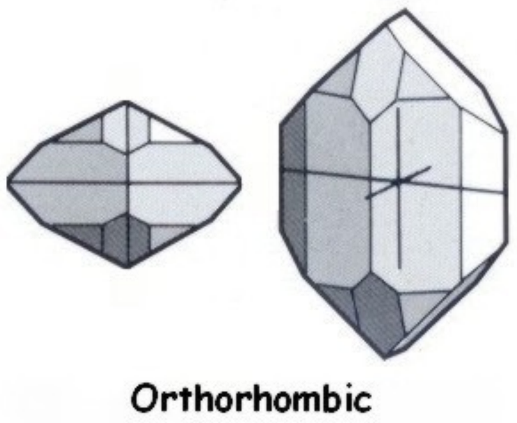


# ORThorhombic velocity model a new standard for seismic anisotropy

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***ROSE meeting, 25.04.2016***

# Brief history

Schoenberg,M., and K. Helbig, 1997, Orthorhombic media: Modeling elastic wave behavior in a vertically fractured earth: *Geophysics*, **62**, 1954–1974.

Tsvankin, I., 1997, Anisotropic parameters and P-wave velocity for orthorhombic media: *Geophysics*, **62**, 1292-1309.

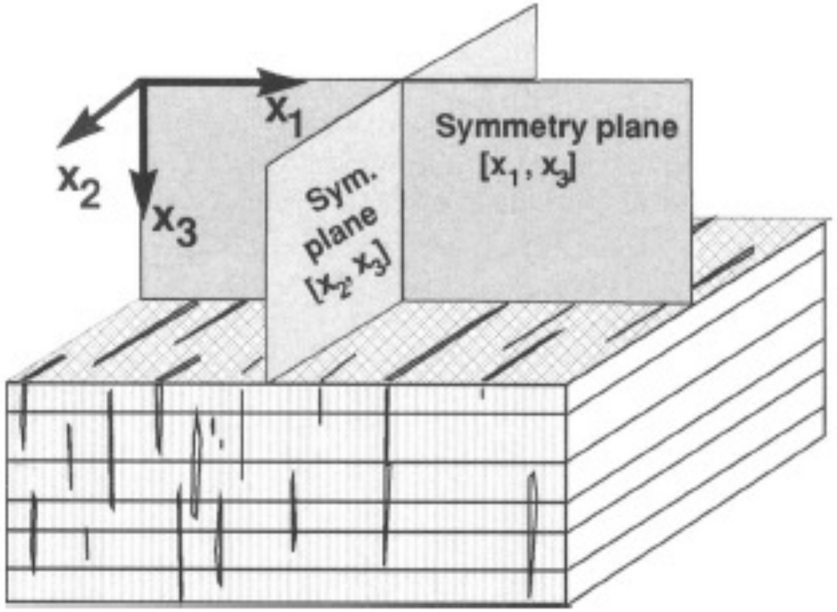



FIG. 1. An orthorhombic model caused by parallel vertical cracks embedded in a medium composed of thin horizontal layers. Orthorhombic media have three mutually orthogonal planes of mirror symmetry.

*Courtesy Tsvankin (1997)*

# P&S waves in ORT

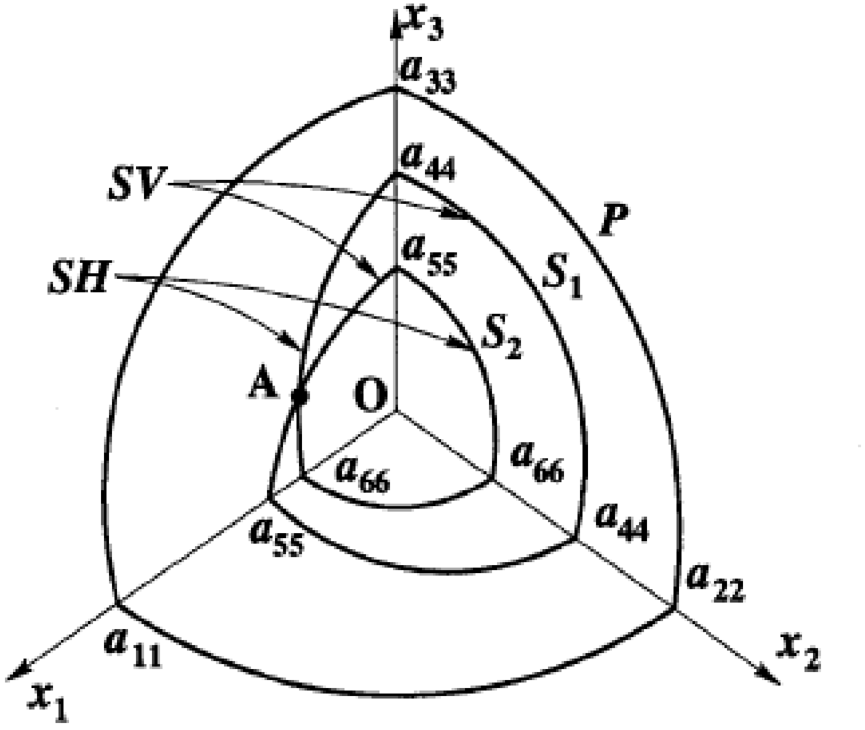



FIG. 1. Sketch of body-wave phase velocity surfaces in orthorhombic media. The value $a_{ij} = \sqrt{c_{ij}/\rho}$, where $c_{ij}$ are the elastic stiffness coefficients and $\rho$ is the density.

Courtesy Grechka et al., (1999).

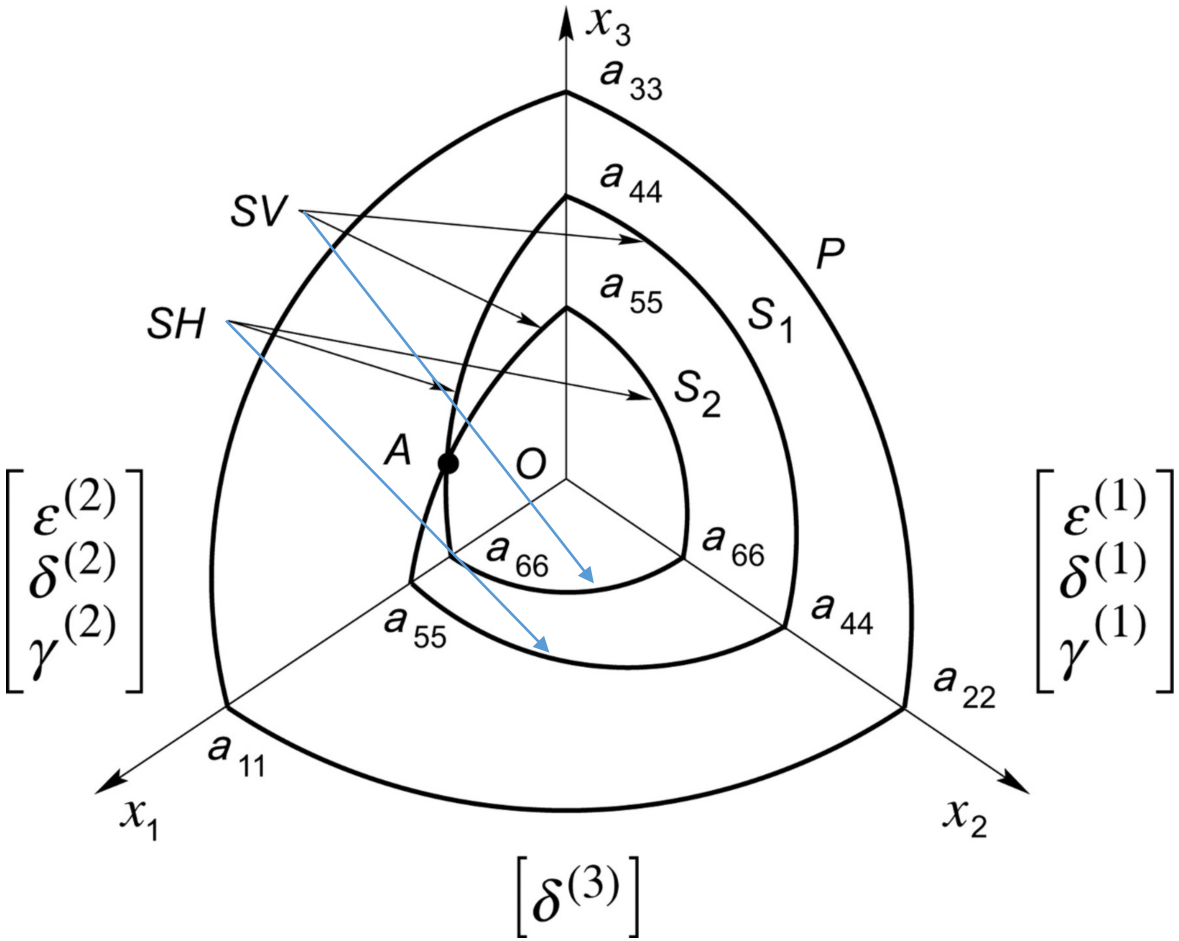
$$\mathbf{C}_{ORT} = \begin{pmatrix} c_{11} & c_{12} & c_{13} & & & \\ & c_{22} & c_{23} & & & \\ & & c_{33} & & & \\ & & & c_{44} & & \\ & & & & c_{55} & \\ & & & & & c_{66} \end{pmatrix}$$

9 independent parameters

$$c_{ij} \Leftrightarrow V_{P0}, V_{S0}, \begin{matrix} \varepsilon_1, \delta_1, \gamma_1 \\ \varepsilon_2, \delta_2, \gamma_2 \\ \delta_3 \end{matrix}$$

*Tsvankin (1997)*

# Parameterization in ORT




S-waves in ORT – ***a headache***

1. Coupling
2. Triplications
3. Singularities

# P-waves in ORT

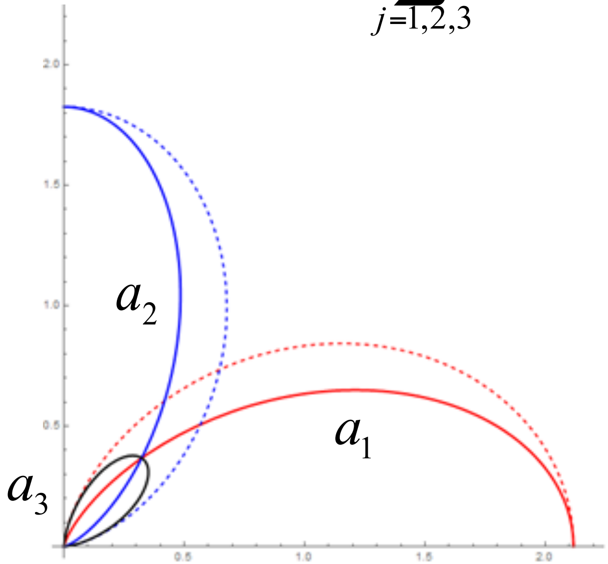
Alkhalifah, T., 2003, An acoustic wave equation for orthorhombic anisotropy: Geophysics, **68**, N4, 1169-1172.

Stovas, A., 2015, Azimuthally dependent kinematic properties of orthorhombic media: *Geophysics*, **80**, C107-C122.

$$\tau(x,y,z) \approx \tau_0(x,y,z) + \sum_{j=1,2,3} a_j(x,y,z)\eta_j$$

$$V_{P0},\ \begin{matrix} V_{nmo1} \\ V_{nmo2} \end{matrix},\ \begin{matrix} \eta_1 \\ \eta_2 \\ \eta_3 \end{matrix}$$

$$\eta_{xy} = \sqrt{\frac{(1+2\eta_1)(1+2\eta_2)}{(1+2\eta_3)}} - 1$$



# From 2D (VTI) with 3 parameters

P-wave processing parameters: $t_0\left(V_0\right), V_{nmo}^2, \eta$

$$t^2\left(x\right) = t_0^2 + \frac{x^2}{V_{nmo}^2} - \frac{2\eta x^4}{V_{nmo}^4 t_0^2} + \ldots$$

# To 3D (ORT) with 6 parameters

P-wave processing parameters: $t_0\left(V_0\right), V_{nmo1}^2, V_{nmo2}^2, \eta_1, \eta_2, \eta_{xy}\left(\eta_3\right)$

$$t^2\left(x,y\right) = t_0^2 + \frac{x^2}{V_{nmo1}^2} + \frac{y^2}{V_{nmo2}^2} - \frac{2\eta_1 x^4}{V_{nmo1}^4 t_0^2} - \frac{2\eta_2 y^4}{V_{nmo2}^4 t_0^2} - \frac{2\eta_{xy} x^2 y^2}{V_{nmo1}^2 V_{nmo2}^2 t_0^2} + \ldots$$

# Hierarchy of Wave Equations (acoustic)

$$\frac{\partial^2 P}{\partial t^2} = a\left(\frac{\partial^2 P}{\partial z^2} + \frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2}\right) \qquad I$$

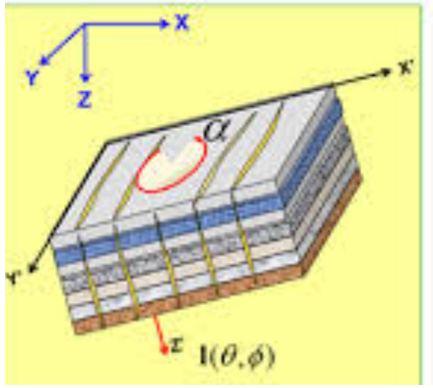
$$\frac{\partial^2 P}{\partial t^2} = a\frac{\partial^2 P}{\partial z^2} + b\left(\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2}\right) \quad or \quad \frac{\partial^2 P}{\partial t^2} = a\frac{\partial^2 P}{\partial z^2} + b_1\frac{\partial^2 P}{\partial x^2} + b_2\frac{\partial^2 P}{\partial y^2} \qquad EI$$

$$\frac{\partial^4 P}{\partial t^4} = a\frac{\partial^4 P}{\partial t^2 \partial z^2} + b\left(\frac{\partial^4 P}{\partial t^2 \partial x^2} + \frac{\partial^4 P}{\partial t^2 \partial y^2}\right) + c\left(\frac{\partial^4 P}{\partial x^2 \partial z^2} + \frac{\partial^4 P}{\partial y^2 \partial z^2}\right) \qquad VTI$$

$$\frac{\partial^6 P}{\partial t^6} = a\frac{\partial^6 P}{\partial t^4 \partial z^2} + \left(b_1\frac{\partial^6 P}{\partial t^4 \partial x^2} + b_2\frac{\partial^6 P}{\partial t^4 \partial y^2}\right) + \left(c_1\frac{\partial^6 P}{\partial t^2 \partial x^2 \partial z^2} + c_2\frac{\partial^6 P}{\partial t^2 \partial y^2 \partial z^2}\right) + d_1\frac{\partial^6 P}{\partial t^2 \partial x^2 \partial y^2} + d_2\frac{\partial^6 P}{\partial z^2 \partial x^2 \partial y^2} \qquad ORT$$

$$d_2 = d_2\left(a, b_1, b_2, c_1, c_2, d_1\right)$$

# TORT




Azimuthal rotation (cross-term coefficients):

$$t^2(x,y) = t_0^2 + \frac{x^2}{V_{nmo1}^2} + axy + \frac{y^2}{V_{nmo2}^2}$$

$$-\frac{2\eta_1 x^4}{V_{nmo1}^4 t_0^2} + bx^3 y - \frac{2\eta_{xy} x^2 y^2}{V_{nmo1}^2 V_{nmo2}^2 t_0^2} + cxy^3 - \frac{2\eta_2 y^4}{V_{nmo2}^4 t_0^2} + \ldots$$

Inclination (odd-order coefficients):

$$t^2(x,y) = t_0^2 + ax + by + \frac{x^2}{V_{nmo1}^2} + \frac{y^2}{V_{nmo2}^2} + \ldots$$

# Azimuthal dependence

$$t^2\left(x,y\right)=t_0^2+\frac{x^2}{V_{nmo1}^2}+\frac{y^2}{V_{nmo2}^2}-\frac{2\eta_1 x^4}{V_{nmo1}^4 t_0^2}-\frac{2\eta_2 y^4}{V_{nmo2}^4 t_0^2}-\frac{2\eta_{xy} x^2 y^2}{V_{nmo1}^2 V_{nmo2}^2 t_0^2}+\dots$$

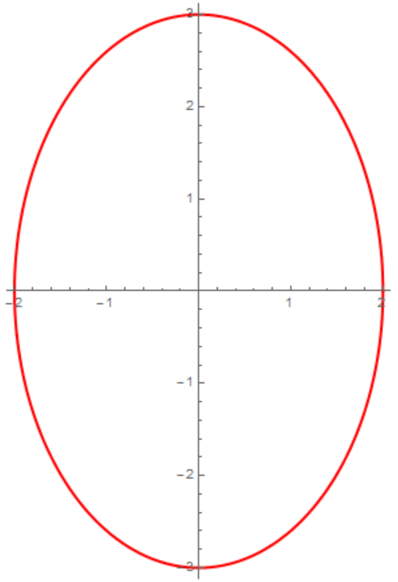
$$\Updownarrow$$

$$t^2\left(r,\Theta\right)=t_0^2+\frac{r^2}{V_n^2\left(\Theta\right)}-\frac{2\eta_r\left(\Theta\right)r^4}{V_n^4\left(\Theta\right)t_0^2}+\dots$$

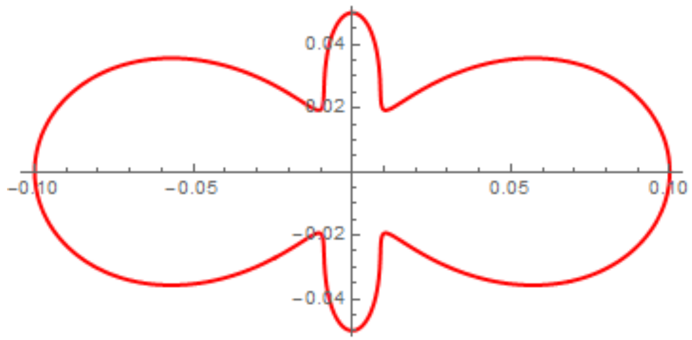
# Azimuthal dependence

$$\frac{1}{V_n^2(\Theta)} = \frac{\cos^2\Theta}{V_{nmo1}^2} + \frac{\sin^2\Theta}{V_{nmo2}^2}$$

$$\eta_r(\Theta) = \frac{\dfrac{\eta_1\cos^4\Theta}{V_{nmo1}^4} + \dfrac{\eta_2\sin^4\Theta}{V_{nmo2}^4} + \dfrac{\eta_{xy}\sin^2\Theta\cos^2\Theta}{V_{nmo1}^2 V_{nmo2}^2}}{\left(\dfrac{\cos^2\Theta}{V_{nmo1}^2} + \dfrac{\sin^2\Theta}{V_{nmo2}^2}\right)^2}$$



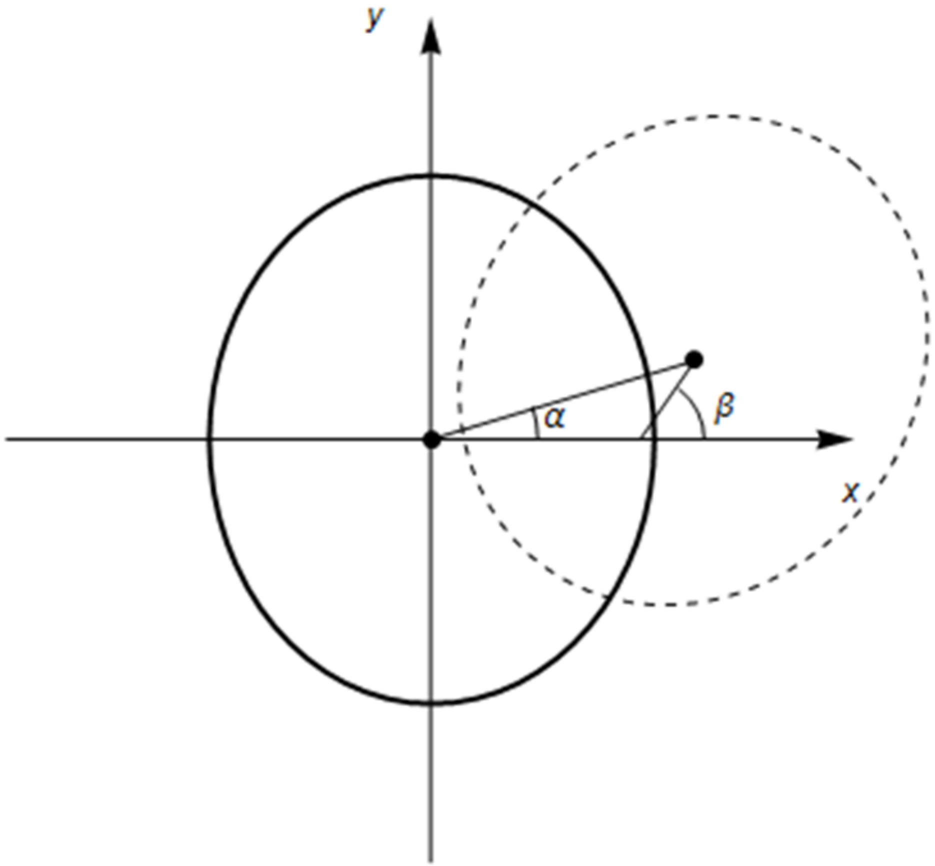
NMO velocity ellipse (Grechka and Tsvankin, 1998)




Azimuthally dependent anellipticity (Stovas, 2015)

# Interpretation of NMO ellipse

$$\frac{1}{V_n^2(\Theta)} = \frac{\cos^2(\Theta - \beta)}{V_{nmo1}^2} + \frac{\sin^2(\Theta - \beta)}{V_{nmo2}^2}$$




$$t^2(x, y) = t_0^2 + \frac{(x - x_0)^2}{V_{nmo1}^2} + \frac{(y - y_0)^2}{V_{nmo2}^2} + \ldots$$

$$\tan\alpha = \frac{y_0}{x_0}$$

NMO ellipse remains *ellipse* for a stack of azimuthally oriented ORT layers

# Effective ORT (from the stack of azimuthally dependent ORT layers)

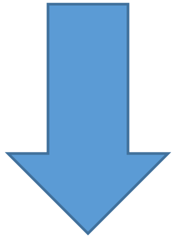
*1. Dix (Grechka&Tsvankin, 1999)* $V_0, V_{nmo1}, V_{nmo2}, \mathbf{\Phi}$

*2. LS solution for anellipticity*

$$\mathbf{UN} = \mathbf{DS}$$

$$\mathbf{N} = \left(N_1, N_2, N_{xy}\right)^T$$

$$\mathbf{S} = \left(1/V_{nmo1}^4, 1/V_{nmo2}^4, 1/V_{nmo1}^2 V_{nmo2}^2\right)^T$$



$$\mathbf{N} = \left(\mathbf{U}^T\mathbf{U}\right)^{-1}\mathbf{U}^T\mathbf{DS}$$

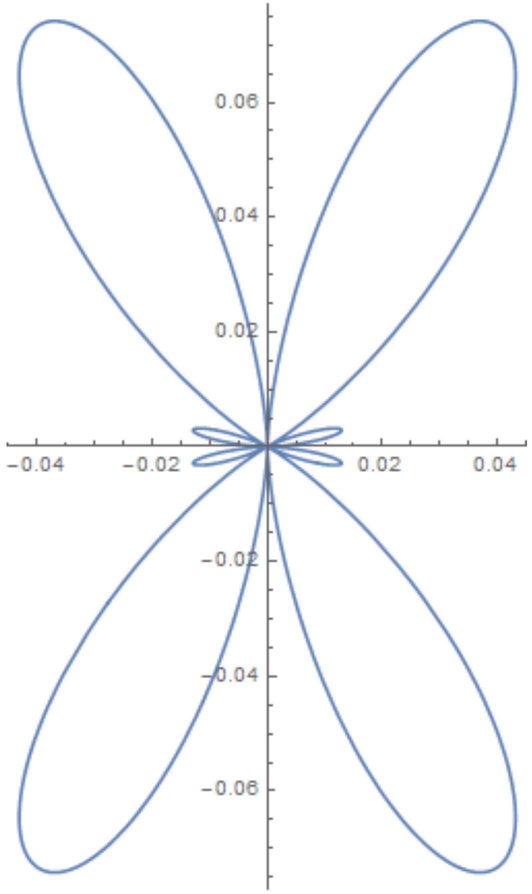
$$\mathbf{U} = \mathbf{U}\left(\Phi\right)$$

*Stovas, 2015*

# ORT = VTI($\phi$)?

$\delta(\phi)$ *and* $\varepsilon(\phi)$

Error in $v_{h(\phi)}$



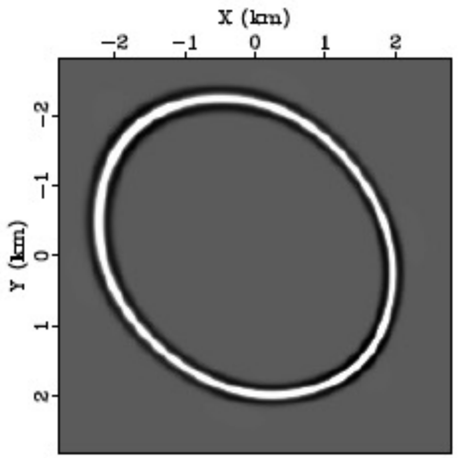

$$v_h^2(\phi) = \frac{1}{2}\left( E(\phi) + \sqrt{E^2(\phi) - \frac{8\eta_3(1+2\eta_1)(1+2\eta_2)V_{nmo1}^2 V_{nmo2}^2 \sin^2\phi\cos^2\phi}{(1+2\eta_3)}} \right)$$

$$E(\phi) = V_{nmo1}^2(1+2\eta_1)\cos^2\phi + V_{nmo2}^2(1+2\eta_2)\sin^2\phi$$

$$v_h^2(\phi) = v_n^2(\phi)(1+2\eta_r(\phi))$$

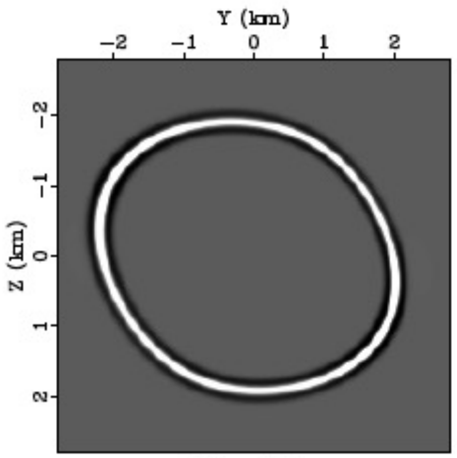
$$= \frac{V_{nmo1}^4(1+2\eta_1)\cos^4\phi + V_{nmo2}^4(1+2\eta_2)\sin^4\phi + 2V_{nmo1}^2 V_{nmo2}^2(1+\eta_{xy})\sin^2\phi\cos^2\phi}{V_{nmo1}^2\cos^2\phi + V_{nmo2}^2\sin^2\phi}$$

# Wave propagation in TORT



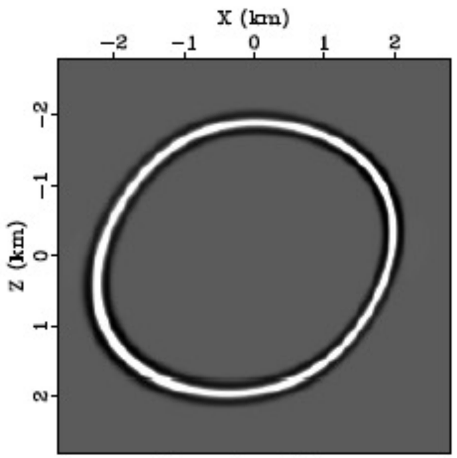

Depth Slice

(a)



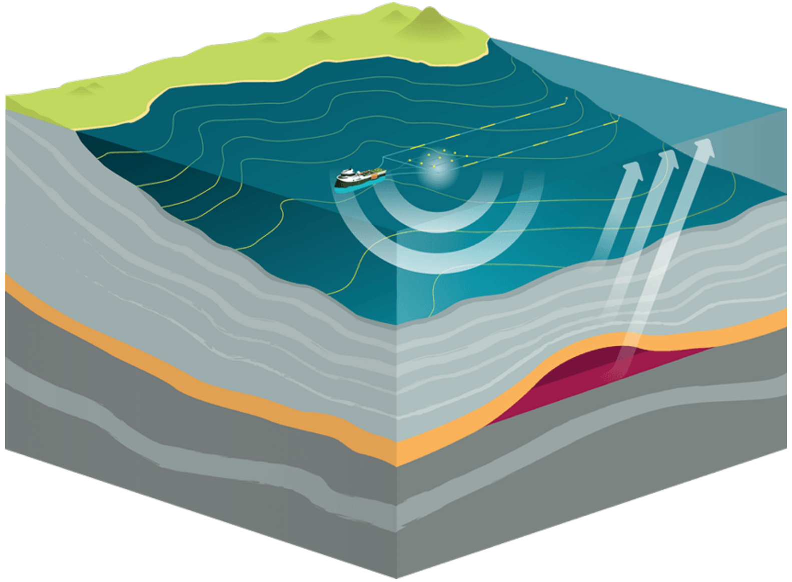

Inline Slice

(b)



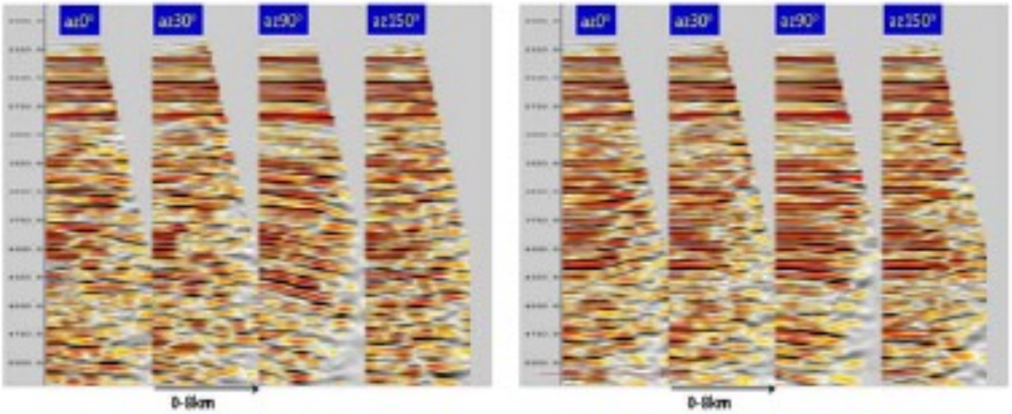

Crossline Slice

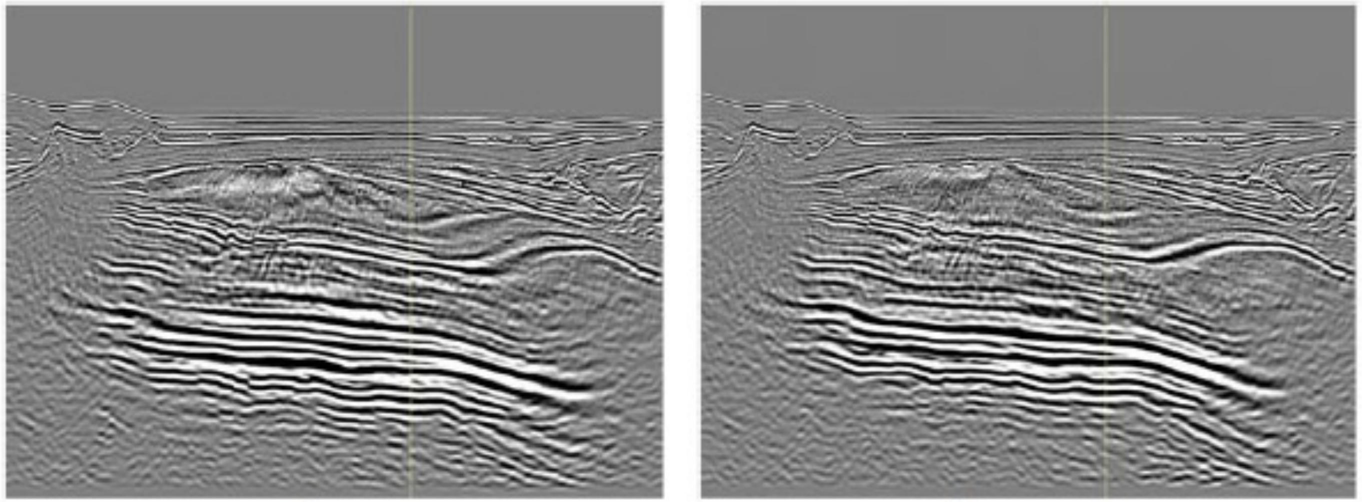
(c)



*Song and Alkhalifah, 2013*

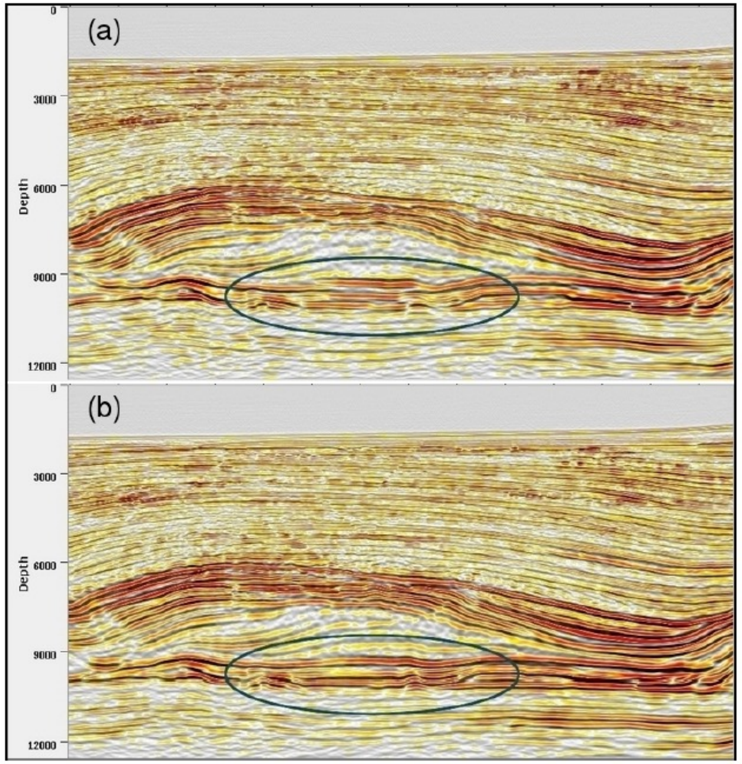
# TTI versus TORT



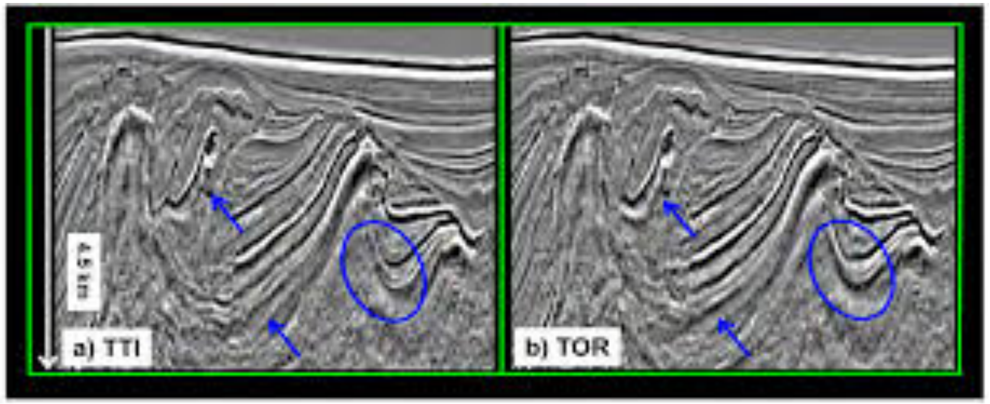



a b

*Zhang et al, 2012*




*Suh, 2014*

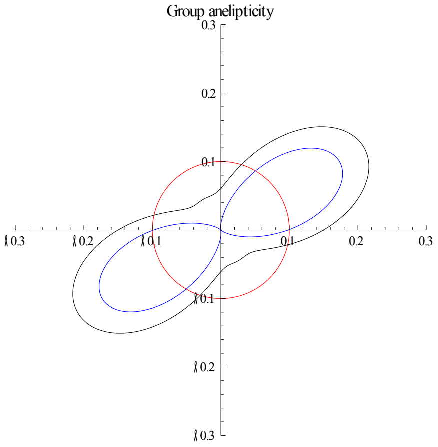


# Conclusions

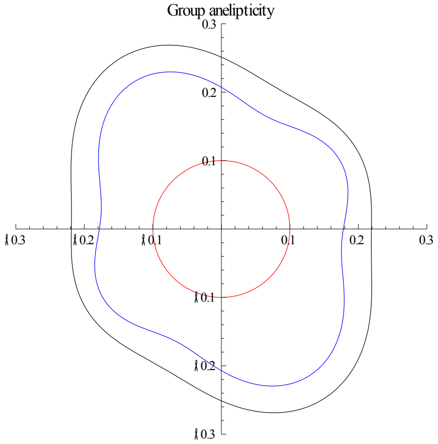
- ORT reflects anisotropy in 3D
- 9 parameters(elastic)/6 parameters(acoustic) + 3 angles
- Azimuthal dependence of kinematic properties
- Extension for multilayered medium
- Better focusing

# Acknowledgement

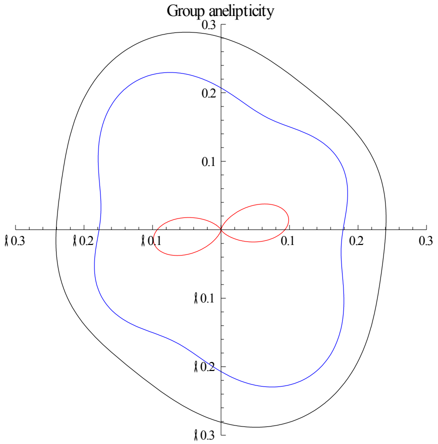
# Anellipticity from multilayered model



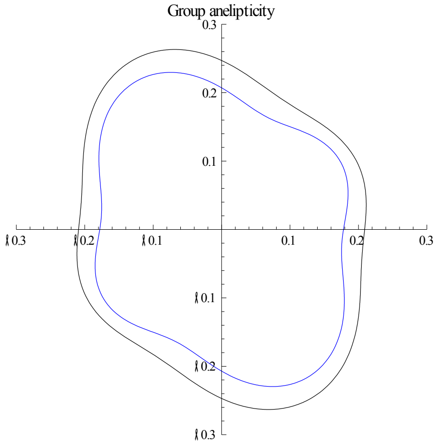

VTI&HTI




VTI&ORT




HTI&ORT




EI&ORT