Aperiodic signal apparition – De-aliased sim-source separation

Dirk-Jan van Manen, Kurt Eggenberger, Johan Robertsson
Fredrik Andersson, Lasse Amundsen

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Periodic signal apparition

- Activate e.g. every second shot with filter A

- Modulation function:

$$m(n) = \frac{1}{2}[1 + (-1)^n] + \frac{1}{2}A[1 - (-1)^n]$$
$$= \{1, A, 1, A, 1, A, \cdots\} \ (n \in \mathrm{N})$$

- Examples of A:

$$A(\omega) \in \left\{1, -1, 0, \tfrac{1}{2}, 1, e^{i\omega T}, 1 + e^{i\omega T}, \cdots\right\}$$

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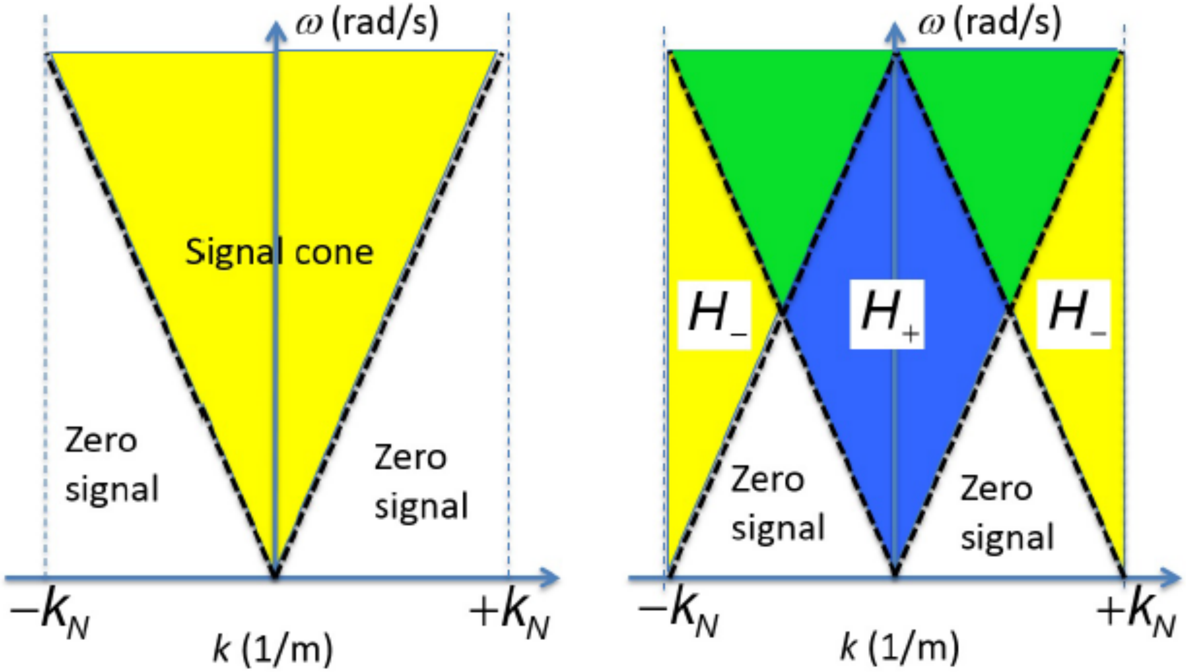
Effect of modulation in FK



$$H(k) = \frac{1}{N}\sum_{n=0}^{N-1} m(n) f(n) e^{-i2\pi kn/N}$$

$$= \frac{1}{2}[1 + A]F(k) + \frac{1}{2}[1 - A]F(k - k_N)$$

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Apparition solves Sim-source!

- Acquire 1st source conventionally
- Acquire 2nd (3rd, 4th,..) source(s) modulated

Observations:

- 2nd (3rd, 4th,...) source(s) are apparated and become «ghostly apparent» in FK
- Remaining signal in central cone for apparated sources predictable
- Choosing e.g. $A(\omega) = e^{i\omega T}$ apparition can be achieved with conventional technology

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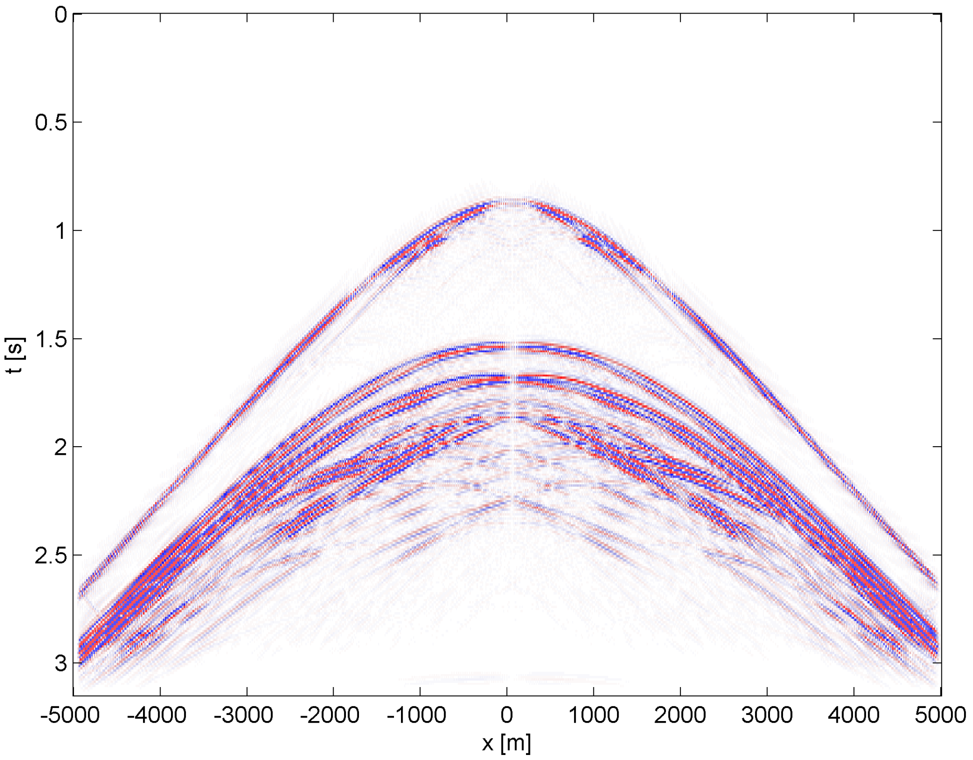
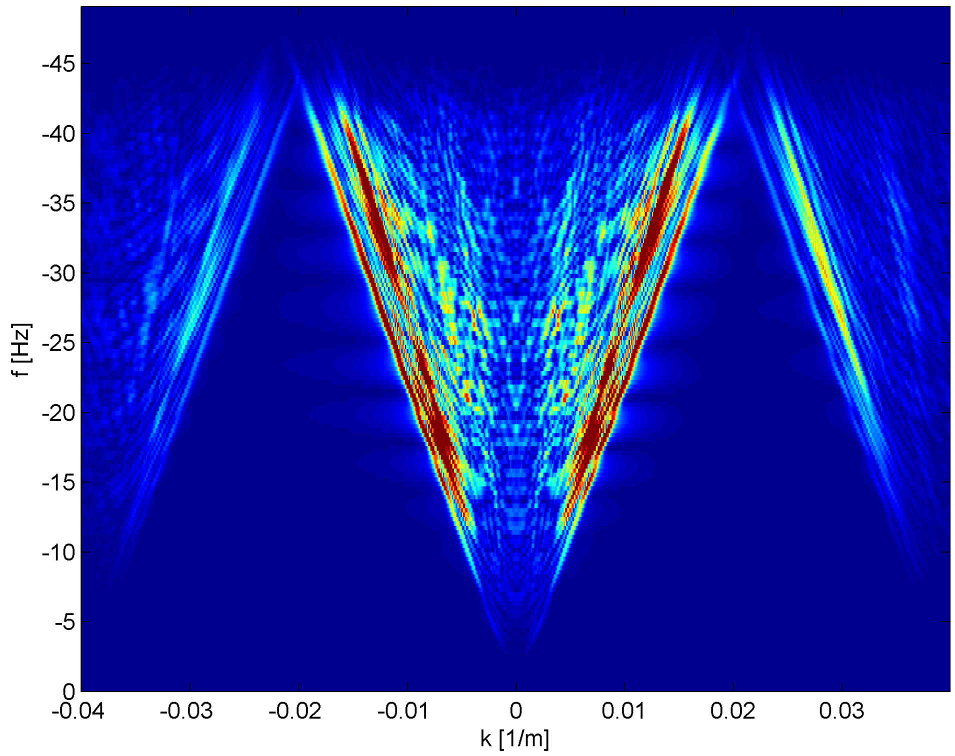
Input data – 2 simultaneous sources

dx = 12.5m





m(n) = 1, $e^{i\omega T}$, 1, $e^{i\omega T}$, 1, $e^{i\omega T}$, …

T=0.005 s

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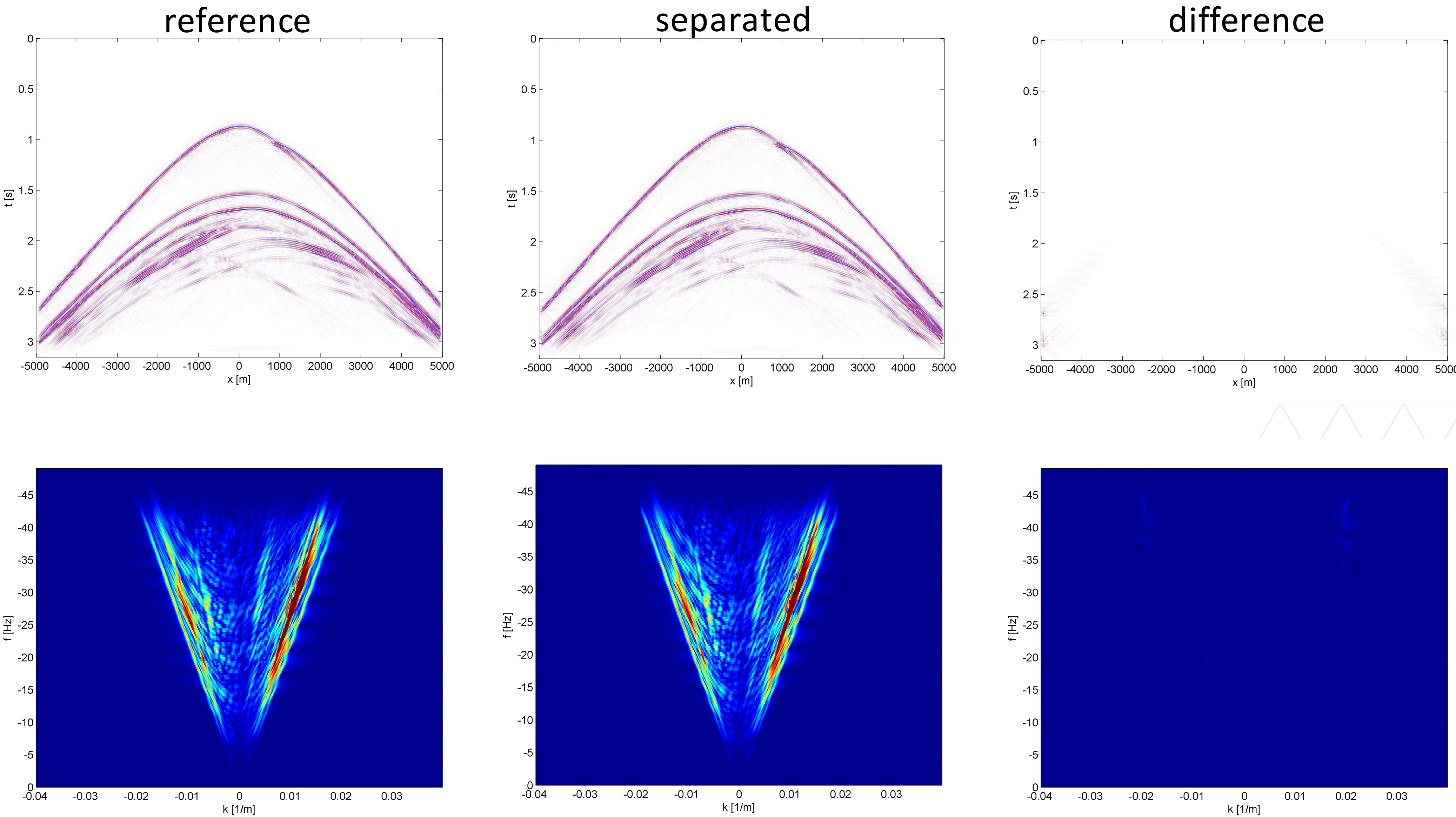
Separated data – Source 1



OK, great, but what about...

- Position perturbations stemming from shooting on time?
- Timing perturbations stemming from shooting on position?
- General timing inaccuracies / deviations from perfectly periodic modulation?
- Aliasing / non-realistic source spacing?

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Aperiodic apparition

Lesser-known dual of the convolution theorem:

$$m(n) \cdot s(n) = \mathcal{F}^{-1}\{M(k) \underset{c}{*} S(k)\}$$

where:

$M(k)$	denotes the wavenumber transform of the modulation function $m(n)$
$S(k)$	denotes the wavenumber transform of the source wavefield $s(n)$
$\mathcal{F}^{-1}$	denotes the inverse discrete Fourier transform
$\underset{c}{*}$	denotes cyclic (circular) convolution

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Aperiodic apparition

Cyclic convolution written as matrix multiplication with matrix:

$$\boldsymbol{C_M} = \begin{bmatrix} M(0) & M(N-1) & \cdots & M(2) & M(1) \\ M(1) & M(0) & & M(3) & M(2) \\ \vdots & & \ddots & & \vdots \\ M(N-2) & M(N-3) & \cdots & M(0) & M(N-1) \\ M(N-1) & M(N-2) & & M(1) & M(0) \end{bmatrix}$$

I.e.:

$$\mathbf{m} \odot \mathbf{s} = \mathcal{F}^{-1}\{\mathbf{C_M}\boldsymbol{S}\}$$

where:

$\mathbf{S} = [S(0), S(1), \dots, S(N-1)]^T$ is wavenumber transform of src wavefield

$\mathbf{s} = [s(0), s(1), \dots, s(N-1)]^T$ is the source wavefield

$\mathbf{m} = [m(0), m(1), \dots, m(N-1)]^T$ is the modulation function

$\odot$ is element-wise multiplication

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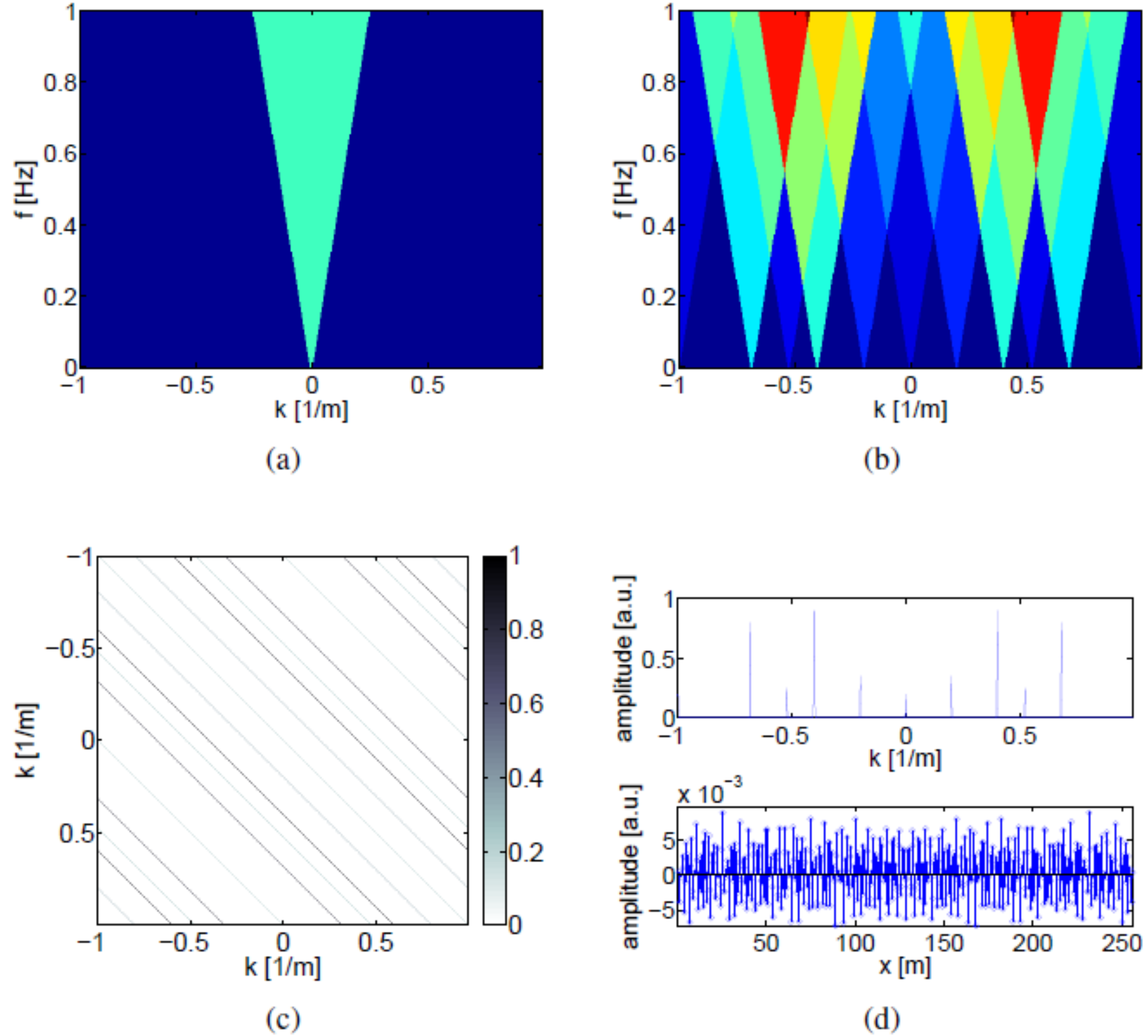
Cyclic convolution example



Aperiodic apparition

Effect of general non-periodic modulation function:

Introduce additional, scaled replications of the signal cones of the sources along the wavenumber axis

Position and the scaling factor of the replications exactly determined by the DFT of the aperiodic modulation function, $M(k)$, which cyclically convolves the signal cones

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Aperiodic apparition

Effect of encoding on s_2 (in s_1 times) described by: $\boldsymbol{C}_{\boldsymbol{M}_{\mathbf{12}}}$

Data in s_2 times: $d_2(t,n) = \mathcal{F}^{-1}\{\mathcal{F}\{d_1(t,n)\}e^{-i\omega T(n)}\}$

Effect of encoding on s_1 (in s_2 times) described by: $\boldsymbol{C}_{\boldsymbol{M}_{\mathbf{21}}}$

General linear k-domain forward model: $\mathbf{D} = \mathbf{GS}$

where: $\boldsymbol{G} = \begin{pmatrix} \boldsymbol{C}_{\boldsymbol{M}_{\mathbf{11}}} & \boldsymbol{C}_{\boldsymbol{M}_{\mathbf{12}}} \\ \boldsymbol{C}_{\boldsymbol{M}_{\mathbf{21}}} & \boldsymbol{C}_{\boldsymbol{M}_{\mathbf{22}}} \end{pmatrix} = \begin{pmatrix} \boldsymbol{I} & \boldsymbol{C}_{\boldsymbol{M}_{\mathbf{12}}} \\ \boldsymbol{C}_{\boldsymbol{M}_{\mathbf{21}}} & \boldsymbol{I} \end{pmatrix}$

and: $\mathbf{S} = [S_1(0), \dots, S_1(N-1), S_2(0), \dots, S_2(N-1)]^T$

$\mathbf{D} = [D_1(0), \dots, D_1(N-1), D_2(0), \dots, D_2(N-1)]^T$

Aperiodic apparition

Variable count: $2N$ unknowns and $2N$ equations.

Half the data vector was constructed from other half by simple time-shifting operation → data do not provide independent information

Act as regularization by making the system of equations symmetric.

→ Use cone constraint to reduce # unknowns

→ LSQ solution: $\hat{\mathbf{S}}^{\mathbf{LSQ}} = (\hat{\mathbf{G}}^{\mathbf{H}}\hat{\mathbf{G}} + \lambda^2\hat{\mathbf{I}})^{-1}\hat{\mathbf{G}}^{\mathbf{H}}\mathbf{D},$

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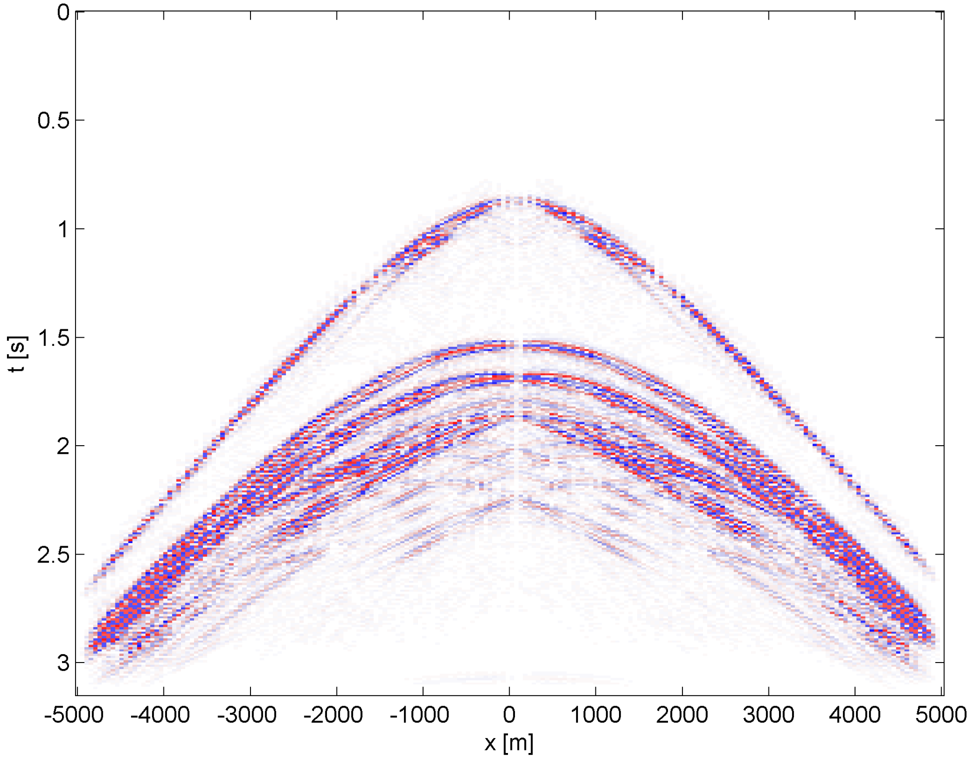
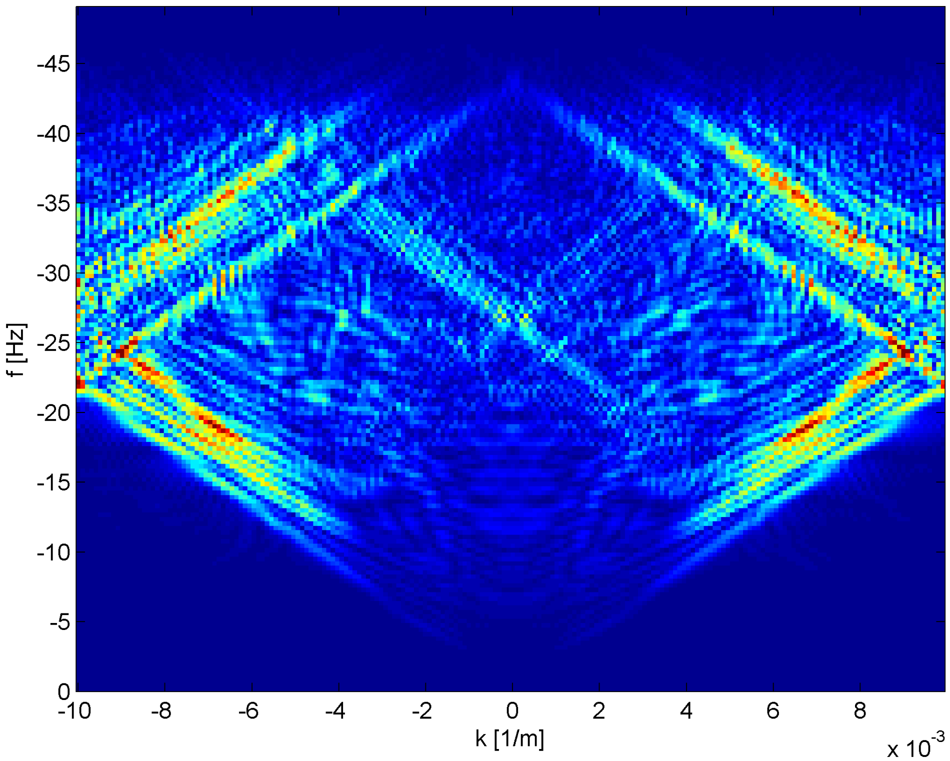
Input data – 2 simultaneous sources

dx = 50m





m(n) = 1, $e^{i\omega T}$, 1, $e^{i\omega T}$, 1, $e^{i\omega T}$, …

T=0.005 s

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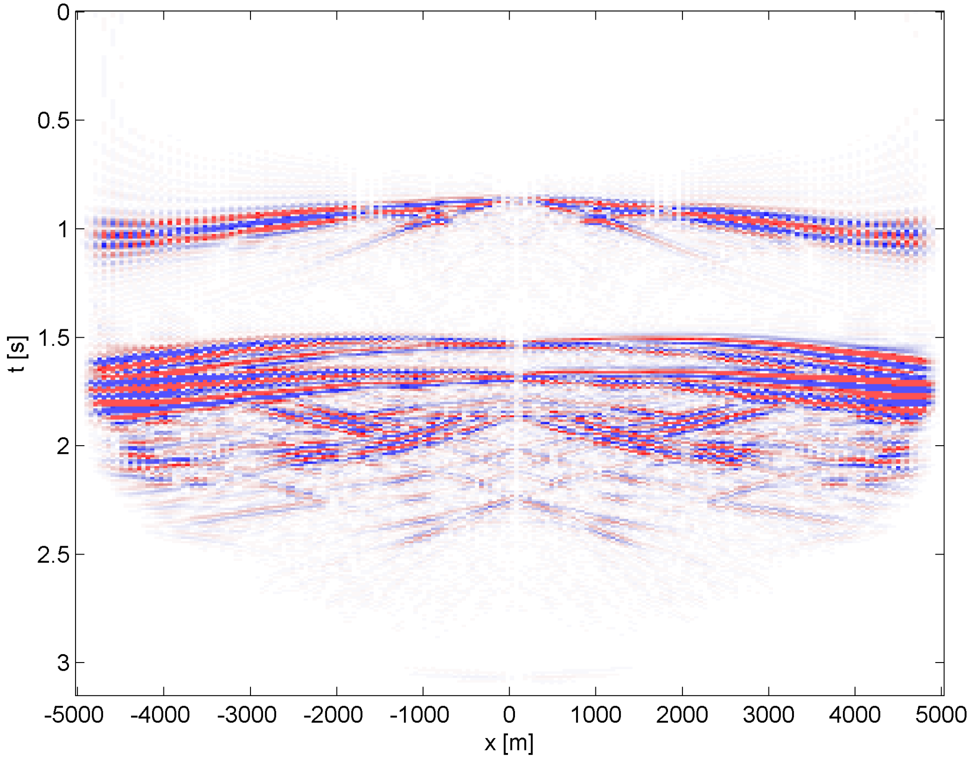
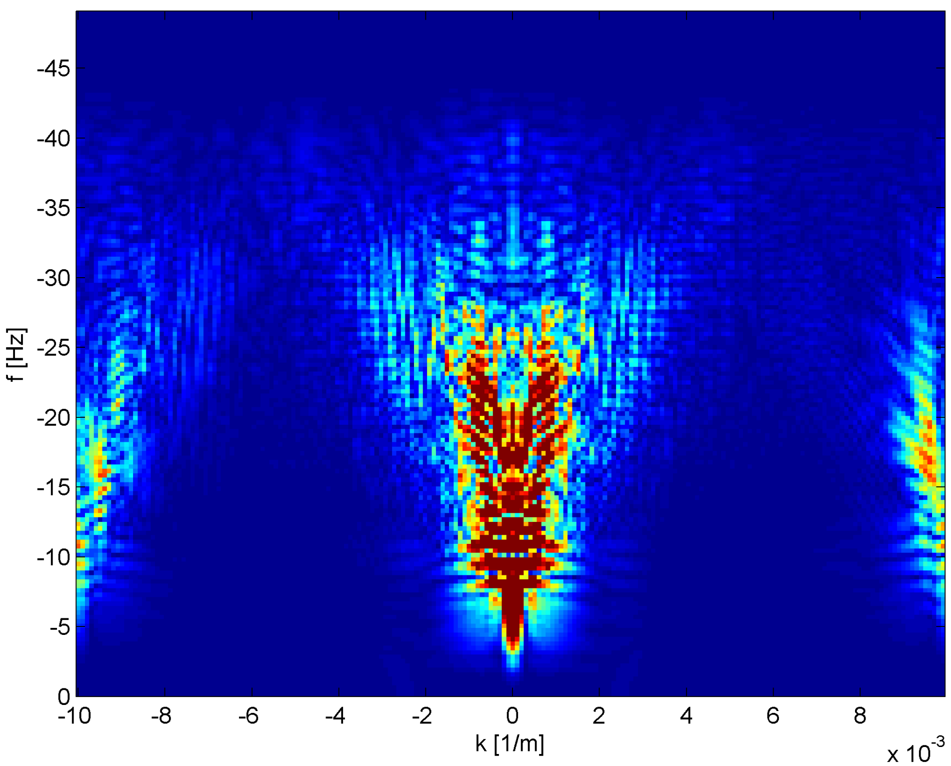
Input data – NMO corrected





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Observations and workflow

- NMO correction dealiases the data ...BUT...
- NMO stretch makes the encoding time delay space and time variant → can be predicted
- Aperiodic apparition can be applied to handle the space-variant time delay
- Need to process in a time-variant manner → S-transform
- $\beta = \dfrac{t_x}{t_0 - \dfrac{x^2 v'(t_0)}{v^3(t_0)}}$ $\left[t_x = \sqrt{{t_0}^2 - \frac{x^2}{{v_{rms}}^2(t_0)}}\right]$

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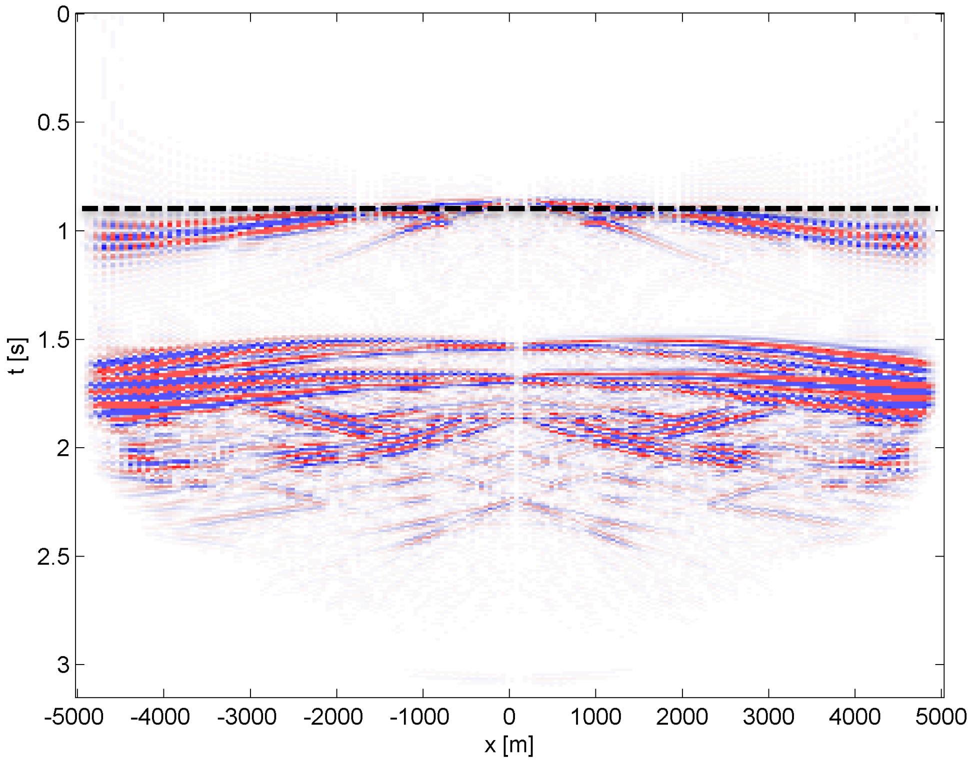
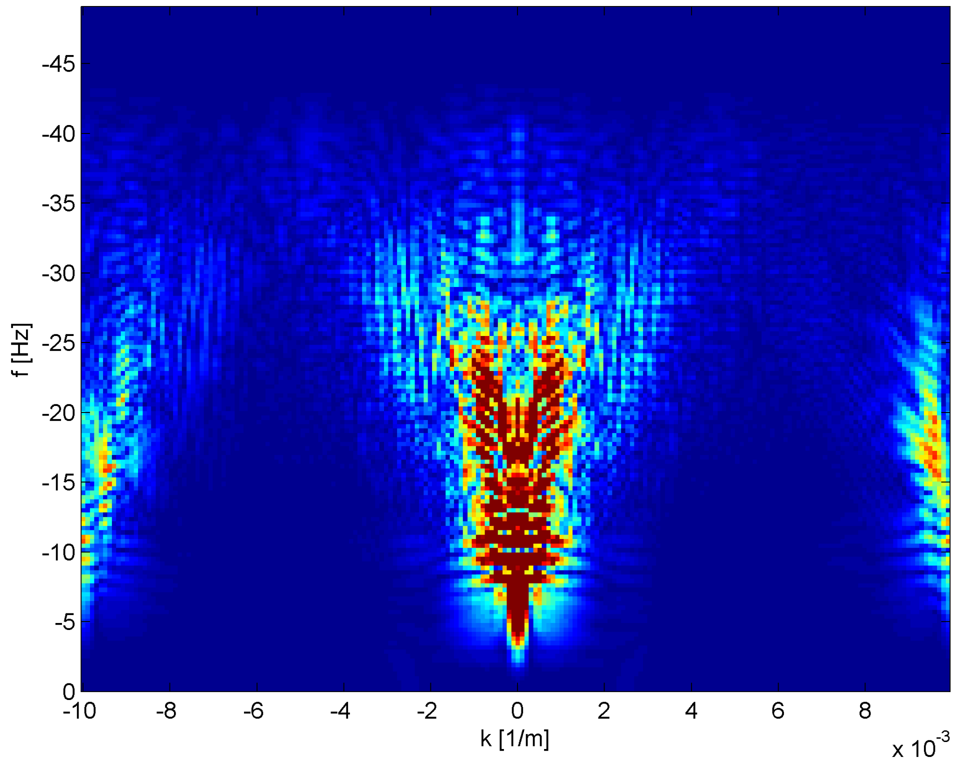
Input data – NMO corrected





→ Look at: separation for S-transform slice at t=0.9s

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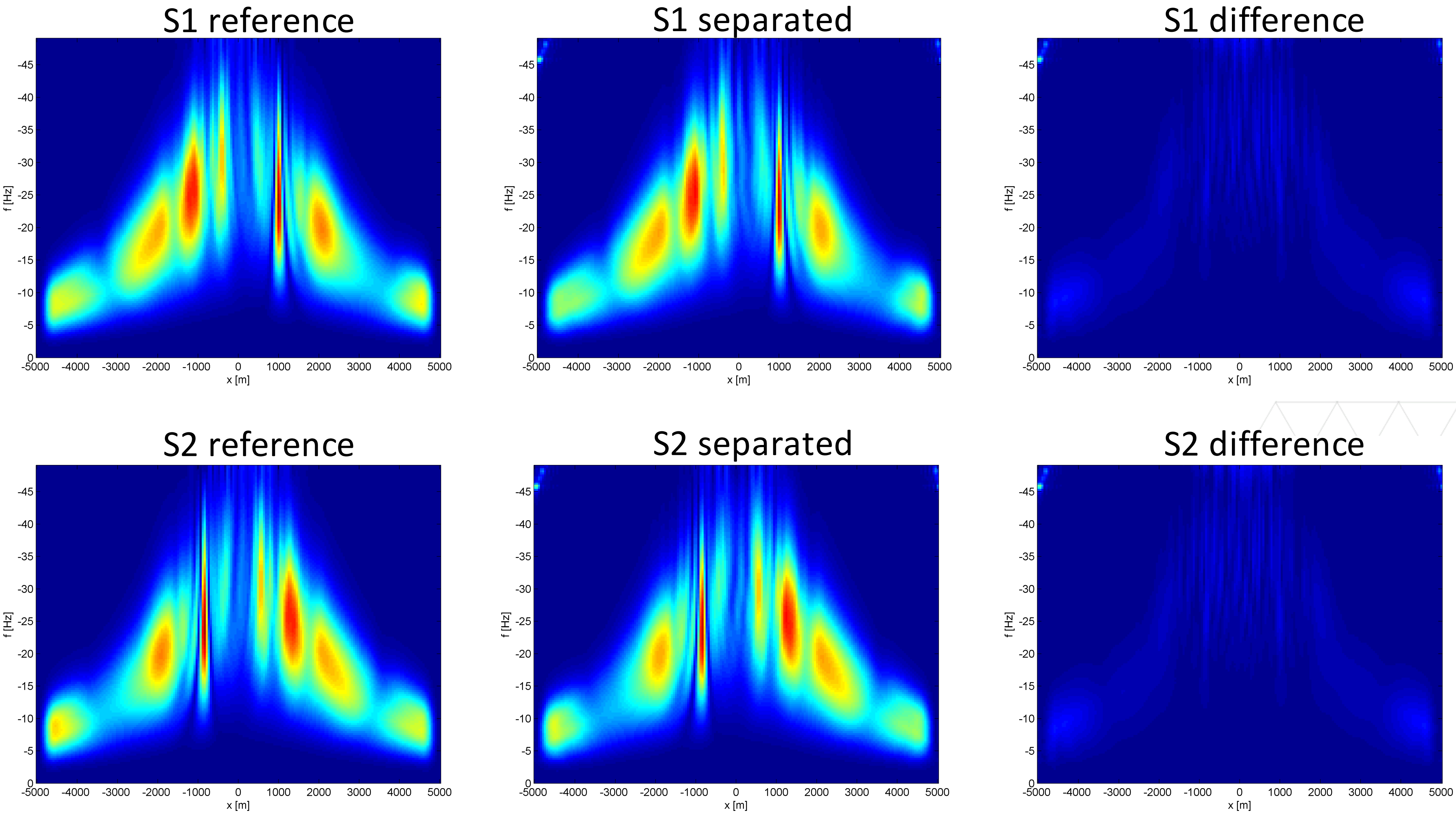
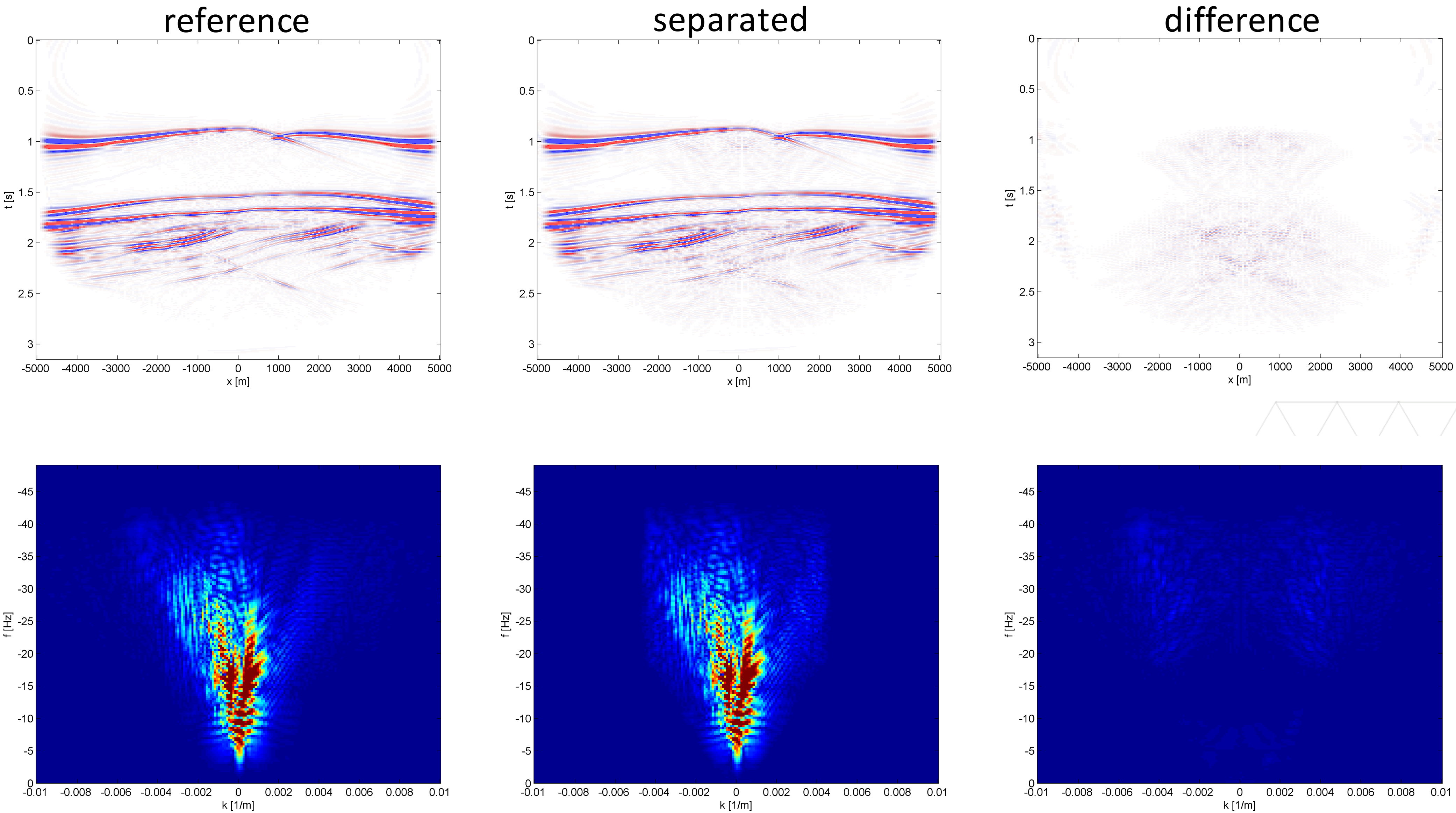
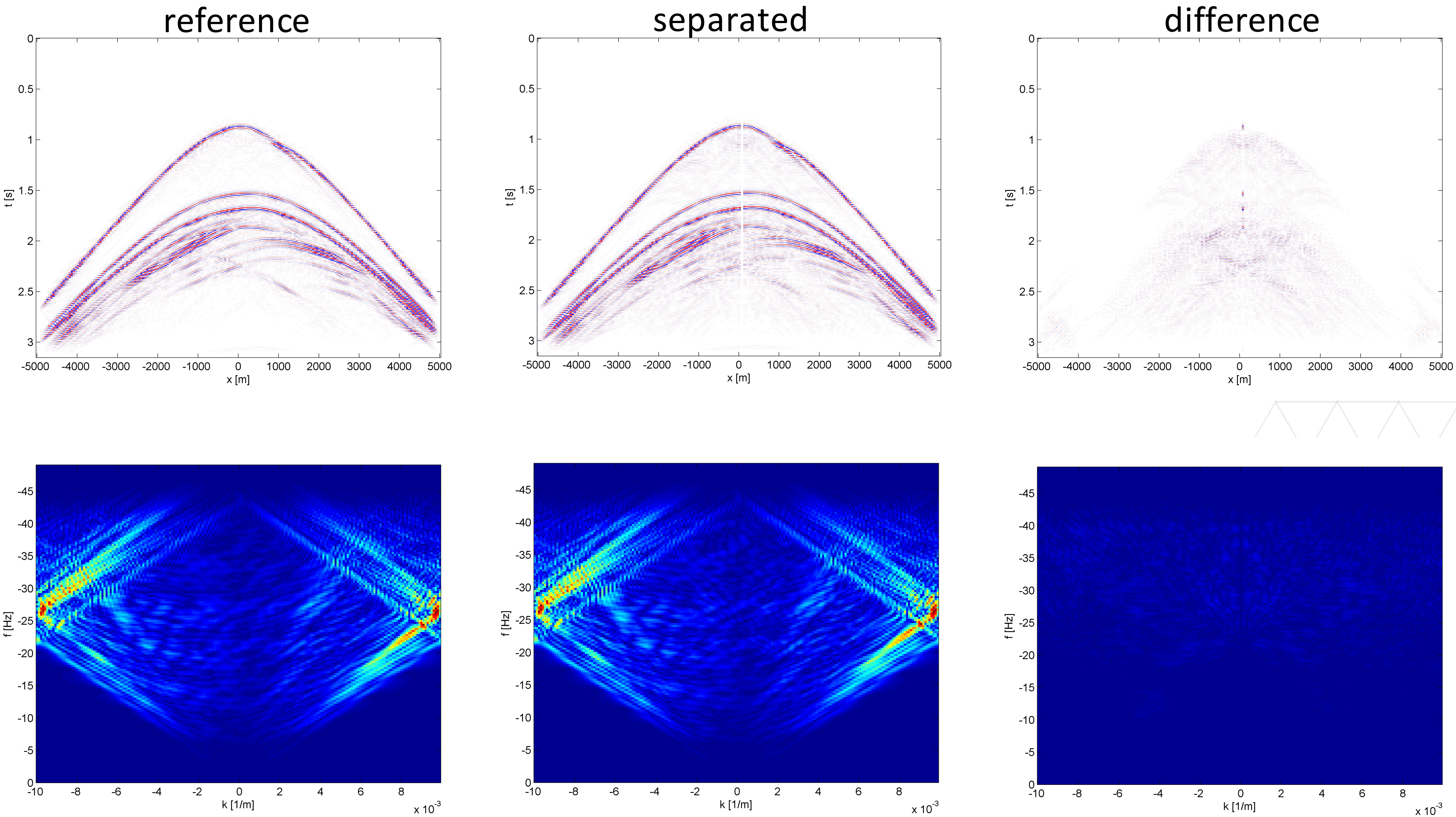
Separated data – S-transform slice



Separated data – NMO corrected



Separated data – INMO corrected



Conclusion

- Wavefield signal apparition offers fundamentally new approach for simultaneous source separation
- Wavefield signal apparition can be generalized to non-periodic apparition patterns:
  - cyclic convolution with transform of the modulation function models replication and scaling in wavenumber domain
  - least-squares solution of system of equations with source signals modeled by cones in FK domain
- Generalization to non-periodic acquisition is major advance:
  - enables separation of simultaneous source data with realistic perturbations during acquisition (firing time, position, etc.)
  - enables signal separation out after de-aliasing transforms such as NMO/INMO

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