P- and SV-wave reflections by a thin VTI layer

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Outline

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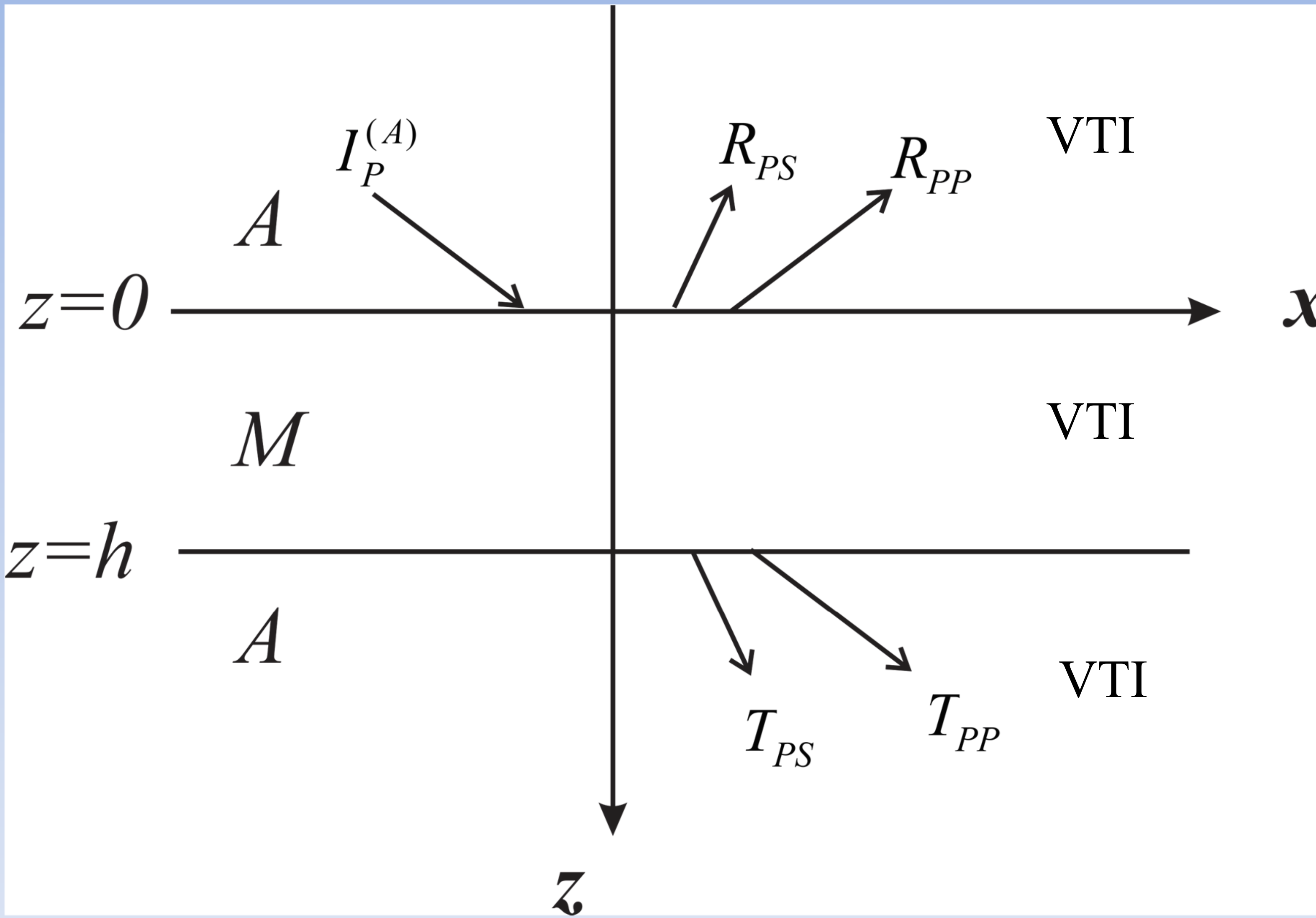
- Thomsen's notation
- Reflection and transmission of waves by a VTI layer
- Transfer matrix for a VTI layer
- An approximate formula for the transfer matrix
- AVO-type formulae for reflection coefficients
- Summary

The statement of problem



Assumption: $h << \lambda_{P0}^{(M)}$ and $\lambda_{S0}^{(M)}$

Thomsen's notation

Velocity parameters along *z*-axis:

$$\upsilon_{P0} = \sqrt{\frac{c_{33}}{\rho}}$$

$$\upsilon_{s0} = \sqrt{\frac{c_{55}}{\rho}}$$

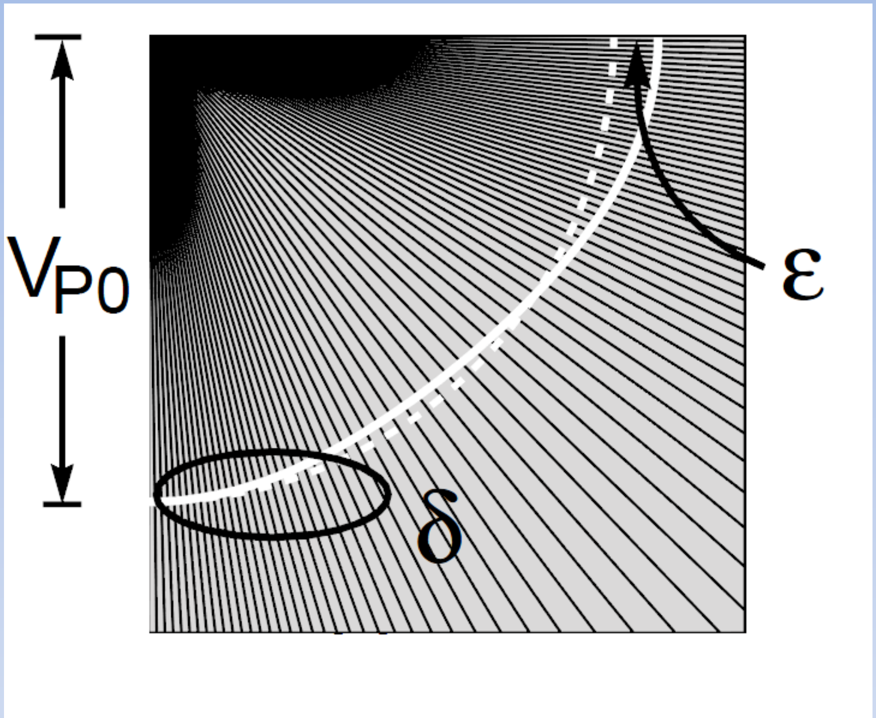
Anisotropy parameters:

$$\varepsilon = \frac{c_{11} - c_{33}}{2c_{33}}$$

$$\delta = \frac{(c_{13} + c_{44})^2 - (c_{33} - c_{44})^2}{2c_{33}(c_{33} - c_{44})}$$

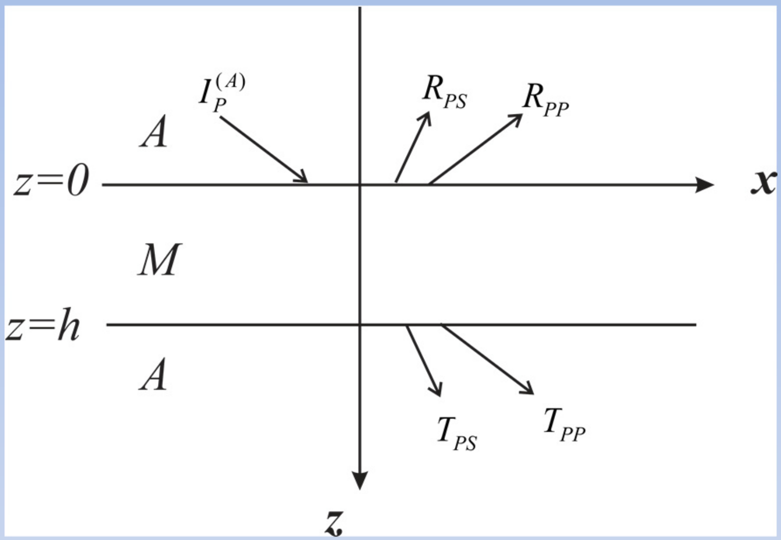
$$\gamma = \frac{c_{66} - c_{44}}{2c_{44}}$$

where c_{ij} and ρ denote stiffness coefficients and density.



P-wave ray propagation in the [x, z] plane of a homogeneous VTI medium (after Ruger, 1996). The solid line denotes the wavefront of P-waves in VTI media. The dashed line denotes the wavefront in the isotropic reference.

Reflection and transmission of waves by a VTI layer



Displacement of an incident P-wave:

$$\boldsymbol{u}_P^{(I)} = \boldsymbol{g}_P^{(I)} \exp(-i\omega(t - px - q_P^{(I)} z))$$

Displacements of reflected P- and S-waves:

$$\boldsymbol{u}_P^{(A-)} = R_{PP} \boldsymbol{g}_P^{(A-)} \exp(-i\omega(t - px + q_P^{(A)} z))$$

$$\boldsymbol{u}_S^{(A-)} = R_{PS} \boldsymbol{g}_S^{(A-)} \exp(-i\omega(t - px + q_S^{(A)} z))$$

Displacement of transmitted P- and S-waves:

$$\boldsymbol{u}_P^{(A+)} = T_{PP} \boldsymbol{g}_P^{(A+)} \exp(-i\omega(t - px - q_P^{(A)} (z - h)))$$

$$\boldsymbol{u}_S^{(A+)} = T_{PS} \boldsymbol{g}_S^{(A+)} \exp(-i\omega(t - px - q_S^{(A)} (z - h)))$$

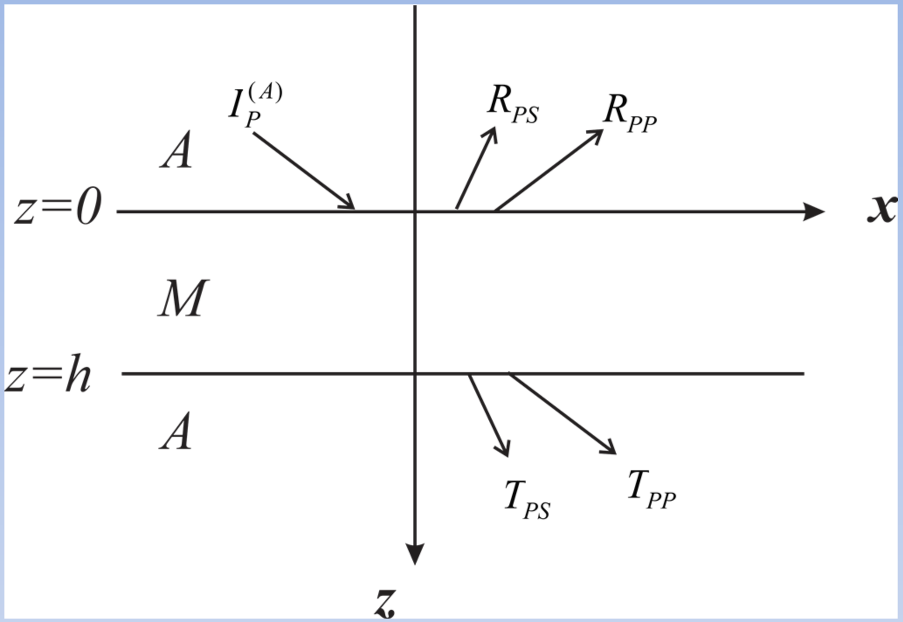
Convention: the x-component of polarization vector of all waves is always positive.

Transfer matrix for a VTI layer

Transfer matrix *B*

$$\boldsymbol{w}\big|_{z=0} = \boldsymbol{B}\,\boldsymbol{w}\big|_{z=h}$$

where $\boldsymbol{w} = (\upsilon_x, \upsilon_z, \tau_{zz}, \tau_{zx})^T$



$$\begin{pmatrix} \upsilon_x \\ \upsilon_z \\ \tau_{zz} \\ \tau_{zx} \end{pmatrix}\Bigg|_{z=0} = \begin{pmatrix} B_{11} & B_{12} & B_{13} & B_{14} \\ B_{21} & B_{22} & B_{23} & B_{24} \\ B_{31} & B_{32} & B_{33} & B_{34} \\ B_{41} & B_{42} & B_{43} & B_{44} \end{pmatrix} \begin{pmatrix} \upsilon_x \\ \upsilon_z \\ \tau_{zz} \\ \tau_{zx} \end{pmatrix}\Bigg|_{z=h}$$

$B_{13} = B_{24}$ $\quad B_{12} = B_{34}$ $\quad B_{11} = B_{44}$ $\quad B_{22} = B_{33}$ $\quad B_{21} = B_{43}$ $\quad B_{31} = B_{42}$

When h=0,

$$\begin{pmatrix} B_{11} & B_{12} & B_{13} & B_{14} \\ B_{21} & B_{22} & B_{23} & B_{24} \\ B_{31} & B_{32} & B_{33} & B_{34} \\ B_{41} & B_{42} & B_{43} & B_{44} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Transfer matrix for *N*-layer media

$$\boldsymbol{w}\big|_{z=0} = \boldsymbol{B}_1\boldsymbol{B}_2\ldots\boldsymbol{B}_n\,\boldsymbol{w}\big|_{z=h_1+h_2+\ldots+h_N}$$

An approximate formula for the transfer matrix

Assumption: $h << \lambda_{P0}^{M}$ and λ_{S0}^{M}

$$\begin{pmatrix} B_{11} & B_{12} & B_{13} & B_{14} \\ B_{21} & B_{22} & B_{23} & B_{24} \\ B_{31} & B_{32} & B_{33} & B_{34} \\ B_{41} & B_{42} & B_{43} & B_{44} \end{pmatrix} \approx \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} + (ih\omega) \left(\begin{array}{cc|cc} 0 & A_{12} & 0 & A_{14} \\ A_{21} & 0 & A_{23} & 0 \\ \hline 0 & A_{32} & 0 & A_{12} \\ A_{41} & 0 & A_{21} & 0 \end{array}\right)$$

$$+ (ih\omega)^2 \left(\begin{array}{cc|cc} C_{11} & 0 & C_{13} & 0 \\ 0 & C_{22} & 0 & C_{13} \\ \hline C_{31} & 0 & C_{22} & 0 \\ 0 & C_{31} & 0 & C_{11} \end{array}\right)$$

$$A_{ij} = A_{ij}(\rho^{(M)}, \upsilon_{P0}^{(M)}, \upsilon_{S0}^{(M)}, \varepsilon^{(M)}, \delta^{(M)}, p) \quad B_{ij} = B_{ij}(\rho^{(M)}, \upsilon_{P0}^{(M)}, \upsilon_{S0}^{(M)}, \varepsilon^{(M)}, \delta^{(M)}, p)$$

$$C_{ij} = C_{ij}(\rho^{(M)}, \upsilon_{P0}^{(M)}, \upsilon_{S0}^{(M)}, \varepsilon^{(M)}, \delta^{(M)}, p)$$

where p denotes the horizontal slowness component.

Reflection and transmission coefficients: normal incidence of P- and SV-waves

$$R_{PP} = \frac{2r_{PP}\sin k_{P0}^{(M)}}{(1+r_{PP}^2)\sin k_{P0}^{(M)} + i(1-r_{PP}^2)\cos k_{P0}^{(M)}}$$

$$T_{PP} = \frac{i(1-r_{PP}^2)}{(1+r_{PP}^2)\sin k_{P0} + i(1-r_{PP}^2)\cos k_{P0}}$$

$$r_{PP} = \frac{Z_{P0}^{(M)} - Z_{P0}^{(A)}}{Z_{P0}^{(M)} + Z_{P0}^{(A)}} \qquad k_{P0}^{(M)} = \frac{h\omega}{\upsilon_{P0}^{(M)}}$$

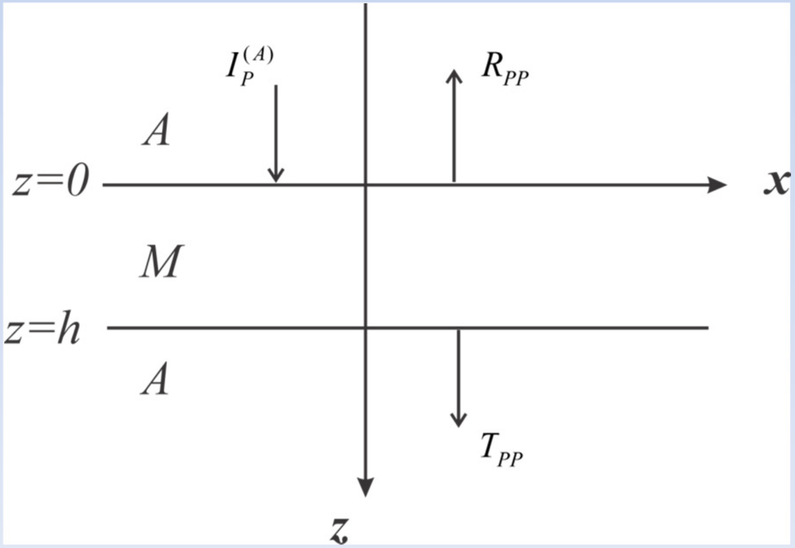
$$Z_{P0} = \rho\upsilon_{P0}$$

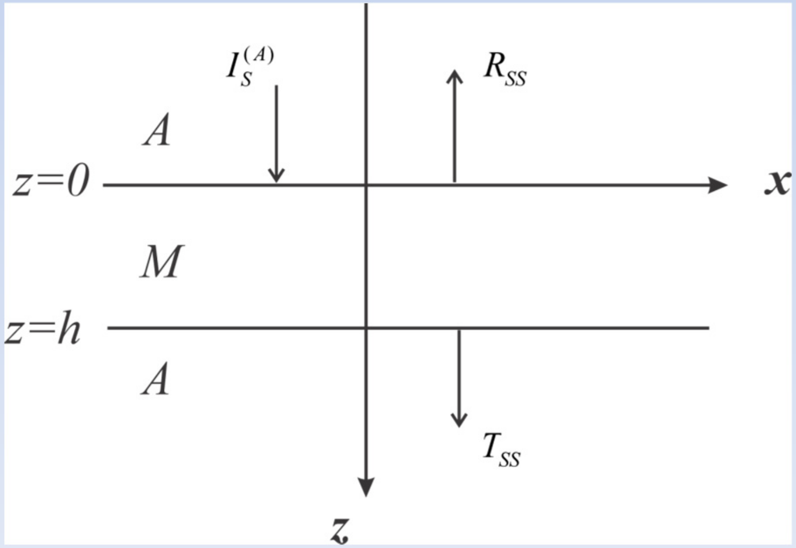
$$R_{SS} = \frac{2r_{SS}\sin k_{P0}^{(M)}}{(1+r_{SS}^2)\sin k_{S0}^{(M)} + i(1-r_{SS}^2)\cos k_{S0}^{(M)}}$$

$$T_{SS} = \frac{i(1-r_{SS}^2)}{(1+r_{SS}^2)\sin k_{S0} + i(1-r_{SS}^2)\cos k_{S0}}$$

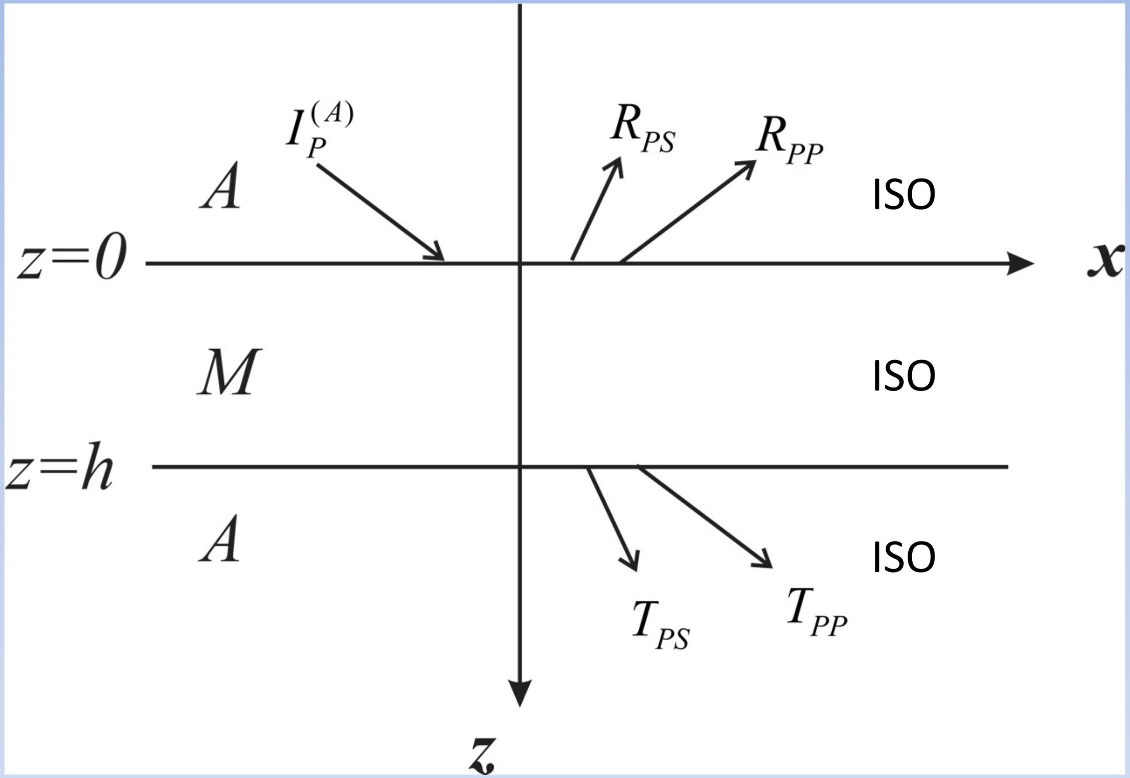
$$r_{SS} = \frac{Z_{S0}^{(M)} - Z_{S0}^{(A)}}{Z_{S0}^{(M)} + Z_{S0}^{(A)}} \qquad k_{S0}^{(M)} = \frac{h\omega}{\upsilon_{S0}^{(M)}}$$

$$Z_{S0} = \rho\upsilon_{S0}$$





Example 1: exact reflection coefficients by a thin isotropic layer

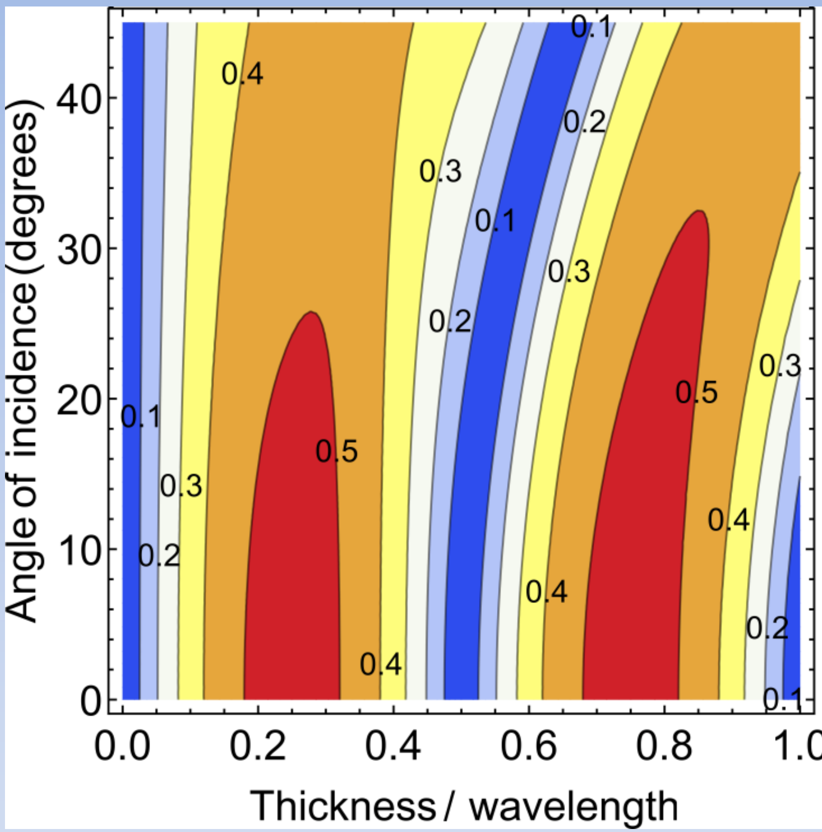


$\omega = 30Hz$

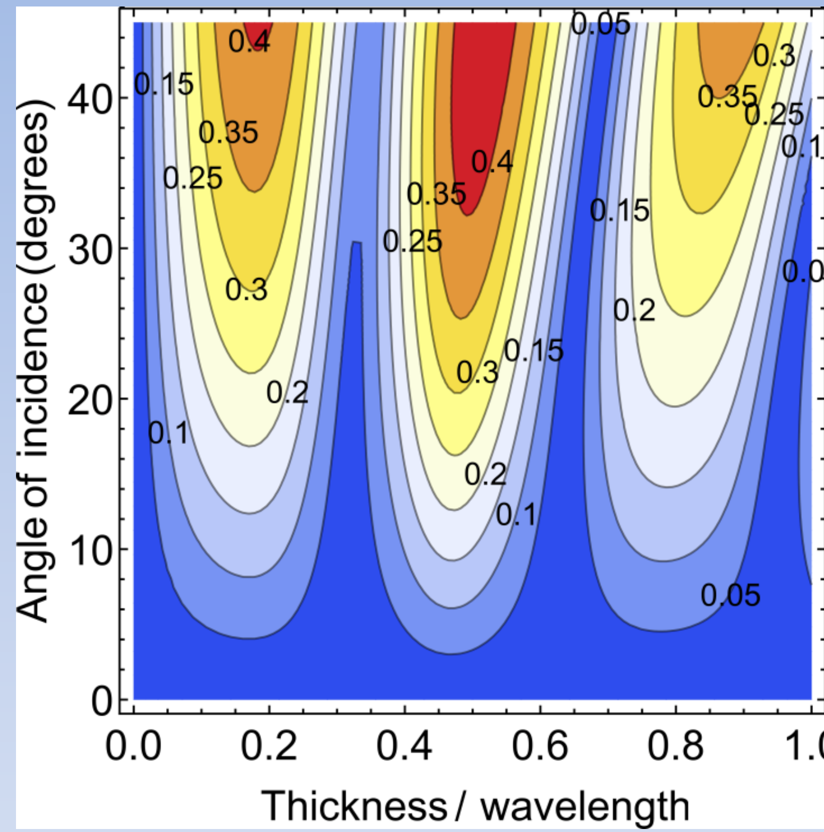
$$\upsilon_{P0}^{(A)} = 2.5km/s \quad \upsilon_{S0}^{(A)} = 1.2km/s \quad \rho^{(A)} = 2.3g/cm^3$$

$$\upsilon_{P0}^{(M)} = 3.5km/s \quad \upsilon_{S0}^{(M)} = 1.6km/s \quad \rho^{(M)} = 3.0g/cm^3$$

$|R_{PP}|$



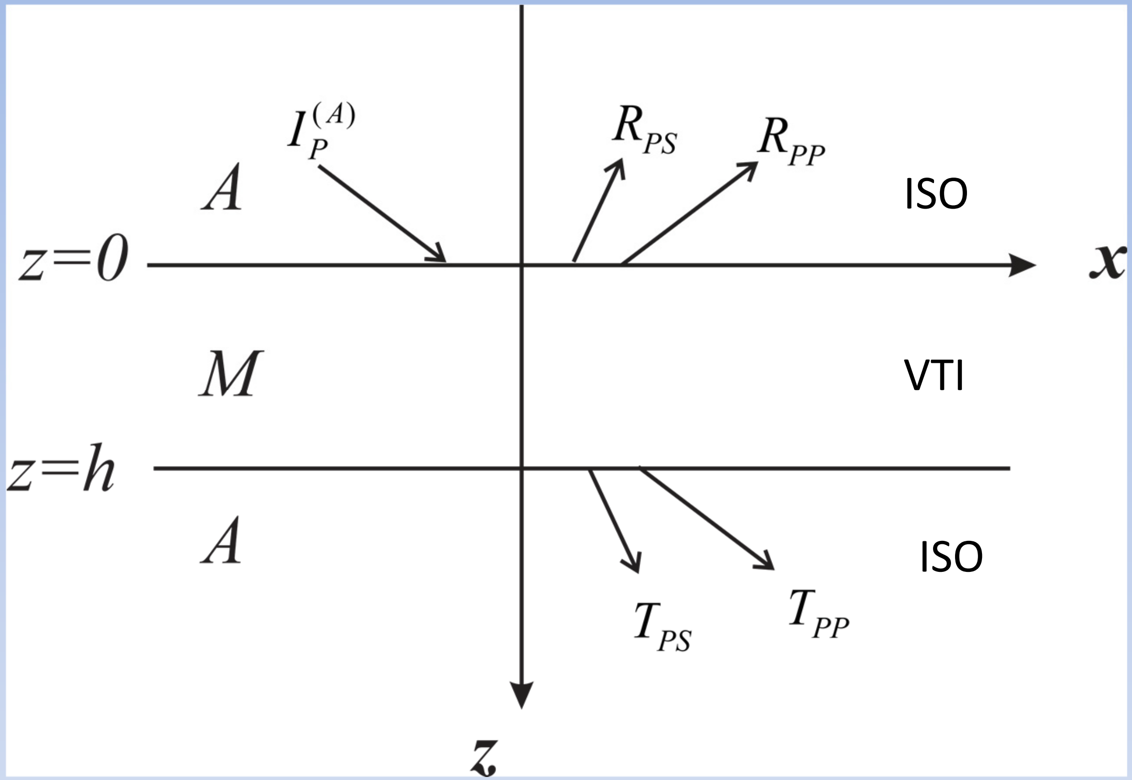
$|R_{PS}|$



Here, wavelength means the wavelength of P-waves inside the isotropic layer.

Example 2: A test on the first-order approximation for transfer matrix

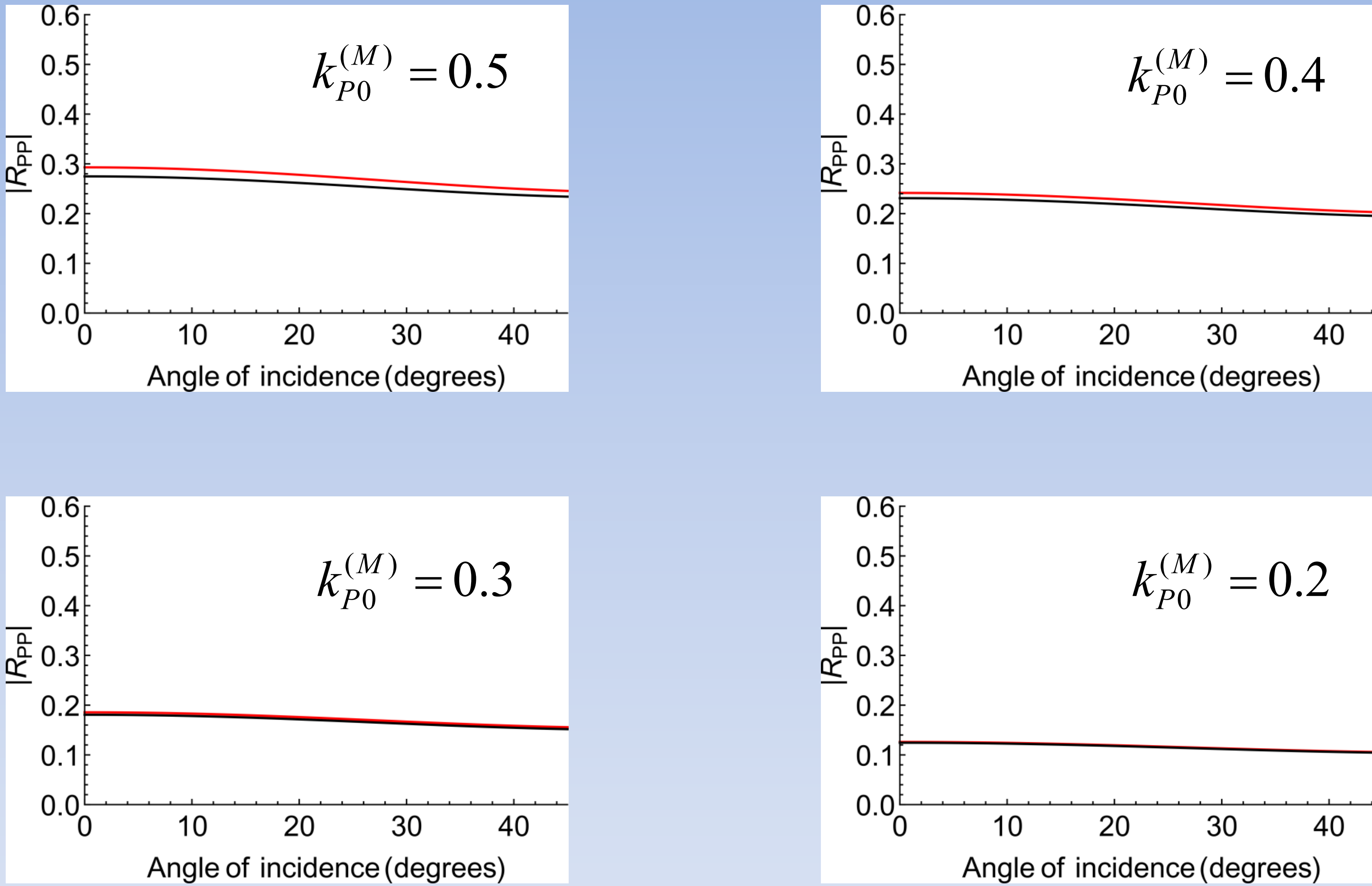
$\omega = 30Hz$



$\upsilon_{P0}^{(A)} = 2.5km/s \quad \upsilon_{S0}^{(A)} = 1.2km/s \quad \rho^{(A)} = 2.3g/cm^3 \quad \varepsilon^{(A)} = 0.0 \quad \delta^{(A)} = 0.0$

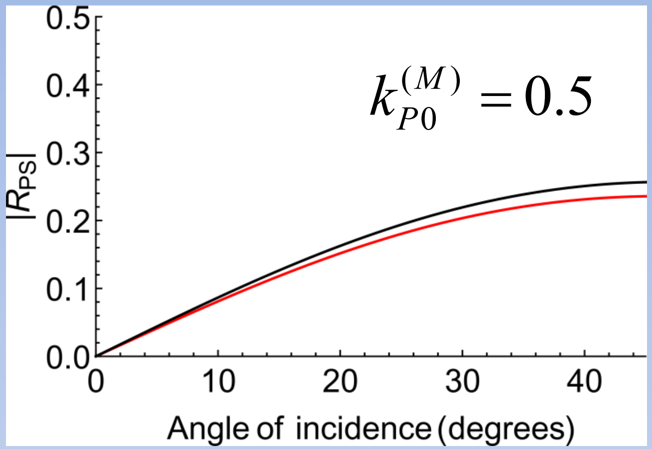
$\upsilon_{P0}^{(M)} = 3.5km/s \quad \upsilon_{S0}^{(M)} = 1.6km/s \quad \rho^{(M)} = 3.0g/cm^3 \quad \varepsilon^{(M)} = 0.1 \quad \delta^{(M)} = 0.05$

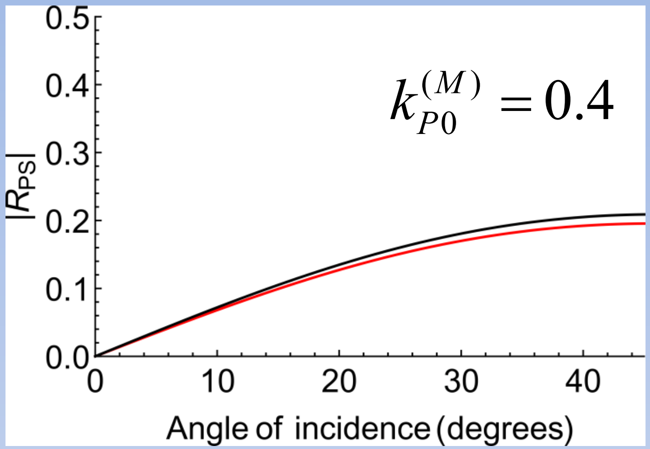
$\left| R_{PP} \right|$

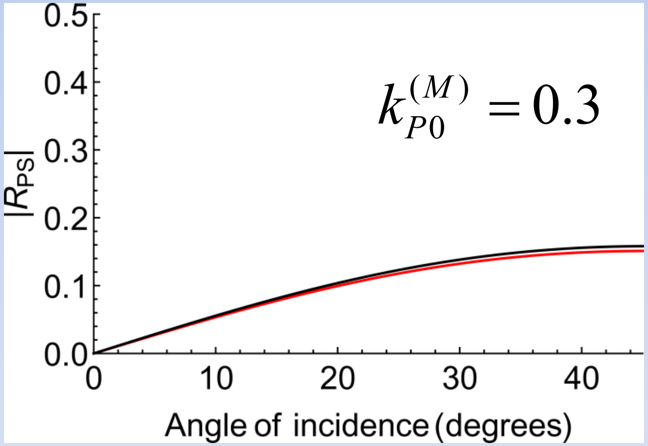


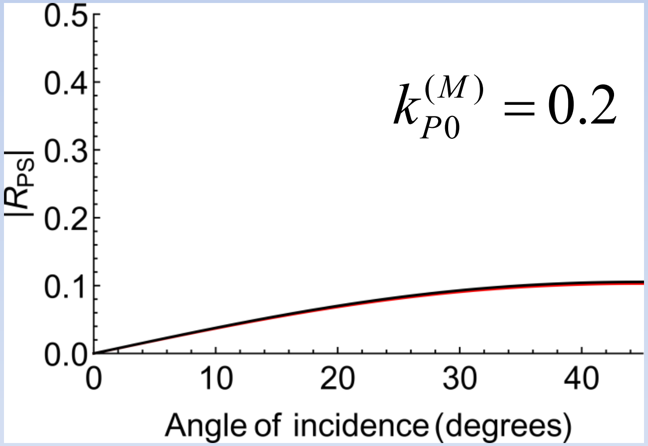
$k_{P0}^{(M)} \equiv \omega h / \upsilon_{P0}^{(M)}$ Red: exact ; black: approximate.

$|R_{PS}|$









$k_{P0}^{(M)} \equiv \omega h / \upsilon_{P0}^{(M)}$ Red: exact ; black: approximate.

AVO-type formulae for reflection coefficients - assumptions

Weak contrast in velocity parameters and density:

$$\left|\frac{\Delta\upsilon_{p0}}{\bar{\upsilon}_{p0}}\right| \ll 1 \qquad \left|\frac{\Delta\upsilon_{s0}}{\bar{\upsilon}_{s0}}\right| \ll 1 \qquad \left|\frac{\Delta\rho}{\bar{\rho}}\right| \ll 1$$

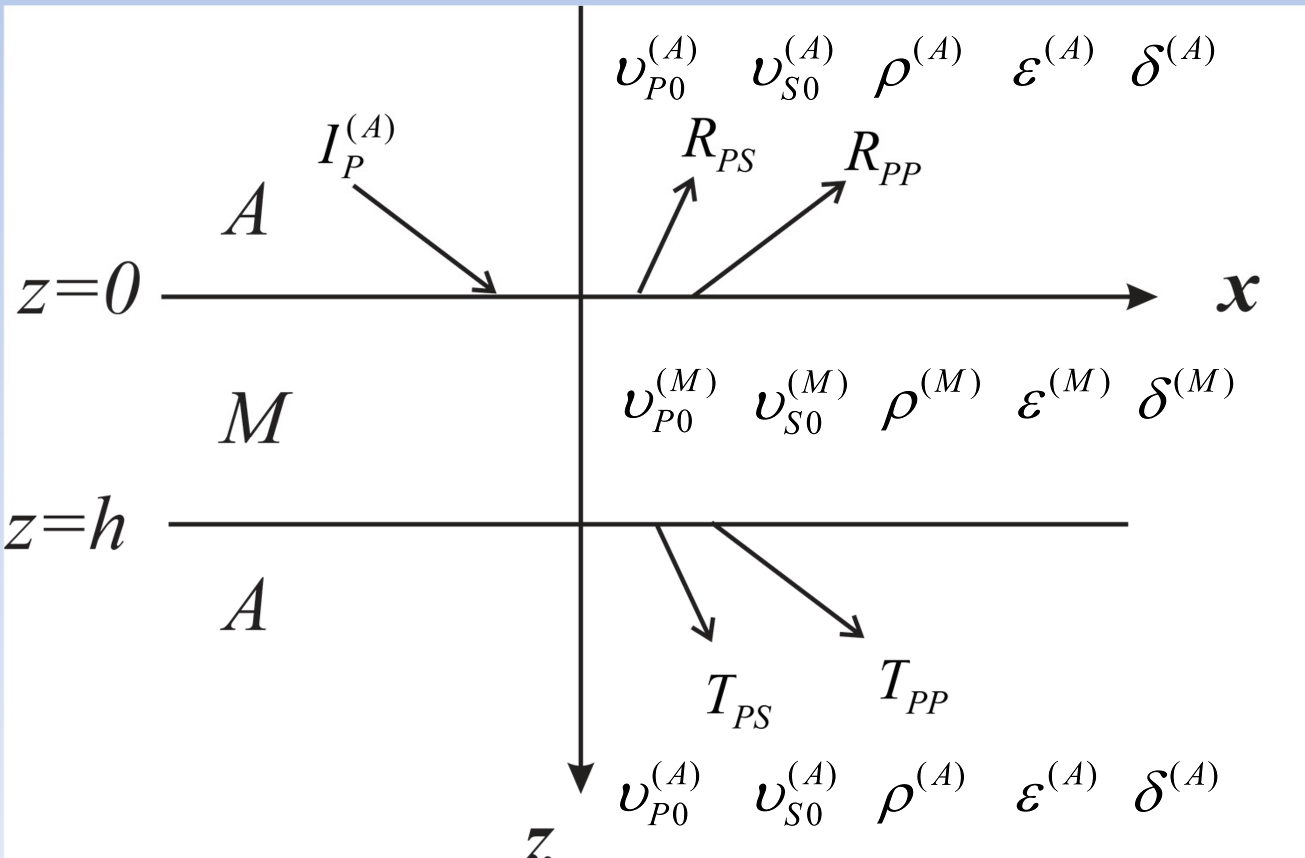
$$\Delta\upsilon_{P0} = \upsilon_{P0}^{(M)} - \upsilon_{P0}^{(A)}$$

$$\bar{\upsilon}_{P0} = \frac{1}{2}(\upsilon_{P0}^{(M)} + \upsilon_{P0}^{(A)})$$

and weak anisotropy:

$$\left|\varepsilon^{(A)}\right| \ll 1 \qquad \left|\delta^{(A)}\right| \ll 1 \qquad \left|\varepsilon^{(M)}\right| \ll 1 \qquad \left|\delta^{(M)}\right| \ll 1$$

Thin layer: $h << \lambda_{P0}^{(M)}$ and $\lambda_{S0}^{(M)}$



AVO-type formulae for reflection coefficients – PP-waves

The first-order formula

$$R_{PP} = \frac{i\omega h}{\bar{\upsilon}_{p0}} R_{PP}^{(1)}$$

where

$$R_{PP}^{(1)} = -\cos\theta_P \left(\frac{\Delta Z_{P0}}{\bar{Z}_{P0}} + \tan^2\theta_P \frac{\Delta\upsilon_{P0}}{\bar{\upsilon}_{P0}} - 4\sin^2\theta_P \frac{\bar{\upsilon}_{S0}^2}{\bar{\upsilon}_{P0}^2} \frac{\Delta G_0}{\bar{G}_0} + \sin^2\theta_P (\tan^2\theta_P \Delta\varepsilon + \Delta\delta) \right)$$

θ_P denotes the angle of incidence of P-waves.

$Z_{P0} = \rho\upsilon_{P0}$ denotes P-wave impedance in the isotropic reference.

$G_0 = \rho\upsilon_{S0}^2$ denotes shear modulus in the isotropic reference.

The second-order formula

$$R_{PP} = \frac{i\omega h}{\bar{\upsilon}_{p0}} R_{PP}^{(1)} - \frac{\omega^2 h^2}{\bar{\upsilon}_{P0}^2} R_{PP}^{(2)}$$

where

$$R_{PP}^{(2)} = \cos\theta_P R_{PP}^{(1)}$$

AVO-type formulae for reflection coefficients – SS-waves

The first-order formula

$$R_{SS} = \frac{i\omega h}{\bar{\upsilon}_{s0}} R_{SS}^{(1)}$$

where

$$R_{SS}^{(1)} = -\cos\theta_S \left((1 - 2\cos(2\theta_S)) \frac{\Delta G_0}{\bar{G}_0} + \frac{\cos(2\theta_S)}{\cos^2\theta_S} \frac{\Delta\upsilon_{S0}}{\bar{\upsilon}_{S0}} + \sin^2\theta_S \frac{\bar{\upsilon}_{S0}^2}{\bar{\upsilon}_{P0}^2} (\Delta\varepsilon - \Delta\delta) \right)$$

θ_S denotes the angle of incidence of S-waves in the isotropic reference.

$G_0 = \rho\upsilon_{S0}^2$ denotes shear modulus in the isotropic reference.

The second-order formula

$$R_{SS} = \frac{i\omega h}{\bar{\upsilon}_{p0}} R_{SS}^{(1)} - \frac{\omega^2 h^2}{\bar{\upsilon}_{S0}^2} R_{SS}^{(2)}$$

where

$$R_{SS}^{(2)} = \cos\theta_S R_{SS}^{(1)}$$

AVO-type formulae for reflection coefficient – PS-waves

The first-order formula

$$R_{PS} = \frac{i\omega h}{\bar{\upsilon}_{P0}} R_{PS}^{(11)} + \frac{i\omega h}{\bar{\upsilon}_{S0}} R_{PS}^{(12)}$$

where

$$R_{PS}^{(11)} = \sin\theta_S \frac{\Delta\rho}{\bar{\rho}} + 2\cos(2\theta_S)\sin\theta_S \frac{\Delta\upsilon_{S0}}{\bar{\upsilon}_{S0}}$$

$$R_{PS}^{(12)} = \frac{\sin(\theta_P + 3\theta_S)}{\cos\theta_S}\frac{\Delta\rho}{\bar{\rho}} + 2\cos\theta_P\cos(2\theta_S)\sin\theta_S \frac{\Delta\upsilon_{S0}}{\bar{\upsilon}_{S0}} - \frac{\bar{\upsilon}_{P0}}{2\bar{\upsilon}_{S0}}\sin\theta_S(2\sin^2\theta_P\Delta\varepsilon - \cos(2\theta_P)\Delta\delta)$$

θ_S denotes the reflection angle of PS-waves in the isotropic reference.

The second-order formula

$$R_{PS} = \frac{i\omega h}{\bar{\upsilon}_{P0}} R_{PS}^{(11)} + \frac{i\omega h}{\bar{\upsilon}_{S0}} R_{PS}^{(12)} - \frac{\omega^2 h^2}{\bar{\upsilon}_{P0}^2} R_{PS}^{(21)} - \frac{\omega^2 h^2}{\bar{\upsilon}_{P0}\bar{\upsilon}_{S0}} R_{PS}^{(22)} - \frac{\omega^2 h^2}{\bar{\upsilon}_{S0}^2} R_{SS}^{(23)}$$

Example 3: A Numerical test on the AVO-type formulae

$$\upsilon_{P0}^{(A)} = 3.0km/s \quad \upsilon_{S0}^{(A)} = 1.6km/s \quad \rho^{(A)} = 2.6g/cm^3 \quad \varepsilon^{(M)} = 0.1 \quad \delta^{(M)} = 0.05$$

$$\upsilon_{P0}^{(M)} = 2.5km/s \quad \upsilon_{S0}^{(M)} = 1.2km/s \quad \rho^{(M)} = 2.3g/cm^3 \quad \varepsilon^{(M)} = 0.2 \quad \delta^{(M)} = -0.1$$

$$\omega = 30Hz \qquad k_{P0}^{(M)} = \omega h / \upsilon_{P0}^{(M)} = 0.2 \quad \Rightarrow \quad h = 16.7m$$

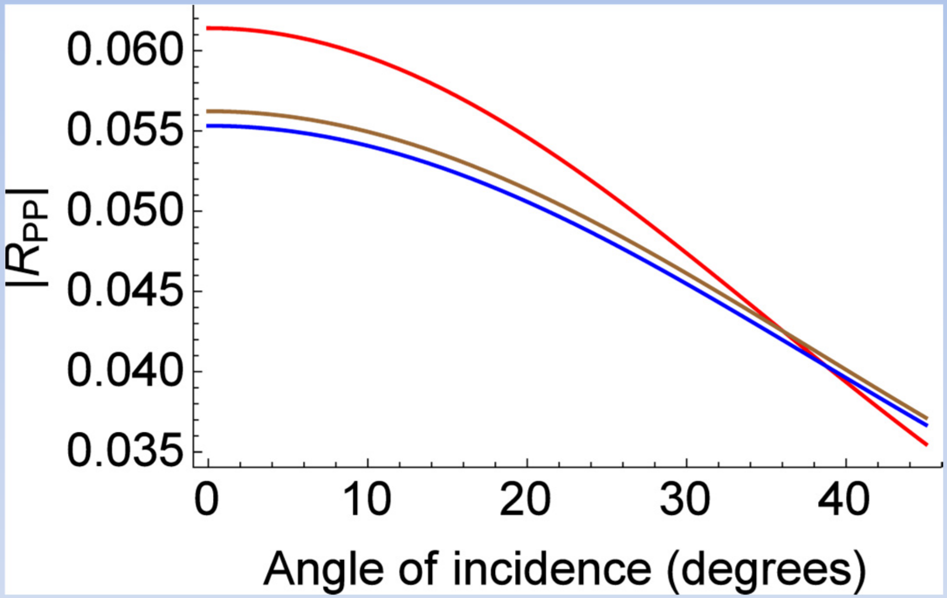
PP-wave reflection coefficient: $R_{PP} = |R_{PP}| \exp(i\phi_{PP})$ $\quad \phi_{PP} \in (-\pi, \pi]$

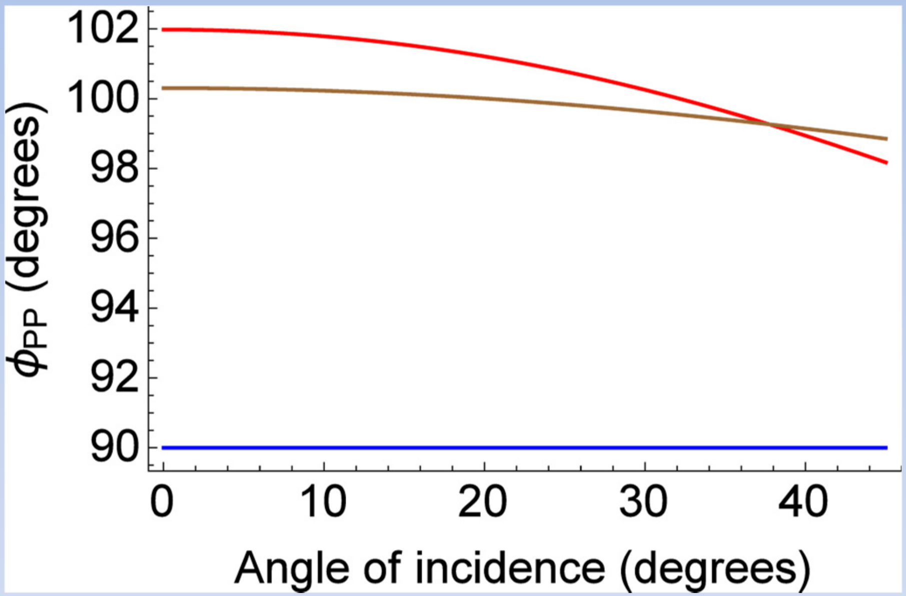
PS-wave reflection coefficient: $R_{PS} = |R_{PS}| \exp(i\phi_{PS})$ $\quad \phi_{PS} \in (-\pi, \pi]$

SS-wave reflection coefficient: $R_{SS} = |R_{SS}| \exp(i\phi_{SS})$ $\quad \phi_{SS} \in (-\pi, \pi]$

PP-waves

$\left| R_{PP} \right|$ ϕ_{PP}

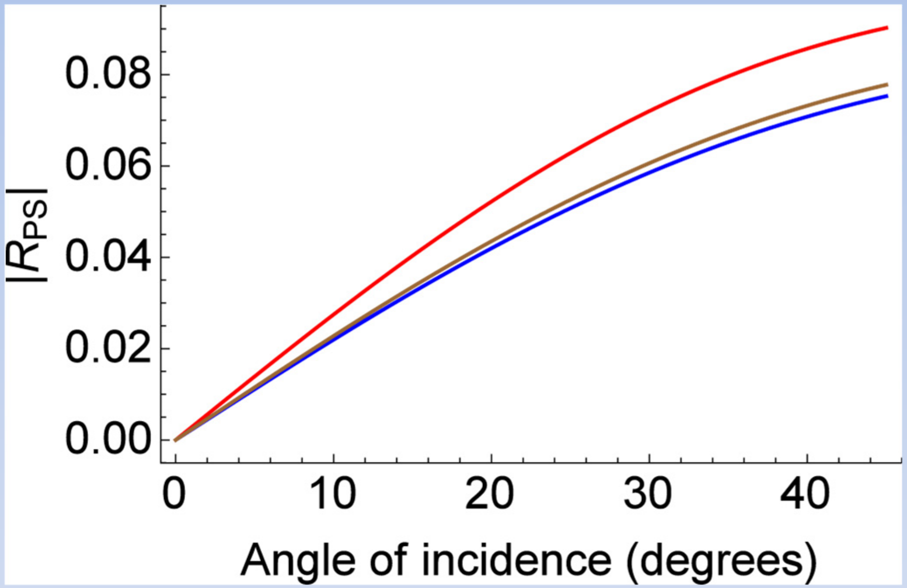




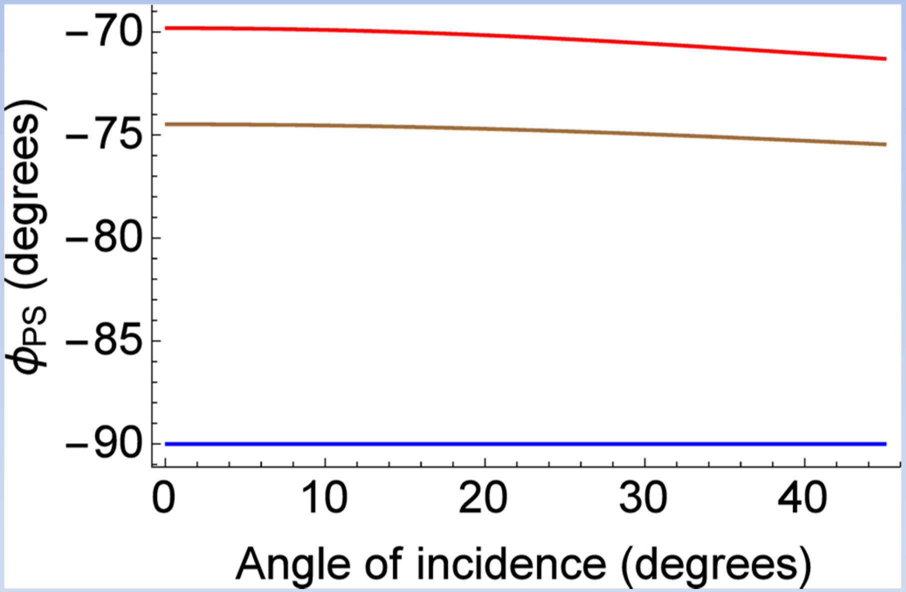
Red: exact result. Blue: the first-order AVO-type formula. Brown: the second-order AVO-type formula

PS-waves

$\left|R_{PS}\right|$



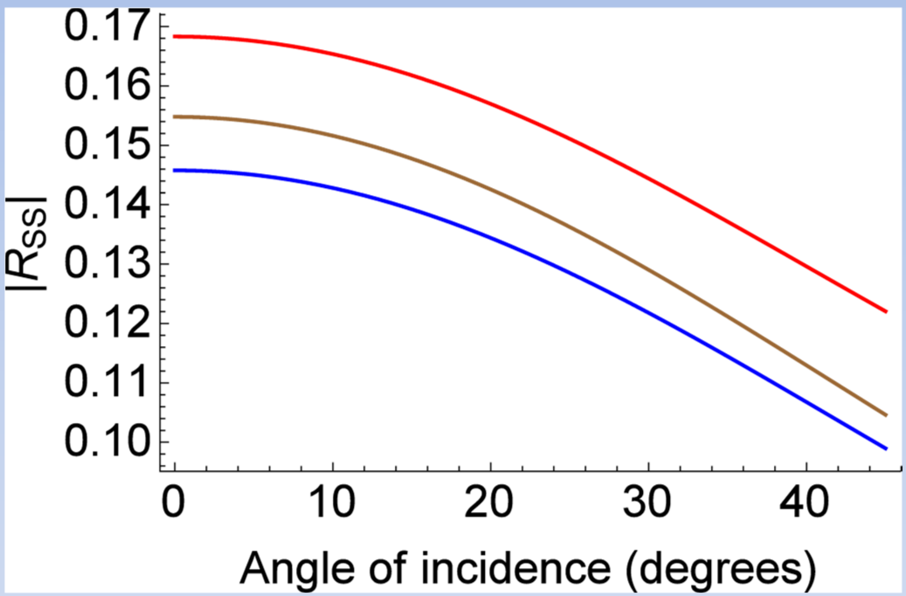
ϕ_{PS}



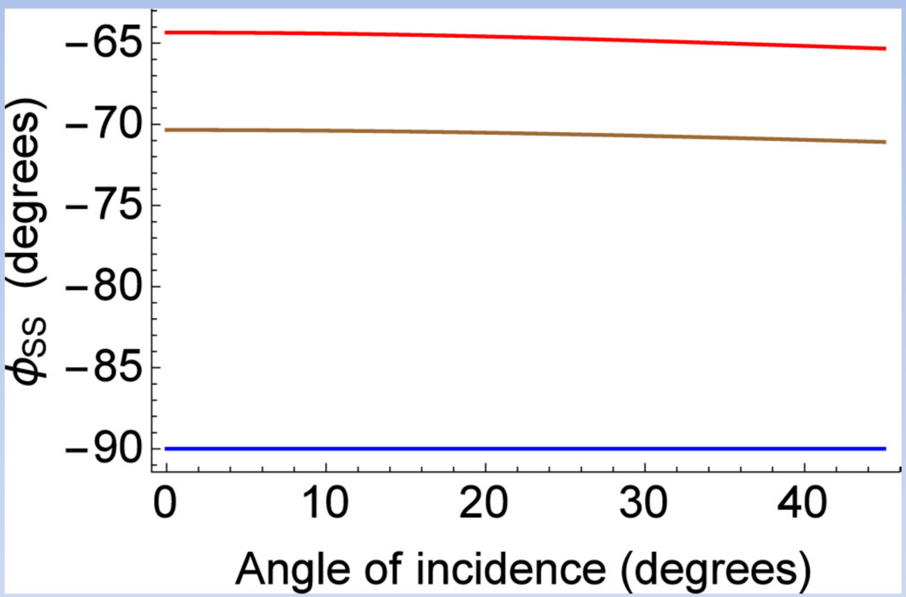
Red: exact result. Blue: the first-order AVO-type formula. Brown: the second-order AVO-type formula

SS-waves

$|R_{SS}|$



ϕ_{SS}



Red: exact result. Blue: the first-order AVO-type formula. Brown: the second-order AVO-type formula

Summary

1. The approximate transfer matrix for a homogenous thin VTI layer.
2. The factor $k_{P0}^{(M)} \equiv \omega h / \upsilon_{P0}^{(M)}$ describing the thin layer in the context of the approximate transfer matrix.
3. AVO-type formulae for PP-, SS- and PS-waves reflections coefficients.
4. The first- and second-order AVO-type formulae have almost the same accuracy for the magnitude of PP-, PS- and SS-wave reflection coefficients. The second-order formulae may be used to calculate the phase of the reflection coefficients.

Acknowledgement

We thank ROSE for financial support. Also, thank you for attention.