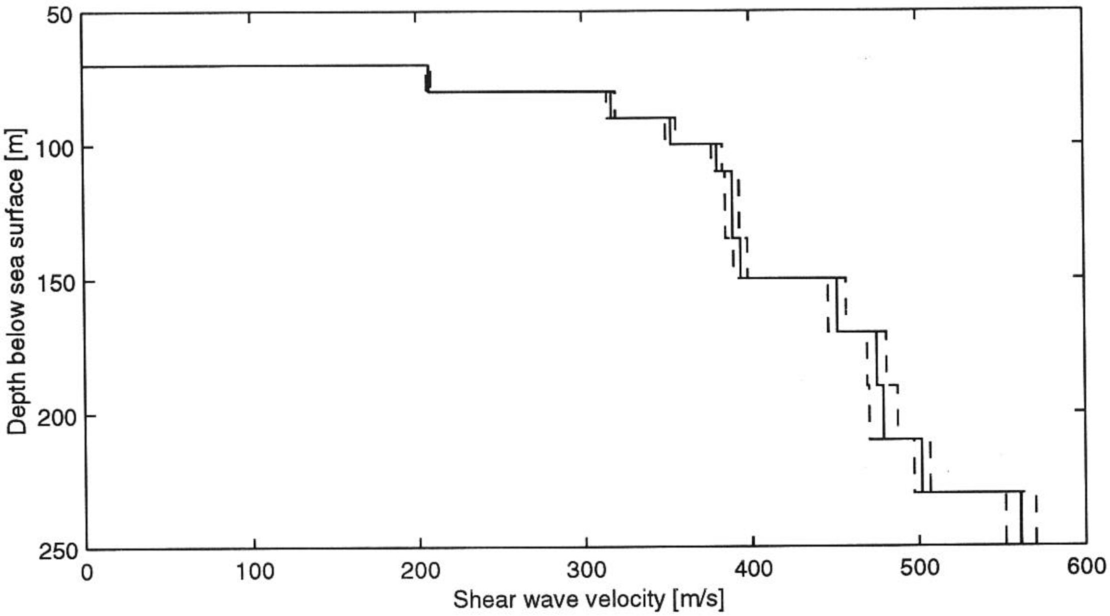

FIG. 3. Resulting estimates of shear wave velocity in sea floor. Stapled lines are error bounds on estimate.

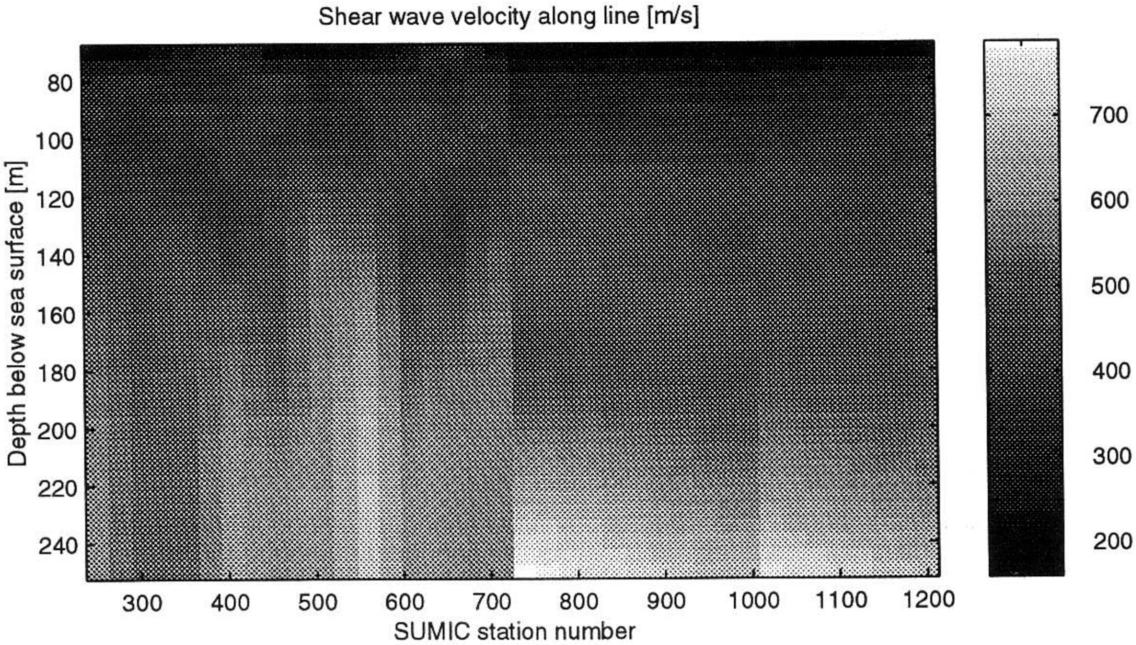

FIG. 4. Resulting estimates of shear wave velocity in sea floor along survey line.