Ensemble based seismic inversion

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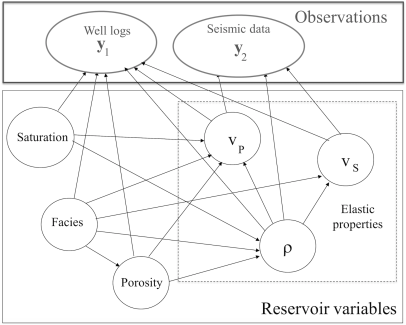
Uncertainty in reservoir evaluation (URE) project

Project initiated and led by Henning Omre, Dept of Math Sciences, NTNU, since 1992. Currently Håkon Tjelmeland, Henning Omre and Jo Eidsvik.

Working with Per Avseth, Børge Arntsen, Bjørn Ursin, Martin Landrø, ++. And with partners (currently 7 companies).

Vision: Provide creative mathematically-based solutions to recognized challenges in reservoir evaluation. Develop methodologies for analysis of spatio-temporal phenomena.

URE project

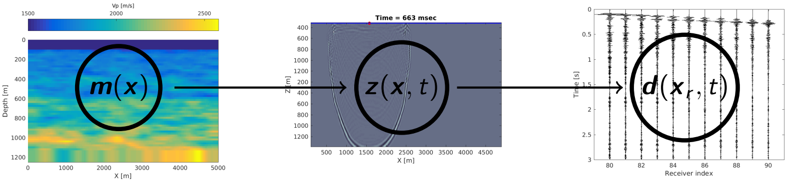


- Spatio-temporal statistics. Computational statistics.
- Use geophysics and geology to contrain models.
- Lithology and fluid prediction. AVO inversion. History matching.

Inversion

Elastic parameters

Measurements



Propagating Wavefield

Michael's PhD goal: *Prediction of elastic properties from seismic (waveform) data, with uncertainties.*
(new ground for URE)

Statistical model formulation

Velocity $\boldsymbol{m}$ with prior $p(\boldsymbol{m})$, usually Gaussian process in (lateral, depth) coordinates.

Seismic data $\boldsymbol{d}$. Likelihood

$$\boldsymbol{d}(\boldsymbol{x}_r, t) = g(\boldsymbol{x}_r, t; \boldsymbol{m}, \boldsymbol{\theta}) + \boldsymbol{e}$$

with forward model $g(\cdot)$ and errors $\boldsymbol{e} \sim \text{Normal}(0, \boldsymbol{\Sigma}_{\boldsymbol{e}})$.

Bayes' rule:

$$p(\boldsymbol{m}|\boldsymbol{d}) \propto p(\boldsymbol{d}|\boldsymbol{m})p(\boldsymbol{m})$$

Approximate solution

Bayes' rule:

$$p(\boldsymbol{m}|\boldsymbol{d}) \propto p(\boldsymbol{d}|\boldsymbol{m})p(\boldsymbol{m})$$

Optimize objective function

$$\boldsymbol{O}(\boldsymbol{m}) = (\boldsymbol{m}-\boldsymbol{\mu})^t\boldsymbol{\Sigma}^{-1}(\boldsymbol{m}-\boldsymbol{\mu})+(\boldsymbol{d}-g(\boldsymbol{x}_r, t; \boldsymbol{m}, \theta))^t\boldsymbol{\Sigma}_{\boldsymbol{e}}^{-1}(\boldsymbol{d}-g(\boldsymbol{x}_r, t; \boldsymbol{m}, \boldsymbol{\theta}))$$

Prediction:

$$\hat{m} = \text{argmax}\,[O(\boldsymbol{m})]$$

Uncertainty from Hessian:

$$\hat{\text{Var}}(\boldsymbol{m}|\boldsymbol{d}) = \left[-\frac{d^2O(\boldsymbol{m})}{d\boldsymbol{m}^2}\right]^{-1}$$

Assumption - Gaussian approximation at mode. Derivatives available.

Sequential formulation

Partitioning the entire data record $\boldsymbol{d}$ into N_c sets

$$\begin{aligned} p(\boldsymbol{m}|\boldsymbol{d}) &\propto p(\boldsymbol{d}|\boldsymbol{m})p(\boldsymbol{m}) \\ &= p(\boldsymbol{m}) \prod_{i=1}^{N_c} p(\boldsymbol{d}_i|\boldsymbol{d}_{1:i-1}, \boldsymbol{m}) \end{aligned}$$

Sequential model updating (in offset, time and/or frequency).

$$p(\boldsymbol{m}) \rightarrow p(\boldsymbol{m}|\boldsymbol{d}_1) \rightarrow p(\boldsymbol{m}|\boldsymbol{d}_1, \boldsymbol{d}_2) \rightarrow p(\boldsymbol{m}|\boldsymbol{d}_1, \boldsymbol{d}_2, \boldsymbol{d}_3)$$

Possibly a robust path to solution.

Kalman-like sequential updating

Linear state-space model, initialised with $\boldsymbol{\mu}_0$ and $\boldsymbol{P}_0$:

$$\begin{aligned} \text{dynamics:} \quad \boldsymbol{m}_t &= \boldsymbol{m}_{t-1} \qquad\qquad , \\ \text{observation:} \quad \boldsymbol{d}_t &= \boldsymbol{H}_t \boldsymbol{m}_t + \boldsymbol{e}_t, \ \boldsymbol{e}_t \sim \text{Normal}(\boldsymbol{0}, \boldsymbol{R}) \end{aligned}$$

Forecast step for mean and variance:

$$\begin{aligned} \boldsymbol{\mu}_t &= \boldsymbol{\mu}_{t-1} \\ \boldsymbol{P}_t &= \boldsymbol{P}_{t-1} \end{aligned}$$

Analysis step for mean and variance:

$$\begin{aligned} \boldsymbol{\mu}_t &= \boldsymbol{\mu}_t + \boldsymbol{K}_t \left(\boldsymbol{y}_t - \boldsymbol{H}_t \boldsymbol{\mu}_t \right) \\ \boldsymbol{P}_t &= \left(\boldsymbol{I} - \boldsymbol{K}_t \boldsymbol{H}_t \right) \boldsymbol{P}_t \end{aligned}$$

Kalman gain:

$$\boldsymbol{K}_t = \boldsymbol{P}_t \boldsymbol{H}_t^T \left(\boldsymbol{H}_t \boldsymbol{P}_t \boldsymbol{H}_t^T + \boldsymbol{R} \right)^{-1}$$

Ensemble based approximation

- Prior (and posterior) represented by an ensemble of realisations

$$\{\boldsymbol{m}^f\}_{i=1}^{N_e} \sim p(\boldsymbol{m})$$
$$\{\boldsymbol{m}^a\}_{i=1}^{N_e} \sim p(\boldsymbol{m}|\boldsymbol{d})$$

- Generally $N_e \ll N_m$ and/or $N_e \ll N_d$.
- High dimensional parameter- and data-space. Not many sampling based methods are suitable $\rightarrow$ choice of Ensemble Kalman Filter.
- Can handle (some degree of) nonlinearity.

Approach for parameter estimation with EnKF

- Assimilation cycle update equation:

$$\boldsymbol{m}_i^a = \boldsymbol{m}_i^f + \hat{\boldsymbol{K}} \left(\boldsymbol{d} - \boldsymbol{d}_i^f \right) \text{ with}$$

$$\boldsymbol{d}_i^f = g(\boldsymbol{m}_i^f) + \boldsymbol{e}_i, \quad \boldsymbol{e}_i \sim \text{Normal}(\boldsymbol{0}, \boldsymbol{\Sigma_e}),\ i = 1, \ldots, N_e$$

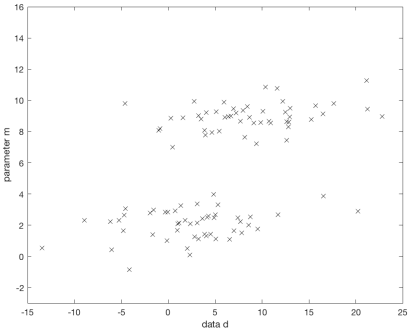
  with $g(\cdot)$ now defining forward evaluation and partitioning of data.
- Estimating Kalman gain matrix $\boldsymbol{K}$ from forecast models and data:

$$\hat{\boldsymbol{K}} = \hat{\boldsymbol{\Sigma}}_{\boldsymbol{m},\boldsymbol{d}} \hat{\boldsymbol{\Sigma}}_{\boldsymbol{d}}^{-1}$$

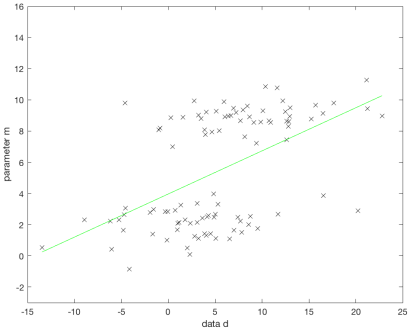
- Linear correlations can identify (most influential) subset of parameters.

Univariate example - forecast samples

$$m_i \sim p(m), \qquad d_i = m_i + N(0, 5^2)$$

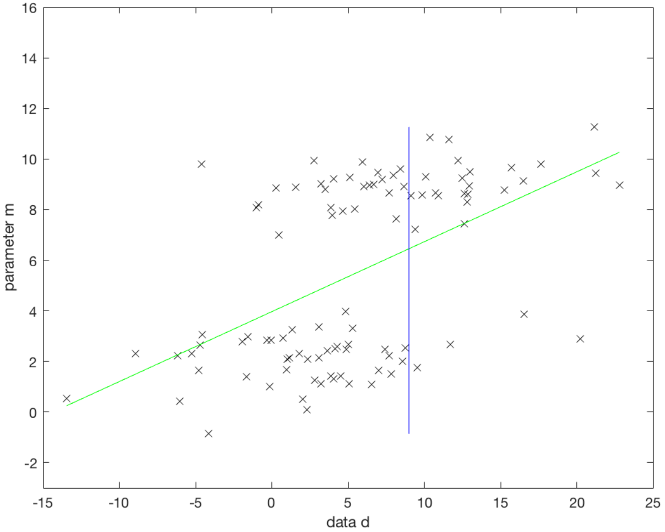


Univariate example - regression fit

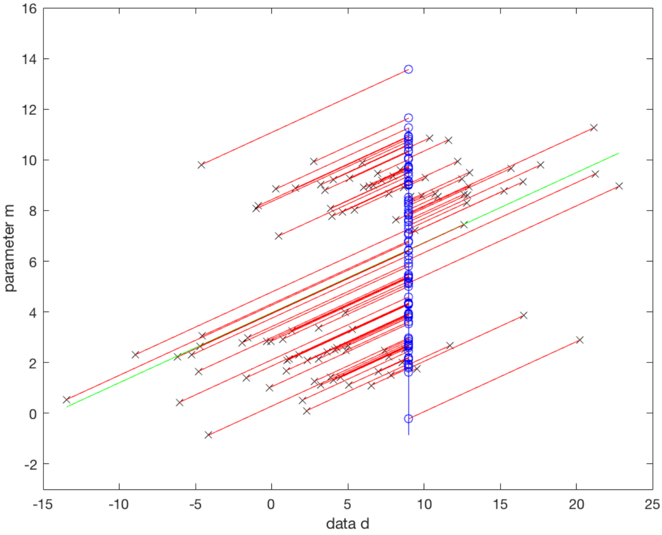


Univariate example - observation

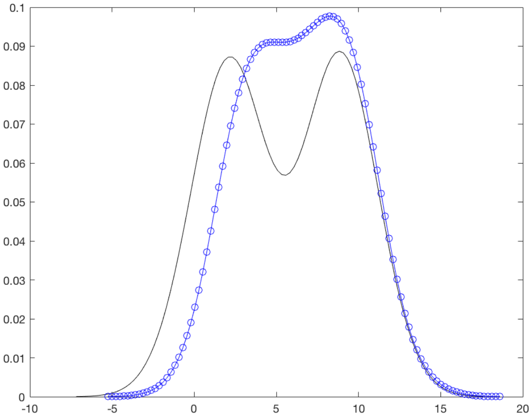
$d = 9$.



Univariate example - analysis or update step

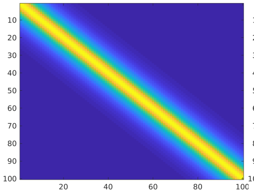


Univariate example - prior and posterior

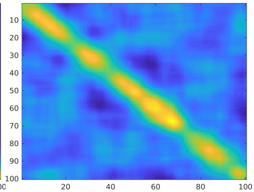


Sample covariance matrix

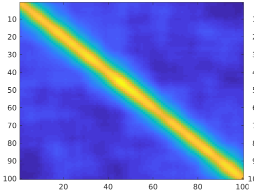
$\hat{K}$ based on Monte Carlo samples.



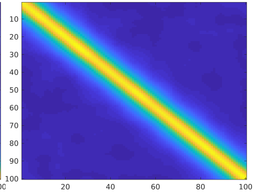
(a) True



(b) $N_e \ll N_x$



(c) $N_e = N_x$



(d) $N_e \gg N_x$

Properties and challenges

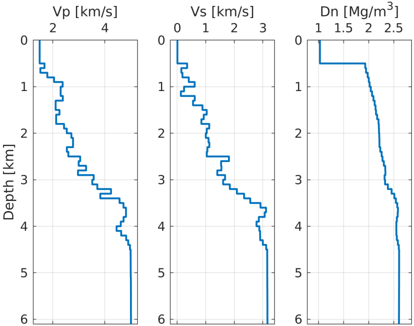
- Forward model evaluation is costly. And data is non-linear function of parameters.
- Seismograms are non-stationary time series.
- Multicollinearity in data.
- Combination of large data space and small ensemble space $\rightarrow$ ensemble collapse.
- Observations are non-local.

Case

- Project with data from BP.
- 1D (layered subsurface) assumptions $\rightarrow$ simpler and faster model.
- Forward model can be implemented in various ways, but here using Erzsol3, an old open source F77 code.

Data

- True model based on well log data.
- Observations (synthetic) are CMP gather.
- Data partitioning, blocks of offset-time intervals.



(e)

Data

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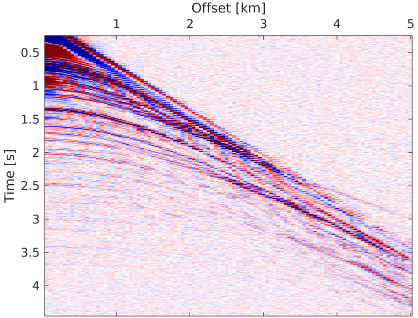

(f)

Data

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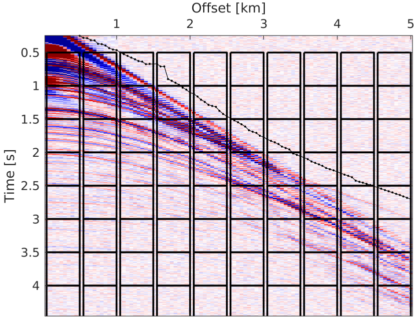
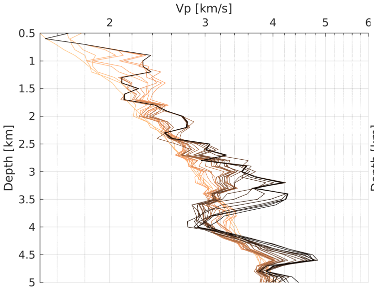


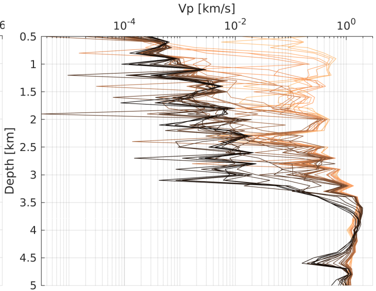
(g)

Inversion

Sequential updating (yellow-black)



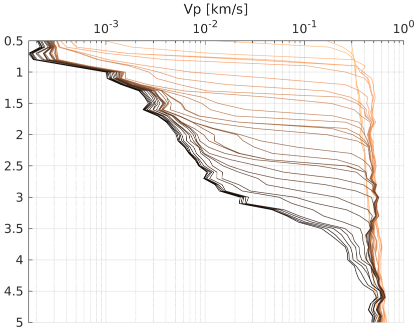
(h) Mean



(i) Error

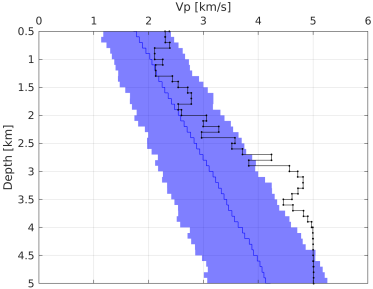
Inversion

Sequential updating (yellow to black)

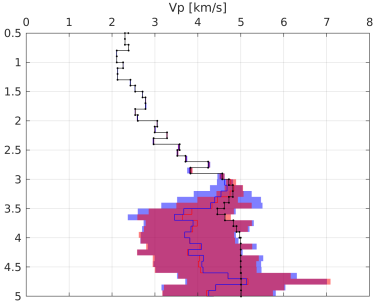


(j) Standard deviation

Inversion results



(k) Initial prior



(l) Final posterior

Ongoing work

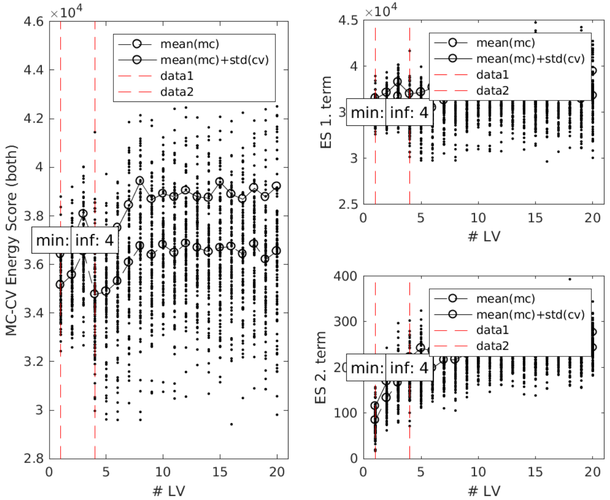
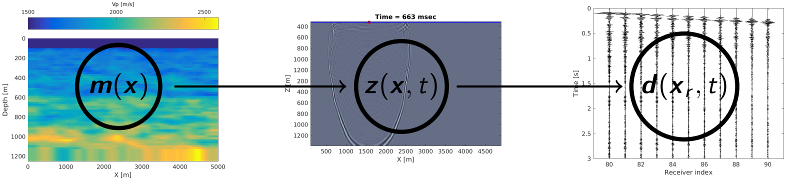


Figure: Example of Partial least squares for data reduction

Ongoing work

Elastic parameters

Measurements



Propagating Wavefield

Summary

- Uncertainty quantification in seismic inversion. Very costly forward model, approximate prediction and uncertainties via ensembles, with useful properties.
- Synthetic models and CMP data showing good results.
- Data partitioning approach required to reduce data size and avoid collapse. Now studying different blocking schemes in time, offset and frequency.
- Standard techniques like localization for Ensemble Kalman filter are difficult for seismic data.