ROSE

Rock Seismic Research Project

A new parameterization in acoustic orthorhombic media

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The parameterization matters !

Seismic modeling

Traveltime approximation

Velocity analysis

Full waveform inversion

Sensitivity & trade-off

Tomography

Objective

A perturbation-based approximation for traveltime using different parameterization

Outline

1 A new parameterization for an acoustic orthorhombic model

2 Perturbation based method for traveltime approximation

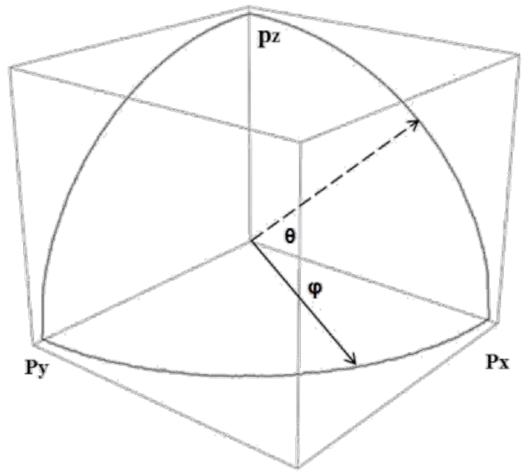
3 Sensitivity analysis of anellipticity parameters

4 Numerical examples

5 Conclusions

Seismic Anisotropy

The dependence of the seismic velocity on the propagation angle



$$\frac{1}{V(\theta,\phi)}$$

Isotropic model

ISO

$$\begin{bmatrix} \lambda+2\mu & \lambda & \lambda & & & \\ \lambda & \lambda+2\mu & \lambda & & & \\ \lambda & \lambda & \lambda+2\mu & & & \\ & & & \mu & & \\ & & & & \mu & \\ & & & & & \mu \end{bmatrix}$$

2 independent coefficients

Transverse isotropic model with a vertical axis

VTI

$$\begin{bmatrix} C_{11} & C_{11}-2C_{66} & C_{13} & & & \\ C_{11}-2C_{66} & C_{11} & C_{13} & & & \\ C_{13} & C_{13} & C_{33} & & & \\ & & & C_{55} & & \\ & & & & C_{55} & \\ & & & & & C_{66} \end{bmatrix}$$

5 independent coefficients

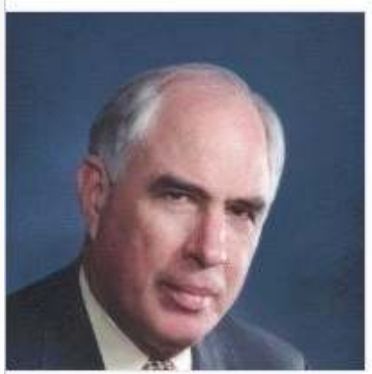
Orthorhombic

ORT

$$\begin{bmatrix} C_{11} & C_{12} & C_{13} & & & \\ C_{12} & C_{22} & C_{23} & & & \\ C_{13} & C_{23} & C_{33} & & & \\ & & & C_{44} & & \\ & & & & C_{55} & \\ & & & & & C_{66} \end{bmatrix}$$

9 independent coefficients

Elastic anisotropy model



Leon Thomsen (1986)

V_{P0}, V_{S0}, ε, δ, γ

$$V_{P0} = \sqrt{\frac{C_{33}}{\rho}},$$

$$V_{S0} = \sqrt{\frac{C_{44}}{\rho}},$$

$$\varepsilon = \frac{C_{11} - C_{33}}{2C_{33}},$$

$$\delta = \frac{(C_{13} + C_{44})^2 - (C_{33} - C_{44})^2}{2C_{33}(C_{33} - C_{44})},$$

$$\gamma = \frac{C_{66} - C_{44}}{2C_{44}},$$

Simplify

VTI

Acoustic assumption for anisotropy model

Tariq Alkhalifah (1998)

Vp, ε, δ P wave

$$V_{nmo} = V_{p0}\sqrt{1+2\delta},$$

$$\eta = \frac{\varepsilon - \delta}{1+2\delta}$$

Alkhalifah and Tsvankin (1995)

Parameterization ORT model

ORT

$$\begin{bmatrix} C_{11} & C_{12} & C_{13} & & & \\ C_{12} & C_{22} & C_{23} & & & \\ C_{13} & C_{23} & C_{33} & & & \\ & & & C_{44} & & \\ & & & & C_{55} & \\ & & & & & C_{66} \end{bmatrix}$$

Tsvankin (1997)

Velocity:

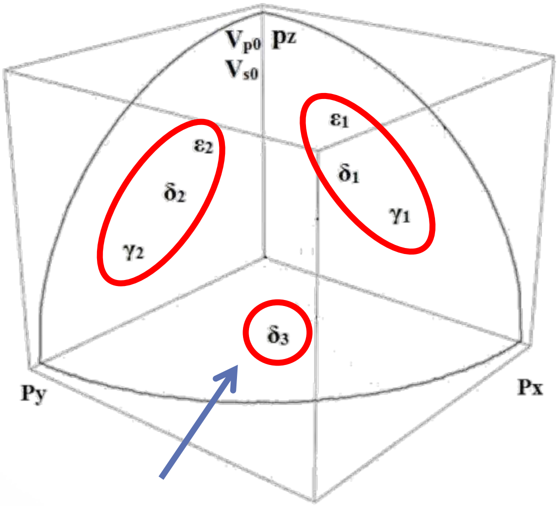
$V_{p0,} V_{s0}$

Anisotropy parameter

$[XOZ]$ plane: $\varepsilon_1, \delta_1, \gamma_1$

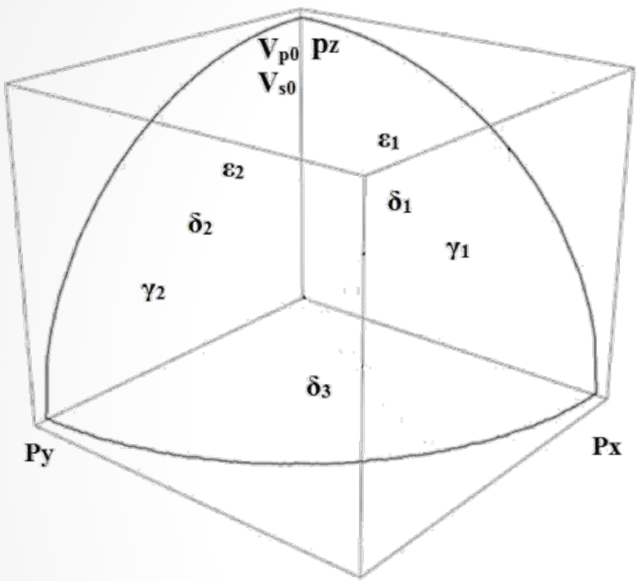
$[YOZ]$ plane: $\varepsilon_2, \delta_2, \gamma_2$

$[XOY]$ plane: δ_3



Acoustic ORT model

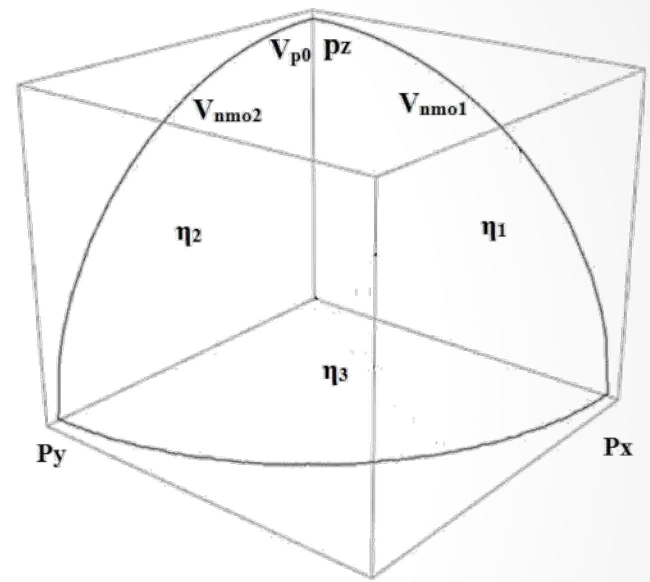
Elastic ORT **P & S waves**



Tsvankin 1997

$$\mathbf{9 \Rightarrow 6}$$

Acoustic ORT P wave

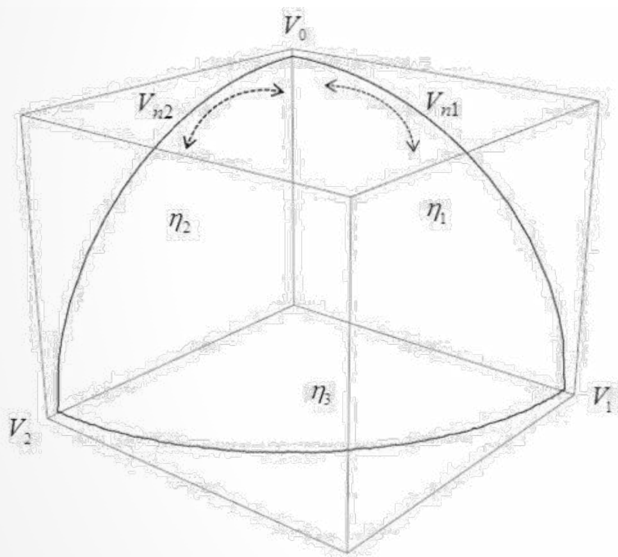


Alkhalifah 2003

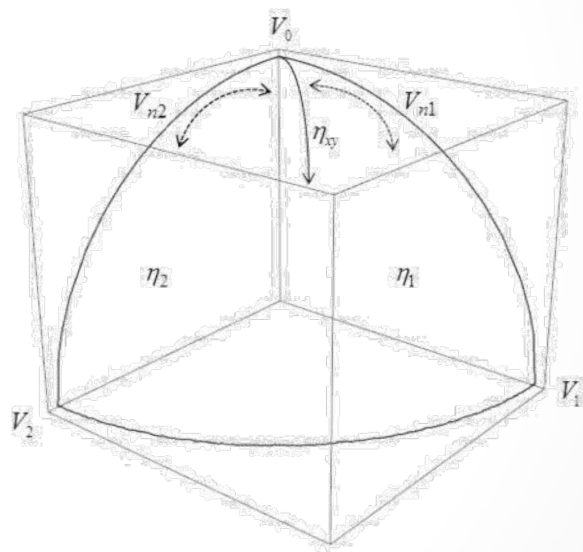
The parameterization for acoustic ORT model

P wave

Alkhalifah 2003



Stovas 2015



$$\eta_{xy} = \sqrt{\frac{(1+2\eta_1)(1+2\eta_2)}{1+2\eta_3}} - 1$$

$$\eta_{xy} = \sqrt{\frac{(1+2\eta_1)(1+2\eta_2)}{1+2\eta_3}} - 1,$$

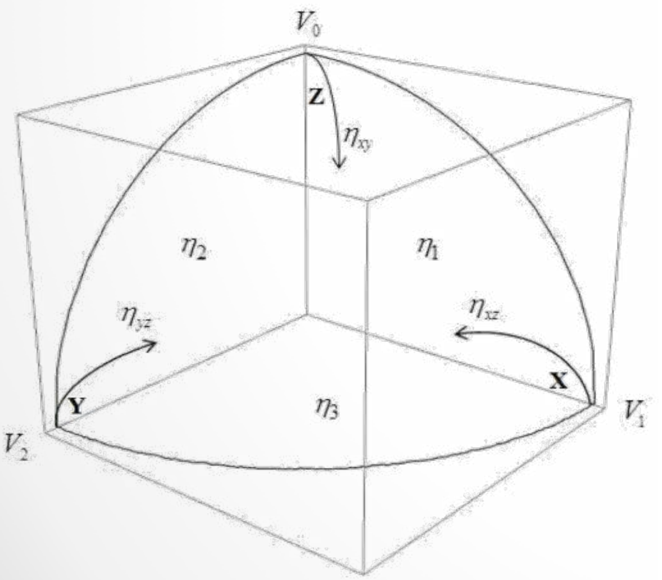
$$\eta_{xz} = \sqrt{\frac{(1+2\eta_1)(1+2\eta_3)}{1+2\eta_2}} - 1,$$

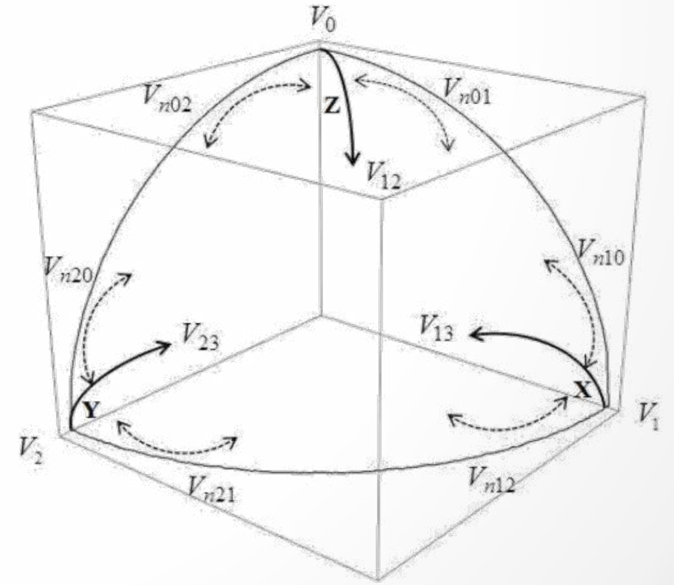
$$\eta_{yz} = \sqrt{\frac{(1+2\eta_2)(1+2\eta_3)}{1+2\eta_1}} - 1.$$

$$V_{12}^2 = V_{n01}V_{n02},$$

$$V_{13}^2 = V_{n10}V_{n12},$$

$$V_{23}^2 = V_{n20}V_{n21}$$





The parameterizations

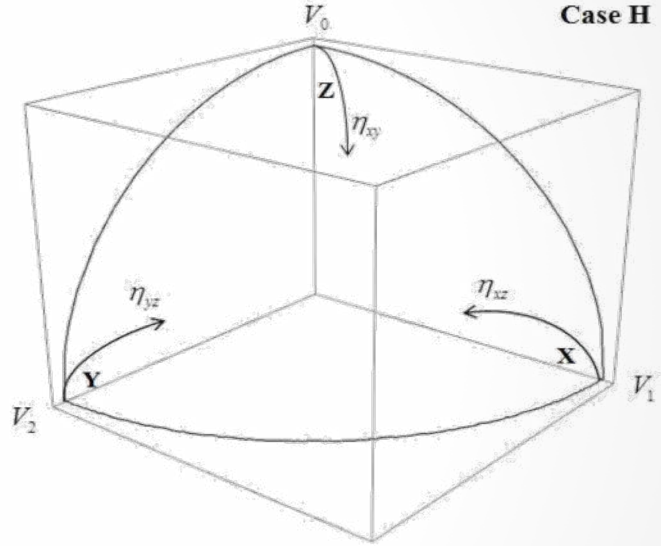
Parameterization	Elliptical background	Anellipticity parameters
Non-symmetric parameterizations		
Case A	V_0, V_{n1}, V_{n2}	η_1, η_2, η_3
Case B	V_0, V_{n1}, V_{n2}	$\eta_1, \eta_2, \eta_{xy}$
Case C	V_0, V_{n1}, V_{n2}	$\eta_{xy}, \eta_{xz}, \eta_{yz}$
Case D	V_0, V_{h1}, V_{h2}	$\eta_1, \eta_2, \eta_{xy}$
Symmetric parameterizations		
Case E	V_{12}, V_{13}, V_{23}	η_1, η_2, η_3
Case F	V_{12}, V_{13}, V_{23}	$\eta_{xy}, \eta_{xz}, \eta_{yz}$
Case G	V_0, V_{h1}, V_{h2}	η_1, η_2, η_3
Case H	V_0, V_{h1}, V_{h2}	$\eta_{xy}, \eta_{xz}, \eta_{yz}$

Table 1. Eight types of parameterizations with different background model and different set of anellipticity parameters.

Perturbation-based approximation

Acoustic Eikonal equation in ORT model

Alkhalifah 2003



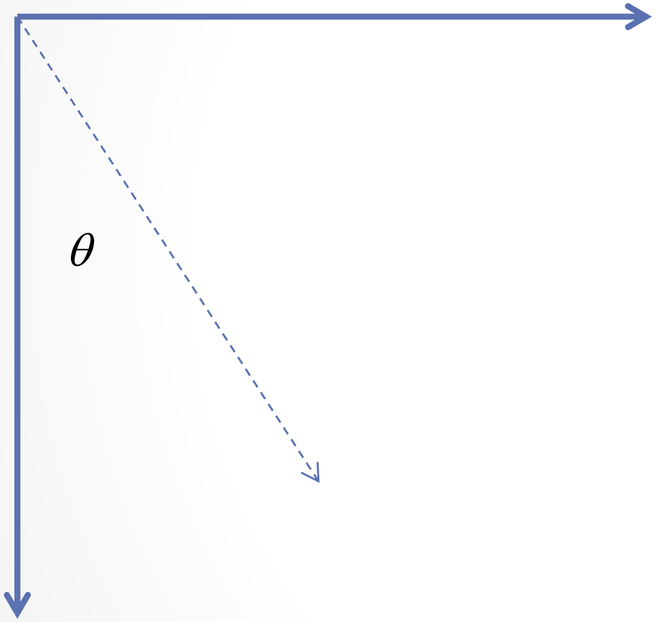
$$\tau = \tau_0 + \sum_i a_i \eta_i + \sum_{i,j} b_{ij} \eta_i \eta_j, (j = j = 1,2,3)$$

$$1 \equiv xy, 2 \equiv xz, 3 \equiv yz$$

$$\tau_0 = \sqrt{t_0^2 + \frac{x^2}{V_{h1}^2} + \frac{y^2}{V_{h2}^2}},$$

Sensitivity analysis

Group velocity related coefficients



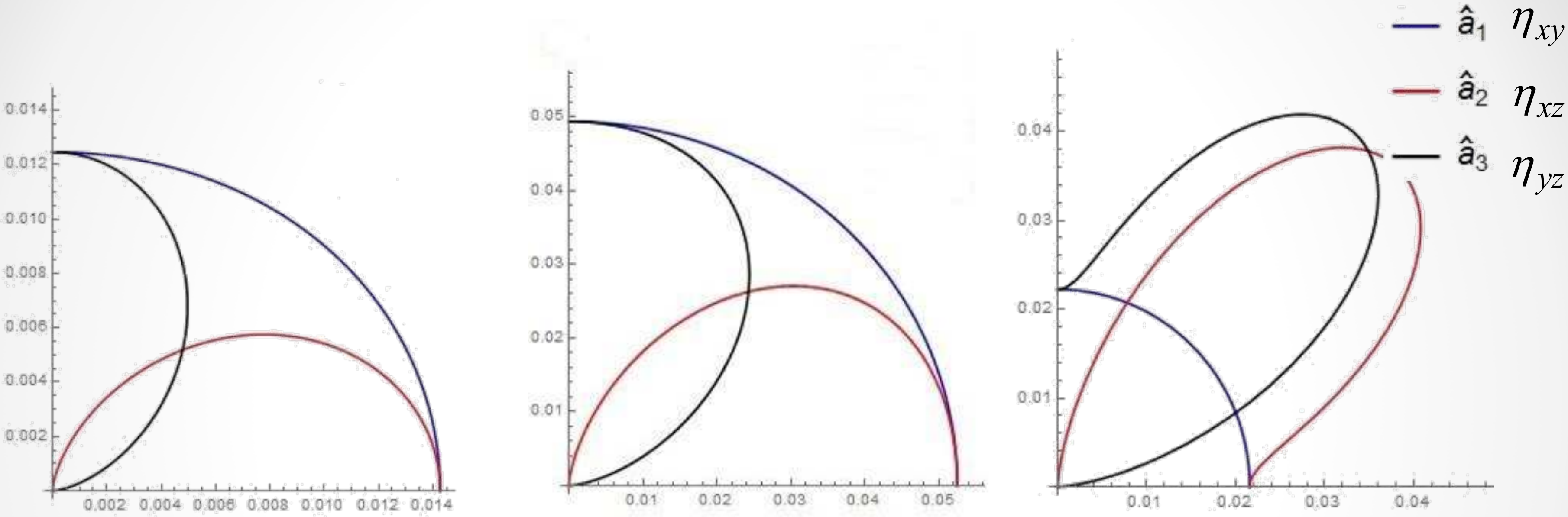
$$\theta \in (0,30^\circ) \qquad (30^\circ,60^\circ) \qquad (60^\circ,90^\circ)$$

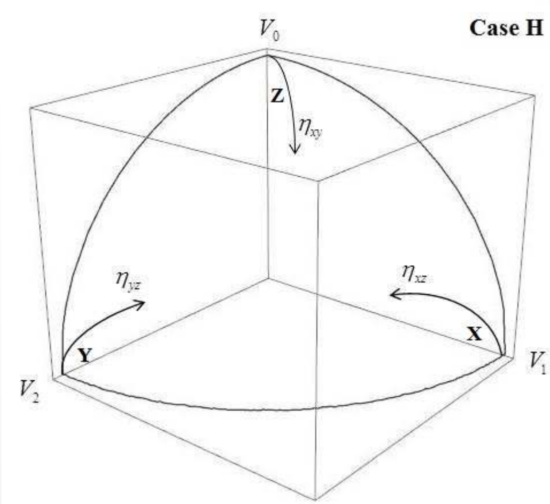
$$\hat{a}_i(\phi) = \frac{1}{\theta_2 - \theta_1}\int_{\theta_1}^{\theta_2} \tilde{a}_i(\theta,\phi)d\theta,$$

$$\hat{b}_{ij}(\phi) = \frac{1}{\theta_2 - \theta_1}\int_{\theta_1}^{\theta_2} \tilde{b}_{ij\,i}(\theta,\phi)d\theta.$$

Sensitivity analysis

$\theta \in (0, 30^\circ)$ $(30^\circ, 60^\circ)$ $(60^\circ, 90^\circ)$





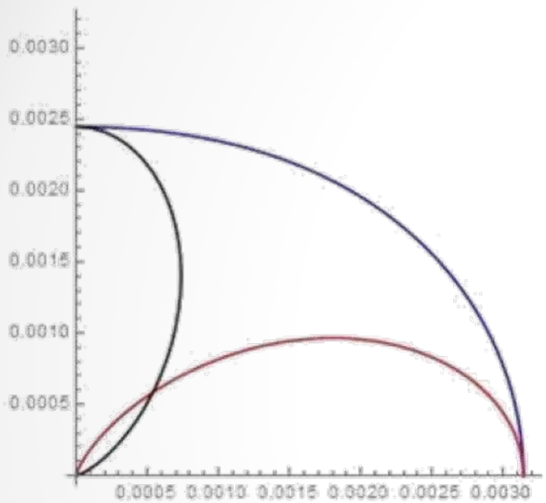
elliptical background : V_0, V_{h1}, V_{h2}

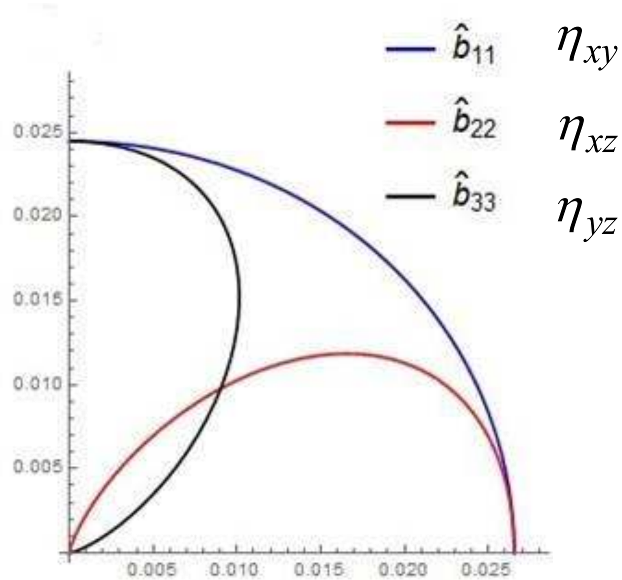
Sensitivity analysis

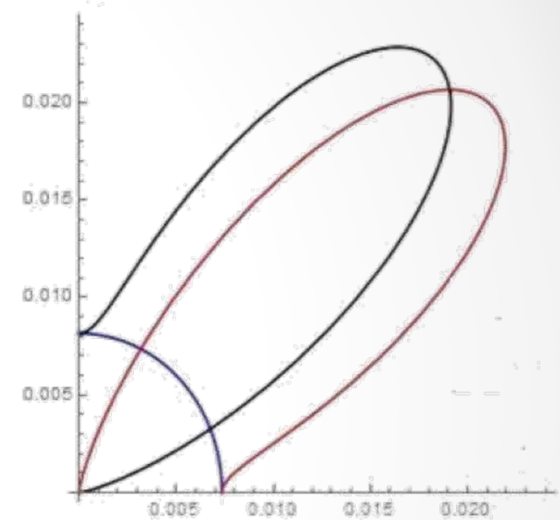
$\theta \in (0, 30^\circ)$

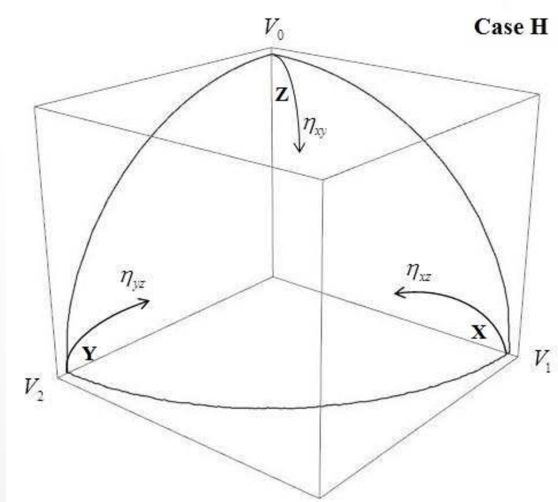
$(30^\circ, 60^\circ)$

$(60^\circ, 90^\circ)$





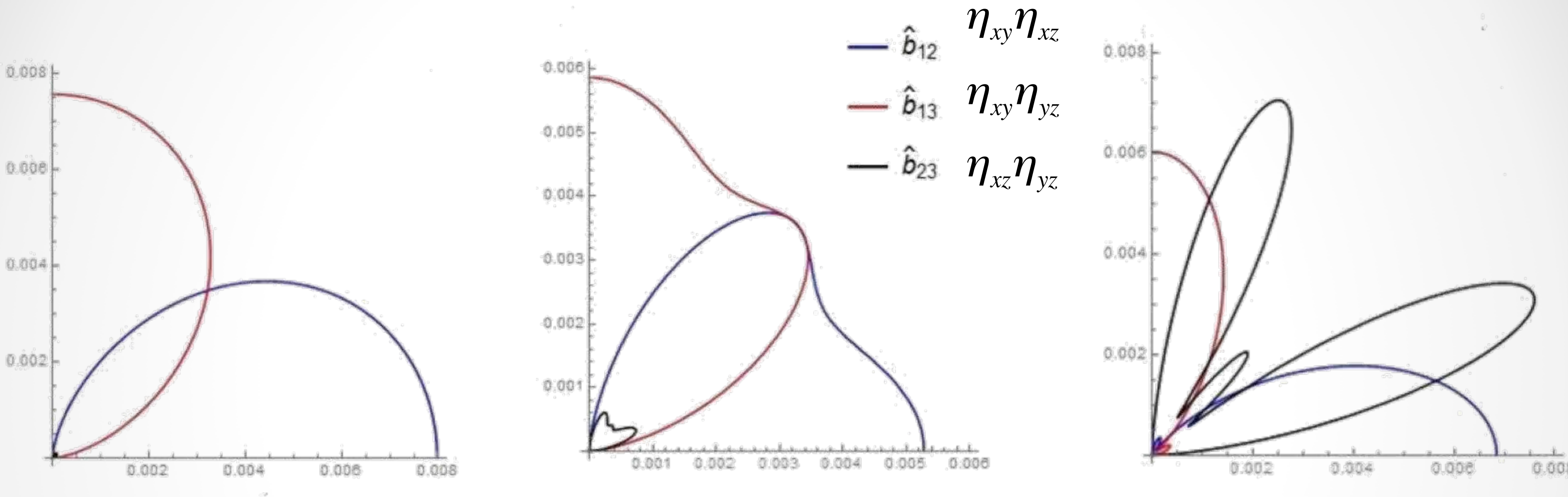


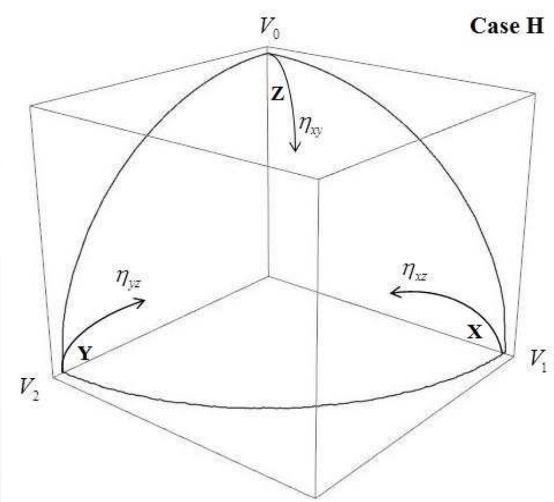


elliptical background : V_0, V_{h1}, V_{h2}

Sensitivity analysis

$\theta \in (0,30^{\circ})$ $(30^{\circ},60^{\circ})$ $(60^{\circ},90^{\circ})$

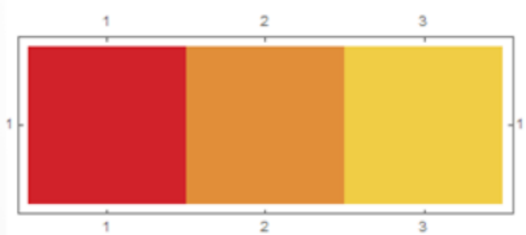




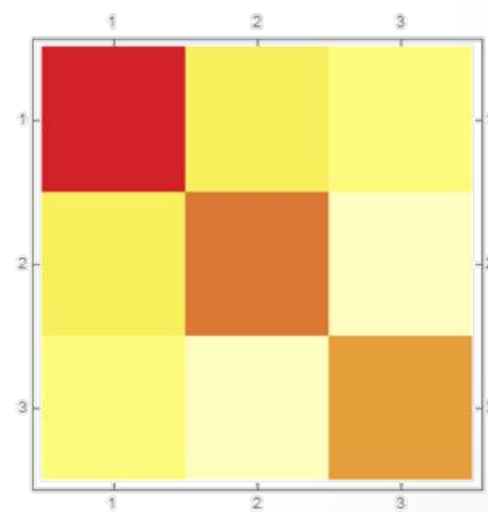
elliptical background : V_0, V_{h1}, V_{h2}

Overall sensitivity (case H)

$$\hat{A}_i = \frac{4}{\pi^2}\int_0^{\pi/2}\int_0^{\pi/2}\tilde{a}_i(\theta,\phi)d\theta d\phi, \qquad \hat{B}_{ij} = \frac{4}{\pi^2}\int_0^{\pi/2}\int_0^{\pi/2}\tilde{b}_{ij}(\theta,\phi)d\theta d\phi,$$



A_1 A_2 A_3

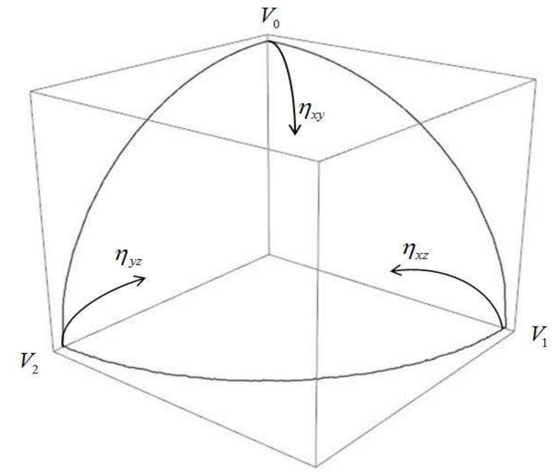


$1 \equiv \eta_{xy}, 2 \equiv \eta_{xz}, 3 \equiv \eta_{yz}$

Numerical examples (model parameters)

Velocities (km/s)	V_0	V_{h1}	V_{h2}	V_{n1}	V_{n2}	V_{12}	V_{13}	V_{23}
	2	2.4	2.6	2.1	2.23	2.17	2.04	1.94

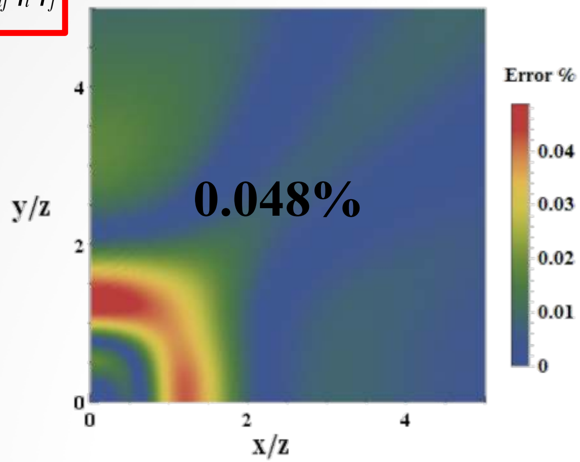
Anellipticity parameters	η_1	η_2	η_3	η_{xy}	η_{xz}	η_{yz}
	0.15	0.18	0.1	0.214	0.07	0.12



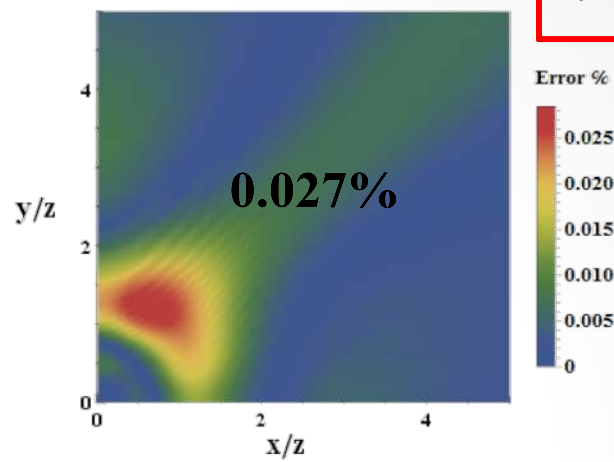
Numerical examples-Case H

$$\tau = \tau_0 + \sum_i a_i \eta_i + \sum_{i,j} b_{ij} \eta_i \eta_j$$

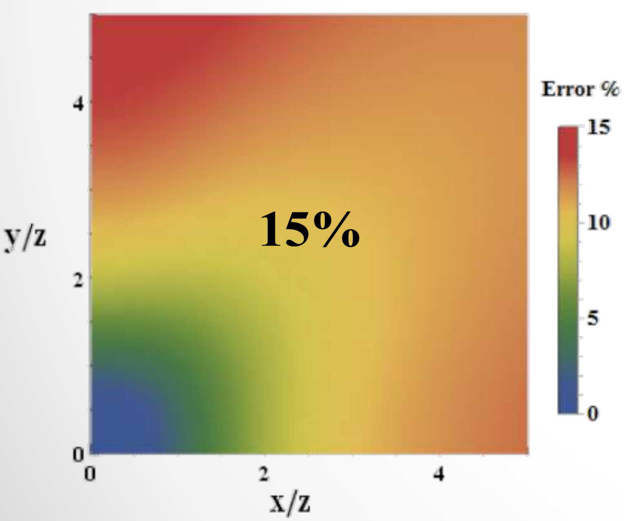
$$\tau_3 = \frac{\tau_0 \tau_2 - \tau_1^2}{\tau_0 + \tau_2 - 2\tau_1},$$



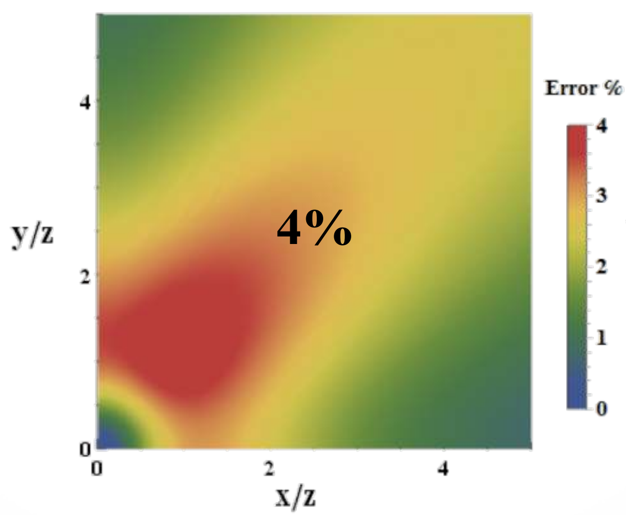
before the Shanks



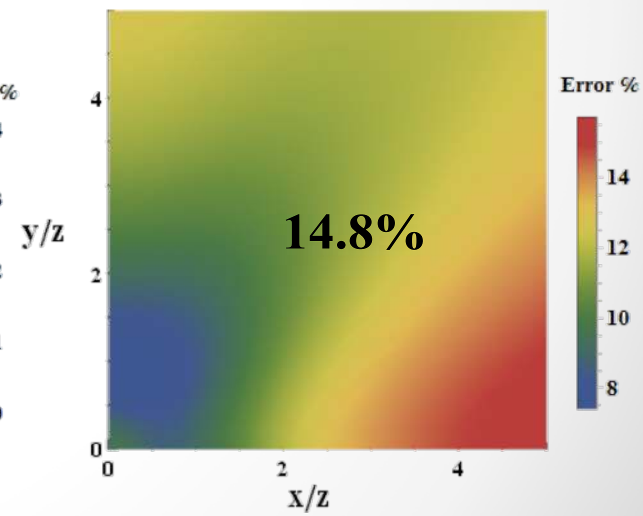
after the Shanks



V_0, V_{n1}, V_{n2}



V_0, V_{h1}, V_{h2}



V_{12}, V_{13}, V_{23}

Numerical examples

Parameterization	Elliptical background	Anellipticity parameters
Non-symmetric parameterizations		
Case A	V_0, V_{n1}, V_{n2}	η_1, η_2, η_3
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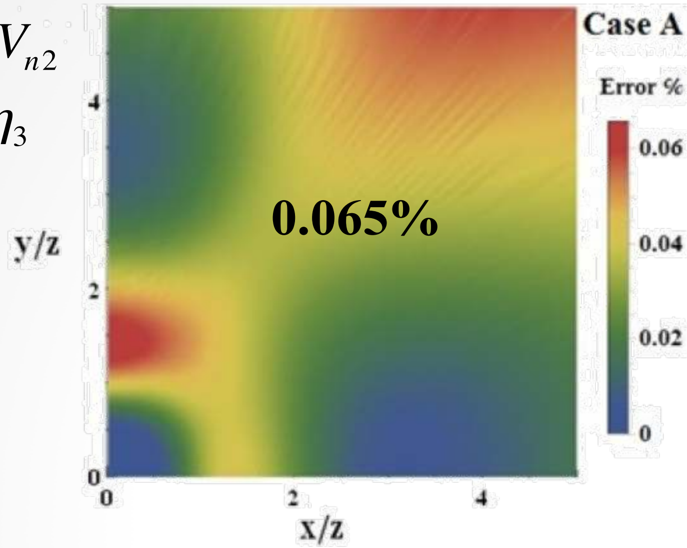
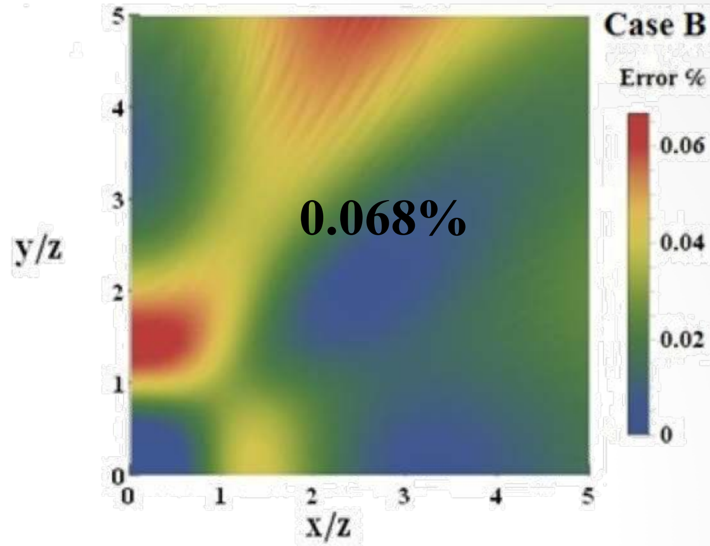
Numerical examples

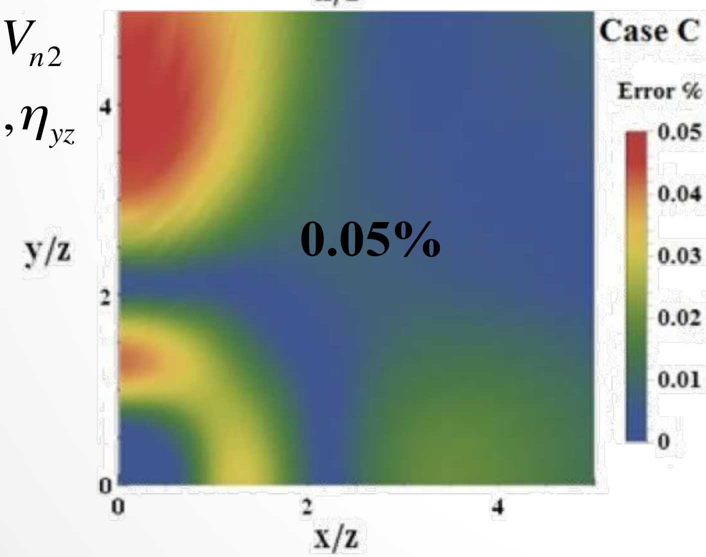
V_0, V_{n1}, V_{n2}
η_1, η_2, η_3

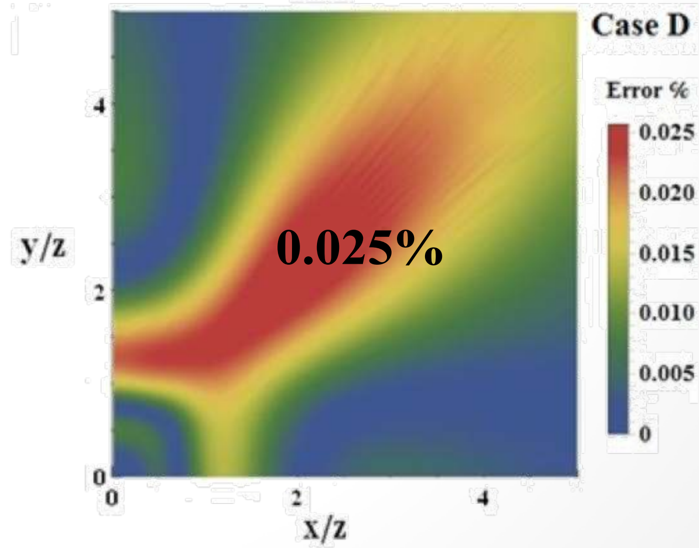




V_0, V_{n1}, V_{n2}
$\eta_1, \eta_2, \eta_{xy}$

V_0, V_{n1}, V_{n2}
$\eta_{xy}, \eta_{xz}, \eta_{yz}$

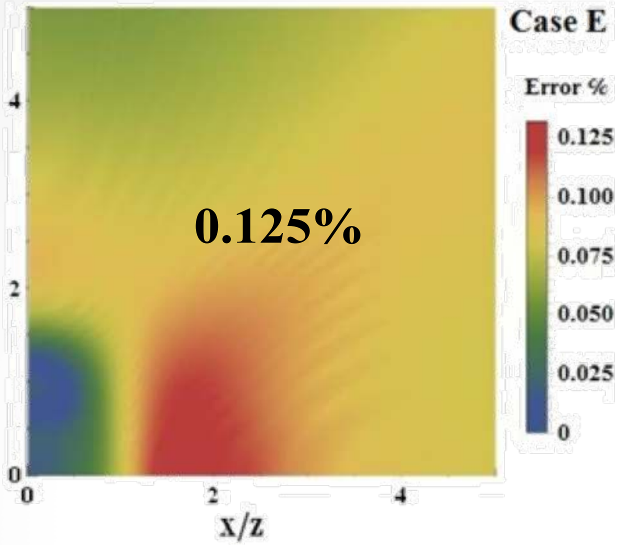




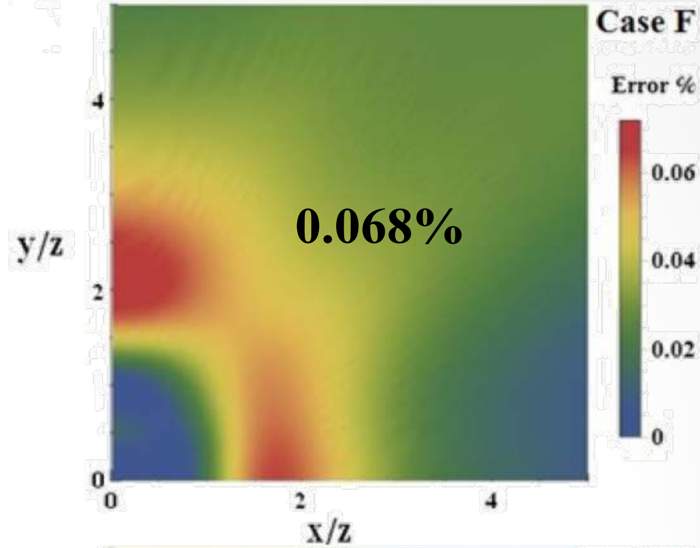
V_0, V_{h1}, V_{h2}
$\eta_1, \eta_2, \eta_{xy}$

Numerical examples

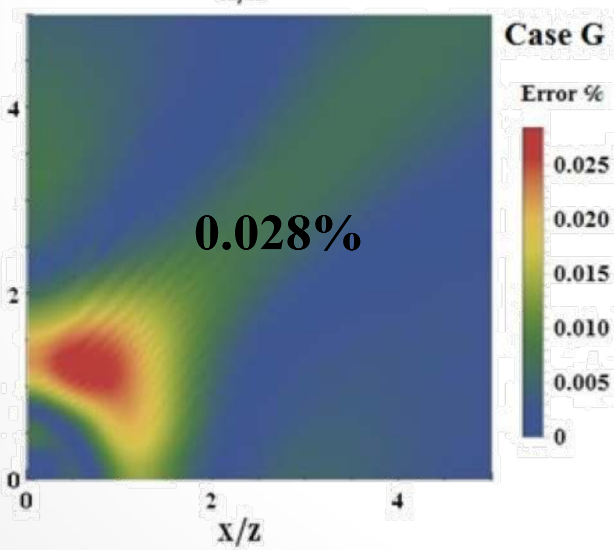
V_{12}, V_{13}, V_{23}
η_1, η_2, η_3



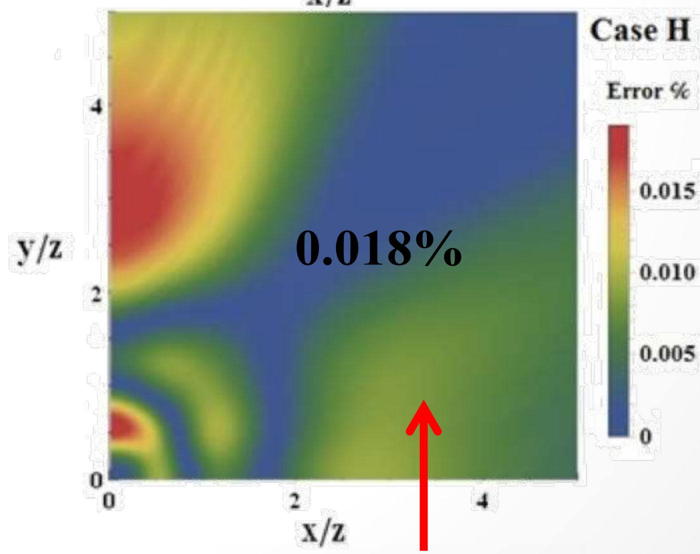
V_{12}, V_{13}, V_{23}
$\eta_{xy}, \eta_{xz}, \eta_{yz}$



V_0, V_{h1}, V_{h2}
η_1, η_2, η_3



V_0, V_{h1}, V_{h2}
$\eta_{xy}, \eta_{xz}, \eta_{yz}$



Most accurate
model dependent

Conclusions

1. A group of new parameterizations for a homogeneous ORT model.

2. The parameterization with vertical and two horizontal velocities and three cross-term anellipticity parameters results in best accuracy (model dependent).

3. More accurate traveltime results in better accuracy in the anisotropy estimation of the velocity analysis.

End

Thanks for attention !

Xu S. and Stovas A. 2017, A new parameterization for acoustic orthorhombic media, *Geophysics*, 82, C229-C240