ROSE
Rock Seismic Research Project

ROSE MEETING

Shear waves in acoustic anisotropic media

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Outline

- Background
- New parameters defined from the slowness surface
- Group velocity surface
- Traveltime equation
- Relative geometrical spreading
- Conclusion

Background

- The acoustic VTI medium was firstly proposed by setting the $v_{S0} = 0$ (Alkhalifah, 1998).

- During modeling for acoustic VTI medium, diamond shaped waves propagate and was initially considered as artefacts (Alkhalifah, 2000).

- This artefacts were S-waves, and acoustic VTI medium can also be practical from the upscaling point of view (Grechka et al., 2004).

- Anomalously low S-wave velocity (10-50m/s) was observed in unconsolidated ocean-bottom sediments (Ayres and Theilen, 1999).

Background

According to the “long wave equivalent” medium theory (Backus, 1962), a stack of thin isotropic or transversely isotropic layers can be replaced with an effective medium.

$$C_{44} = \left\langle (C_{44}^{k})^{-1} \right\rangle^{-1}$$

→ If any layer has $C_{44}^{k} = 0$ (fluid layer), the effective $C_{44} = 0$ ($V_{S0} = 0$).

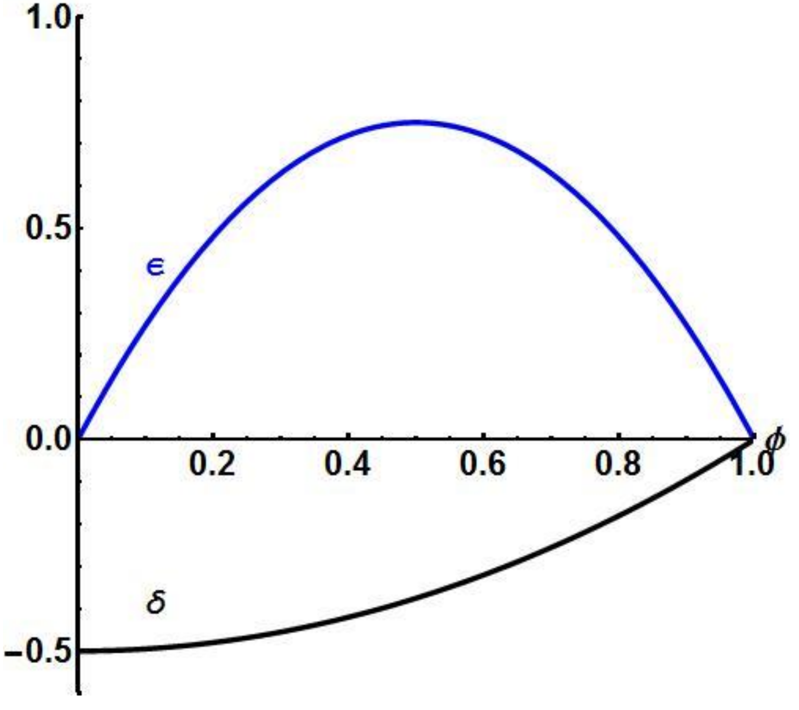
Model: A binary medium composed of interlayering plane solid and fluid layers

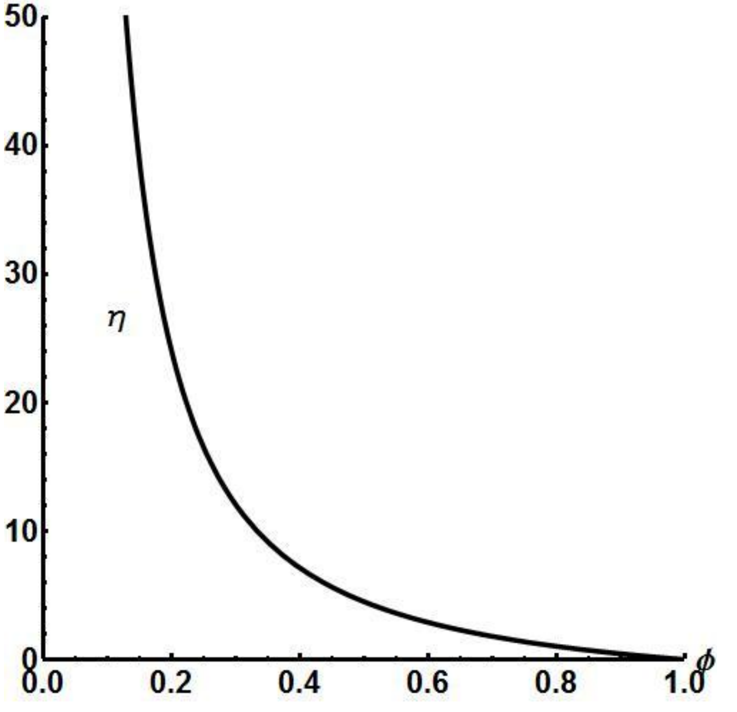
Solid layer $V_p = 3.00 km/s$ $V_S = 2.12 km/s$ $\rho = 2.00 g/cm^3$

Fluid layer $V_{Pf} = 1.50 km/s$ $V_{Pf} = 0.00 km/s$ $\rho_f = 1.00 g/cm^3$

Background

Anisotropic parameters as the function of fluid volume fraction





The acoustic VTI medium can be strongly anisotropic.

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New parameters defined from the slowness surface

For the acoustic VTI medium, the slowness surface is

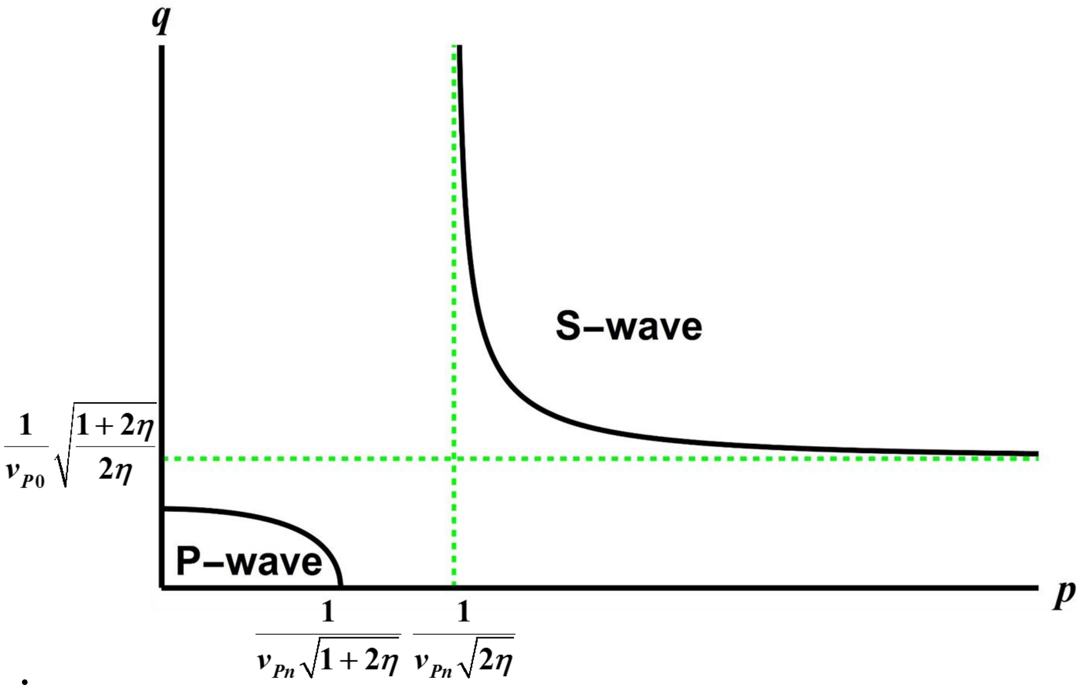
$$q = \frac{1}{v_{P0}} \sqrt{\frac{(1+2\eta)v_{Pn}^2 p^2 - 1}{2\eta v_{Pn}^2 p^2 - 1}}$$

This equation works for both P- and S-waves, but separate branches.

S-waves:

$$|p| \geq \frac{1}{v_{Pn}\sqrt{2\eta}}$$

$$|q| \geq \frac{\sqrt{1+2\eta}}{v_{P0}\sqrt{2\eta}}$$



There is no wave mode conversion

New parameters defined from the slowness surface

The vertical and horizontal S-wave group velocities V_{S0} and V_{SX} can be defined from the asymptotes as

$$V_{S0} = v_{P0}\sqrt{\frac{2\eta}{1+2\eta}}, \quad V_{SX} = v_{Pn}\sqrt{2\eta}$$

The new set of parameters to describe the S-wave propagation:

$$V_{S0}, \quad V_{SX} \quad \text{and} \quad \eta$$

The slowness surface in acoustic VTI medium can be rewritten as

$$q = \frac{1}{V_{S0}}\sqrt{\frac{p^2V_{SX}^2 + 2\eta(p^2V_{SX}^2 - 1)}{(1+2\eta)(p^2V_{SX}^2 - 1)}}$$

When $\eta = 0$, $\quad q = \frac{1}{V_{S0}}\sqrt{\frac{p^2V_{SX}^2}{p^2V_{SX}^2 - 1}},$

This is a reference medium for the S-wave propagation.

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Group velocity surface

V_G S-wave group velocity

φ group angle (from the vertical symmetry axis)

the slowness surface → *the group velocity surface*

$$q = \frac{1}{V_{S0}} \sqrt{\frac{p^2 V_{SX}^2 + 2\eta (p^2 V_{SX}^2 - 1)}{(1+2\eta)(p^2 V_{SX}^2 - 1)}}$$

$$\frac{1}{V_G} = \frac{q - p\dfrac{dq}{dp}}{\sqrt{1+(\dfrac{dq}{dp})^2}}, \quad \tan\varphi = -\frac{dq}{dp}.$$

When $\eta = 0$

$$q = \frac{1}{V_{S0}} \sqrt{\frac{p^2 V_{SX}^2}{p^2 V_{SX}^2 - 1}},$$

$$\frac{1}{V_G^{2/3}} = \frac{\sin^{2/3}(\varphi)}{V_{SX}^{2/3}} + \frac{\cos^{2/3}(\varphi)}{V_{S0}^{2/3}}$$

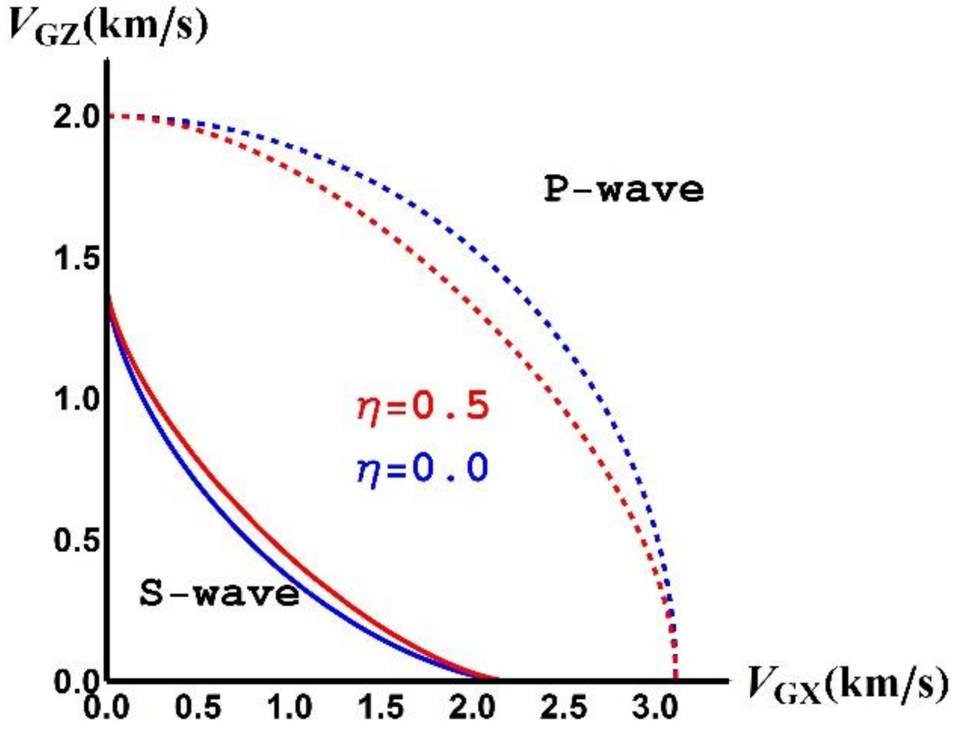
Astroid equation

Group velocity surface

Model

v_{P0} (km/s)	v_{Pn} (km/s)	η	V_{S0} (km/s)	V_{SX} (km/s)	z ·(km)
2.00	2.20	0.50	1.40	2.20	1.00

Group velocity surface



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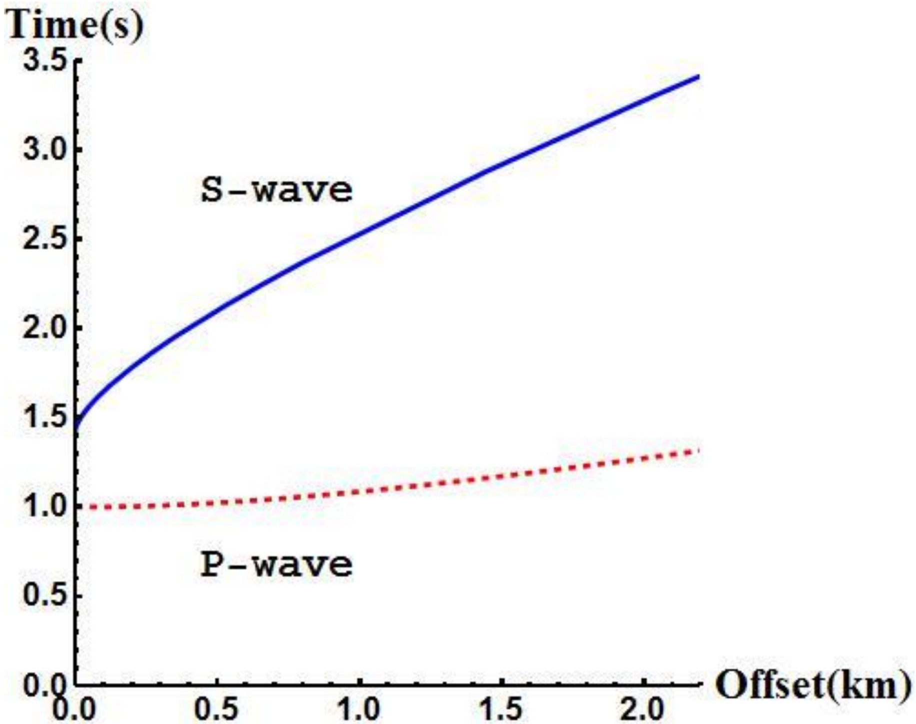
Traveltime equation

S-wave traveltime equation

$$t^{2/3}(x) = t_{S0}^{2/3}(1 + \tilde{x}^{2/3} + \frac{2}{3}\eta\tilde{x}^{4/3} + \ldots)$$, where $\tilde{x} = x / t_{S0}V_{Sn}$ and $V_{Sn} = V_{SX}\sqrt{1+2\eta}$

when $\eta = 0,$ $t^{2/3}(x) = t_{S0}^{2/3}(1 + \tilde{x}^{2/3})$

traveltime curve

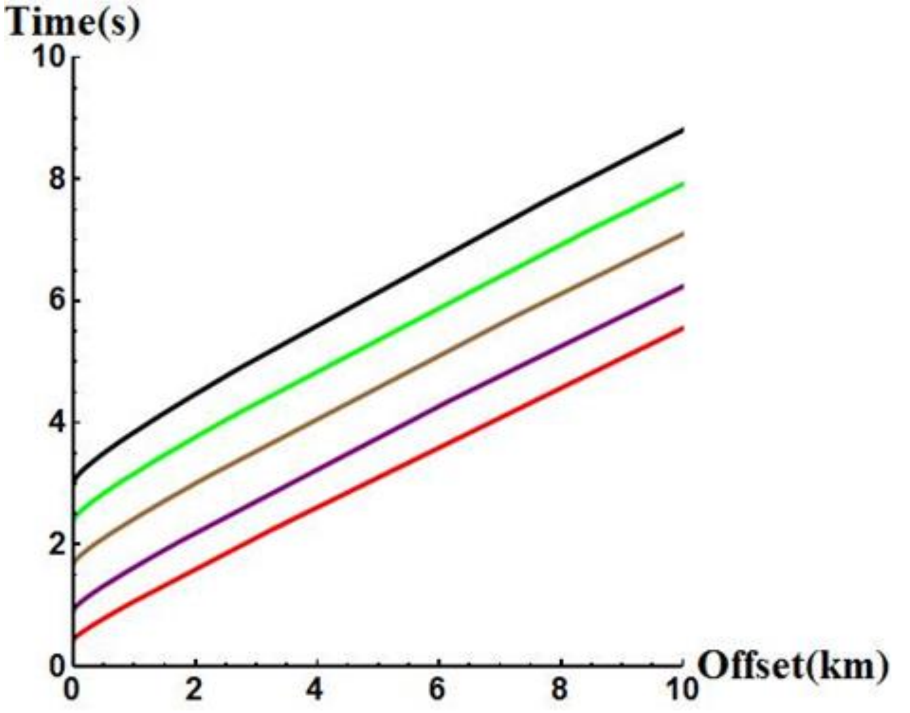


Traveltime equation

multi-layered model

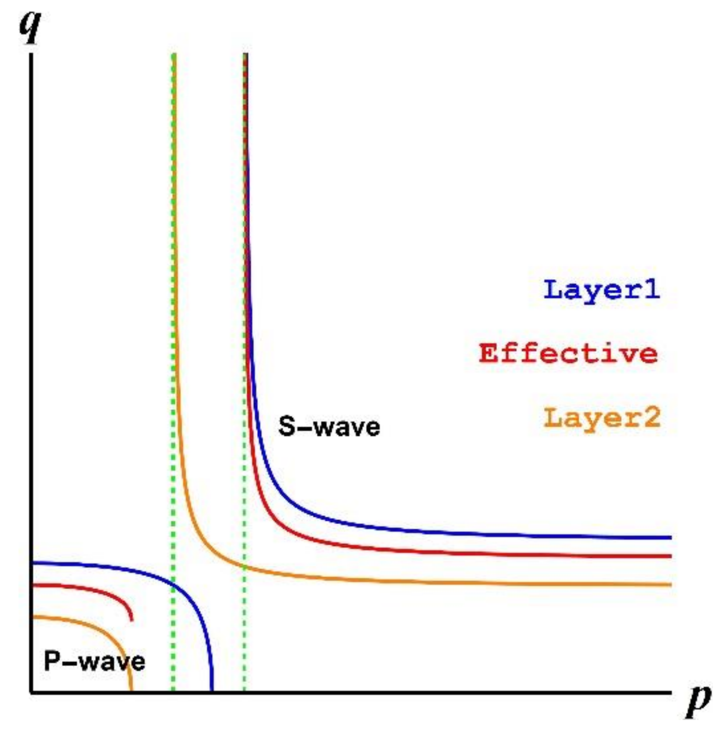
layer	z (km)	η	V_{S0} (km/s)	V_{Sn} (km/s)
1	0.30	1.92	1.40	4.8
2	0.40	2.53	1.63	5.4
3	0.70	4.06	1.85	7.2
4	0.60	5.31	1.68	7.8
5	0.50	3.42	1.60	5.8

reflected wave traveltime curves from the bottom of each layer



S-wave in acoustic VTI medium avoids the events crossing at the large offset

Traveltime equation



two-layered model slowness surface

at large offset

P-wave: $p_{eff} \le p^{k}_{\min}$

S-wave: $p_{eff} \ge p^{k}_{\max}$

$$p = \frac{dt}{dx}$$

P-wave has crossing events at large offset

In acoustic VTI, S-wave avoids the events crossing at the large offset

Traveltime equation

P-wave Dix type equations $V^{\Sigma}_{P0},\ V^{\Sigma}_{Pn},\ \eta^{\Sigma}_{P}$

$$t^{\Sigma}_{P0}V^{\Sigma}_{P0} = \sum t_{P0,j}v_{P0,j},$$
$$t^{\Sigma}_{P0}(V^{\Sigma}_{Pn})^2 = \sum t_{P0,j}v^2_{Pn,j},$$
$$t^{\Sigma}_{P0}(V^{\Sigma}_{Pn})^4(1+8\eta^{\Sigma}_{P}) = \sum t_{P0,j}v^4_{Pn,j}(1+8\eta_j)$$

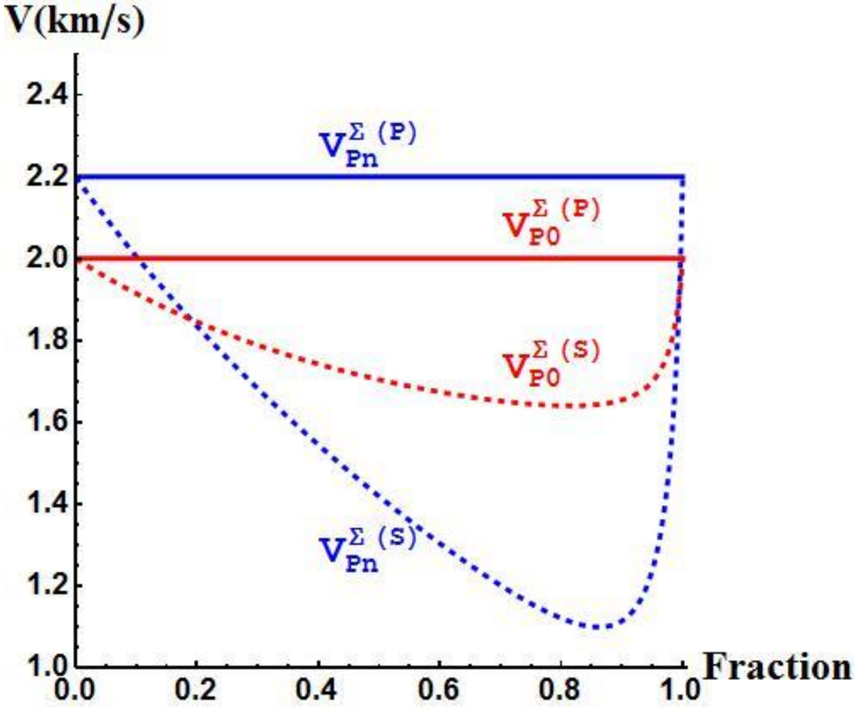
S-wave Dix type equations $V^{\Sigma}_{S0},\ V^{\Sigma}_{Sn},\ \eta^{\Sigma}_{S}$

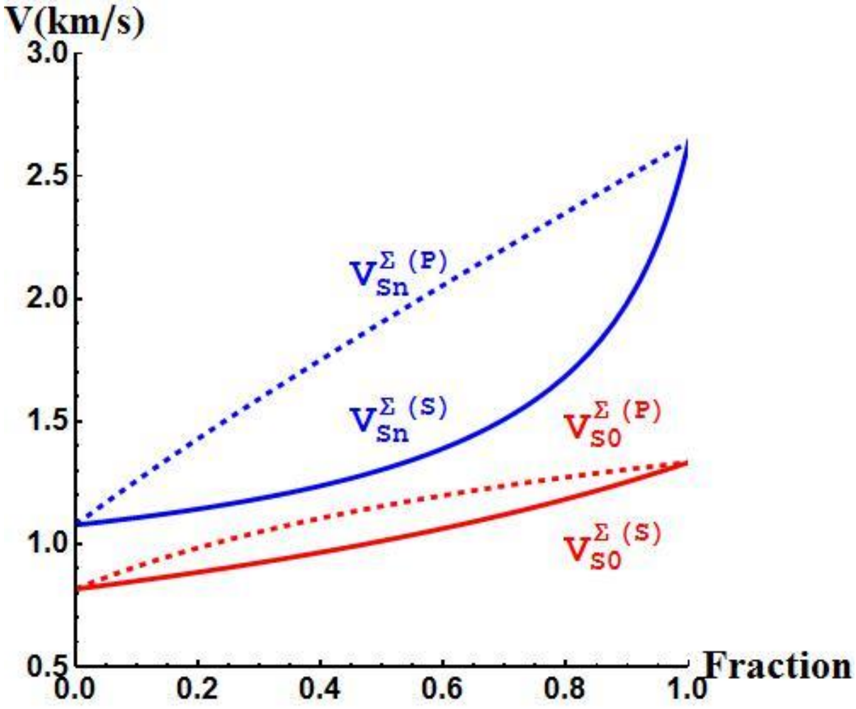
$$t^{\Sigma}_{S0}V^{\Sigma}_{S0} = \sum t_{S0,j}V_{S0,j},$$
$$\frac{t^{\Sigma}_{S0}}{(V^{\Sigma}_{Sn})^2} = \sum \frac{t_{S0,j}}{V^2_{Sn,j}},$$
$$\frac{t^{\Sigma}_{S0}(3+8\eta^{\Sigma}_{S})}{(V^{\Sigma}_{Sn})^4} = \sum \frac{t_{S0,j}(3+8\eta_j)}{v^4_{Sn,j}}$$

Two-layered model

layer	v_{P0} (km/s)	v_{Pn} (km/s)	η	V_{S0} (km/s)	V_{Sn} (km/s)
1	2.00	2.20	0.10	0.82	1.08
2	2.00	2.20	0.40	1.34	2.64

Traveltime equation

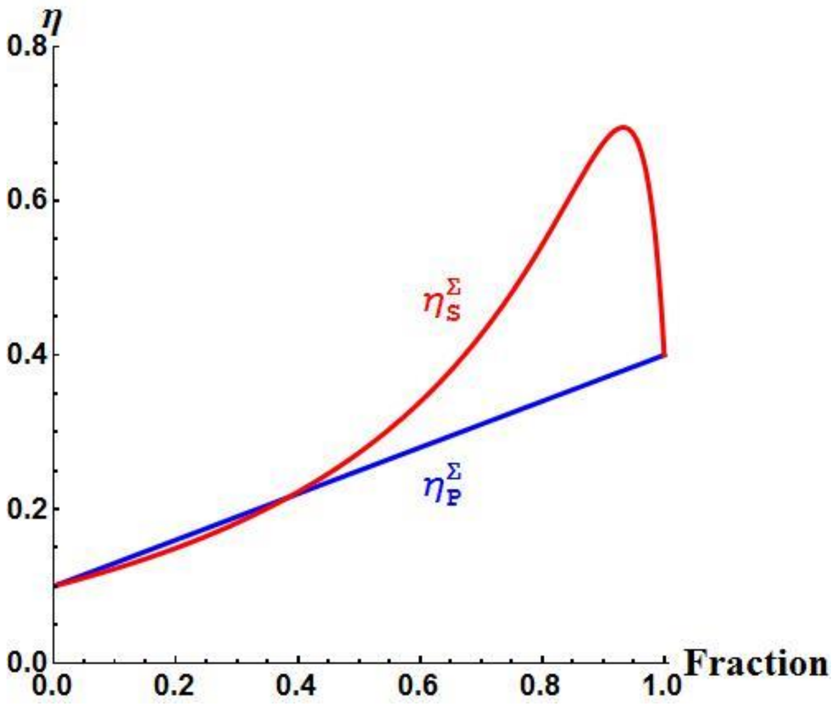




P-wave effective parameters
Solid line: P-wave Dix equation
Dashed line: S-wave Dix equation

S-wave effective parameters
Dashed line: P-wave Dix equation
Solid line: S-wave Dix equation

Traveltime equation



effective η
Red line: P-wave Dix equation
Blue line: S-wave Dix equation

Effective acoustic VTI layer $V_{S0}^\Sigma,\ V_{Sn}^\Sigma,\ \eta_S^\Sigma \neq V_{P0}^\Sigma,\ V_{Pn}^\Sigma,\ \eta_P^\Sigma$

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Relative geometrical spreading

Relative geometrical spreading $L = \Omega * Ln$

radiation pattern $\Omega = \cos\varphi$

geometrical spreading factor $Ln = (Abs[\frac{x}{p}\frac{dx}{dp}])^{1/2}$

geometrical spreading factor as function of normalized offset

P-wave $Ln = t_{P0}V_{Pn}^2[1 + (1+8\eta)\tilde{x}^2 + ...]$

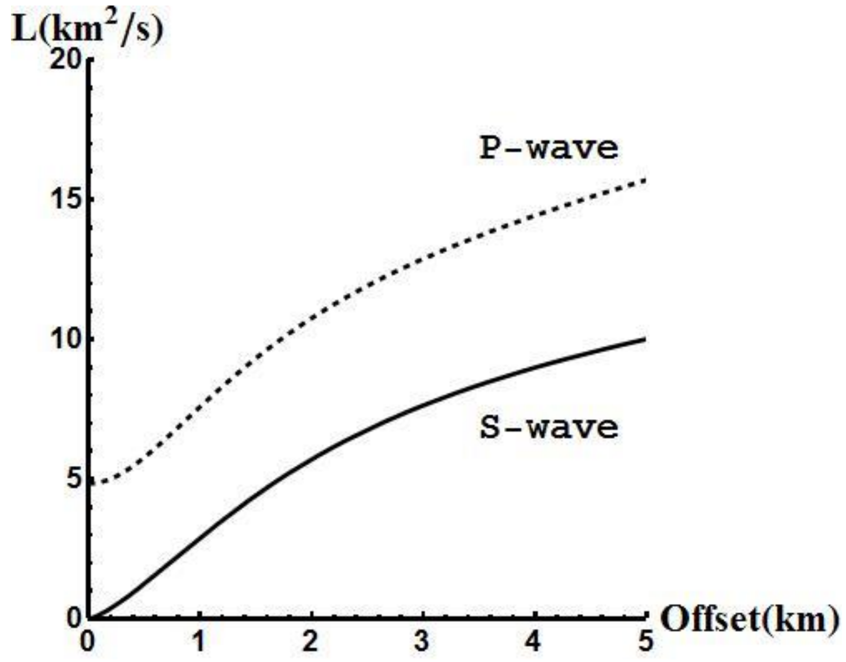
S-wave $Ln = t_{S0}V_{Sn}^2[\sqrt{3}\tilde{x}^{4/3} - \frac{\eta(3+4\eta)\tilde{x}^{8/3}}{3^{4/3}} + ...]$

Relative geometrical spreading

Model

v_{P0} (km/s)	v_{Pn} (km/s)	η	V_{S0} (km/s)	V_{SX} (km/s)	z ·(km)
2.00	2.20	0.50	1.40	2.20	1.00

relative geometrical spreading



the zero offset point represents a cusp point

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Conclusion

- In acoustic VTI medium, P- and S-waves are defined by the same slowness surface equation
- New parameters are defined for S-wave
- S-wave group velocity surface is quasi-astroid shaped.
- We derive the S-wave traveltime equation
- S-wave relative geometrical spreading is zero at the zero offset

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Thanks for your attention