HIGH-RESOLUTION COMPLEX TIME-FREQUENCY ANALYSIS

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IGEO

Overview

- Introduction
- Classical Methods
- Wigner-Ville Distribution
- Complex Short Term Auto-Regressive Methods
- Numerical Results
- Conclusions
- Selected References

Introduction

- Time-Frequency Analysis in
  - Seismics
  - Seismology
  - Volcanology
  - Astrophysics
- Given a time signal, create a time-varying frequency field

- Classical methods use the real signal.
- Here:
  - Use the analytic signal or complex trace;
  - Short-time window;
  - Burg, Marple and Morf algorithms.

Classical Methods

- Spectrogram or Short-Term Fourier Tranform(STFT):
  - DFT, possibly weighed, of a short segment of a real signal;
  - No model for the data, but low resolution and leakage

Classical Methods

- Short-Term Autoregressive model (STAR):
  - Estimate an AR model from a short segment of a real signal spectrum;
  - Use inverse of the AR model;
  - Attain higher resolution;
  - Must determine AR model order.

Wigner-Ville Distribution

- Works with analytic signal (complex trace).
- Shows interference between harmonic signals due to the quadratic nature of the method.
- To reduce interference use a maximum entropy method (Burg) to compute a complex prediction error filter to predict the kernel in WV distribution (MEWV).

High-Resolution Complex Time-Frequency Analysis

- Uses the analytic signal;
- Computes a complex AR model in a short-time window (CSTAR);
- Uses only data inside the window.
- Makes no assumptions about the signal outside the window.

High-Resolution Complex Time-Frequency Analysis

- Algorithms:
  - Burg,
  - Marple
  - Morf
- All minimize the sum of forward and backward prediction error energies.

About Algorithms

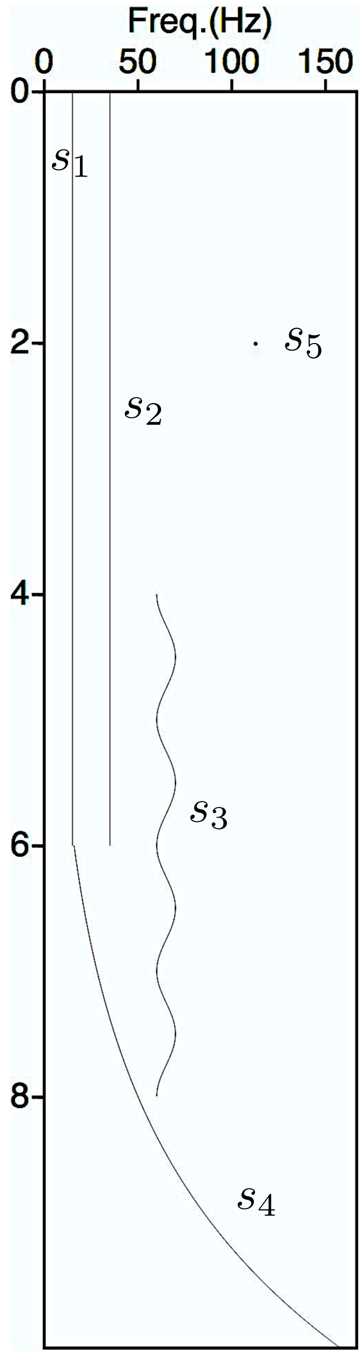
- Burg and Marple set the forward and backward prediction error operator(PEO) equal.
- Burg minimizes with respect to reflection coefficient in the Levinson algorithm.
- Marple minimizes with respect to the coefficients in the PEO.
- Morf minimizes with respect to separate forward and backward PEO's.

Synthetic Data

- Sum of five Signals:

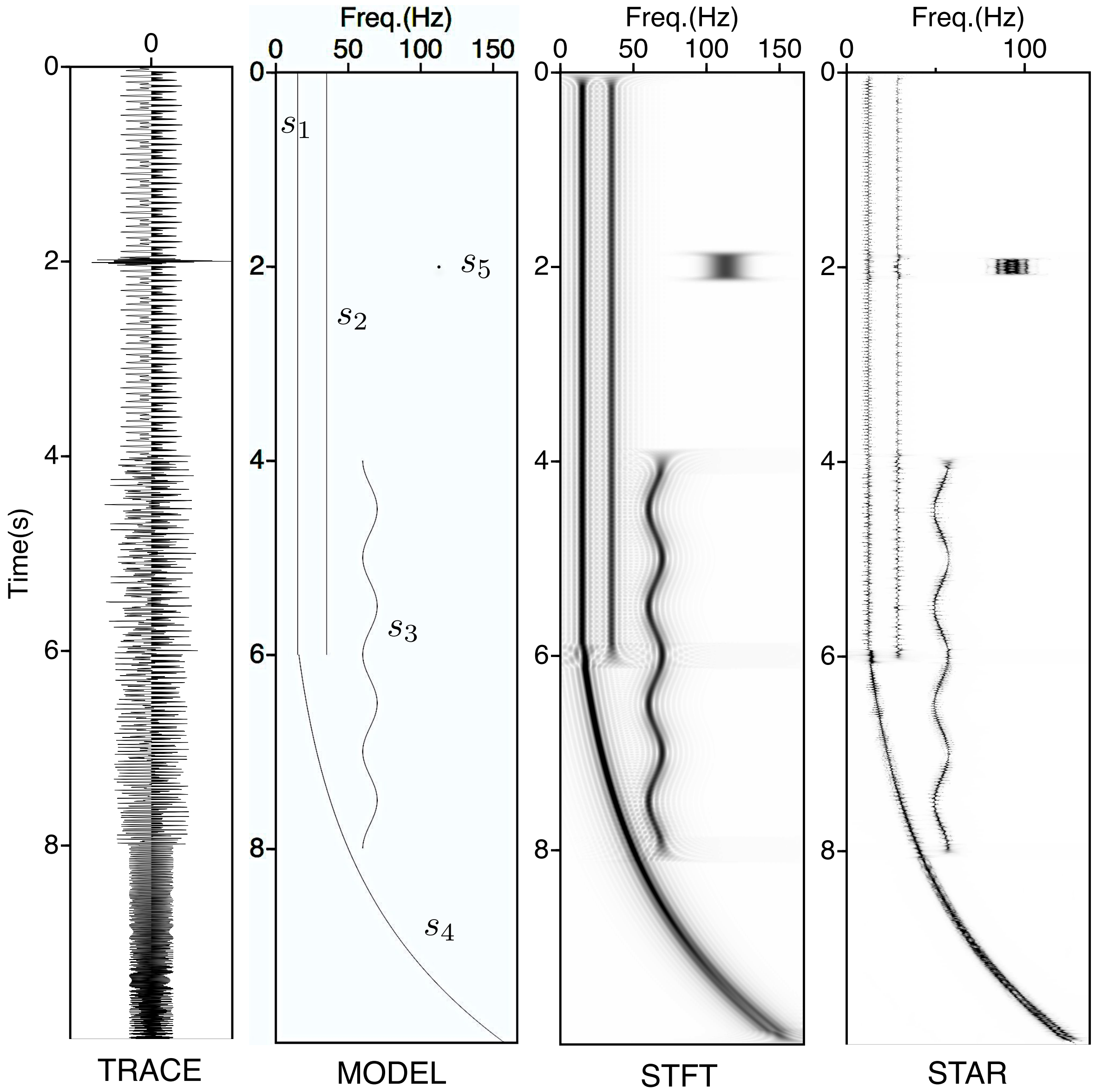
$$
\begin{aligned}
s_1(t) &= 0.8\cos(30\pi t) & 0 \le t \le 6s \\
s_2(t) &= 0.6\cos(70\pi t) & 0 \le t \le 6s \\
s_3(t) &= 0.7\cos(130\pi t + 5\sin(2\pi t)) & 4s \le t \le 8s \\
s_4(t) &= \sin\left(\frac{8\pi 100^{t/8}}{\log(100)}\right) & 6s < t \le 10s \\
s_5(t) &= 3e^{-1250(t-2)^2}\cos 710(t-2) & 0 \le t \le 10s
\end{aligned}
$$

- Two harmonic signals s_1 and s_2 (*f=15 and 35Hz*);
- Frequency-modulated harmonic, s_3 (*f=65Hz*);
- Gliding harmonic, s_4 (*f=35 to 158Hz*);
- A Morlet wavelet, s_5, at *113Hz*.

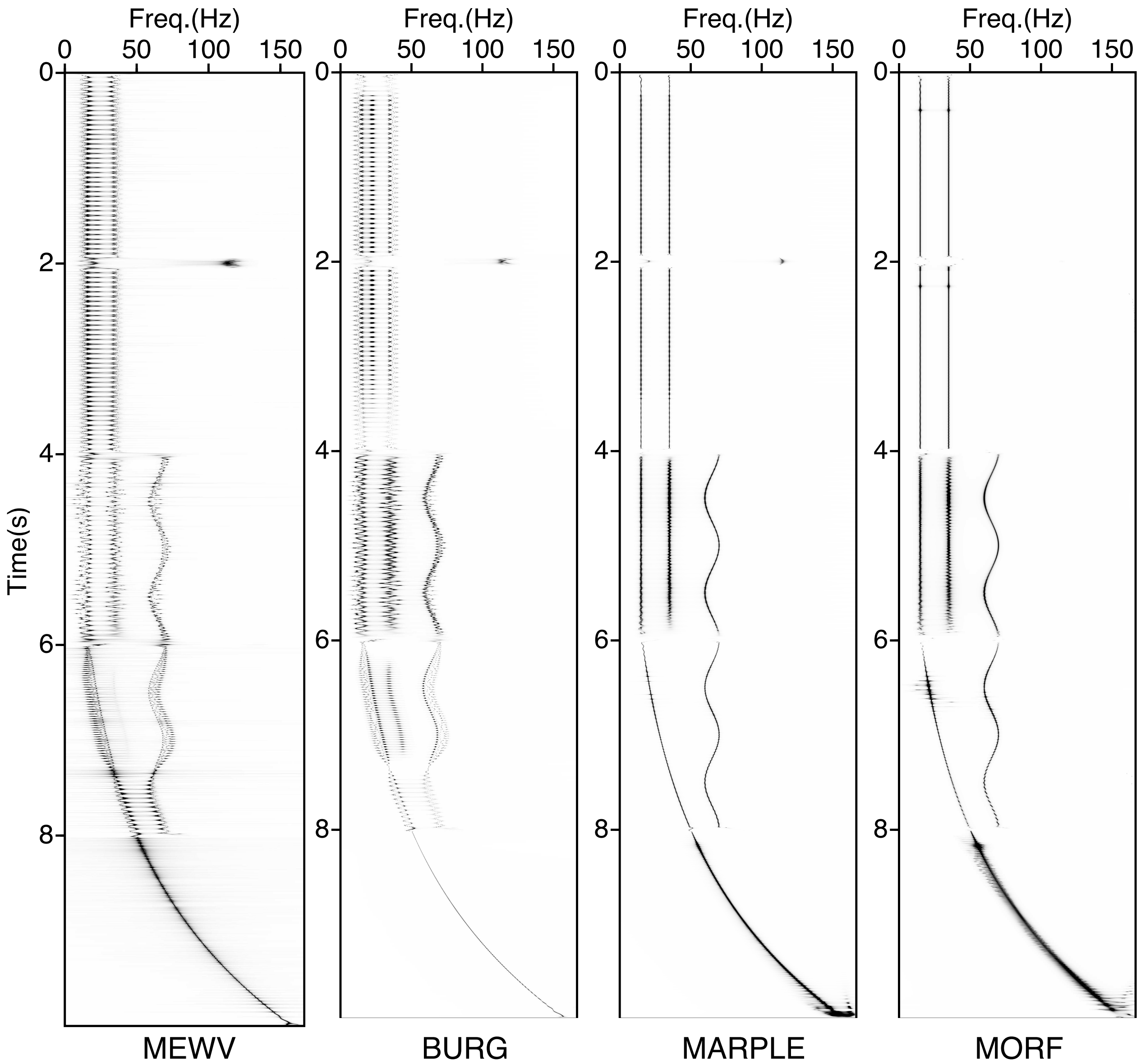


MODEL

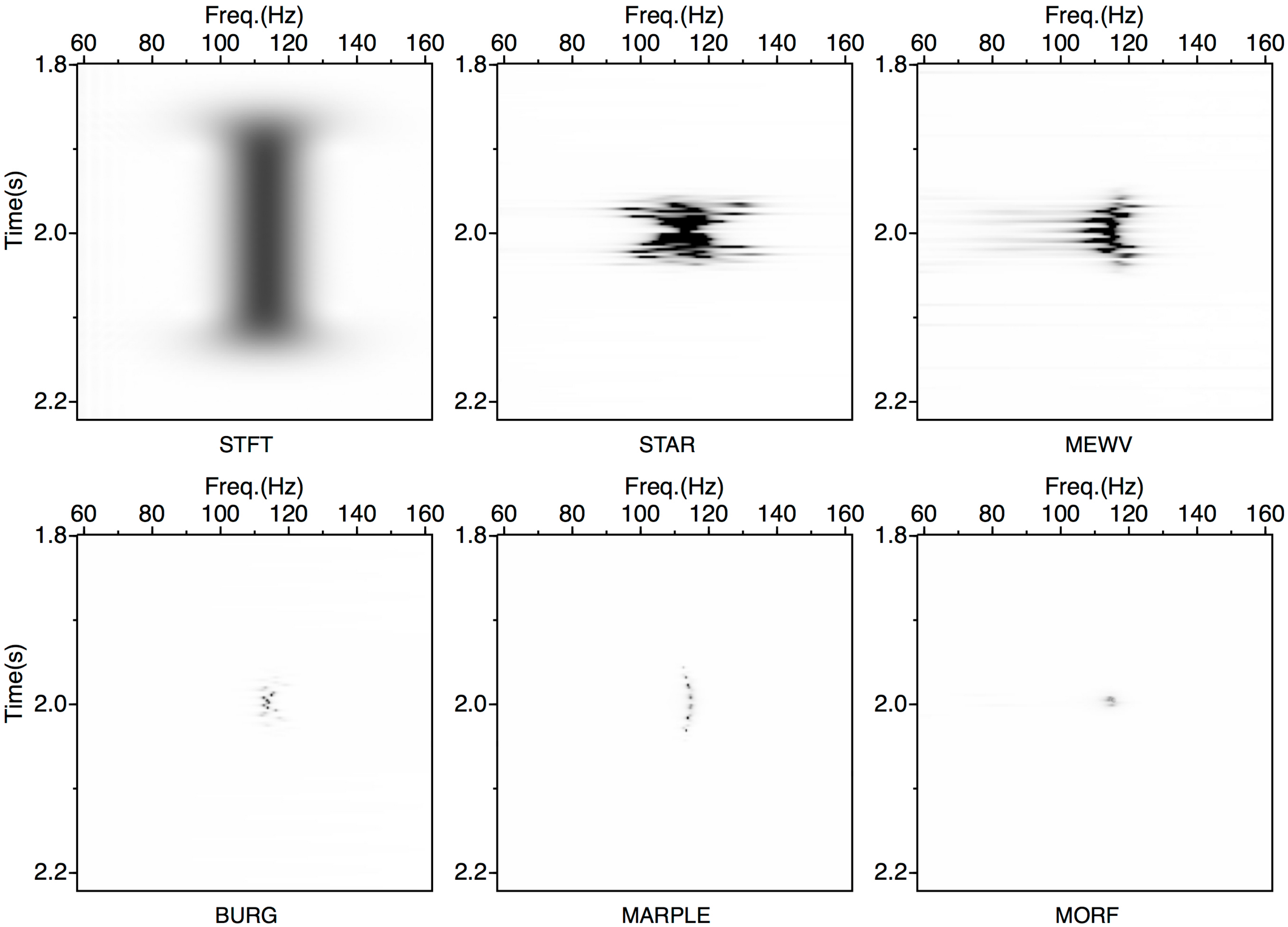
Synthetic Data - Classical Methods



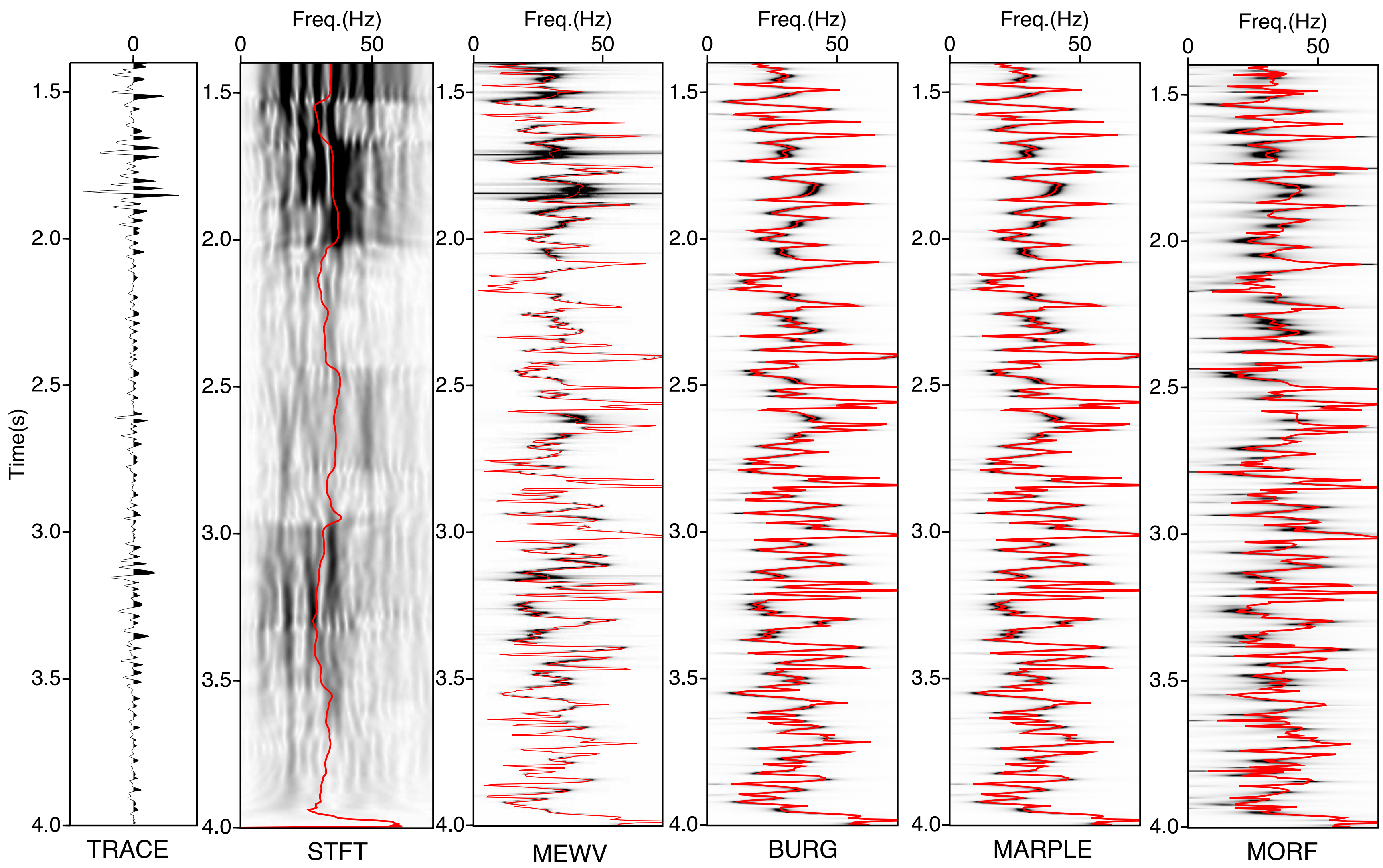
Synthetic Data - CSTAR



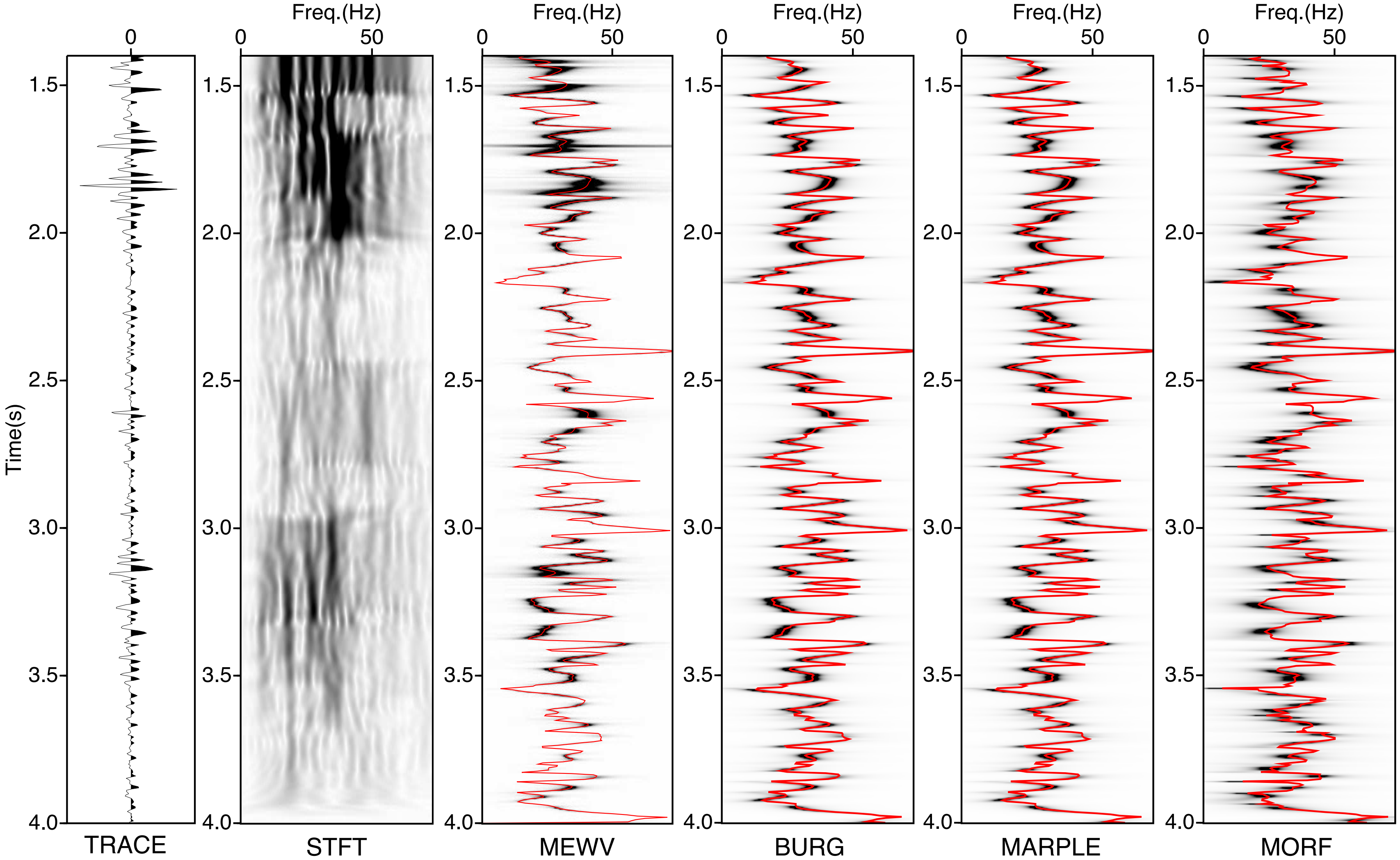
Enlarged Morlet wavelet



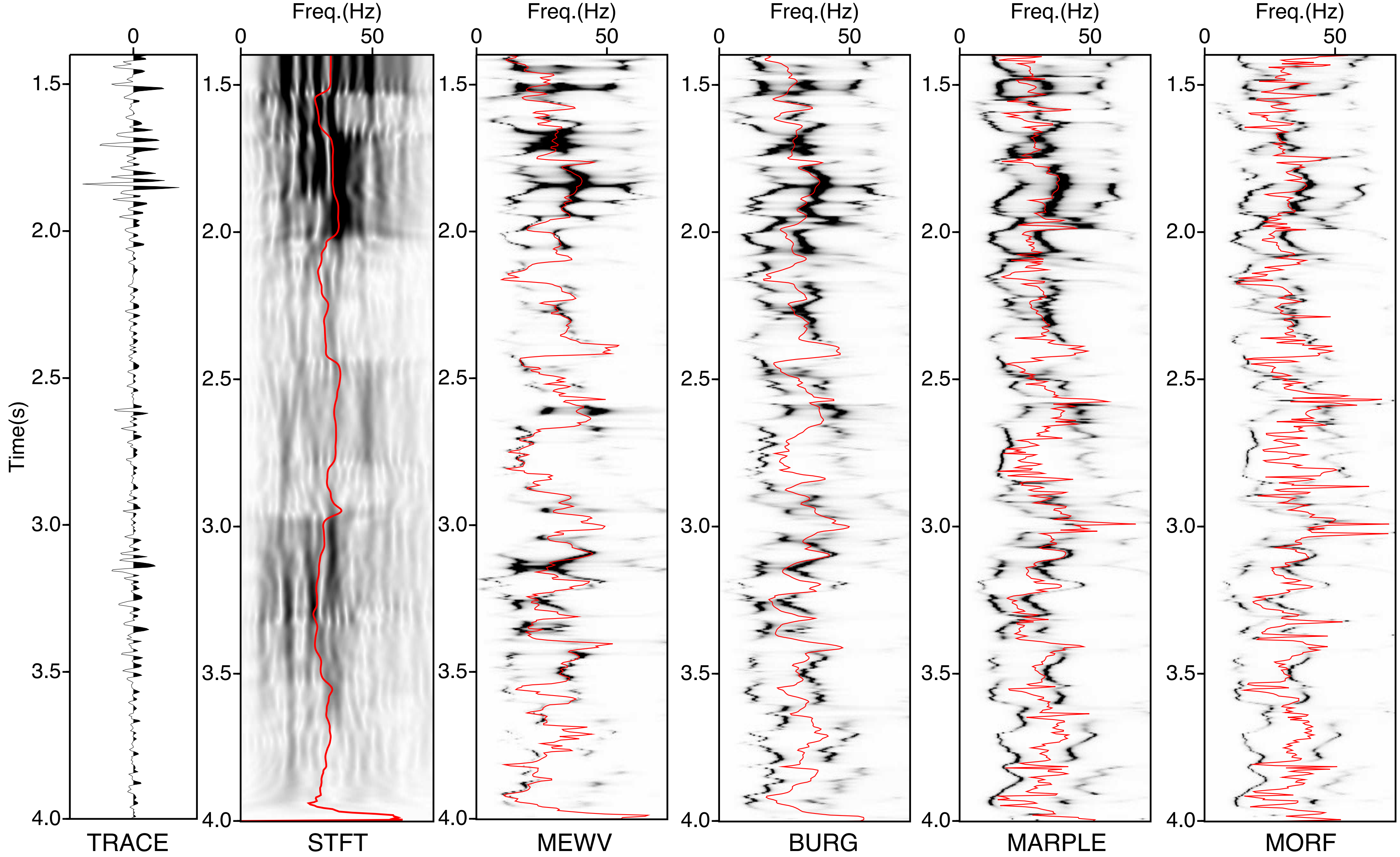
Marine Data, $L_W=3$, $L_C=1$



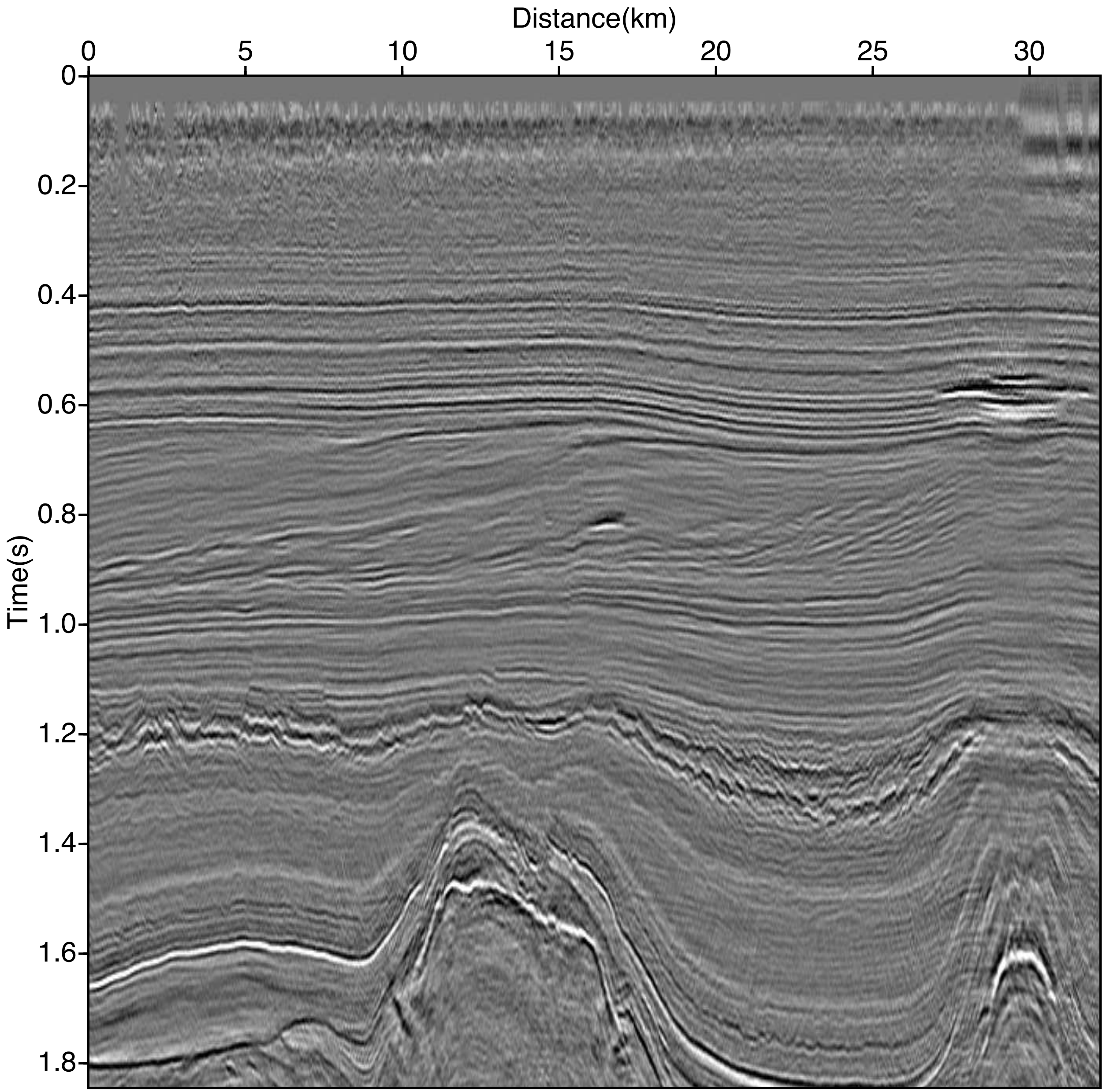
Marine Data, $L_W=5$, $L_C=1$



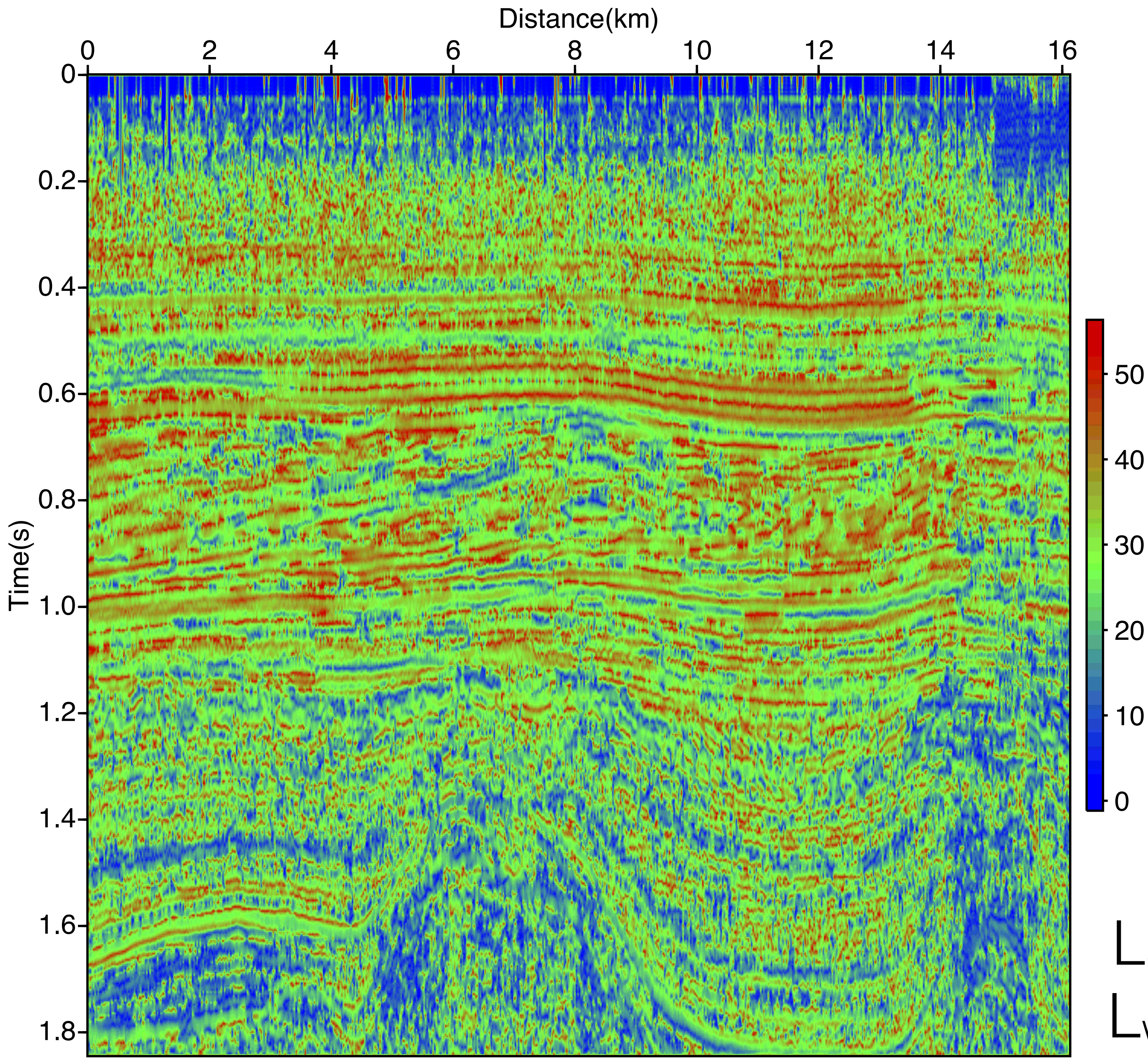
Marine Data, L_W=13, L_C=3



Marine Seismic Section



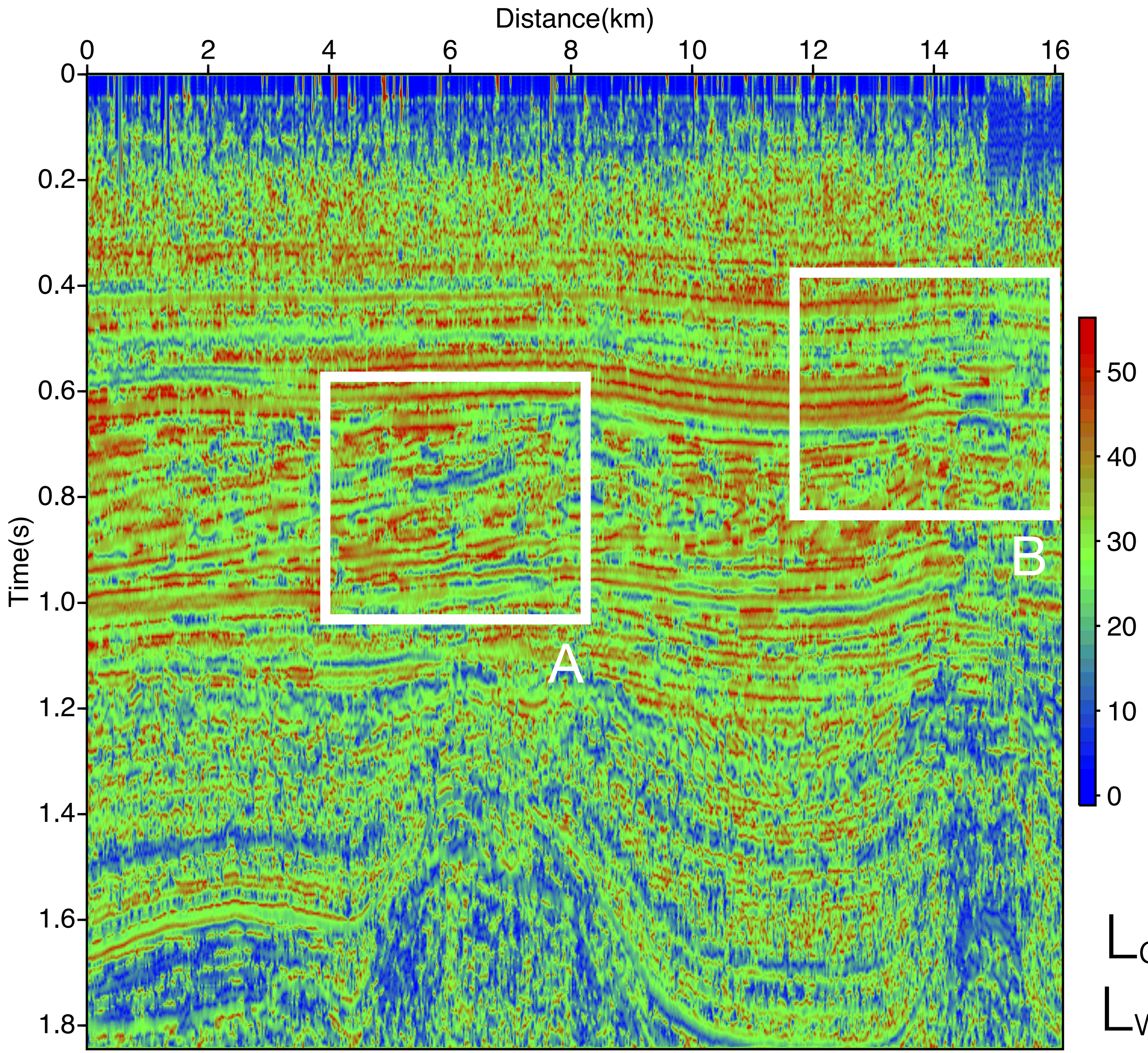
Frequency Field for Marine Seismic



$L_C=1$

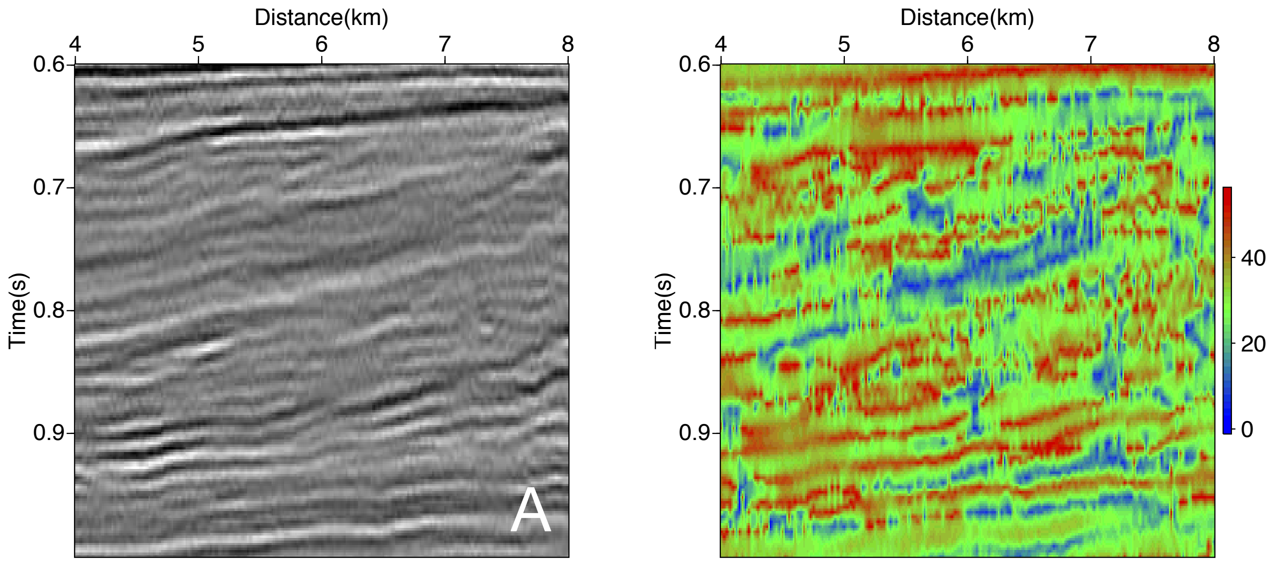
$L_W=3$

Frequency Field for Marine Seismic

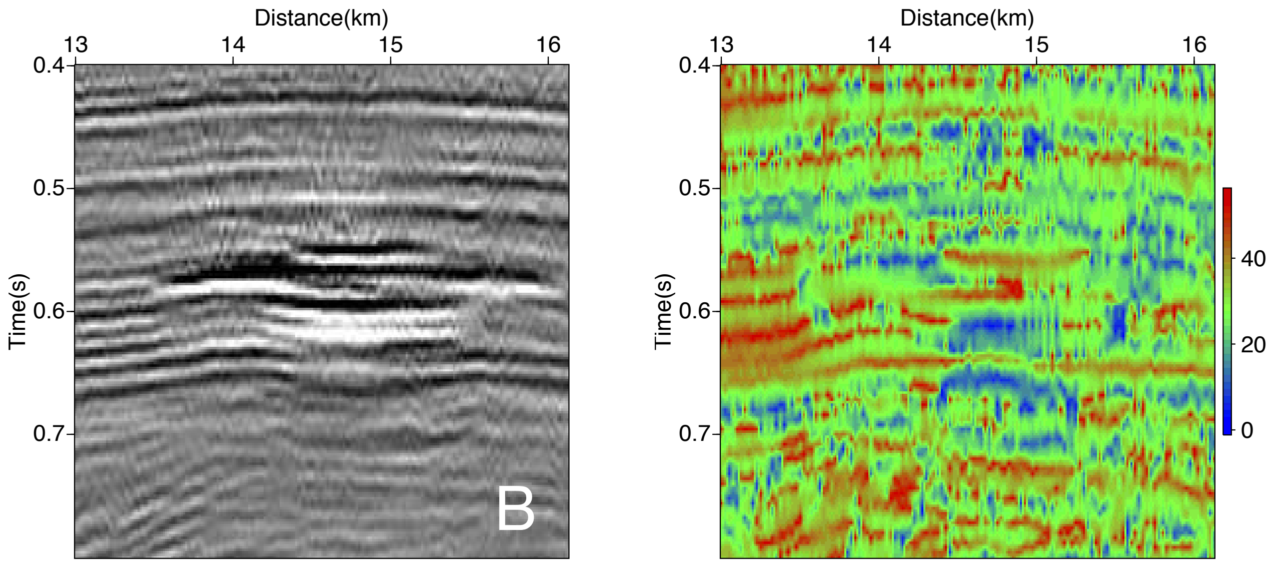


$L_C=1$
$L_W=3$

Detail of Seismic Section

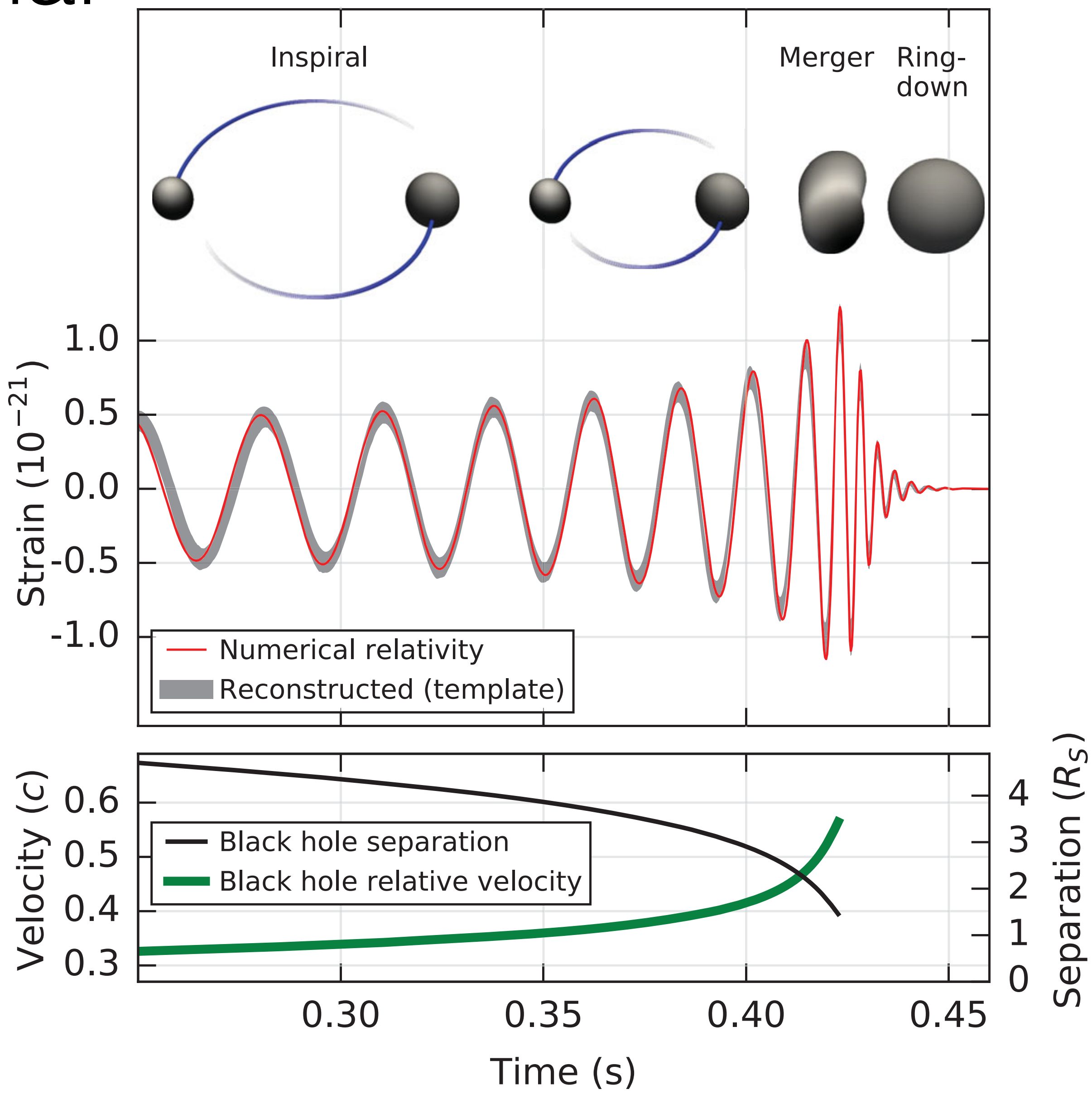


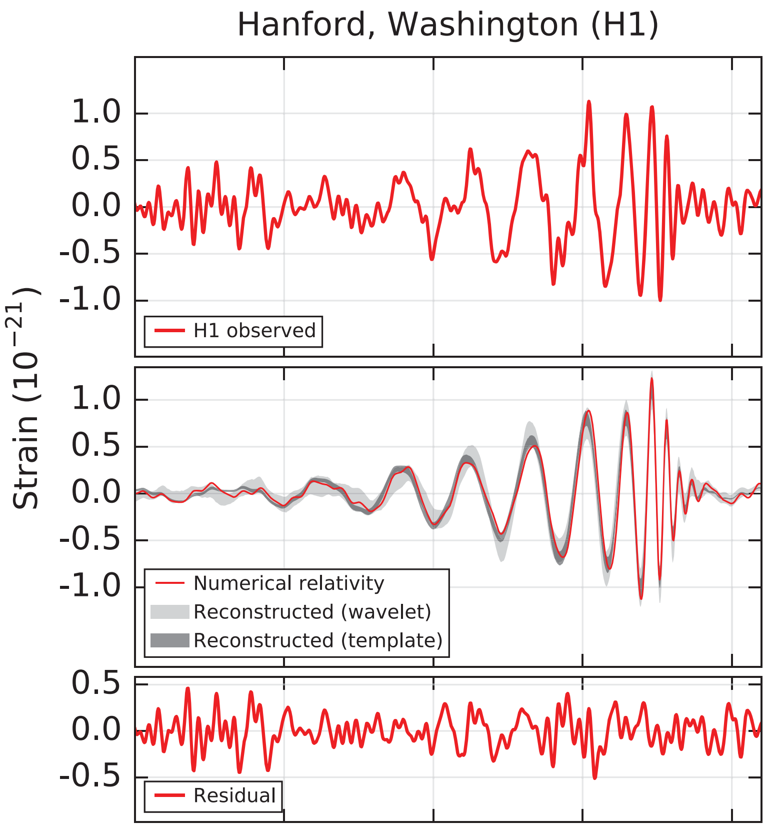
$L_C=1$
$L_W=3$



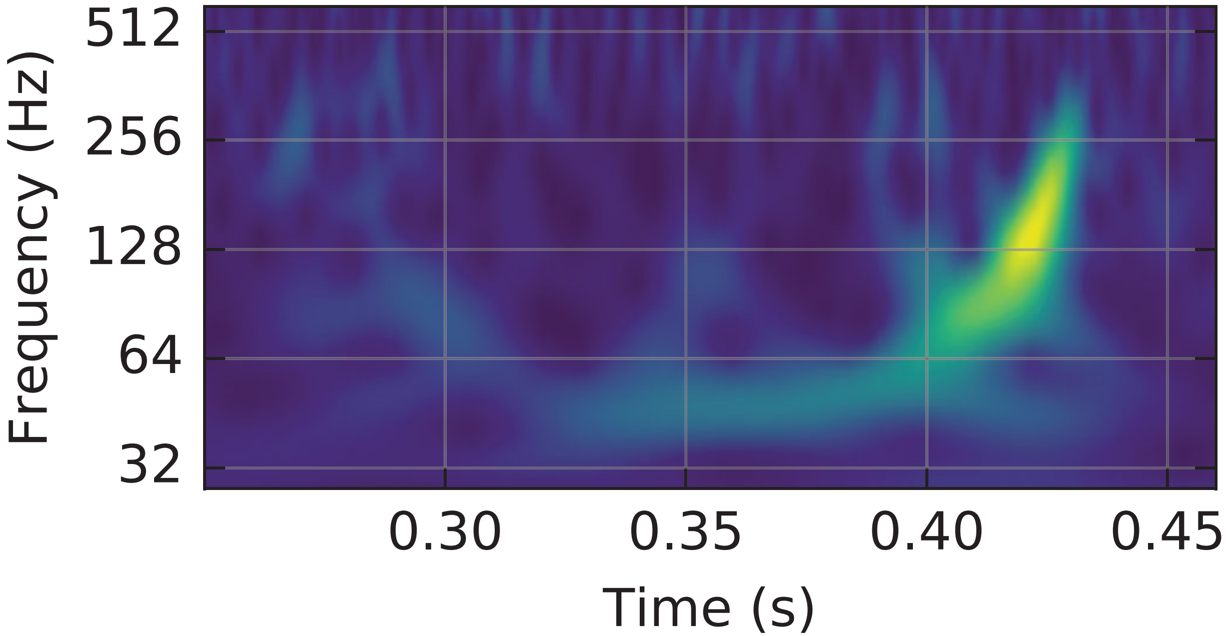
$L_C=1$
$L_W=3$

LIGO Gravitational Wave Model

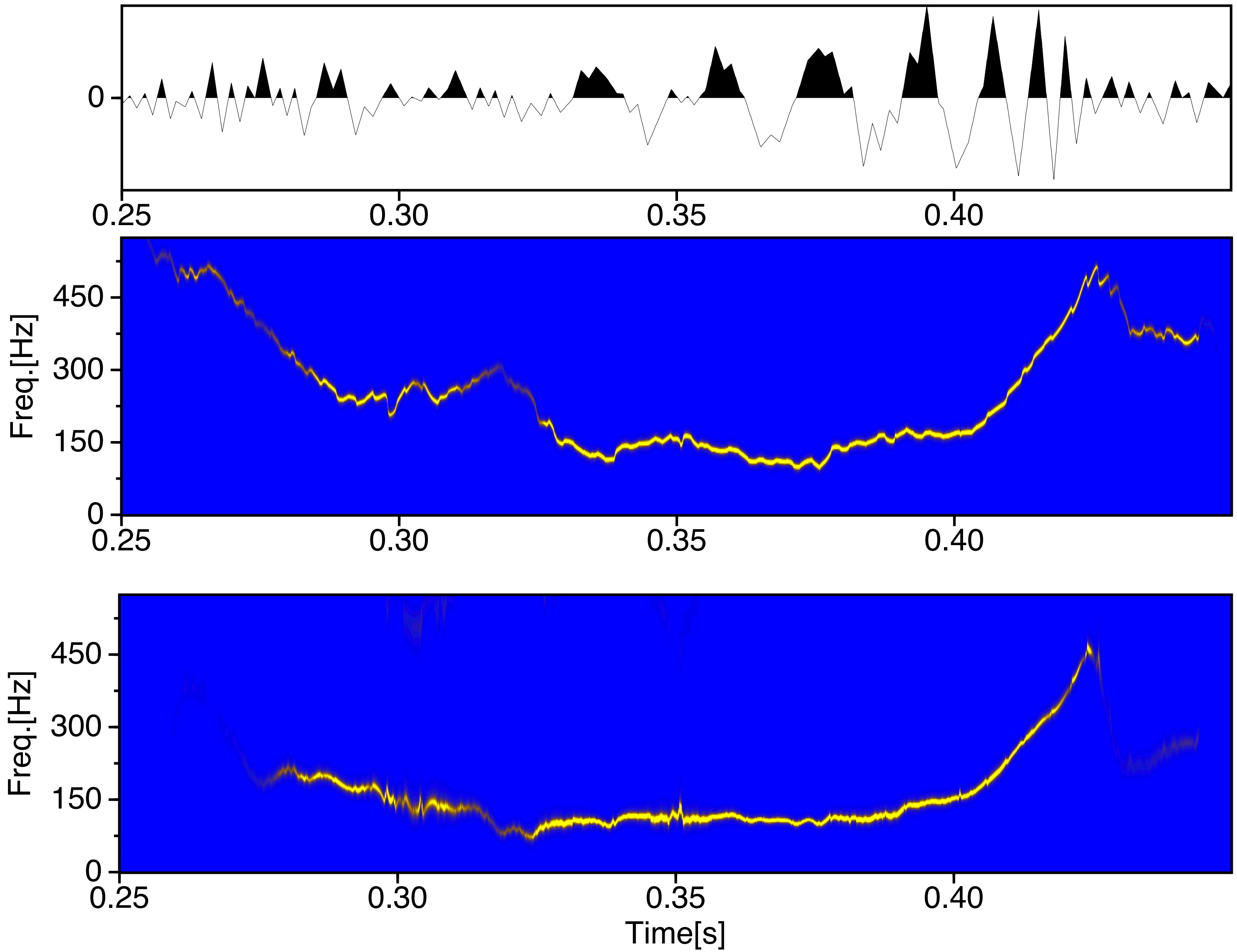




LIGO
Hanford data



Power Spectrum of LIGO Grav. Wave Detection with CSTAR



$L_C=1$
$L_W=241$

$L_C=5$
$L_W=241$

Conclusions

- CSTAR is a high-resolution method for time-frequency analysis;
- The Marple algorithm is recommended.
- Good results for:
  - Synthetic data,
  - Marine seismic data,
  - LIGO data.

Selected References

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