Normal modes in an orthorhombic medium

Yuriy Ivanov* and Alexey Stovas

IGP, NTNU

April 23rd, 2018
RoSe meeting

Outline

Normal modes

Normal modes?

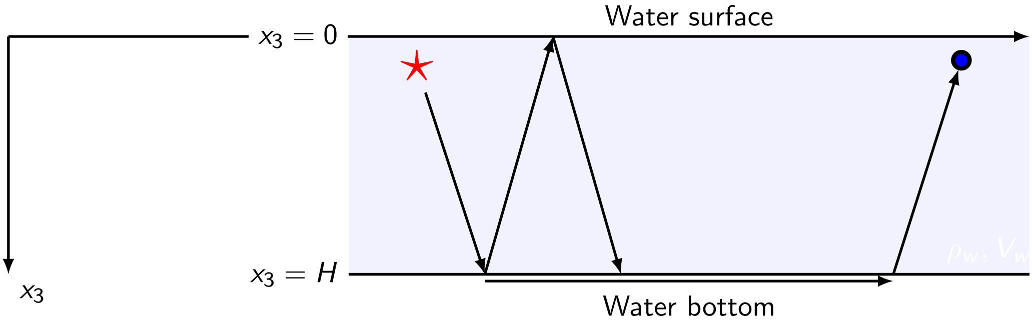

Figure 1: Sketch of a guided wave propagation. Vertical scale exaggerated.

Normal modes!

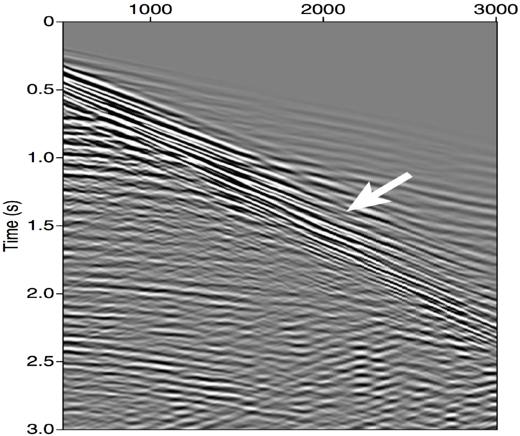

Figure 2: Typical marine common-shot gather (Wang et al., 2016).

Normal modes

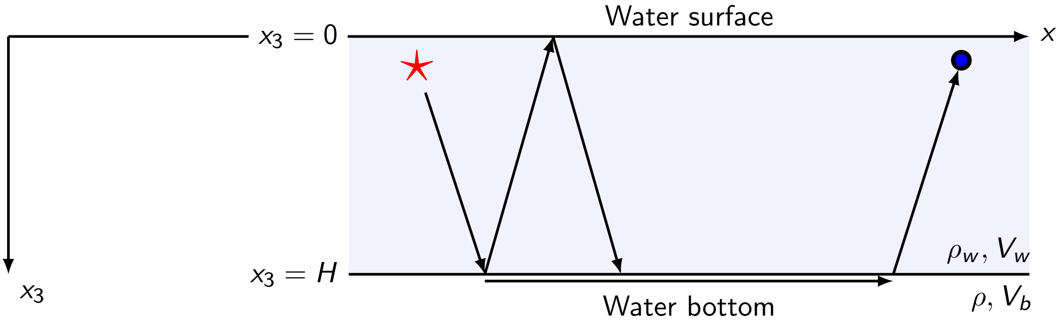

Figure 3: Sketch of a guided wave propagation. Vertical scale exaggerated.

Normal modes

Period equation: phase velocity

Acoustic case[1]:

$$\tan kH\sqrt{\frac{c^2}{V_w^2}-1} = -\frac{\rho}{\rho_w}\frac{\sqrt{\frac{c^2}{V_w^2}-1}}{\sqrt{1-\frac{c^2}{V_b^2}}}, \tag{1}$$

[1] Pekeris (1948); Press and Ewing (1950); Landrø and Hatchell (2012)

Normal modes

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$$\boxed{c = c(\omega).}$$

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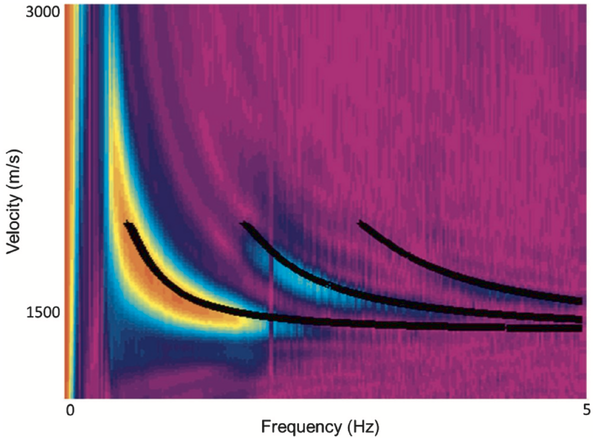

Figure 4: Estimated phase velocity dispersion curves (Hatchell and Mehta, 2010).

Normal modes

Period equation: group velocity

Group (envelope or modulation) velocity:

$$U = \frac{d\omega}{dk}, \qquad (2)$$

Normal modes

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Solving the period equation (5),

$$\boxed{U = U(\omega).}$$

Normal modes

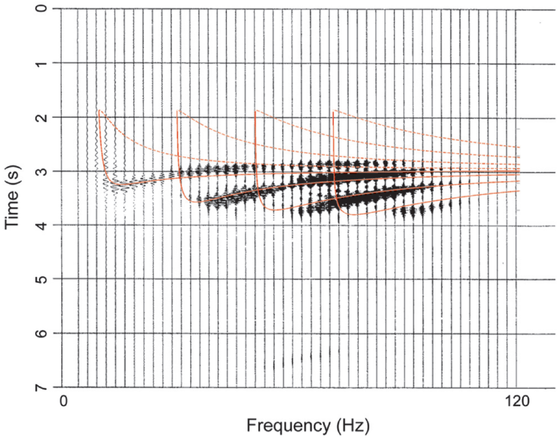

Figure 5: Frequency analysis: group and phase velocity dispersion (Landrø and Hatchell, 2012).

Normal modes

Signal or noise?

- Shallow marine sediments characterization,

Normal modes

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- Better denoising.

Orthorhombic media

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- Anisotropic symmetry class,

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  $\rightarrow$ $V = V(\alpha, \theta)$,

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- 9 (nine) independent parameters + density,

Orthorhombic media?

- Anisotropic symmetry class,
  - → $V = V(\alpha, \theta)$,
  - → 1x P-wave, 2x S-waves,
- Three mutually orthogonal planes of mirror symmetry,
- 9 (nine) independent parameters + density,
- Suitable for description of fractured (and layered) rocks.

Orthorhombic media!

Figure 6: Fractured sandstone: orthorhombic symmetry.

Orthorhombic media

Hooke's law

$$\boldsymbol{\sigma} = \boldsymbol{c}\boldsymbol{\epsilon}, \tag{3}$$

Orthorhombic media

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stress tensor

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stress tensor strain tensor

Orthorhombic media

Hooke's law

$$\boldsymbol{\sigma} = \boldsymbol{c}\boldsymbol{\epsilon}, \tag{3}$$

stress tensor strain tensor stiffness tensor

Orthorhombic media

Stiffness tensor

$$\mathbf{C} = \begin{pmatrix} c_{11} & c_{12} & c_{13} & & & \\ & c_{22} & c_{23} & & & \\ & & c_{33} & & & \\ & \text{SYM} & & c_{44} & & \\ & & & & c_{55} & \\ & & & & & c_{66} \end{pmatrix}. \qquad (4)$$

Normal modes in orthorhombic media

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Motivation

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- Sub-vertical fractures in gas-hydrate bearing sediments[3],
- Intersecting fracture systems[4],
- 15% azimuthal anisotropy[5].

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[3] Lee and Collett (2009); Cook et al. (2010)
[4] Sriram et al. (2013)
[5] Kumar et al. (2006)

Normal modes

Acoustic case

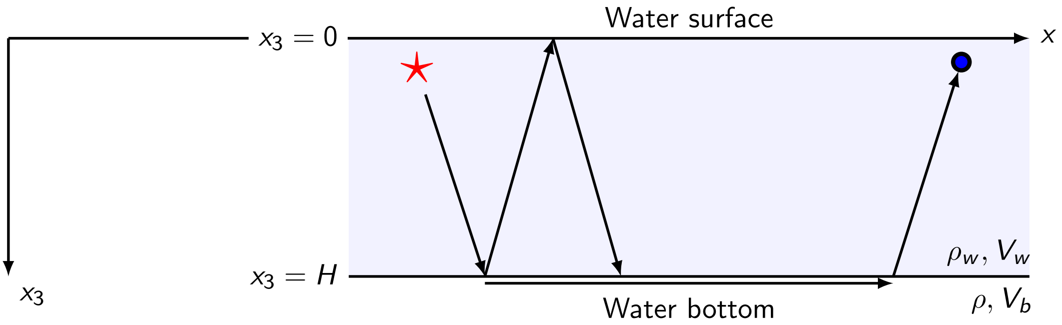

Figure 7: Sketch of a guided wave propagation. Vertical scale exaggerated.

Normal modes

Orthorhombic case

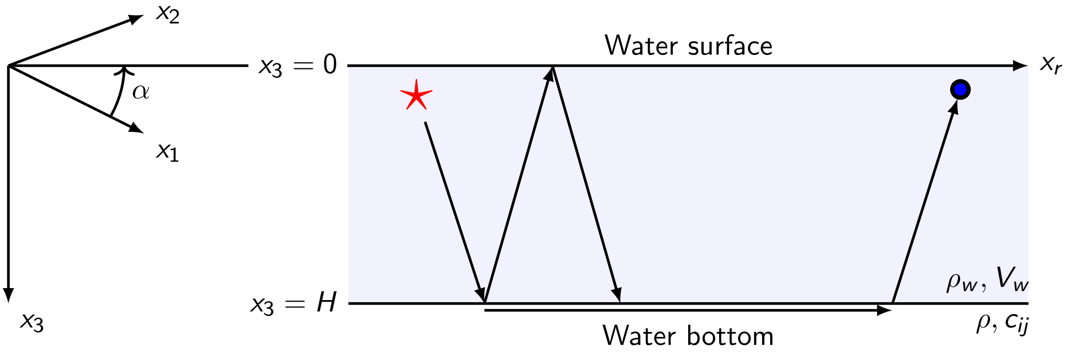

Figure 8: Sketch of a guided wave propagation. Vertical scale exaggerated.

Period equation

Elastic orthorhombic media: phase velocity

Ivanov and Stovas (2017):

$$\tan k_r H\sqrt{\frac{c^2}{V_w^2}-1}=\frac{\sqrt{\frac{c^2}{V_w^2}-1}}{c^2\rho_w}\left(\rho c^2-c_{44}\sin\alpha^2-c_{55}\cos\alpha^2\right)\times \frac{c_{13}\cos\alpha^2+c_{23}\sin\alpha^2+\rho c^2-c_{33}(\nu_1\nu_3+\nu_2\nu_3+\nu_1\nu_2)}{(\nu_1+\nu_2+\nu_3)\left(\rho c^2-c_{44}\sin^2\alpha-c_{55}\cos^2\alpha\right)-c_{33}\nu_1\nu_2\nu_3}. \quad (5)$$

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radial wavenumber

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radial wavenumber phase azimuth

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radial wavenumber phase azimuth P/S1/S2-waves attenuation coefficients

Numerical example

Model

$$\mathbf{C} = \begin{pmatrix} 9 & 2.25 & 3.6 & & & \\ & 5.94 & 2.4 & & & \\ & & 9.84 & 0 & & \\ & \text{SYM} & & 2 & & \\ & & & & 2.182 & \\ & & & & & 1.6 \end{pmatrix}. \quad (6)$$

$\rho/\rho_w = 1.56,\ V_w = 1.485$ km s^{-1}, $H = 0.075$ km.

Numerical example

Dispersion curves

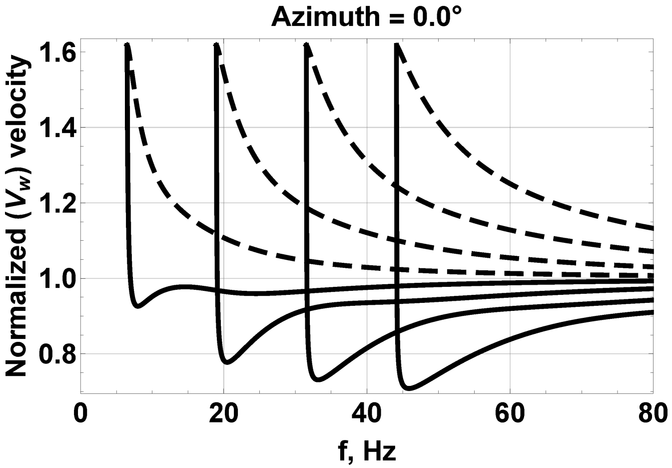
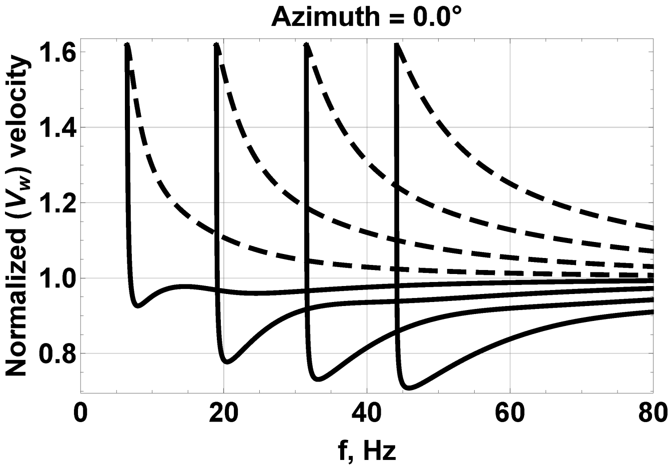


Figure 9: Phase (- - -) and group (——) velocities.

Numerical example

Dispersion curves



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Period equation

Elastic orthorhombic media: group velocity

Group velocity vector:

$$\mathbf{U} = \frac{d\omega}{d\mathbf{k}} \tag{7}$$

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Steering angle:

$$\beta = \tan^{-1}\frac{U_\alpha}{U_r}, \tag{8}$$

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Group azimuth:

$$\phi = \alpha + \beta. \tag{9}$$

Period equation

Group velocity

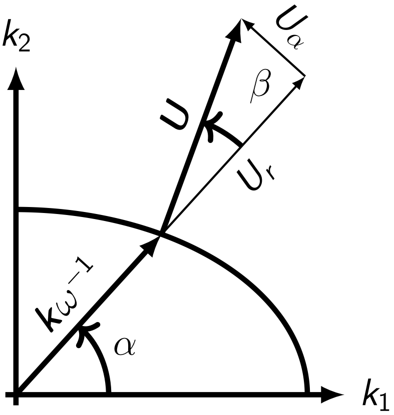

Figure 10: Schematic of the phase and group velocity vectors relation (horizontal plane).

Numerical example

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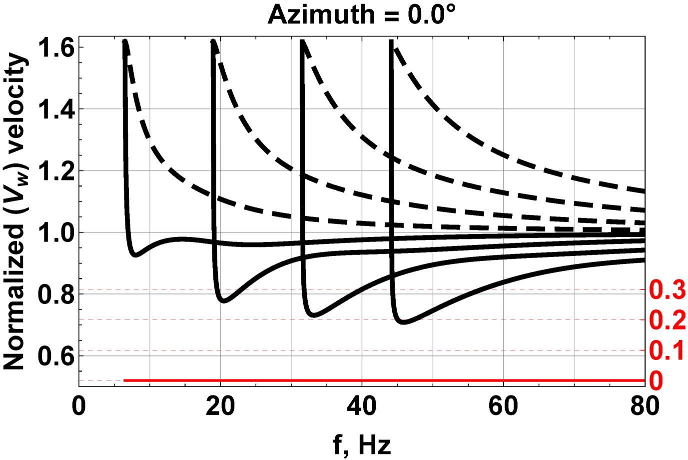
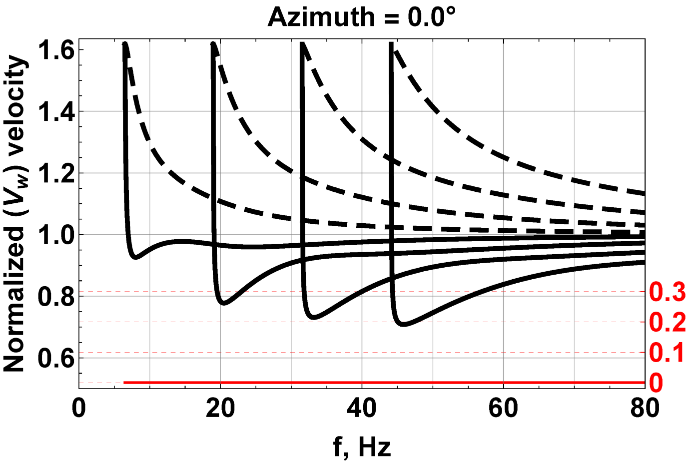


Figure 11: Phase (- - -), radial group (——), and tangential group velocities.

Numerical example

Dispersion curves



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Numerical example

Azimuthal dependence

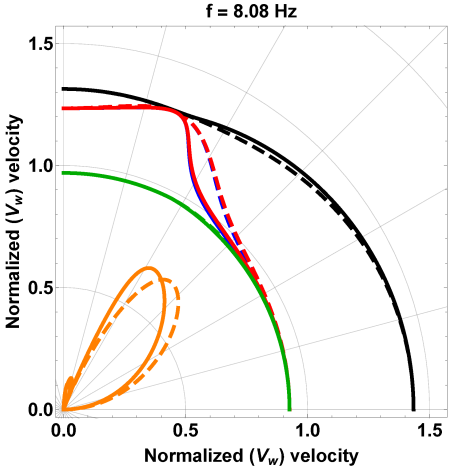
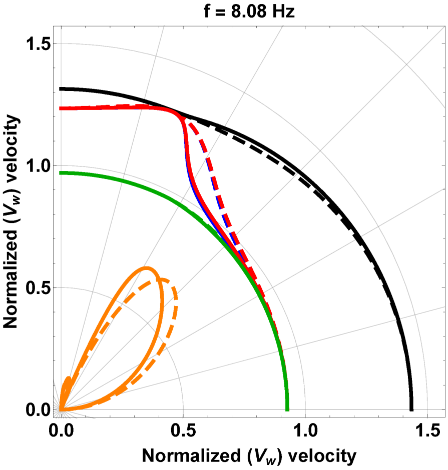


Figure 12: Azimuthal (α: - - -, ϕ: ——) dependence: c, U_r, $|\boldsymbol{U}|$, $U_\alpha \times 5$, $\min_\omega U_r$.

Numerical example

Azimuthal dependence



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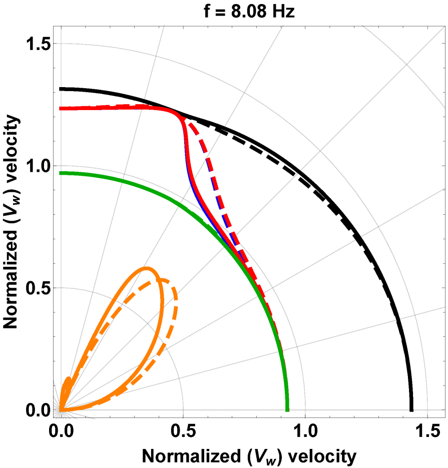

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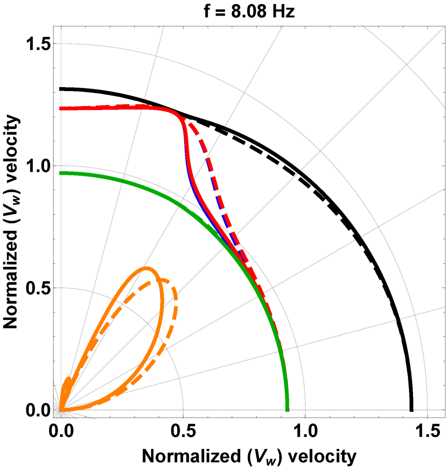

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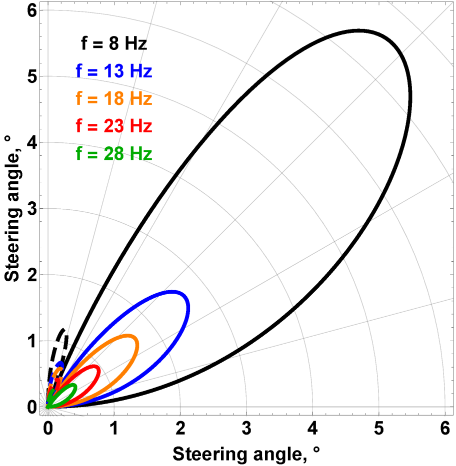

Figure 14: Steering angle.

Useful simplification

Orthorhombic media

Stiffness tensor

$$\mathbf{C} = \begin{pmatrix} c_{11} & c_{12} & c_{13} & & & \\ & c_{22} & c_{23} & & & \\ & & c_{33} & & & \\ & \text{SYM} & & c_{44} & & \\ & & & & c_{55} & \\ & & & & & c_{66} \end{pmatrix}. \tag{10}$$

Ellipsoidal orthorhombic media

Stiffness tensor

$$\mathbf{C} = \rho \begin{pmatrix} V_1^2 & V_1 V_2 & V_1 V_3 & & & \\ & V_2^2 & V_1 V_2 & & & \\ & & V_3^2 & & & \\ & \text{SYM} & & 0 & & \\ & & & & 0 & \\ & & & & & 0 \end{pmatrix}. \tag{11}$$

Ellipsoidal orthorhombic media

Period equation

$$\tan k_r H \sqrt{\frac{c^2}{V_w^2} - 1} = -\frac{\rho}{\rho_w} \frac{\sqrt{\frac{c^2}{V_w^2} - 1}}{\sqrt{\frac{V_2^2 \sin^2\alpha + V_1^2 \cos^2\alpha}{V_3^2} - \frac{c^2}{V_3^2}}}. \tag{12}$$

Acoustic media

Period equation

$$\tan kH\sqrt{\frac{c^2}{V_w^2}-1} = -\frac{\rho}{\rho_w}\frac{\sqrt{\frac{c^2}{V_w^2}-1}}{\sqrt{1-\frac{c^2}{V_b^2}}}. \qquad (13)$$

Ellipsoidal orthorhombic media

Group-velocity limits

$$\{U_r, U_\alpha\}|_{\omega\to\infty} = \{V_w, 0\} \, ,$$

Ellipsoidal orthorhombic media

Group-velocity limits

$$\{U_r, U_\alpha\}\big|_{\omega\to\infty} = \{V_w, 0\}\,,$$

$$\{U_r, U_\alpha\}\big|_{\omega\to\omega_{\text{cut-off}}} = \left\{\sqrt{V_2^2\sin^2\alpha + V_1^2\cos^2\alpha}, \frac{\left(V_1^2 - V_2^2\right)\sin\alpha\cos\alpha}{\sqrt{V_2^2\sin^2\alpha + V_1^2\cos^2\alpha}}\right\}, \tag{14}$$

Conclusions

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- Potential use for water-bottom sediments characterization.

Acknowledgments

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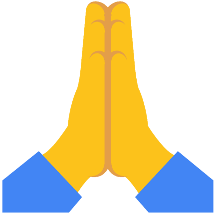

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