Sewing the Born Approximation into the Finite Difference Scheme with Interpolation to Model Small-Scale Structures

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April 24, 2018
ROSE meeting, Trondheim

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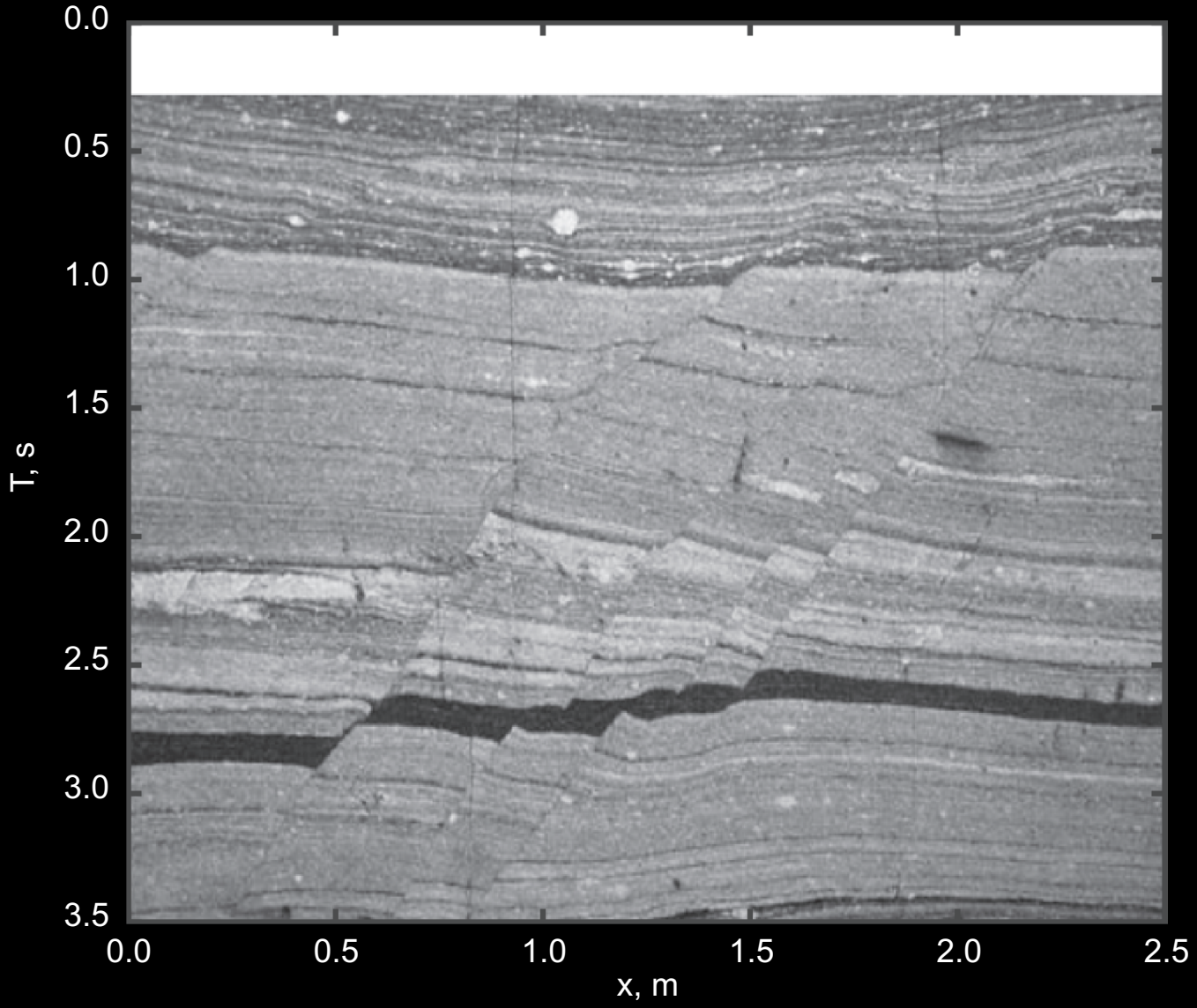
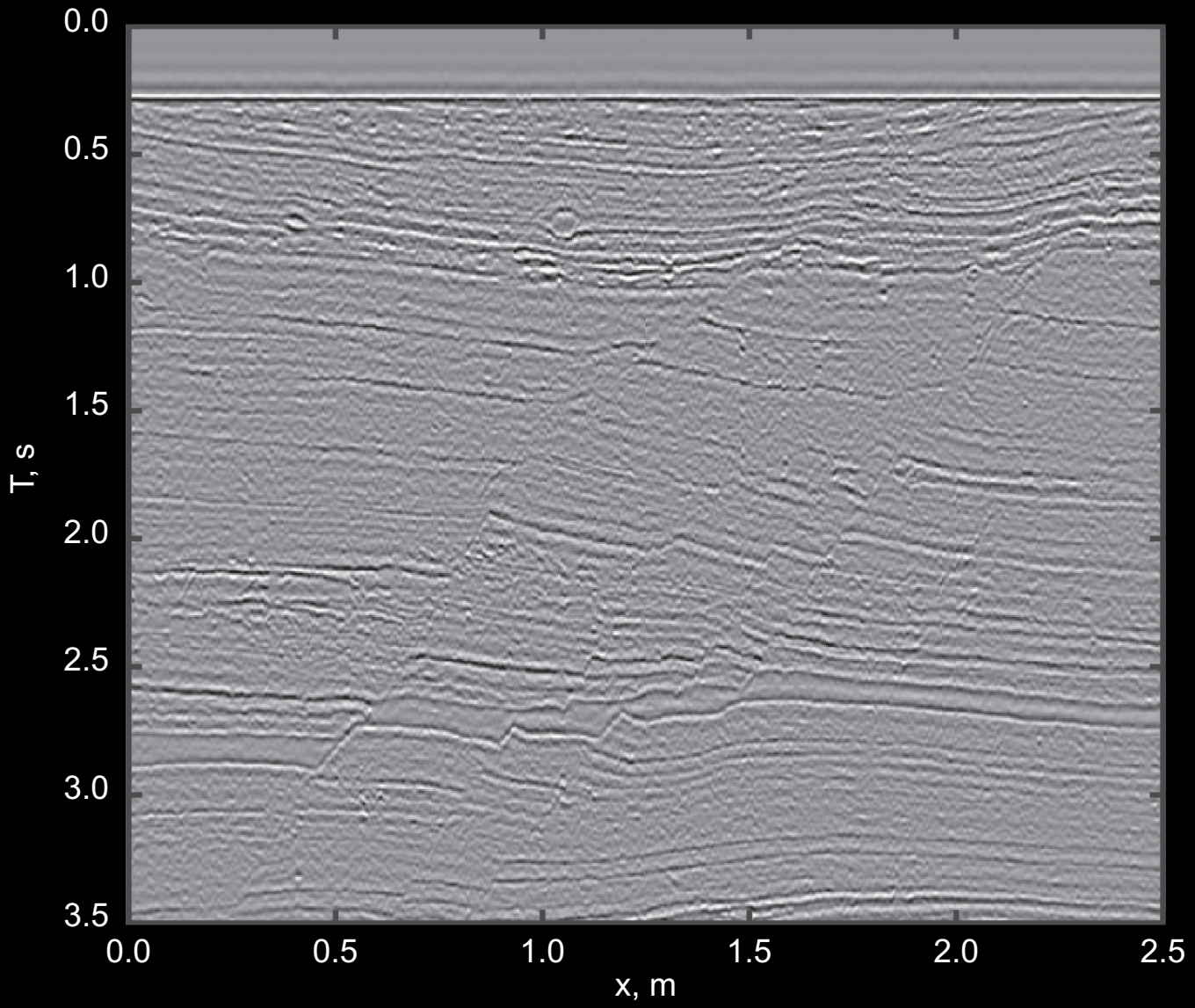
True velocity model



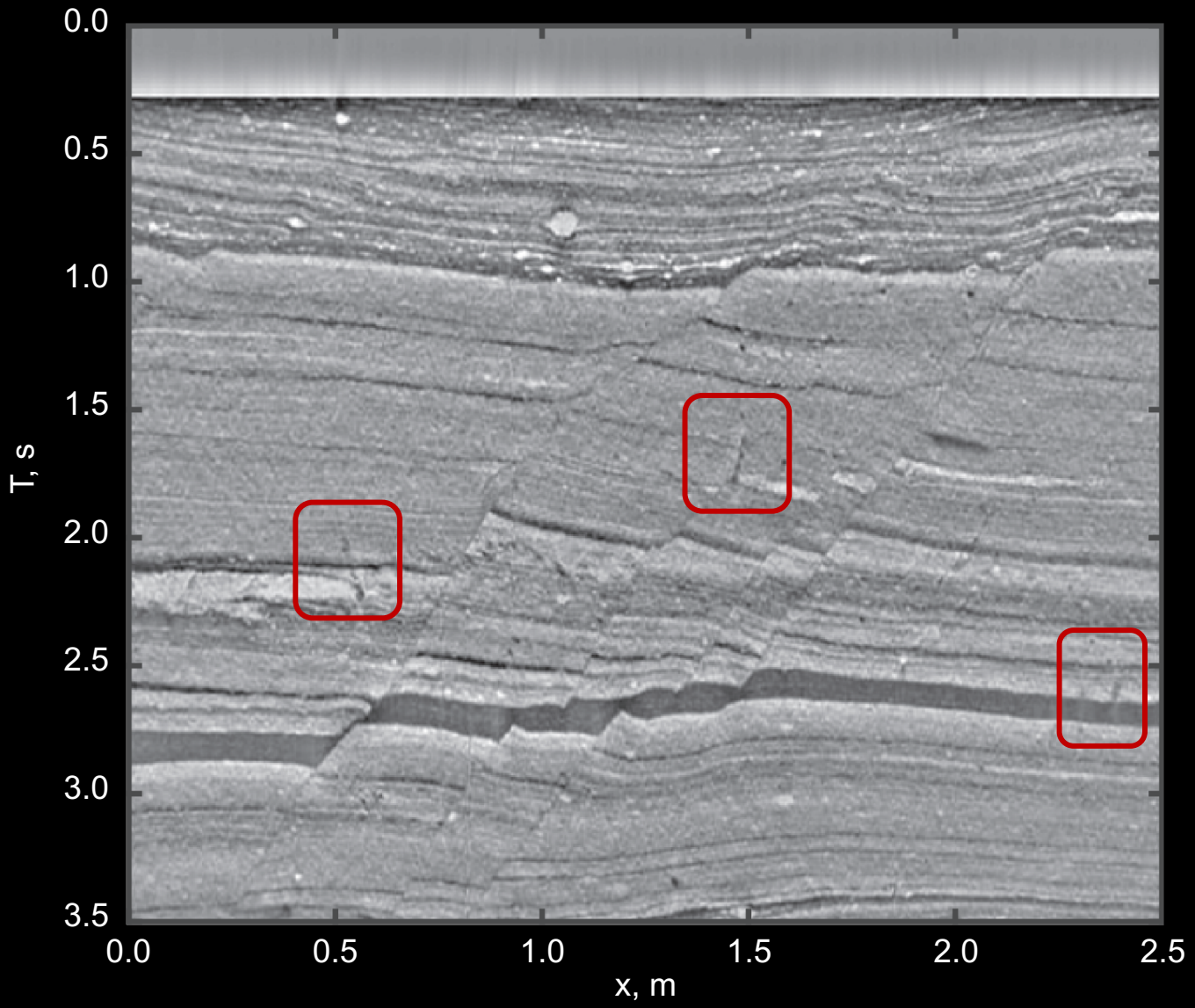
courtesy of Kjetil Haavik

Inverted velocity model, 15 – 50 Hz



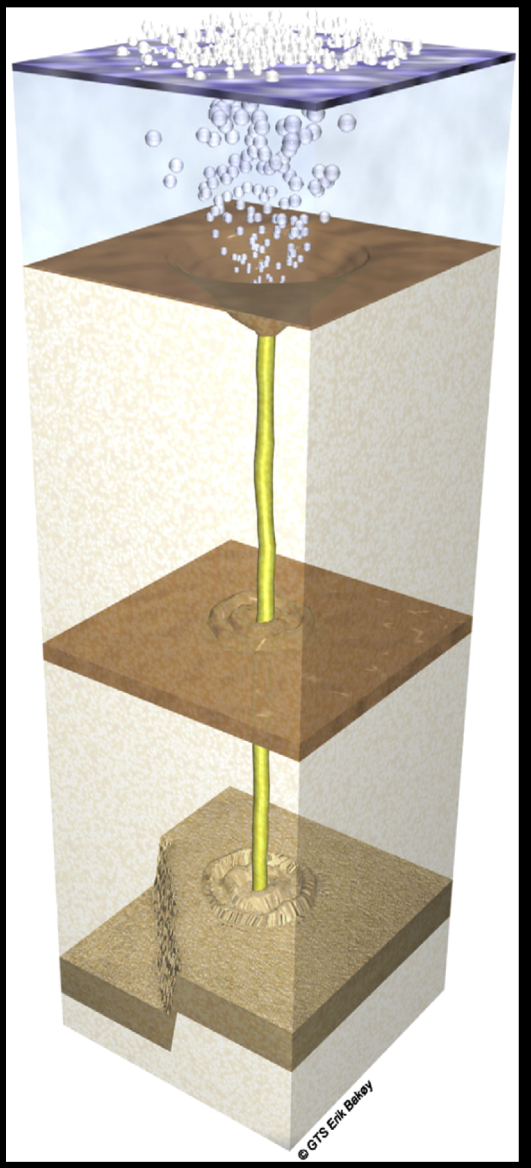
courtesy of Kjetil Haavik

Inverted velocity model, 3 – 50 Hz



courtesy of Kjetil Haavik

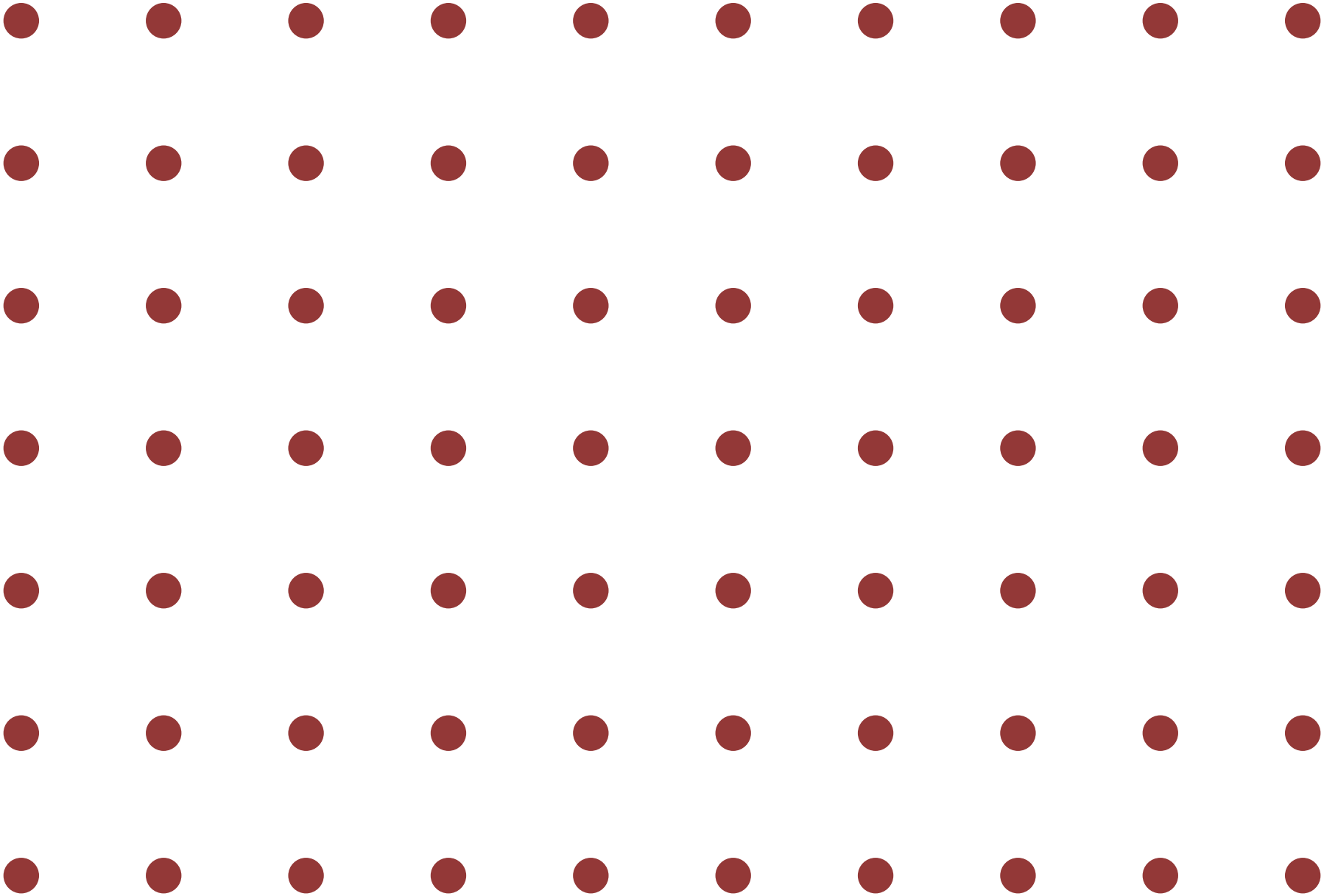
http://www.geosci.usyd.edu.au

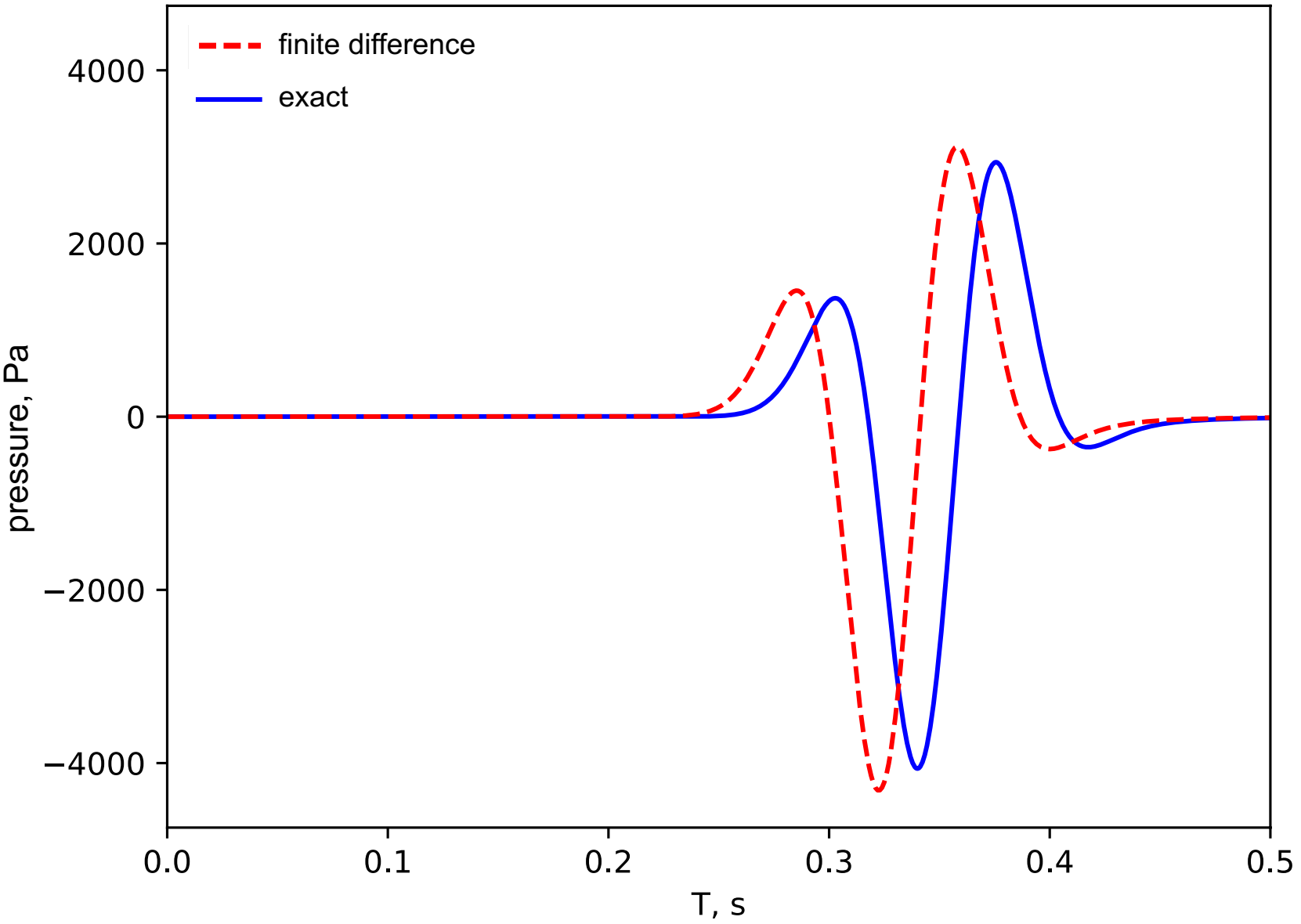


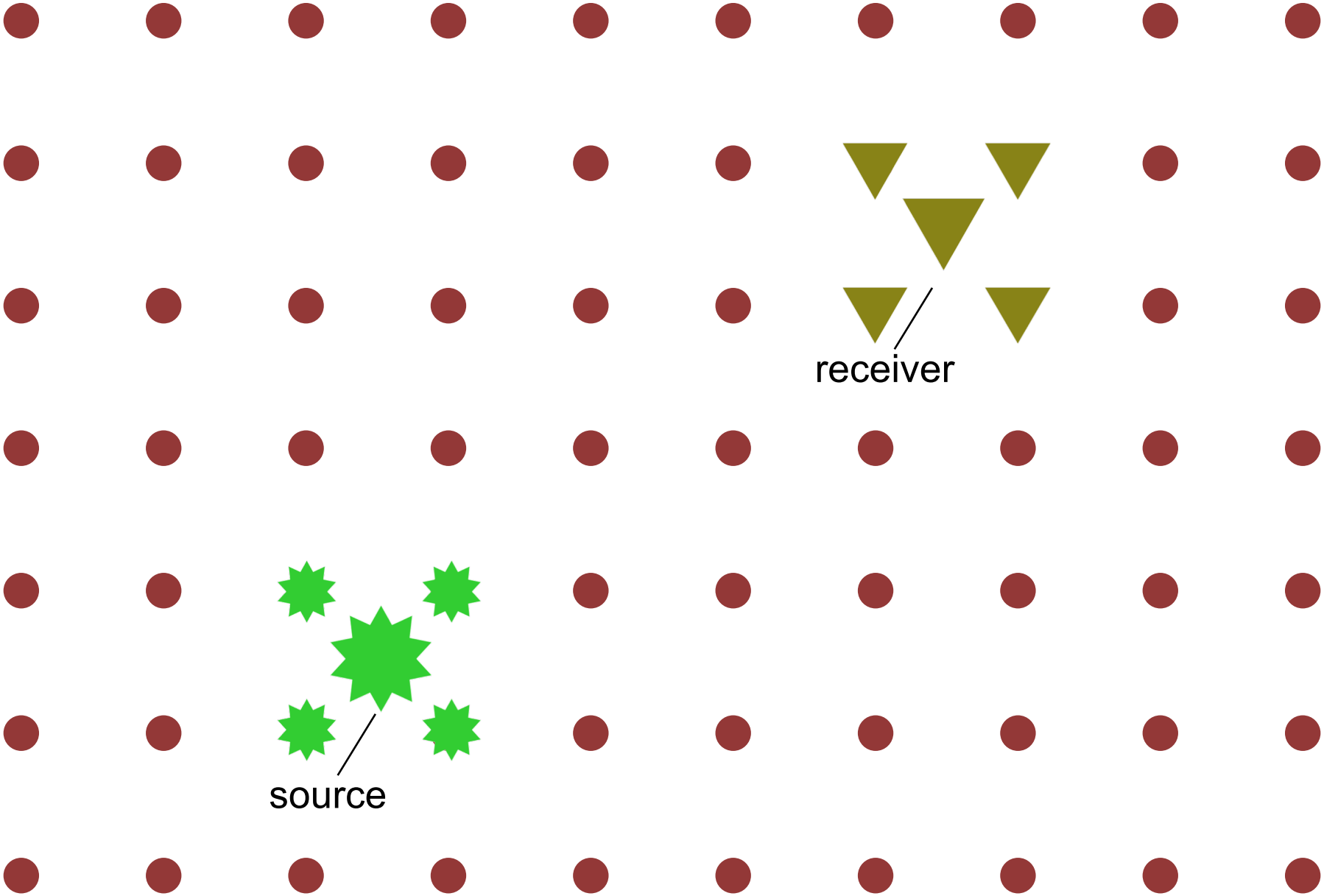
from Løseth et al., 2011

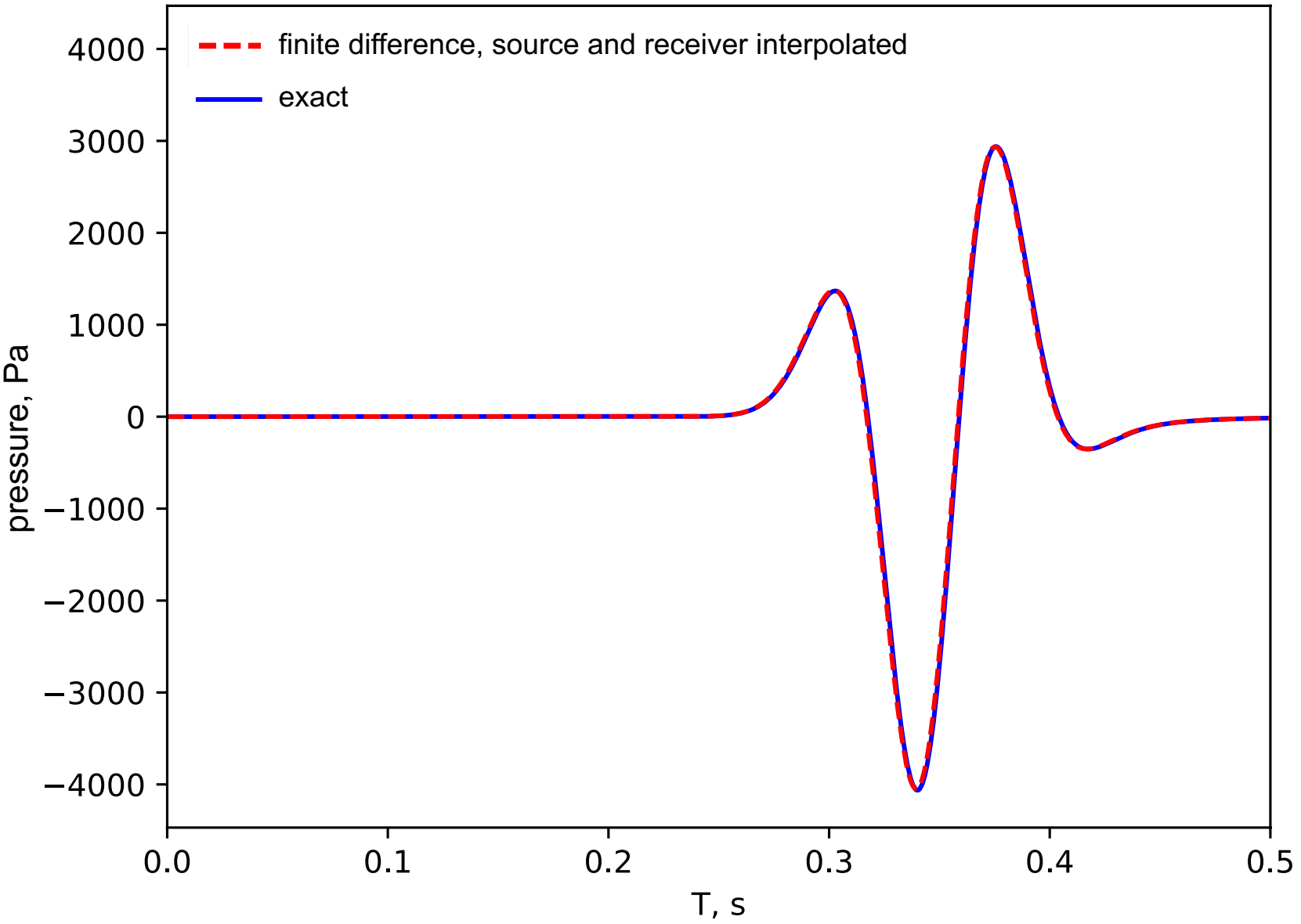
$$\rho \dot{v}_i = \partial_i p + f_i$$

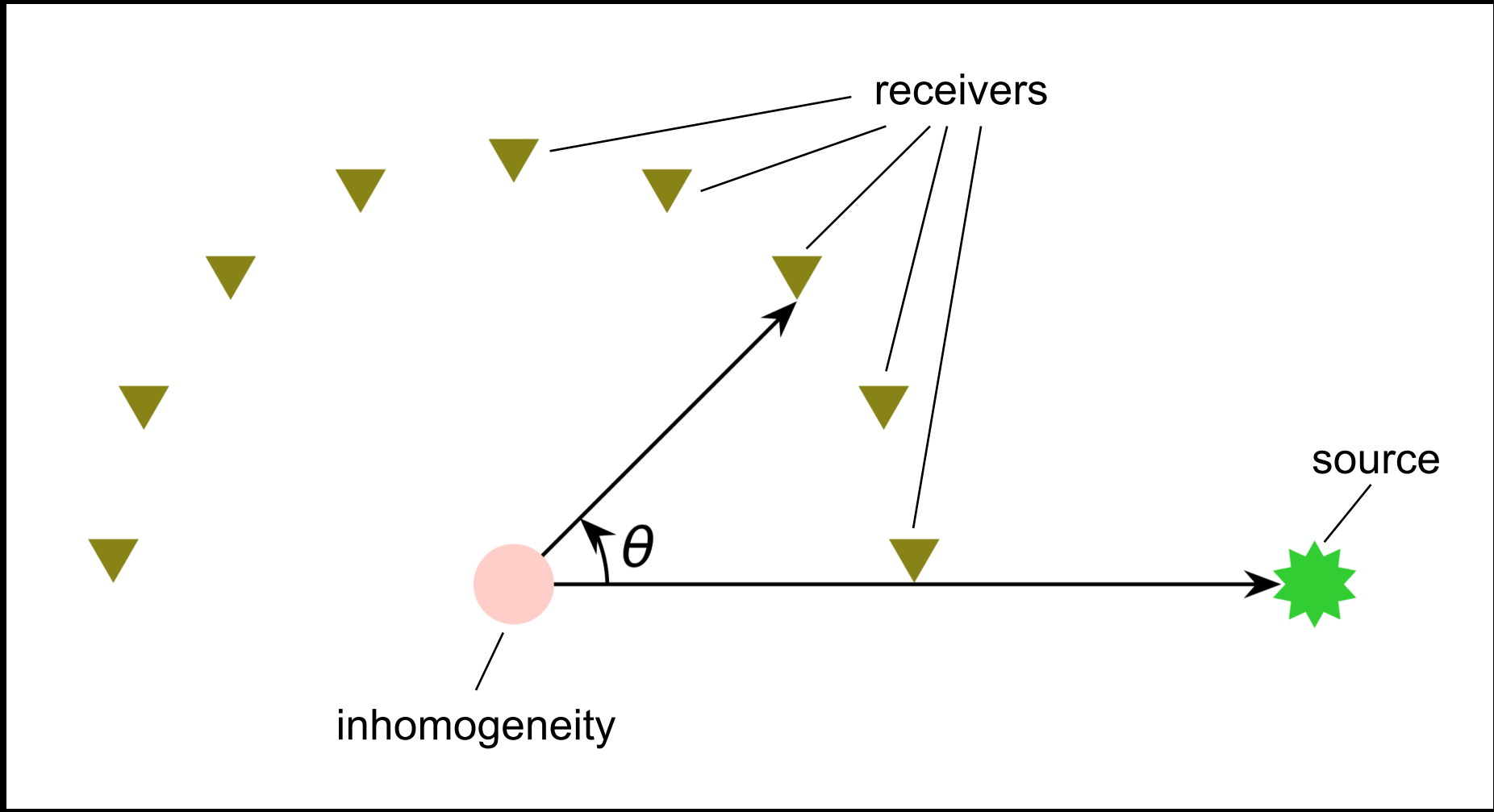
$$\dot{p} = \kappa \partial_i v_i + \dot{s}$$

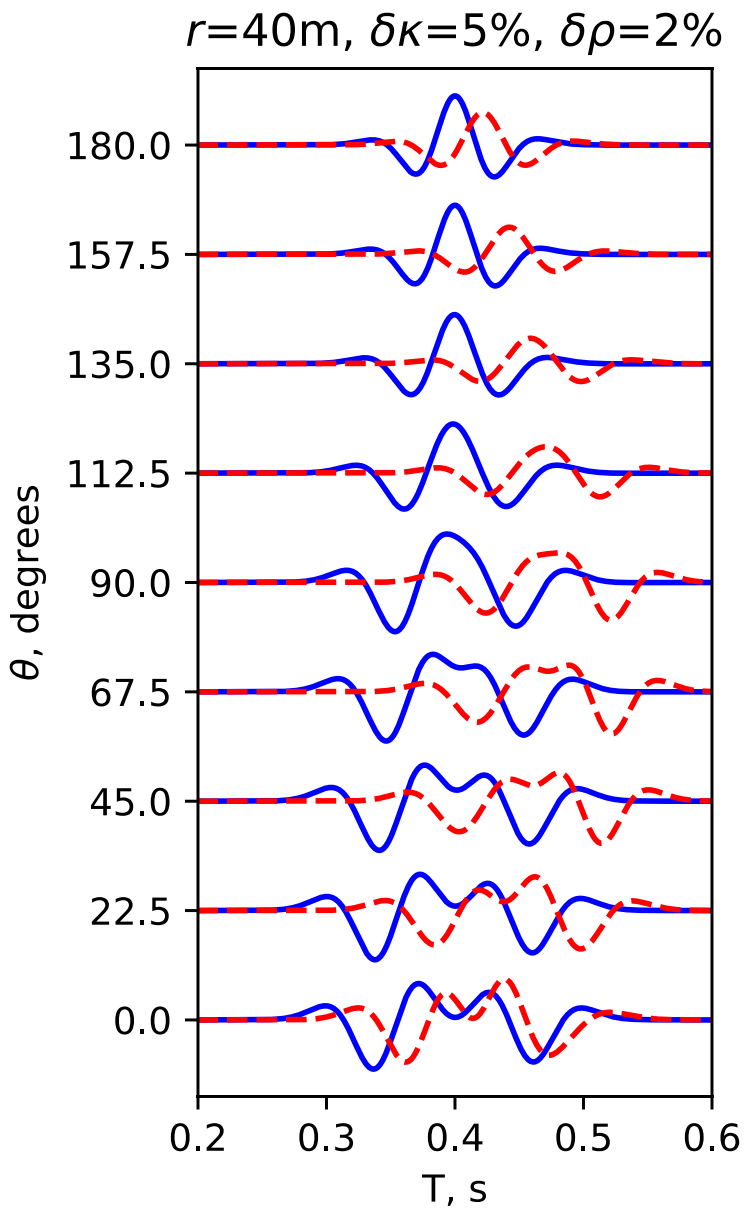



finite difference
exact
4000
2000
0
−2000
−4000
pressure, Pa
0.0
0.1
0.2
0.3
0.4
0.5
T, s


receiver
source


finite difference, source and receiver interpolated
exact
4000
3000
2000
1000
0
−1000
−2000
−3000
−4000
pressure, Pa
0.0
0.1
0.2
0.3
0.4
0.5
T, s

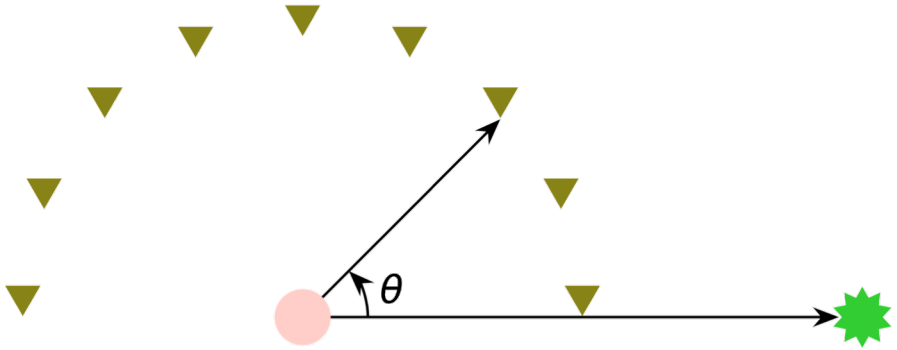

receivers
source
θ
inhomogeneity

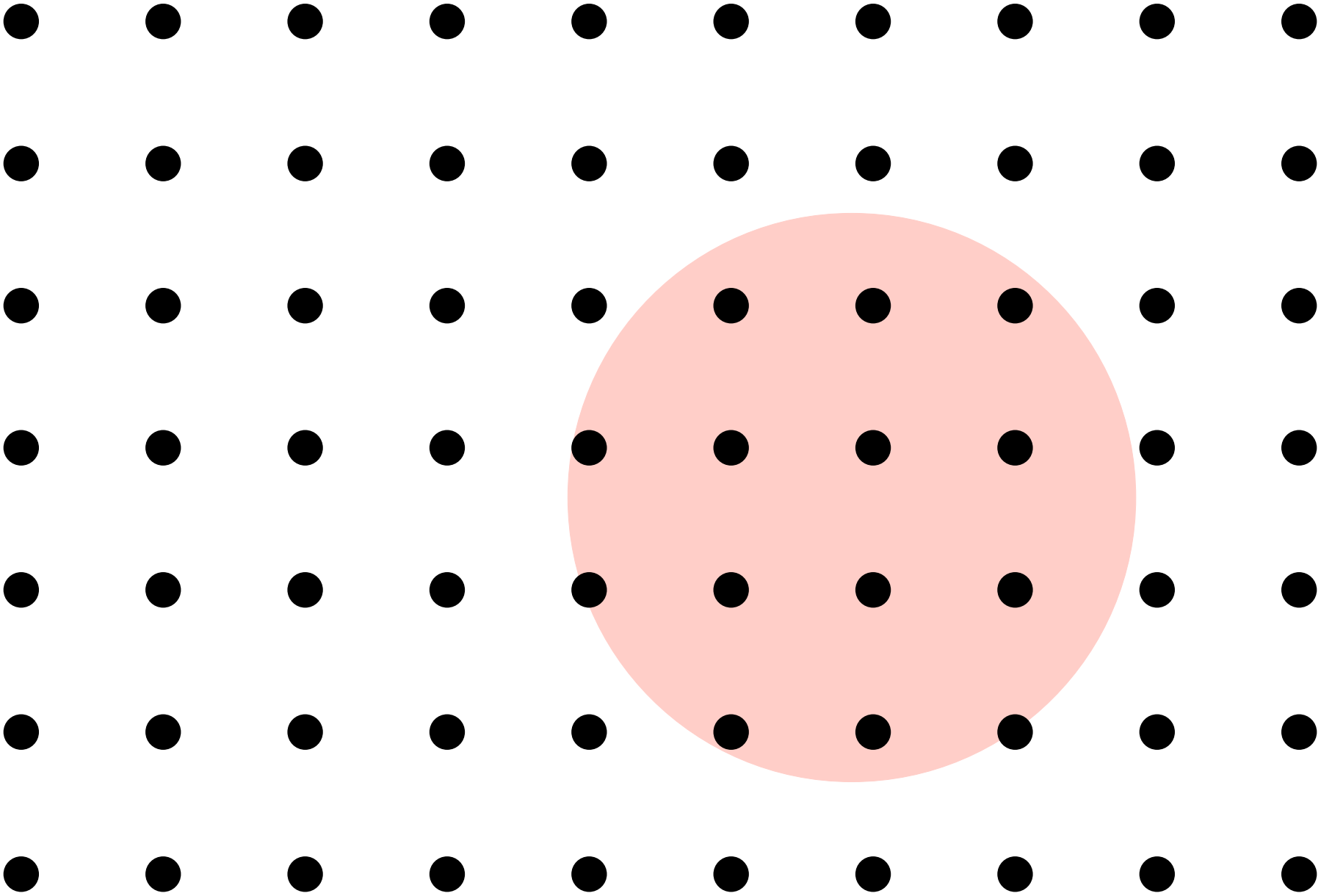


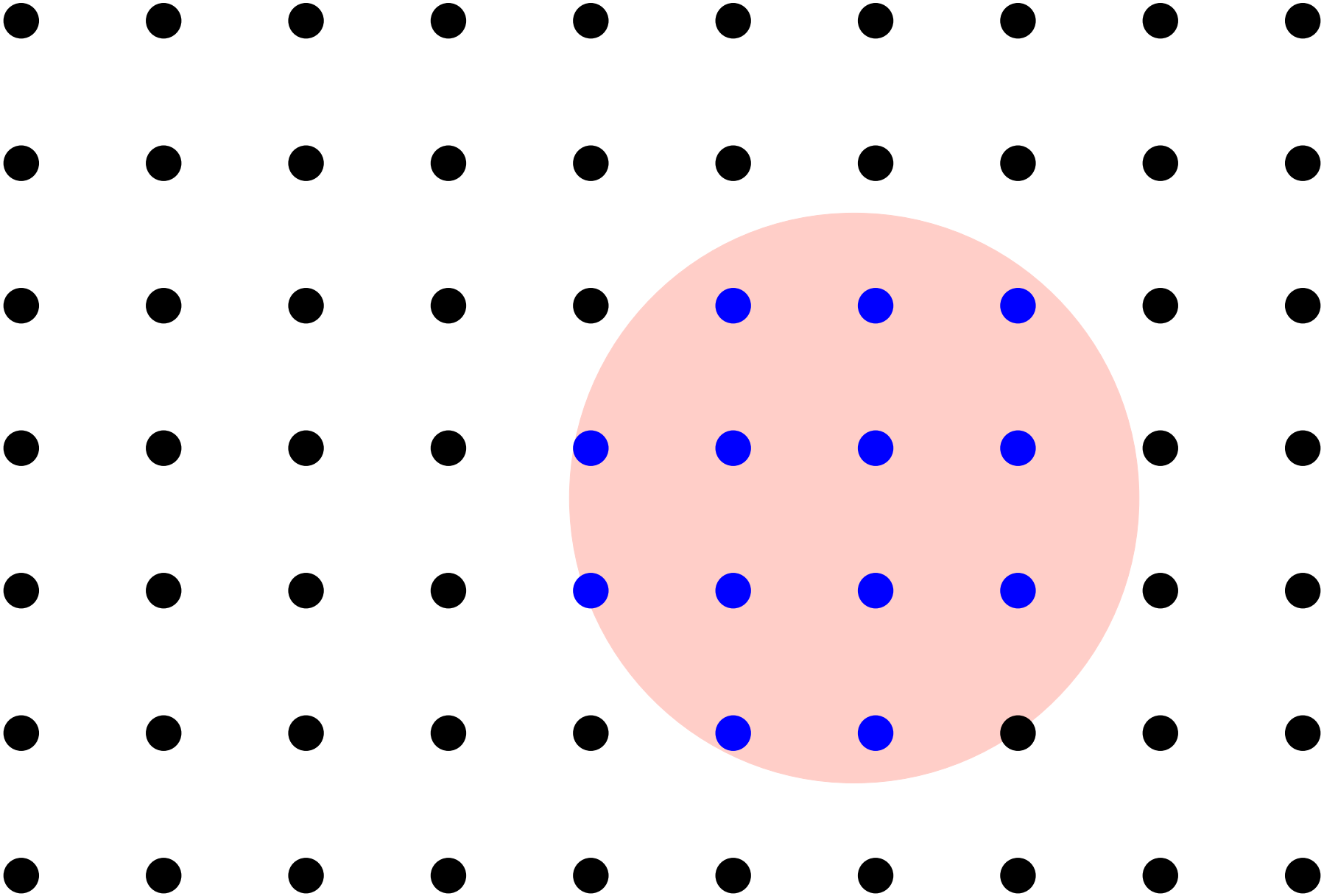
finite difference

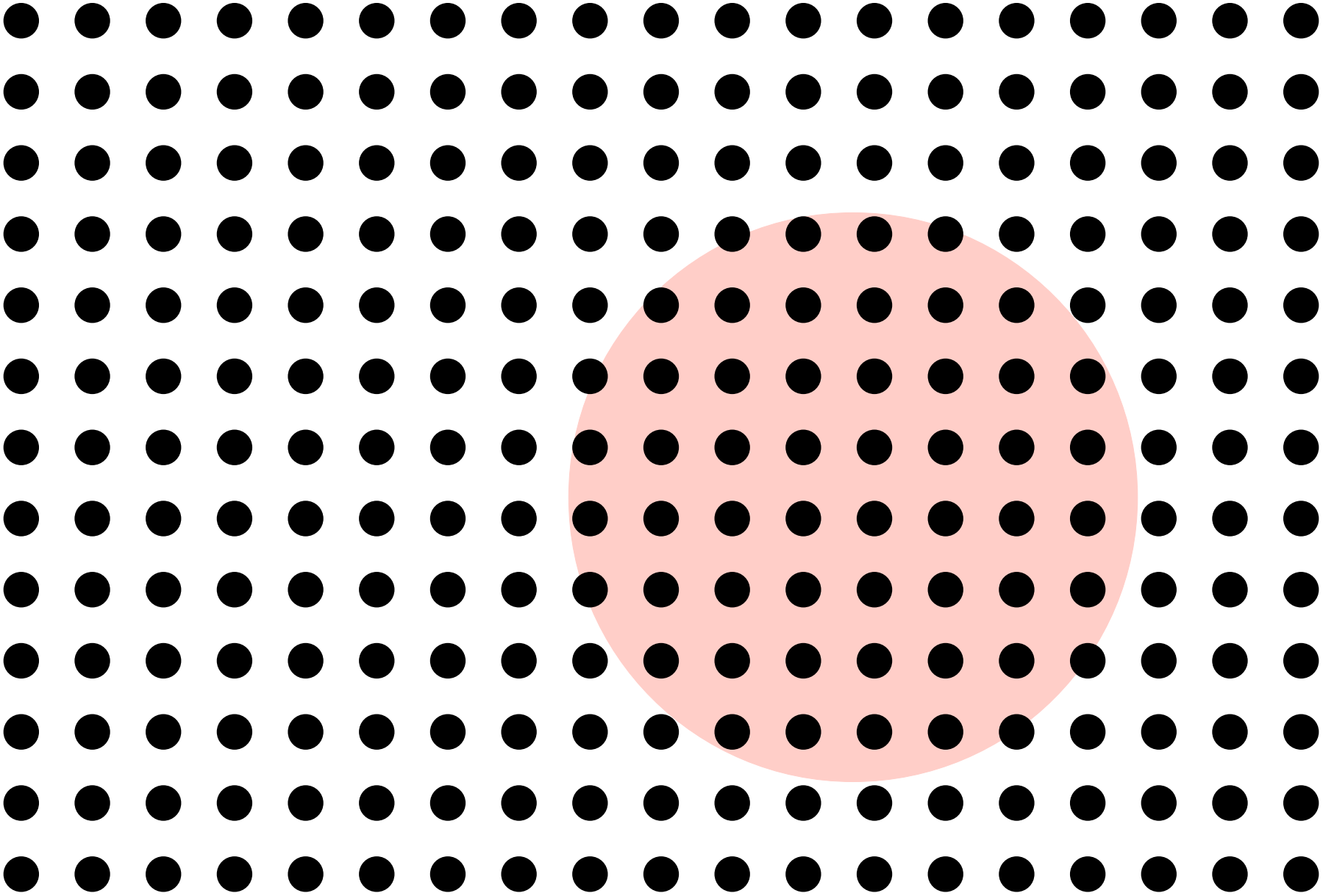
exact

$$\frac{\text{diameter}}{\text{wavelength}} \approx 0.5$$







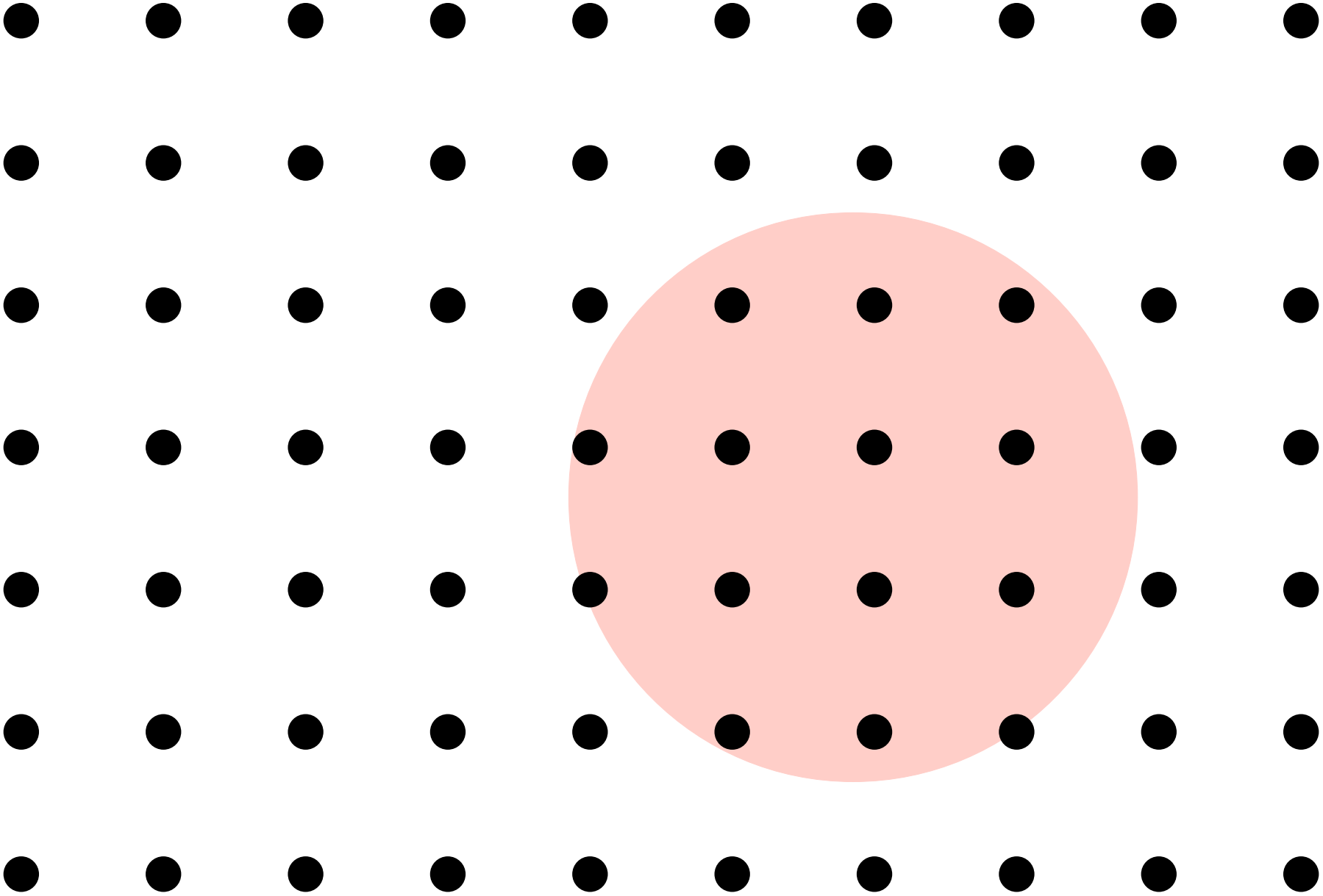


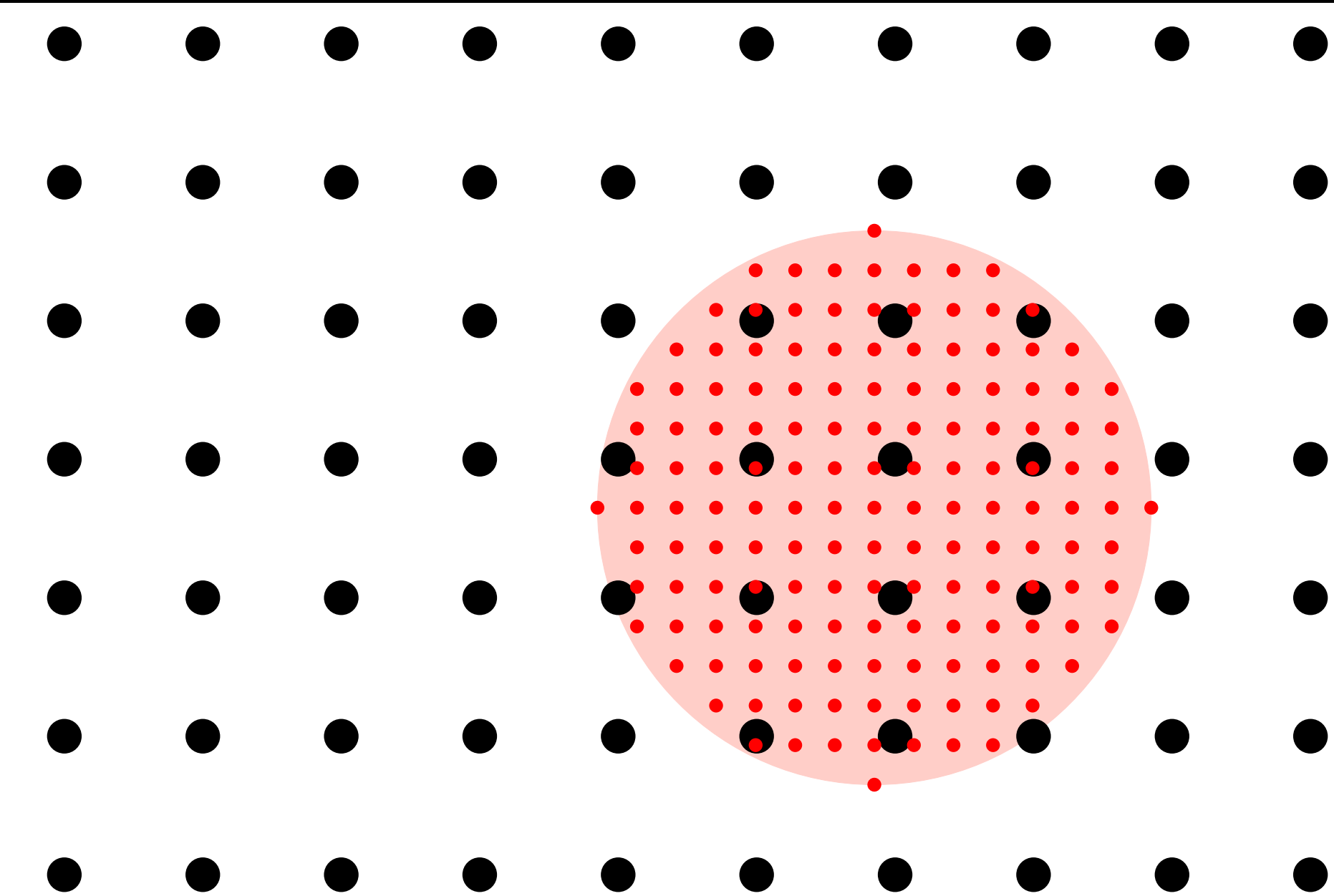
$$\rho = \rho^0 + \rho^1 \qquad \rho^1 \ll \rho^0$$
$$\kappa = \kappa^0 + \kappa^1 \qquad \kappa^1 \ll \kappa^0$$

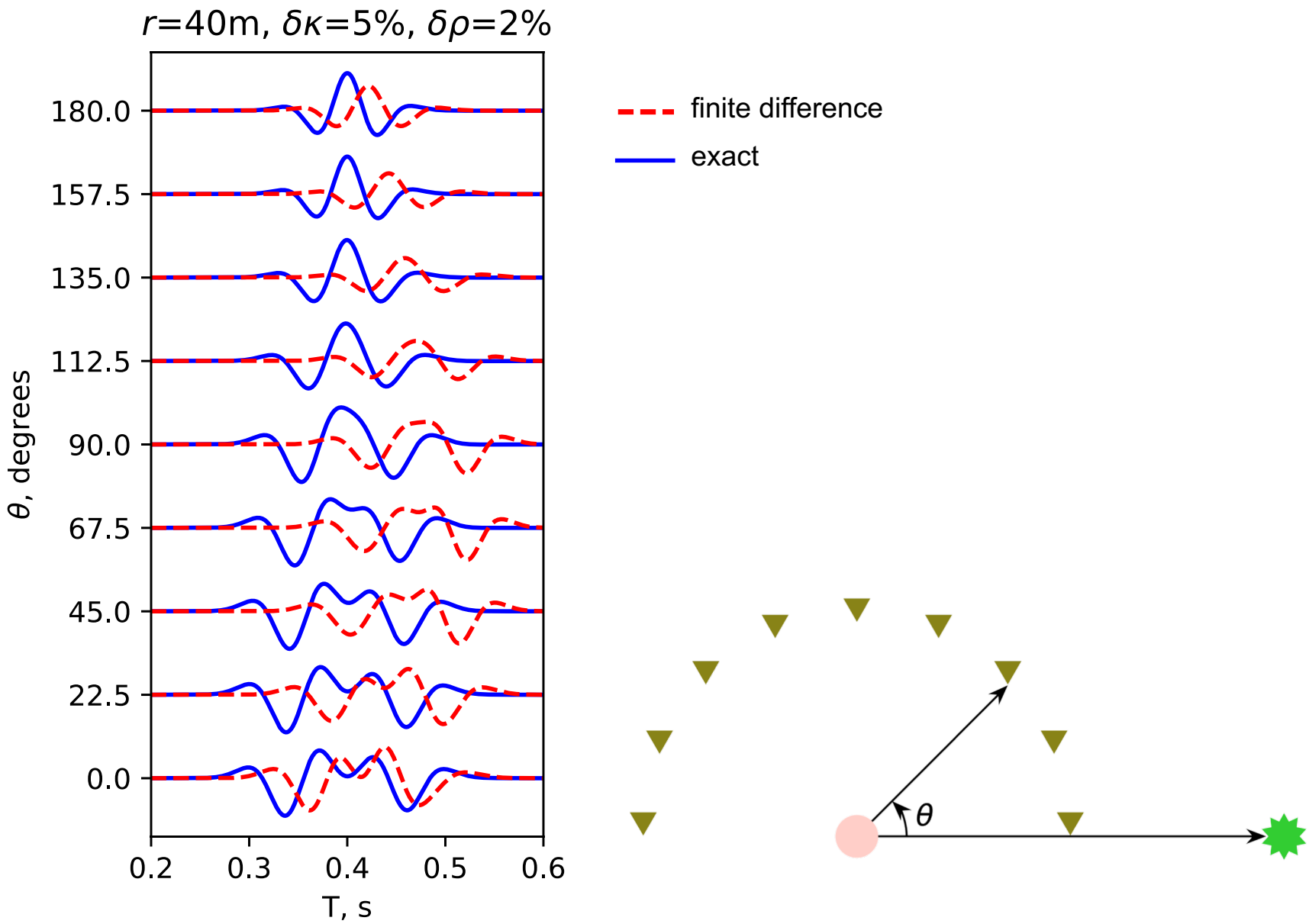
$$p = p^0 + p^1 \qquad p^1 \ll p^0$$
$$v_i = v_i^0 + v_i^1 \qquad v_i^1 \ll v_i^0$$

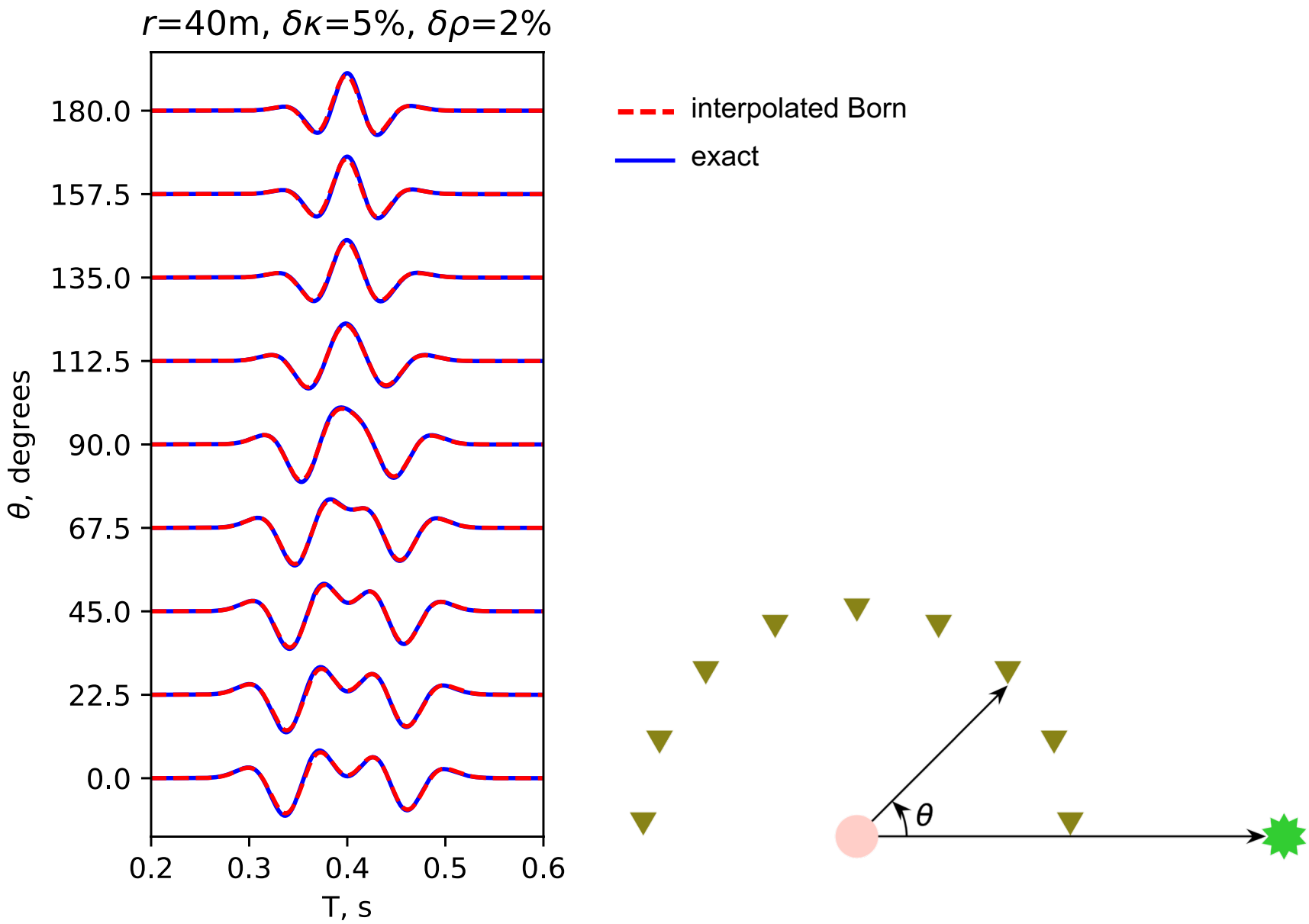
$$\rho^0 \dot{v}_i^0 = \partial_i p^0 + f_i$$
$$\dot{p}^0 = \kappa^0 \partial_i v_i^0 + \dot{s}$$

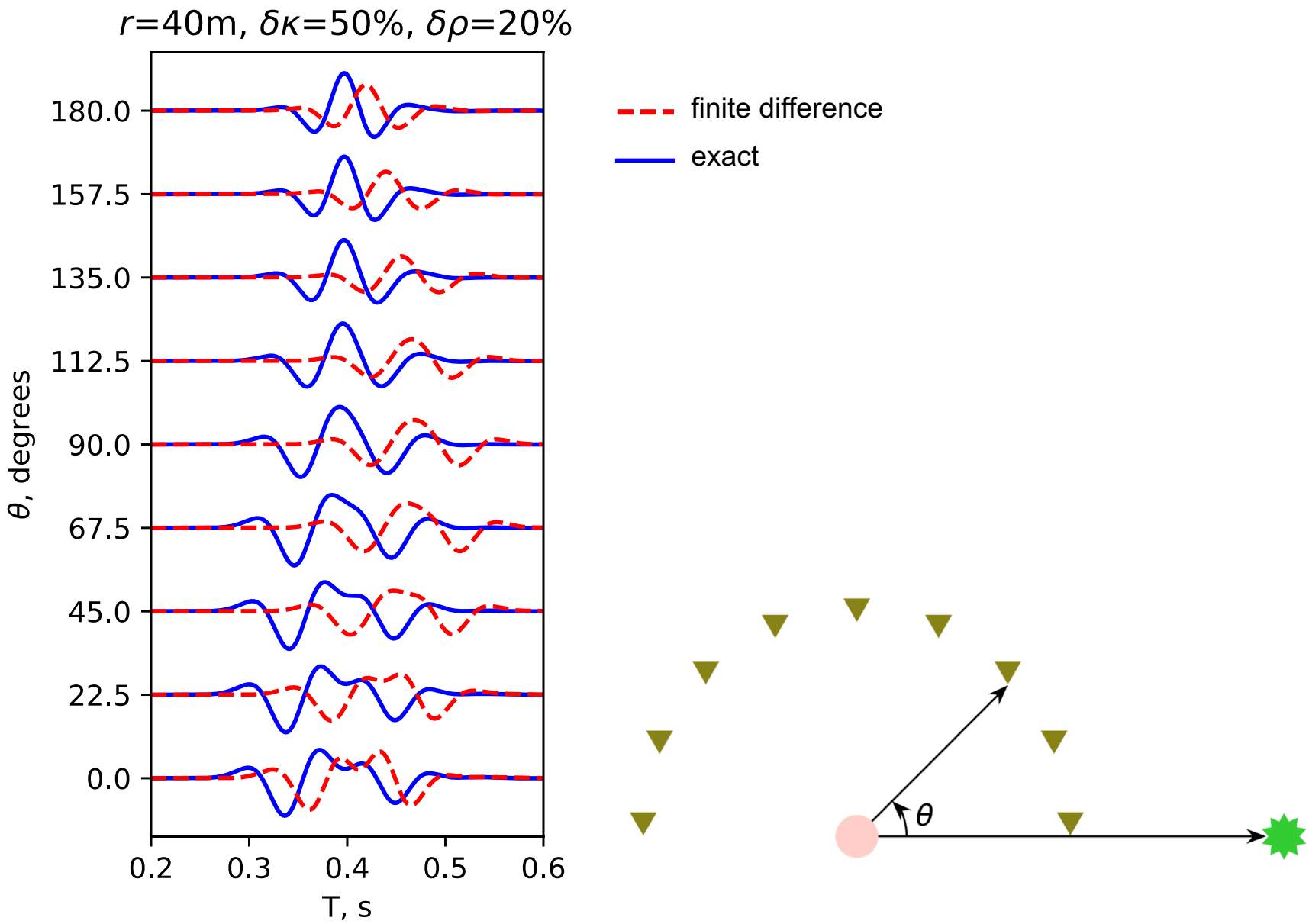
$$\rho^0 \dot{v}_i^1 = \partial_i p^1 - \rho^1 \dot{v}_i^0$$
$$\dot{p}^1 = \kappa^0 \partial_i v_i^1 + \kappa^1 \partial_i v_i^0$$

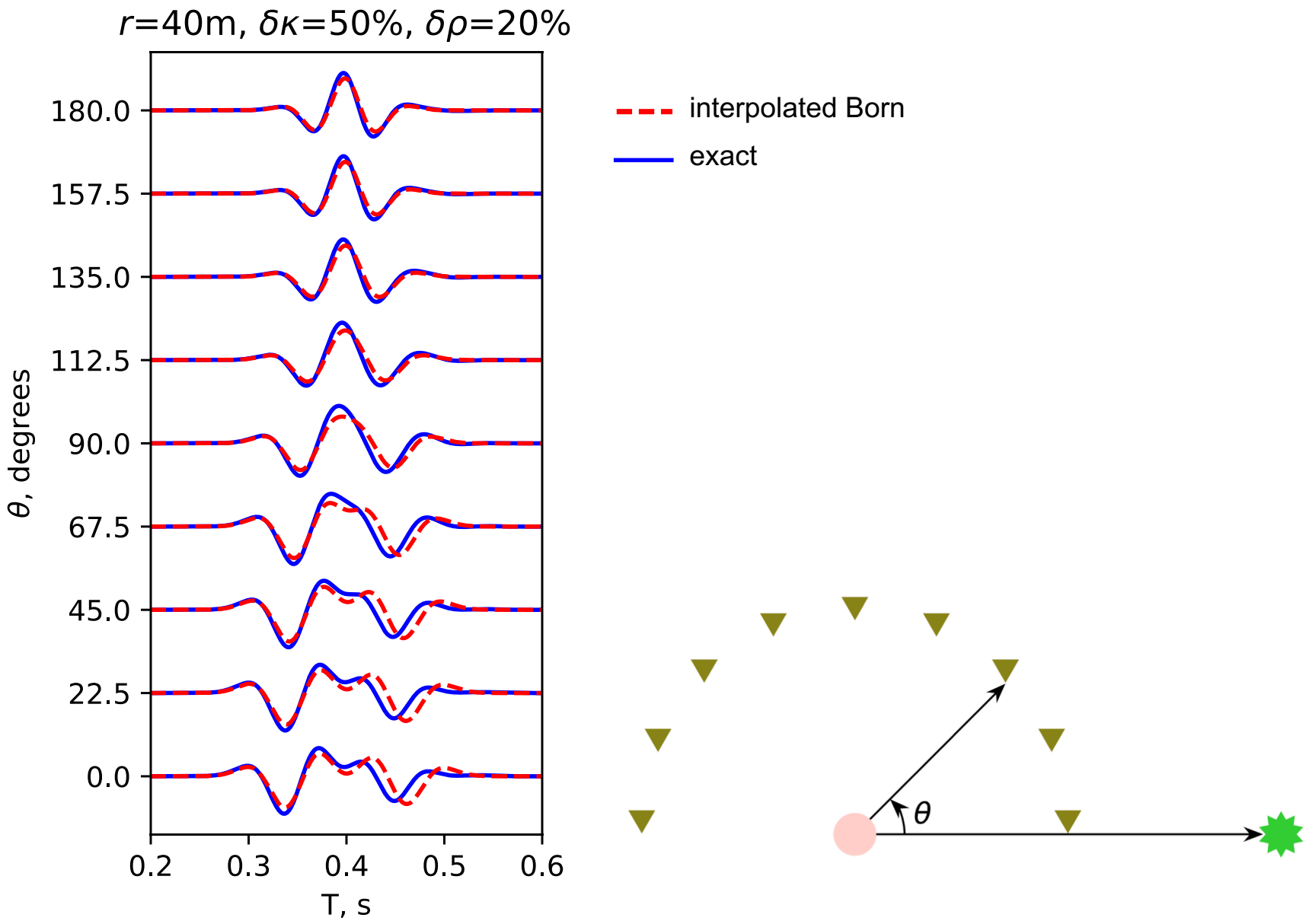





r=40m, δκ=5%, δρ=2%
180.0
157.5
135.0
112.5
90.0
67.5
45.0
22.5
0.0
θ, degrees
0.2
0.3
0.4
0.5
0.6
T, s
finite difference
exact
θ


r=40m, δκ=5%, δρ=2%
180.0
157.5
135.0
112.5
90.0
67.5
45.0
22.5
0.0
θ, degrees
0.2
0.3
0.4
0.5
0.6
T, s
interpolated Born
exact
θ


r=40m, δκ=50%, δρ=20%
finite difference
exact
θ, degrees
180.0
157.5
135.0
112.5
90.0
67.5
45.0
22.5
0.0
0.2
0.3
0.4
0.5
0.6
T, s
θ


r=40m, δκ=50%, δρ=20%
180.0
157.5
135.0
112.5
90.0
67.5
45.0
22.5
0.0
θ, degrees
0.2
0.3
0.4
0.5
0.6
T, s
interpolated Born
exact
θ

$$\rho \dot{v}_i = \partial_i p + f_i$$
$$\dot{p} = \kappa \partial_i v_i + \dot{s}$$

$$\rho = \rho^0 + \rho^1$$

$$\kappa = \kappa^0 + \kappa^1$$

$$\rho^1 \ll \rho^0$$

$$\kappa^1 \ll \kappa^1$$

$$p = p^0 + p^1$$

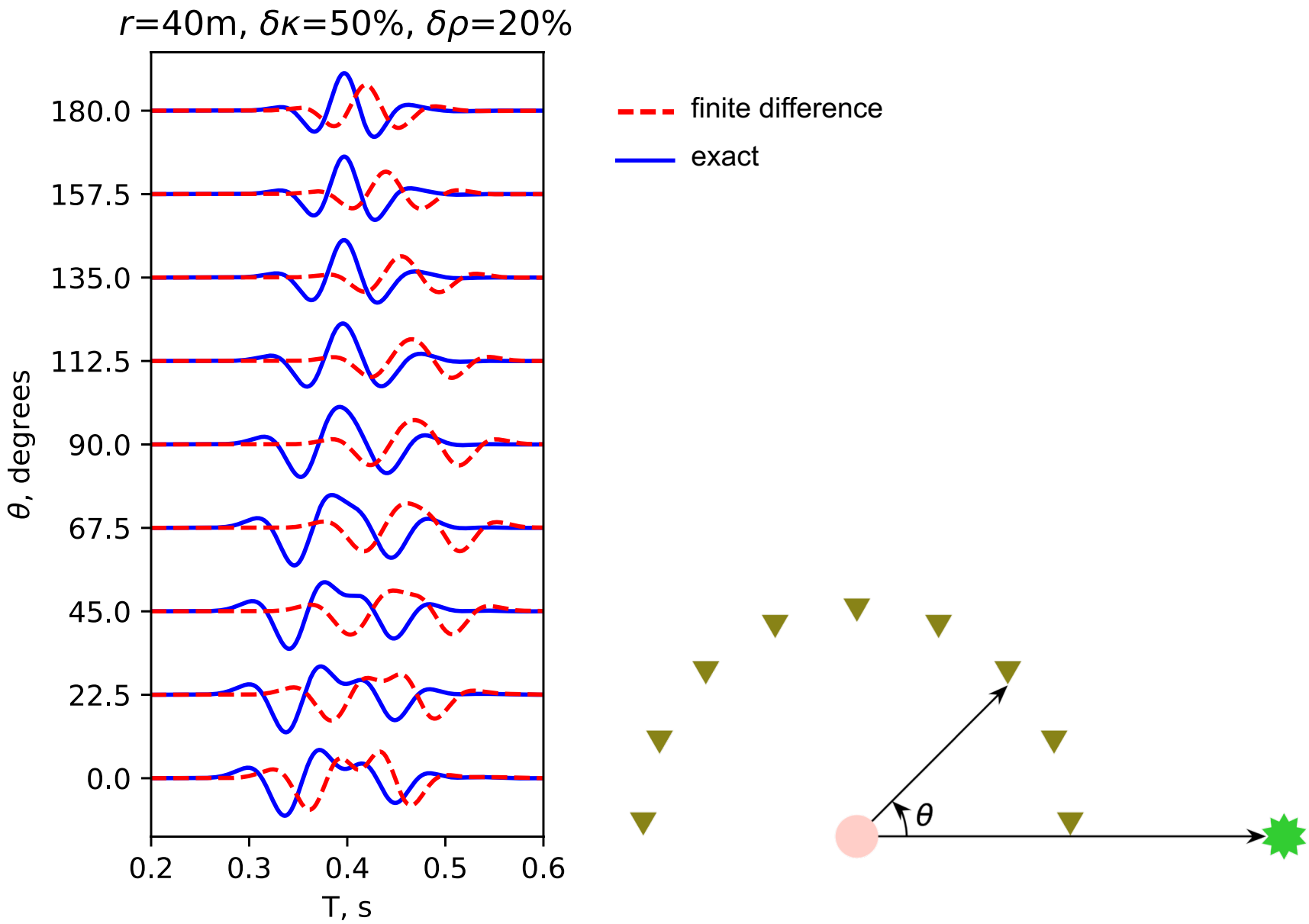
$$v_i = v_i^0 + v_i^1$$

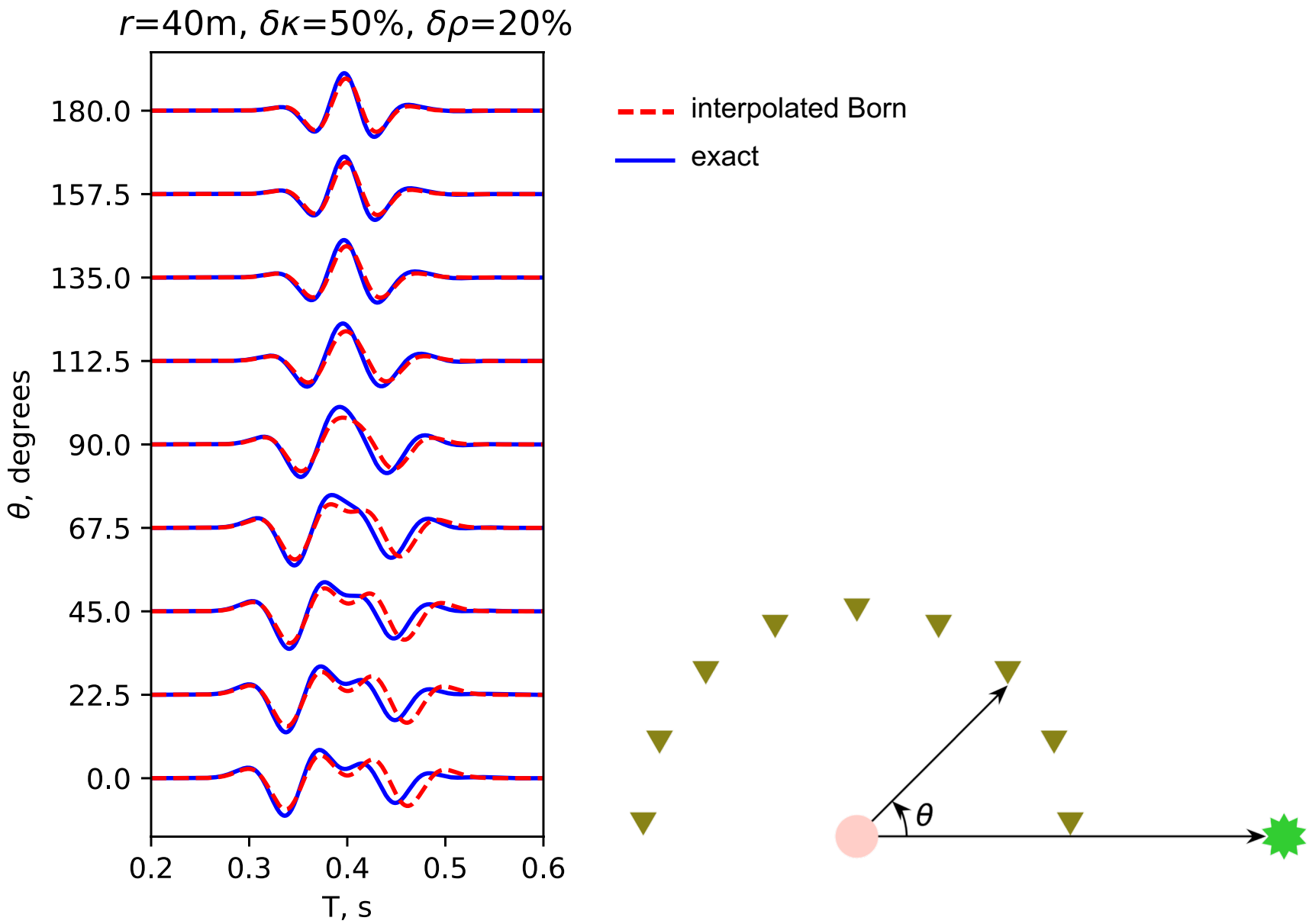
$$p^1 \ll p^0$$

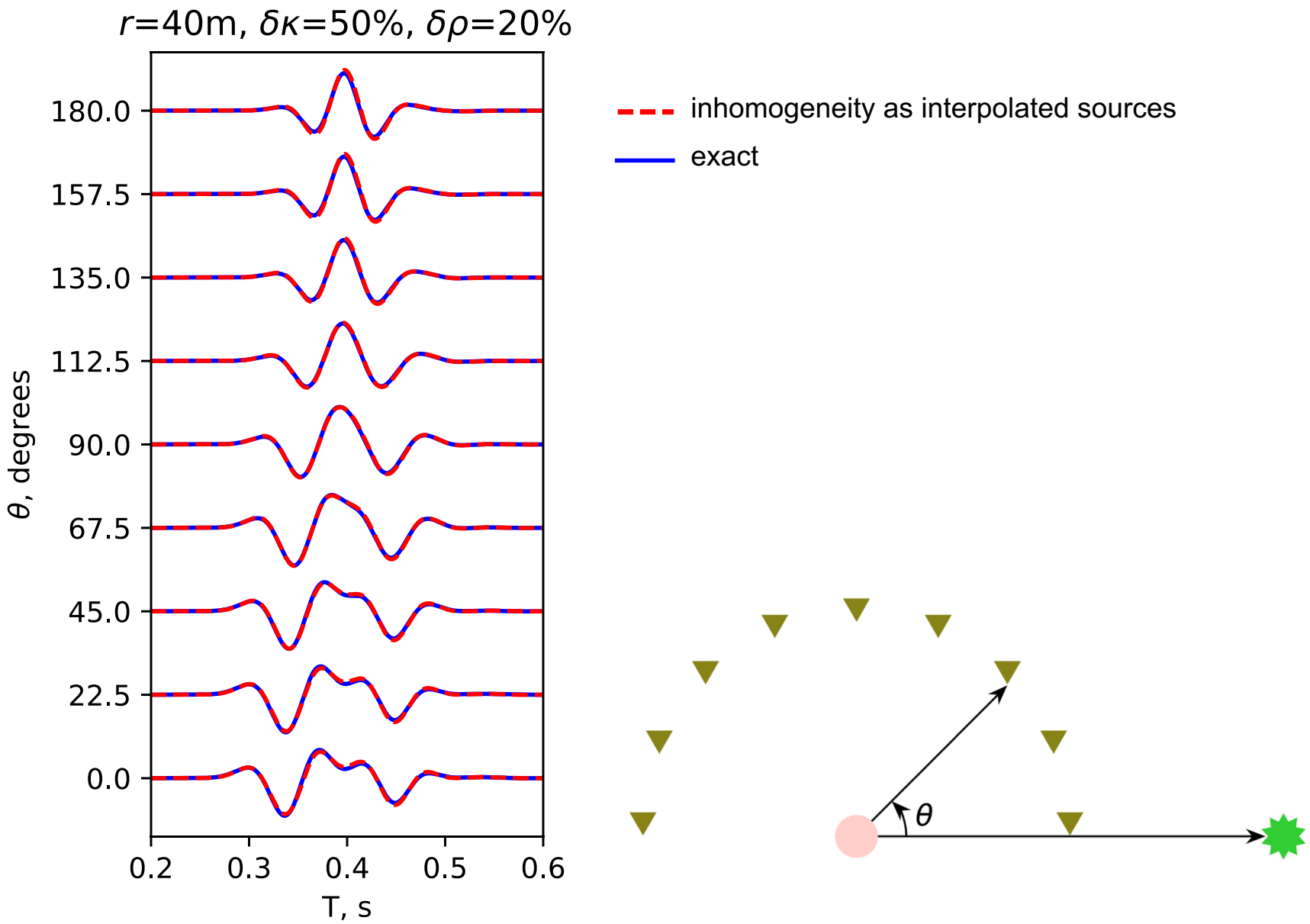
$$v_i^1 \ll v_i^0$$

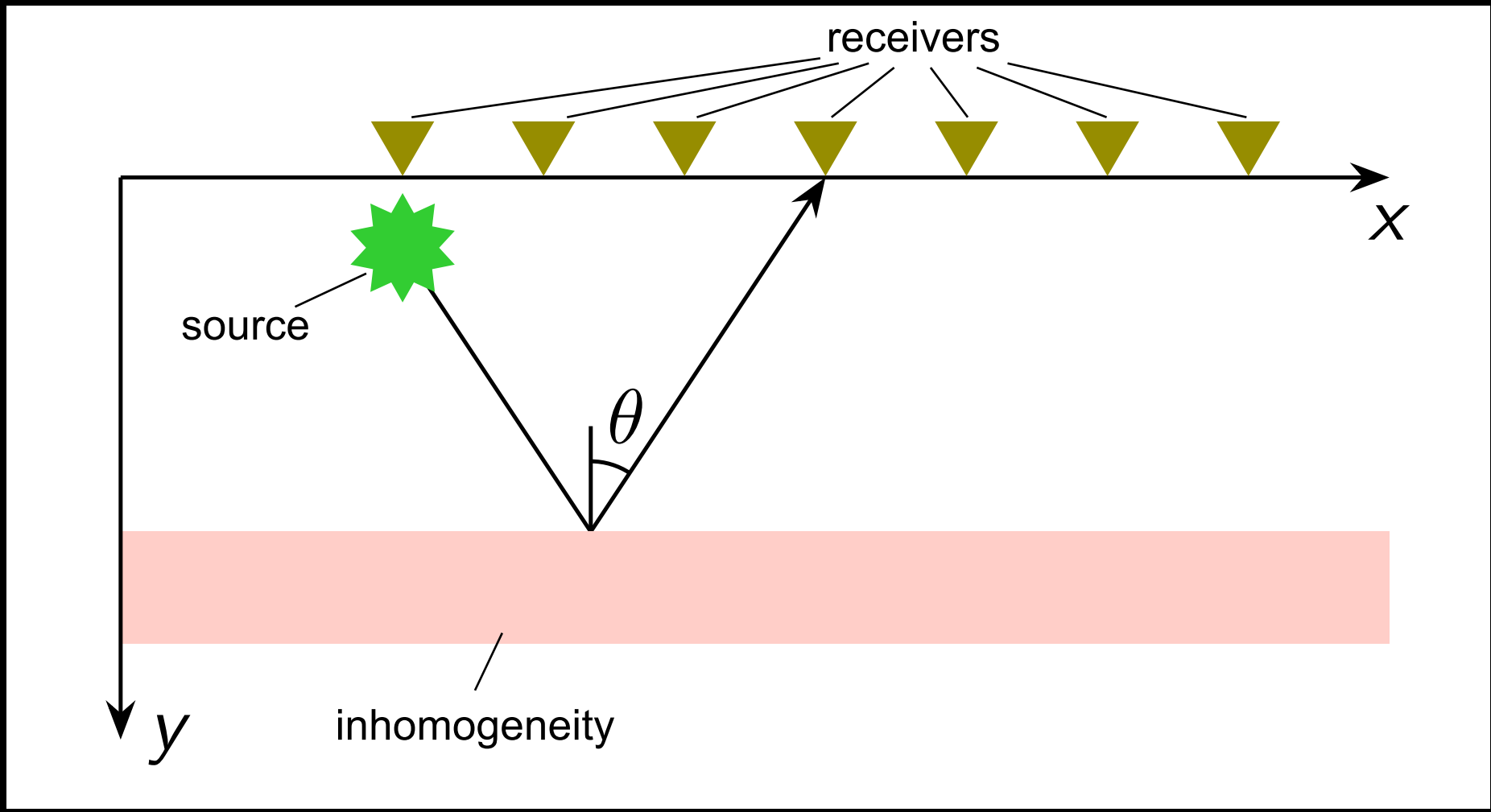
$$\rho^0 \dot{v}_i = \partial_i p + f_i - \rho^1 \dot{v}_i$$

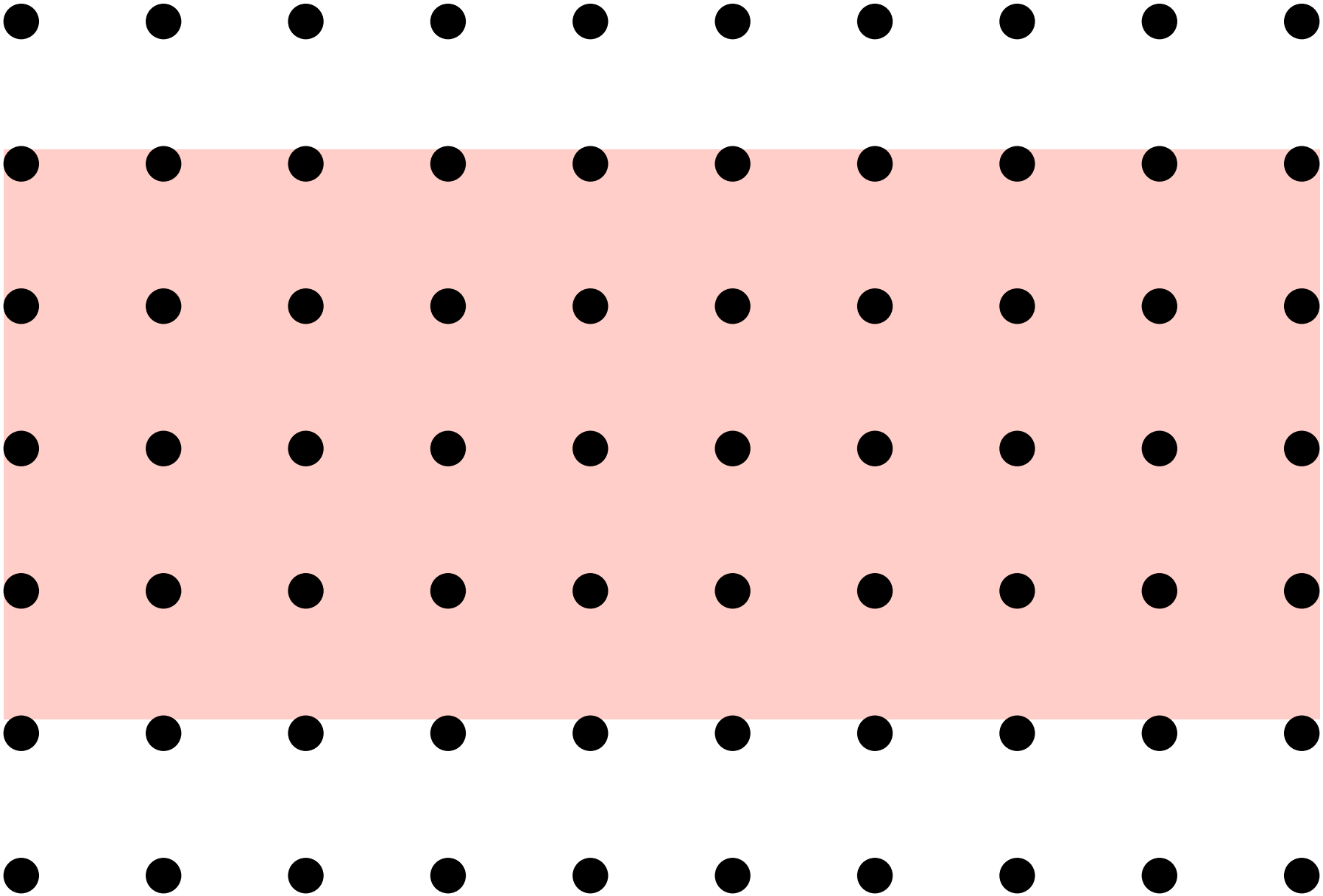
$$\dot{p} = \kappa^0 \partial_i v_i + \dot{s} + \kappa^1 \partial_i v_i$$

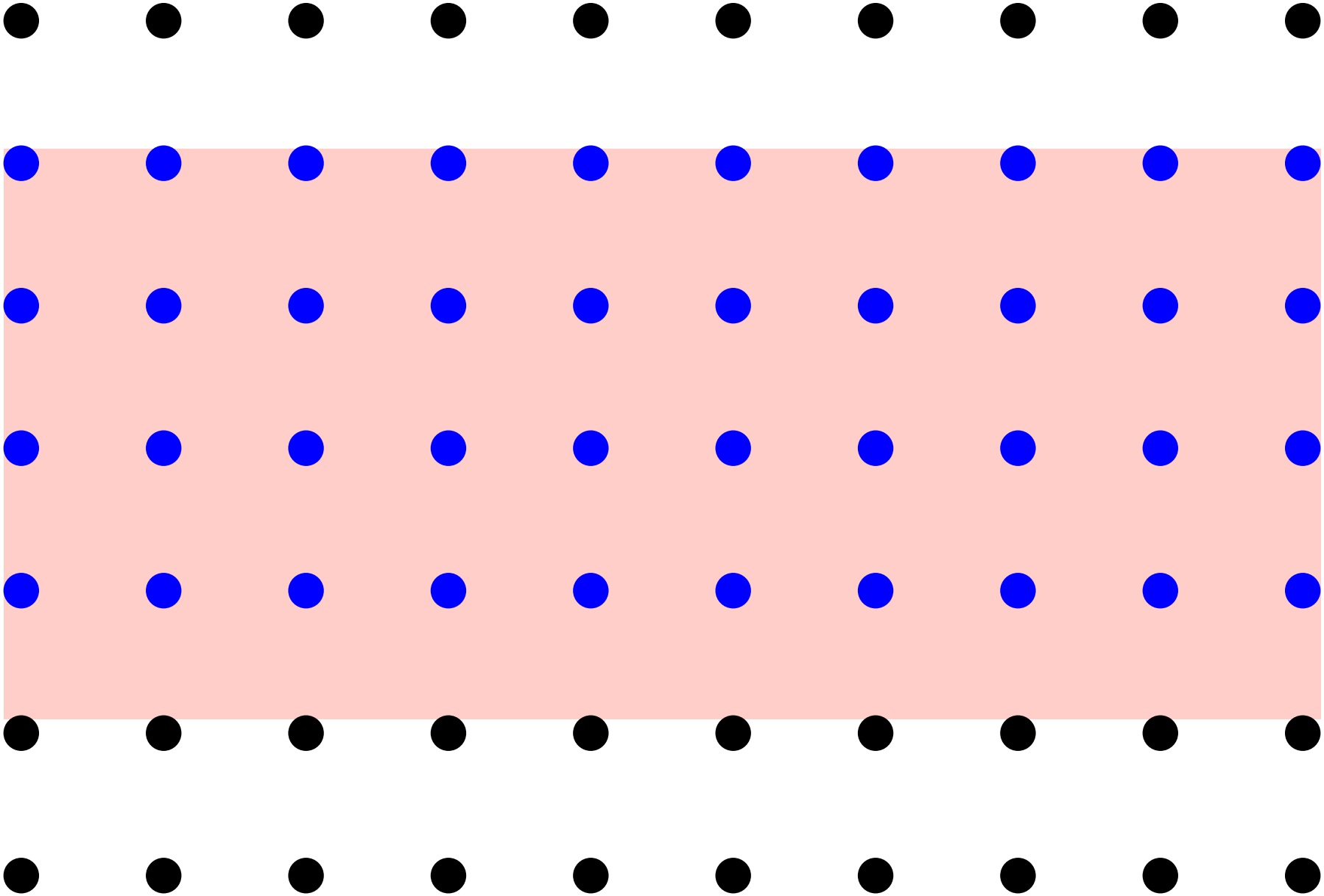

r=40m, δκ=50%, δρ=20%
finite difference
exact
θ, degrees
180.0
157.5
135.0
112.5
90.0
67.5
45.0
22.5
0.0
0.2
0.3
0.4
0.5
0.6
T, s
θ

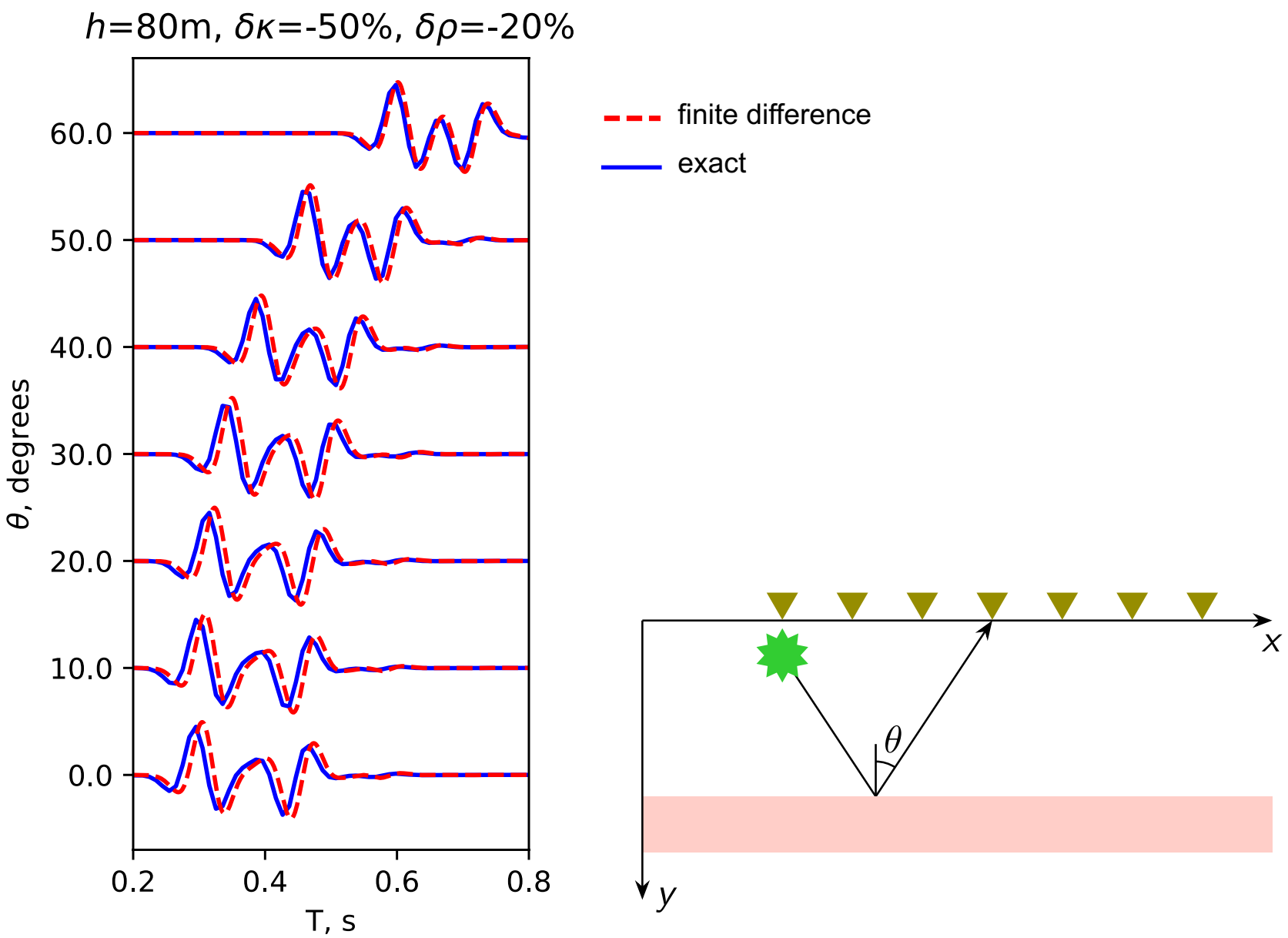

r=40m, δκ=50%, δρ=20%
180.0
157.5
135.0
112.5
90.0
67.5
45.0
22.5
0.0
θ, degrees
0.2
0.3
0.4
0.5
0.6
T, s
interpolated Born
exact
θ

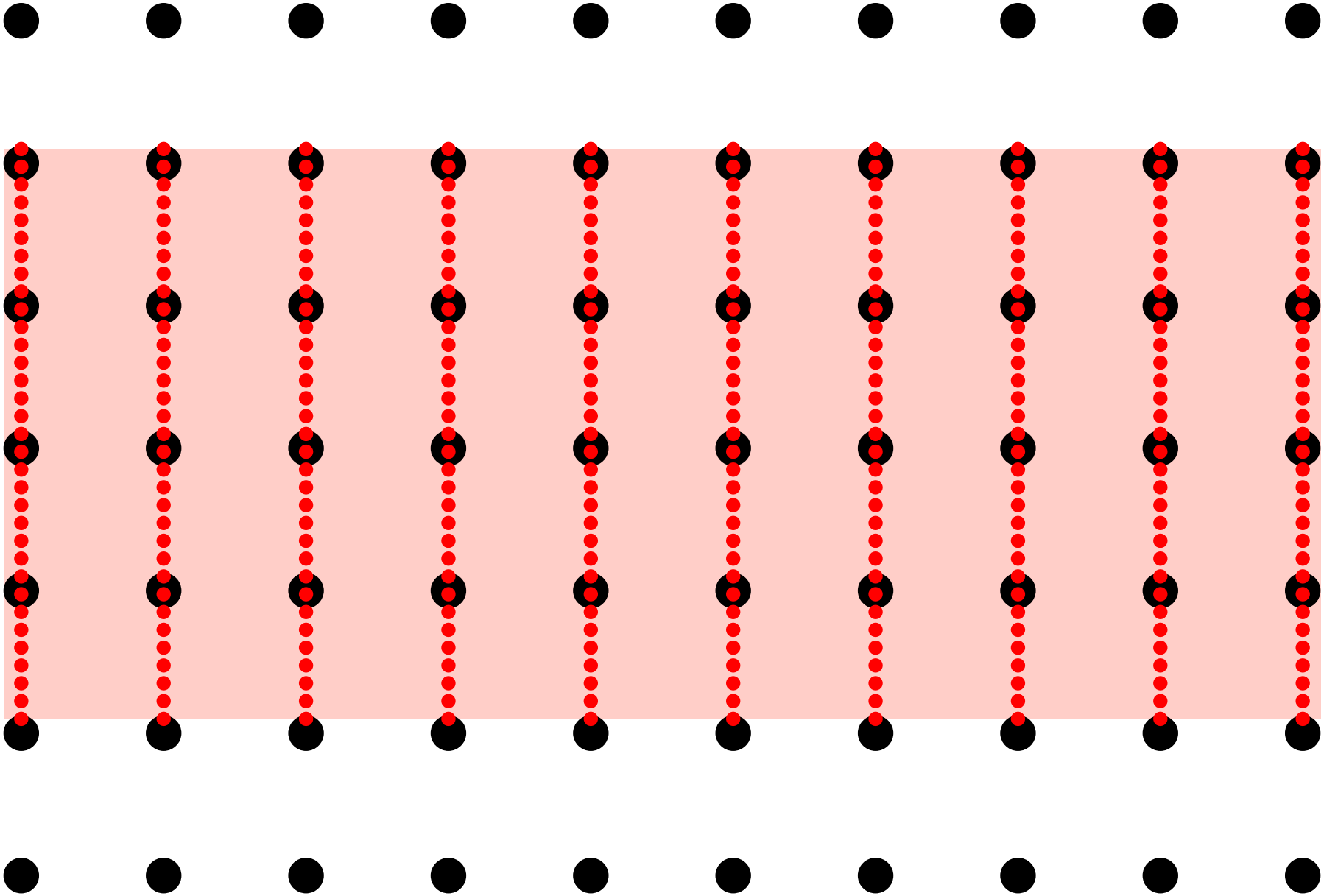

r=40m, δκ=50%, δρ=20%
180.0
157.5
135.0
112.5
90.0
67.5
45.0
22.5
0.0
θ, degrees
0.2
0.3
0.4
0.5
0.6
T, s
inhomogeneity as interpolated sources
exact
θ

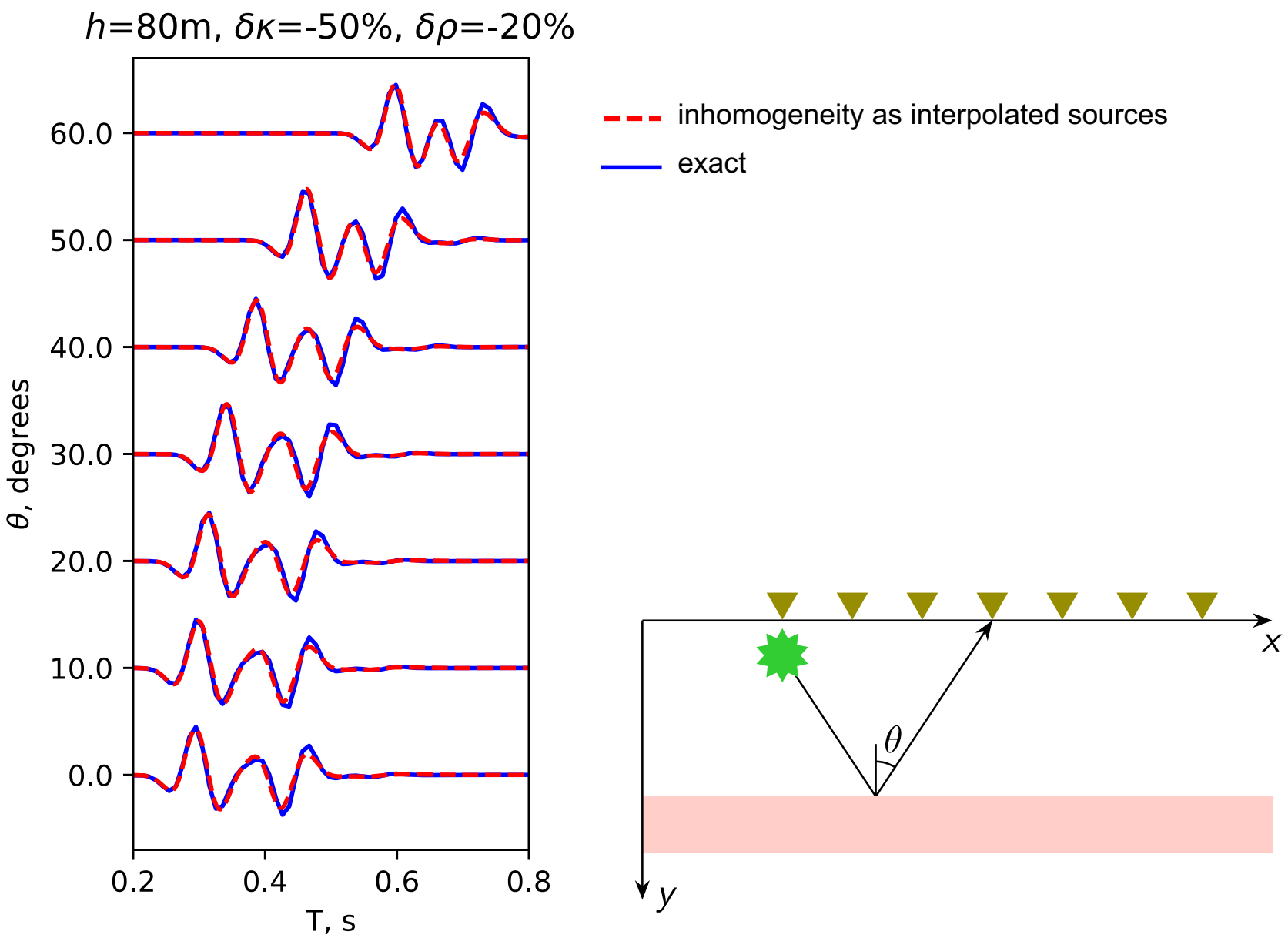

receivers
x
source
θ
y
inhomogeneity






h=80m, δκ=-50%, δρ=-20%
finite difference
exact
60.0
50.0
40.0
30.0
20.0
10.0
0.0
θ, degrees
0.2
0.4
0.6
0.8
T, s
x
y
θ




h=80m, δκ=-50%, δρ=-20%
60.0
50.0
40.0
30.0
20.0
10.0
0.0
θ, degrees
0.2
0.4
0.6
0.8
T, s
inhomogeneity as interpolated sources
exact
x
y
θ

Conclusions

1. Discretization $\Longrightarrow$ errors
2. Sources/receivers with interpolation
3. Born on an auxiliary grid
4. Inhomogeneities as secondary sources

Discussion

1. Interpolation near boundaries
2. Stability