# Reverse-time true amplitude migration 

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## Incorrect AVO behavior




## Correct reflection coefficients


[Arntsen et al., 2013] [Zhang et al., 2014]

## Overview

1. Introduction
2. True Amplitude Imaging condition
3. Numerical examples
4. Conclusions

## Introduction

1. AVO analysis important for exploration and reservoir characterization
2. Output gathers from migration should ideally be equal to angle-dependent reflection coefficient
3. The most commonly used imaging condition in Reverse-time migration gives gathers with incorrect angle-dependence
4. Simple modification of Claerbouts (1971) imaging condition gives correct angle dependence
5. Theoretically sound

## Imaging condition

$\longleftarrow$ Offset $\longrightarrow$


## Imaging condition



## Imaging condition



Depth

## Imaging condition



## Imaging condition



## Imaging condition

Using an approach described by f.ex [Vasconcelos et al., 2010] one finds: New imaging condition for multicomponent streamer data:

$$
\begin{align*}
r\left(\mathbf{x}, \mathbf{x}^{\prime}\right) & =\sum_{\mathbf{x}_{s}} \int d t \partial_{z} p_{0}\left(\mathbf{x}, \mathbf{x}_{s}, t\right) p_{s c}\left(\mathbf{x}^{\prime}, \mathbf{x}_{s}, t\right) \\
& -\sum_{\mathbf{x}_{s}} \int d t p_{0}\left(\mathbf{x}, \mathbf{x}_{s}, t\right) \partial_{z} p_{s c}\left(\mathbf{x}^{\prime}, \mathbf{x}_{s}, t\right) \tag{1}
\end{align*}
$$

New imaging condition for conventional streamer data:

$$
\begin{equation*}
r\left(\mathbf{x}, \mathbf{x}^{\prime}\right)=\sum_{\mathbf{x}_{s}} \int d t \partial_{z} p_{0}\left(\mathbf{x}, \mathbf{x}_{s}, t\right) p_{s c}\left(\mathbf{x}^{\prime}, \mathbf{x}_{s}, t\right) \tag{2}
\end{equation*}
$$

[Ordoñez et al., 2014] Old imaging condition: (Rickett and Sava, 2002)

$$
\begin{equation*}
r_{c}\left(\mathbf{x}, \mathbf{x}^{\prime}\right)=\sum_{\mathbf{x}_{s}} \int d t p_{0}\left(\mathbf{x}, \mathbf{x}_{s}, t\right) p_{s c}\left(\mathbf{x}^{\prime}, \mathbf{x}_{s}, t\right) \tag{3}
\end{equation*}
$$

## Imaging condition



## Imaging condition

Plane wave reflection coefficient:

$$
\begin{equation*}
r\left(\mathbf{x}, \mathbf{k}_{h}, z\right)=\int d \mathbf{h} \exp \left(i \mathbf{k}_{h} \cdot \mathbf{h}\right) r\left(\mathbf{x}, \mathbf{h}=\mathbf{x}-\mathbf{x}^{\prime}\right) \tag{4}
\end{equation*}
$$

Numerical example


Numerical example


Numerical example


Data

## Imaging condition

Data at depth:


Depth

## Numerical example

$p_{s c}\left(\mathbf{x}, \mathbf{x}_{s}, t\right)$ at depth of 1000 m.


## Numerical example

$\hat{p}_{0}\left(\mathbf{x}, \mathbf{x}_{s}, t\right)$ at depth of 1000 m


## Imaging condition

New imaging condition:

$$
\begin{equation*}
r\left(\mathbf{x}, \mathbf{x}^{\prime}, t=0\right)=\sum_{\mathbf{x}_{s}} \int d t \partial_{z} p_{0}\left(\mathbf{x}, \mathbf{x}_{s}, t\right) p_{s c}\left(\mathbf{x}^{\prime}, \mathbf{x}_{s}, t\right) \tag{5}
\end{equation*}
$$

Old imaging condition:

$$
\begin{equation*}
r_{c}\left(\mathbf{x}, \mathbf{x}^{\prime}, t=0\right)=\sum_{\mathbf{x}_{s}} \int d t p_{0}\left(\mathbf{x}, \mathbf{x}_{s}, t\right) p_{s c}\left(\mathbf{x}^{\prime}, \mathbf{x}_{s}, t\right) \tag{6}
\end{equation*}
$$

## Numerical example

Reflectivity $p_{s c}\left(\mathbf{x}-\mathbf{x}^{\prime}, t=0\right)$ at depth of 1000 m using new imaging condition


## Numerical example

Reflectivity $p_{s c}\left(\mathbf{x}-\mathbf{x}^{\prime}, t=0\right)$ at depth of 1000 m using old imaging condition


## Numerical example

Reflectivity $p_{s c}\left(\mathbf{x}-\mathbf{x}^{\prime}, t=0\right)$ at all depths using new imaging condition


Offset CIP gather

## Numerical example

Full section $p_{s c}\left(\mathbf{x}-\mathbf{x}^{\prime}=0, t\right)$ at all depths using new imaging condition


Zero offset section

## Numerical example

Plane wave reflection coefficient $p_{s c}\left(\mathbf{x}-\mathbf{x}^{\prime}, t\right)$ at depth of 1000 m using new imaging condition


## Numerical example

Plane wave reflection coefficient $p_{s c}\left(\mathbf{x}-\mathbf{x}^{\prime}, t\right)$ at depth of 1000 m using old imaging condition


## Numerical example

Reflection coefficient at three different depths using new imaging condition

Plane wave reflection coefficient


## Numerical example

Reflection coefficient at three different depths using old imaging condition

Plane wave reflection coefficient


## Summary/Conclusion

- New imaging condition gives correct Amplitude-Versus-Angle behavior
- Easy to implement for reverse-time migration
- Simple modification of existing imaging condition


## Acknowledgements

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## References

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## Imaging condition

$$
\begin{align*}
p_{s c}\left(\mathbf{x}, \mathbf{x}_{s}, t\right)= & -\int_{S} d S\left(\mathbf{x}^{\prime}\right) \rho^{-1}\left(\mathbf{x}^{\prime}\right) \partial_{z} p_{s c}\left(\mathbf{x}^{\prime}, \mathbf{x}_{s}, t\right) * \hat{g}\left(\mathbf{x}, \mathbf{x}^{\prime}, t\right) \\
& +\int_{S} d S\left(\mathbf{x}^{\prime}\right) \rho^{-1}\left(\mathbf{x}^{\prime}\right) \partial_{z} \hat{g}\left(\mathbf{x}, \mathbf{x}^{\prime}, t\right) * p_{s c}\left(\mathbf{x}^{\prime}, \mathbf{x}_{s}, t\right)  \tag{7}\\
p_{s c}\left(\mathbf{x}, \mathbf{x}_{s}, t\right) \approx & \int_{S} d S\left(\mathbf{x}^{\prime}\right) \rho^{-1}\left(\mathbf{x}^{\prime}\right) 2 \partial_{z} \hat{g}\left(\mathbf{x}, \mathbf{x}^{\prime}, t\right) * p_{s c}\left(\mathbf{x}^{\prime}, \mathbf{x}_{s}, t\right) \tag{8}
\end{align*}
$$

- $\hat{g}\left(\mathbf{x}, \mathbf{x}_{s}, t\right)$ : Anticausal background Green's function
- $\rho$ : Density
- *: Time convolution


## Imaging condition

$$
\begin{align*}
p_{s c}\left(\mathbf{x}, \mathbf{x}^{\prime}, t\right) * \hat{s}(t) & = \\
& -\int_{S} d S\left(\mathbf{x}_{s}\right) \rho^{-1}\left(\mathbf{x}_{s}\right) \partial_{z} p_{s c}\left(\mathbf{x}^{\prime}, \mathbf{x}_{s}, t\right) * \hat{p}_{0}\left(\mathbf{x}, \mathbf{x}_{s}, t\right) \\
& +\int_{S} d S\left(\mathbf{x}_{s}\right) \partial_{z} \hat{p}_{0}\left(\mathbf{x}^{\prime}, \mathbf{x}_{s}, t\right) * p_{s c}\left(\mathbf{x}^{\prime}, \mathbf{x}_{s}, t\right)  \tag{9}\\
p_{s c}\left(\mathbf{x}, \mathbf{x}^{\prime}, t\right) * \hat{s}(t) & \approx \int_{S} d s\left(\mathbf{x}_{s}\right) \partial_{z} \hat{p}_{0}\left(\mathbf{x}^{\prime}, \mathbf{x}_{s}, t\right) * p_{s c}\left(\mathbf{x}^{\prime}, \mathbf{x}_{s}, t\right) \tag{10}
\end{align*}
$$

- $\hat{p}_{0}\left(\mathbf{x}, \mathbf{x}_{s}, t\right)$ : Anticausal downgoing wavefield

