Recursive-Iterative Zero-phase Filtering Via Singular Spectrum Analysis

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CONTENTS

Introduction

- Normally the singular value decomposition (SVD) filtering is applied in the t-x domain;
- The conventional SVD filtering explore the spacial correlation between a set of seismic traces to reduce noise and enhance coherence of the events present in the seismic data;
- The new method works on single traces decomposing each seismic trace individually;
- Explore the correlation between reflected events along the time variable (temporal correlation).

- One particular way to apply SVD in a single (or multivariate) time series is the Singular Spectrum Analysis (SSA) method;
- The SSA method is apply on constant-frequency slices in one or many spatial dimensions, for random-noise attenuation for 3D seismic data and for data reconstruction via multichannel SSA;
- We explore SSA method in the time direction and we propose a recursive-iterative algorithm, which:
  - uses only the first eigenimage of the data matrix and
  - decompose seismic traces in the low and high frequency parts;
- The SSA method applied in the time domain corresponds to filtering the time series with a symmetric zero-phase filter, which corresponds to the autocorrelation of the first eigenvector associated to the time variable.

The single-channel Singular Spectrum Analysis

- Let matrix $\mathbf{D}$ be formed by shifted of the data,

$$\mathbf{D} = \begin{bmatrix} \mathbf{d} & 0 & \dots & 0 \\ 0 & \mathbf{d} & 0 & \vdots \\ \vdots & 0 & \ddots & 0 \\ 0 & \dots & 0 & \mathbf{d} \end{bmatrix} = [\, \mathbf{E}_0\mathbf{d}, \quad \dots \quad , \mathbf{E}_N\mathbf{d} \,] = [\, \bar{\mathbf{d}}_0, \dots, \bar{\mathbf{d}}_N \,] \ ,$$

- Where

$$\mathbf{E}_k = \begin{bmatrix} 0 & \dots & 0 \\ 1_k & \ddots & 0 \\ 0 & \ddots & 1_k \\ 0 & \dots & 0 \end{bmatrix} \quad \text{and} \quad \bar{\mathbf{d}}_k = \mathbf{E}_k\mathbf{d} = \begin{bmatrix} \mathbf{0}_k \\ \mathbf{d} \\ \mathbf{0}_{N-k} \end{bmatrix} .$$

The single-channel Singular Spectrum Analysis

- The signal restauration is done by

$$\mathbf{d} = \frac{1}{N+1}\sum_{k=0}^{N}\mathbf{E}_k^T\overline{\mathbf{d}}_k$$

- The SVD of the matrix $\mathbf{D}_N$ may be represented as

$$\mathbf{D} = \sum_{\tau=0}^{N}\sigma_\tau \mathbf{u}_\tau \mathbf{v}_\tau^T = \sum_{\tau=0}^{N}\widetilde{\mathbf{D}}_\tau$$

- $\widetilde{\mathbf{D}}_\tau = \sigma_\tau \mathbf{u}_\tau \mathbf{v}_\tau^T$ represents the eigenimage of index τ of the shifted data matrix $\mathbf{D}$

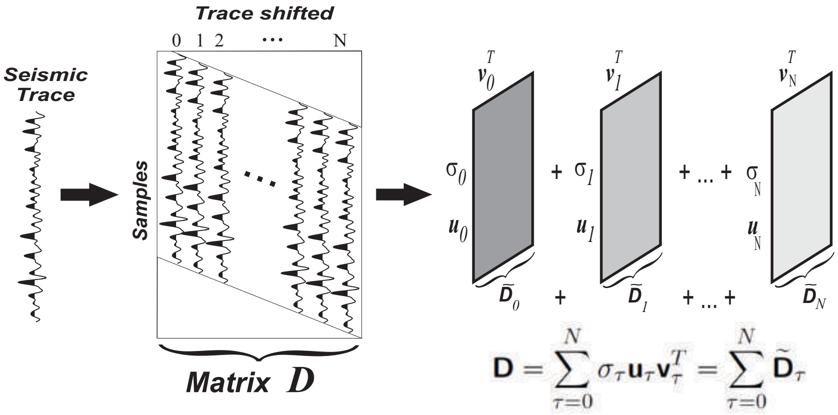

Figure 1: SVD of the data matrix of order N.

The single-channel Singular Spectrum Analysis

- For a given eigenimage, $\widetilde{\mathbf{D}}_\tau$, we can restore a transformed data component

$$\widetilde{\mathbf{d}}_\tau = \sigma_\tau \sum_{k=0}^{N} v_{\tau k} \mathbf{E}_k^T \mathbf{u}_\tau = \sigma_\tau \mathbf{V}_\tau^T \mathbf{u}_\tau \,,$$

- where $\mathbf{V}_\tau^T$ is a banded Toeplitz matrix

$$\mathbf{V}_\tau^T = \sum_{k=0}^{N} v_{\tau k} \mathbf{E}_k^T = \begin{bmatrix} v_{\tau 0} & \dots & v_{\tau N} & \mathbf{0}_{M-1}^T & 0 \\ \mathbf{0}_{M-1} & \ddots & \ddots & \ddots & \mathbf{0}_{M-1} \\ 0 & \mathbf{0}_{M-1}^T & v_{\tau 0} & \dots & v_{\tau N} \end{bmatrix} .$$

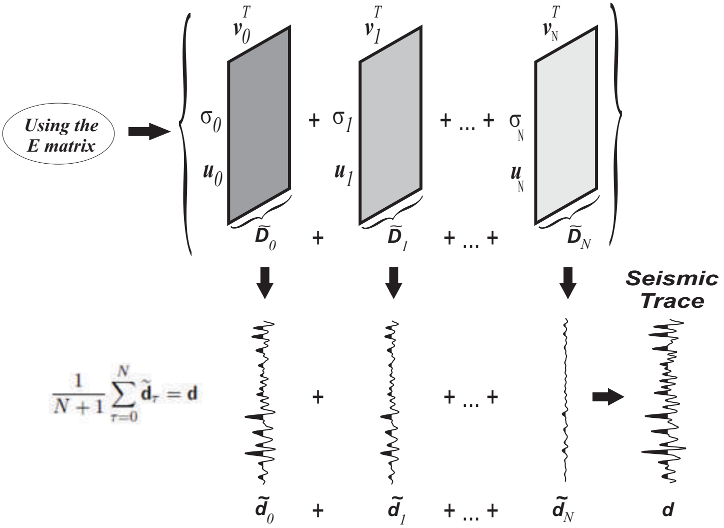

Figure 2: Reconstruction of the data using E Matrix.

Zero-phase property of the SSA Method

- The transformed trace, $\widetilde{\mathbf{d}}_\tau$ is equal to the convolution of the input trace, $\{d_n\} = \{d_0, \ldots, d_M\}$, with the autocorrelation coefficients, $\{r_{\tau n}\} = \{r_{\tau -N}, \ldots, r_{\tau 0}, \ldots, r_{\tau N}\}$, of the eigenvector $\mathbf{v}_\tau$.

- As consequence the transformed trace is not affected by the phase of the operator since the autocorrelation is a zero-phase signal.

- Multiplying the equation the reduced SVD by $\mathbf{v}_\tau$ and considering the orthogonality of the eigenvectors we obtain $\mathbf{u}_\tau = \sigma_\tau^{-1}\mathbf{D}\mathbf{v}_\tau$. So we rewrite:

$$\widetilde{\mathbf{d}}_\tau = \sigma_\tau \mathbf{V}_\tau^T \mathbf{u}_\tau = \mathbf{V}_\tau^T \mathbf{D}\mathbf{v}_\tau \,,$$

Zero-phase property of the SSA Method

- Taking out the index τ, last equation can be written in component form,

$$\tilde{d}_i = \sum_{j=0}^{N} \sum_{k=j}^{M+j} v_{k-i} d_{k-j} v_j$$

- Changing to the new summation variable $n = k - j - i$ gives:

$$\tilde{d}_i = \sum_{j=0}^{N} \sum_{n=i}^{M-i} d_{n+i} v_{n+j} v_j = \sum_{n=i}^{M-i} d_{n+i} r_n \,.$$

- $r_n = \sum v_{n+j} v_j$ is the autocorrelation of the eigenvector $\mathbf{v}_\tau$.

Zero-phase property of the SSA Method

- We see that the output trace of the eigenimage number τ is the cross-correlation of the data vector with the autocorrelation of the eigenvector $\mathbf{v}_\tau$.
- Since the autocorrelation is zero phase, the phase of the output trace is equal to the phase of the data trace.
- The Figure 3 illustrates the decomposition of a seismic trace in 5 eigentraces. The traces in (a) to (e), represent the eigentraces 0 to 4, from low to high frequencies, respectively.

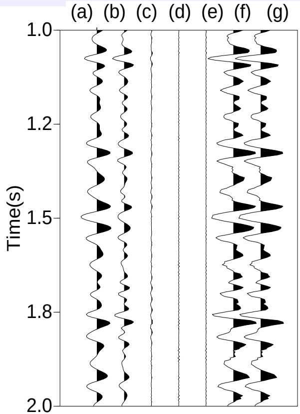

Figure 3: Detail of the decomposition of a seismic trace in 5 eigentraces. The sum of the eigentraces in (f) and the original seismic trace in (g).

A recursive and iterative SSA (RI-SSA) algorithm using the first eigenimage only

- Initial auxiliary vectors: $\widetilde{\mathbf{d}}_0 = \mathbf{d}$ and $\widehat{\mathbf{d}}_0 = \mathbf{d}$

⇒ For $k = 1, \ldots, K$

  ⇒ For $\tau = 1, \ldots, N$

  - Form the matrix $\mathbf{D}_\tau = [\ \bar{\mathbf{d}}_0, \ldots, \bar{\mathbf{d}}_\tau\ ]$ from $\widetilde{\mathbf{d}}_{\tau-1}$
  - Form the first eigenimage $\widetilde{\mathbf{D}}_{\tau 0} = \sigma_0 \mathbf{u}_0 \mathbf{v}_0^T$
  - Form the first eigentrace $\widetilde{\mathbf{d}}_\tau = \sigma_0 \sum_{j=0}^{\tau} v_{0j} \mathbf{E}_j \mathbf{u}_0$

  ⇒ Enddo

  - Update the high frequency part $\widetilde{\mathbf{d}}_0 = \widehat{\mathbf{d}}_{k-1} - \widetilde{\mathbf{d}}_N = \widehat{\mathbf{d}}_k$

⇒ Enddo

- Output:
  - High-frequency part, $\widehat{\mathbf{d}} = \widehat{\mathbf{d}}_K$
  - Low-frequency part, $\widetilde{\mathbf{d}} = \mathbf{d} - \widehat{\mathbf{d}}_K$

Data examples with ground-roll attenuation

⇒ **Processing of a seismic line of the Tacutu basin**

- Total of 179 shot gathers recorded at 4 ms sampling interval;
- The distance between shots is 200 m;
- The distance between the geophones is 50 m and 96 channel per shot in split-spread geometry;
- Maximum and minimum offsets from 2500 m and 150 m respectively;
- Low CMP coverage of 12 fold.

Example of a filtered shot gather - No recursions and no iterations

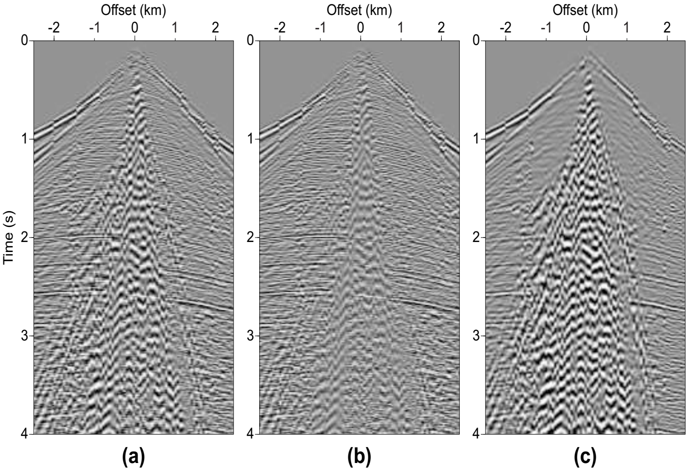

Figure 4: The result of SSA method with $N = 10$. The original shot gather in (a), the high-frequency part in (b), and the low-frequency part (the first eigentraces) in (c).

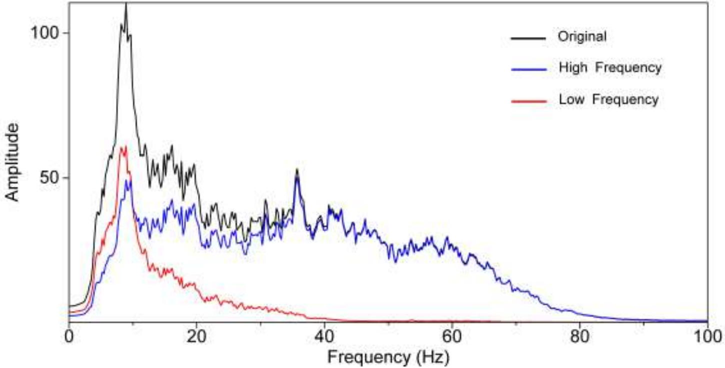

Figure 5: Average amplitude spectra of original and filtered data - SSA method with $N = 10$ no recursions and no iterations.

Example of a filtered shot gather - N=10 recursions and no iterations

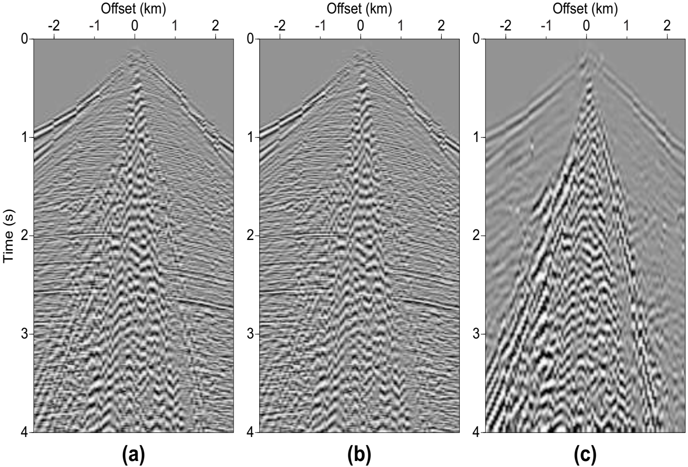

Figure 6: The result of RI-SSA method with $N = 10$ recursions in matrix dimension. The original shot gather in (a), the high-frequency part in (b), and the low-frequency part in (c).

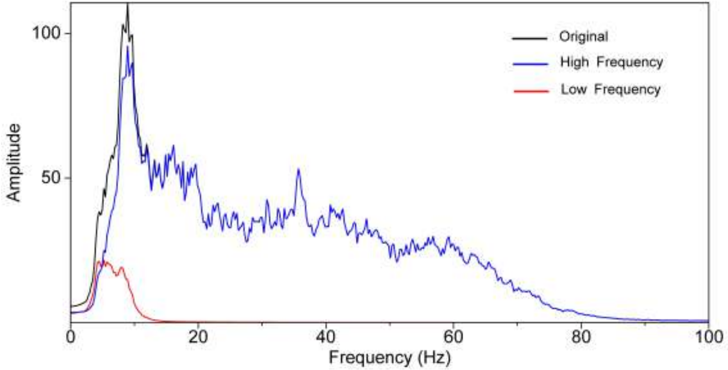

Figure 7: Average amplitude spectra of original and filtered data - RI-SSA method with $N = 10$ recursions in matrix dimension and no iterations.

Example of a filtered shot gather - N=10 recursions and K=20 iterations

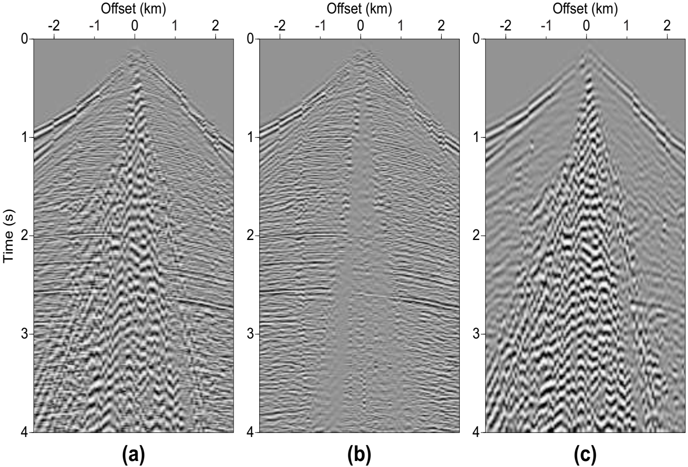

Figure 8: The result of RI-SSA method with $N = 10$ recursions in matrix dimension and $K = 20$ iterations. The original shot gather in (a), the high-frequency part in (b), and the low-frequency part in (c).

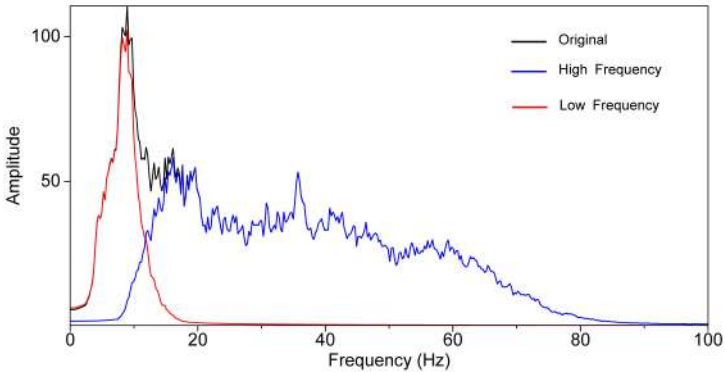

Figure 9: Average amplitude spectra of original and filtered data - RI-SSA method with $N = 10$ recursions in matrix dimension and $K = 20$ iterations.

Comparisons of stacked sections

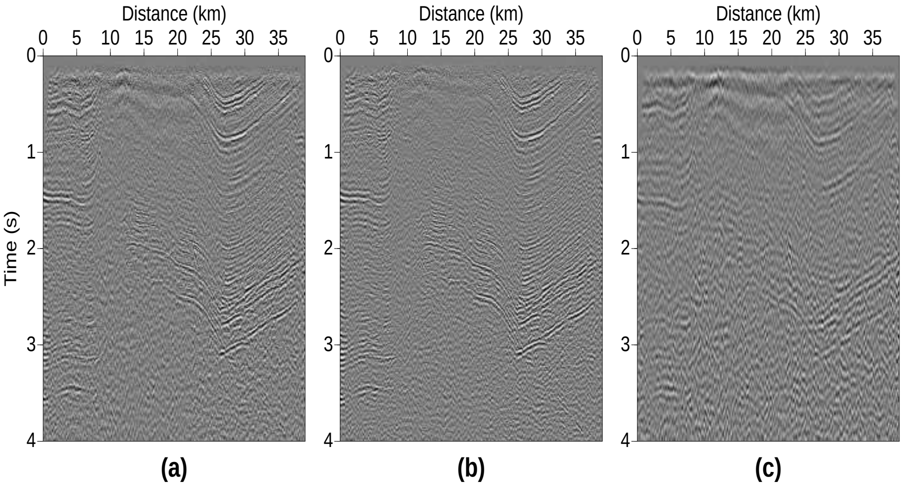

Figure 10: Stacked section of the original data in (a), stacked section of the filtered data in (b) and stacked section of the residual in (c).

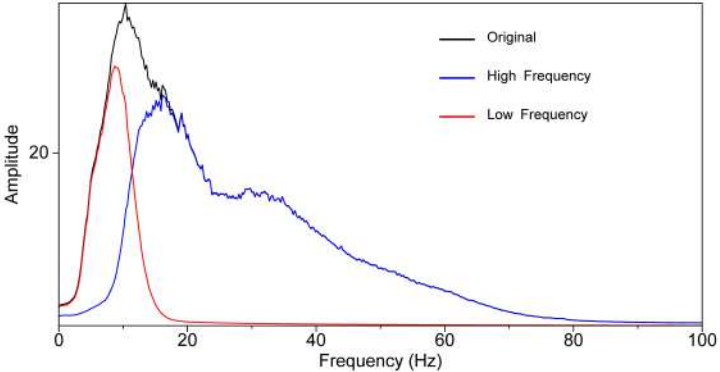

Figure 11: Average amplitude spectra of original and filtered stacked sections in Fig. 10.

Detail of the stacked sections

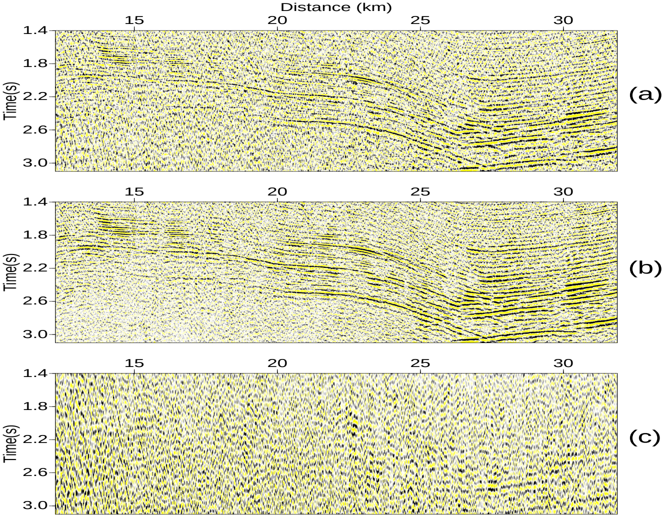

Figure 12: Original stacked section in (a), stacked section of the filtered data in (b) and of the residual in (c).

Conclusions

- The recursive, iterative single-channel RI-SSA method is a zero-phase process which effectively separates high-frequency and low-frequency parts in the data;
- The zero-phase property preserves the traveltime information in the data, and the low-frequency and high-frequency amplitude parts are also preserved;
- The recursive, iterative single-channel RI-SSA algorithm showed excellent results when applied pre-stack to ground-roll attenuation on a seismic line from the Tacutu basin.

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