Acoustic wavefields in the presence of boreholes

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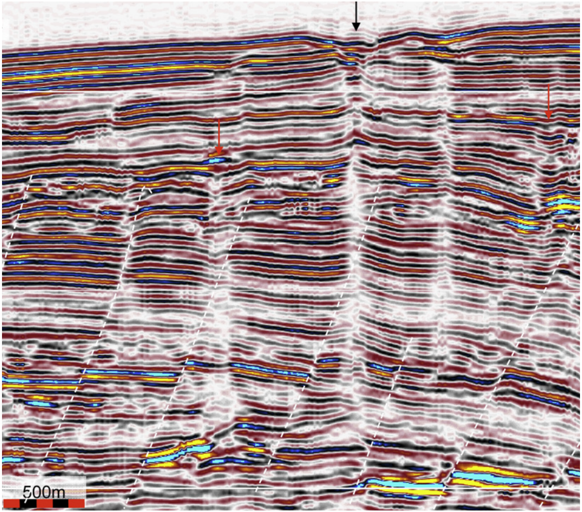
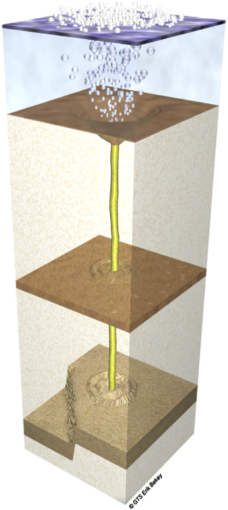
Outline

1. Motivation
2. Theory
   Understanding the modeling method of Rice and Willen (1987)
3. Numerical examples
4. Conclusions

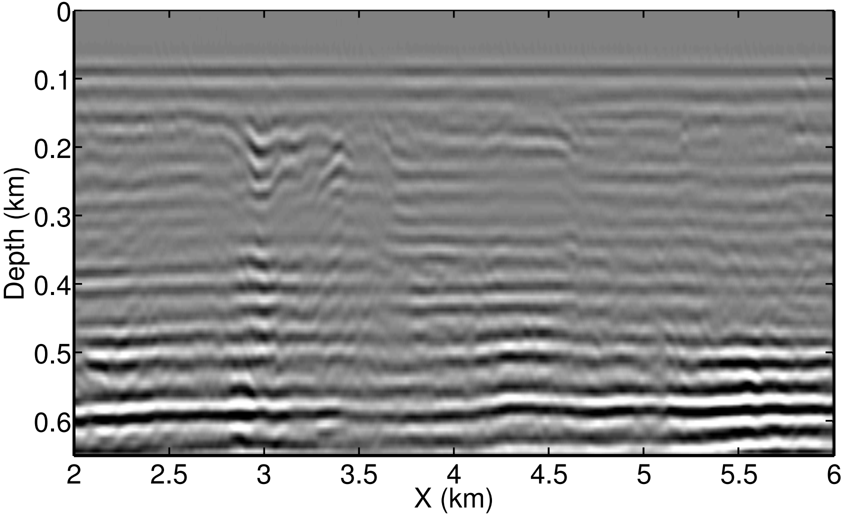
Natural pipes





from Løseth et al. (2011)

Man-made boreholes: before drilling a relief well



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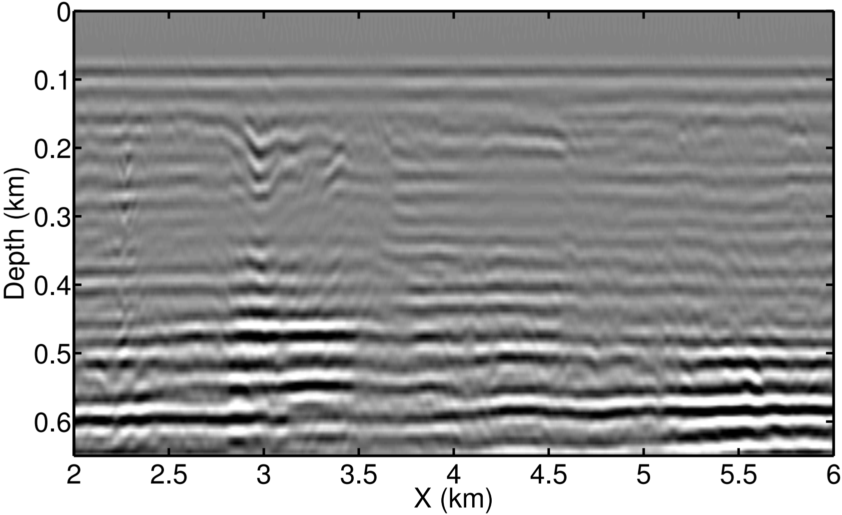

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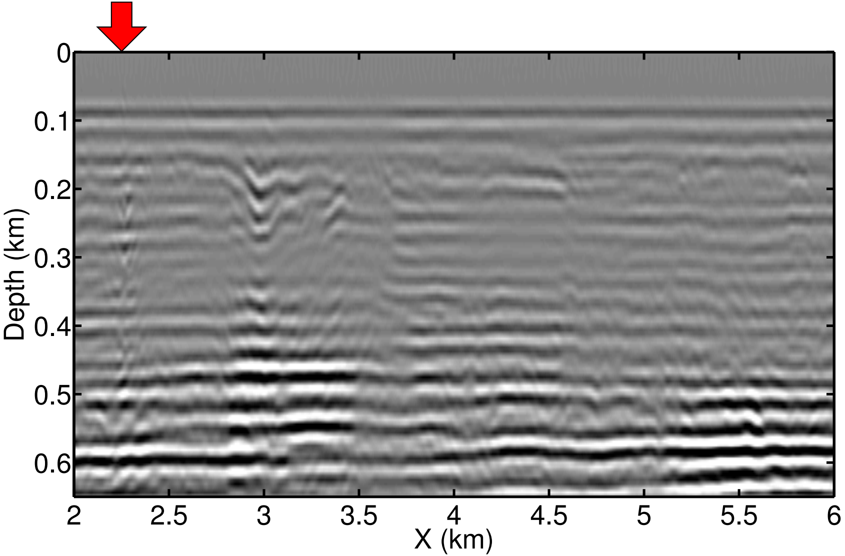
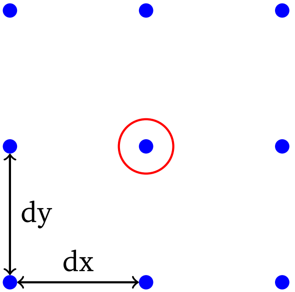
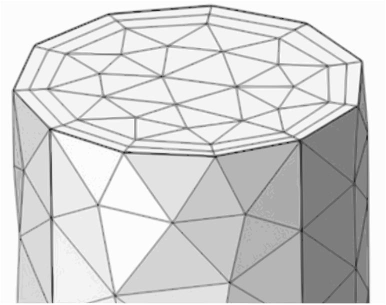


Accuracy of numerical modeling methods

Sampling of space $\Longrightarrow$ loss of accuracy



A small cylinder and a FD grid



A meshed cylinder in FE

Wave equation in time domain

The system of equations in an acoustic medium:

$$\begin{cases} \rho \dfrac{\partial^2 \mathbf{u}}{\partial t^2} = -\nabla p \\ p = -\lambda \nabla \cdot \mathbf{u} + \delta(\mathbf{x}) s(t) \end{cases},$$

ρ, λ – density and bulk modulus of the medium
$\mathbf{u}$ – displacement
p – pressure
$s(t)$ – source signature

Scalar potentials

Let ϕ be the displacement potential: $\mathbf{u} = \nabla\phi \implies$

$$\nabla^2\phi - \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} = \delta(\mathbf{x})\frac{s(t)}{\lambda},$$

$c = \sqrt{\frac{\lambda}{\rho}}$ – wave velocity

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$c = \sqrt{\frac{\lambda}{\rho}}$ – wave velocity

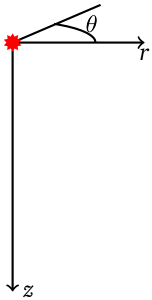
Assuming time-dependence $\phi(\omega, t) = \Phi(\omega)e^{i\omega t} \implies$

$$\nabla^2\Phi + \frac{\omega^2}{c^2}\Phi = \delta(\mathbf{x})\frac{S(\omega)}{\lambda}$$

Solution – in cylindrical coordinates

Solve the equation by separation of variables

$$\Phi = R(r)Z(z)\Theta(\theta)$$

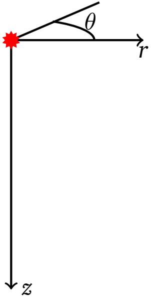


Solution – homogeneous medium

$$\Phi = g(\omega) \int_{-\infty}^{\infty} H_n^{(2)}(kr) \left[Ae^{-\nu z} + Be^{-\nu z} + \frac{1}{\nu} e^{-\nu |z|} \right] kdk$$

$$g(\omega) = -\frac{S(\omega)}{8\pi\lambda}$$

$$\nu = \sqrt{k^2 - \frac{\omega^2}{c^2}}$$

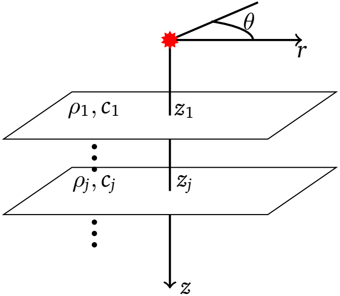


Solution – stratified medium

$$\Phi_j = g(\omega) \int_{-\infty}^{\infty} H_n^{(2)}(kr) \left[A_j e^{-\nu_j z} + B_j e^{-\nu_j z} + \frac{1}{\nu_j} e^{-\nu_j |z|} \right] k dk$$

$$\nu_j = \sqrt{k^2 - \frac{\omega^2}{c_j^2}}$$

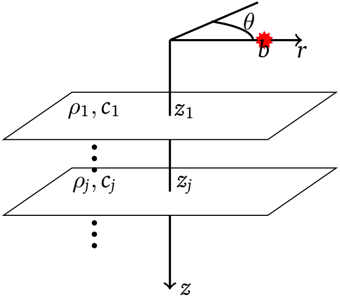
A_j and B_j – from boundary conditions



Solution – offset source

$$\Phi_j = g(\omega) \sum_{n=0}^{\infty} \epsilon_n \cos(n\theta) \int_{-\infty}^{\infty} J_n(kr) H_n^{(2)}(kb) \left[Z_j(z) \right] k dk, \quad r \leqslant b$$

$$\epsilon_n = \begin{cases} 1, n = 0 \\ 2, n > 0 \end{cases}$$



Solution – vertical cylindrical inclusion

$$\Phi_j^{sc}=g(\omega)\sum_{n=0}^{\infty}\epsilon_n\cos(n\theta)\int_{-\infty}^{\infty}S_nH_n^{(2)}(kr)H_n^{(2)}(kb)\left[Z_j(z)\right]kdk$$

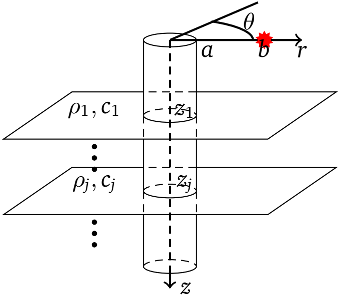
S_n – from boundary conditions:

Empty cylinder: $\left.\Phi_j+\Phi_j^{sc}\right|_{r=a}=0 \implies$

$$\boxed{S_n=-\frac{J_n(ka)}{H_n^{(2)}(ka)}}$$

Rigid cylinder: $\left.\frac{\partial}{\partial r}\Phi_j+\frac{\partial}{\partial r}\Phi_j^{sc}\right|_{r=a}=0 \implies$

$$\boxed{S_n=-\frac{J_n'(ka)}{H_n^{(2)\prime}(ka)}}$$

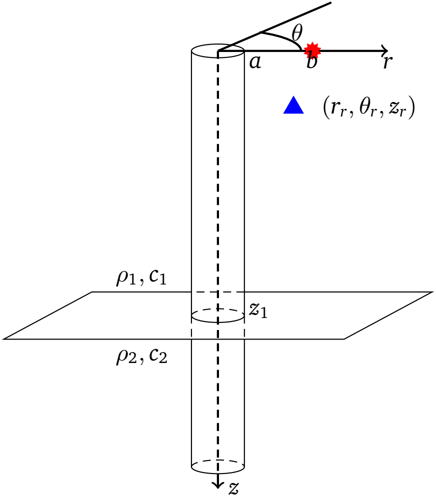


Model parameters

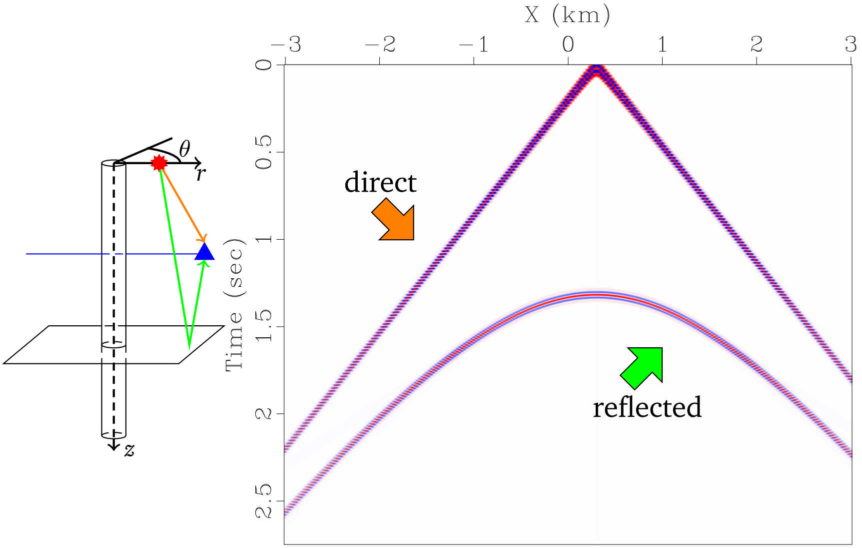
$$\rho_1 = 1.0 \text{ g/cm}^3$$
$$\rho_2 = 2.7 \text{ g/cm}^3$$
$$c_1 = 1500 \text{ m/s}$$
$$c_2 = 2800 \text{ m/s}$$
$$b = 300 \text{ m}$$
$$z_1 = 1000 \text{ m}$$
$$z_s = 0 \text{ m}$$
$$z_r = 25 \text{ m}$$

Acquisition geometry - polar

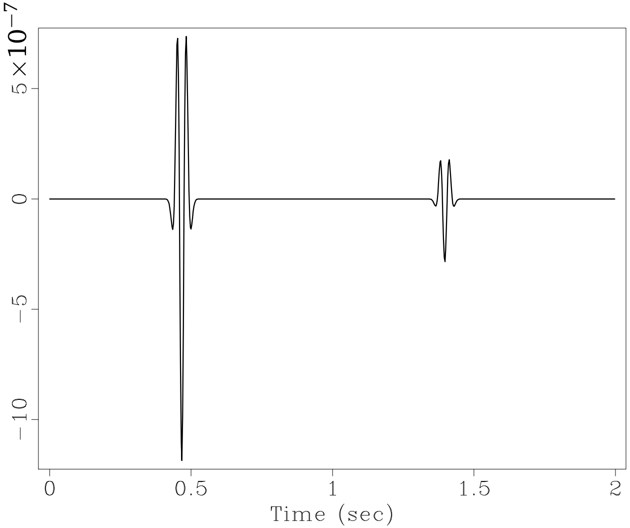
Source signature - Ricker pulse



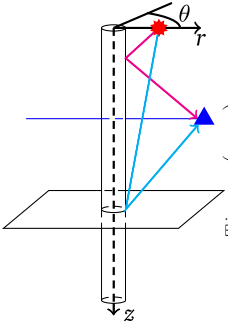
Response of the medium without the well, $f_0 = 20$ Hz, line $Y = 0$ m

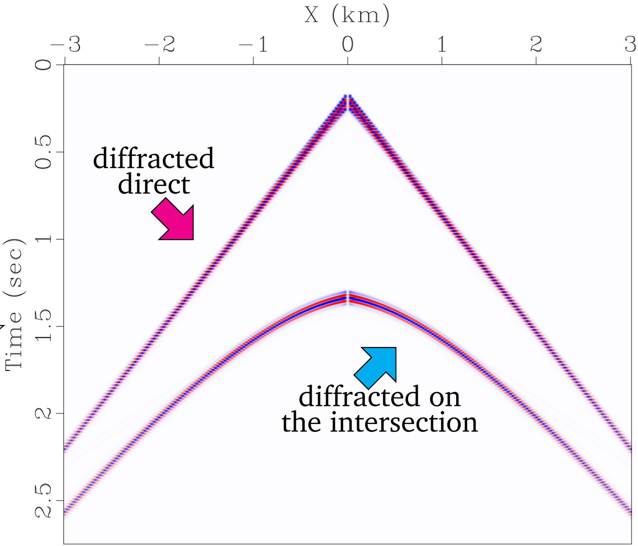


Response of the medium without the well, $f_0 = 20$ Hz, trace $X = 1000$ m, $Y = 0$ m

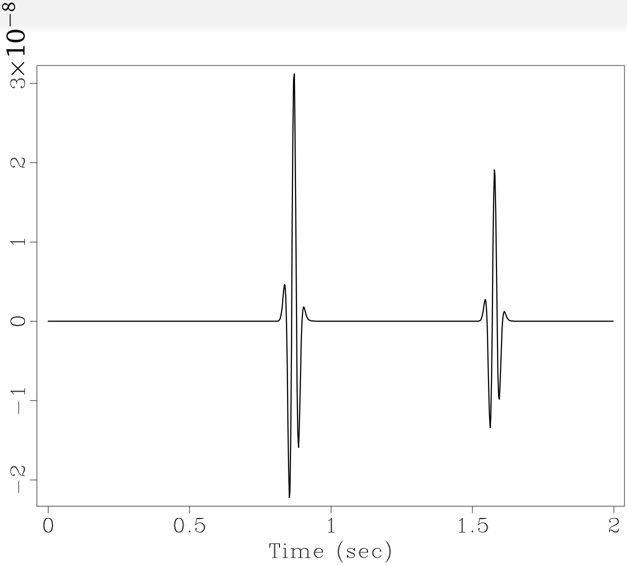


Response of the empty well, $a = 10$ cm, $f_0 = 20$ Hz, line $Y = 0$ m

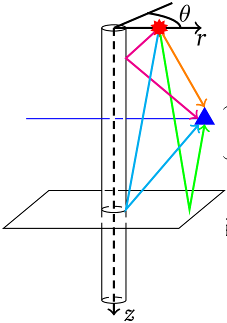


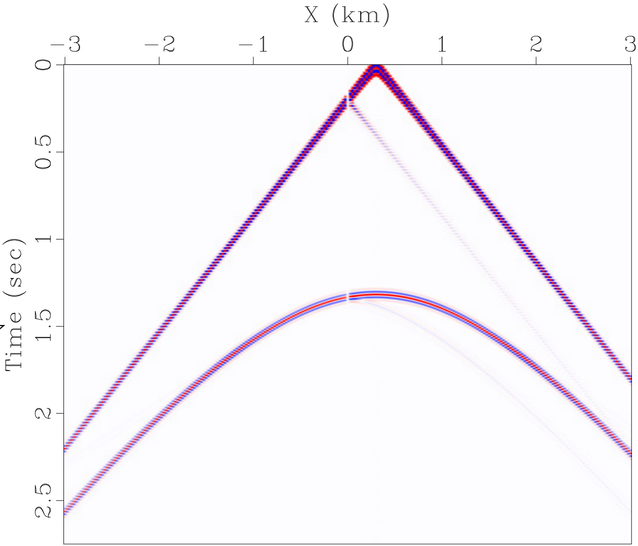


Response of the empty well, $a = 10$ cm, $f_0 = 20$ Hz, trace $X = 1000$ m, $Y = 0$ m

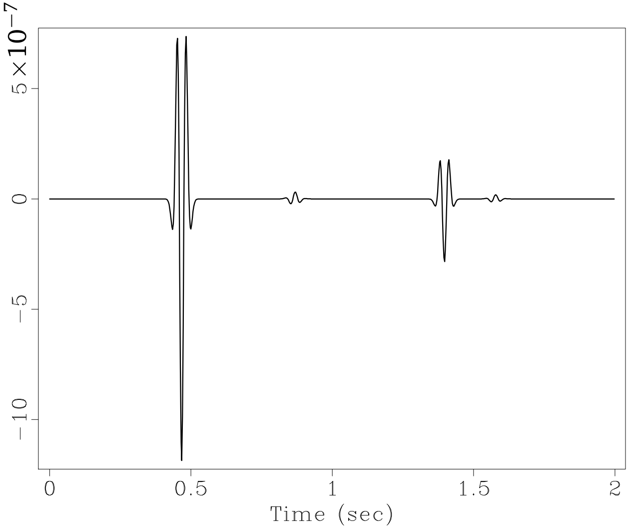


Total wavefield, empty well, $a = 10$ cm, $f_0 = 20$ Hz, line $Y = 0$ m

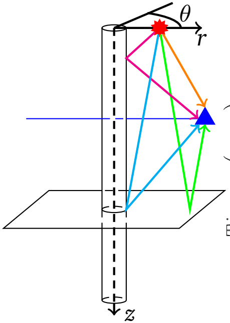


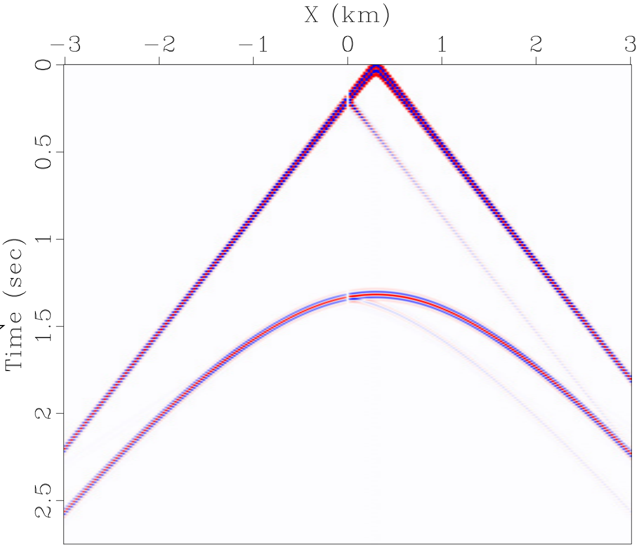


Total wavefield, empty well, $a = 10$ cm, $f_0 = 20$ Hz, trace $X = 1000$ m, $Y = 0$ m

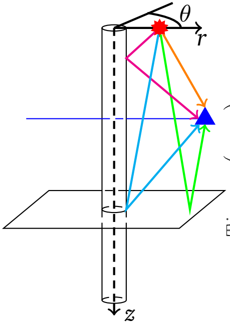


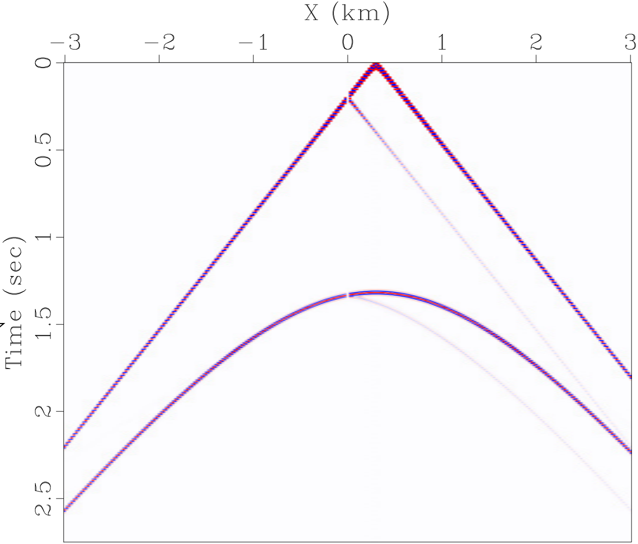
Total wavefield, empty well, $a = 100$ cm, $f_0 = 20$ Hz, line $Y = 0$ m



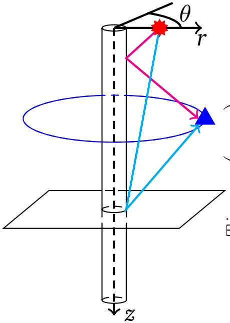


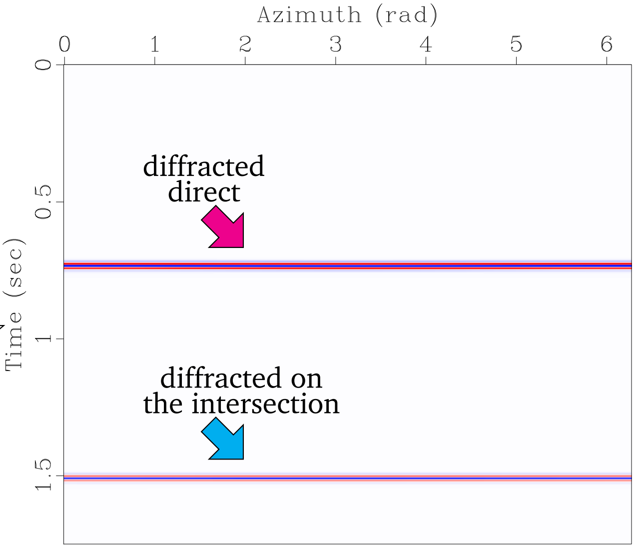
Total wavefield, empty well, $a = 100$ cm, $f_0 = 40$ Hz, line $Y = 0$ m



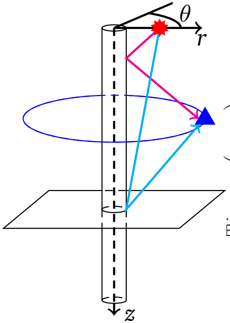


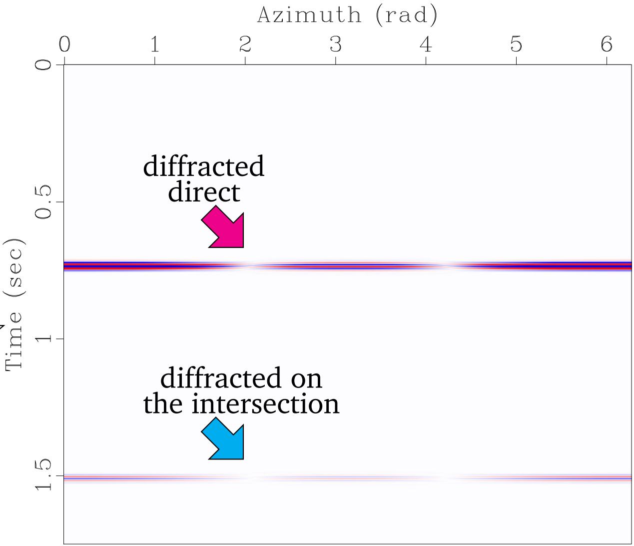
Response of the empty well, $a = 100$ cm, $f_0 = 40$ Hz, line $r = 800$ m





Response of the rigid well, $a = 100$ cm, $f_0 = 40$ Hz, line $r = 800$ m





Scattered wavefields when $ka \to 0$

$$\Phi_j^{sc} = g(\omega) \sum_{n=0}^{\infty} \epsilon_n \cos(n\theta) \int_{-\infty}^{\infty} S_n H_n^{(2)}(kr) H_n^{(2)}(kb) \left[Z_j(z) \right] kdk$$

Empty borehole:

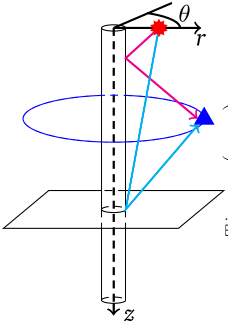
$$Sn \to \begin{cases} -\dfrac{i\pi}{2\ln(ka)}, & n=0 \\ \left(\dfrac{ka}{2}\right)^{2n} \dfrac{i\pi}{n!(n-1)!}, & n>0 \end{cases}$$

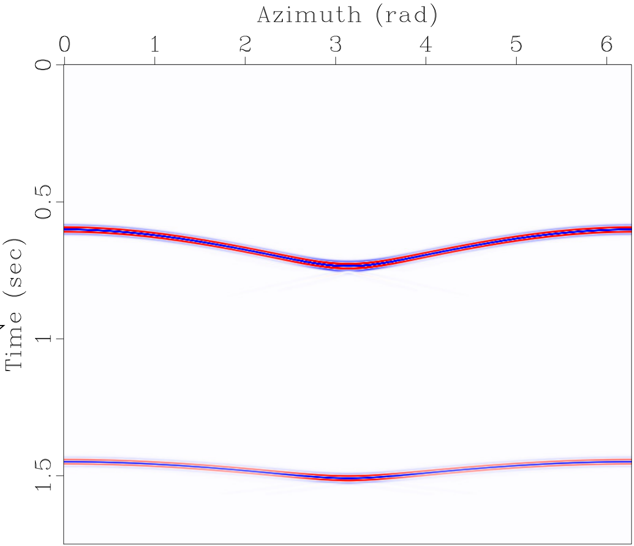
Rigid borehole:

$$Sn \to \begin{cases} \left(\dfrac{ka}{2}\right)^{2} i\pi, & n=0 \\ -\left(\dfrac{ka}{2}\right)^{2n} \dfrac{i\pi}{n!(n-1)!}, & n>0 \end{cases}$$

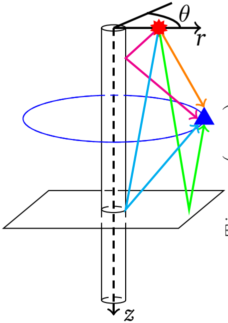
Difference in magnitude only between S_0

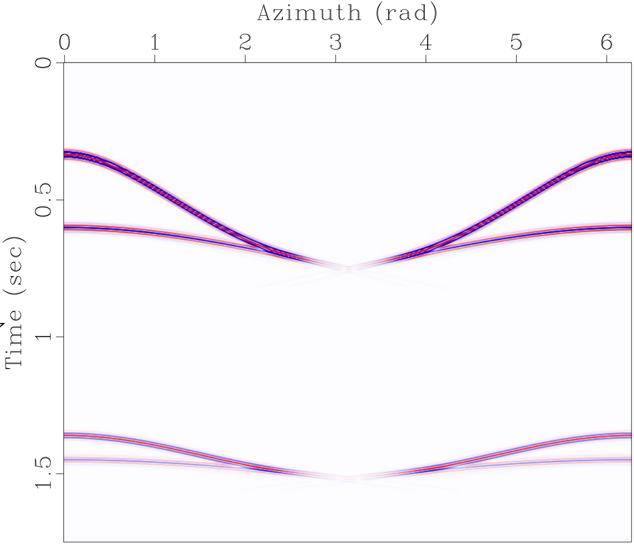
Response of the empty cylinder, $a = 100$ m, $f_0 = 40$ Hz, line $r = 800$ m



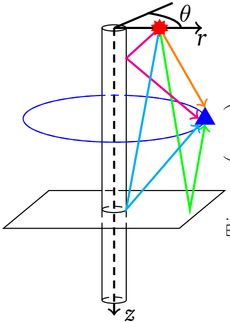


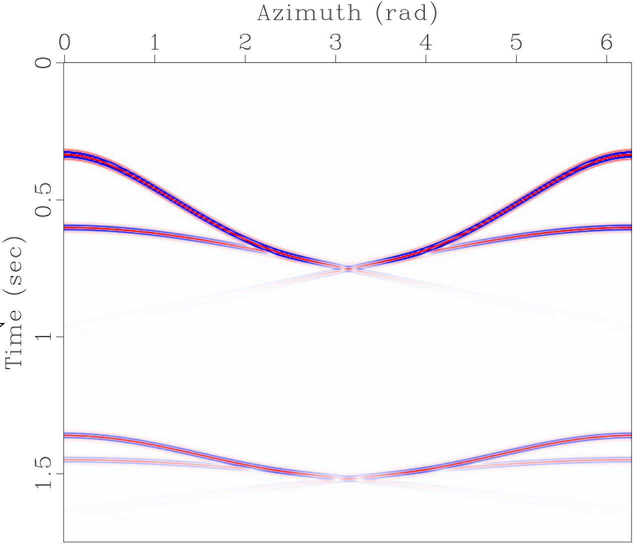
Total wavefield, empty cylinder, $a = 100$ m, $f_0 = 40$ Hz, line $r = 800$ m



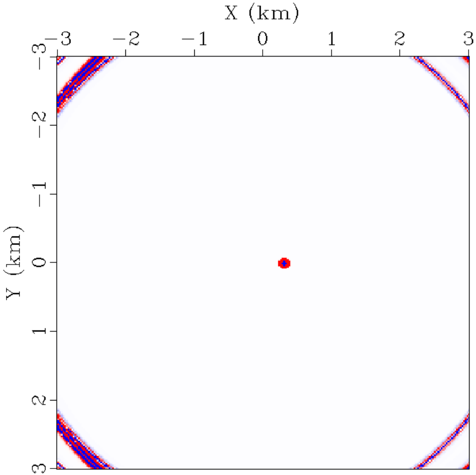


Total wavefield, rigid cylinder, $a = 100$ m, $f_0 = 40$ Hz, line $r = 800$ m





Total wavefield, empty cylinder, $a = 10$ cm, $f_0 = 20$ Hz, areal acquisition



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Conclusions

1. The method provides an exact description of wavefields in stratified acoustic media pierced by empty/rigid cylinders

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1. The method provides an exact description of wavefields in stratified acoustic media pierced by empty/rigid cylinders
2. Scattered wavefield – two parts:
   - Scattered direct wave
   - Scattering from the intersection between the cylinder and the boundaries
3. Even very thin (compared to wavelength) cylinders are visible for seismic frequencies

Discussion

1. The method does not allow both to "fill" cylinders with finite velocity and density and to set an infinite well

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2. Dependence of wavefields on well filling properties should be investigated

Acknowledgements

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References

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Rice, J. A., and D. Willen, 1987, Compressional waves in a layered fluid medium pierced by a right circular cylinder: The Journal of the Acoustical Society of America, **81**, 774.