A practical comparison of different parameterizations for anisotropic elastic full-waveform inversion

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Outline

Introduction

Theory

Models and survey setup

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Acknowledgments

Introduction

- Choice of parameterization can affect results in FWI.
- Apply FWI using different parameterizations to the same problem and examine differences.
- Parameterizations:
  1. V_{P0}, V_{S0}, ε, δ
  2. V_{P0}, V_{S0}, V_{hor}, V_{nmo}
  3. V_{nmo}, V_{S0}, η, δ
- Inverted for V_{P0} in a complex model.
- Multiparameter inversion in a simple model.

Theory

- In FWI we want to find a parameter model $\mathbf{m}$ that can produce modeled data $\mathbf{u}$ which is close to some measured data $\mathbf{d}$.
- Apply a numerical wave operator that maps $\mathbf{m}$ from the model domain into the data domain:

$$\mathcal{L}(\mathbf{m}) = \mathbf{u}. \tag{1}$$

- Ideally, find an inverse operator to map $\mathbf{d}$ from the data domain to the model domain:

$$\mathbf{m} = \mathcal{L}^{-1}(\mathbf{d}). \tag{2}$$

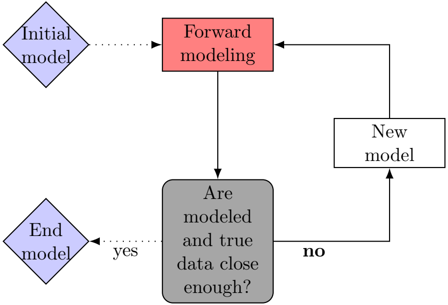
Theory

- Define a misfit functional:

$$\mathcal{F}(\mathbf{m}) = \frac{1}{2}\sum_{j=0}^{n_s}\sum_{i=0}^{n_r} ||\hat{\mathbf{u}}_{i,j}(\mathbf{m}) - \hat{\mathbf{d}}_{i,j}||_2^2. \qquad (3)$$

- The solution is an extreme point of $\mathcal{F}(\mathbf{m})$:

$$\mathbf{m}' = \arg\min_{\mathbf{m}} \mathcal{F}(\mathbf{m}). \qquad (4)$$



Theory

- Update the model iteratively:

$$\mathbf{m}_{k+1} = \mathbf{m}_k - \alpha_k \mathbf{H}_k^{-1} \delta \mathbf{m}_k. \tag{5}$$

- Hessian matrix contains second derivatives of the misfit functional
  - Approximated from previous gradients (L-BFGS)
- Gradients are found via the adjoint method, Mora (1987).

$$\delta \hat{\mathbf{m}}(\mathbf{x}) = \sum_{n_s} \int \mathrm{d}t \sum_{n_r} \frac{\partial u_i(\mathbf{x_S}, \mathbf{x_R}, t)}{\partial \mathbf{m}(\mathbf{x})} \delta u_i(\mathbf{x_S}, \mathbf{x_R}, t). \tag{6}$$

$$\delta u_i(\mathbf{x_S}, \mathbf{x_R}, t) = \int_V \mathrm{d}V \frac{\partial u_i(\mathbf{x_S}, \mathbf{x_R}, t)}{\partial \mathbf{m}(\mathbf{x})} \delta \mathbf{m}(\mathbf{x}). \tag{7}$$

Gradients, 3D

$$\delta\rho = -\sum_{n_s}\int \mathrm{d}t\dot{u}_j\dot{\Psi}_j,$$

$$\delta c_{11} = -\sum_{n_s}\int \mathrm{d}t(u_{1,1}+u_{2,2})(\Psi_{1,1}+\Psi_{2,2}),$$

$$\delta c_{33} = -\sum_{n_s}\int \mathrm{d}t u_{3,3}\Psi_{3,3},$$

$$\delta c_{13} = -\sum_{n_s}\int \mathrm{d}t\Big[\Psi_{3,3}(u_{1,1}+u_{2,2})+(\Psi_{1,1}+\Psi_{2,2})u_{3,3}\Big],$$

$$\delta c_{44} = -\sum_{n_s}\int \mathrm{d}t\Big[(\Psi_{3,1}+\Psi_{1,3})(u_{3,1}+u_{1,3}) + (\Psi_{3,2}+\Psi_{2,3})(u_{3,2}+u_{2,3})\Big],$$

$$\delta c_{66} = -\sum_{n_s}\int \mathrm{d}t\Big[(\Psi_{2,1}+\Psi_{1,2})(u_{2,1}+u_{1,2}) - 2(\Psi_{2,2}u_{1,1}+\Psi_{1,1}u_{2,2})\Big].$$

Gradients for parameterizations, 2D

1)

$$\delta V_{P0} = k_{11}\delta c_{11} + k_{12}\delta c_{13} + k_{13}\delta c_{33},$$
$$\delta V_{S0} = k_{14}\delta c_{13} + k_{15}\delta c_{44},$$
$$\delta\varepsilon = k_{16}\delta c_{11},$$
$$\delta\delta = k_{17}\delta c_{13},$$

2)

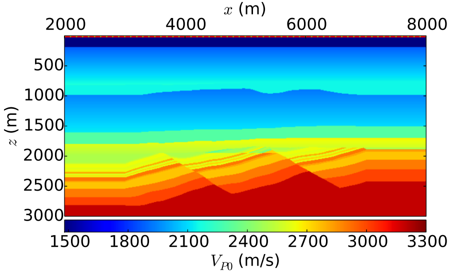
$$\delta V_{P0} = k_{21}\delta c_{13} + k_{22}\delta c_{33},$$
$$\delta V_{S0} = k_{23}\delta c_{13} + k_{24}\delta c_{44},$$
$$\delta V_{\text{hor}} = k_{25}\delta c_{11},$$
$$\delta V_{\text{nmo}} = k_{26}\delta c_{13},$$

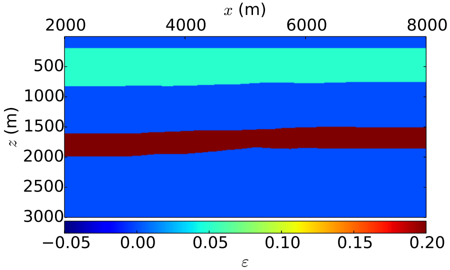
3)

$$\delta V_{\text{nmo}} = k_{31}\delta c_{11} + k_{32}\delta c_{13} + k_{33}\delta c_{33},$$
$$\delta V_{S0} = k_{34}\delta c_{13} + k_{35}\delta c_{44},$$
$$\delta\eta = k_{36}\delta c_{11},$$
$$\delta\delta = k_{37}\delta c_{13} + k_{38}\delta c_{44},$$

Complex model

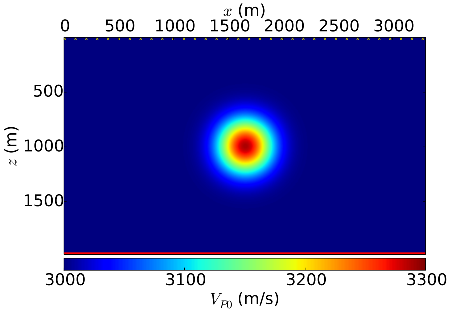
- Synthetic model representative of the Gullfaks field
- 10 km long, 3 km deep
- 1001 × 300 grid points
- Total of 101 shots and 1001 receivers
- Source: 5 Hz Ricker wavelet
- Receivers: Pressure
- Gradient muted in the water layer



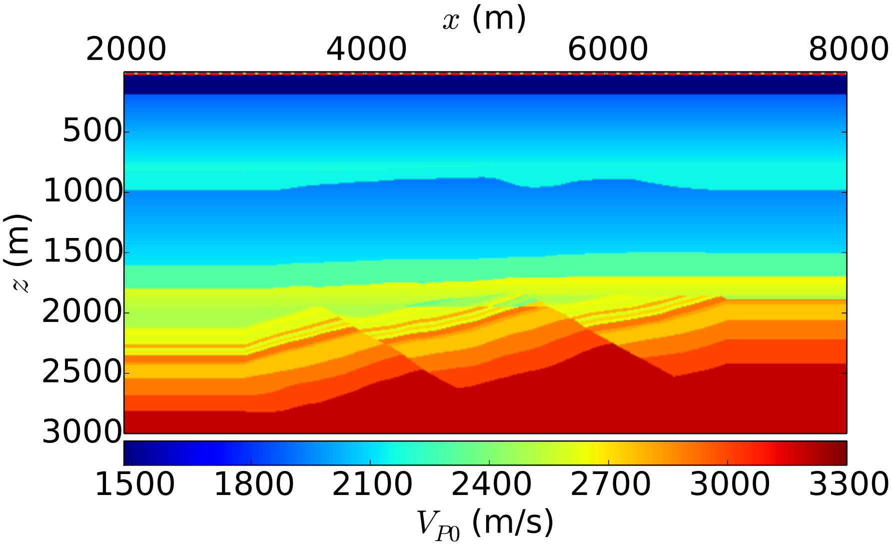


Simple model

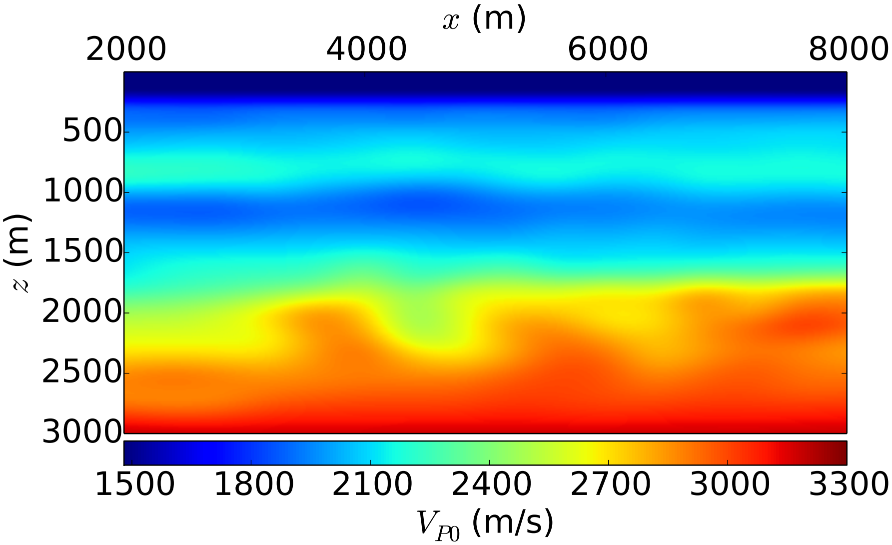
- Homogeneous model with a Gaussian perturbation in V_{P0} or ε
- ~3.5 km long, ~2 km deep
- 500×300 grid points
- Transmission experiment: 34 shots at the top, 500 receivers at the bottom
- Source: 10 Hz Ricker wavelet
- Receivers: P, Vz, Vx
- $V_{P0} = 3000$ m/s, $V_{S0} = 1500$ m/s, $\varepsilon = 0.1$, $\delta = -0.05$.



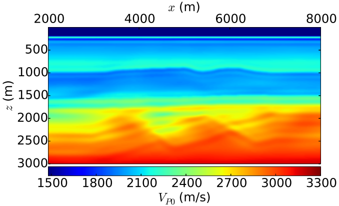
Complex model



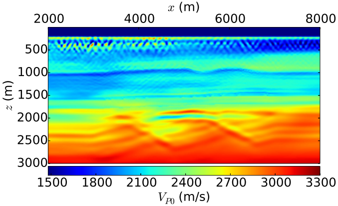
Starting model



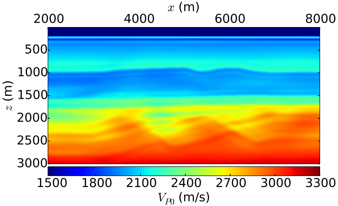
Complex model



(a) Parameterization 1

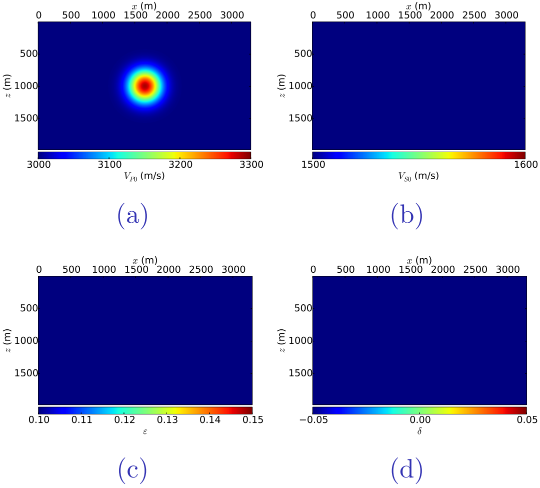


(b) Parameterization 2

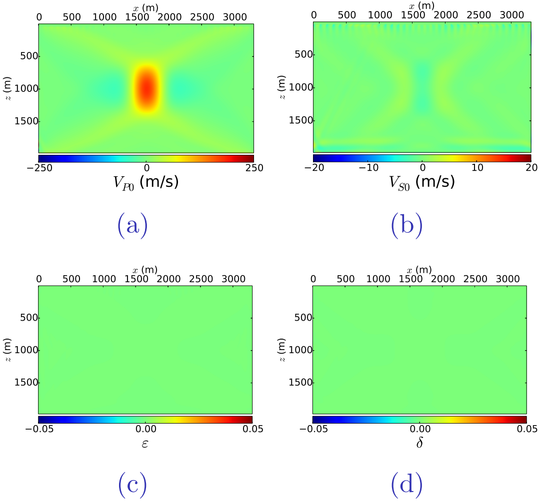


(c) Parameterization 3

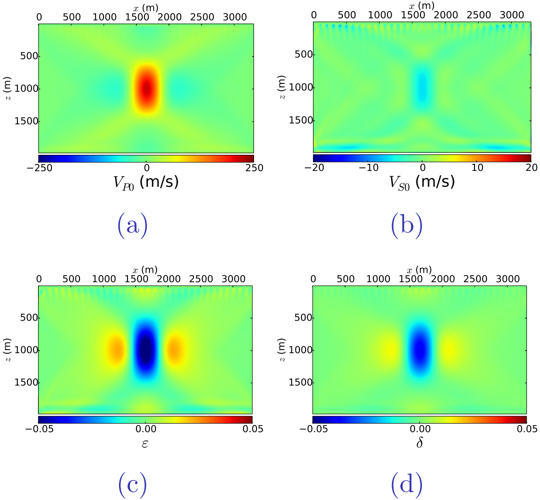
True models, V_{P0} perturbation



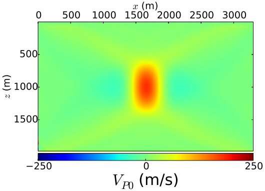
Simple model V_{P0}, parameterization 1



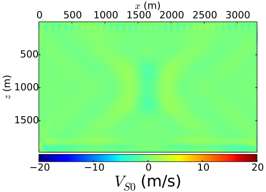
Simple model V_{P0}, parameterization 2



Simple model V_{P0}, parameterization 3



(a)



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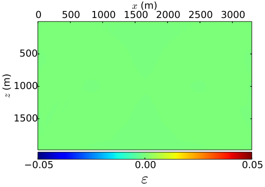

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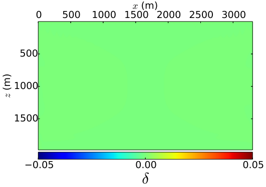
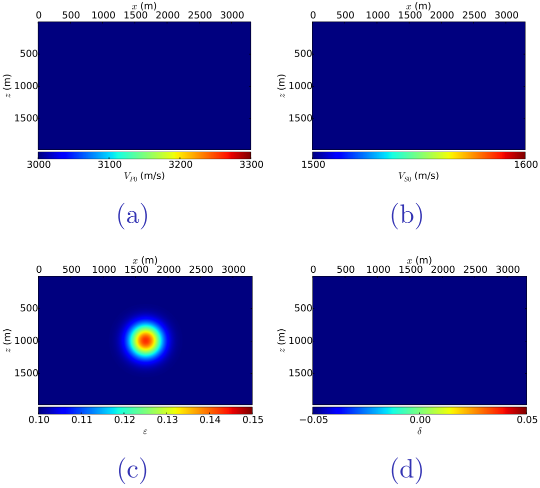
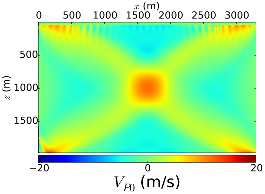


(d)

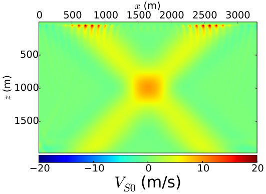
True models, ε perturbation



Simple model ε, parameterization 1



(a)



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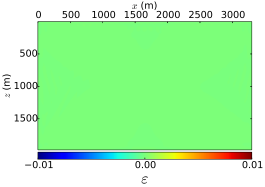

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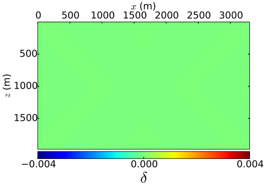
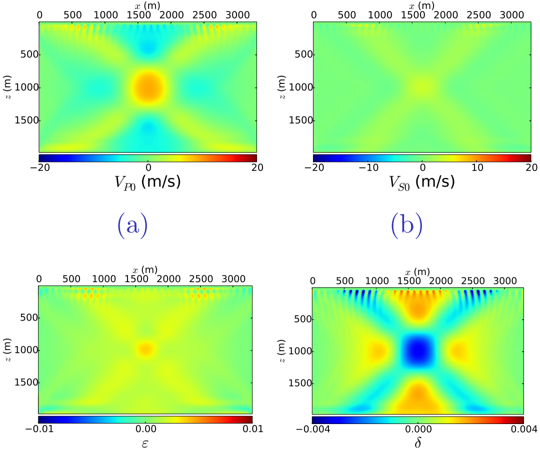


(d)

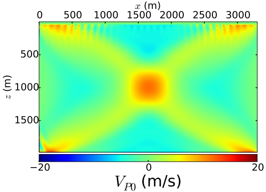
Simple model ε, parameterization 2



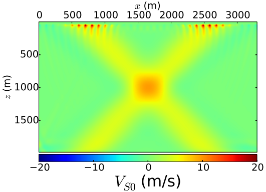
(a) (b)

(c) (d)

Simple model ε, parameterization 3



(a)



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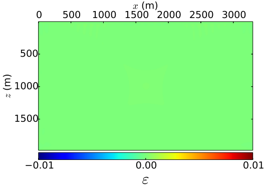

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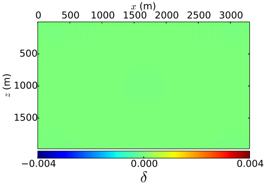
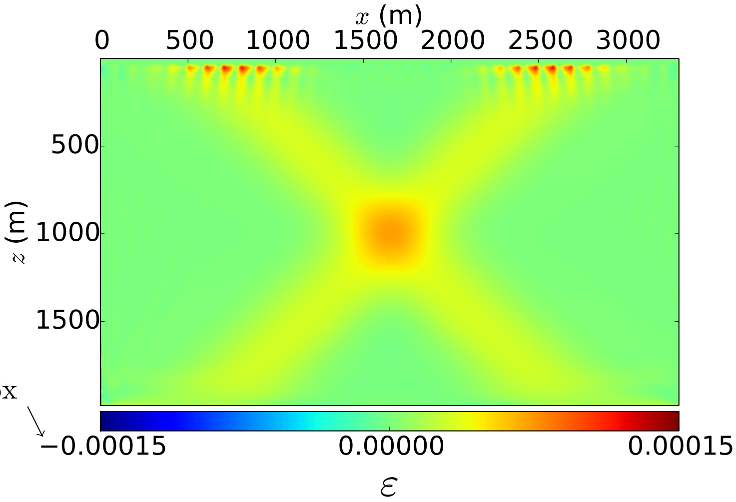


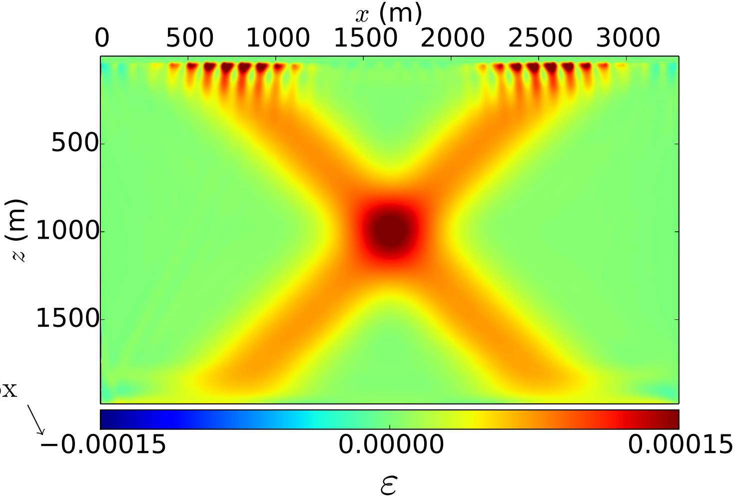
(d)

ε update, parameterization 1



Scale ~26x smaller

ε update, parameterization 3



Scale ~26x smaller

Conclusions

- Multiparameter FWI of multicomponent transmission data in simple models as well as V_{P0} inversion in a complex model, using three different model parameterizations in each example.

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- Simple model experiments demonstrate significant crosstalk in parameterization two.

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- V_{P0} inversions show that using a parameterization that contains as much information as possible about the horizontal component of the wavefields is crucial.
- Simple model experiments demonstrate significant crosstalk in parameterization two.
- Parameterizations one and three perform very similarly, and both have problems updating anisotropy parameters in multiparameter inversion, likely as a consequence of the amplitude difference between velocities and anisotropy parameters.

Acknowledgments

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