Full Waveform Ambient Noise Inversion

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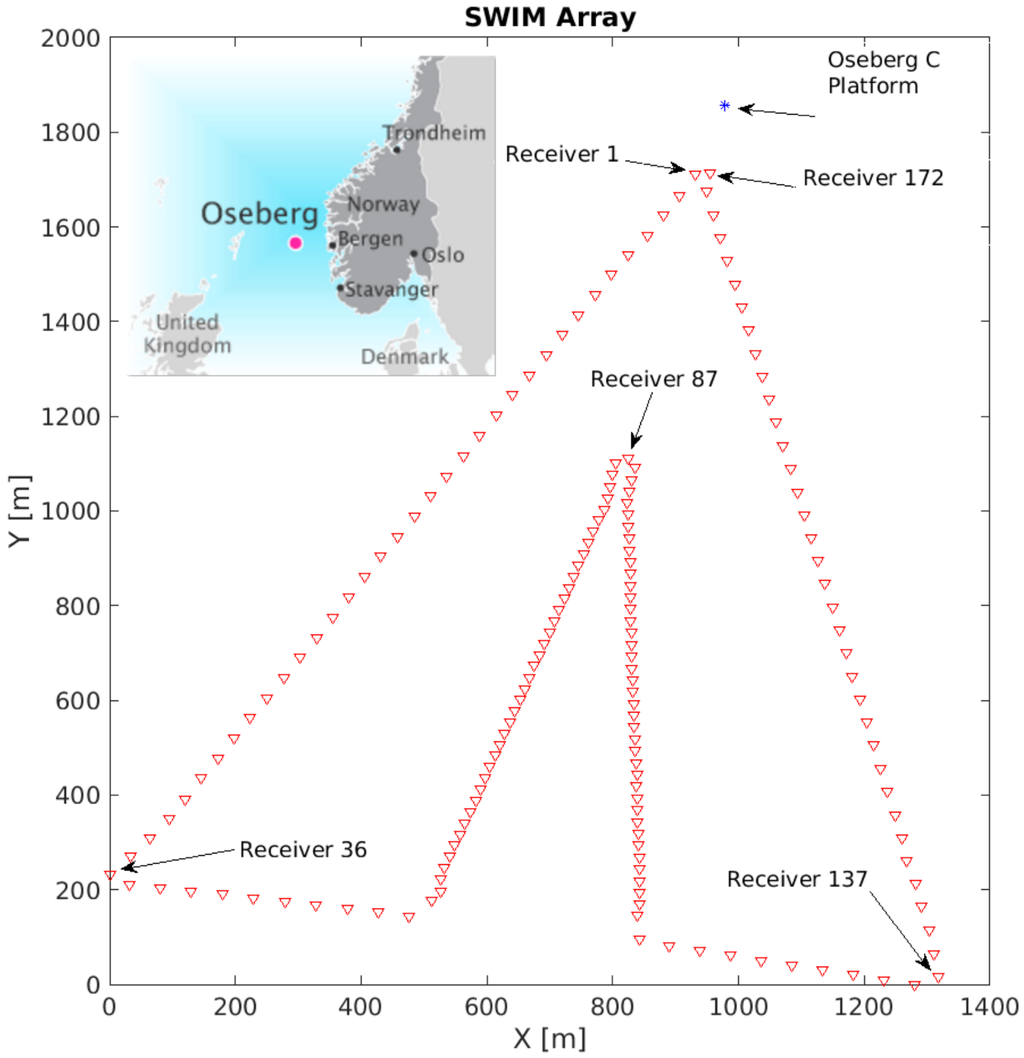
Ambient Noise At The Oseberg Array



Delaney et al. (2017)

Goal:

- Overburden monitoring
- Using ambient seismic noise
- With high temporal resolution [days, hours]

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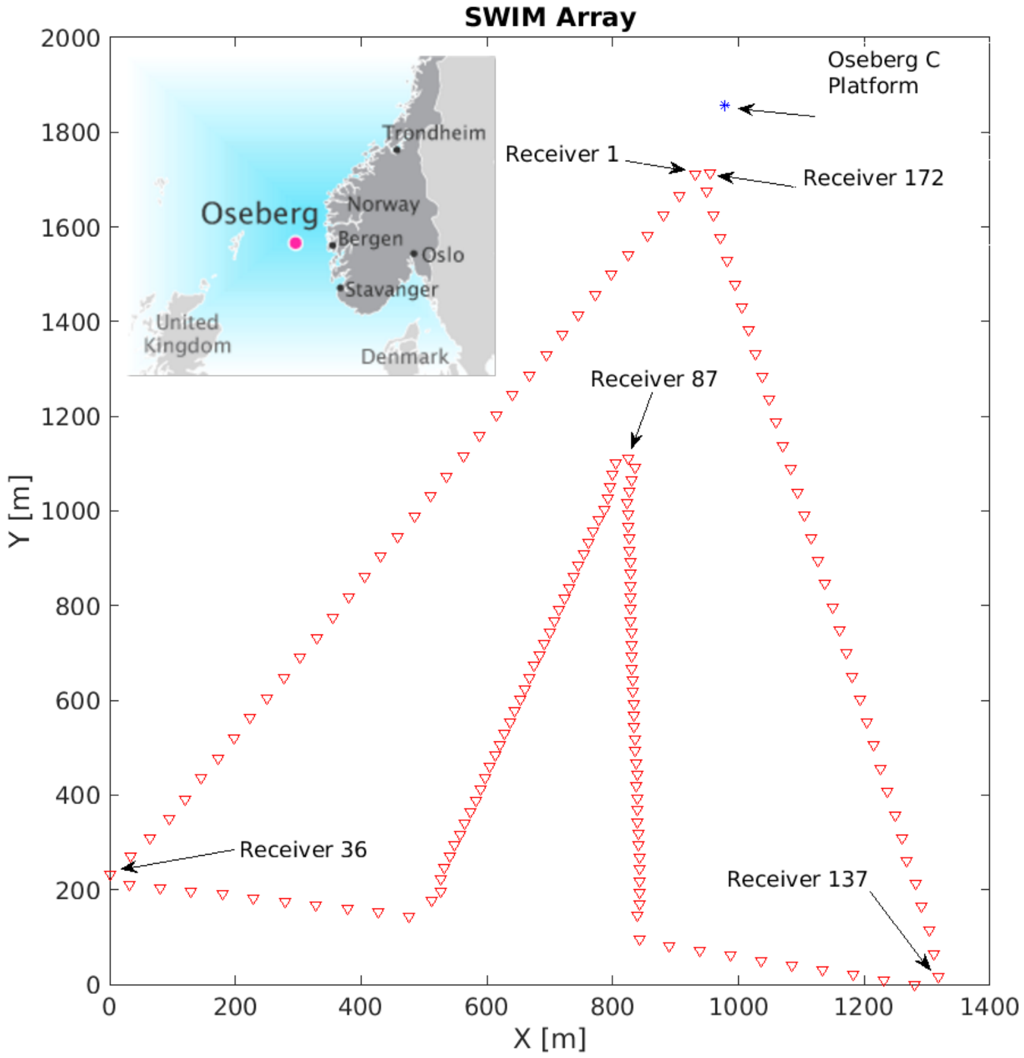

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The challenge is, however, …

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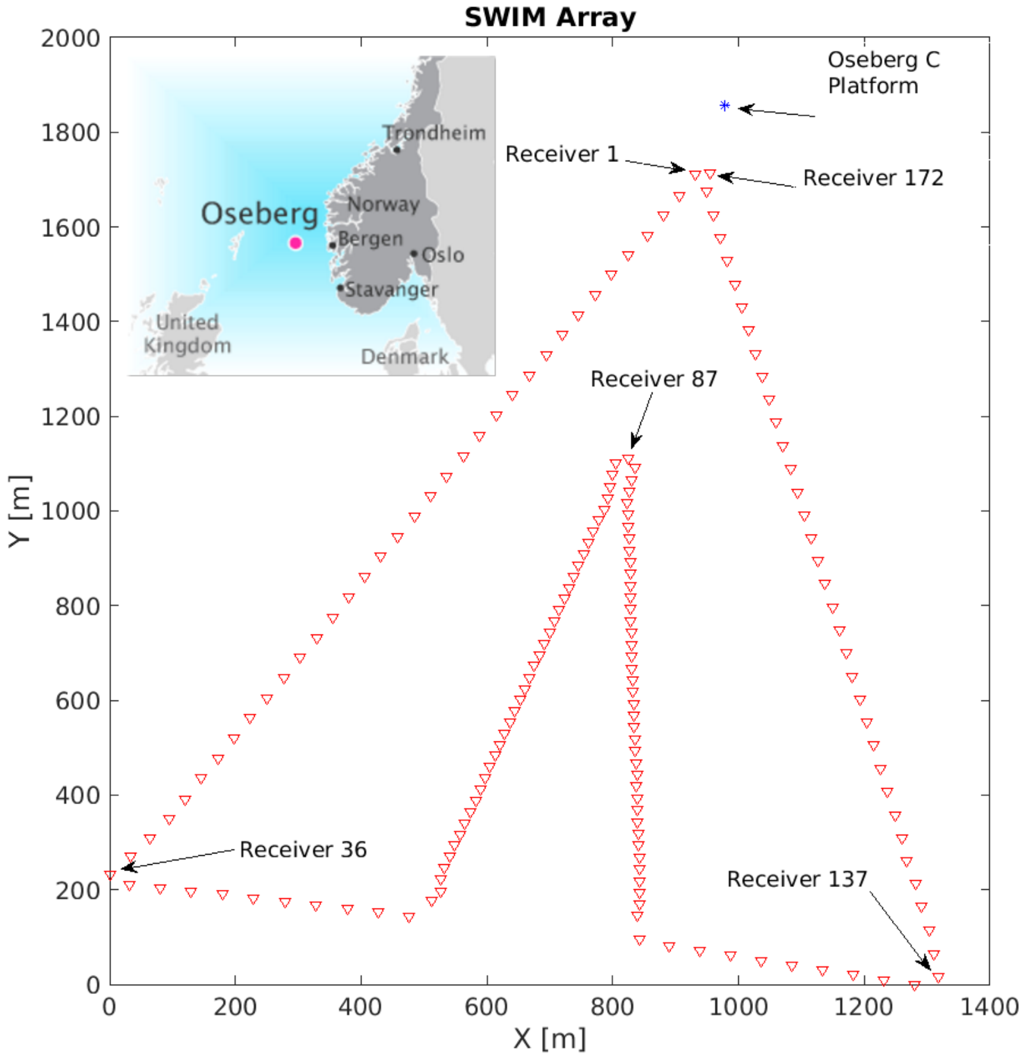

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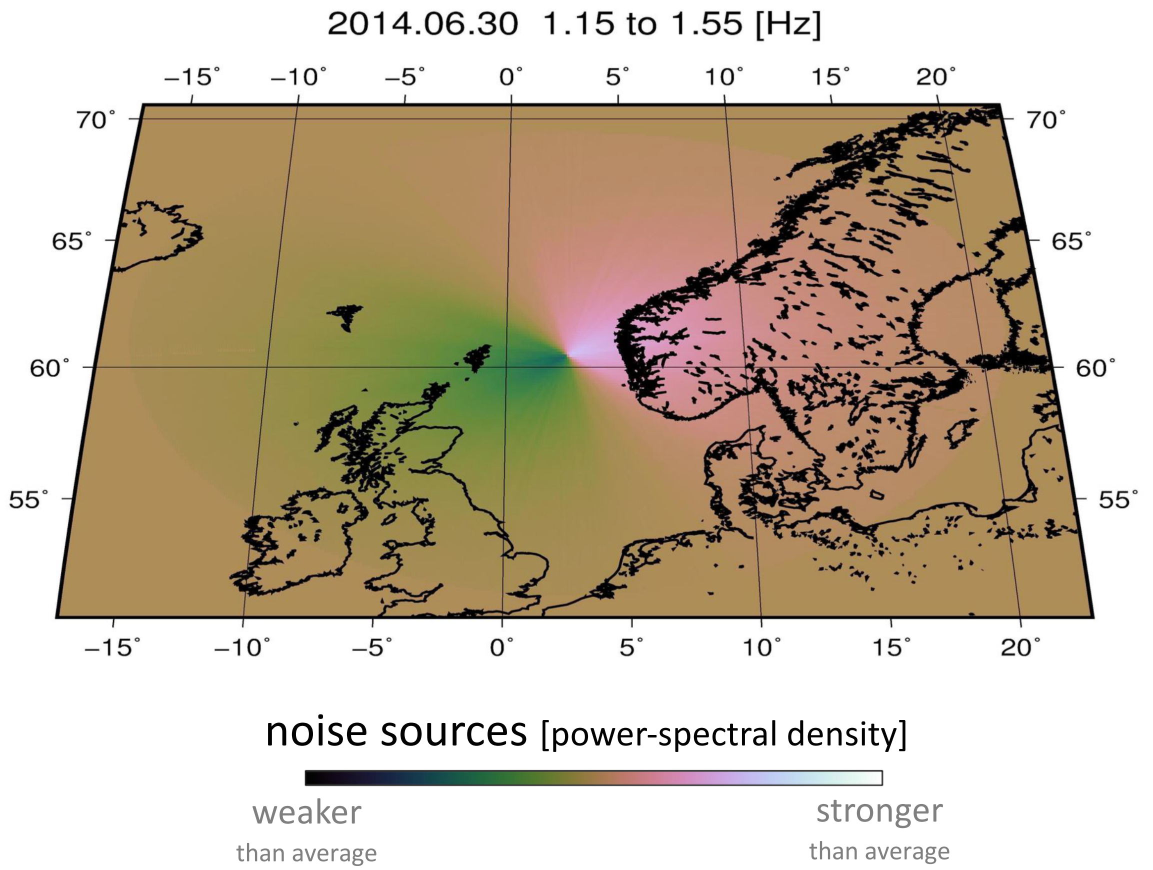
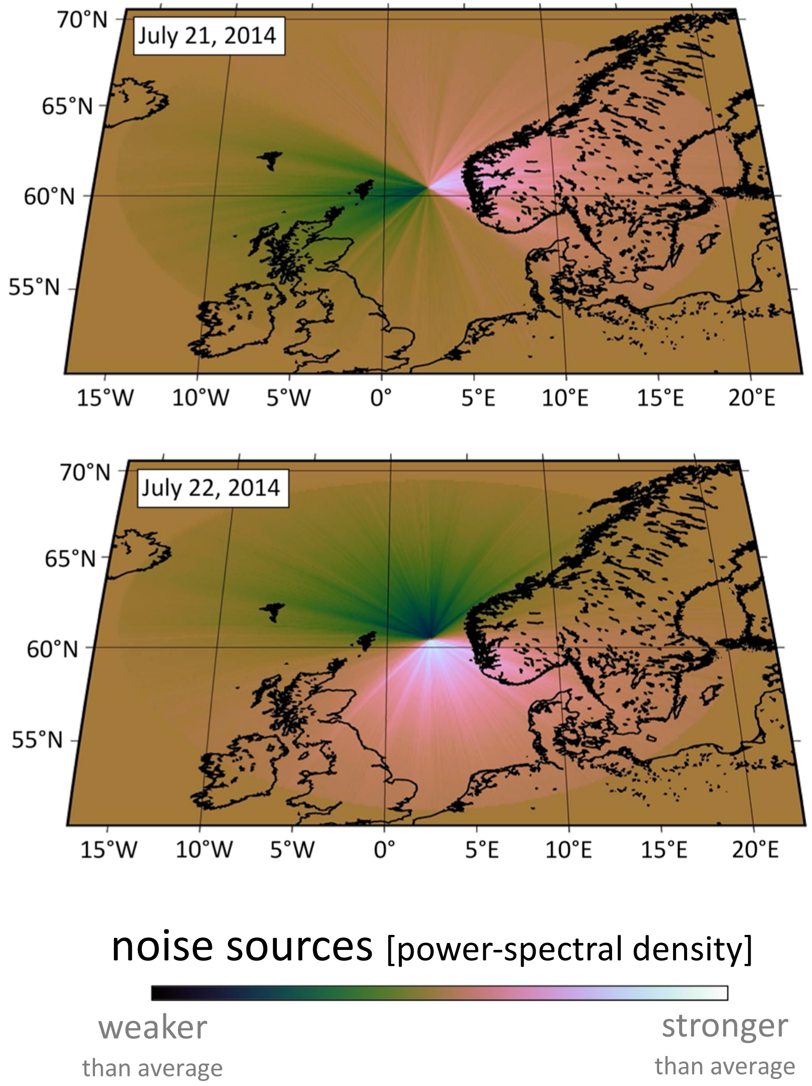
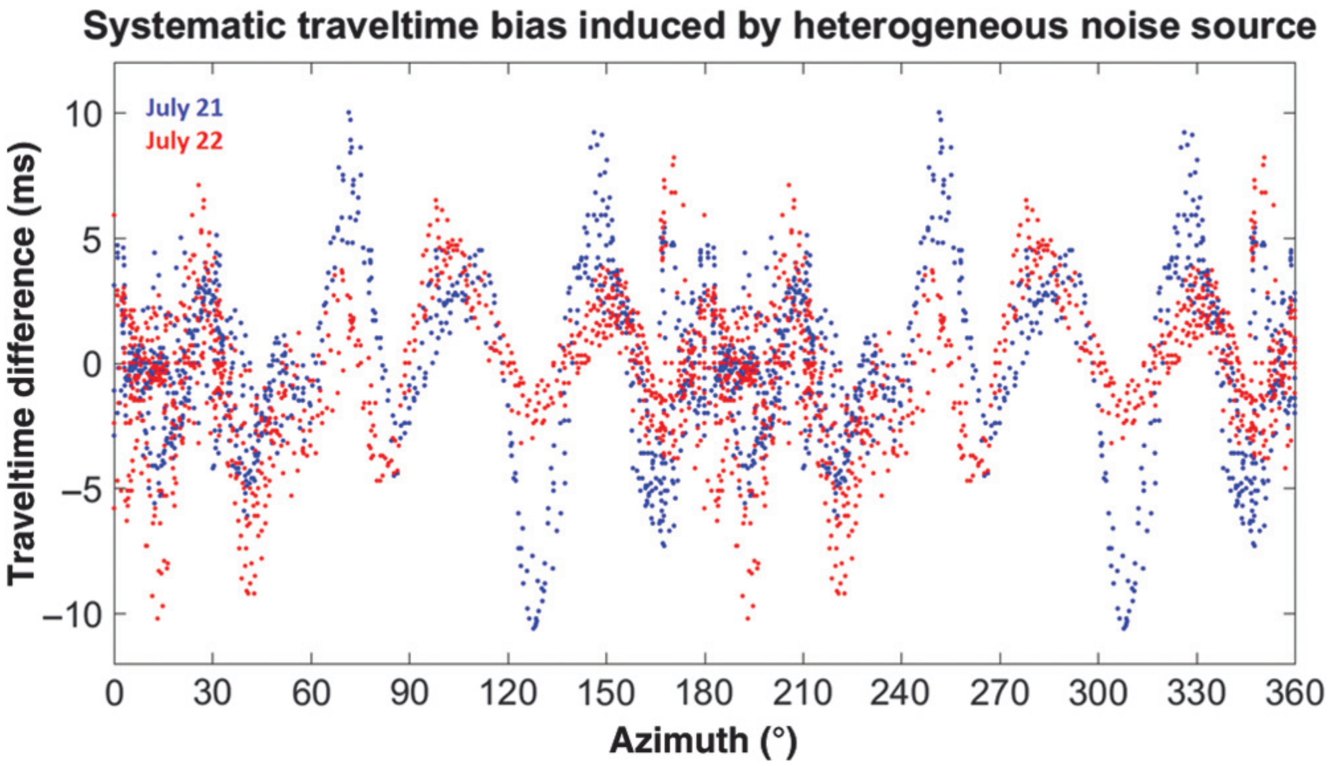


Traveltime Biases





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Objectives

- **Noise-based monitoring with high temporal resolution**
  - Nearly impossible unless sources are stationary [even when only traveltimes are used]

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- **Improve the resolution of ambient noise tomography**
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  - Mostly restricted to fundamental mode Rayleigh waves
  - Spurious arrivals [e.g. Halliday and Curtis 2008, Kimman and Trampert 2010]
  - Amplitudes are difficult to interpret [e.g. Cupillard and Capdeville 2010, Tsai 2011, Stehly and Boué 2017]
  - Data selection / heavy preprocessing [e.g. Bensen et al. 2007 , Mordret et al. 2015]
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Approach: Full Waveform Ambient Noise Inversion

- Develop & apply joint FWI for noise sources and Earth structure

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- **Improve the resolution of tomographic images**

Approach: Full Waveform Ambient Noise Inversion

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Outline

1. Forward problem: modeling correlations
2. Potential of FWANI: synthetic study in 2D
3. Extension to 3D
4. Conclusions & Outlook

Forward Problem

Modeling Correlations

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Omit the principle of Green function retrieval:

- Incorporate heterogeneous noise source distributions
- Account for 3D heterogeneous Earth structure
- Model the full seismic wave propagation physics

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correlation function
between receivers at $\mathbf{x}_1$ and $\mathbf{x}_2$

Green function
source at $\boldsymbol{\xi}_1$

Green function
source at $\boldsymbol{\xi}_2$

noise source
at position $\boldsymbol{\xi}_1$

noise source
at position $\boldsymbol{\xi}_2$

similar in Tromp et al. (2010), Hanasoge (2014), Fichtner (2014)

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$$\Big\langle C(\mathbf{x}_1,\mathbf{x}_2)\Big\rangle = \int G(\mathbf{x}_1,\boldsymbol{\xi}_1)\Big[\int G^*(\mathbf{x}_2,\boldsymbol{\xi}_2)\, S(\boldsymbol{\xi}_1,\boldsymbol{\xi}_2)\; d\boldsymbol{\xi}_1\Big] d\boldsymbol{\xi}_2$$

↕

power-spectral density

$$\langle N(\boldsymbol{\xi}_1)N^*(\boldsymbol{\xi}_2)\rangle = S(\boldsymbol{\xi}_1,\boldsymbol{\xi}_2)$$

similar in Tromp et al. (2010), Hanasoge (2014), Fichtner (2014)

Modeling Correlations

Representation theorem: $u(\mathbf{x}) = \int G(\mathbf{x}, \boldsymbol{\xi}) N(\boldsymbol{\xi}) \, d\boldsymbol{\xi}$

$$\Big\langle C(\mathbf{x}_1, \mathbf{x}_2) \Big\rangle = \int G(\mathbf{x}_1, \boldsymbol{\xi}_1) \left[\int G^*(\mathbf{x}_2, \boldsymbol{\xi}_2) \, S(\boldsymbol{\xi}_1, \boldsymbol{\xi}_2) \; d\boldsymbol{\xi}_1 \right] d\boldsymbol{\xi}_2$$

Step 1: Using source-receiver reciprocity, compute the Green function $G^*(\mathbf{x}_2, \boldsymbol{\xi}_2)$ with source at $\mathbf{x}_2$.

Step 2: Combine its complex conjugate with the power-spectral density $S(\boldsymbol{\xi}_1, \boldsymbol{\xi}_2)$.

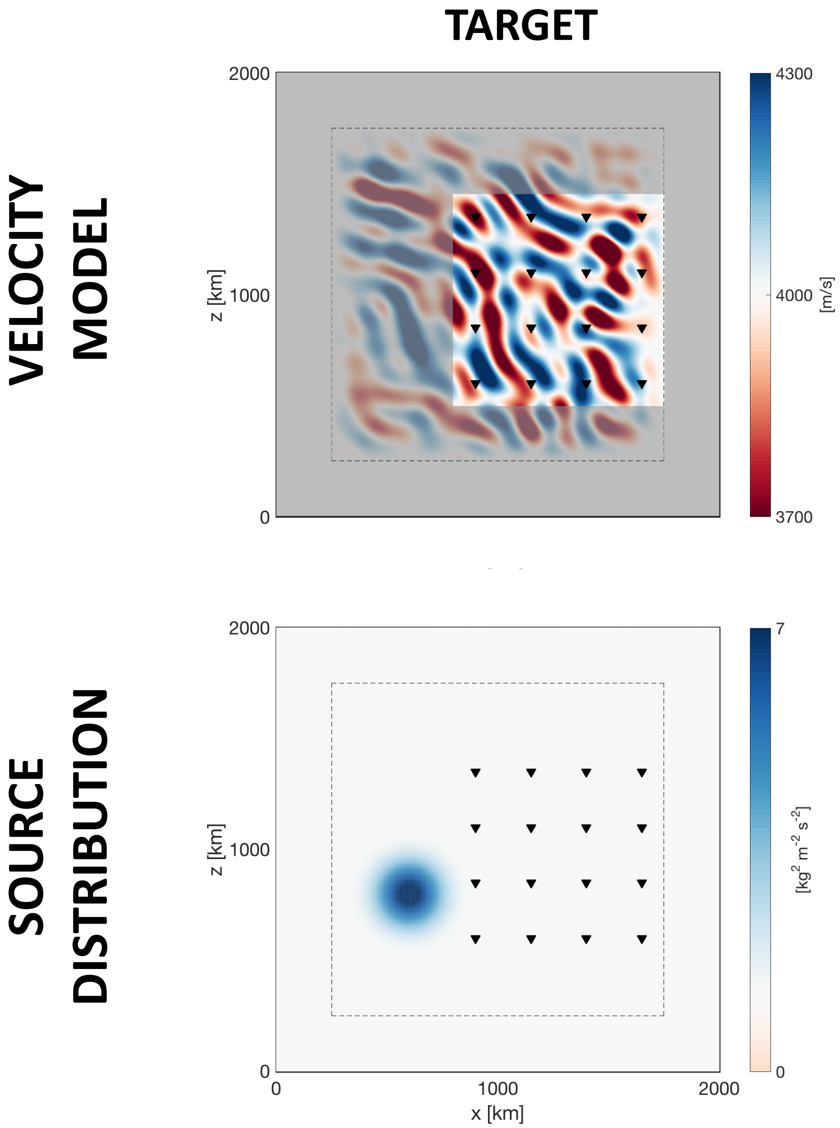
Step 3: Model the correlation wavefield as solution of the wave equation with $\int G^*(\mathbf{x}_2, \boldsymbol{\xi}_2) \, S(\boldsymbol{\xi}_1, \boldsymbol{\xi}_2) \; d\boldsymbol{\xi}_1$ as distributed source.

similar in Tromp et al. (2010), Hanasoge (2014), Fichtner (2014)

SYNTHETIC STUDY IN 2D

POTENTIAL OF FWANI

Potential of Full Waveform Ambient Noise Inversion



Inversion scheme:

- **2D finite-difference discretization of the membrane wave equation** [analogous to fundamental-mode surface wave propagation]
- **Iterative inversion scheme** [based on L-BFGS (Nocedal and Wright 2006)]
- **Stopping criterion:** reduction of the norm of the initial gradient by a factor of 10^{-3}
- **Gaussian smoothing operator as part of the parameterization** [choose the dominant wavelength as standard deviation]
- **Tikhonov regularization term** [Tikhonov 1963]

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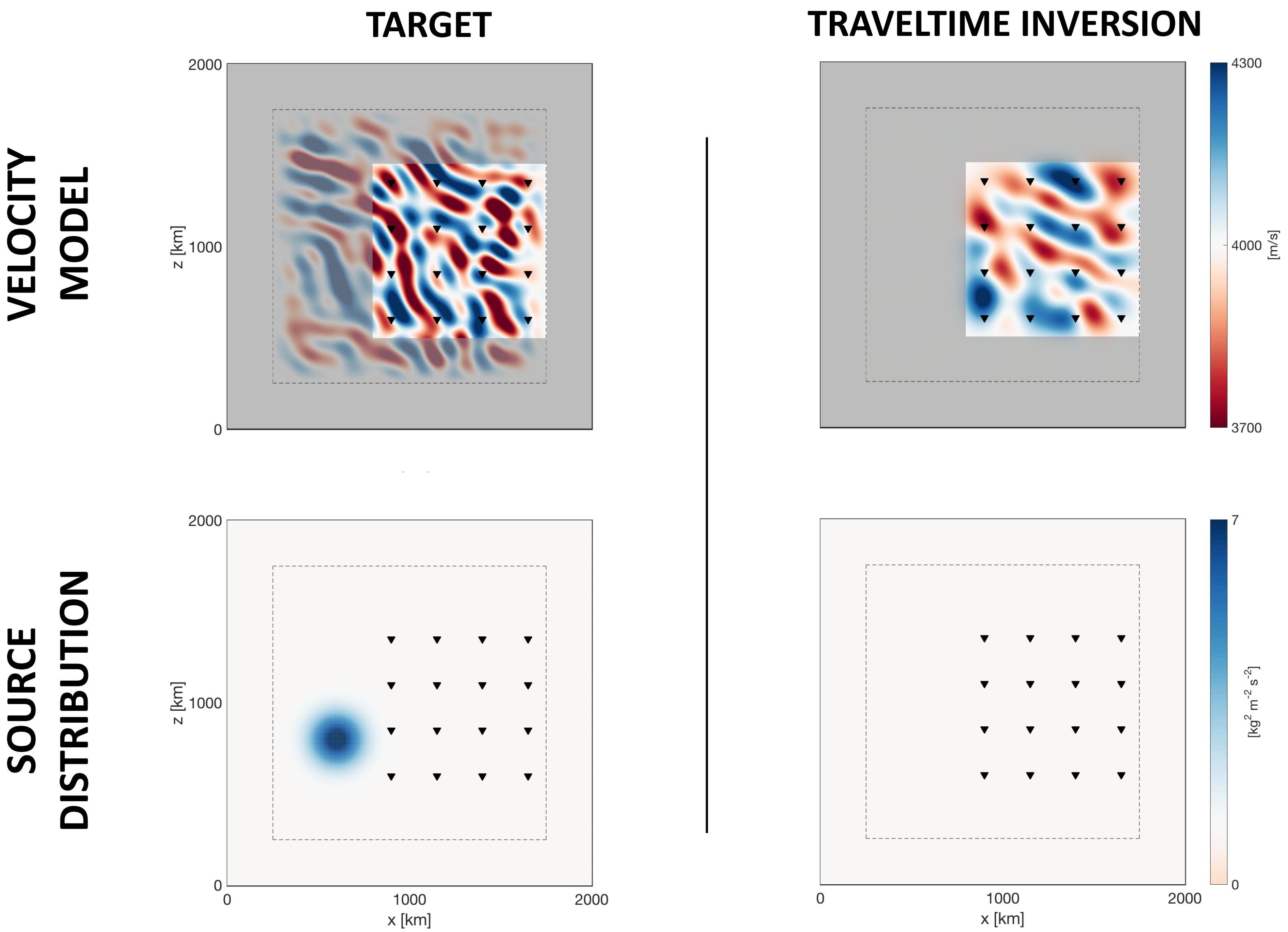

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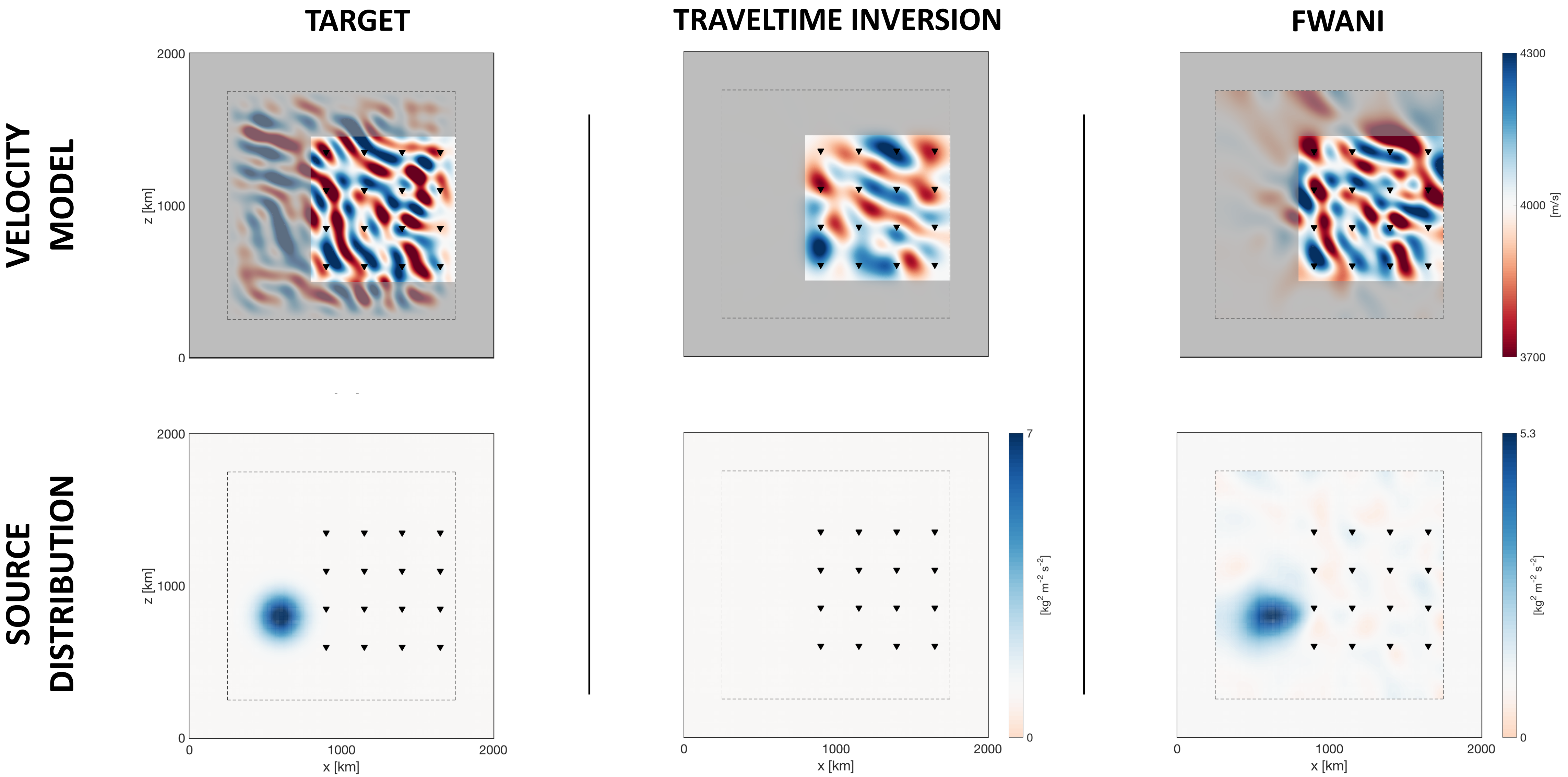
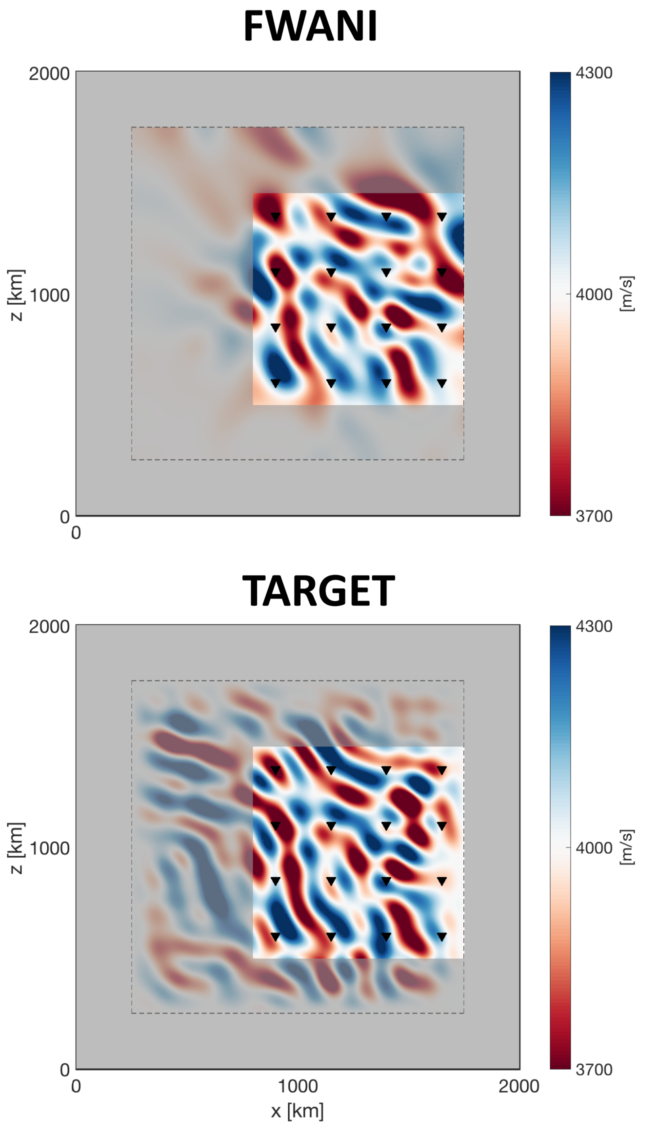


POTENTIAL OF FULL WAVEFORM AMBIENT NOISE INVERSION (FWANI)

- Possible to go **beyond** traditional ambient noise tomography and to **account** for the **noise source distribution**

- Knowledge of the **noise source distribution** is **essential**

- **Trade-offs** between source and structure can be quantified using **Hessian-vector products**
  [Sager et al., GJI, 2018]



Extension to 3D

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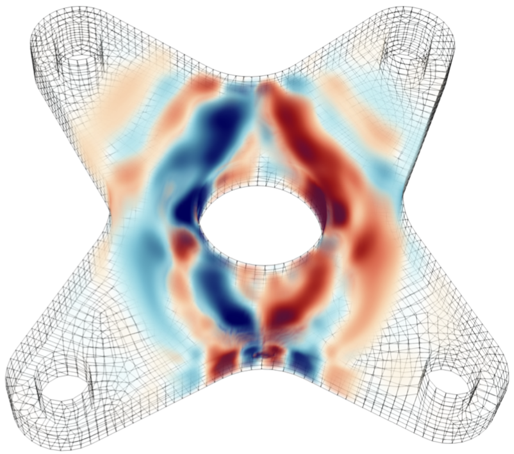
SALVUS

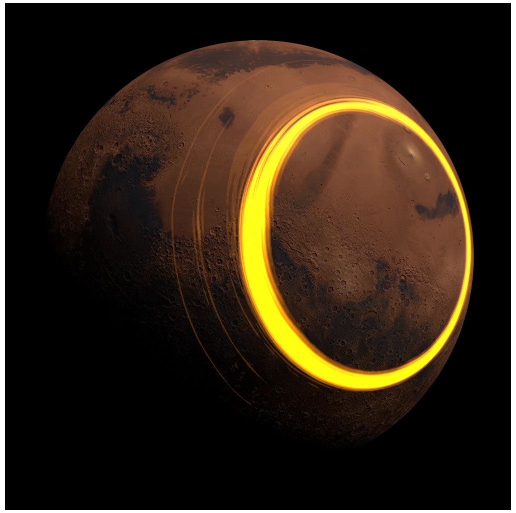
IMAGE EVERYTHING

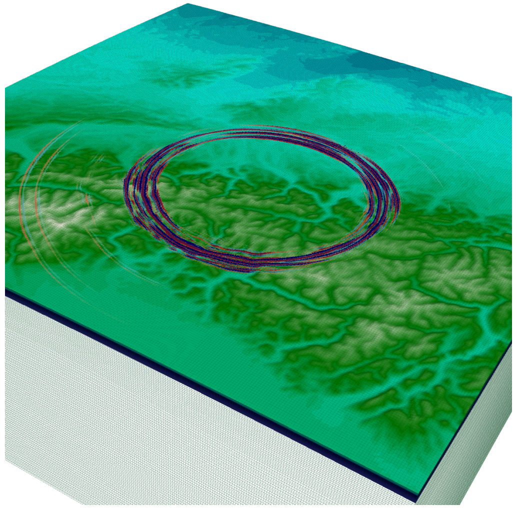
www.salvus.io

A new, high-performance package for FWI with a focus on efficiency, reproducibility, flexibility and scale independence.

- spectral elements
- visco-elastic, acoustic, and coupled
- hexahedra, tetrahedra
- built-in mesher



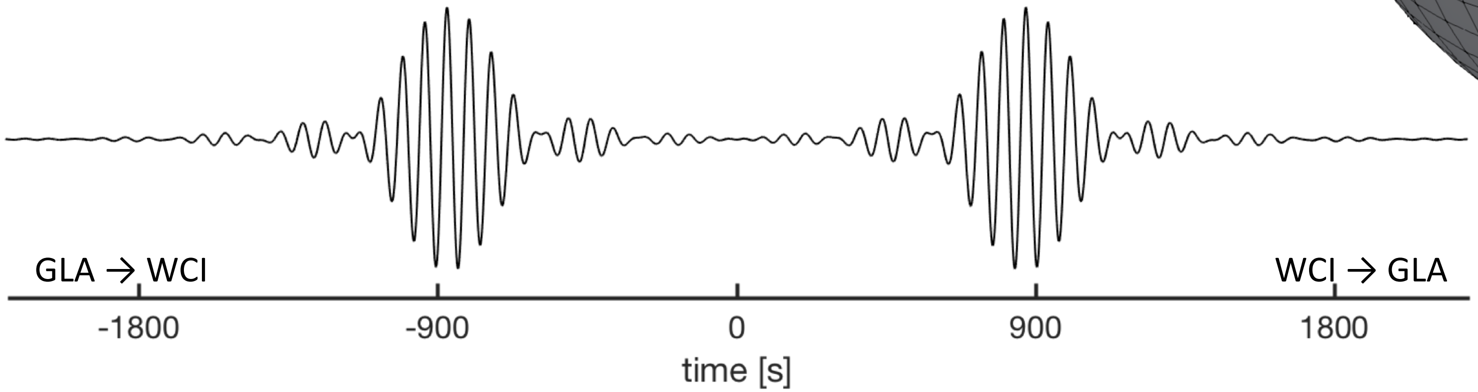




Afanasiev et al., submitted to GJI

GLOBAL FORWARD MODELLING

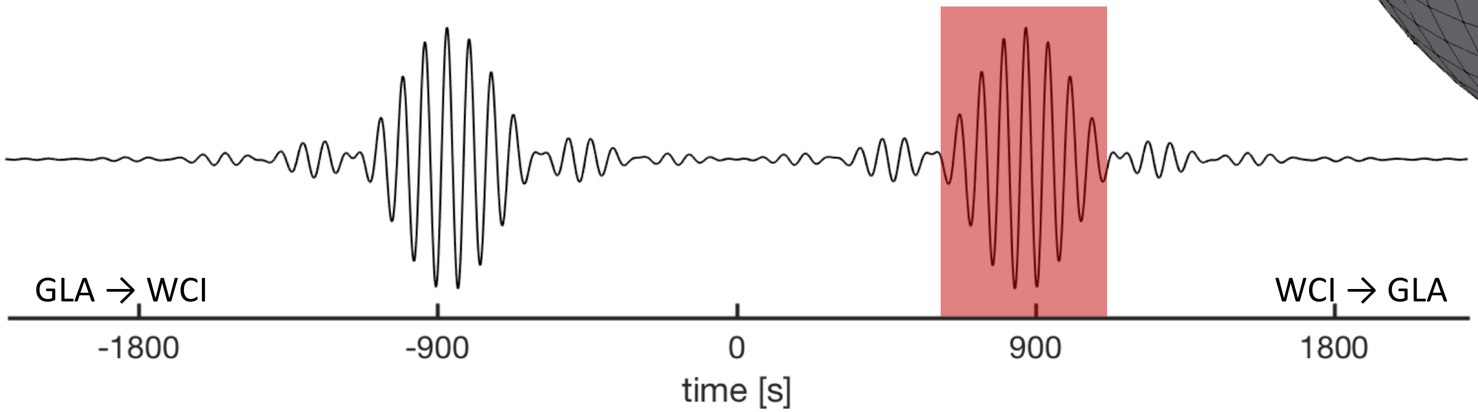
- Velocity model: PREM
- Period band: 55-75 sec
- Homogeneous noise source distribution



Noise Source Sensitivity

Which noise sources in space contribute to the waveform in the **surface wave** window?

[for an energy measurement]



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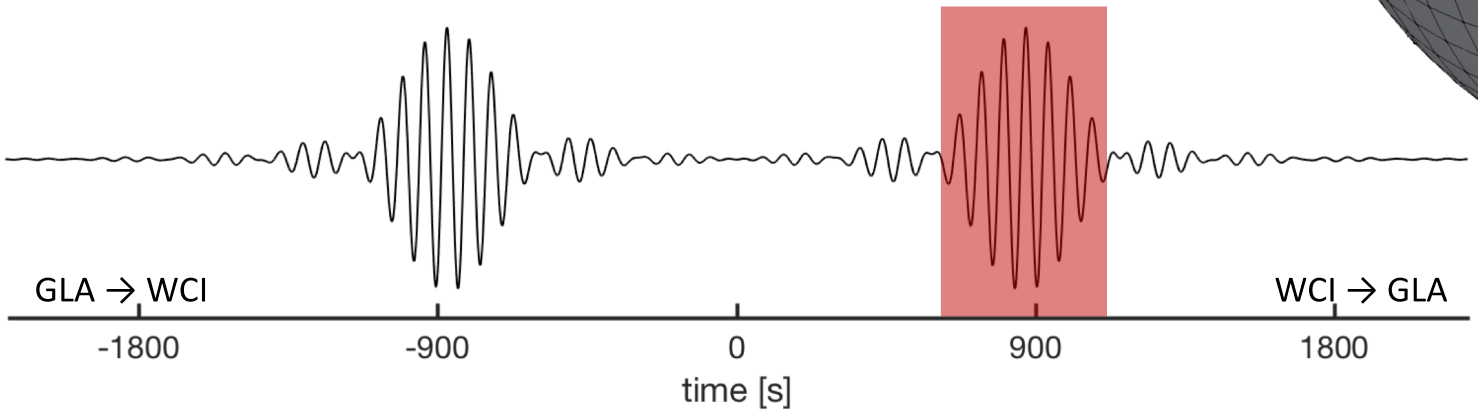

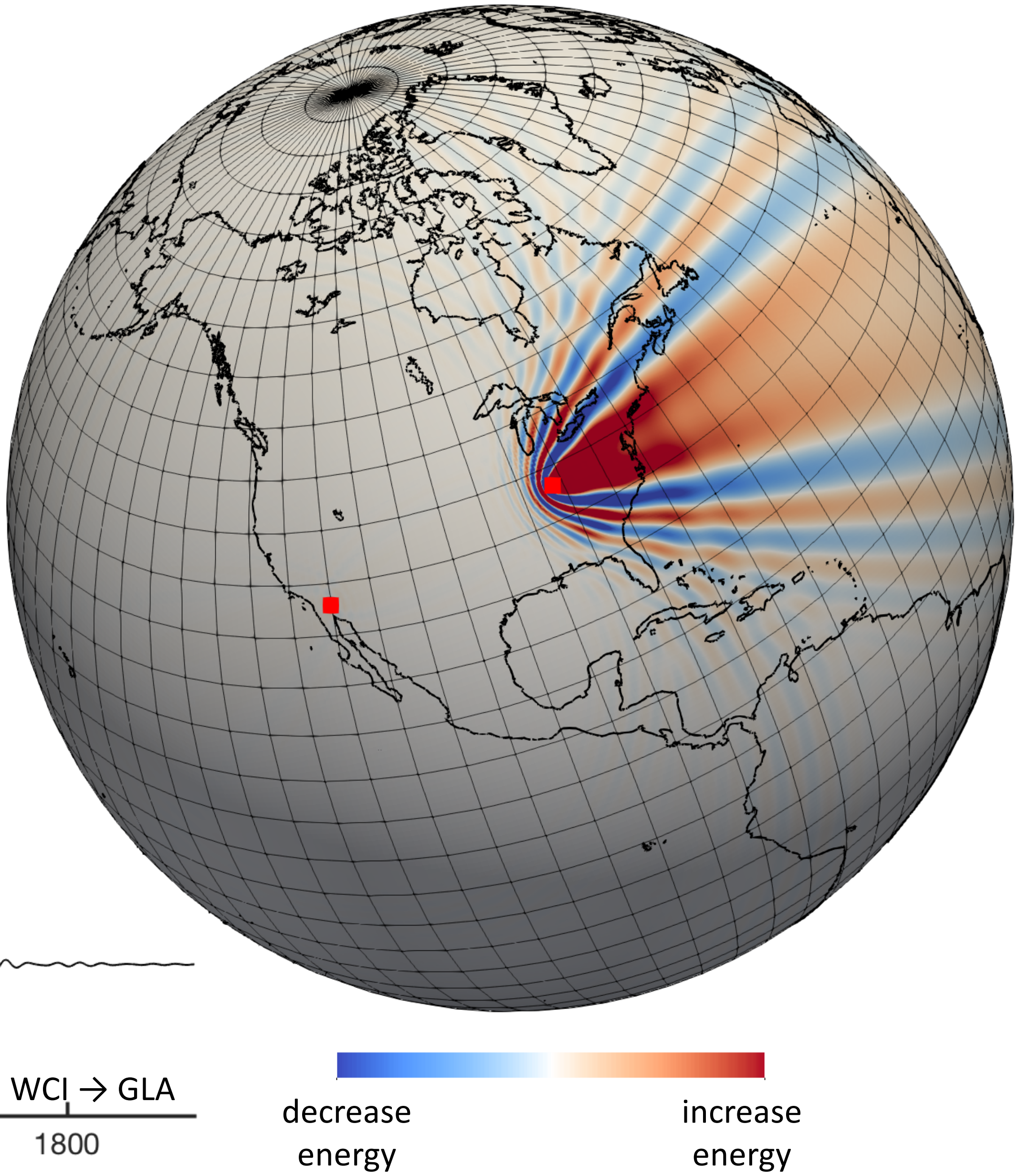

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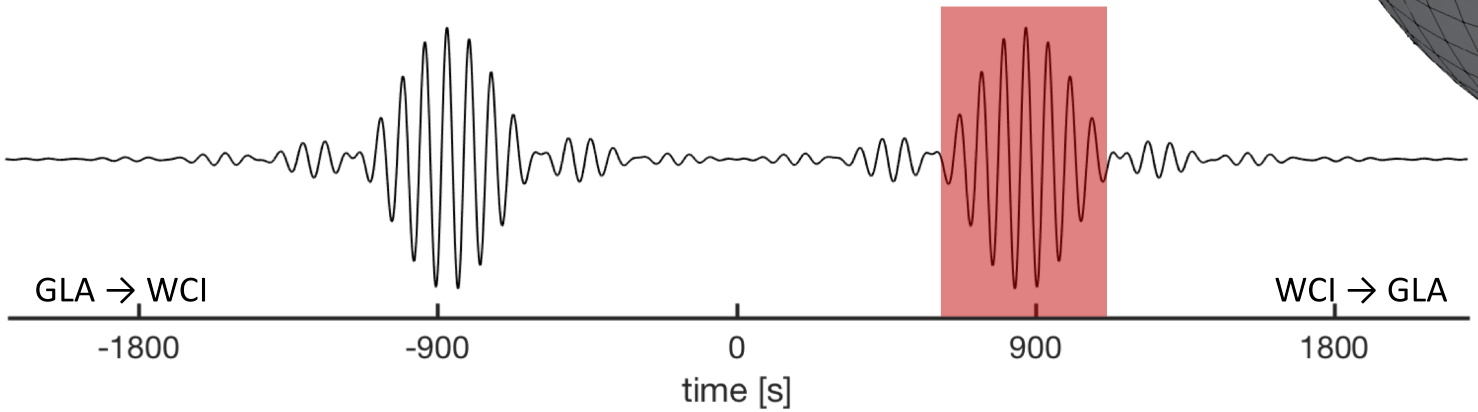

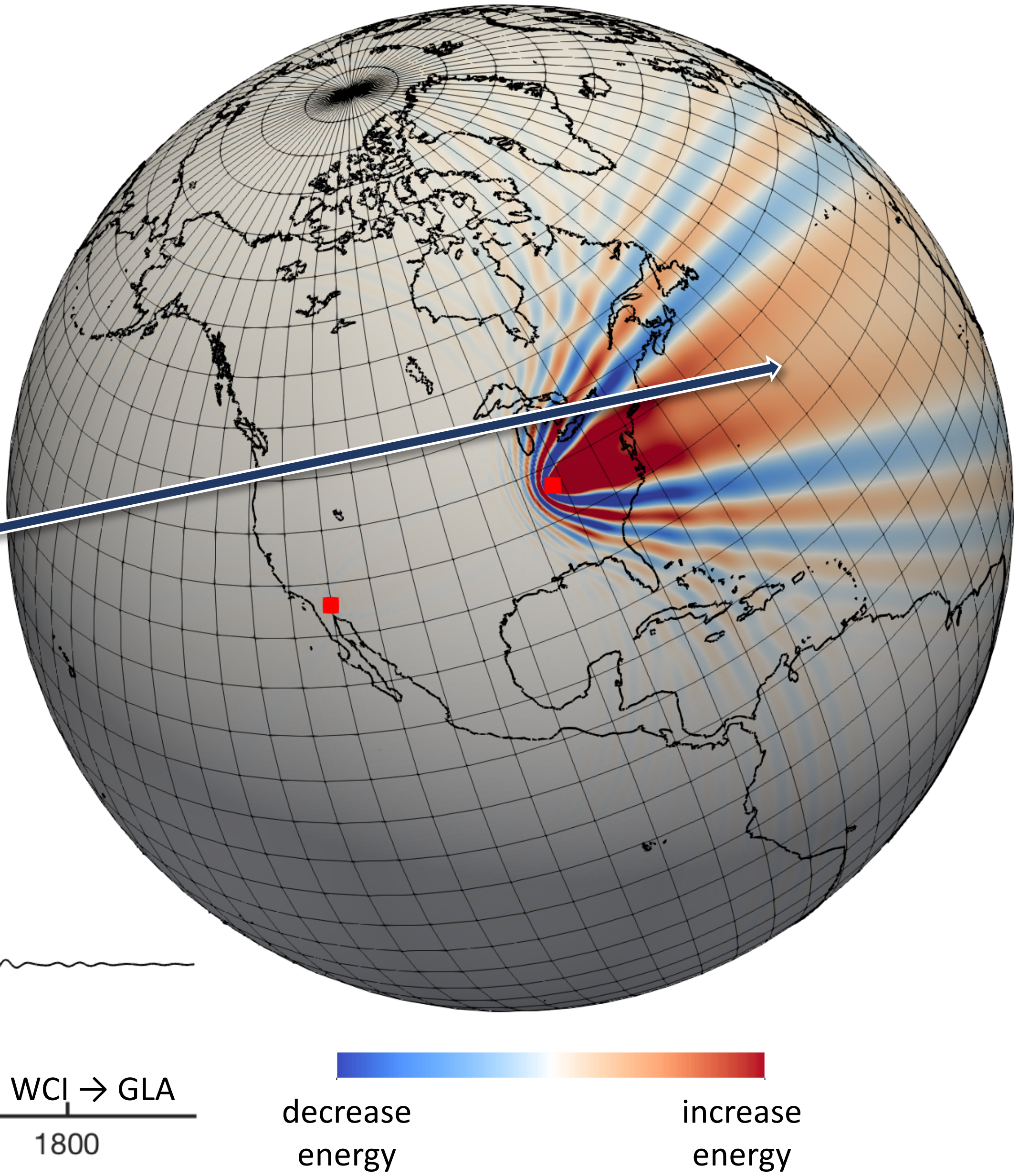

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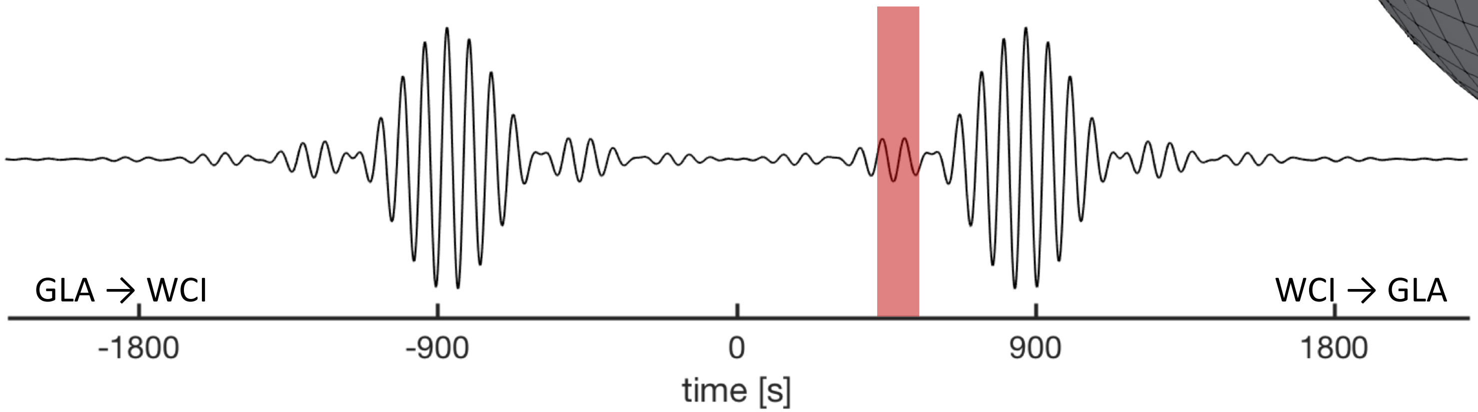

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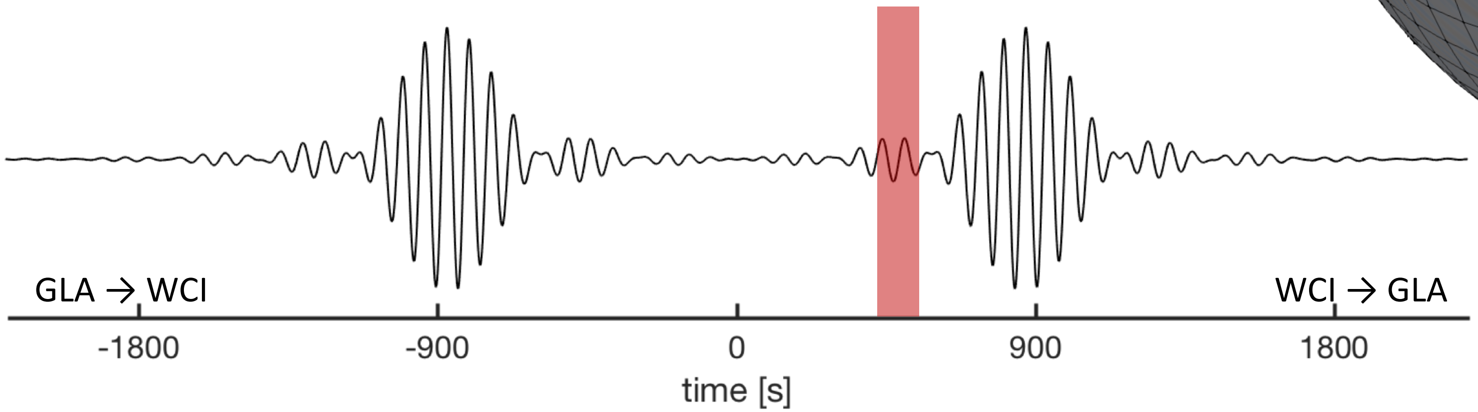

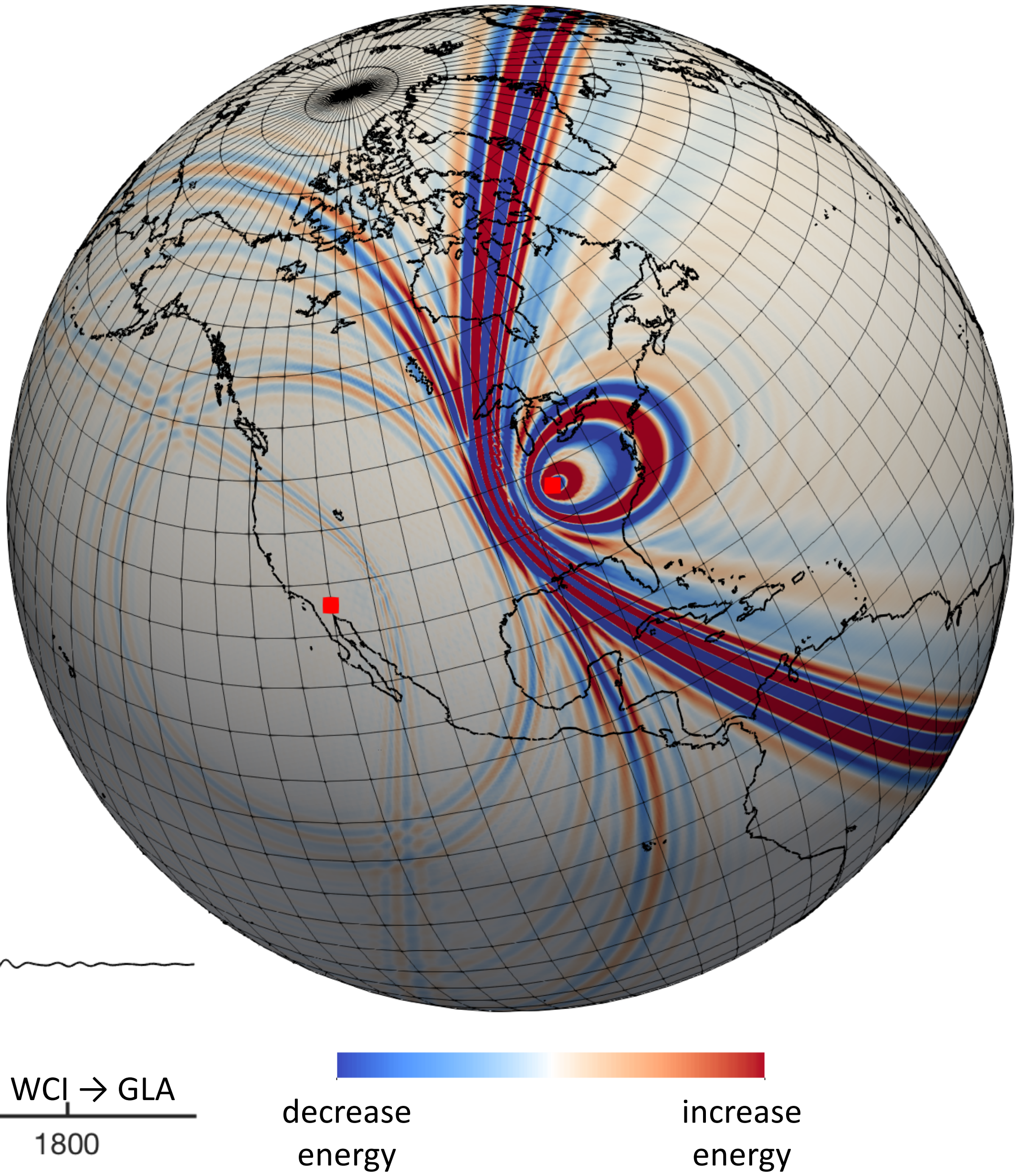

Noise Source Sensitivity

Which noise sources in space contribute to the waveform in the **S-wave** window?

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sensitivity to

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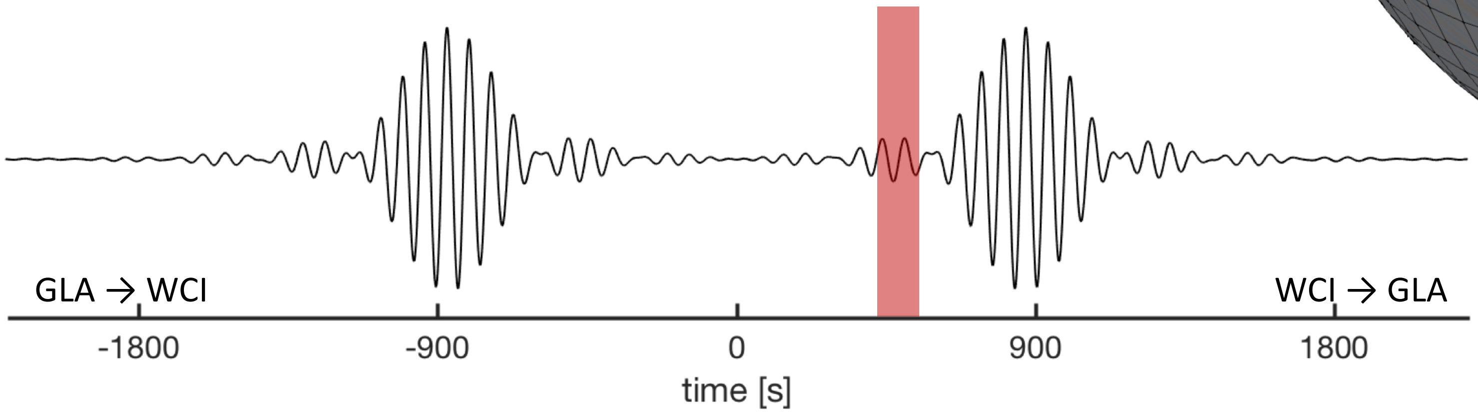

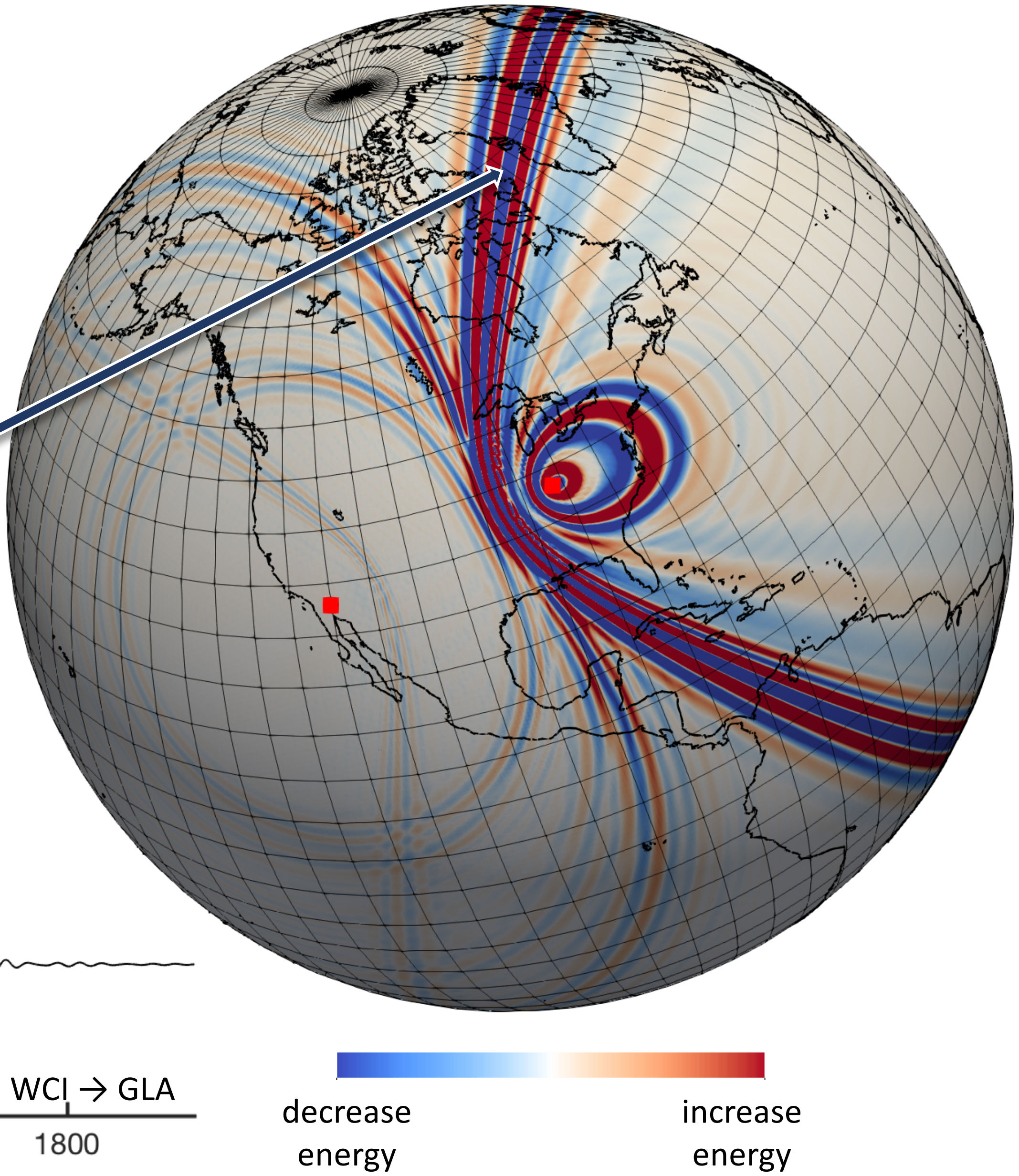

CONCLUSIONS & OUTLOOK

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Synthetic 2D inversions

- Potential to jointly invert for sources and structure

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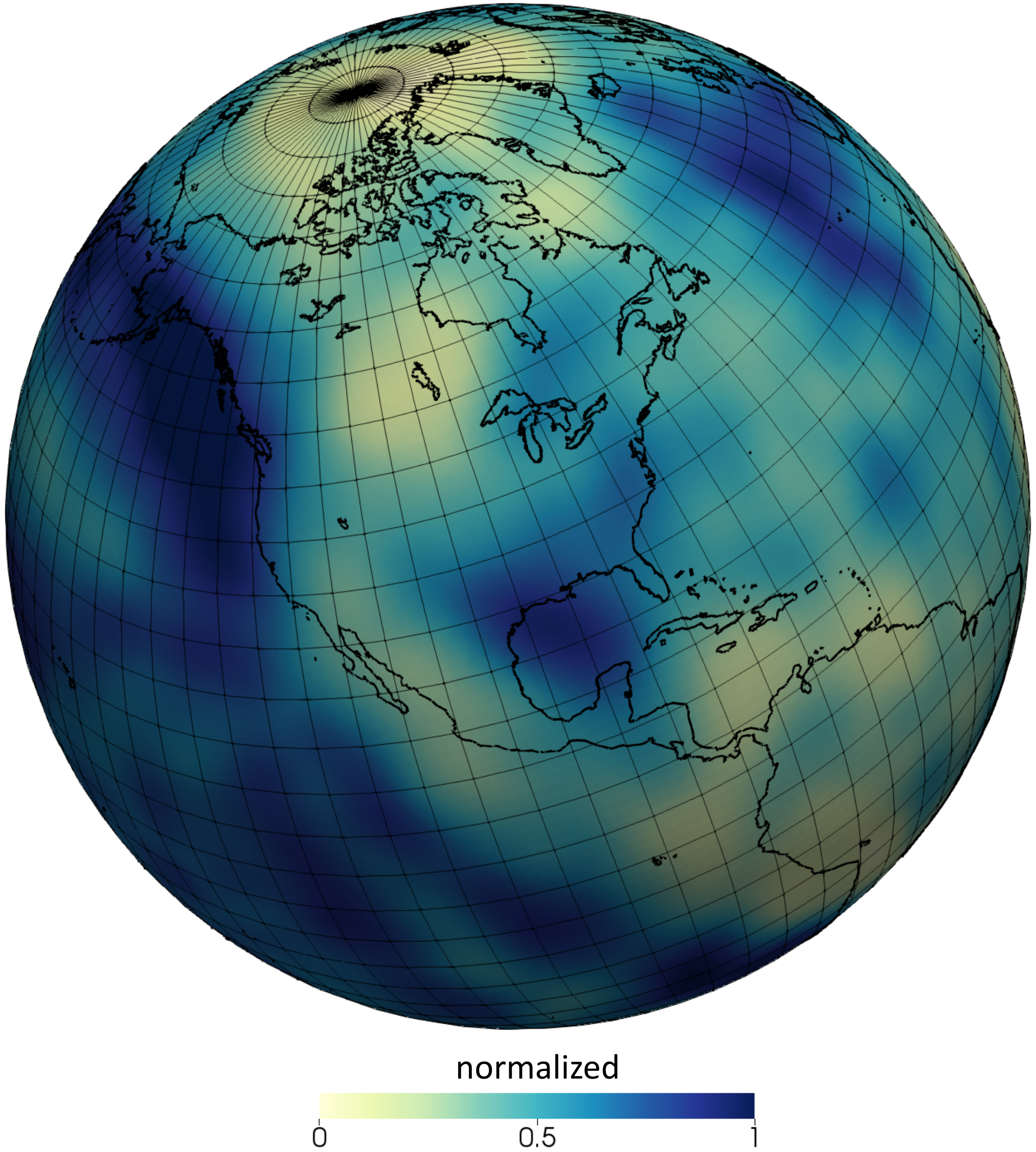
PSD for Earth's hum in winter
[Ermert et al., JGR, 2017]



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3D source-structure inversion

- Framework for 3D media with heterogeneous noise source distributions on the surface
- In progress for the global scale
- Translation to other scales seems easily feasible

Thank you for your attention!

Sager et al., *GJI* 2018. **Towards full-waveform ambient noise inversion**.

Ermert et al., *JGR* 2017. **Ambient noise source inversion in a heterogeneous Earth**.

Delaney et al., *Geophysics* 2017. **Passive seismic monitoring with nonstationary noise sources**.