



# Seismic Data Processing with the Multidimensional Marchenko Equation

*Joost van der Neut (Delft University of Technology)*

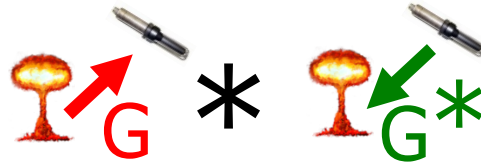
# Contents

## 1. Time-Reversed Wavefields (Green's functions)



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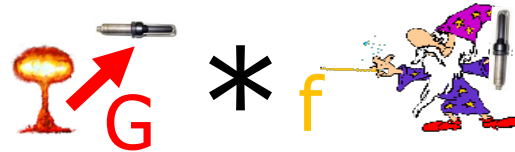


# Contents

## 1. Time-Reversed Wavefields (Green's functions)



## 2. Inverse Wavefields (focusing functions)

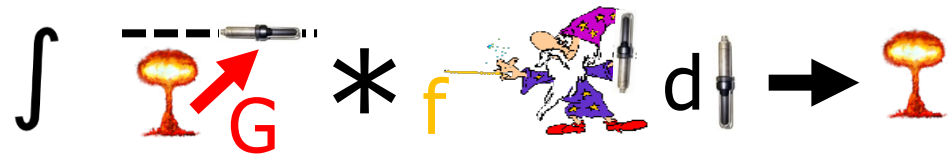


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## 1. Time-Reversed Wavefields (Green's functions)



## 2. Inverse Wavefields (focusing functions)

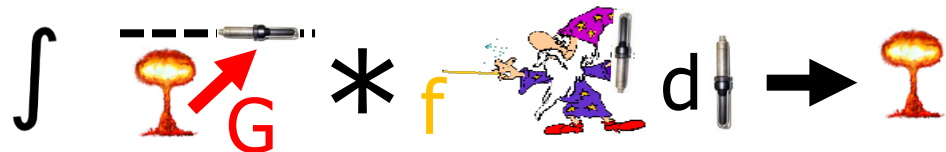


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## 1. Time-Reversed Wavefields (Green's functions)



## 2. Inverse Wavefields (focusing functions)



## 3. The Marchenko Equation (focusing function retrieval)

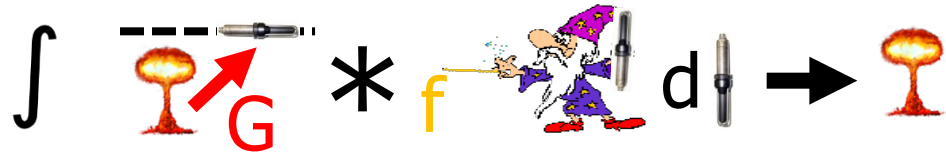
$$y = A f$$
A cartoon wizard character with a purple hat and a white beard, holding a wand that emits a small flame.

# Contents

## 1. Time-Reversed Wavefields (Green's functions)



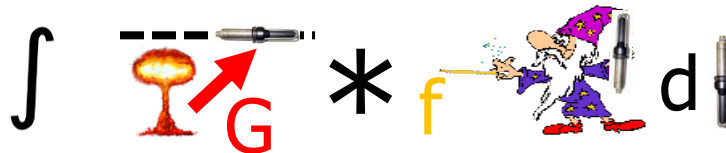
## 2. Inverse Wavefields (focusing functions)



## 3. The Marchenko Equation (focusing function retrieval)



## 4. Source Wavefield Imaging (Single-sided focusing)



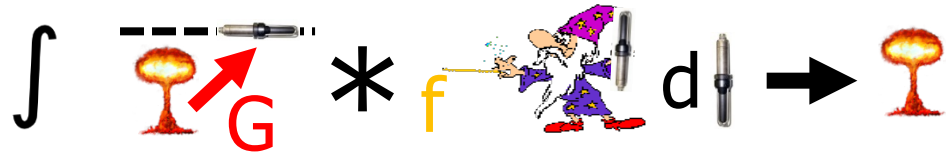


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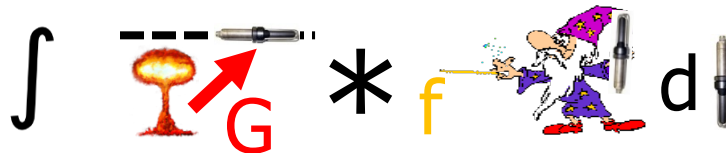
## 2. Inverse Wavefields (focusing functions)



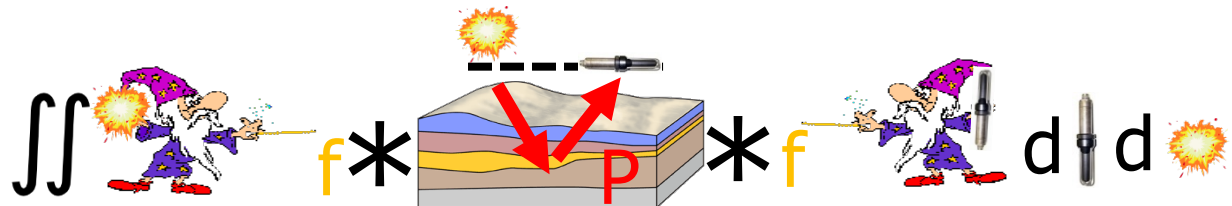
## 3. The Marchenko Equation (focusing function retrieval)



## 4. Source Wavefield Imaging (Single-sided focusing)

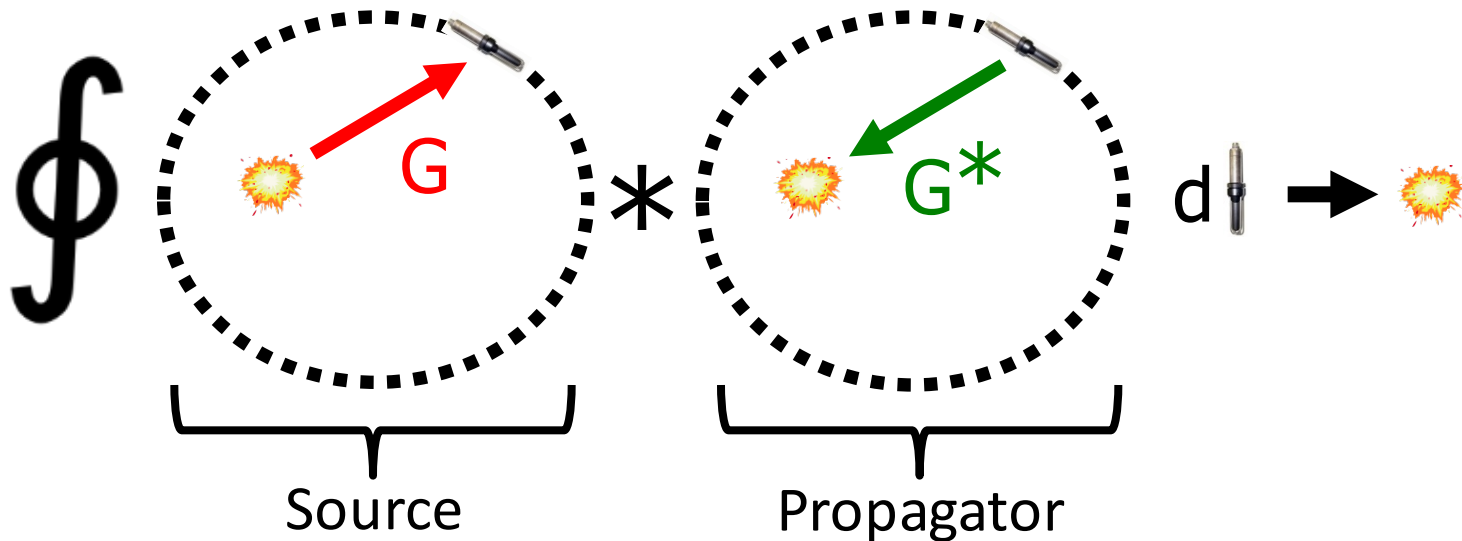
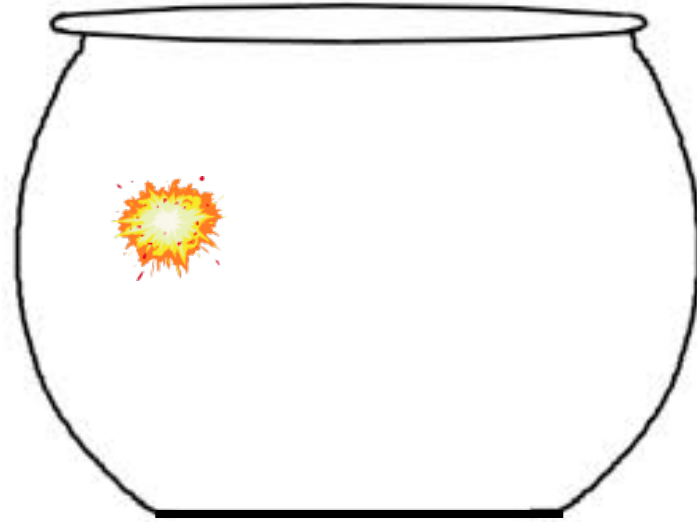


## 5. Reflectivity Imaging (Double-sided focusing)



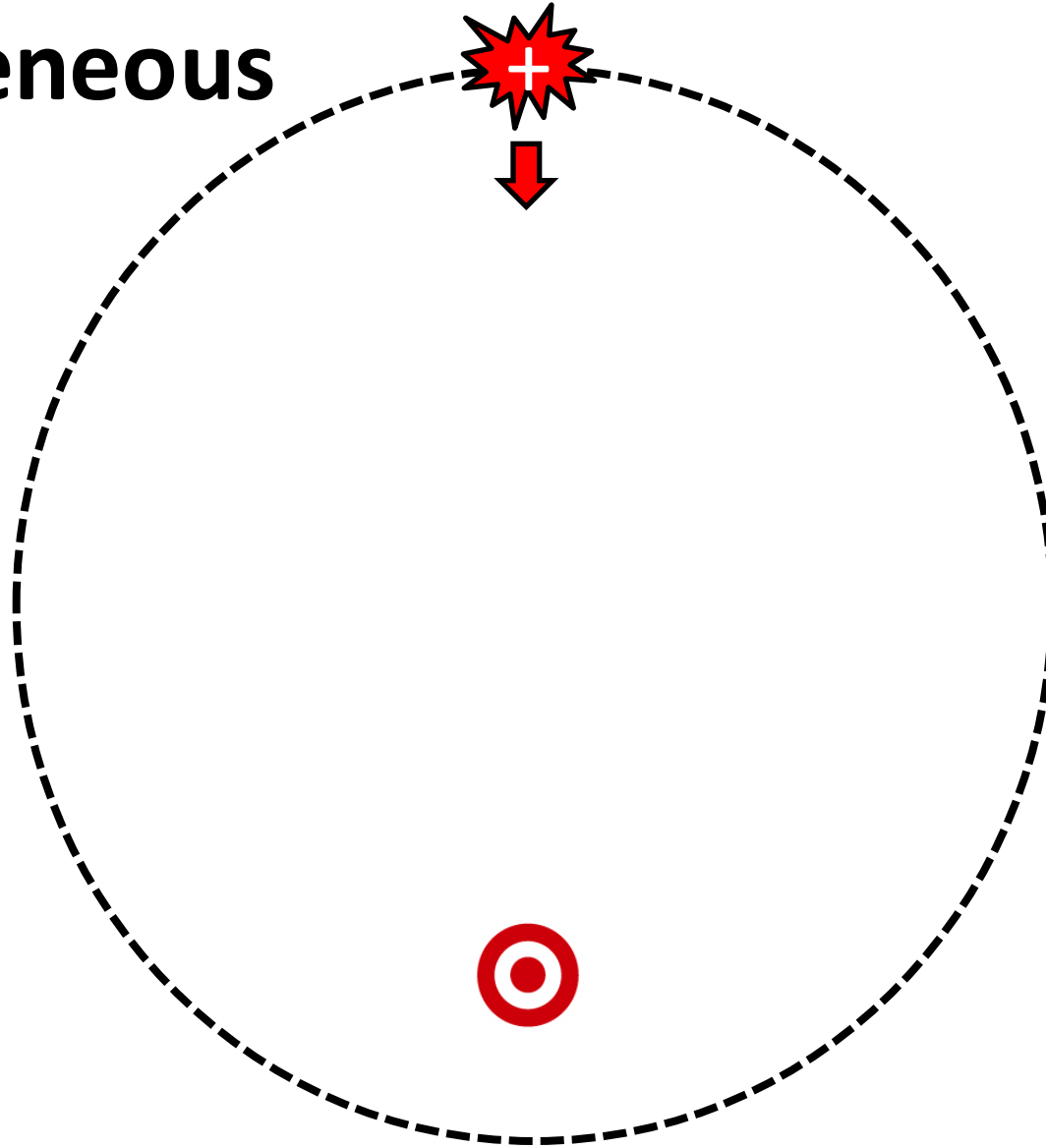
# Time-Reversed Wavefields

Homogeneous  
medium



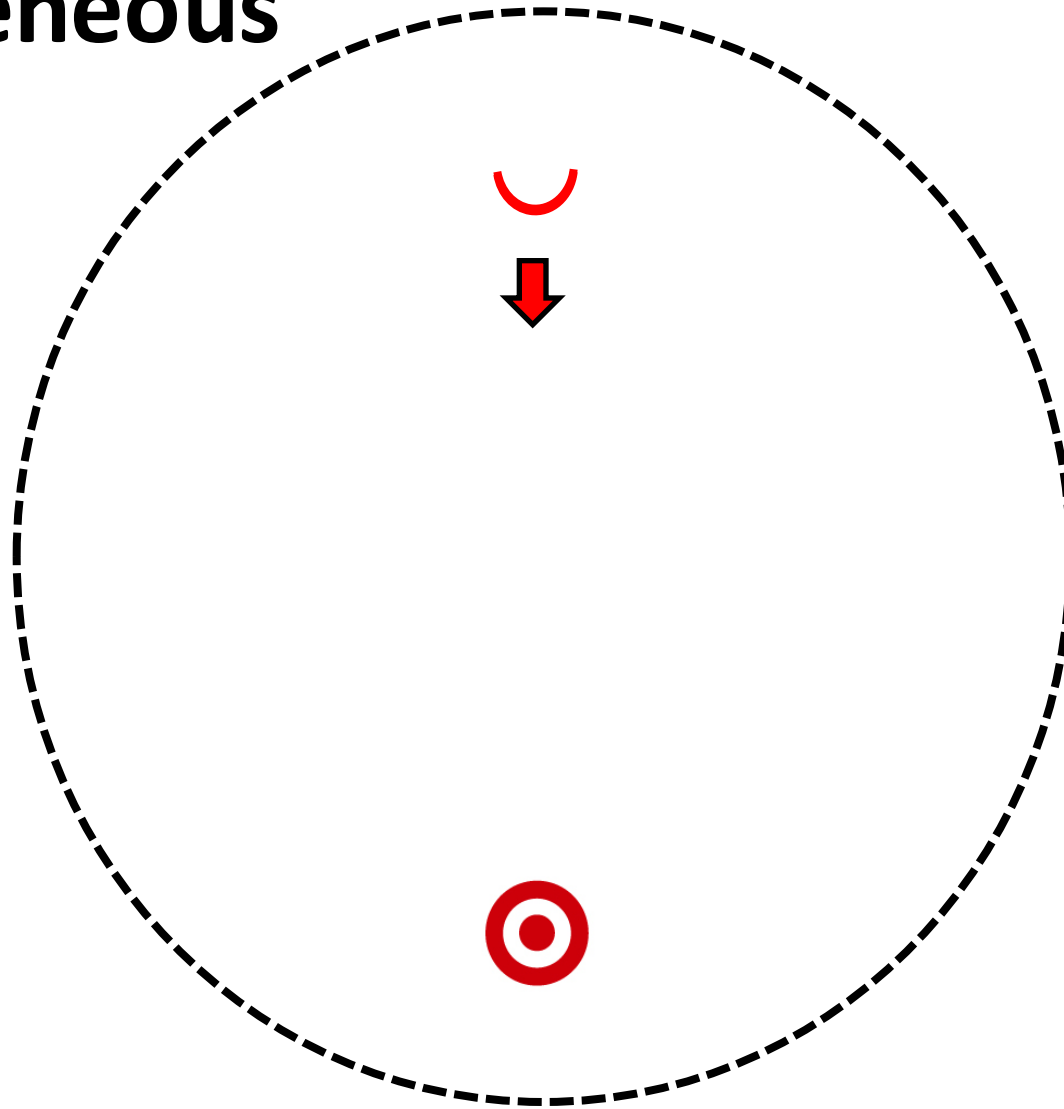
# Time-Reversed Wavefields

Homogeneous  
medium



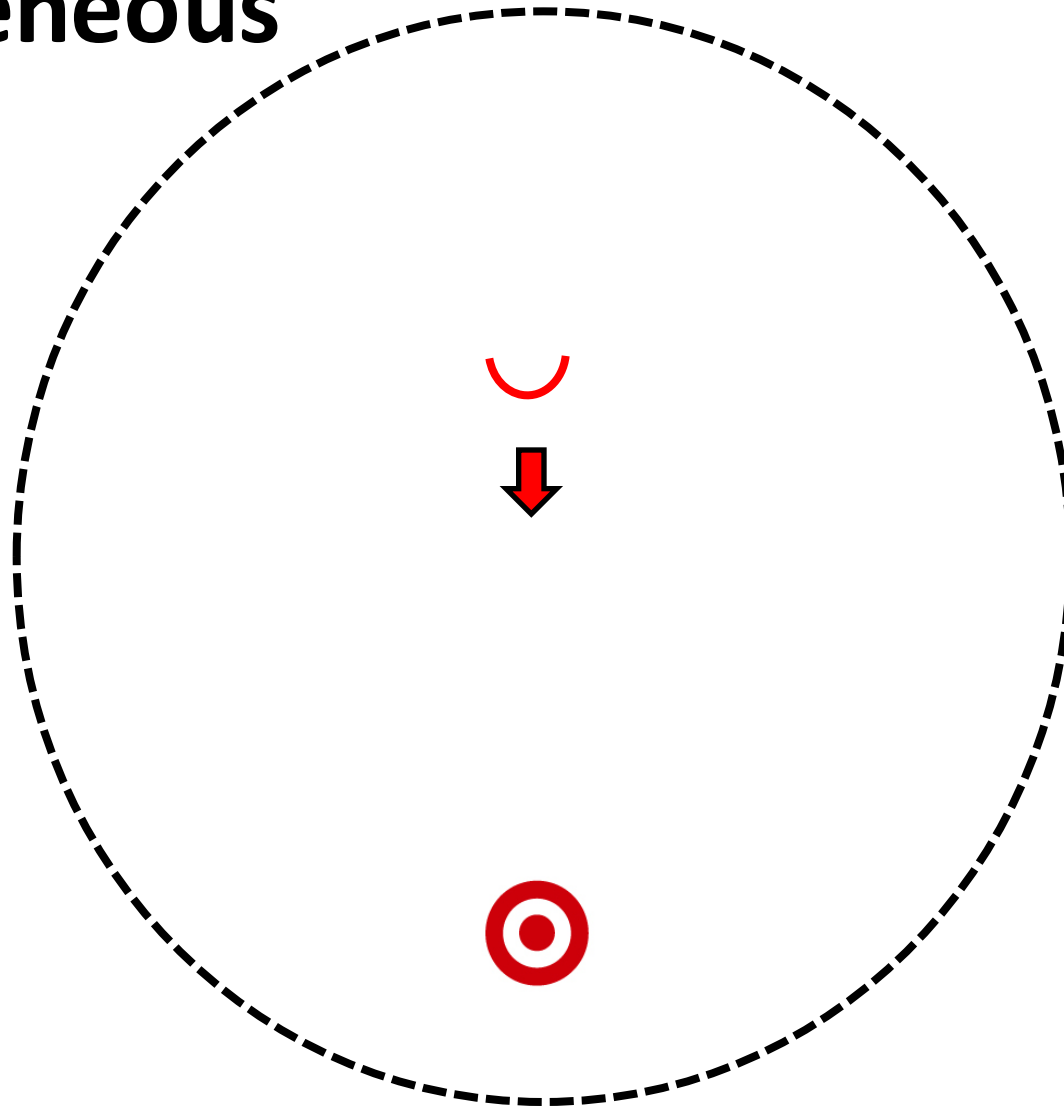
# Time-Reversed Wavefields

Homogeneous  
medium



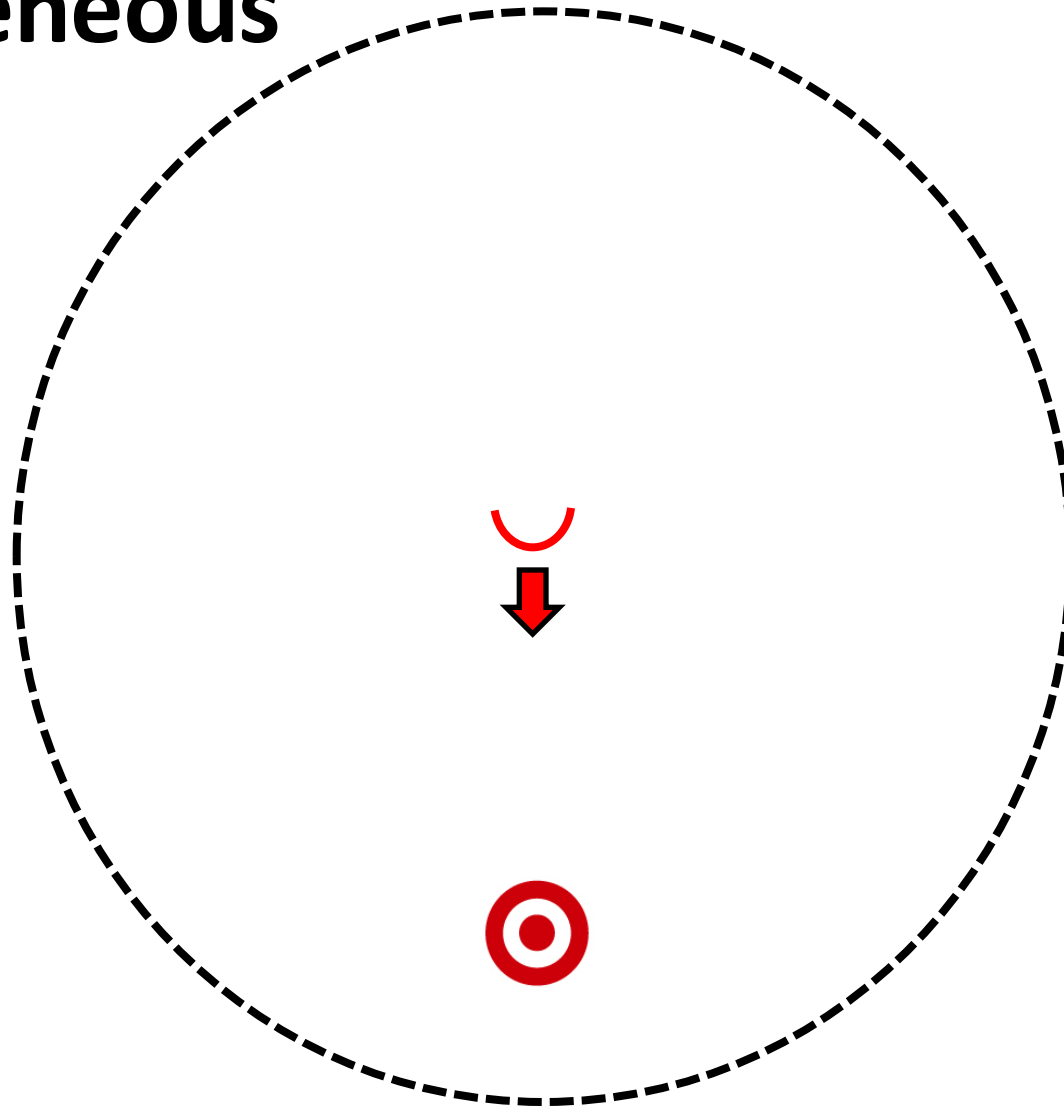
# Time-Reversed Wavefields

Homogeneous  
medium



# Time-Reversed Wavefields

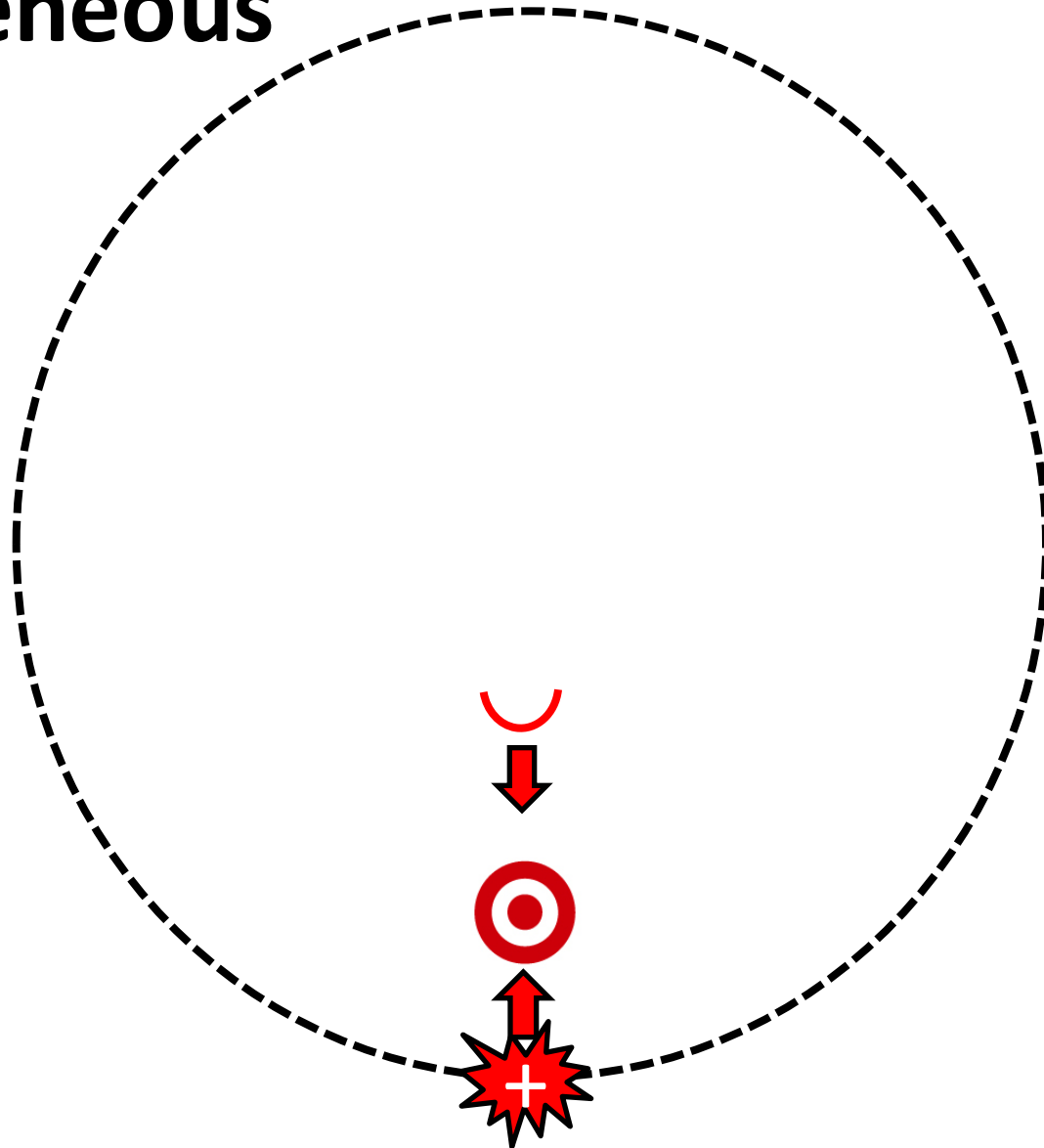
Homogeneous  
medium





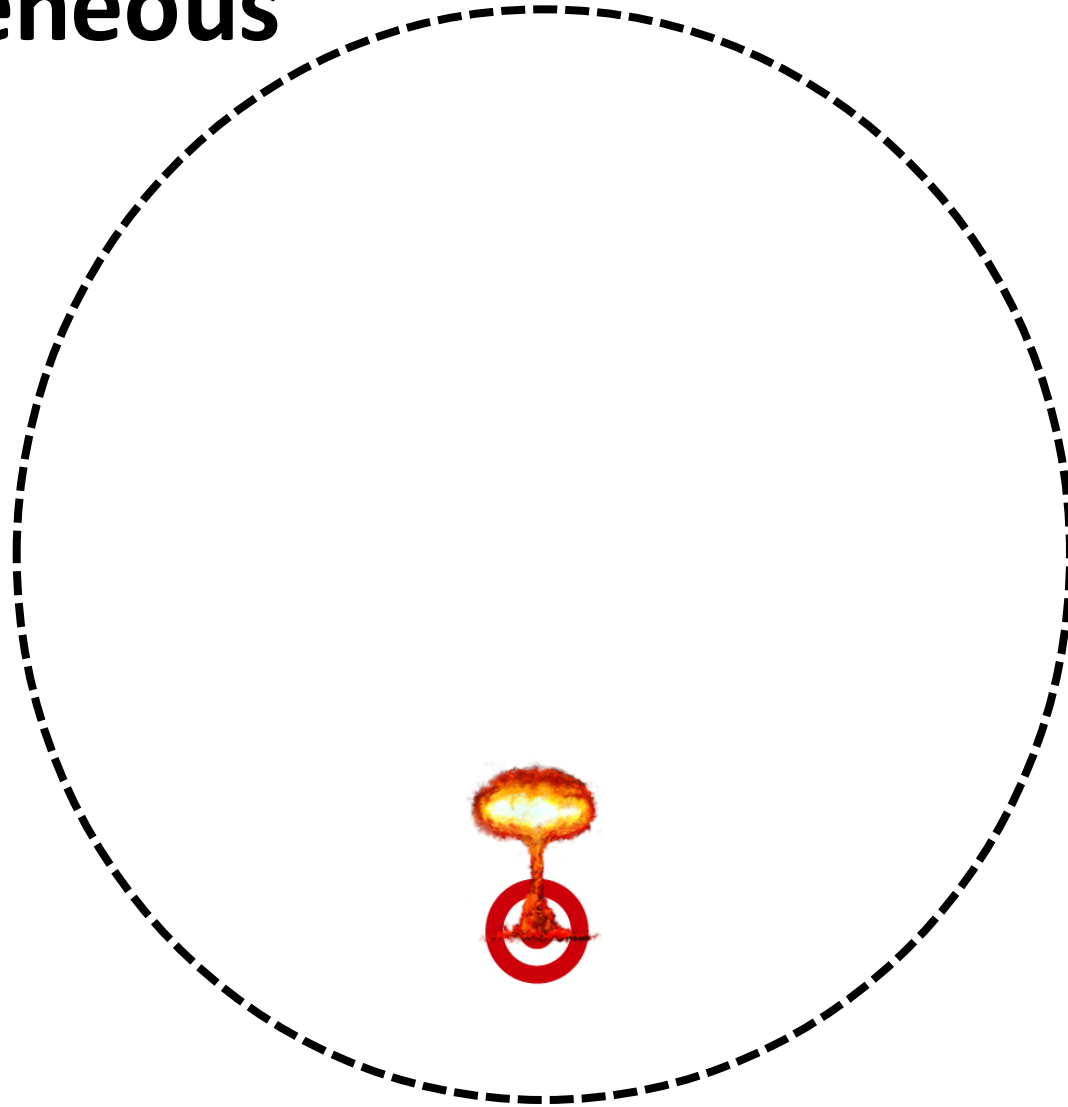
# Time-Reversed Wavefields

Homogeneous  
medium



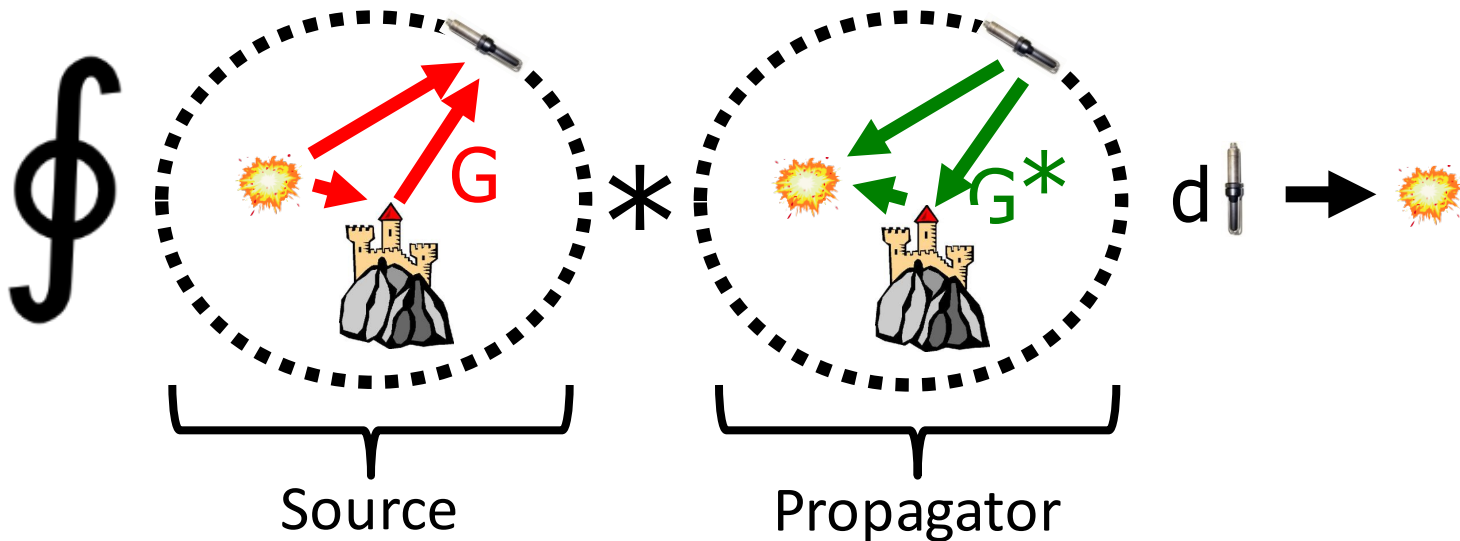
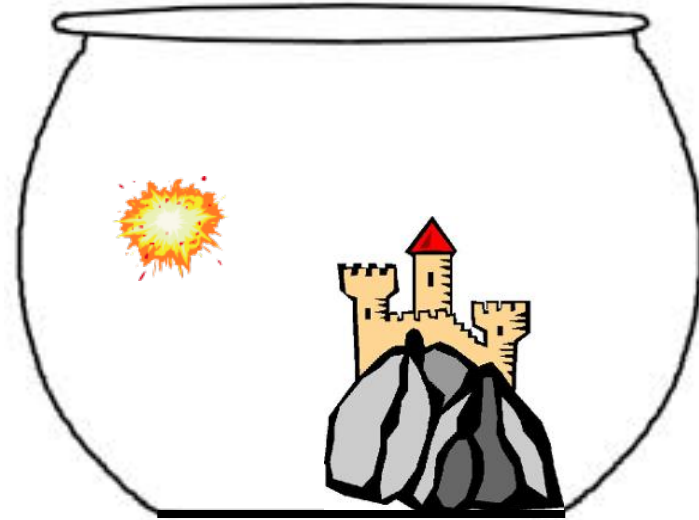
# Time-Reversed Wavefields

Homogeneous  
medium



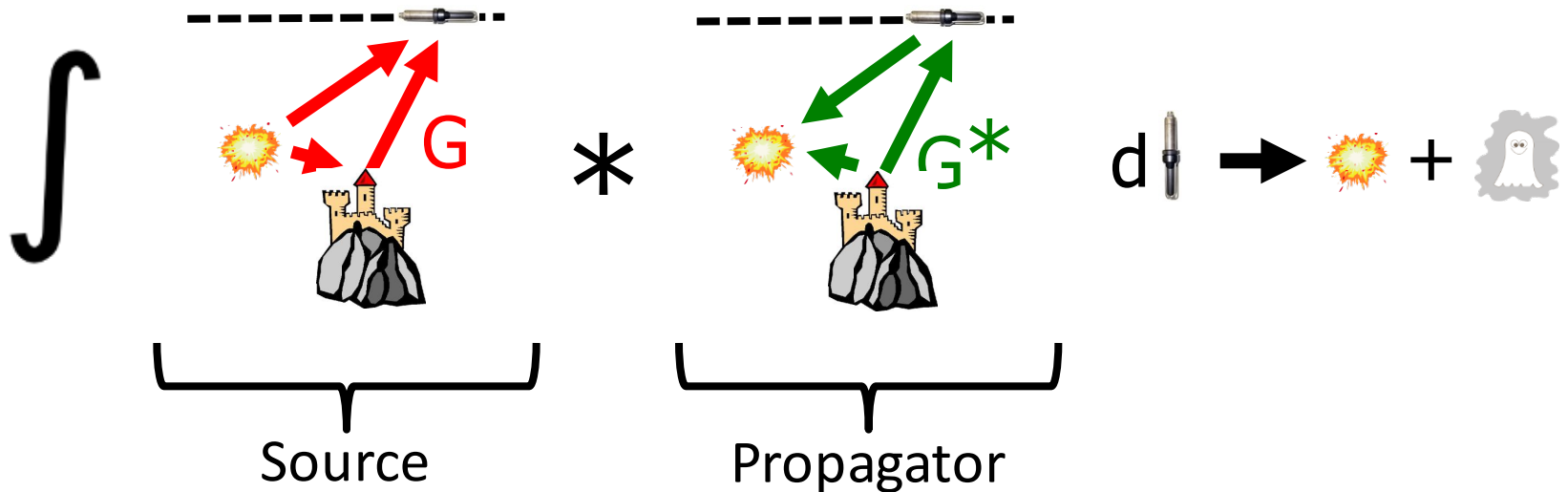
# Time-Reversed Wavefields

Heterogeneous  
medium



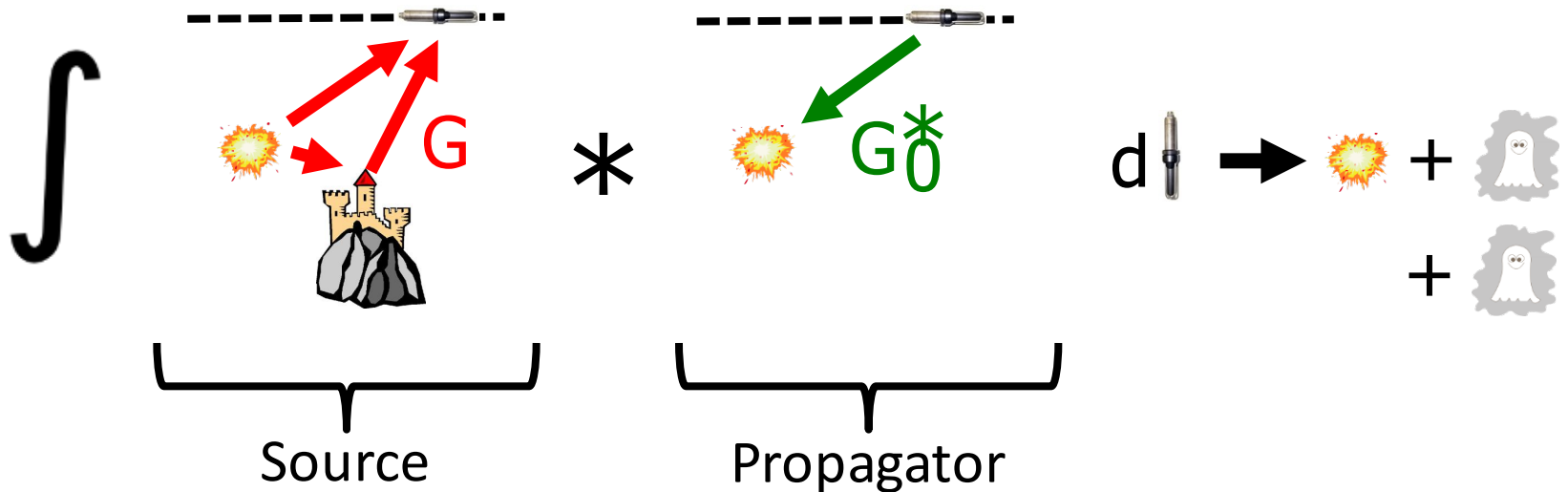
# Time-Reversed Wavefields

## 1. Single-sided illumination



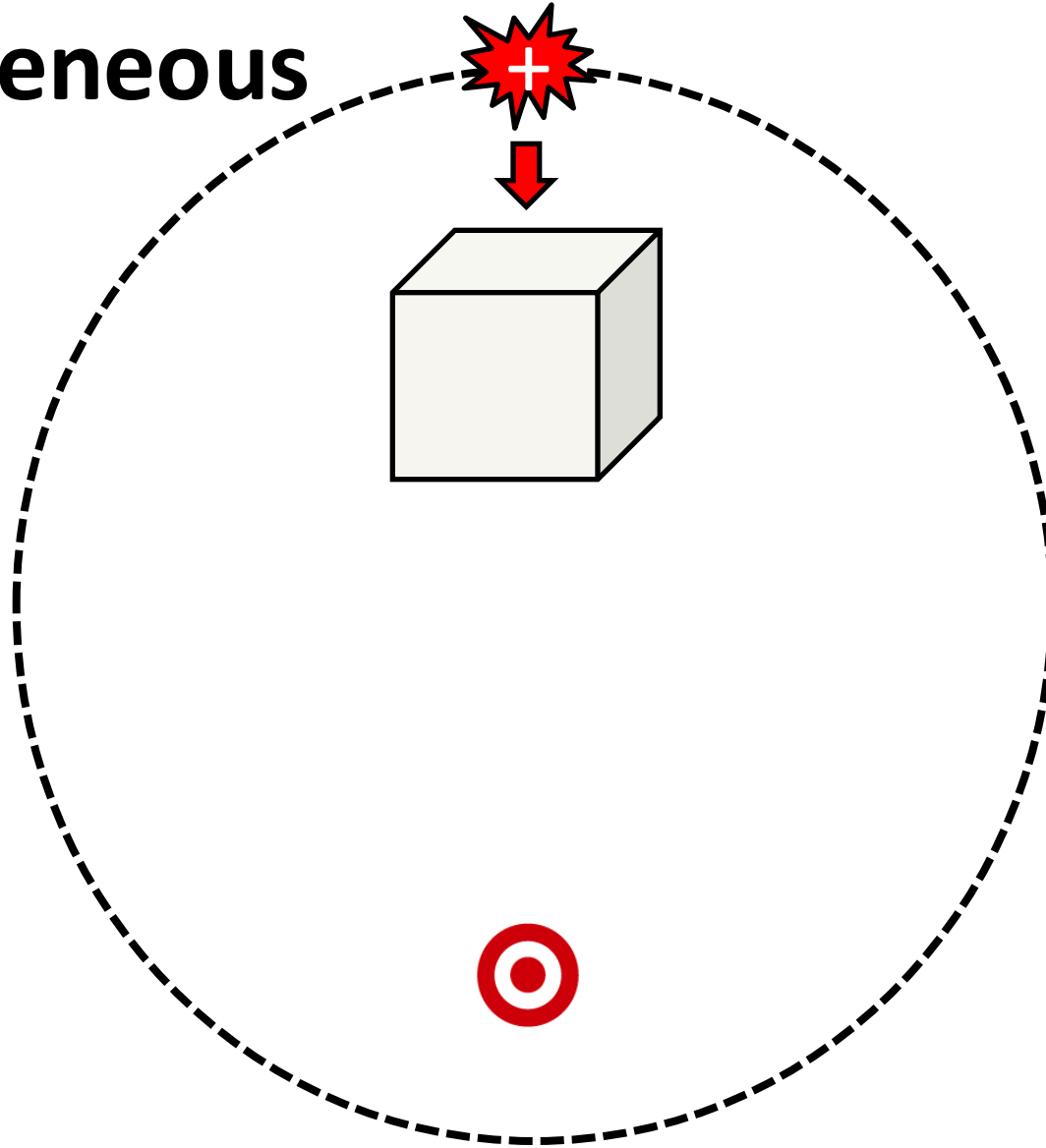
# Time-Reversed Wavefields

1. Single-sided illumination
2. Smooth macro model



# Time-Reversed Wavefields

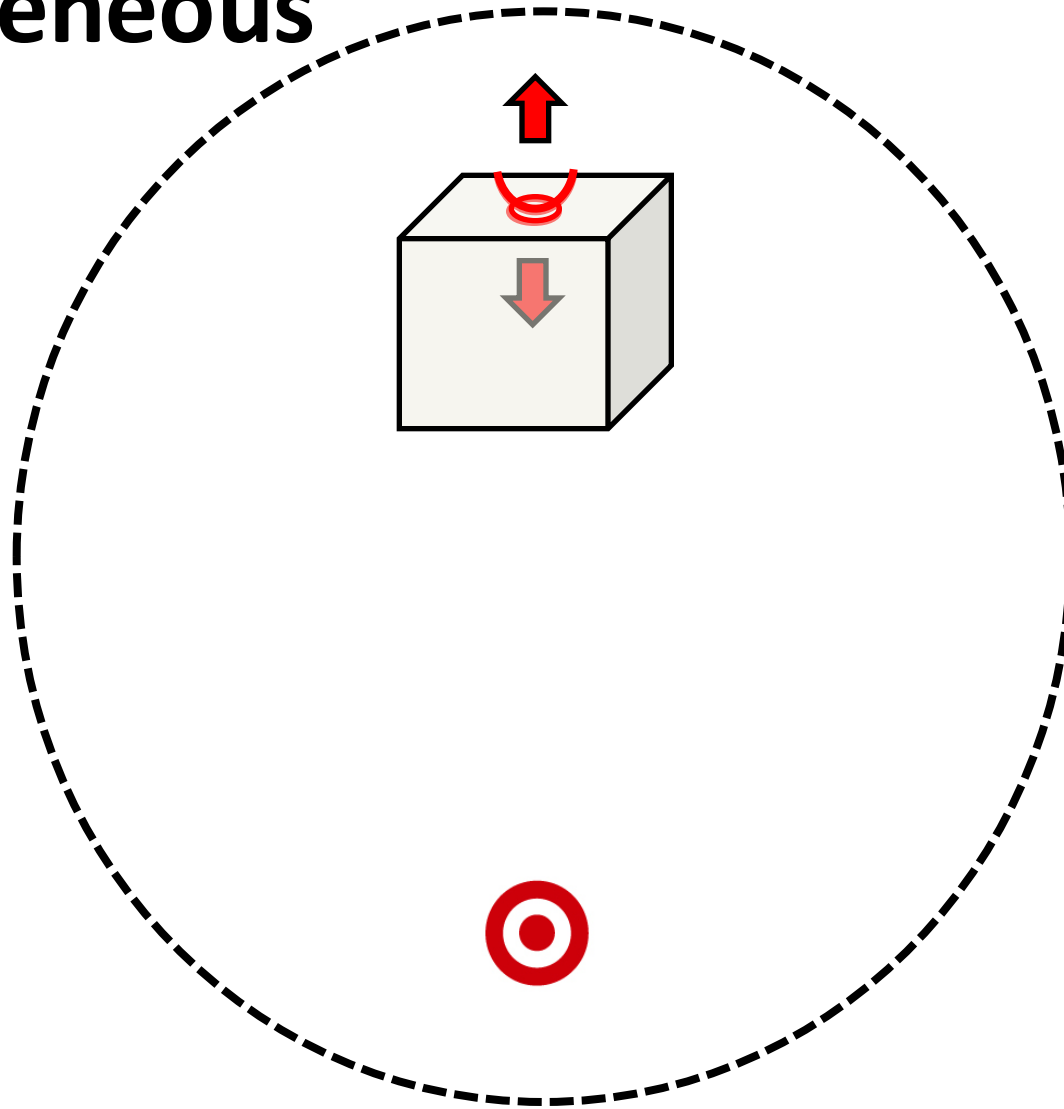
Heterogeneous  
medium





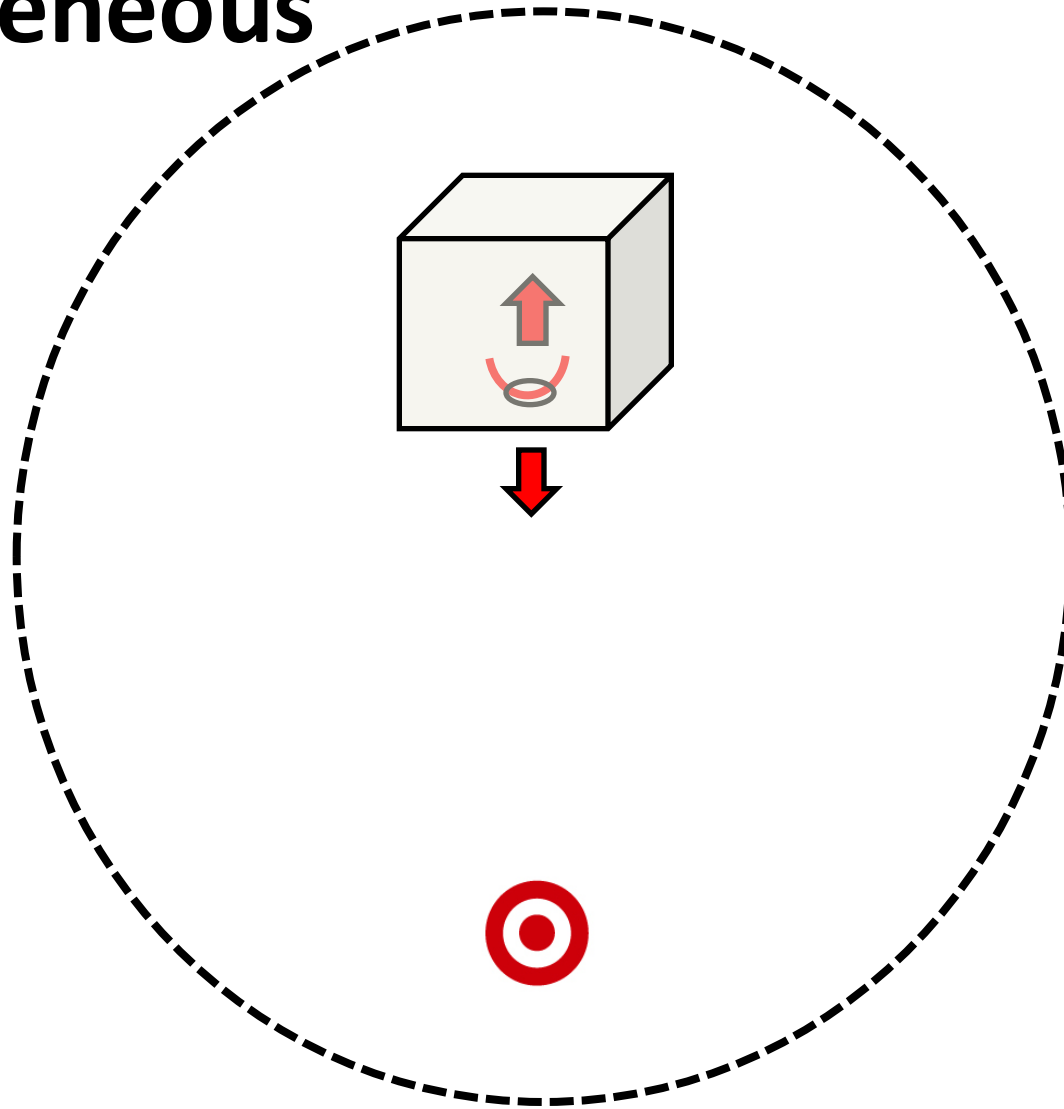
# Time-Reversed Wavefields

Heterogeneous  
medium



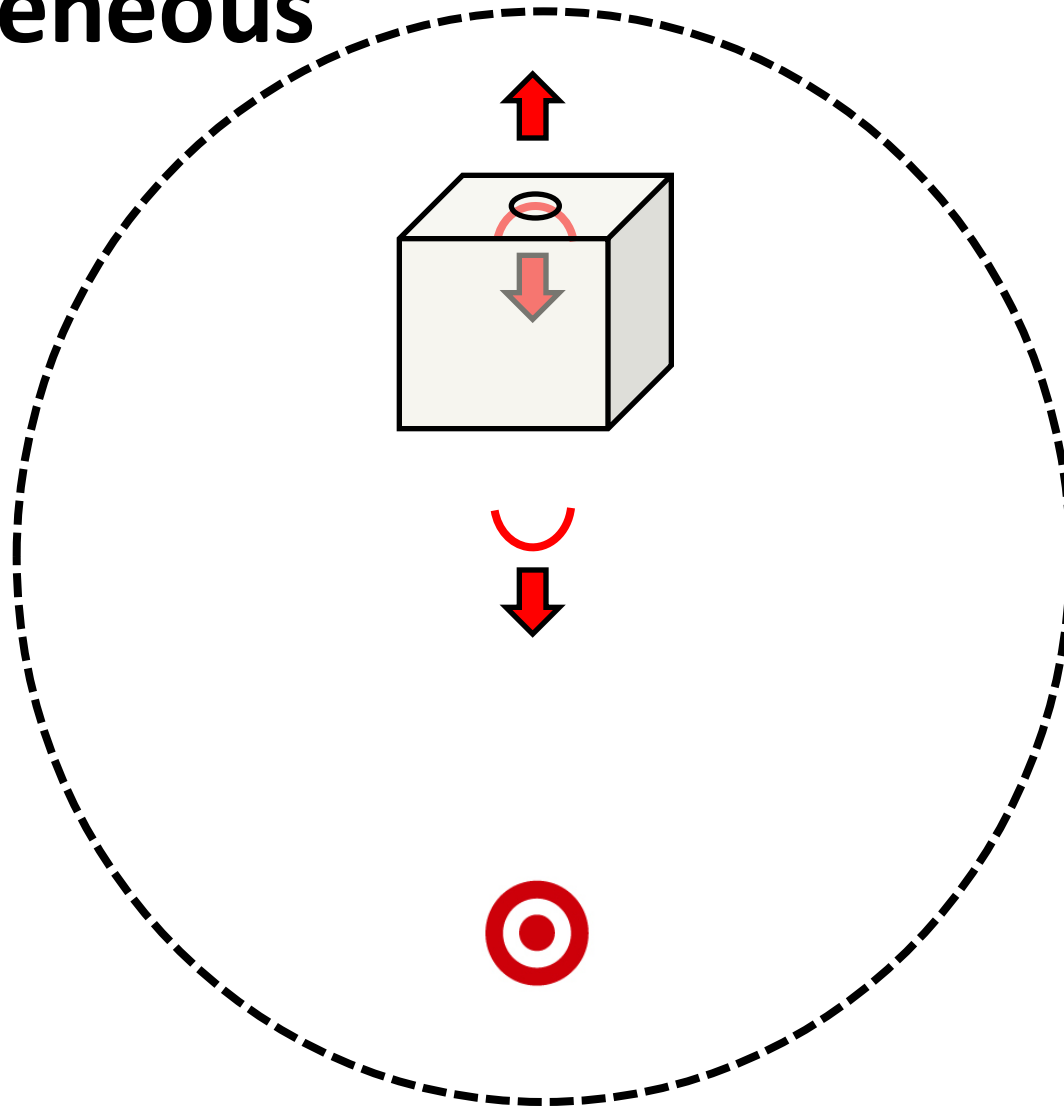
# Time-Reversed Wavefields

Heterogeneous  
medium



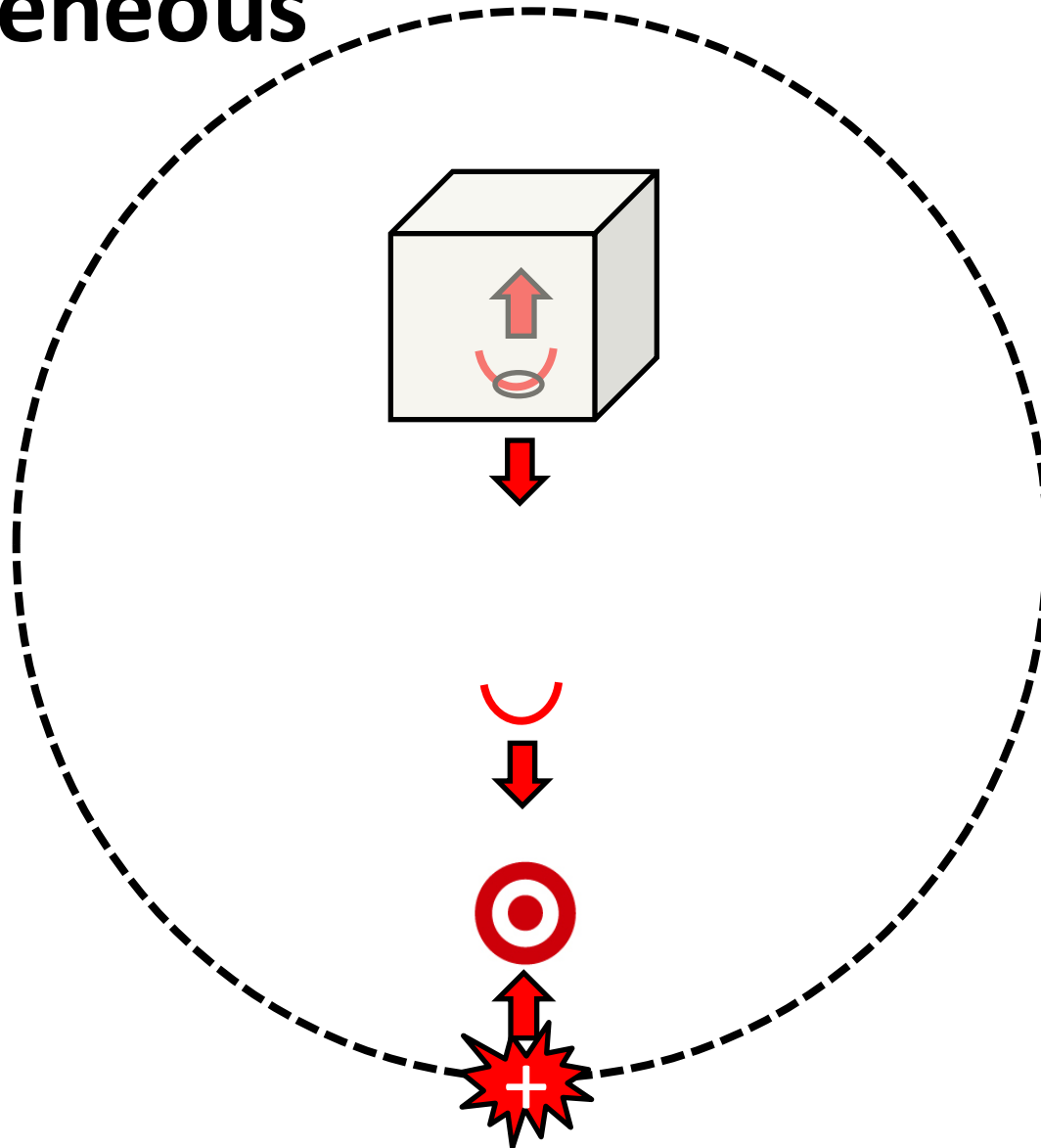
# Time-Reversed Wavefields

Heterogeneous  
medium



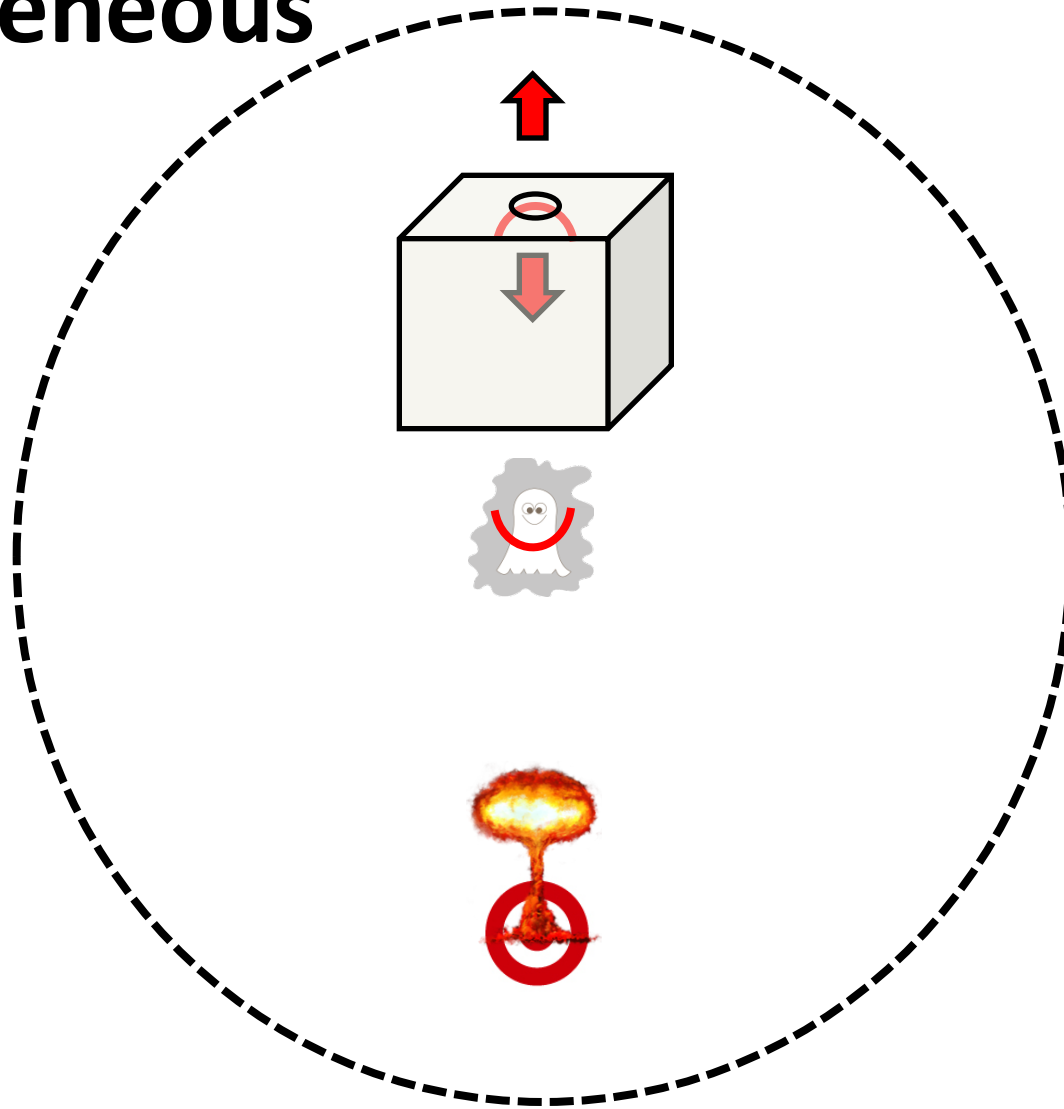
# Time-Reversed Wavefields

Heterogeneous  
medium



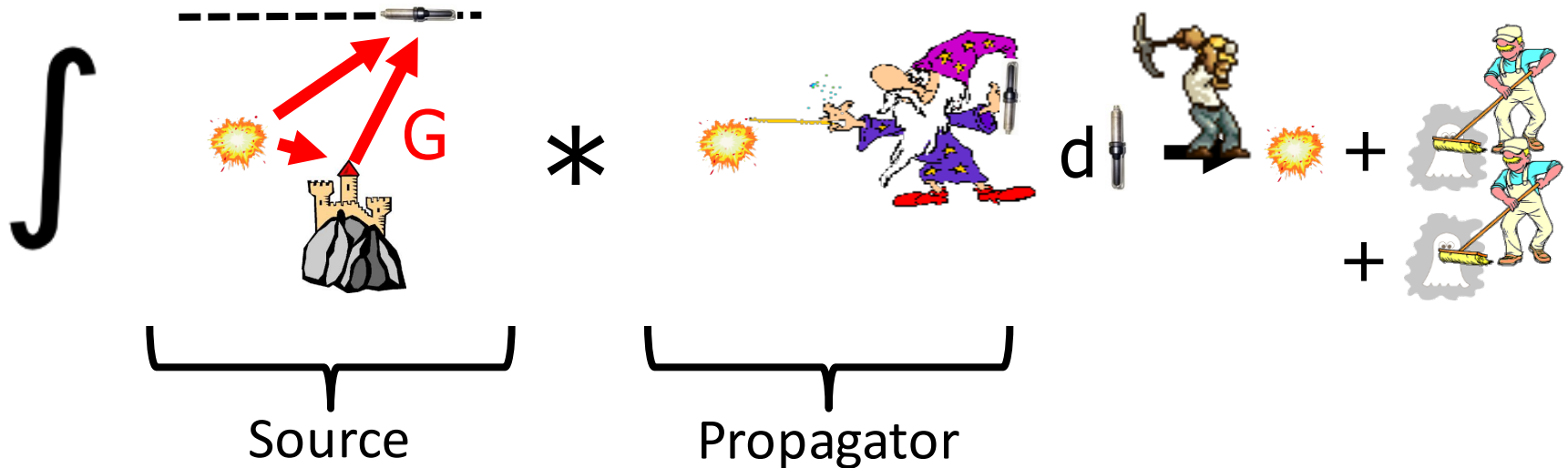
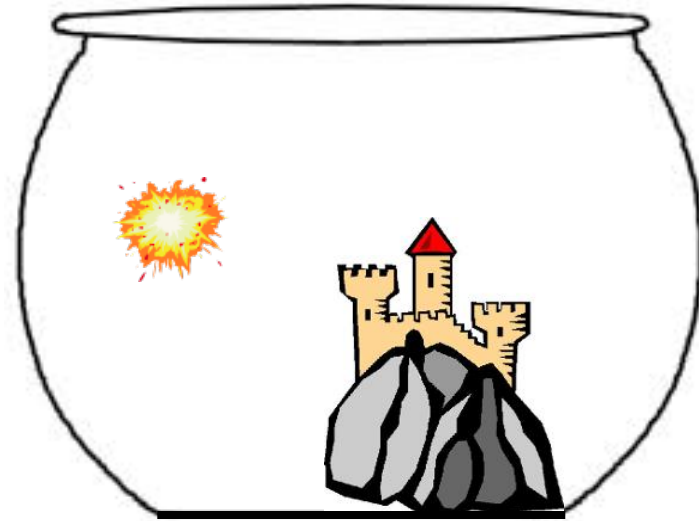
# Time-Reversed Wavefields

Heterogeneous  
medium



# Inverse Wavefields

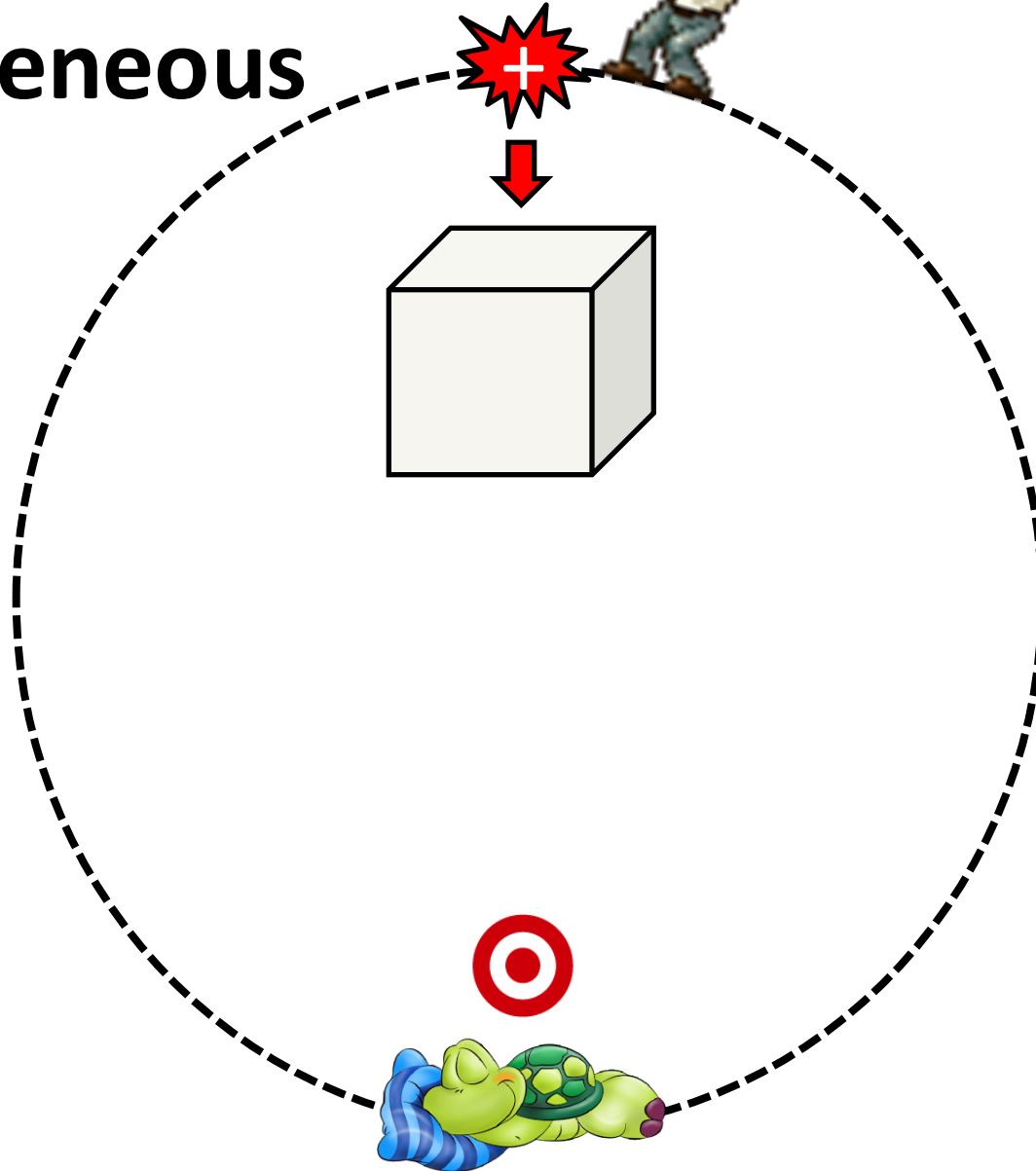
Heterogeneous  
medium





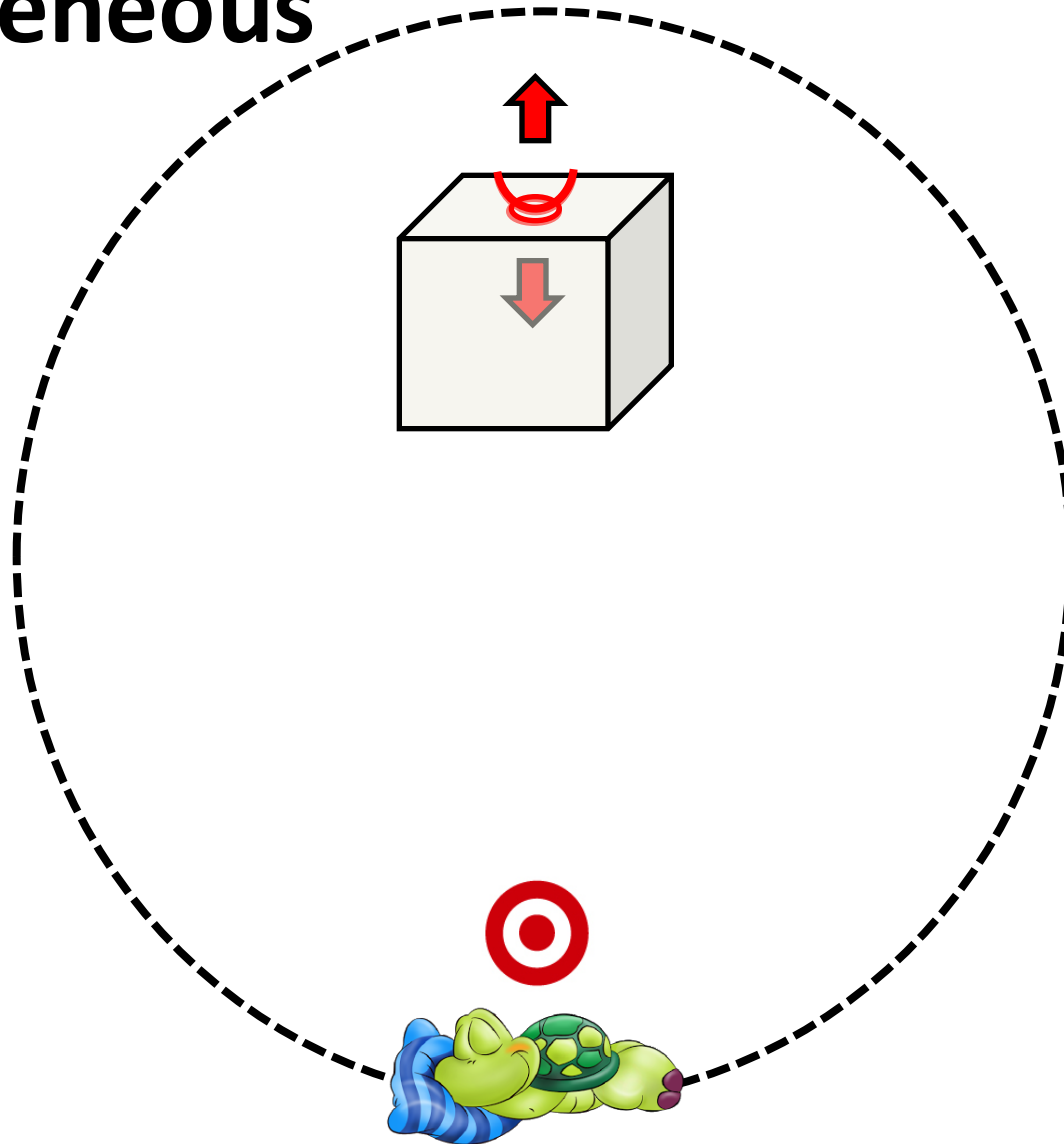
# Inverse Wavefields

Heterogeneous  
medium



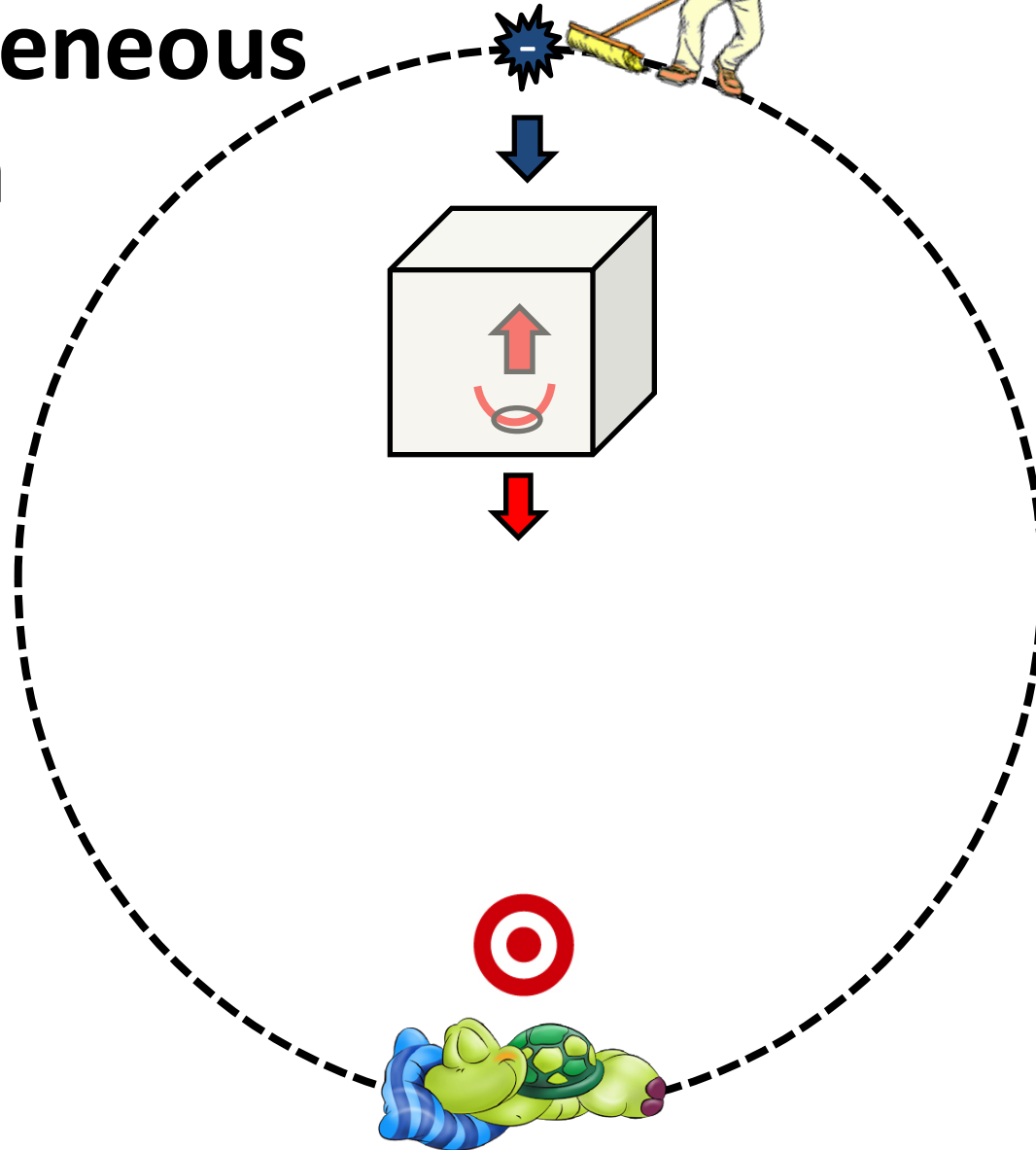
# Inverse Wavefields

Heterogeneous  
medium



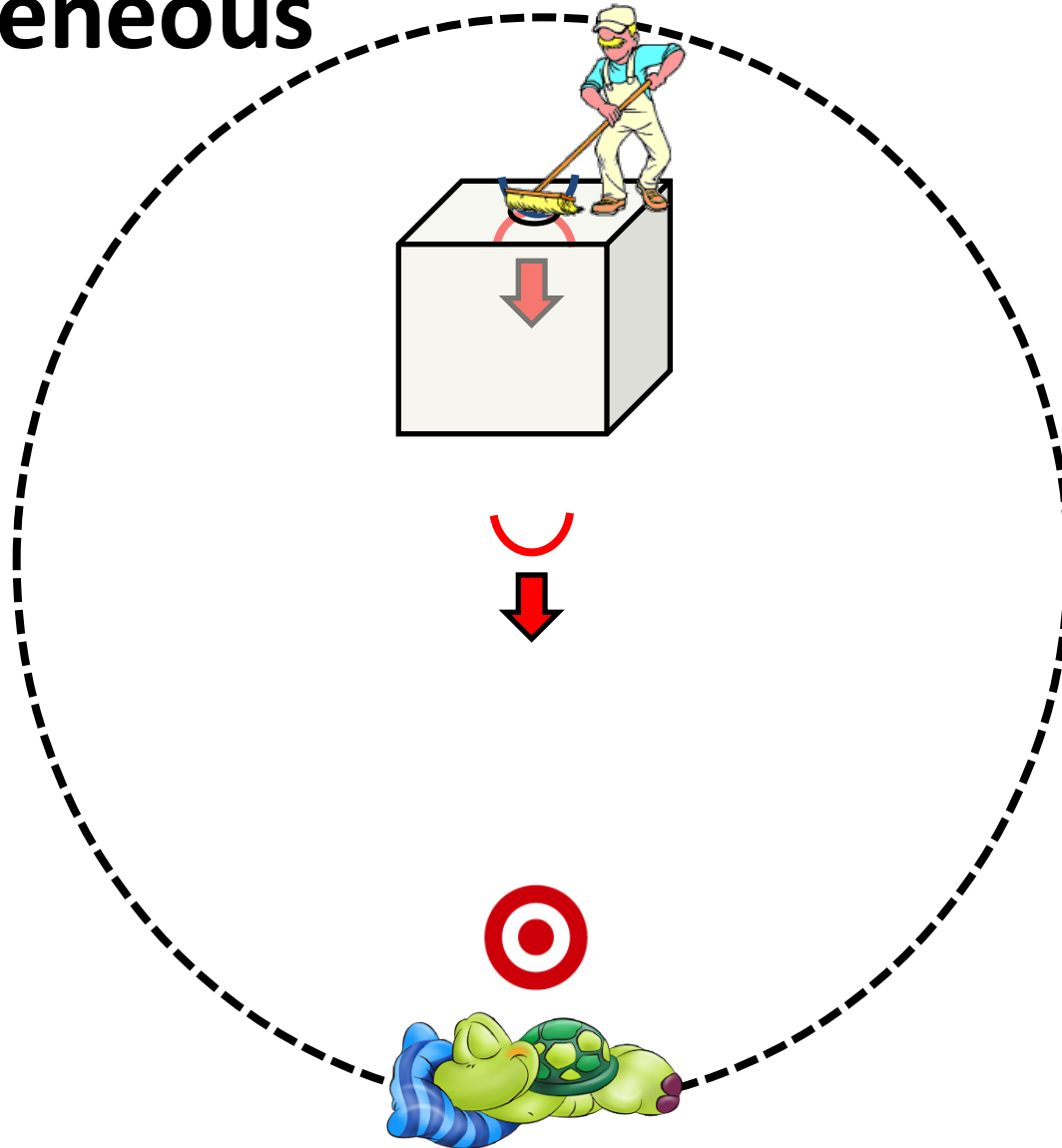
# Inverse Wavefields

Heterogeneous  
medium



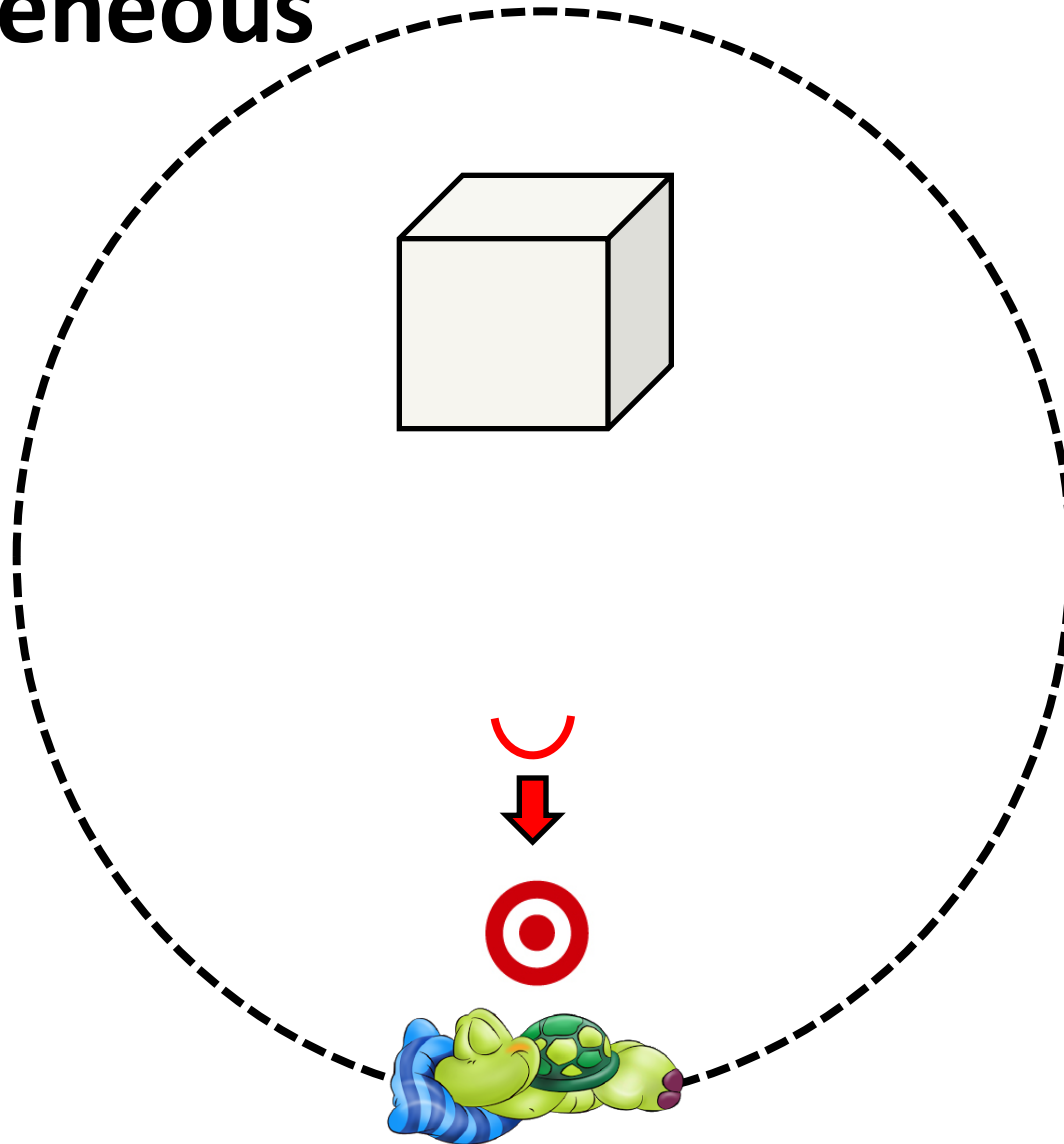
# Inverse Wavefields

Heterogeneous  
medium



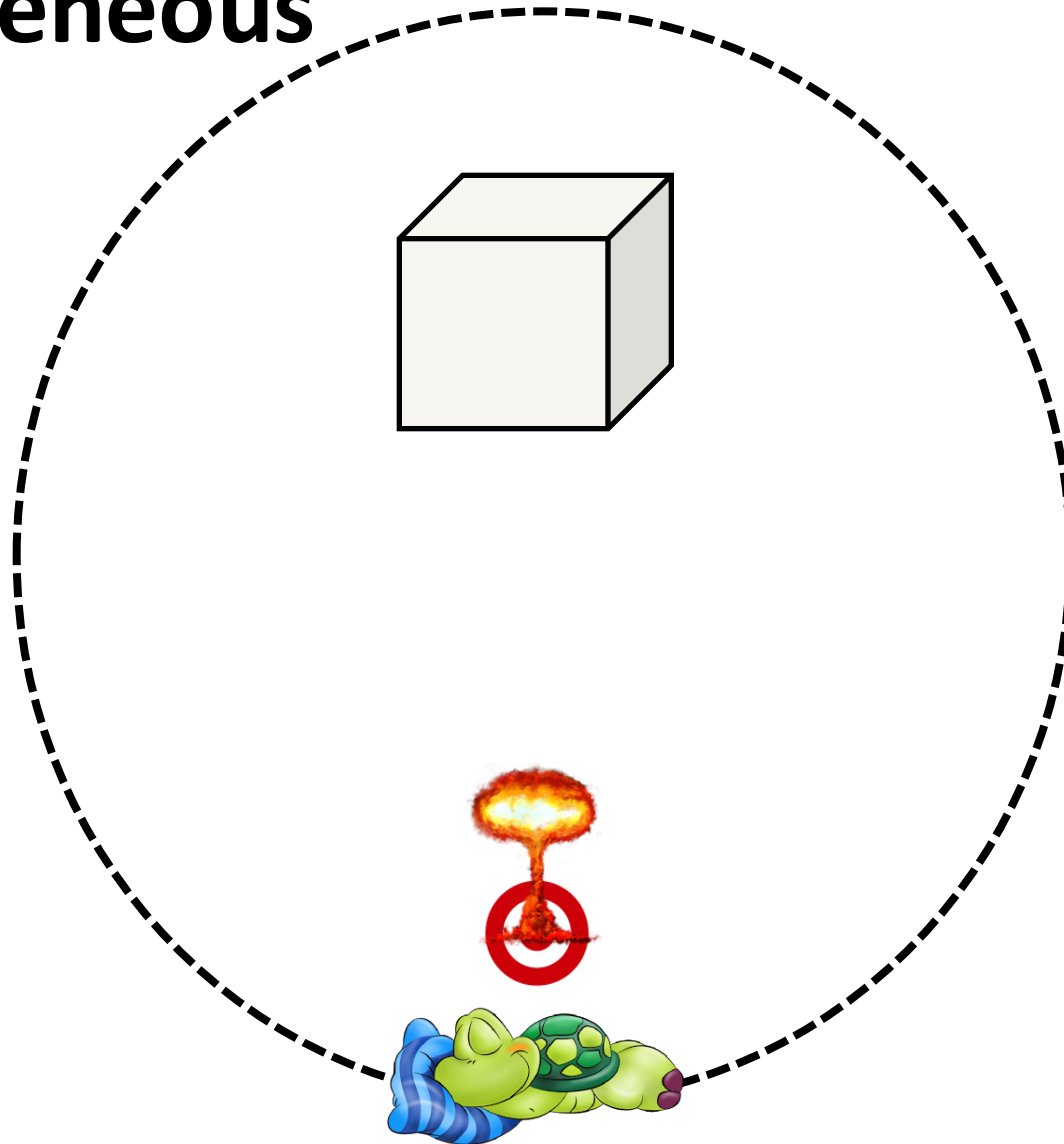
# Inverse Wavefields

Heterogeneous  
medium

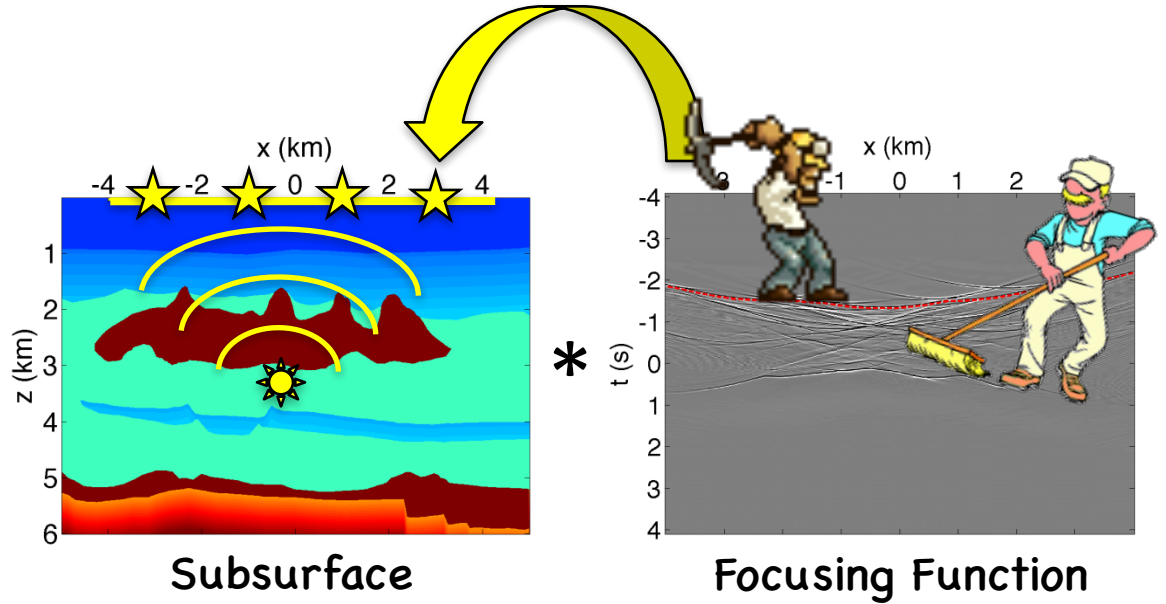


# Inverse Wavefields

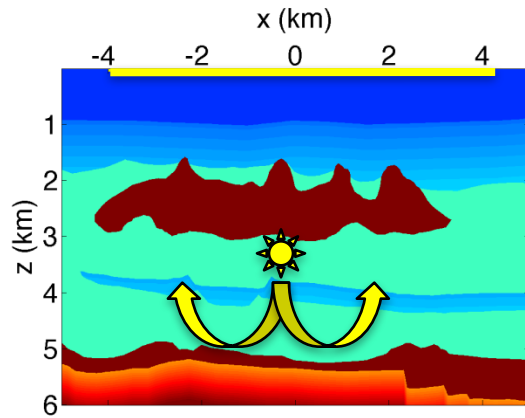
Heterogeneous  
medium



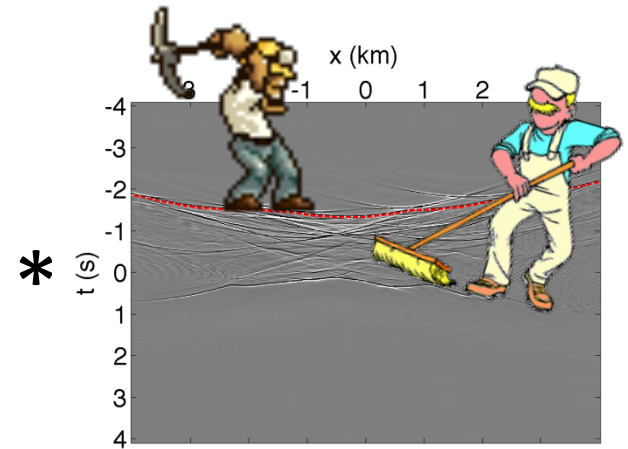
# The Marchenko Equation



# The Marchenko Equation



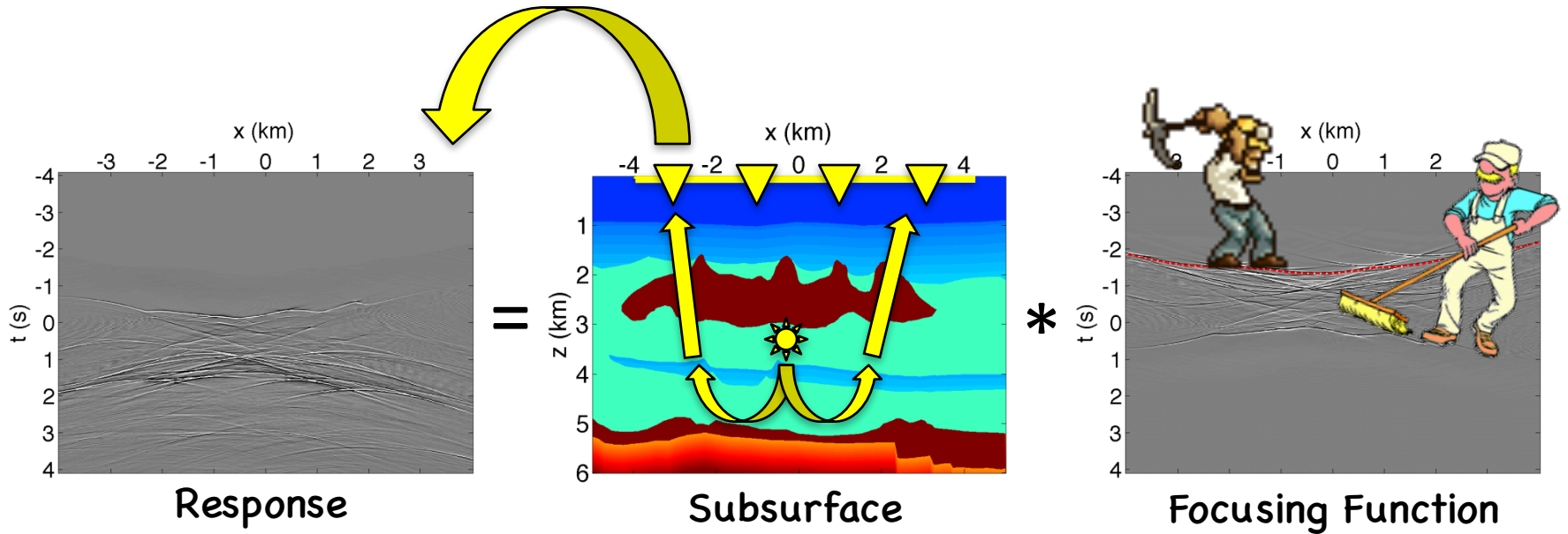
Subsurface



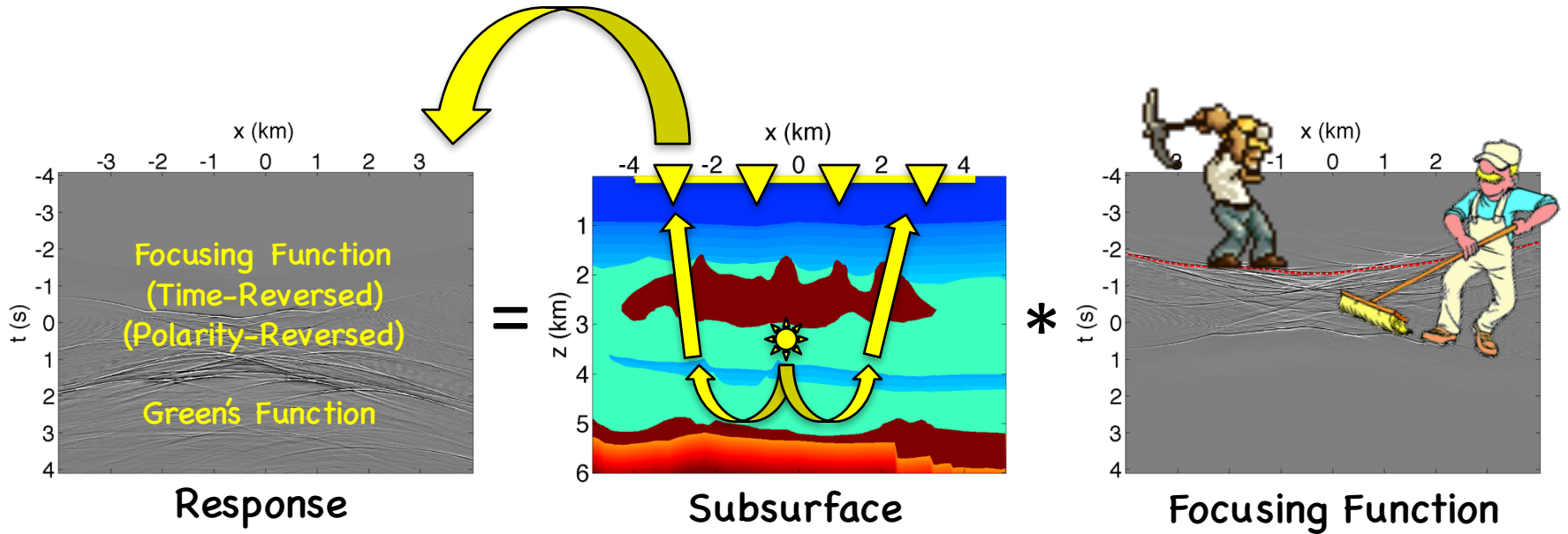
Focusing Function



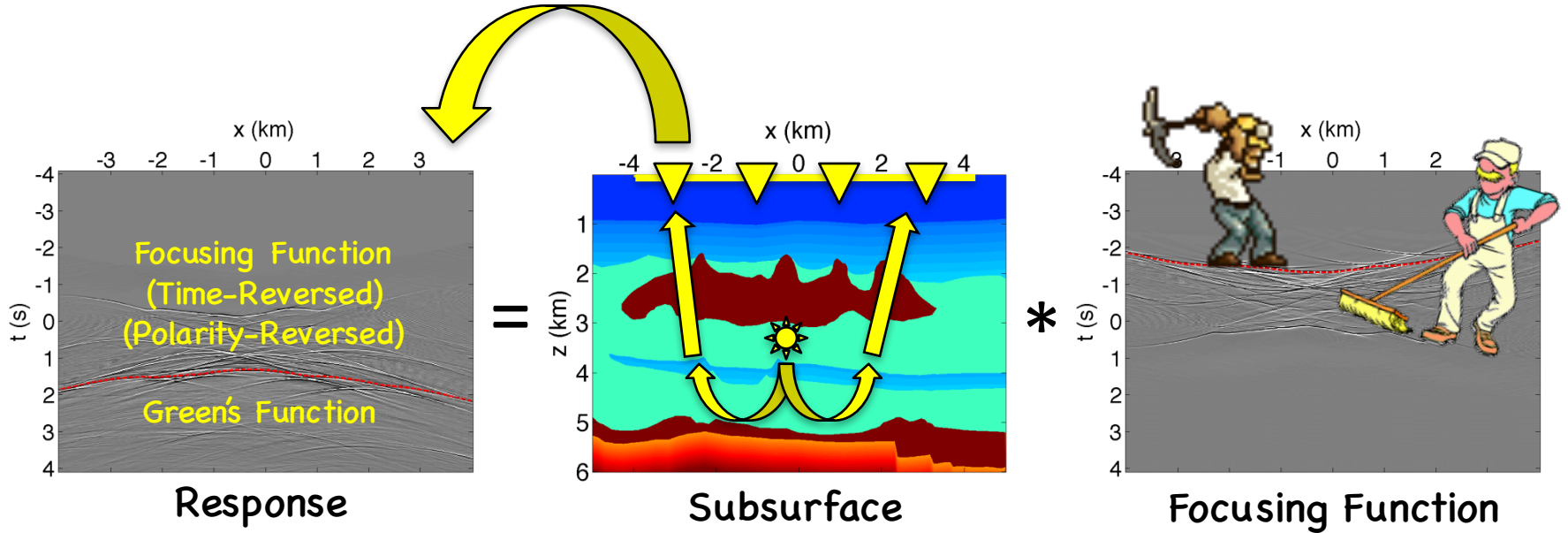
# The Marchenko Equation



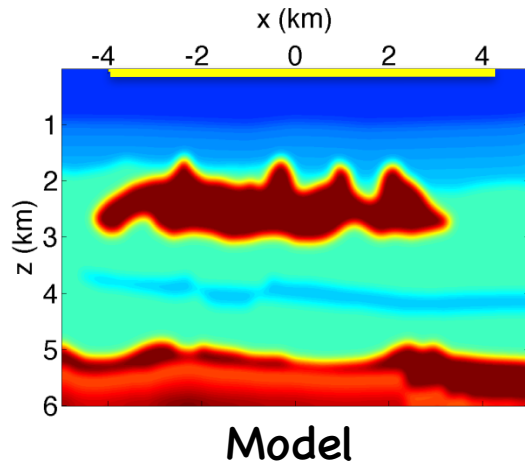
# The Marchenko Equation



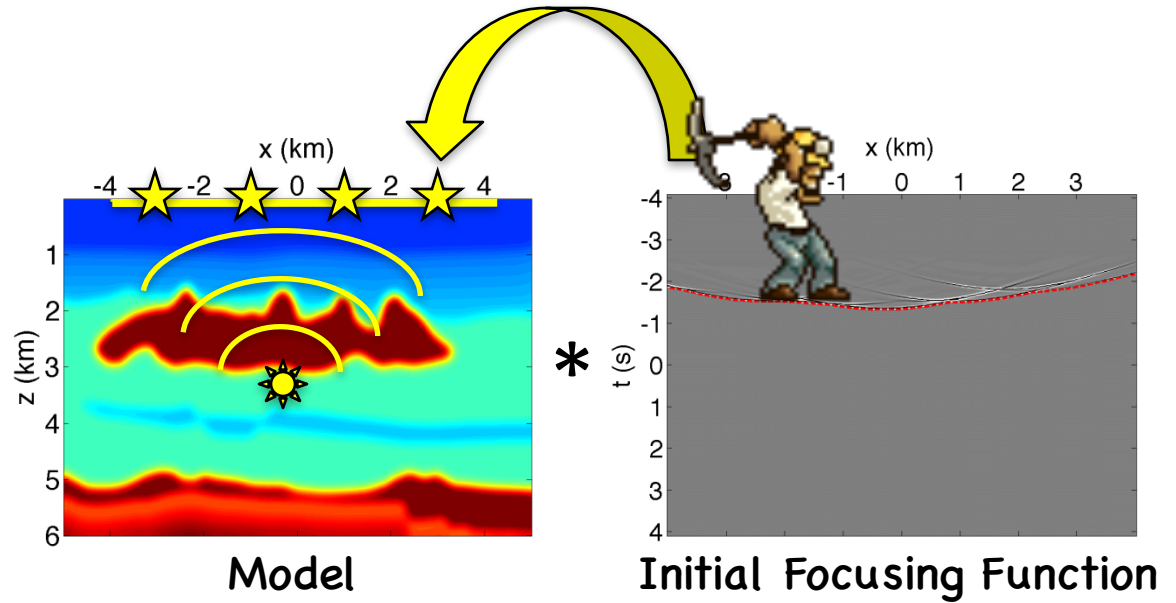
# The Marchenko Equation



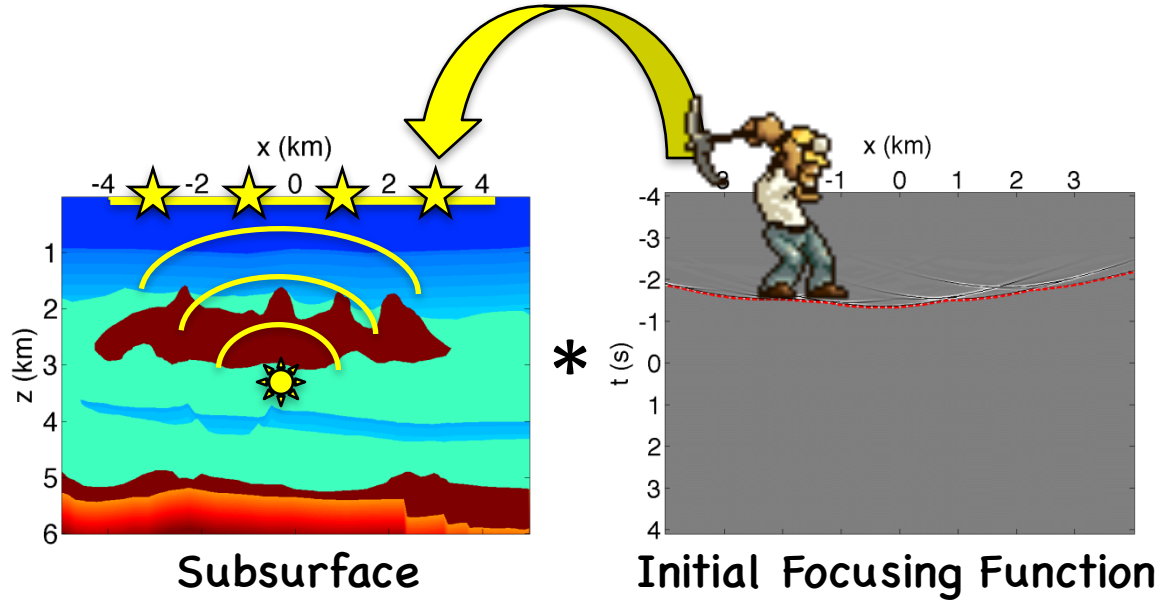
# The Marchenko Equation



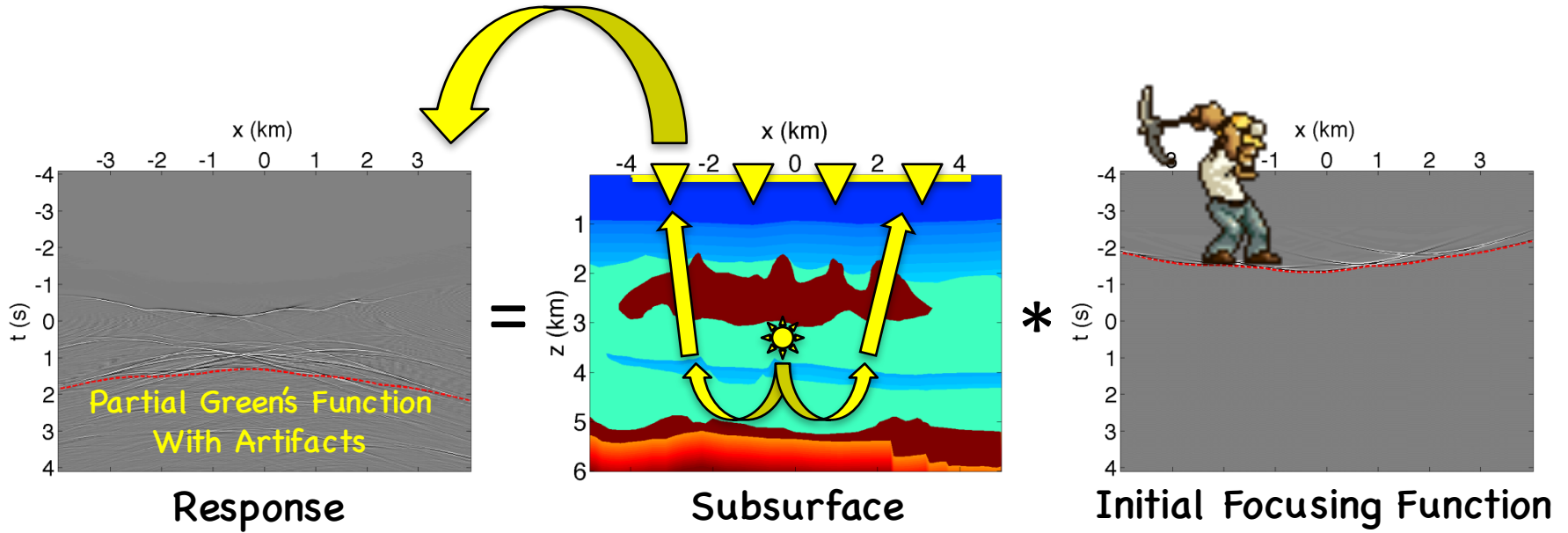
# The Marchenko Equation



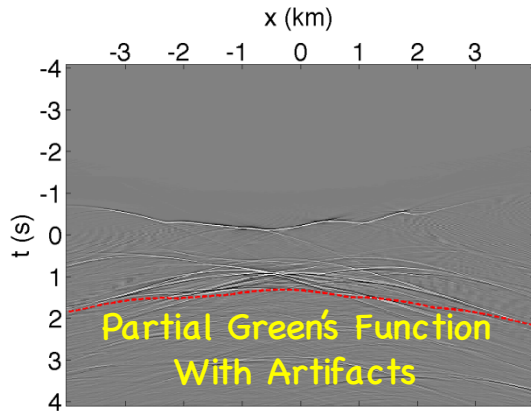
# The Marchenko Equation



# The Marchenko Equation

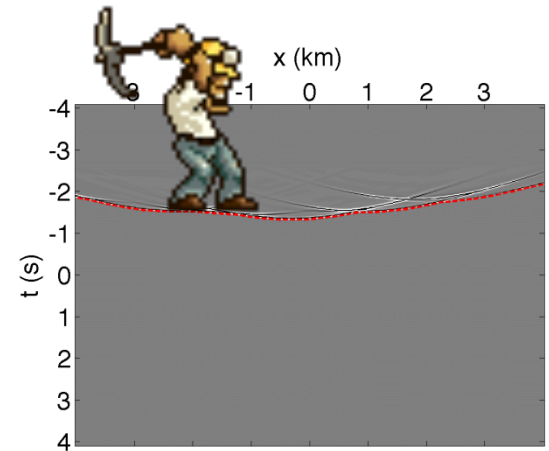


# The Marchenko Equation



Response

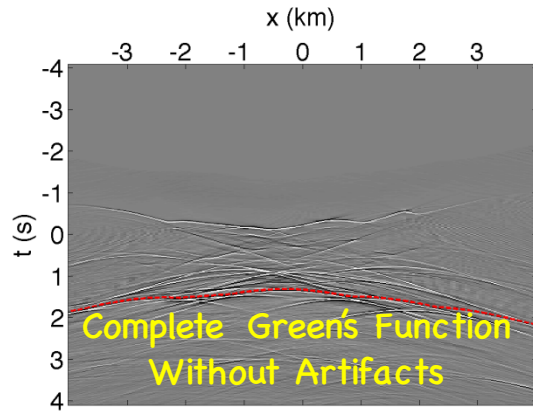
No Time-Reversal  
Mirror



Initial Focusing Function

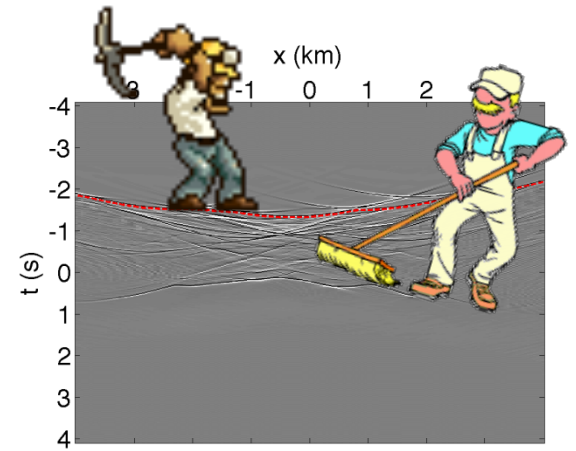


# The Marchenko Equation



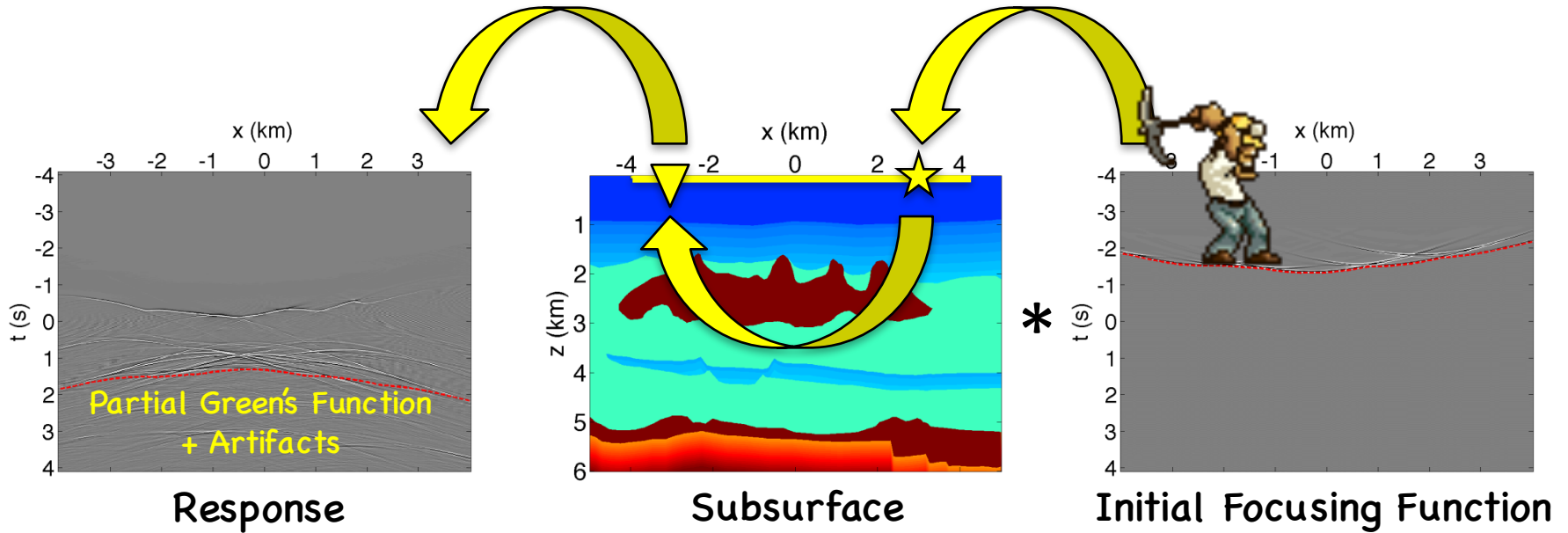
Response

Enforce Time-Reversal  
Mirror



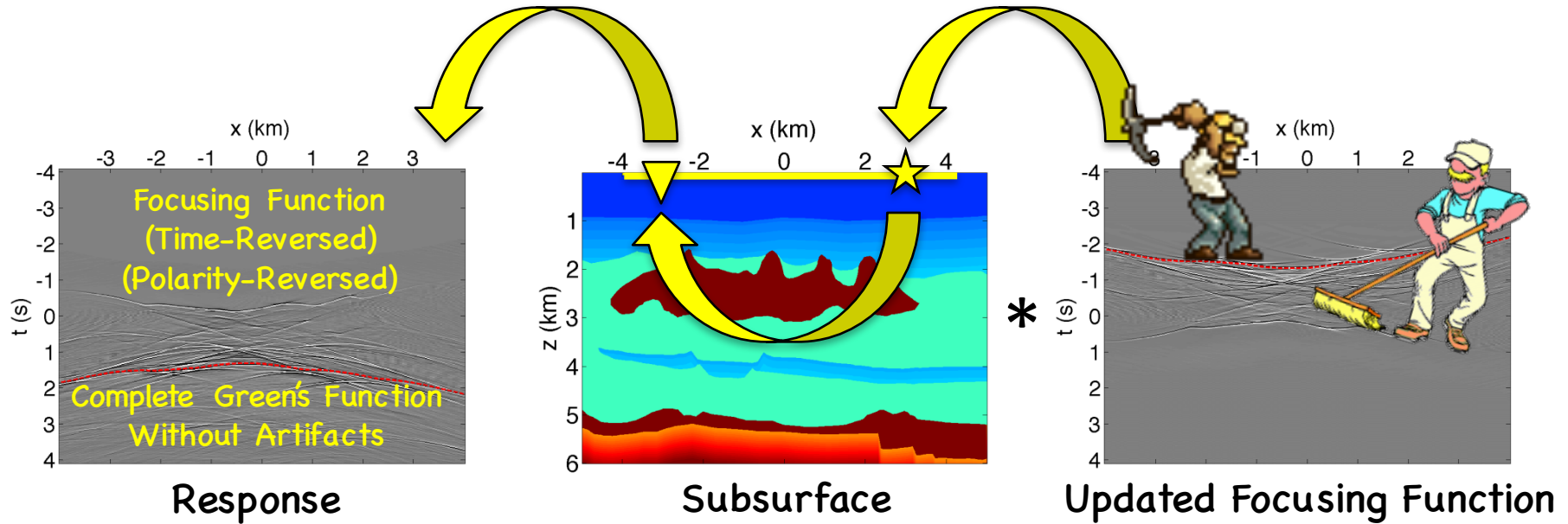
Updated Focusing Function

# The Marchenko Equation



1. Truncate Response
2. Time Reversal
3. Subtract from Focusing Function

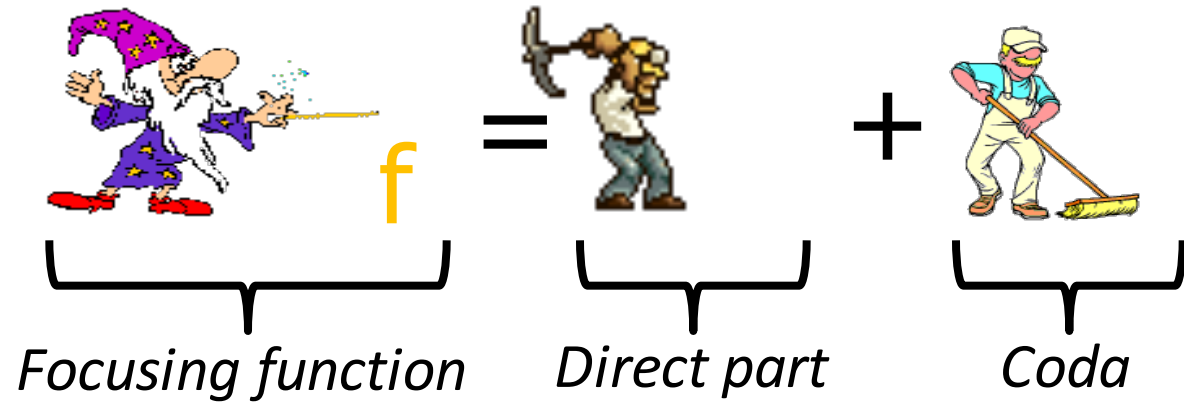
# The Marchenko Equation



1. Truncate Response
2. Time Reversal
3. Subtract from Focusing Function

# The Marchenko Equation

Focusing  
Function:



# The Marchenko Equation

Focusing Function:

$$\underbrace{\text{Wizard}}_f = \underbrace{\text{Miner}}_{\text{Direct part}} + \underbrace{\text{Janitor}}_{\text{Coda}}$$

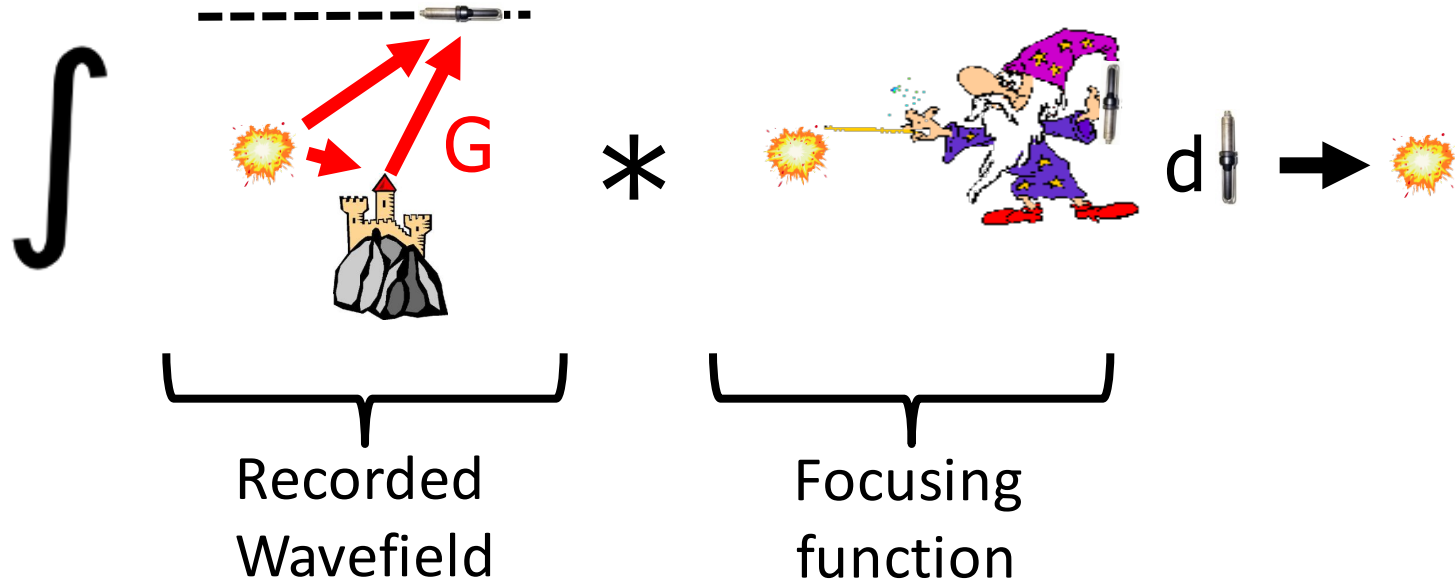
*Focusing function*      *Direct part*      *Coda*

Marchenko Equation:

$$\underbrace{\text{Geophysical Data}}_y = \underbrace{\begin{pmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 1 \end{pmatrix} - \underbrace{\text{Cathedral}}_A \underbrace{\begin{pmatrix} \text{Janitor} \\ \text{Janitor} \\ \dots \end{pmatrix}}_X$$

*y*      *A*      *X*

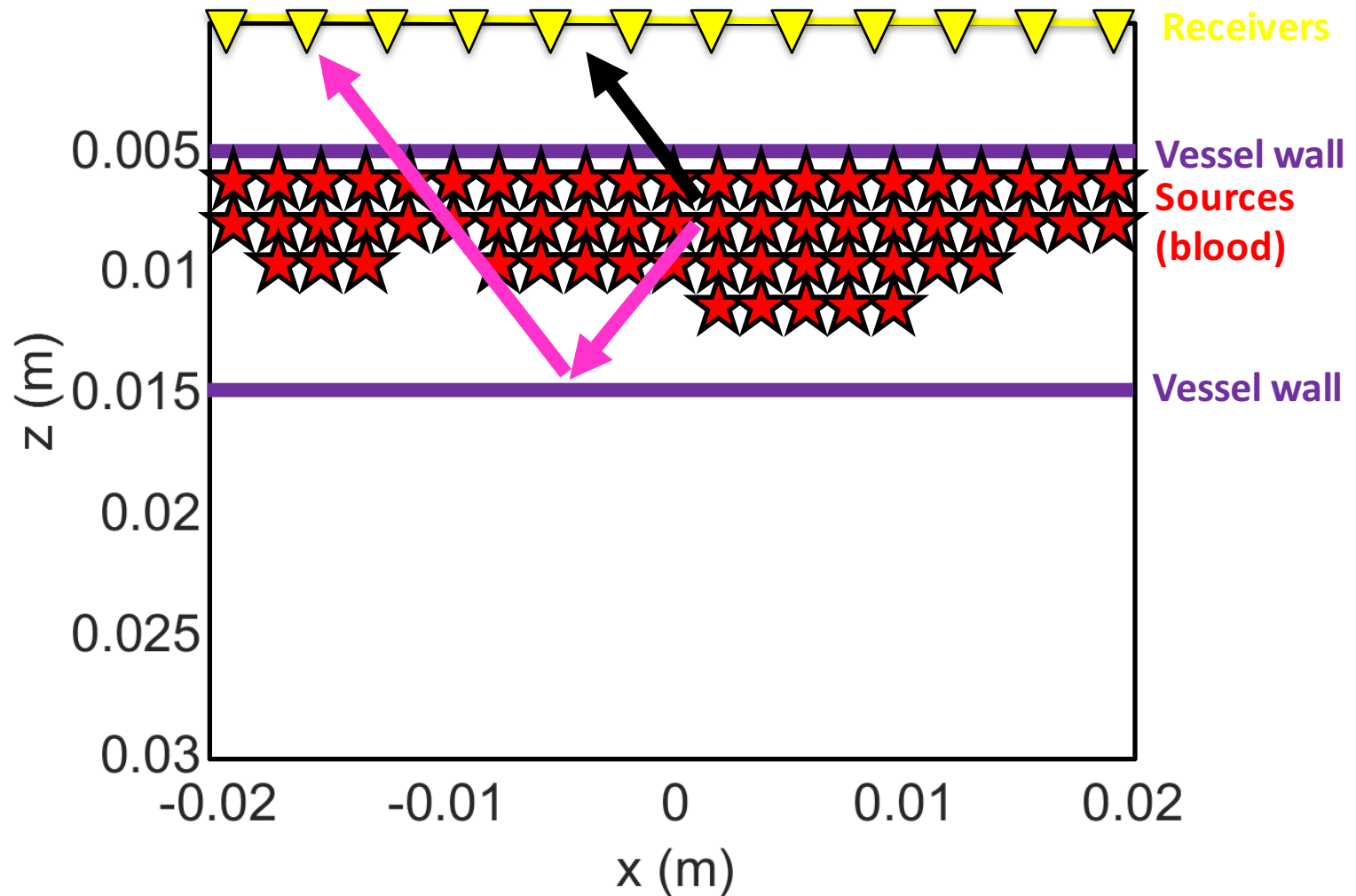
# Source Wavefield Imaging



1. Single-sided detection
2. Focusing function can be obtained from reflection data

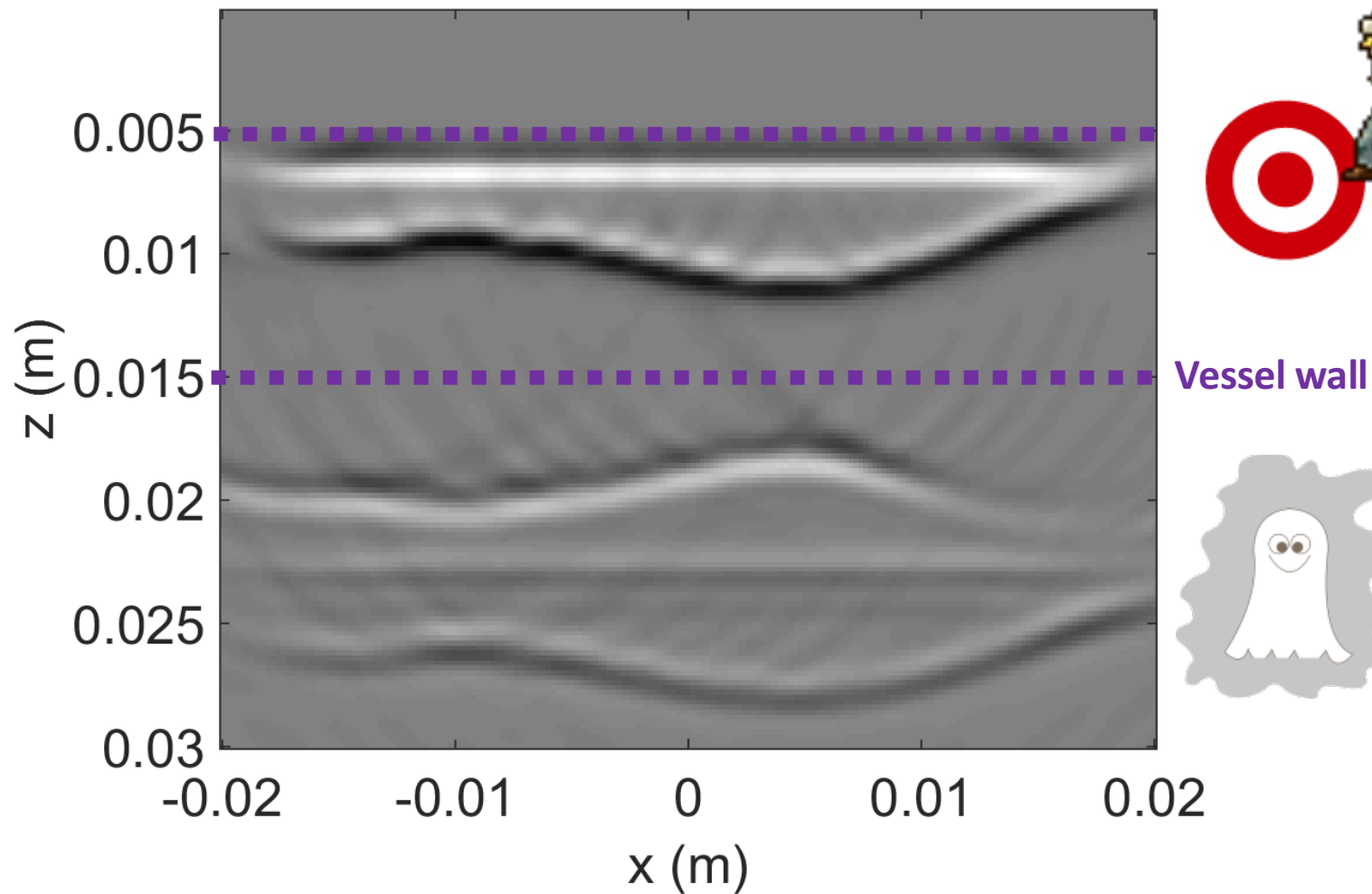
# Source Wavefield Imaging

## Photoacoustic Imaging Model



# Source Wavefield Imaging

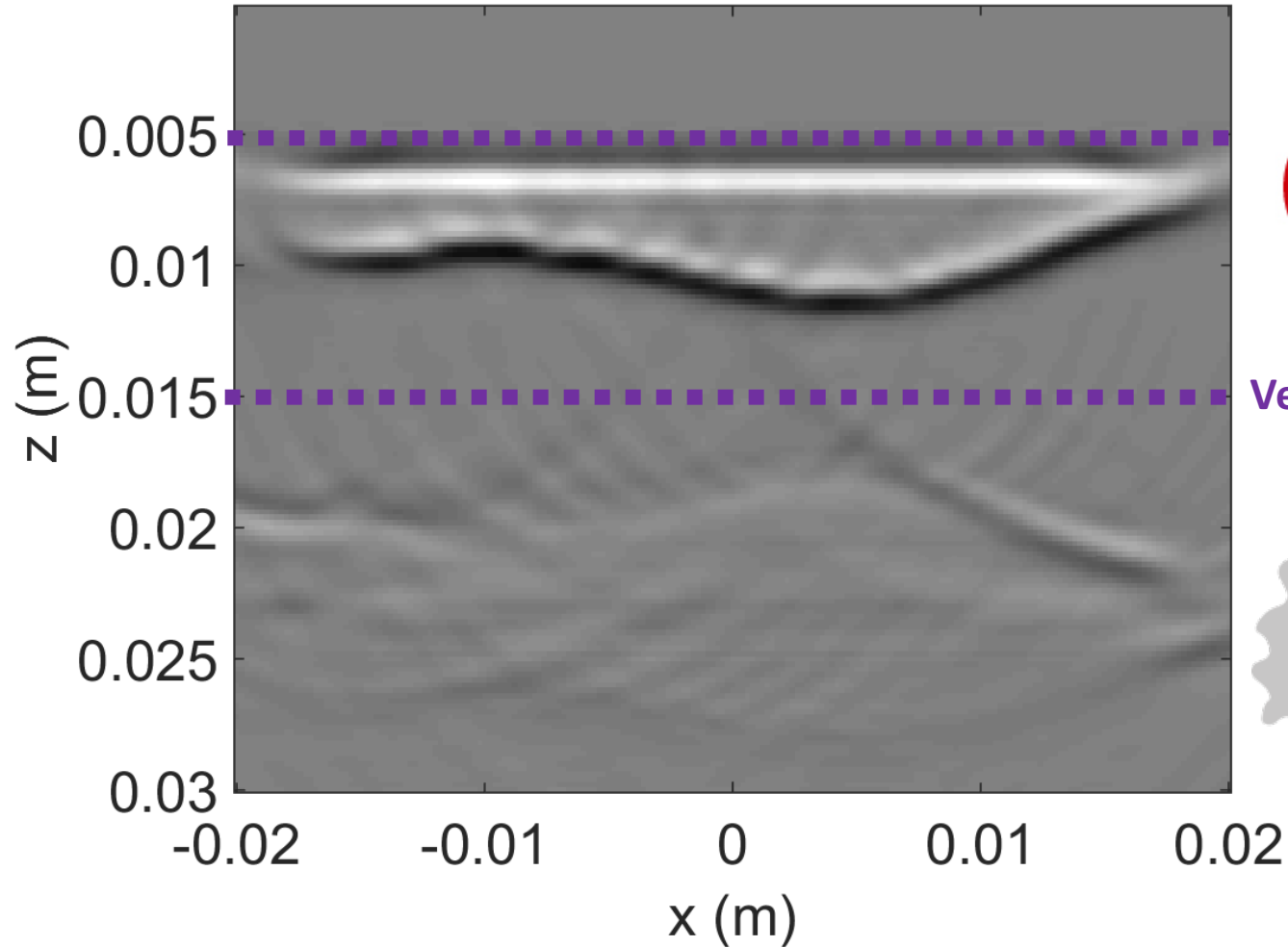
## Conventional Image



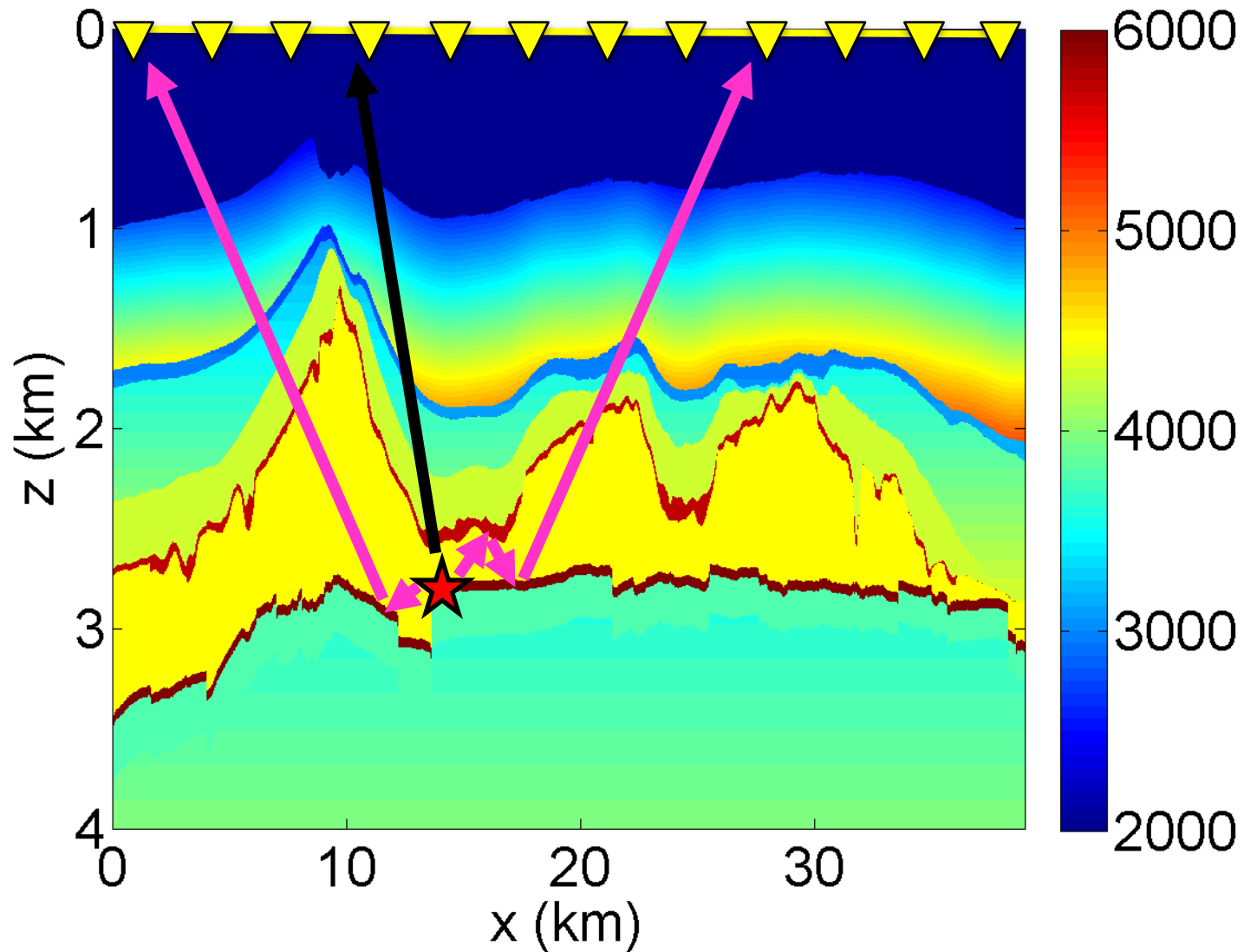


# Source Wavefield Imaging

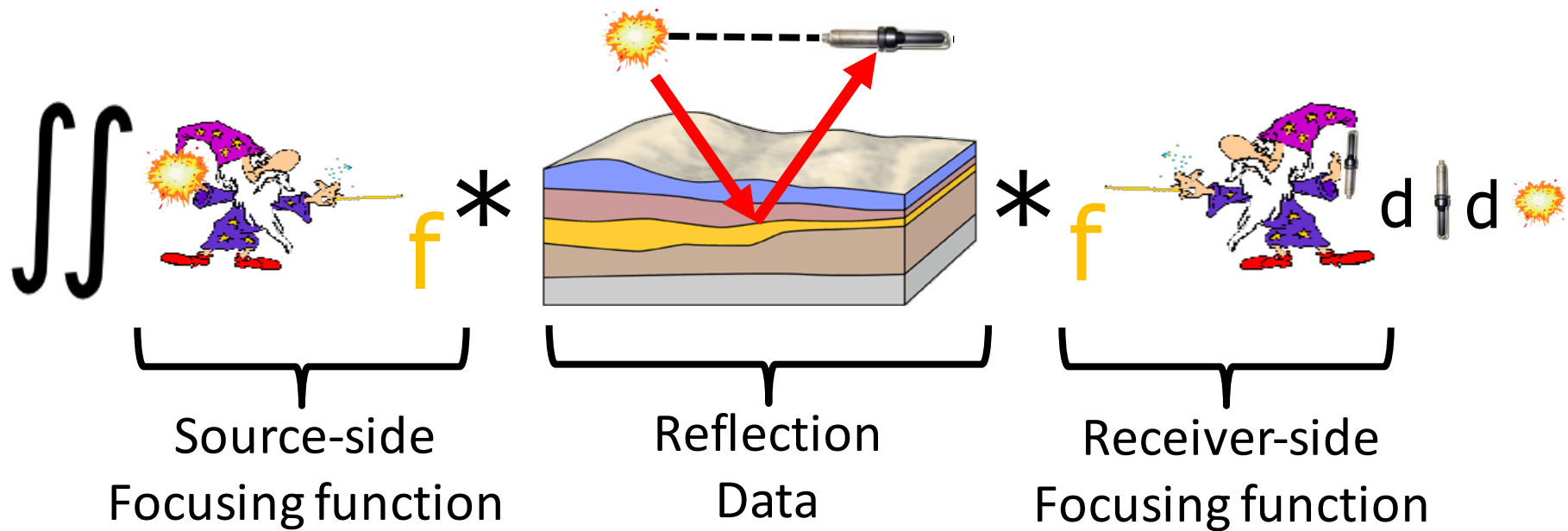
## Marchenko Image



# Source Wavefield Imaging

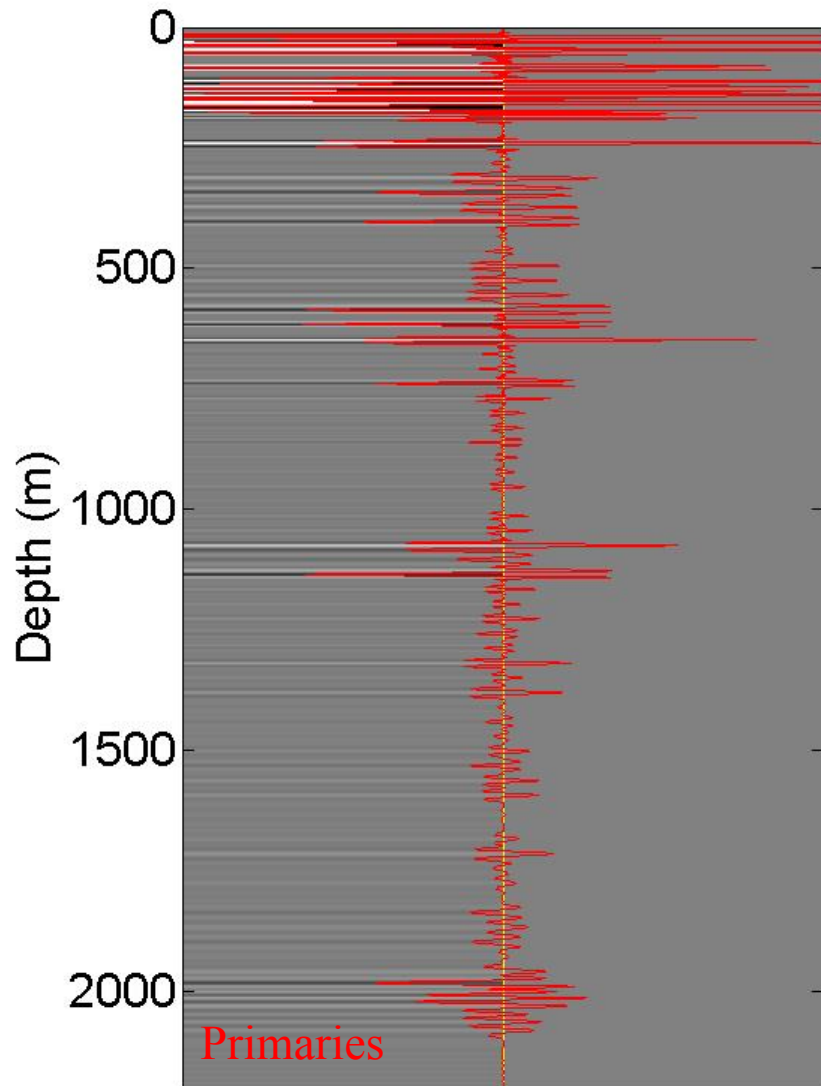


# Reflectivity Imaging

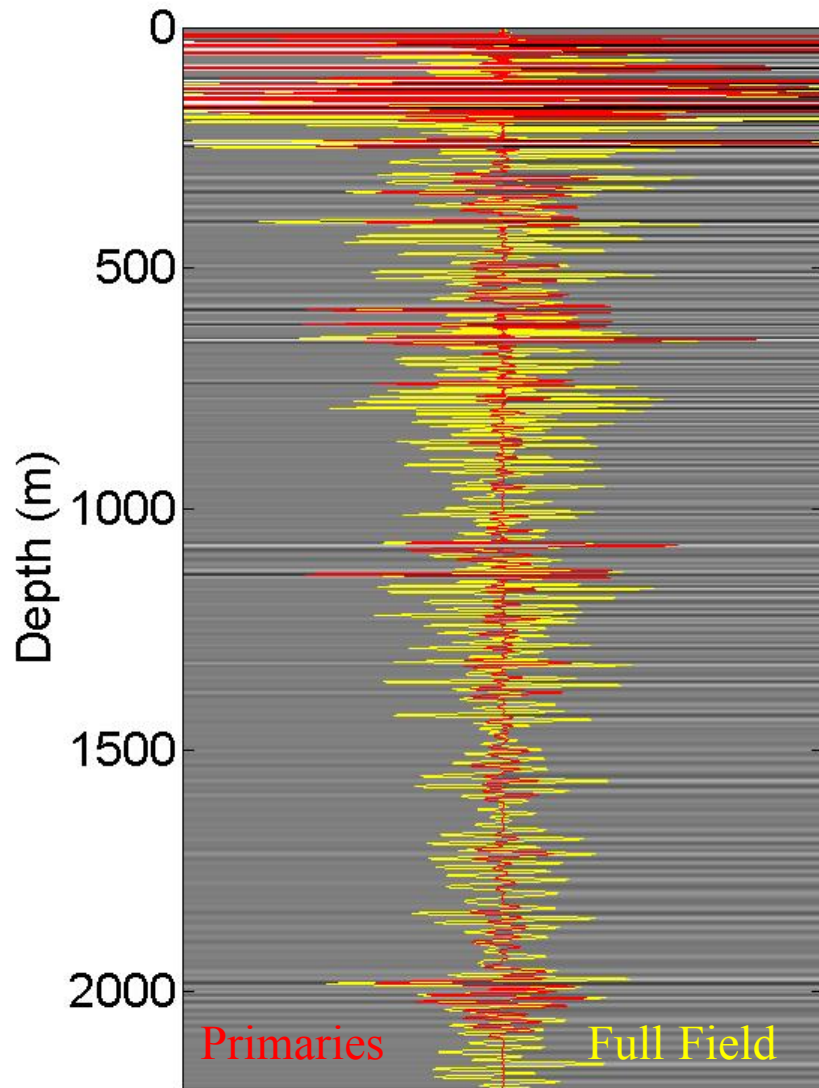


1. **Single-sided reflection data**
2. **Focusing functions can be obtained from reflection data**

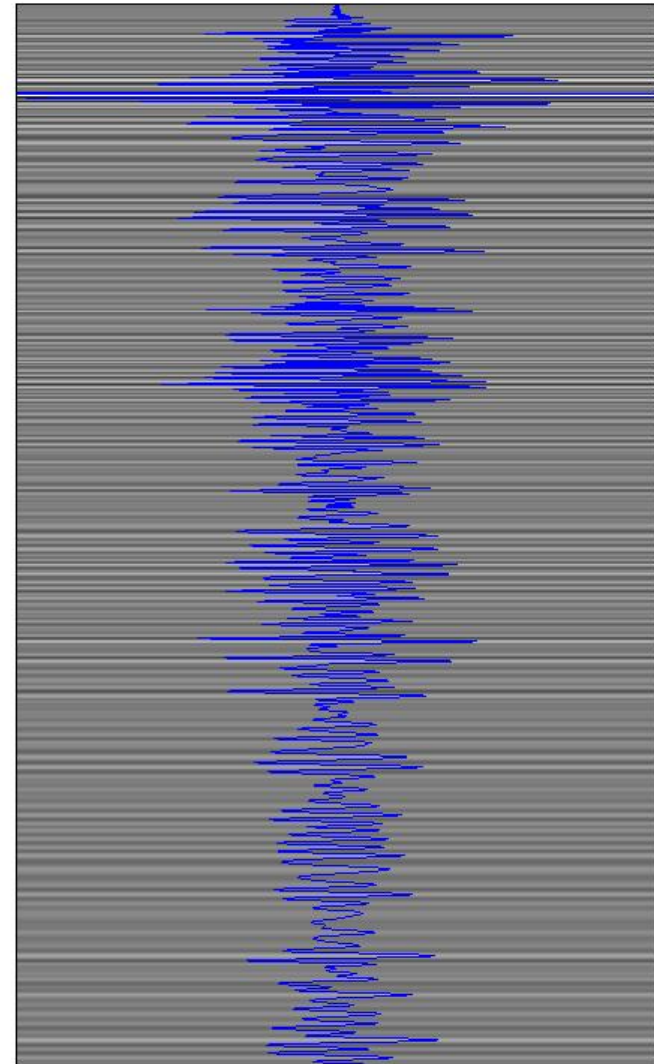
# Reflectivity Imaging



# Reflectivity Imaging

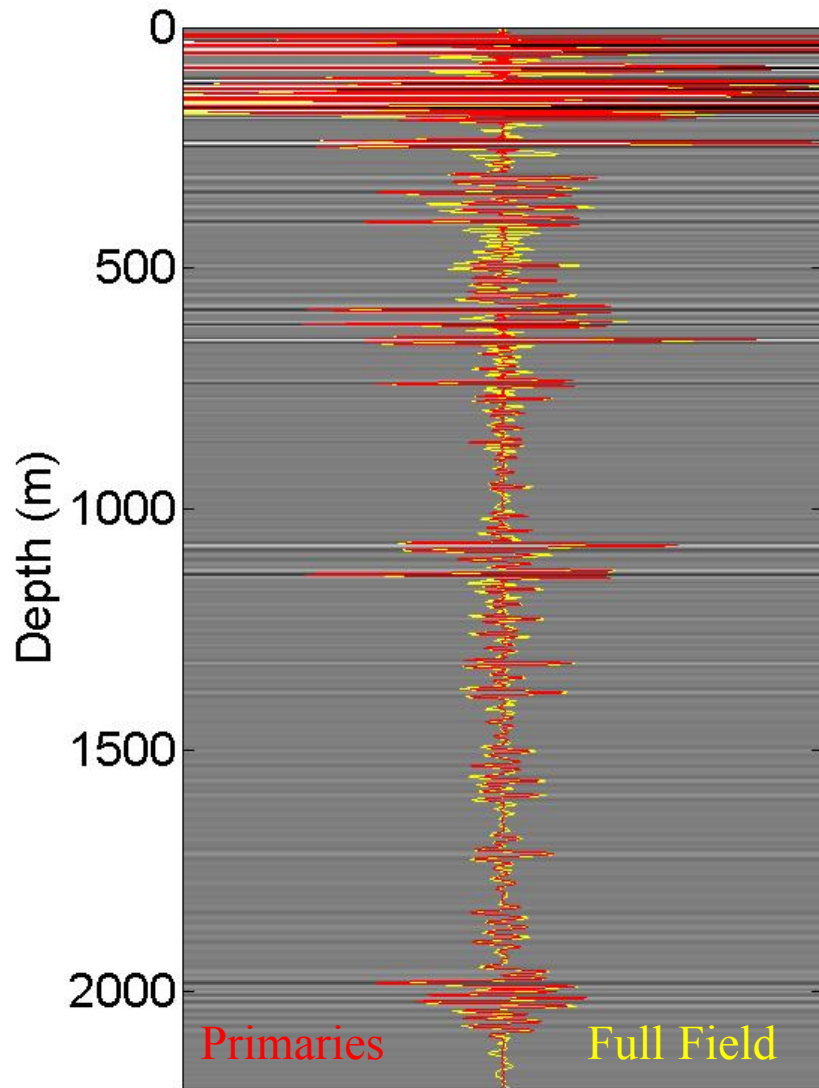


**Conventional Image**

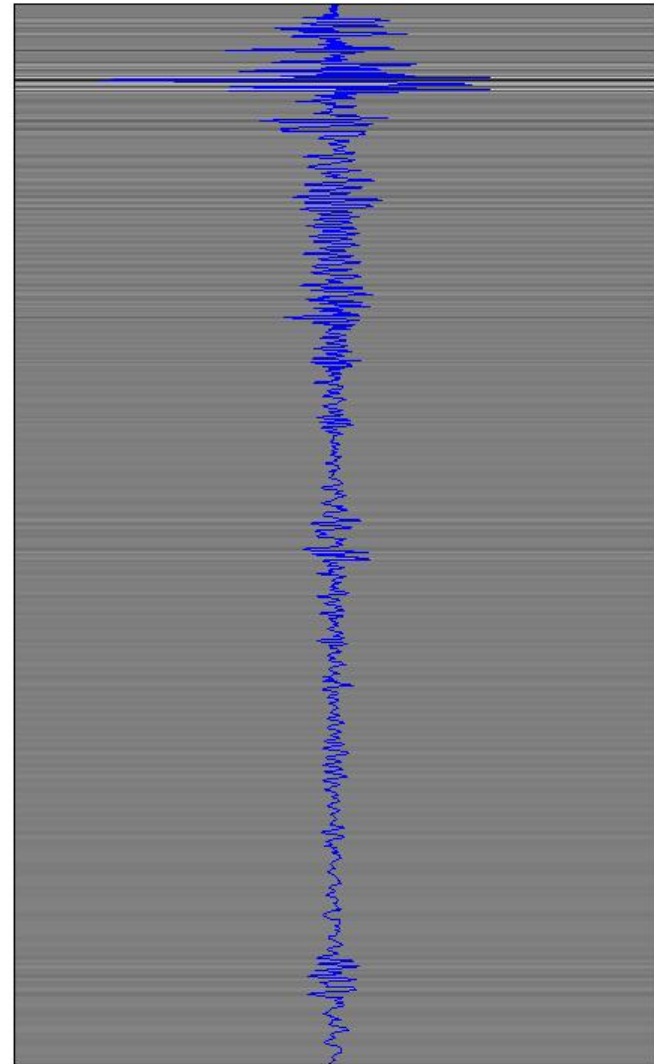


**Full Field - Primaries**

# Reflectivity Imaging



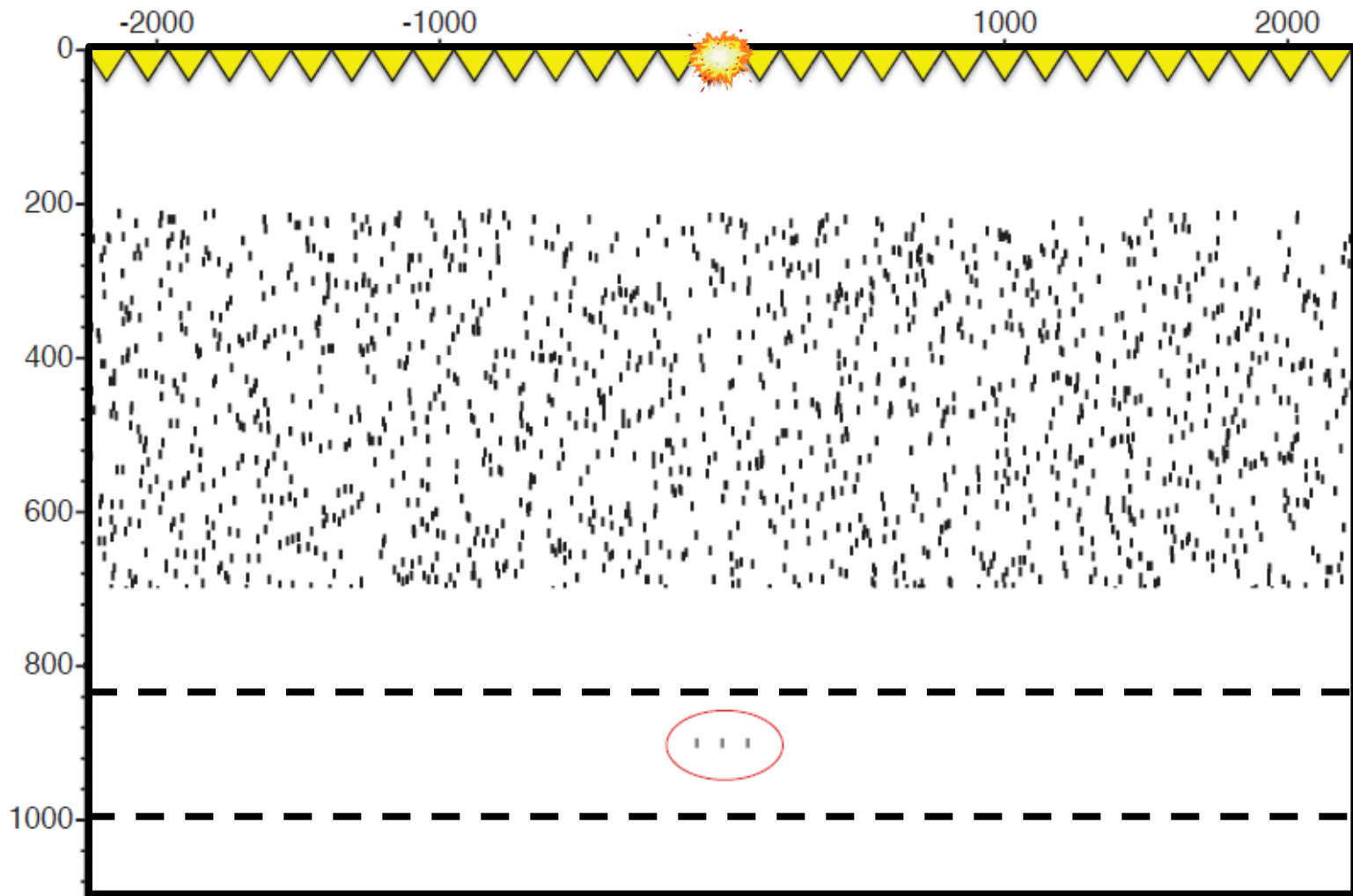
**Marchenko Image**



**Full Field - Primaries**

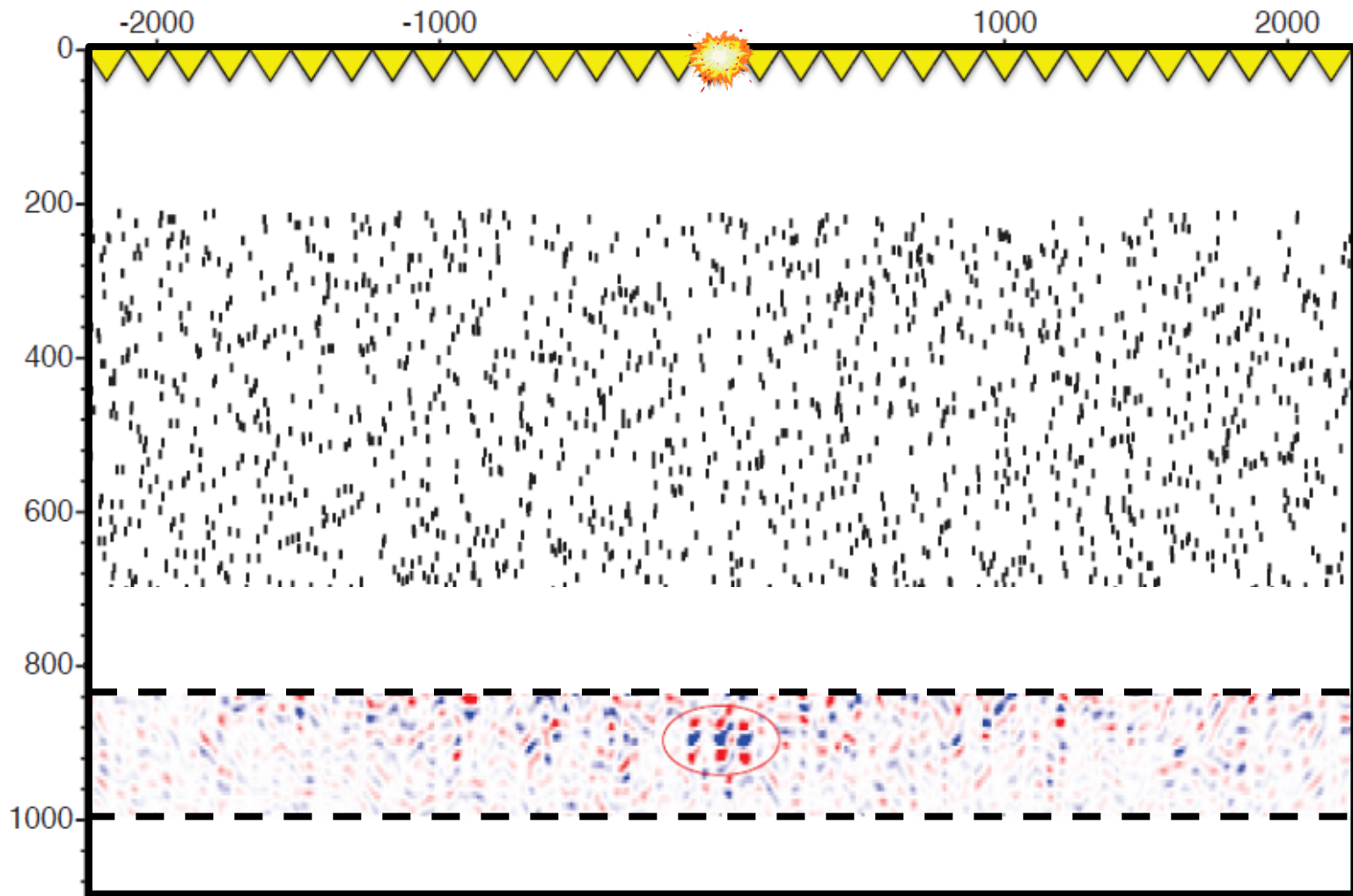
# Reflectivity Imaging

## Model



# Reflectivity Imaging

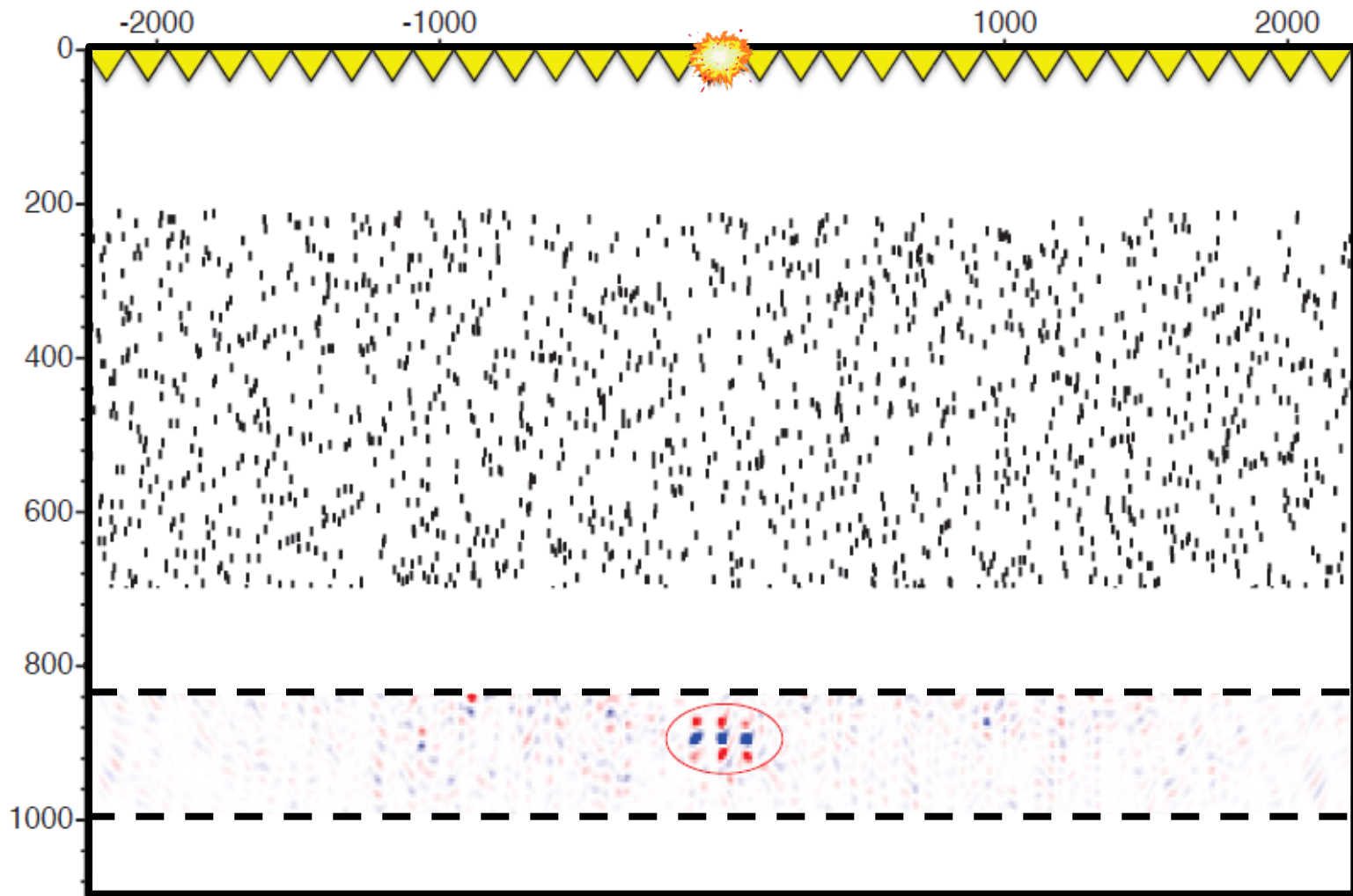
## Conventional Image





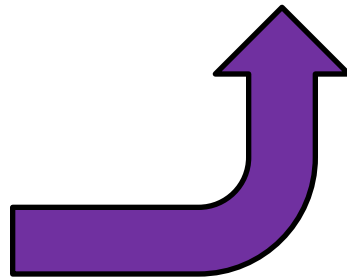
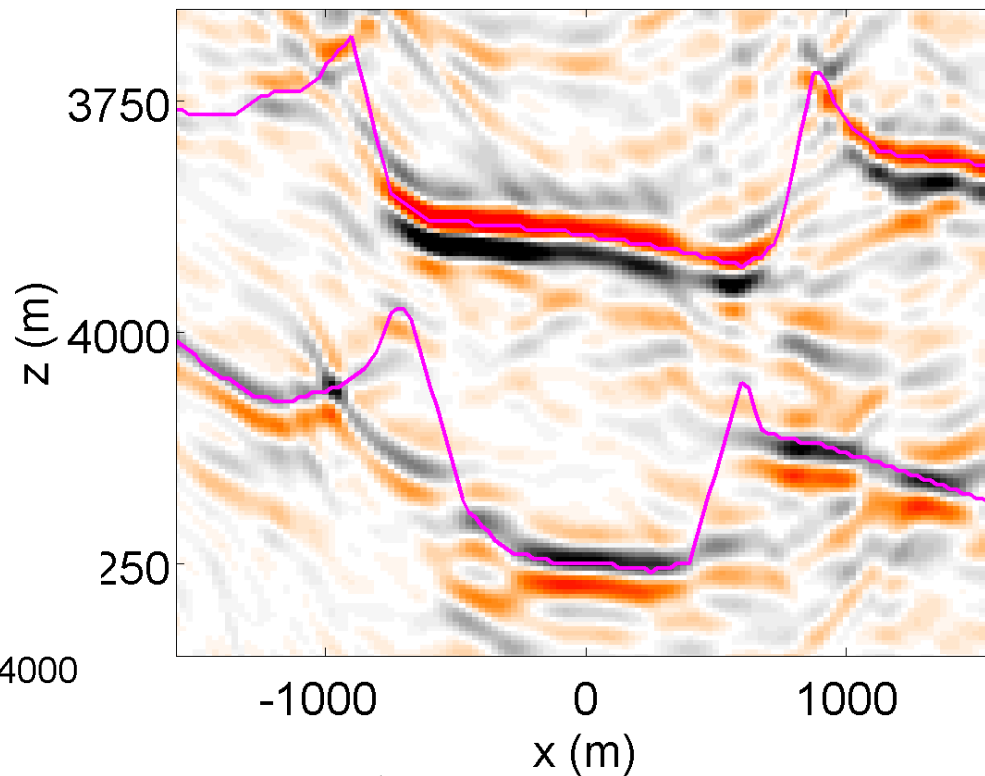
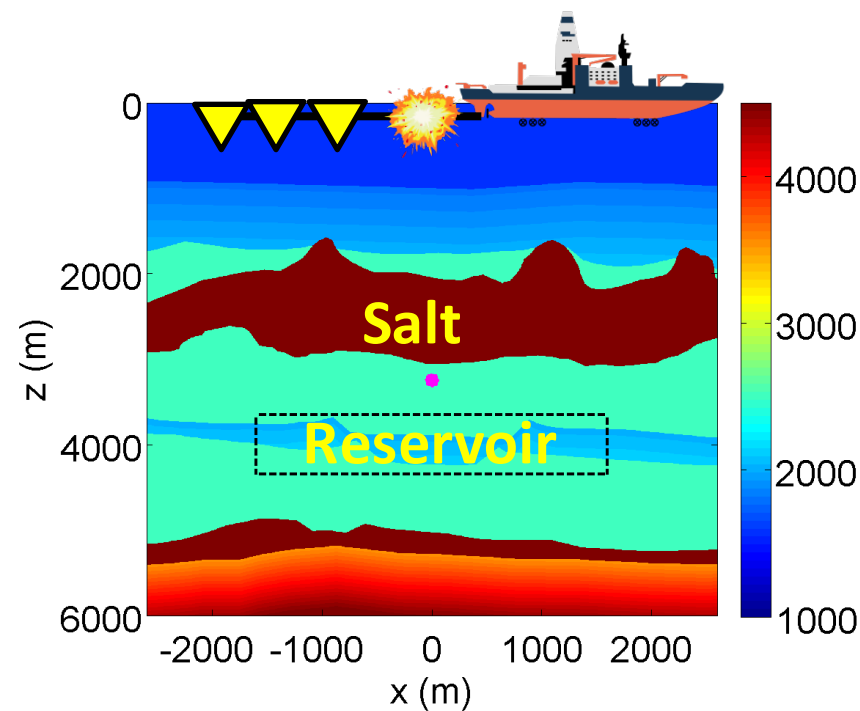
# Reflectivity Imaging

## Marchenko Image



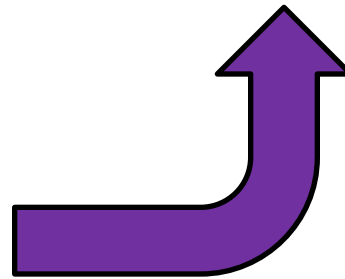
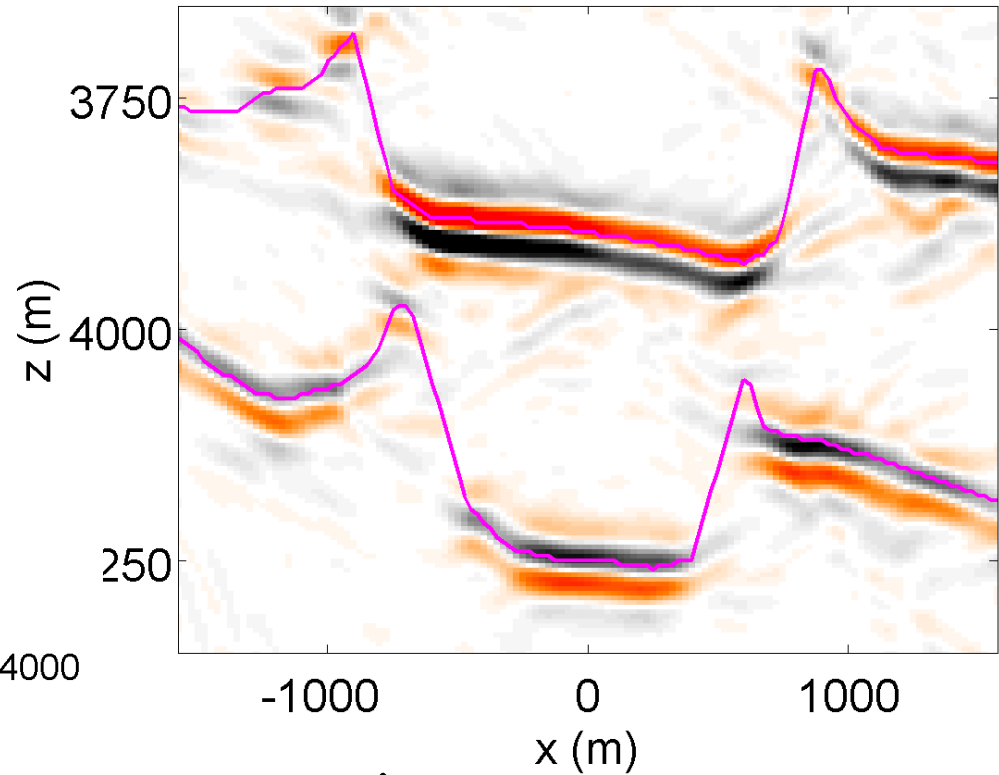
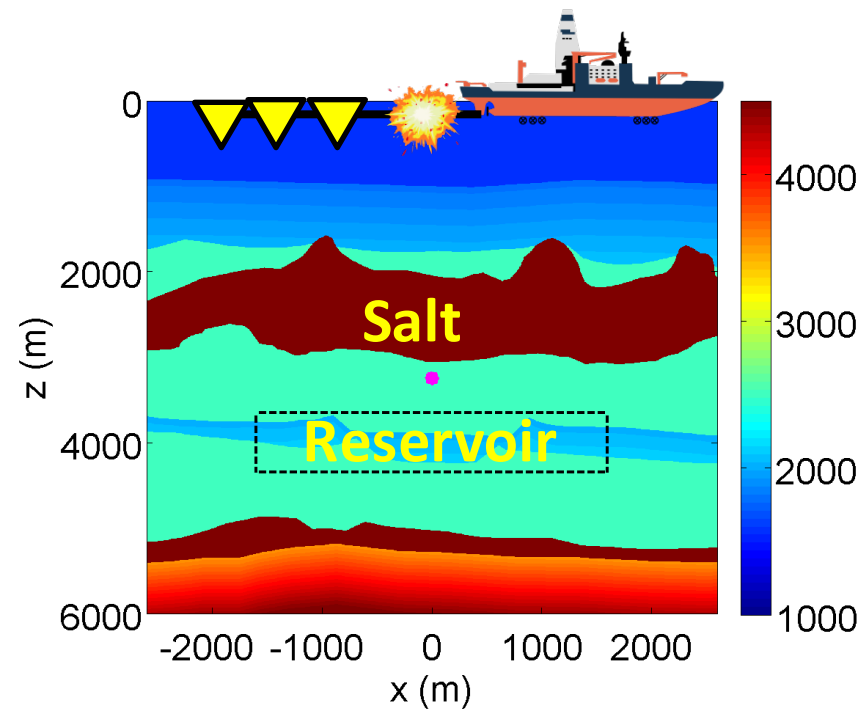
# Reflectivity Imaging

## Conventional Image



# Reflectivity Imaging

## Marchenko Image



# Conclusions

1. Time-Reversed Wavefields
  - a. Exact Green's functions
  - b. Closed boundaries



# Conclusions

## 1. Time-Reversed Wavefields

a. Exact Green's functions

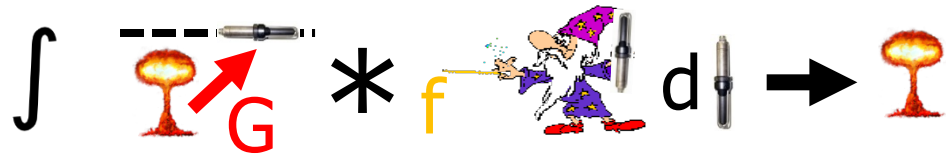
b. Closed boundaries



## 2. Inverse Wavefields

a. Focusing functions

b. Single-sided boundaries



# Conclusions

## 1. Time-Reversed Wavefields

a. Exact Green's functions

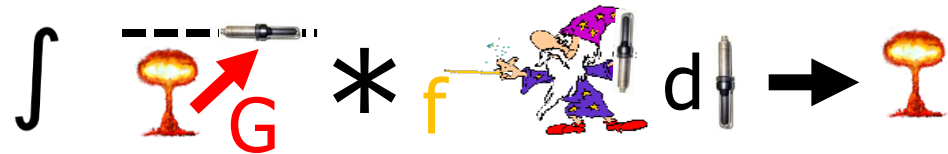
b. Closed boundaries



## 2. Inverse Wavefields

a. Focusing functions

b. Single-sided boundaries



## 3. The Marchenko Equation

a. Iterative substitution

b. Least-squares inversion

$$y = A f$$

# Conclusions

## 1. Time-Reversed Wavefields

a. Exact Green's functions

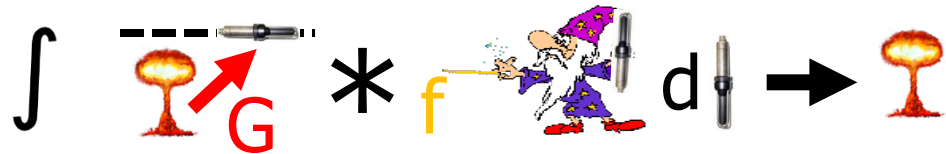
b. Closed boundaries



## 2. Inverse Wavefields

a. Focusing functions

b. Single-sided boundaries



## 3. The Marchenko Equation

a. Iterative substitution

b. Least-squares inversion

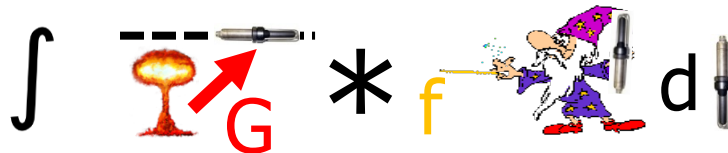
$$y = A f$$

## 4. Source Wavefield Imaging

a. Data

b. Reflection response

c. Macro model



# Conclusions

## 1. Time-Reversed Wavefields

a. Exact Green's functions

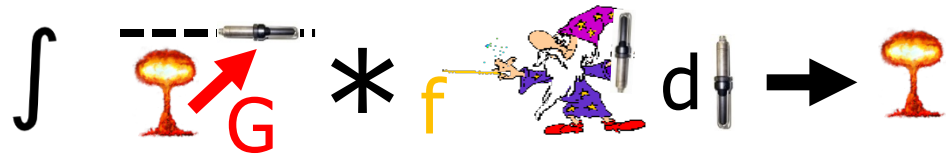
b. Closed boundaries



## 2. Inverse Wavefields

a. Focusing functions

b. Single-sided boundaries



## 3. The Marchenko Equation

a. Iterative substitution

b. Least-squares inversion

$$y = A f$$

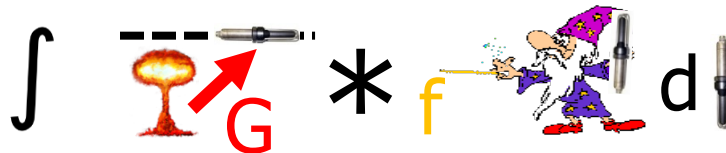

The equation is accompanied by a wizard icon pointing towards the variable 'f'.

## 4. Source Wavefield Imaging

a. Data

b. Reflection response

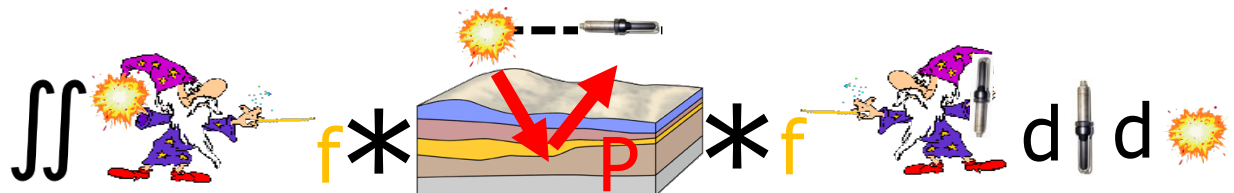
c. Macro model



## 5. Reflectivity Imaging

a. Reflection response

b. Macro model







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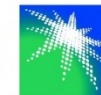
Jesper  
Spetzler



Ning Tu, Tristan van Leeuwen,  
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Kasper van Wijk



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