Combining wave equation velocity analysis and full waveform inversion for improved 3D elastic parameter estimation

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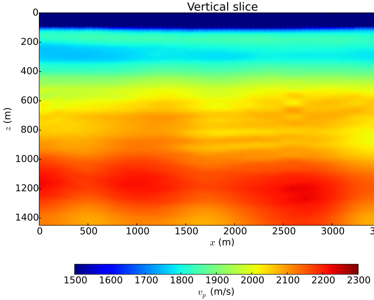
Background

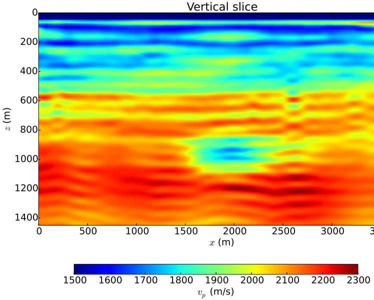
- Compared to other parameter estimation methods, full waveform inversion (FWI) includes different important wave phenomena through the wave equation that potentially can yield solution closer to the real world parameters
- The increase in computational power leads to an increase in possible problem sizes and type of wave phenomena included in the modeling and inversion, as well as the type of method that we can use in an imaging work flow
- The applications of FWI on synthetic and field data the last decade have proved that FWI is a promising method for parameter model estimation [Virieux and Operto, 2009]

Objective

Want a robust method for estimating the initial model for FWI that can yield high resolution inverted subsurface parameter models



Conventional FWI



WEMVA+FWI

Outline

- Theory
- Work flow
- Results
- Conclusion

Theory

Reverse-time migration

Acoustic reverse-time migration (RTM)

Pre stack depth migration = Wave field reconstruction + Imaging condition

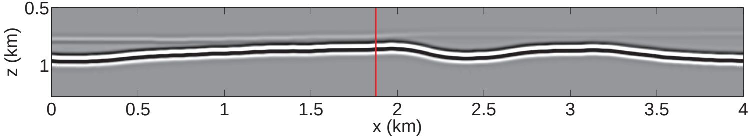
Wave field reconstruction in RTM

$$\left[\frac{1}{v_p^2(\boldsymbol{x})}\frac{\partial^2}{\partial t^2} - \nabla^2\right] w^s(\boldsymbol{x}, t; s) = f(\boldsymbol{x}, t; s)$$

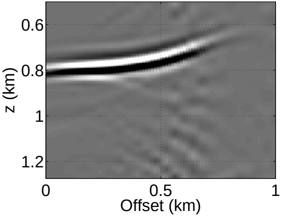
$$\left[\frac{1}{v_p^2(\boldsymbol{x})}\frac{\partial^2}{\partial t^2} - \nabla^2\right] w^r(\boldsymbol{x}, t; s) = d(\boldsymbol{x}, T - t; s)$$

Imaging condition

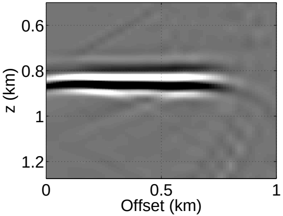
$$\mathcal{R}(\boldsymbol{x}; s) = \int_0^T w^s(\boldsymbol{x}, t; s) w^r(\boldsymbol{x}, T - t; s)$$



Wrong velocities



Correct velocities



Wave equation migration velocity analysis by stack-power

Stack-power

Find a P-wave velocity model that maximizes the stack over sources of the depth migrated image

Define $\mathcal{J}(\boldsymbol{v})$ as the stack of the migrated image. WEMVA is then the problem

$$\arg\max_{\boldsymbol{v}} \mathcal{J}(\boldsymbol{v}) = \sum_{s} \sum_{\boldsymbol{x}} \frac{\partial}{\partial x_3} \left[\mathcal{R}(\boldsymbol{x}, \boldsymbol{v}; s)\right]^2$$

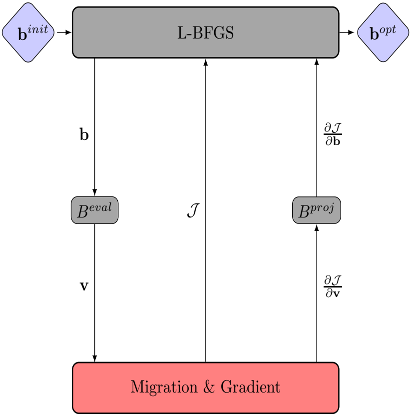
Solved using an iterative method

$$\boldsymbol{v}_{k+1} = \boldsymbol{v}_k + \alpha_k \boldsymbol{g}_k,$$

$\boldsymbol{v}_k$	model at iteration k
$\boldsymbol{g}_k$	gradient of $\mathcal{J}(\boldsymbol{v})$ at iteration k
α_k	step length at iteration k

Organization of WEMVA

- Using an initial model ($\boldsymbol{b}^{init}$), migrate the seismic data, and stack to form an image
- Evaluate $\mathcal{J}$ by computing the stack-power of the stacked image
- Evaluate $\partial\mathcal{J}/\partial\boldsymbol{v}$ by computing the gradient of $\mathcal{J}$
- Finally, project the gradient in a tri-cubic B-spline basis



Full waveform inversion

Goal

Find a parameter model from which it is possible to create synthetic data that is close to some measured data

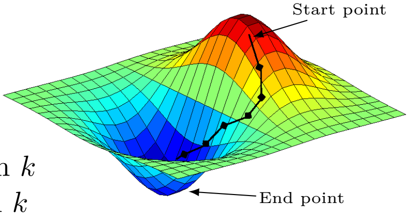
Define $S(\boldsymbol{m})$ as the measure between synthetic and measured data. The FWI is then the problem

$$\arg\min_{\boldsymbol{m}} S(\boldsymbol{m})$$

Solved using an iterative method

$$\boldsymbol{m}_{k+1} = \boldsymbol{m}_k - \alpha_k \boldsymbol{H}_k^{-1} \boldsymbol{g}_k,$$

$\boldsymbol{m}_k$	model at iteration k
$\boldsymbol{g}_k$	gradient of $S(\boldsymbol{m})$ at iteration k
$\boldsymbol{H}_k$	Hessian of $S(\boldsymbol{m})$ at iteration k
α_k	step length at iteration k



FWI: Details

Normalized misfit functional [Raknes and Arntsen, 2014]:

$$S(\boldsymbol{m}) = \frac{1}{2} \sum_{s} \sum_{\boldsymbol{x}} \| \hat{\boldsymbol{u}}_{i,j}(\boldsymbol{x}; \boldsymbol{m}) - \hat{\boldsymbol{d}}_{i,j}(\boldsymbol{x}) \|_2^2$$

Has proved to be favorable when working with streamer data

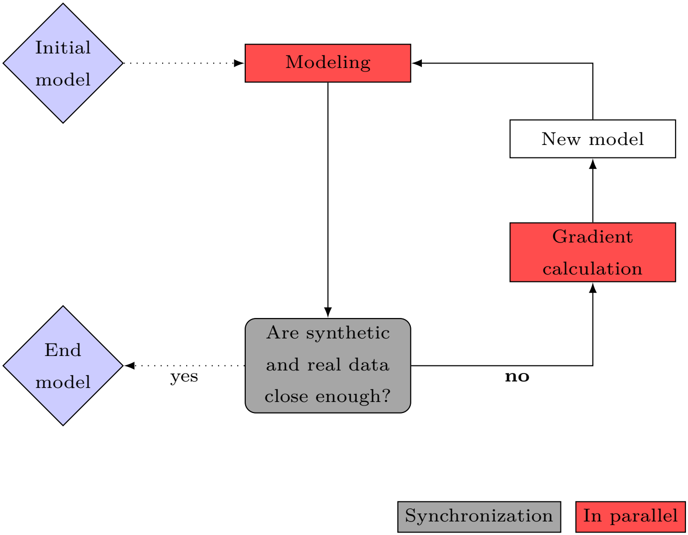
Minimization algorithm [Nocedal and Wright, 2006]:

- L-BFGS (quasi-Newton method)
- Using six gradients in approximating the inverse Hessian

Misc:

- Wave field reconstruction methods are used to reduce the storage needs in the computation of the gradient [Raknes and Weibull, 2016]

FWI: Schematic view



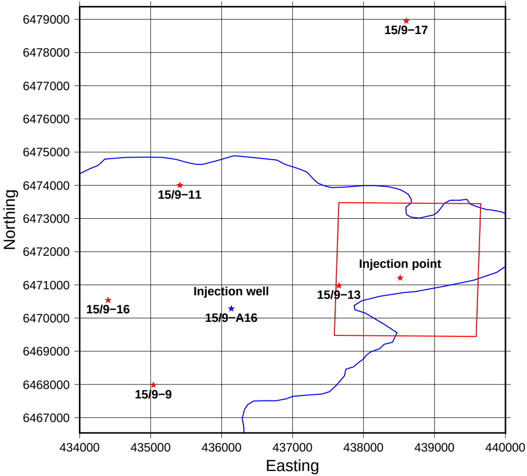
Work flow

Work flow

- Data preprocessing
- Surface related multiple elimination (SRME)
- WEMVA
- FWI

Data preprocessing

- Choosing a local area (red square)
- Regularizing the data into the numerical grid
- Filter the data to the frequency band 10.0–12.0 Hz



SRME

- WEMVA relies on a single scattering assumption
- To help make the data conform to a single scattering assumption we used SRME [Verschuur et al., 1992]
- SRME attenuates the surface related multiples, but does not remove inter bed multiples

WEMVA

- In the 1994 dataset WEMVA was run using pure stack-power maximization objective functional
- In the 2006 dataset WEMVA was constrained to co-depth the Base Utsira reflector to that of the 1994 image
- This produced kinematically accurate velocity models

FWI

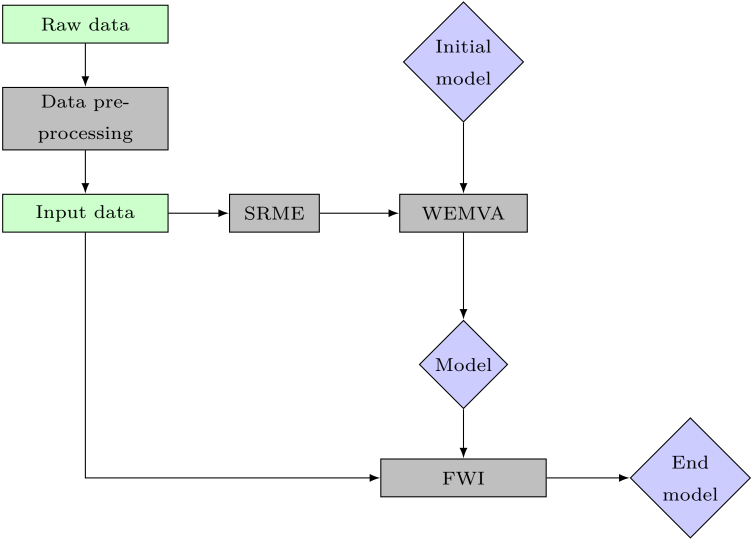
- Elastic isotropic 3D FWI
- Frequency band 10.0–12.0 Hz
- Divide the model into local models, each which covers one shot and the corresponding receivers with sufficient aperture
- Invert for the source wavelet
- Invert for v_p, link v_s and ρ with empirical relationships [Mavko et al., 2009]:

$$\rho = 310 v_p^{1/4}$$

$$v_s = \left(-790000 + 0.287 v_p^2 - (2.89 \times 10^{-8}) v_p^4\right)^{1/2}$$

$[\rho] = \mathrm{kg/m^3}$, $[v_p] = [v_s] = \mathrm{m/s}$

Work flow: Schematic view



Results

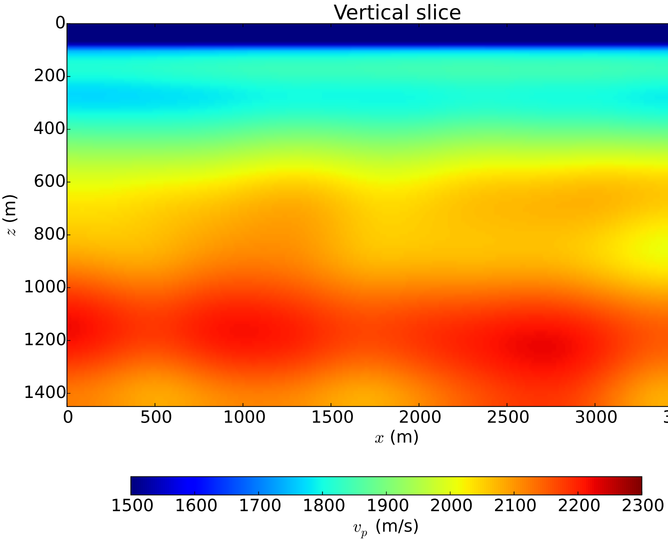
Results

Two approaches

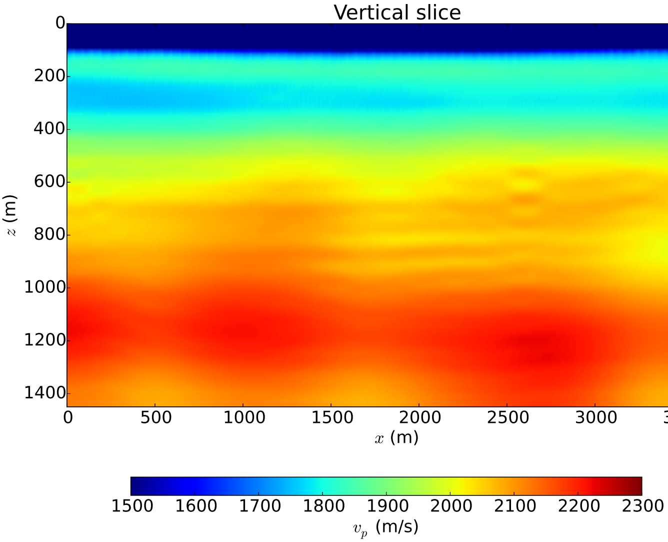
1. **Standard approach**
   The tomographic model is used as initial model for FWI, and a standard inversion run is performed
2. **WEMVA+FWI**
   The tomographic model is used as initial model for WEMVA. The resulting WEMVA model is used as initial model for FWI, and a standard inversion run is performed

The input data and the way the inversion runs are performed are identical, so the *only* difference is the starting model.

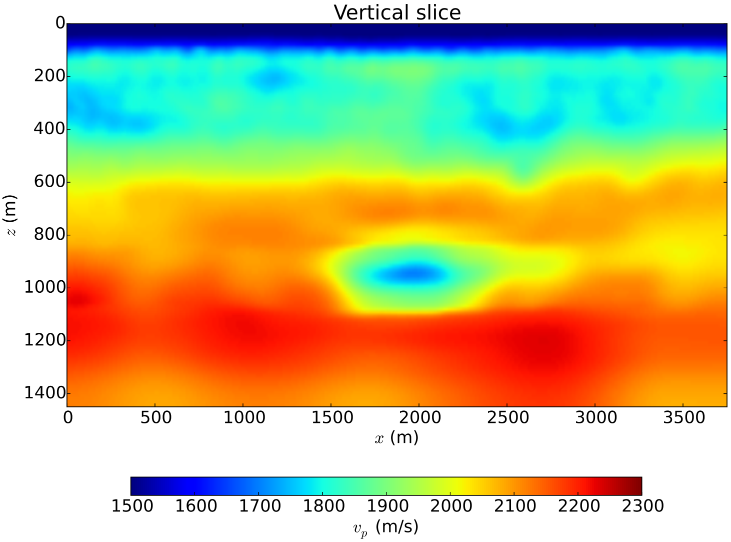
Initial model



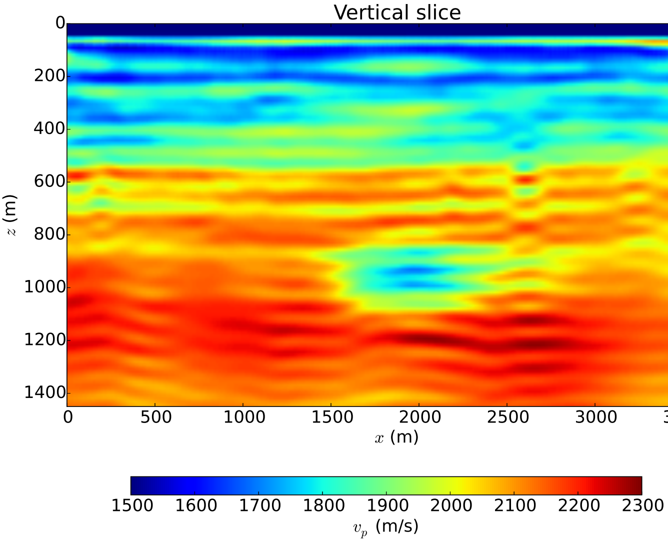
FWI model: Standard approach



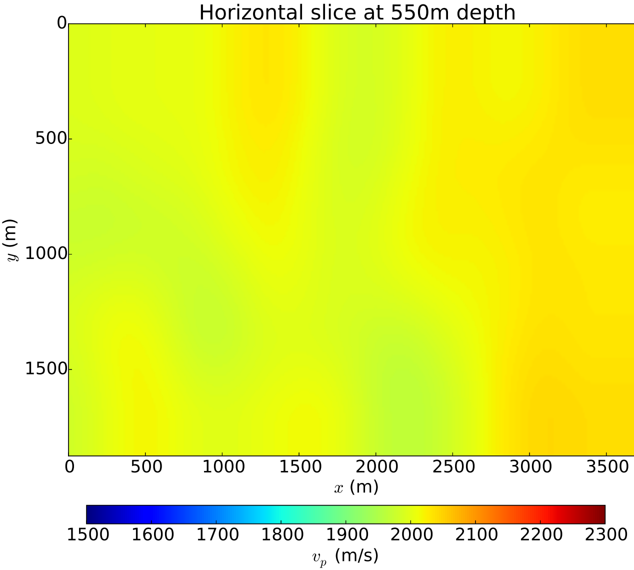
WEMVA model


Vertical slice
z (m)
0
200
400
600
800
1000
1200
1400
0
500
1000
1500
2000
2500
3000
3500
x (m)
1500
1600
1700
1800
1900
2000
2100
2200
2300
v_p (m/s)

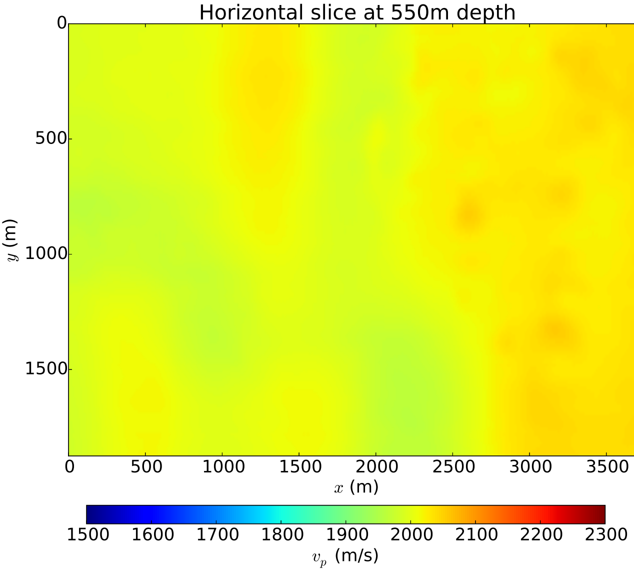
FWI model: WEMVA+FWI



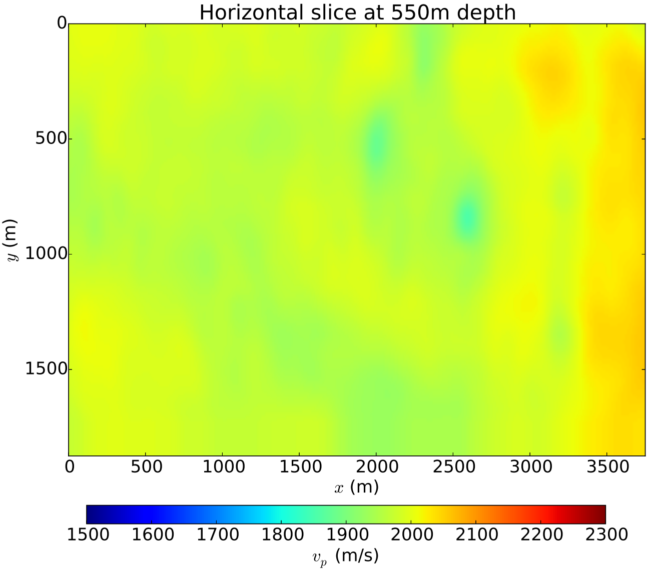
Initial model



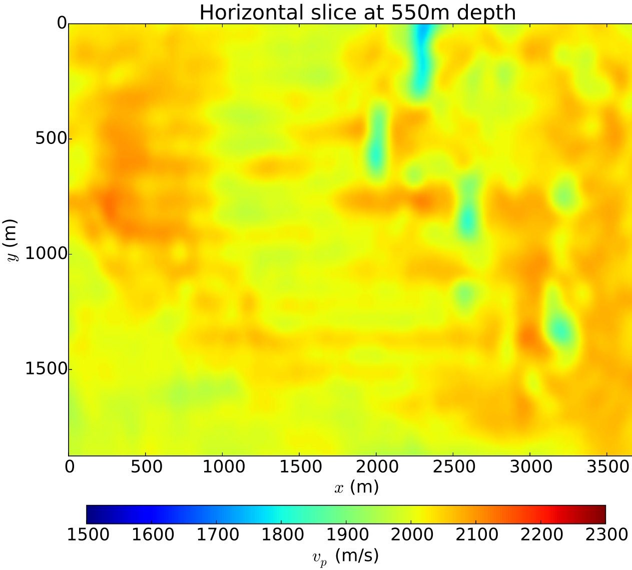
FWI model: Standard approach



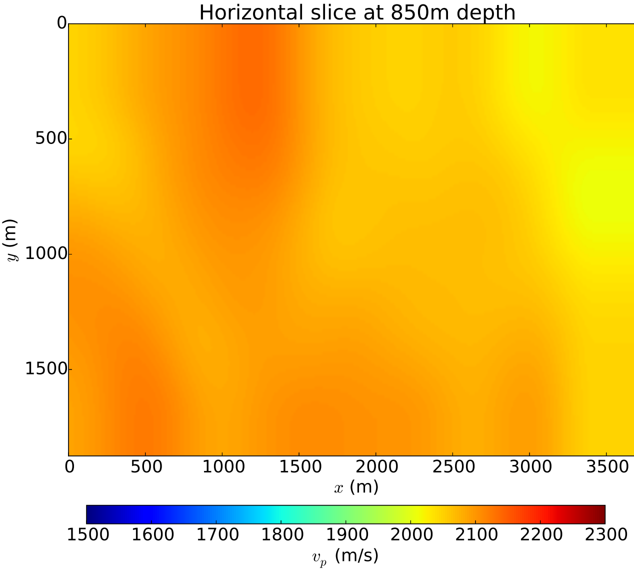
WEMVA model


Horizontal slice at 550m depth
0
500
1000
1500
y (m)
0
500
1000
1500
2000
2500
3000
3500
x (m)
1500
1600
1700
1800
1900
2000
2100
2200
2300
v_p (m/s)

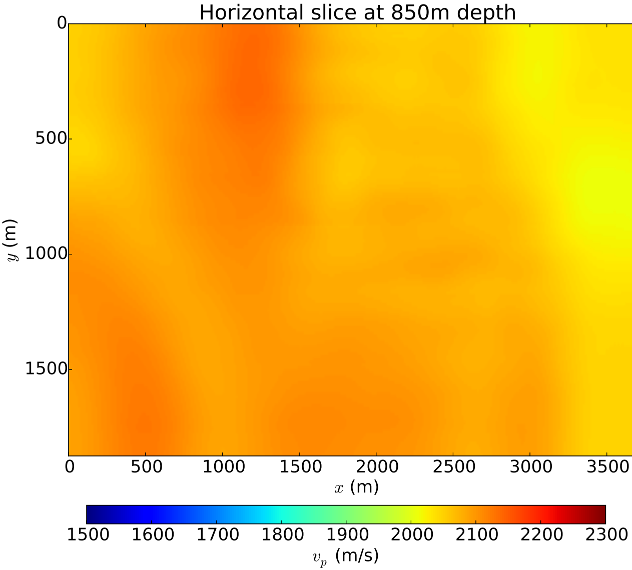
FWI model: WEMVA+FWI



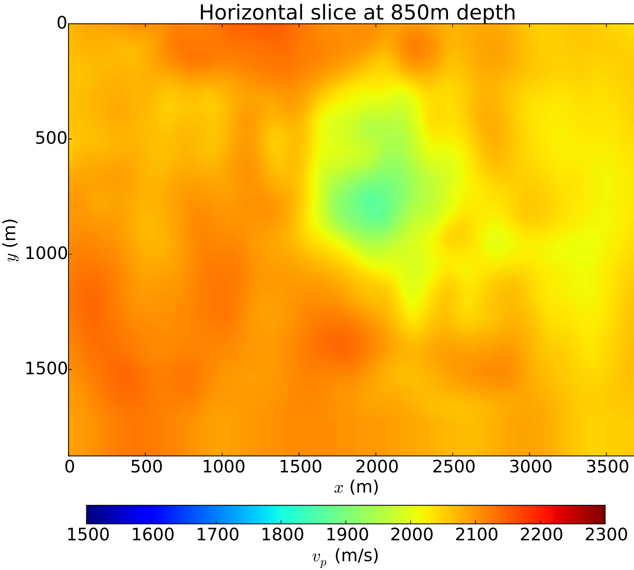
Initial model



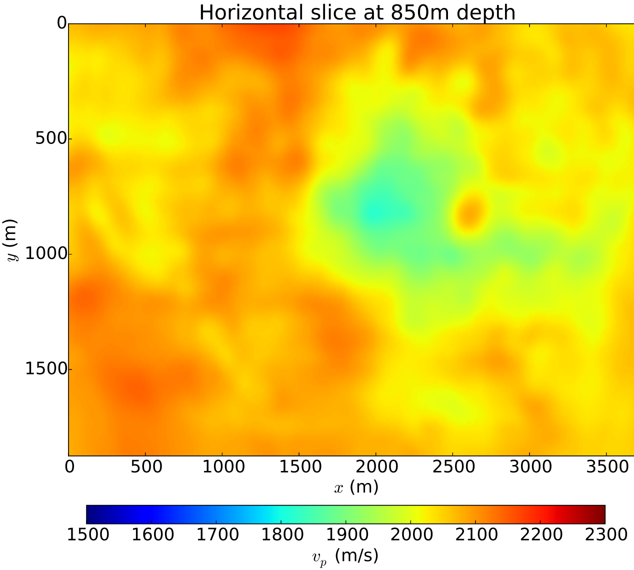
FWI model: Standard approach



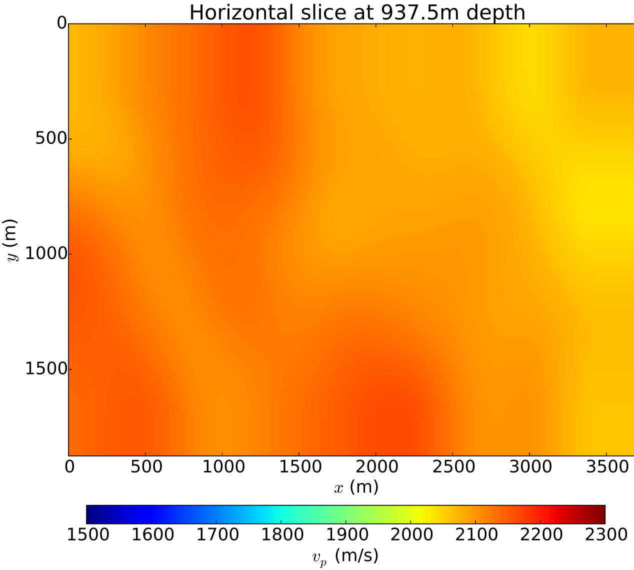
WEMVA model


Horizontal slice at 850m depth
0
500
1000
1500
y (m)
0 500 1000 1500 2000 2500 3000 3500
x (m)
1500 1600 1700 1800 1900 2000 2100 2200 2300
v_p (m/s)

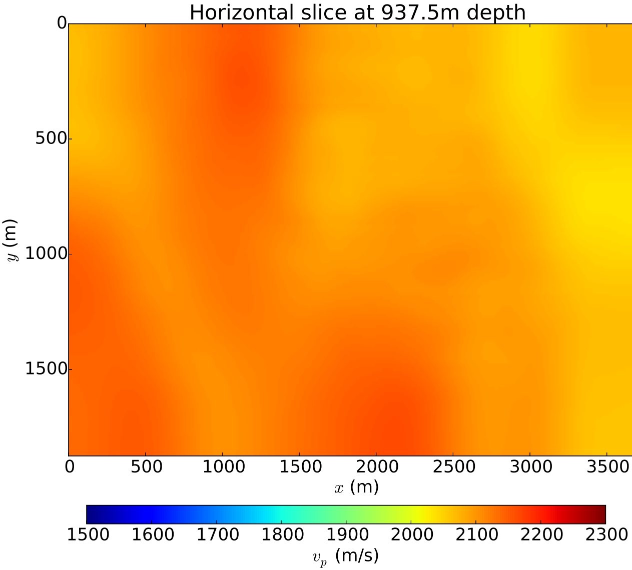
FWI model: WEMVA+FWI



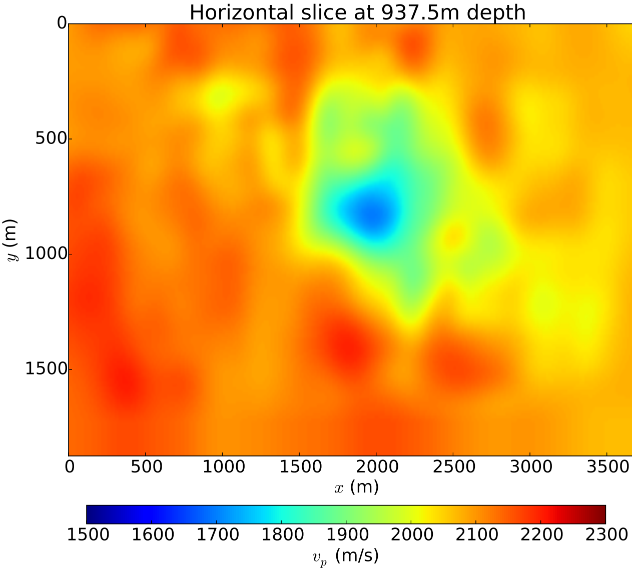
Initial model



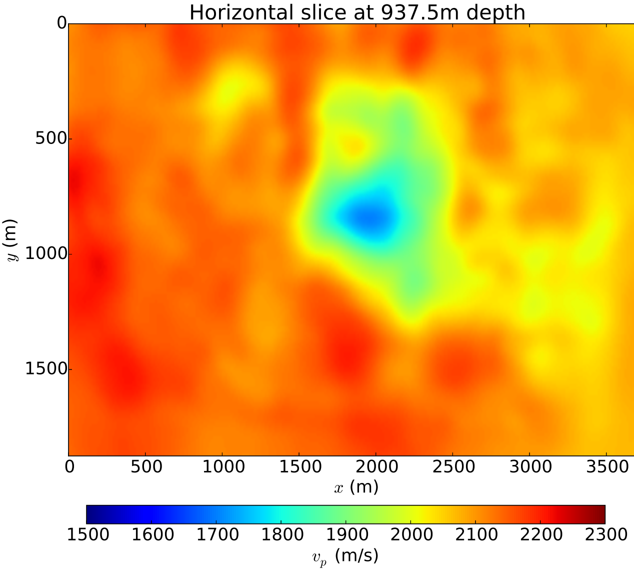
FWI model: Standard approach



WEMVA model


Horizontal slice at 937.5m depth
0
500
1000
1500
y (m)
0
500
1000
1500
2000
2500
3000
3500
x (m)
1500
1600
1700
1800
1900
2000
2100
2200
2300
v_p (m/s)

FWI model: WEMVA+FWI



Conclusions

- Using WEMVA to estimate the initial model for FWI improves the inversion results
- WEMVA improves the kinematics in the estimated model and hence reduce the cycle-skipping issue in FWI
- Very costly (in terms of computational power) work flow that includes some manual work

References

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