Industry-scale finite difference wave modelling on a single GPU using the out-of-core technique

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The ROSE Meeting, 2015

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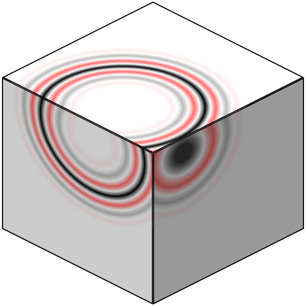
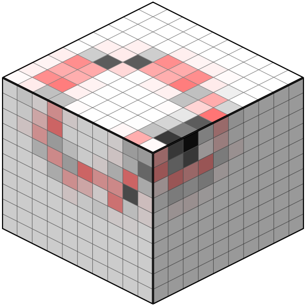
- Used to model physical phenomena, e.g. wave propagation.





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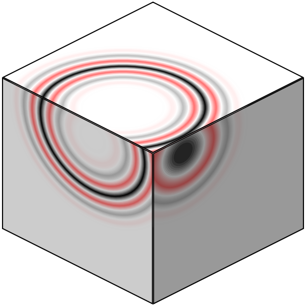

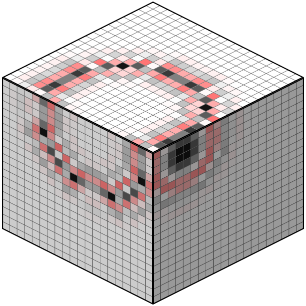

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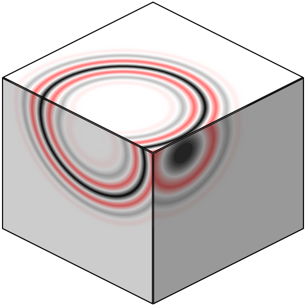

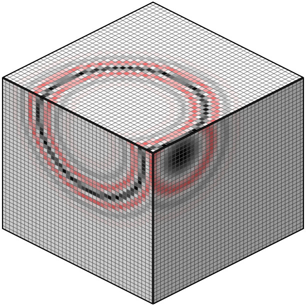

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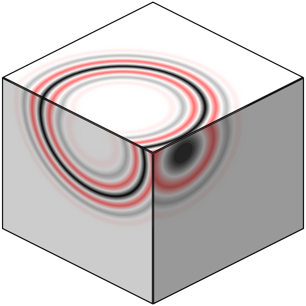

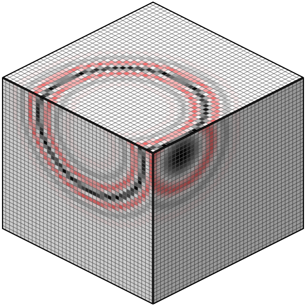

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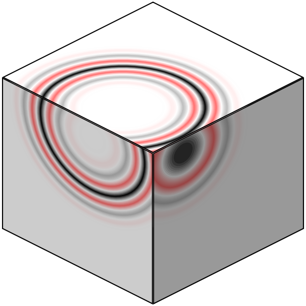
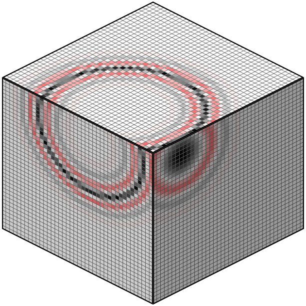
- Circumvent the limitations posed by the size of the GPU.





How to: Finite Difference Modelling

Model a given differential equation, e.g.:

$$\frac{\partial v_i}{\partial t} = \frac{1}{\rho}\left(\frac{\partial \tau_{ij}}{\partial j}\right) \tag{1}$$

$$\frac{\partial \tau_{ij}}{\partial t} = \delta_{ij}\lambda\frac{\partial v_k}{\partial k} + \mu\left(\frac{\partial v_i}{\partial j} + \frac{\partial v_j}{\partial i}\right) \tag{2}$$

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2. Approximate derivatives by weighted sums.
3. Update each variable across a small Δt, many times.

For example ...

$$\frac{\partial}{\partial x} u_{i+\frac{1}{2},j,k} \approx$$

$$\begin{array}{rl}
0.0038 & u_{i-3,j,k} \\
-0.0211 & u_{i-2,j,k} \\
+0.1049 & u_{i-1,j,k} \\
-1.2327 & u_{i,j,k} \\
+1.2327 & u_{i+1,j,k} \\
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-0.0038 & u_{i+4,j,k}
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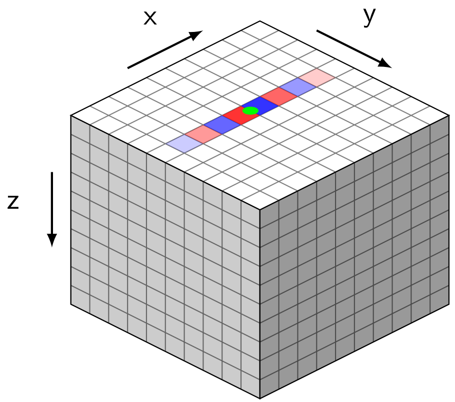

$$\tau_{xy}^{n+\frac{1}{2}} = \tau_{xy}^{n-\frac{1}{2}} + \mu \Delta t \left(\frac{\partial}{\partial x} v_y^n + \frac{\partial}{\partial y} v_x^n \right)$$

Industry scale Finite Difference Modelling?

Model a seismic shot:

- Wavelengths down to 10m.
- 4m$\times$4m$\times$4m cells.
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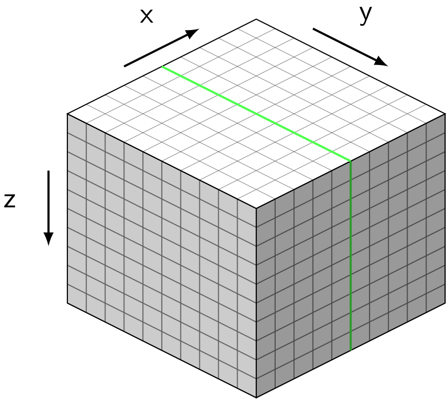
- 48GB of data.
- A consumer GPU typically fits 4GB-8GB of data.

The Memory Barrier – Fitting data onto the GPU



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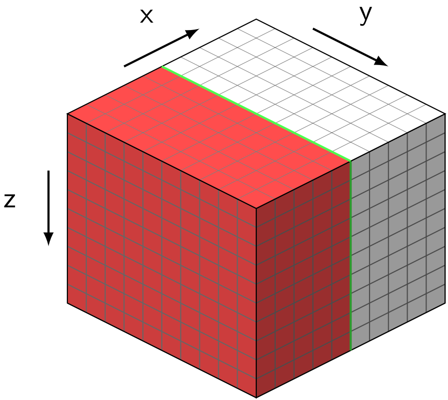

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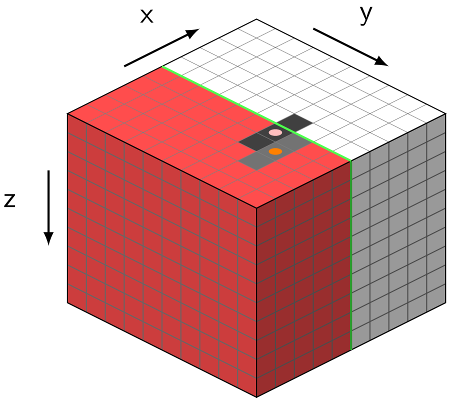

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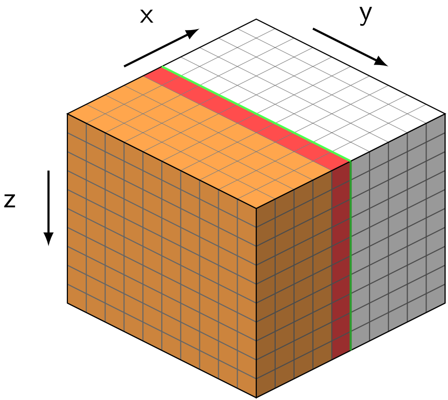

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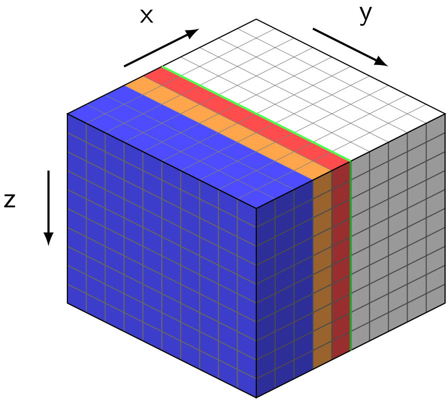

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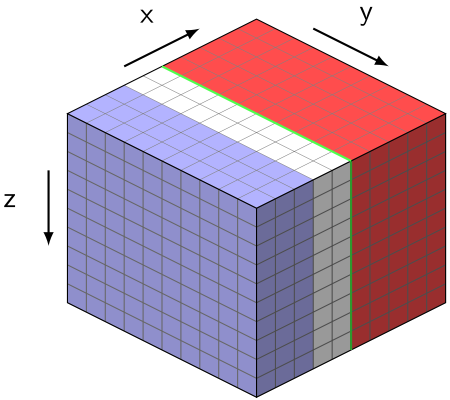

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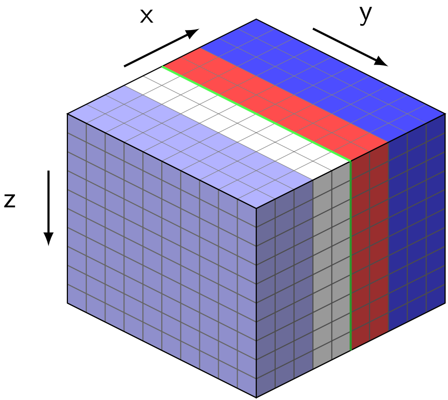

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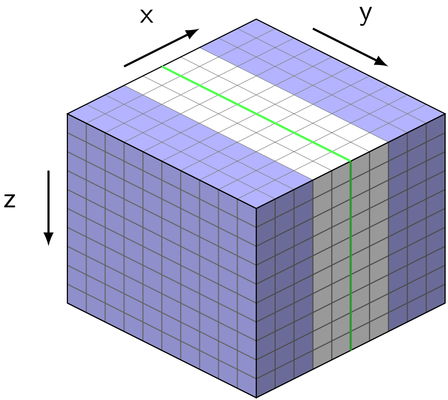

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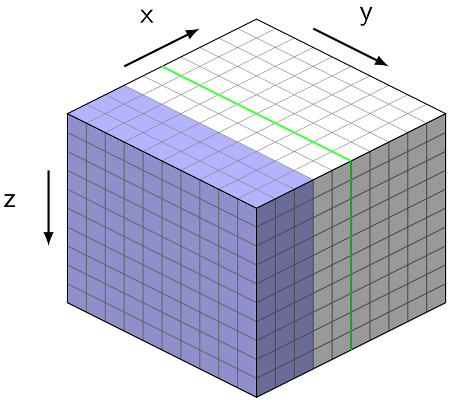

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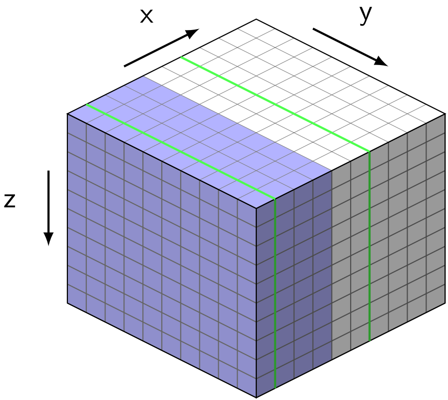

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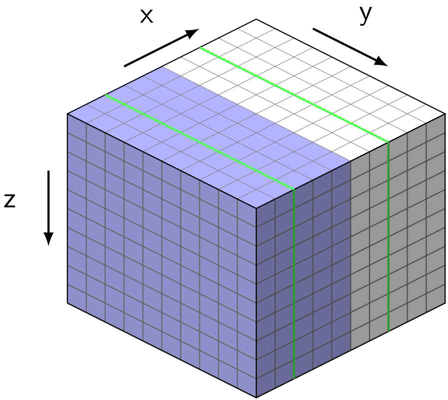

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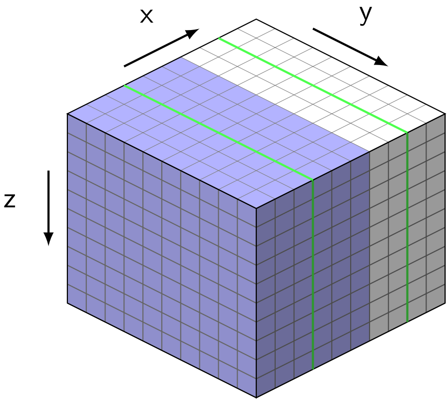

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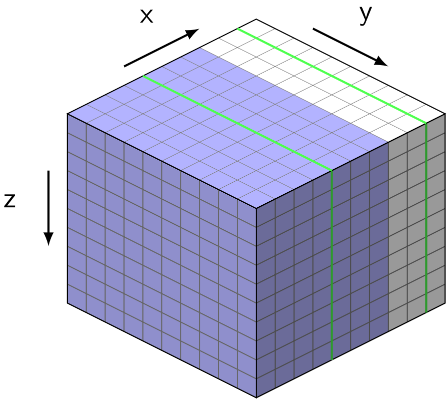

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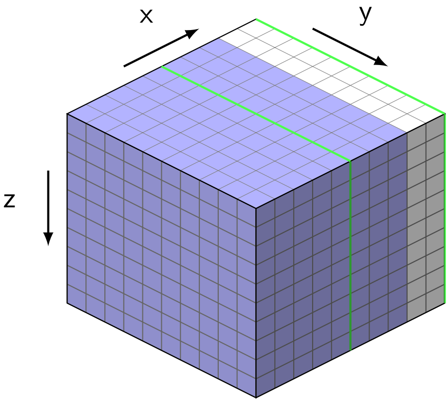

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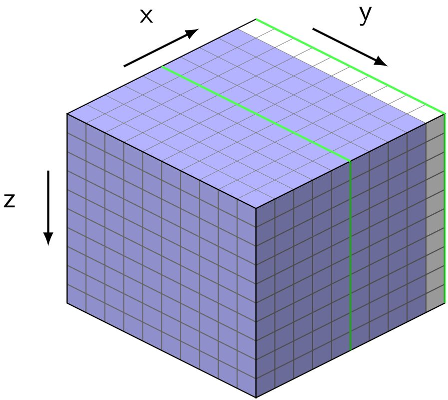

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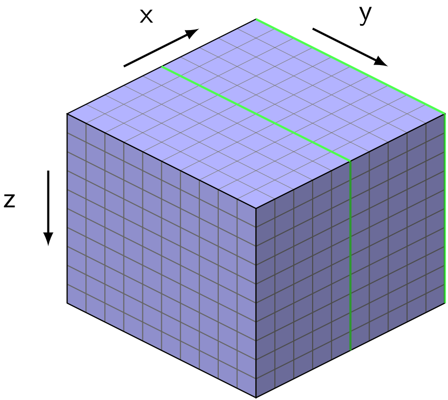

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- $L = 8$ and $Q = 7$ gives $<$ 3GB for the reference model.

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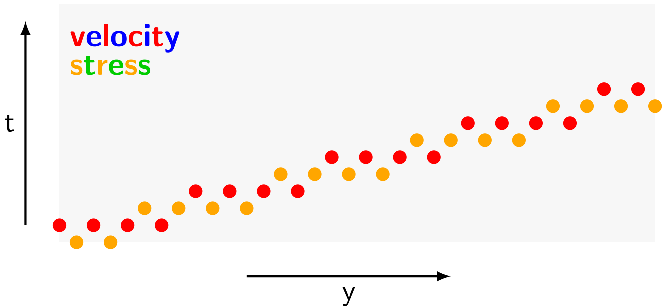

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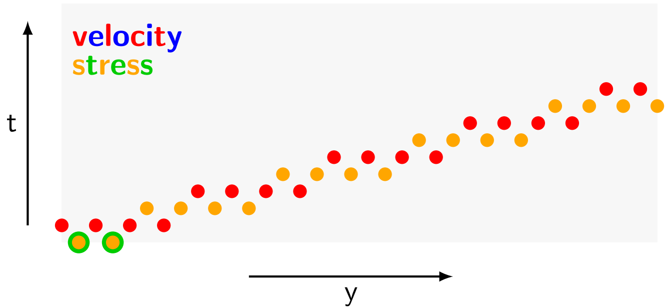

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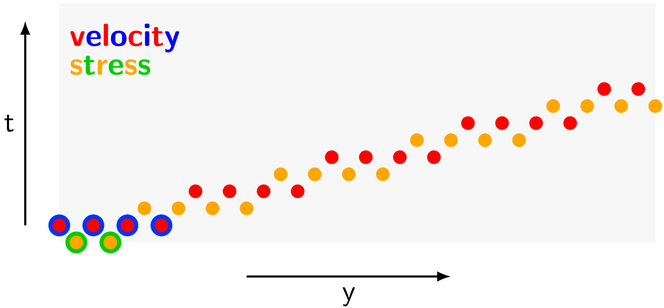

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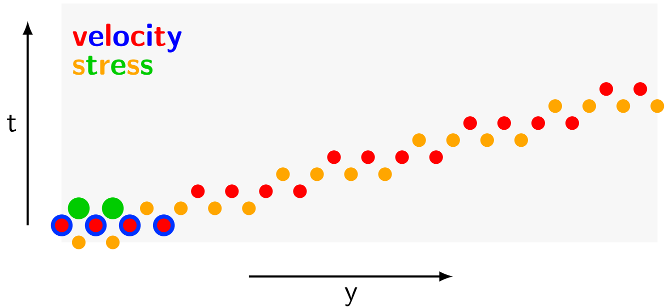

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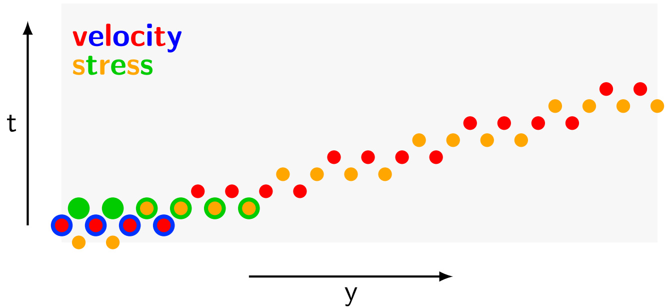

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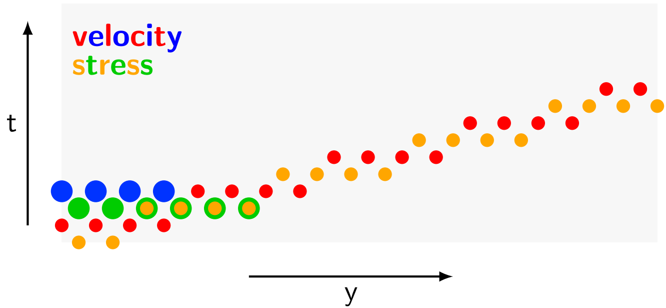

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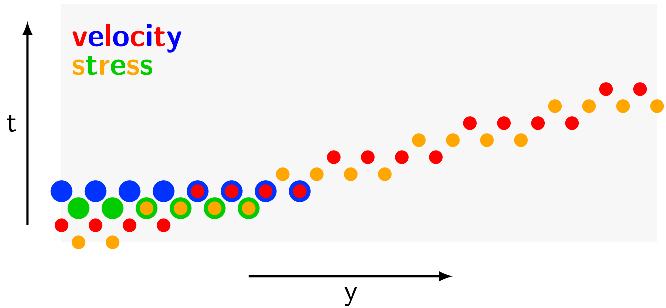

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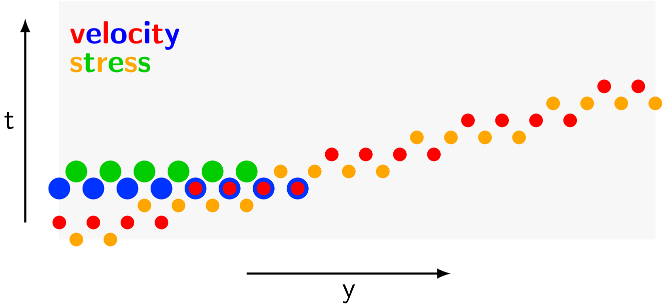

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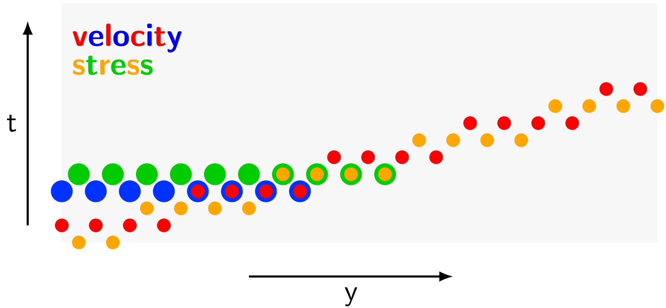

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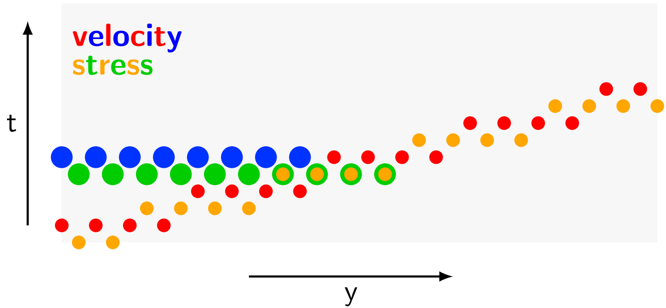

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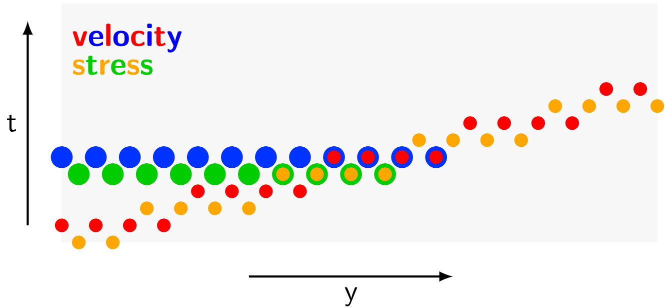

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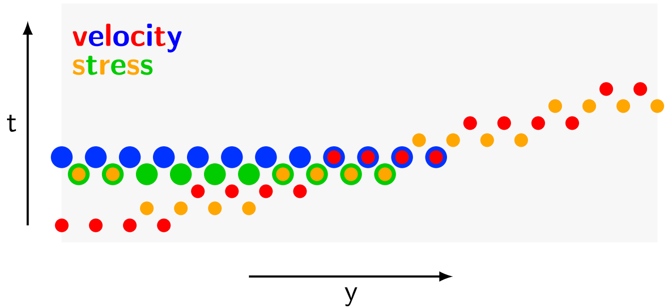

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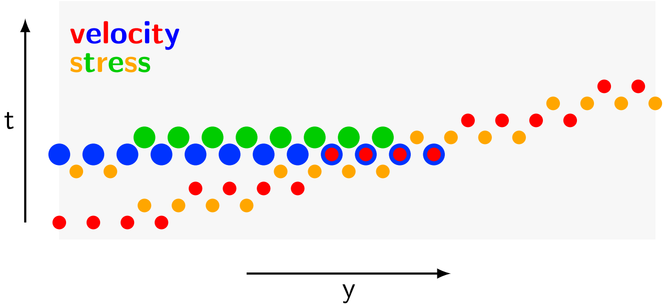

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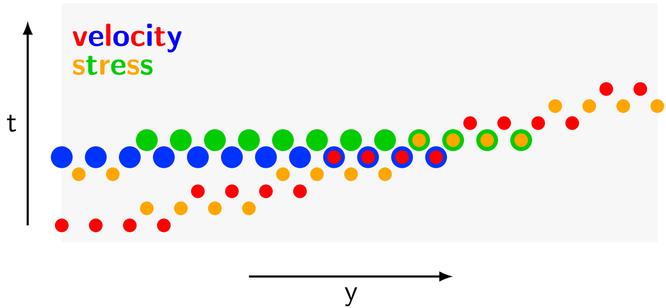

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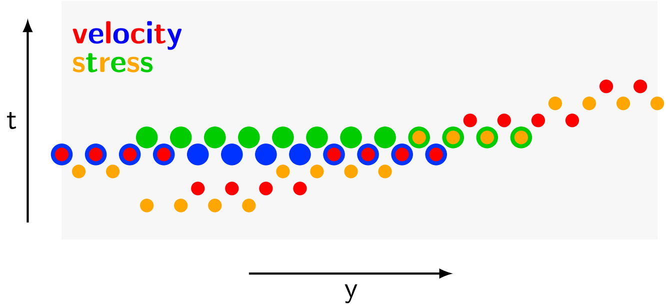

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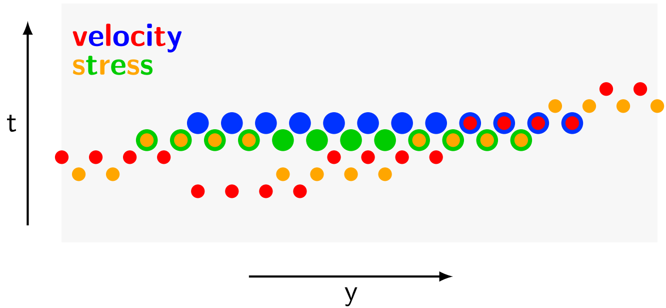

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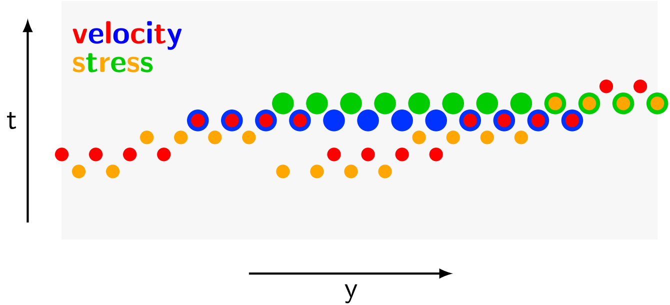

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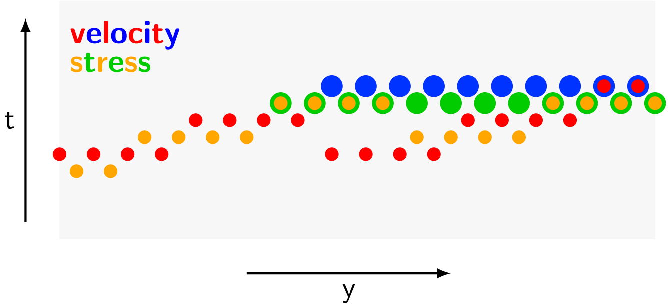

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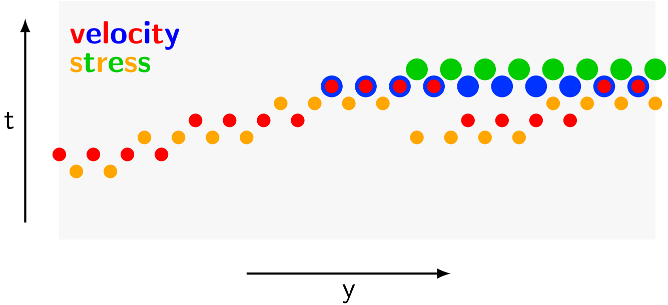

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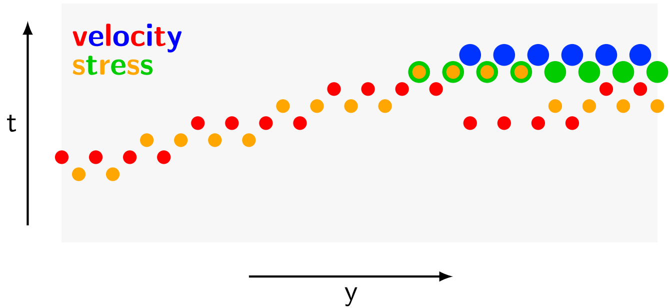

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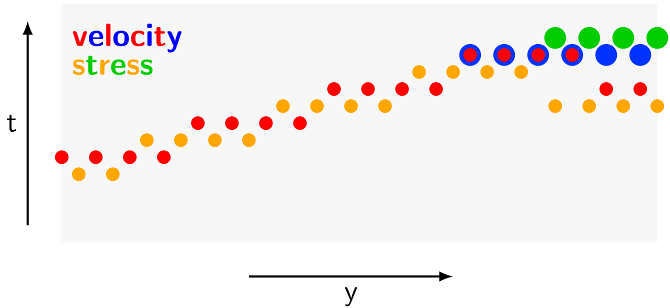

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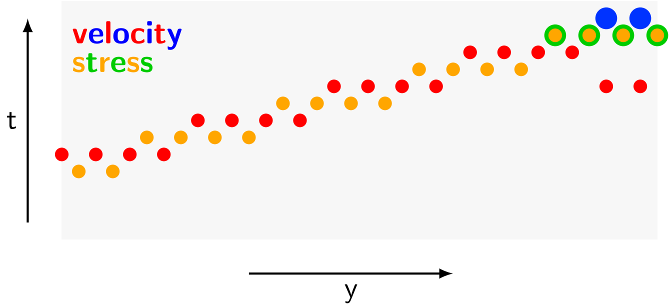

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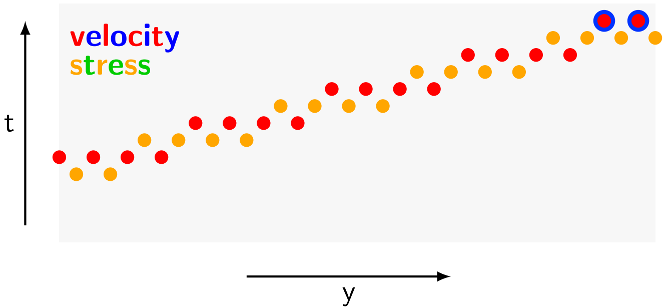

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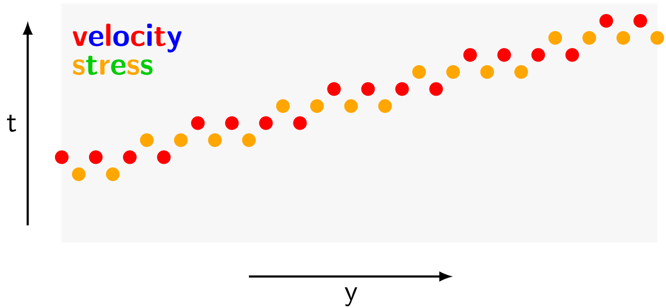
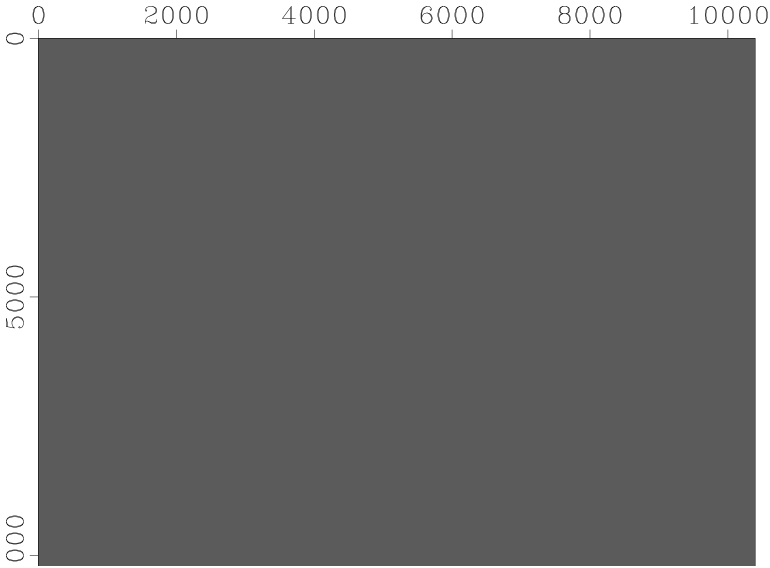


A small example from the Overthrust model



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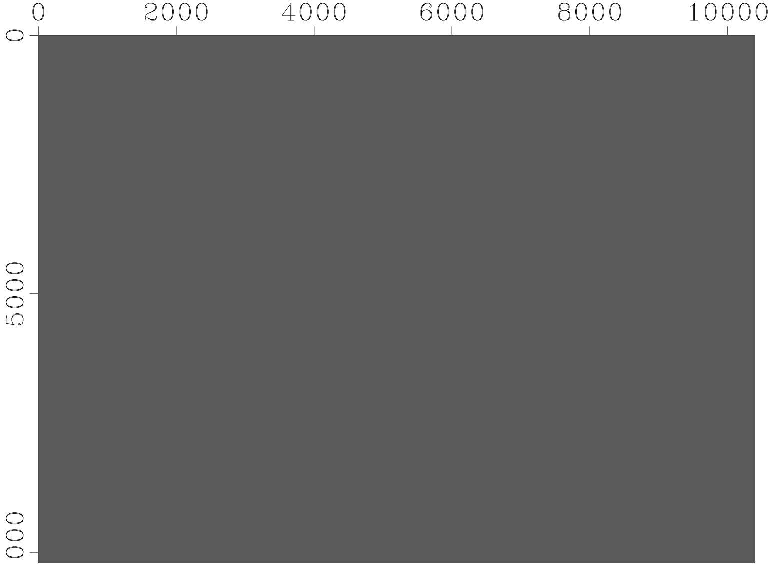

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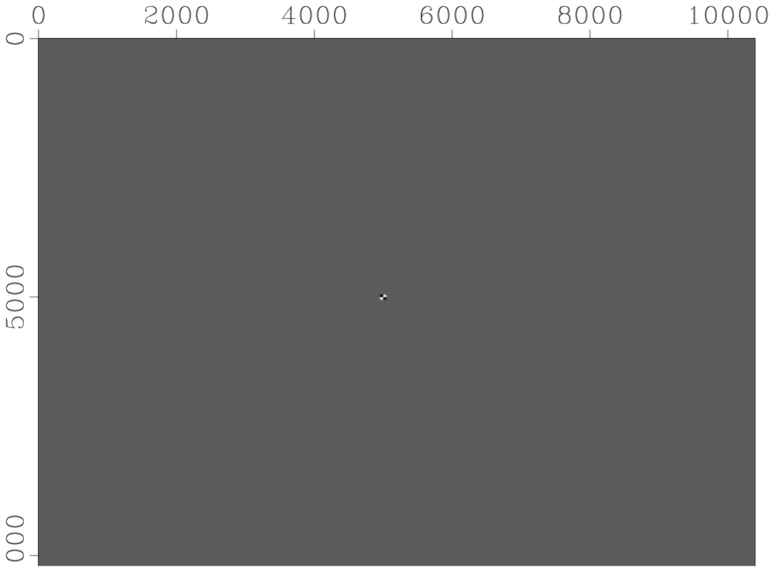

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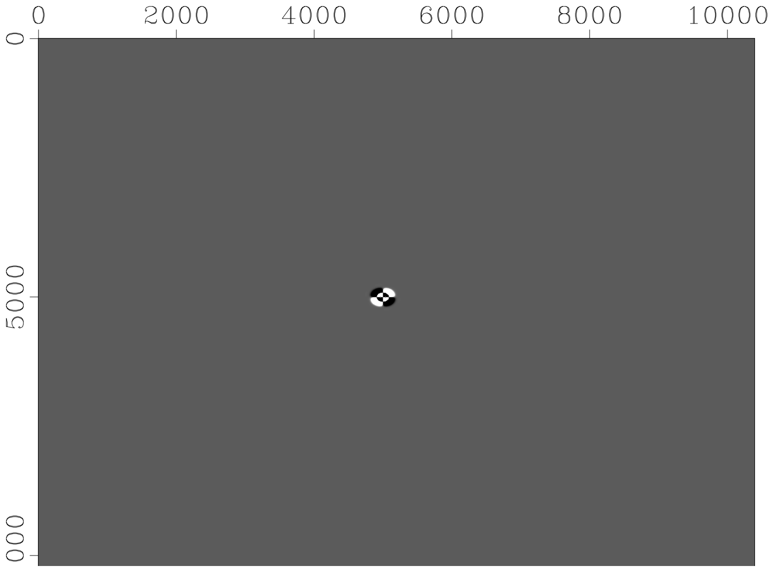

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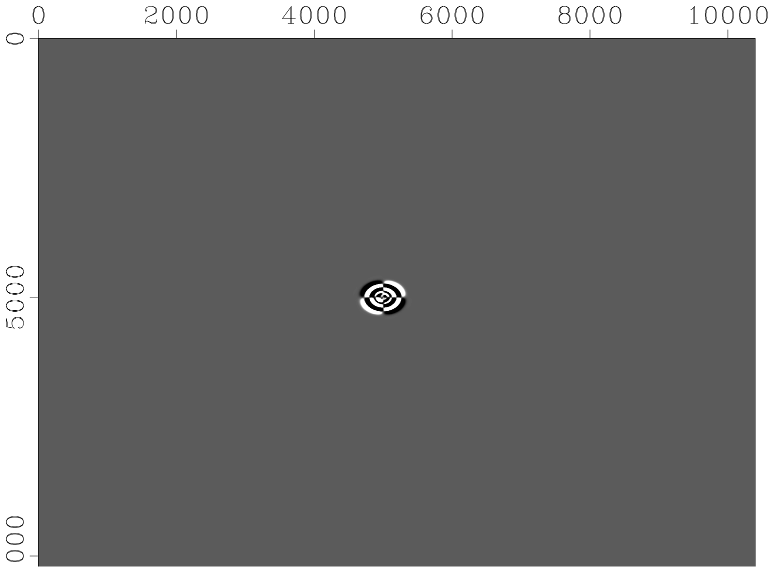

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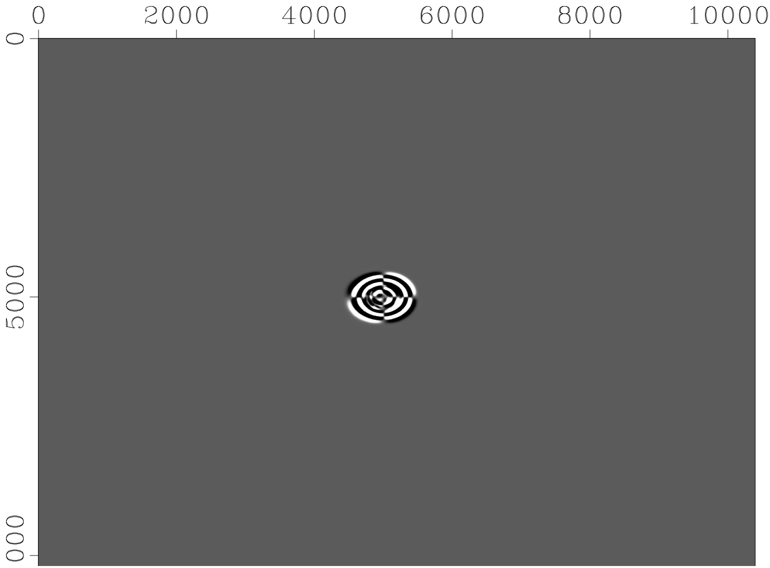

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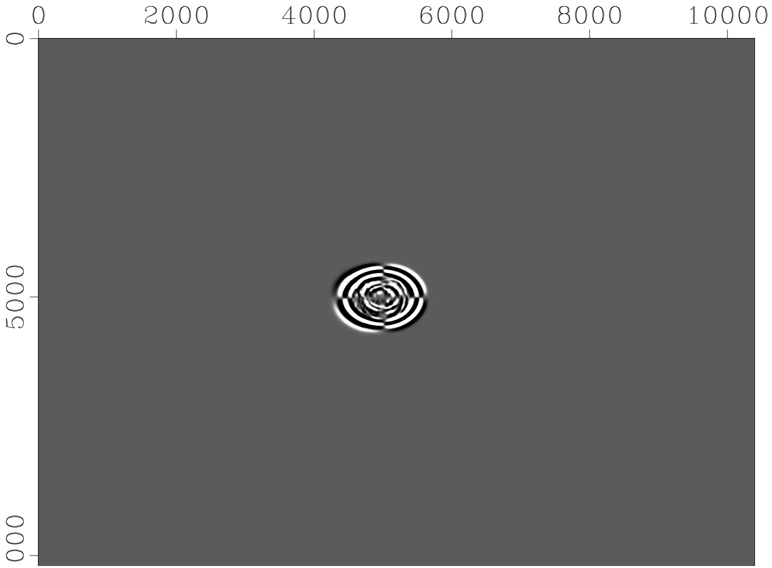

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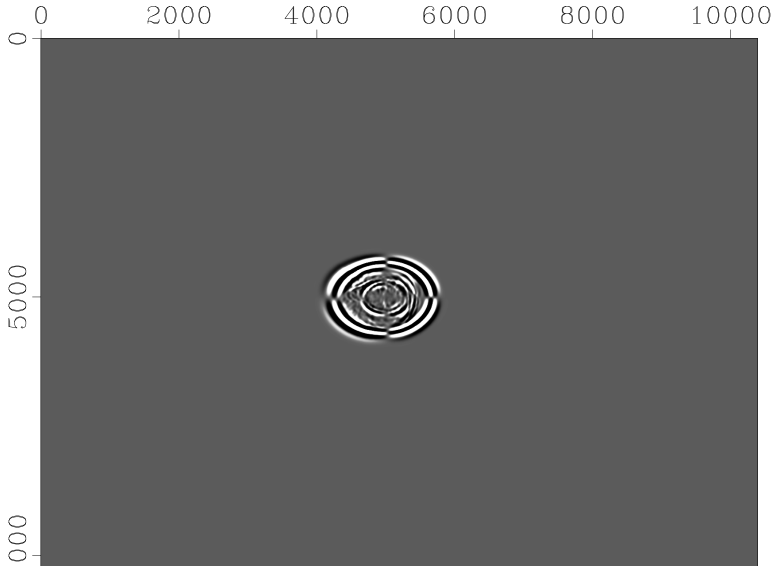

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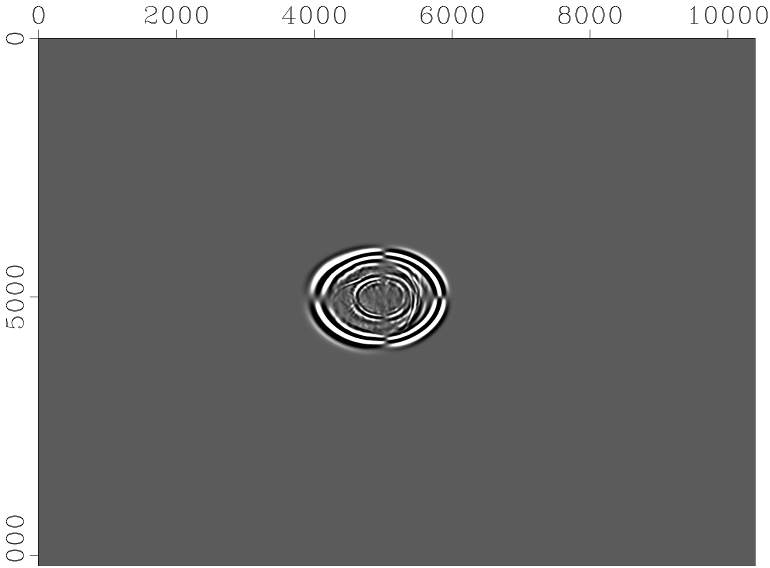

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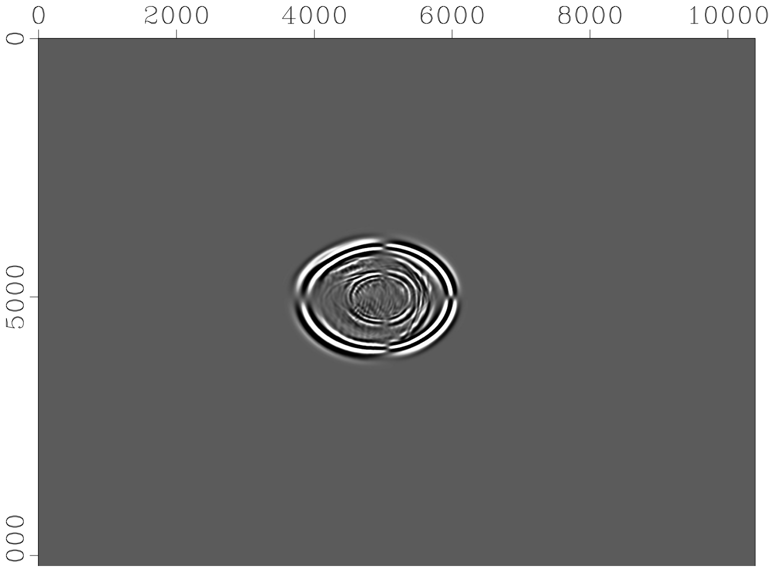

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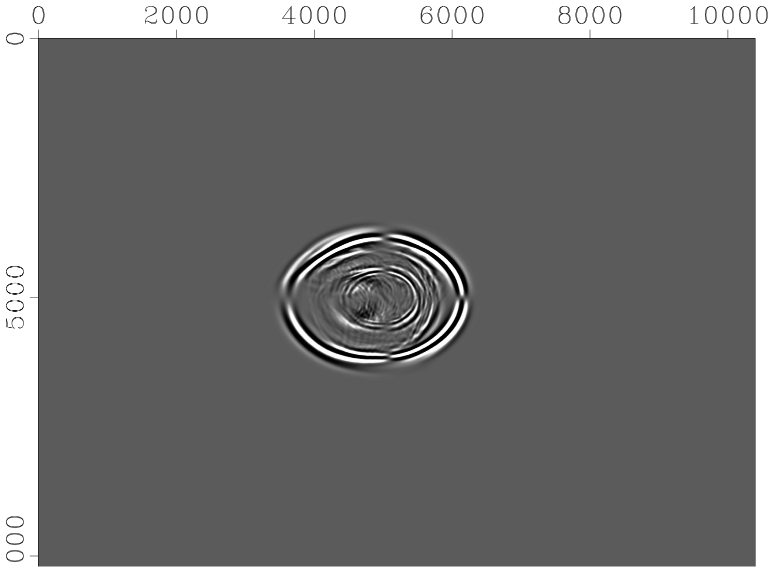

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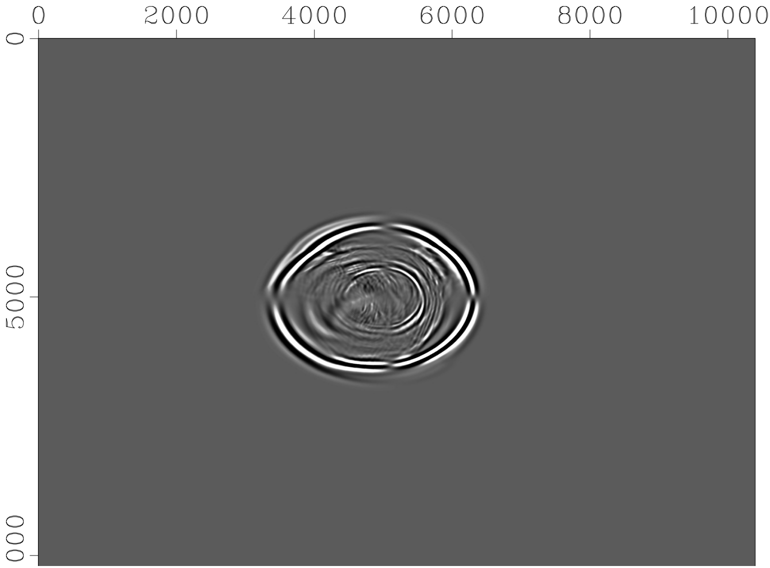

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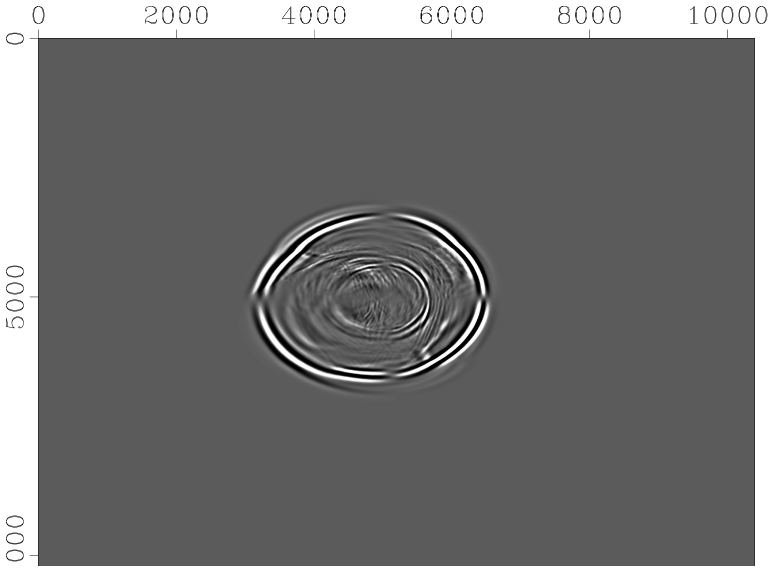

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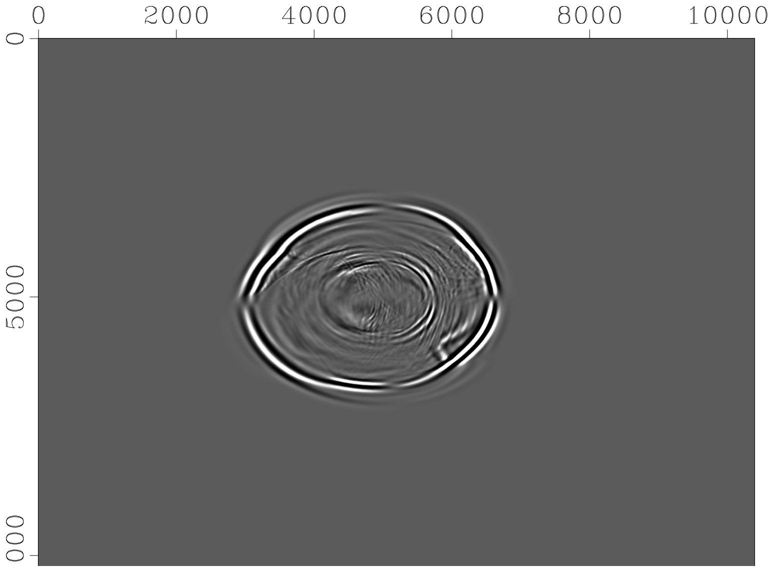

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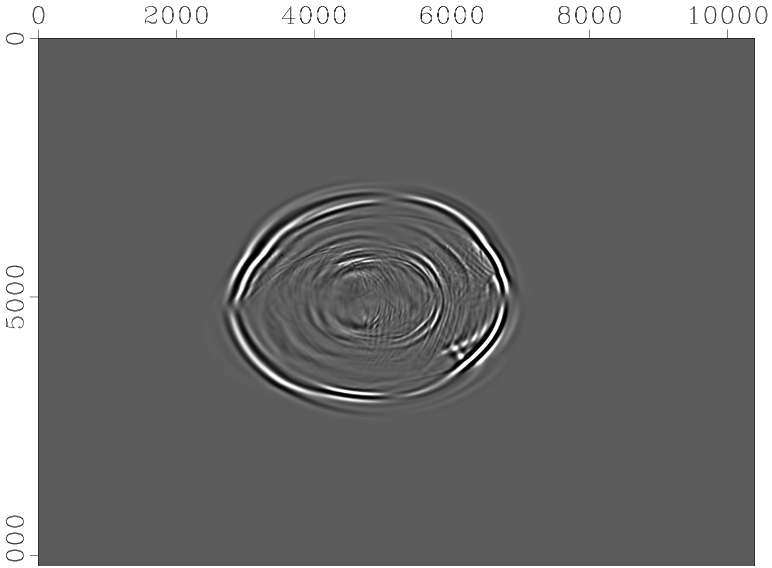

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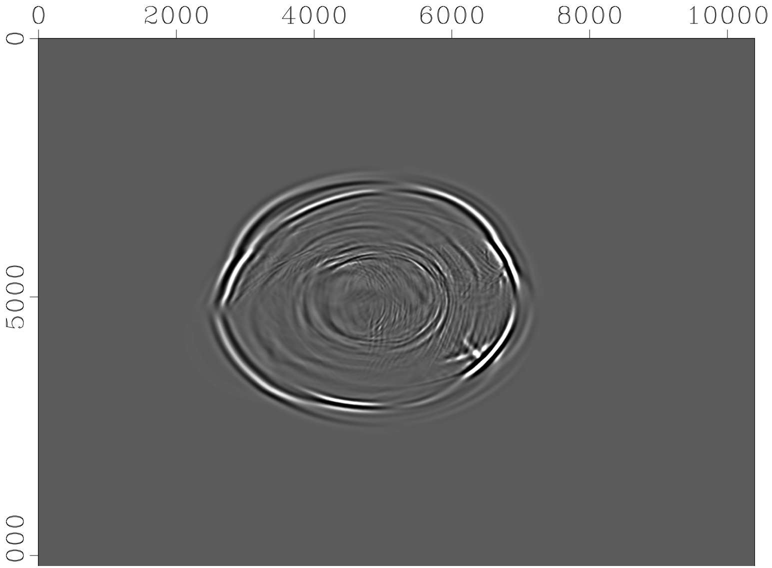

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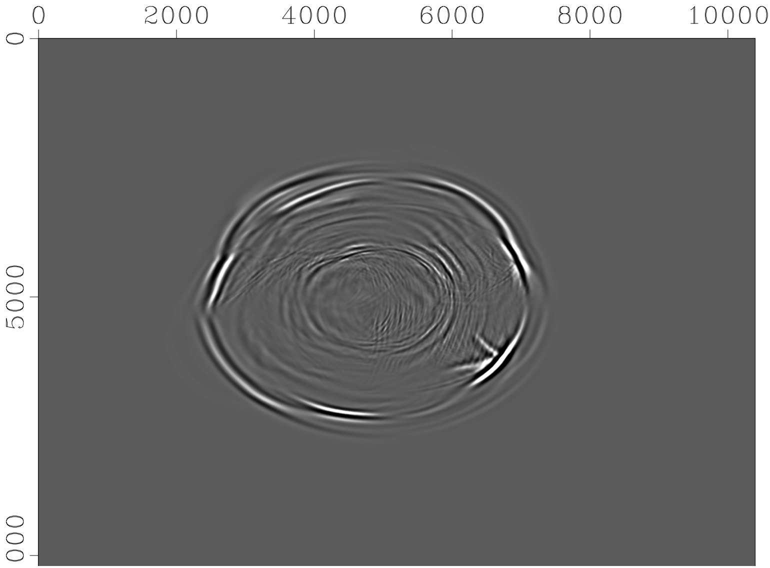

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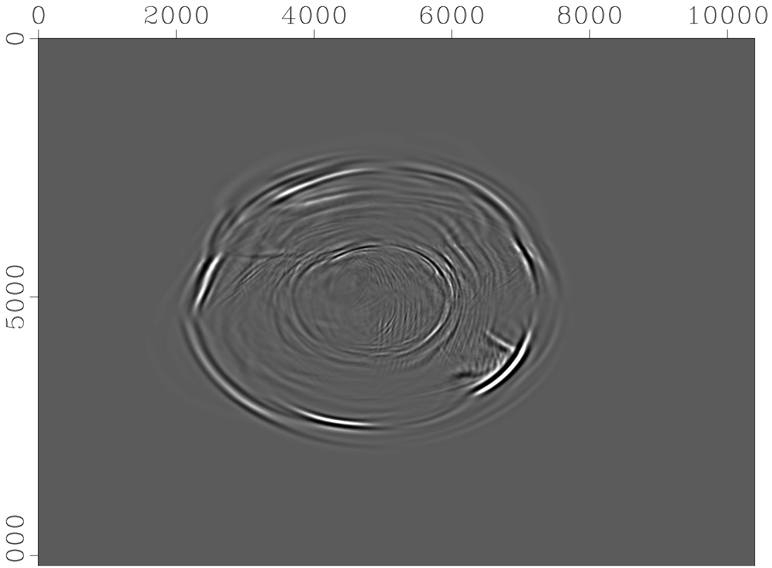

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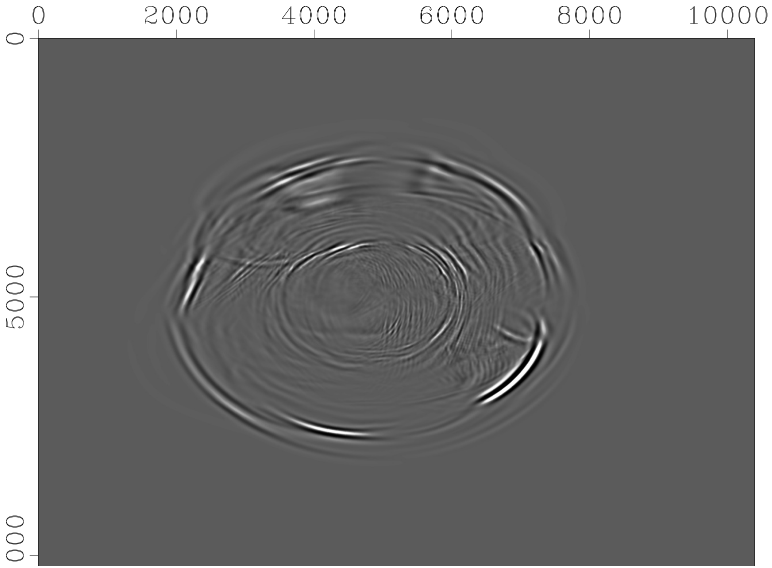

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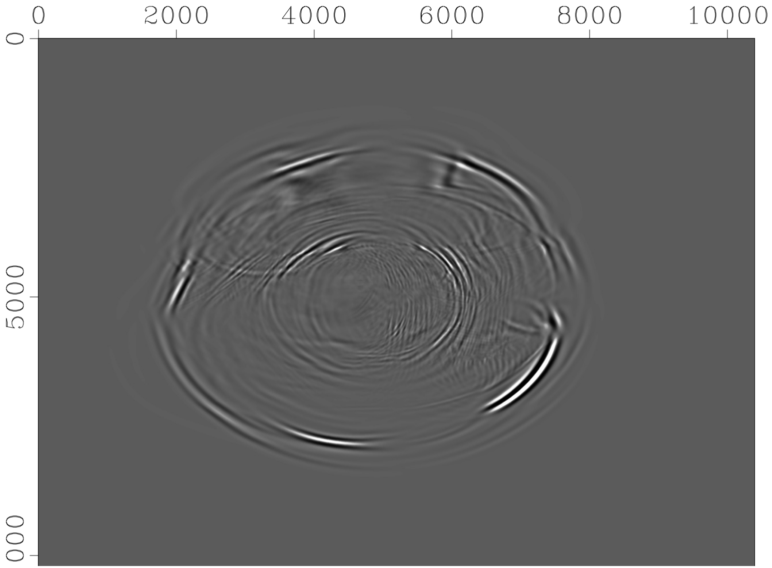

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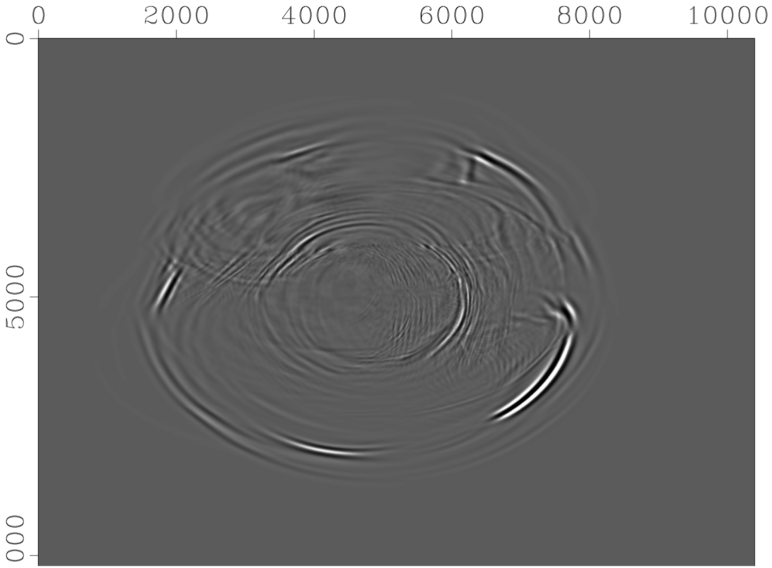

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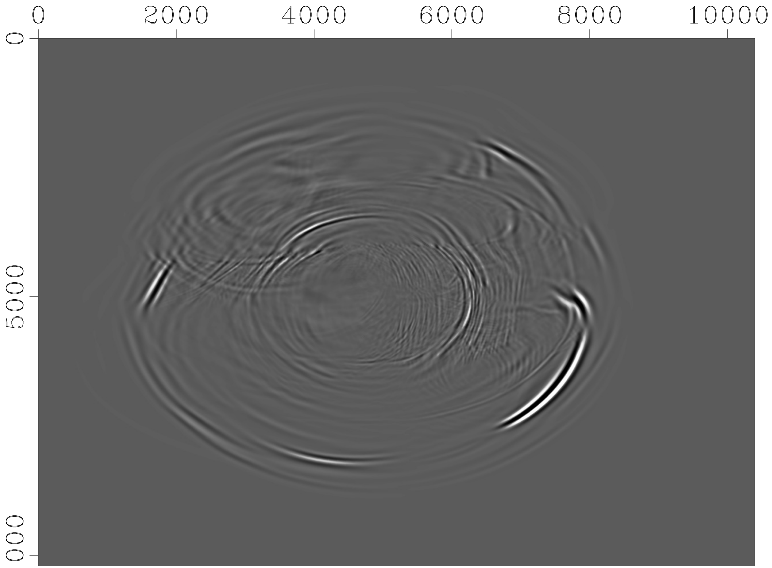

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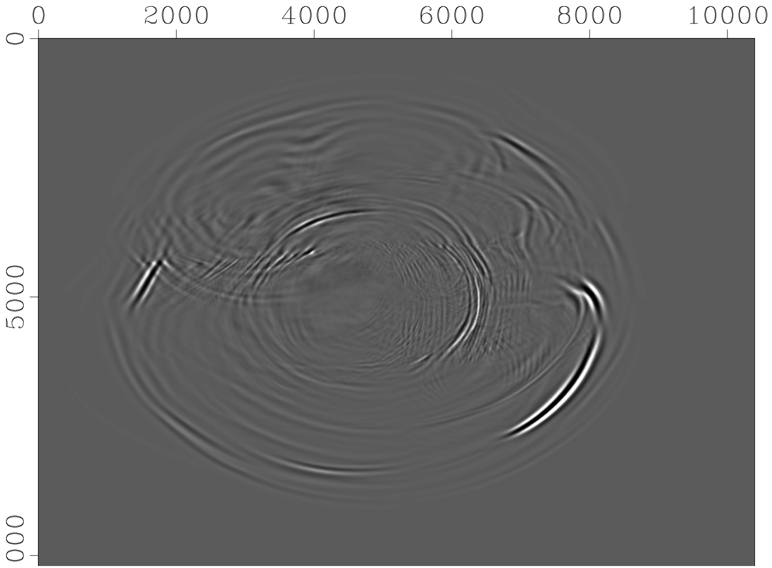

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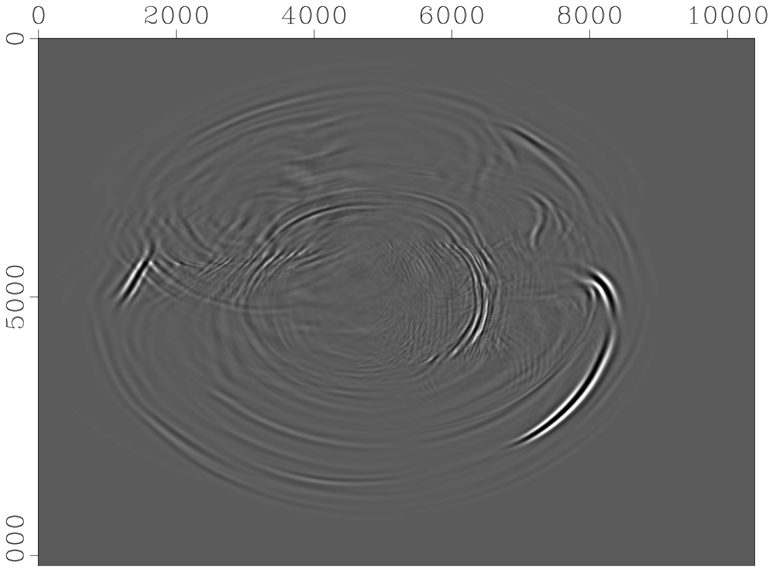

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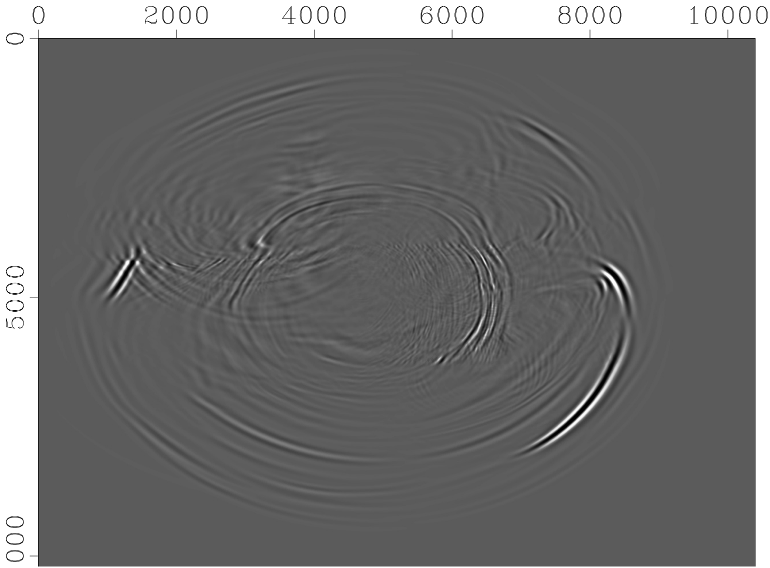

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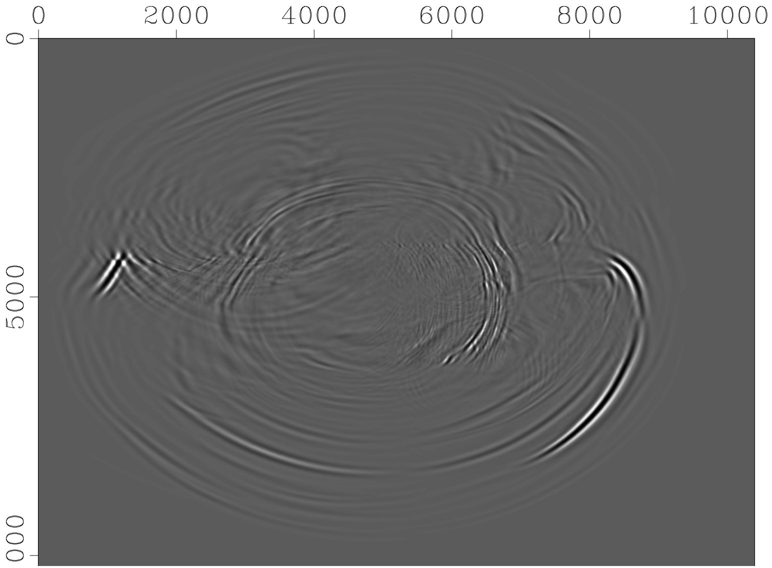

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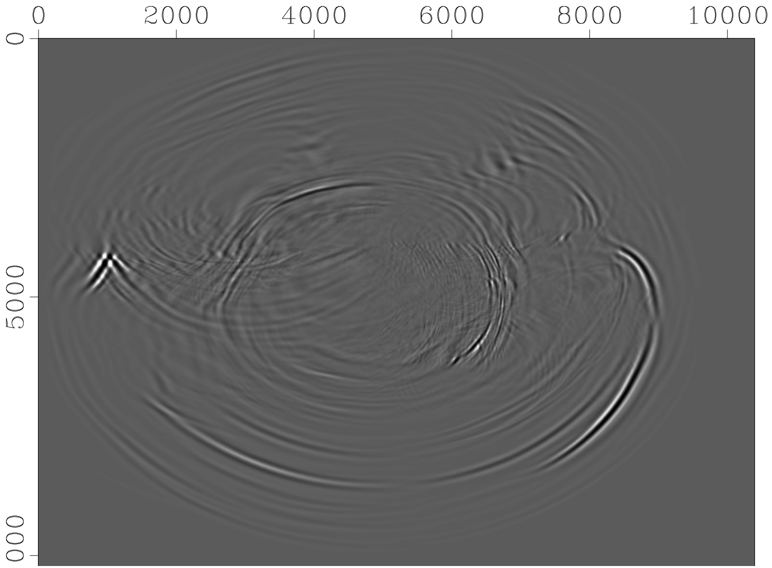

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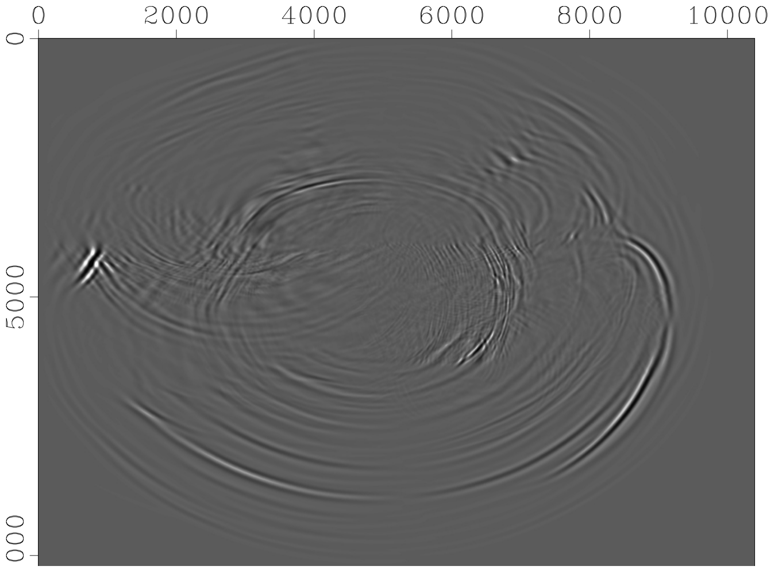

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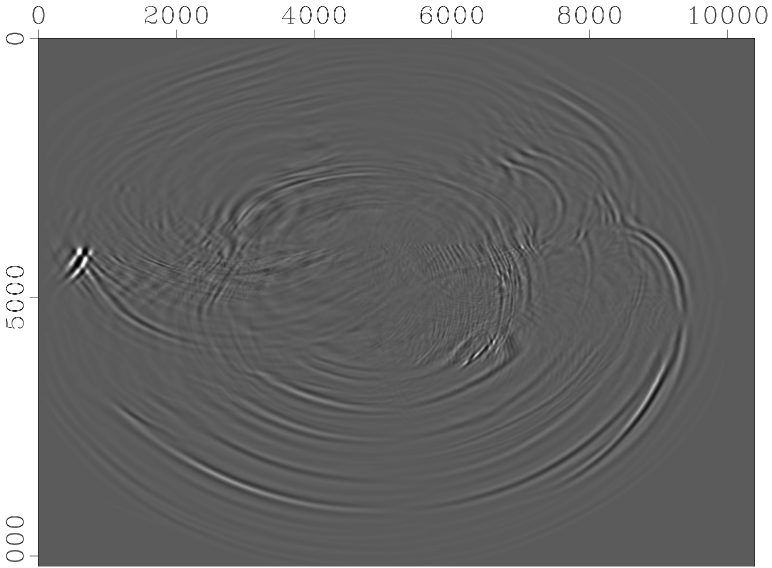

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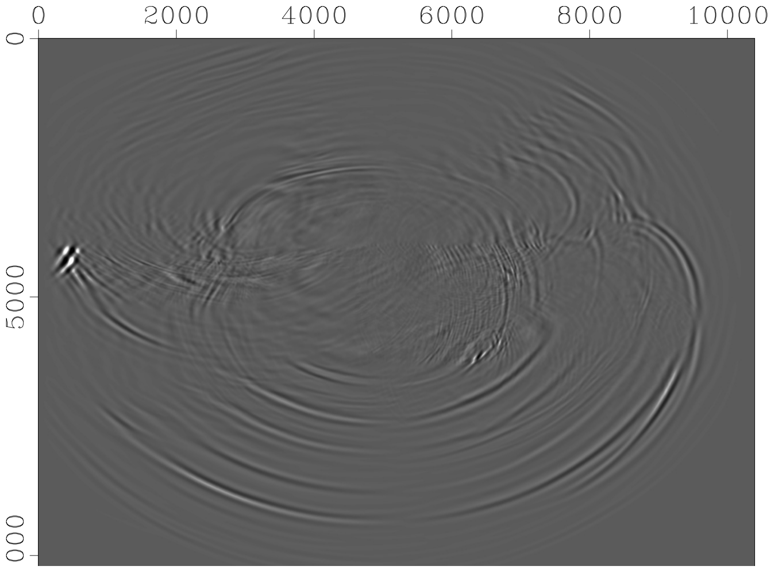

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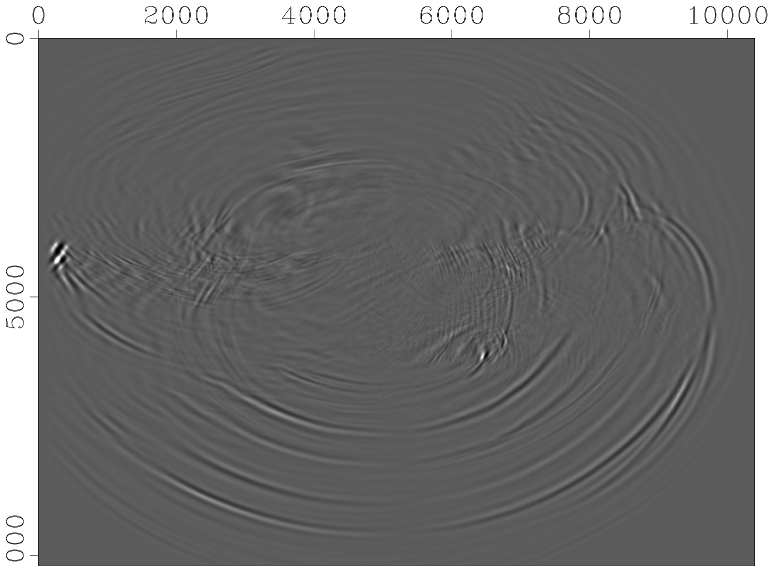

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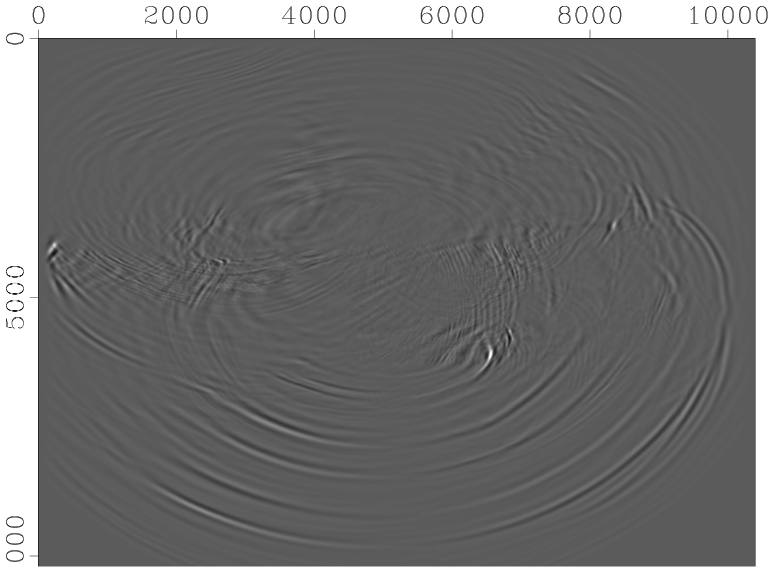

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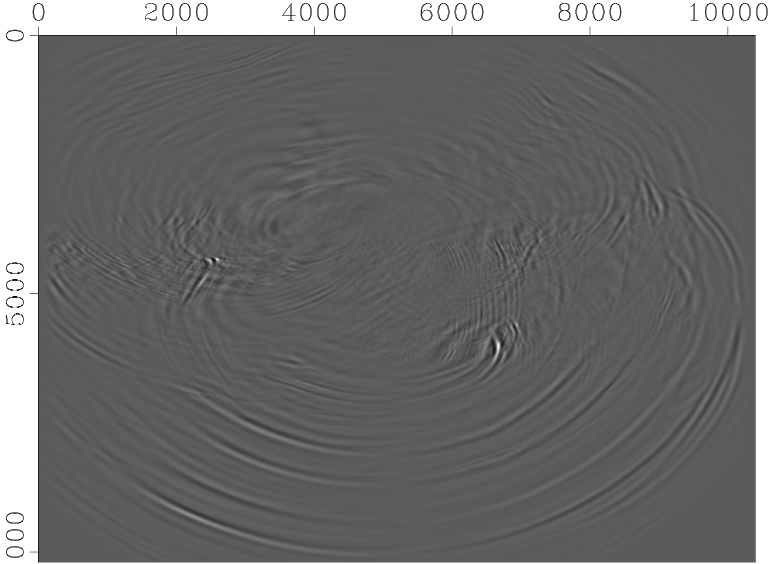

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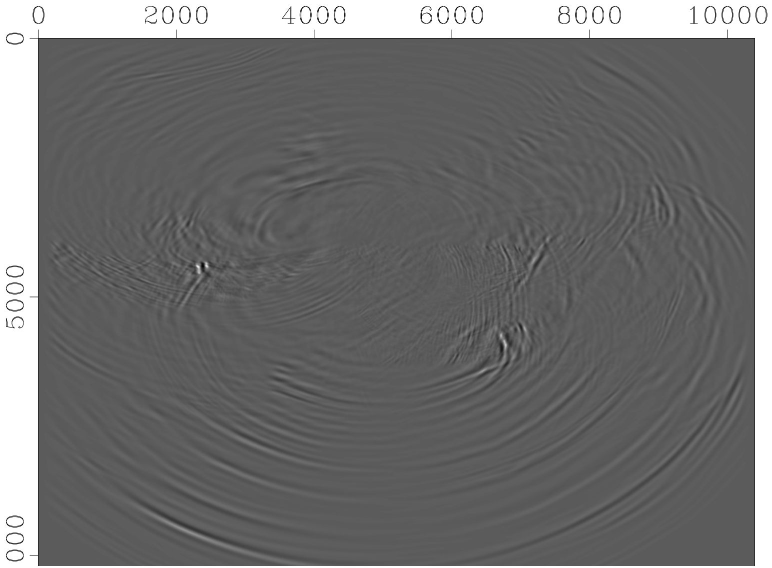

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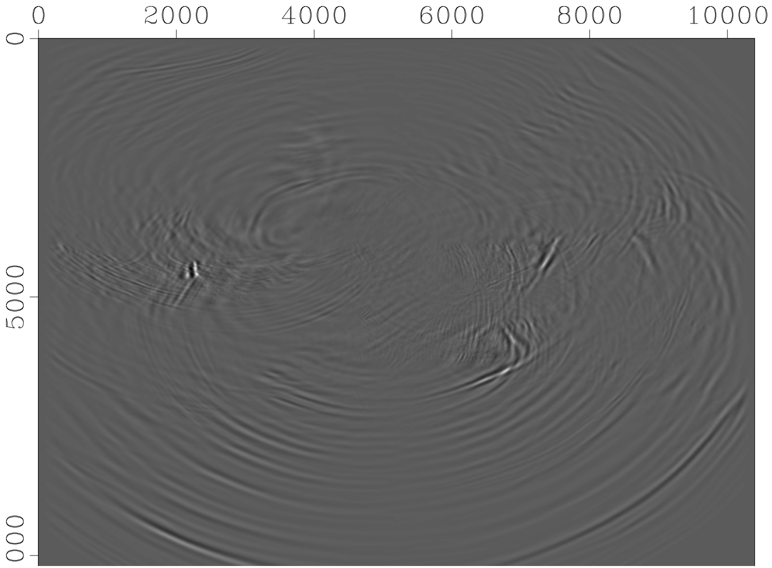

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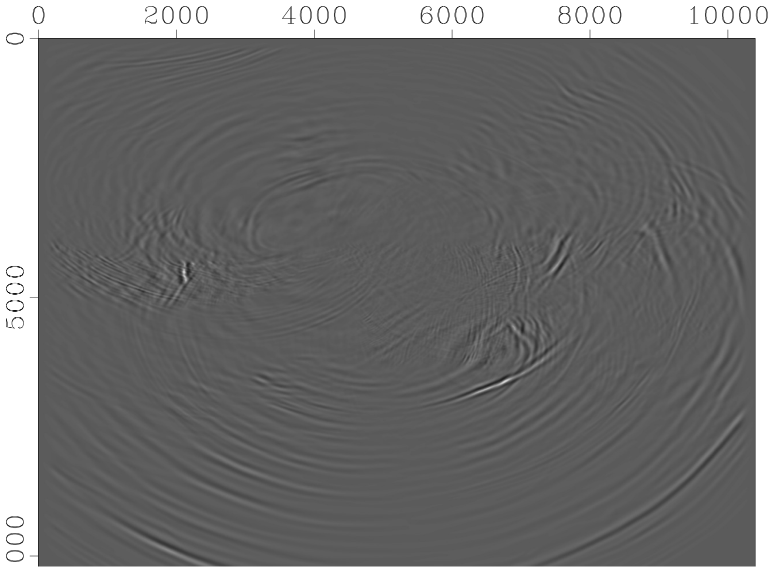

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A big thanks to

I wish to acknowledge Statoil and the sponsors of the ROSE consortium for funding my PhD and the Department of Petroleum Engineering & Applied Geophysics for being a great place to work.

venstad@gmail.com