Recursive stack to zero offset along local slope

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OVERVIEW

- Introduction
- Local data derivatives
- Least squares slopes
- Total least squares slopes
- Stacking along local slopes
- Stacking along the NMO velocity slope
- Automatic slope stack
- Data examples
- Conclusions

INTRODUCTION

- **Standard data processing**
  - Velocity analysis
  - NMO correction
  - Stack
  - Mute to avoid stretch effects
- **Velocities independent seismic processing**
  - Estimation of local slope (Fomel, 2007, Schleicher et al., 2009)

LOCAL DATA DERIVATIVES

$$\frac{\partial d(t_i, x_j)}{\partial t} \approx Dt(t_i, x_j)$$

$$\frac{\partial d(t_i, x_j)}{\partial x} \approx Dx(t_i, x_j)$$

Computed from data points in a small neighboardhood arround (t_i, x_j) (Melo et al., 2007)

LOCAL SLOPE

$$\frac{\partial d}{\partial t}\Delta t + \frac{\partial d}{\partial x}\Delta x = 0 \qquad \text{(plane wave)}$$

or

$$pDt_k + Dx_k = 0 \quad \text{with} \quad p = \frac{dt}{dx}\,.$$

Alternatively,

$$Dt_k + qDx_k = 0 \quad \text{with} \quad q = \frac{dx}{dt}\,.$$

LEAST SQUARES I

Minimize

$$\phi_t = ||e_t||^2 = \sum_k |pDt_k + Dx_k|^2 = ap^2 + 2cp + b$$

with

$$a = \sum_k |Dt_k|^2$$

$$b = \sum_k |Dt_x|^2$$

$$c = \sum_k Dt_k Dx_k$$

The result is

$$\hat{p} = -\frac{c}{a}$$

LEAST SQUARES II

With $q = \dfrac{dx}{dt}$ minimize

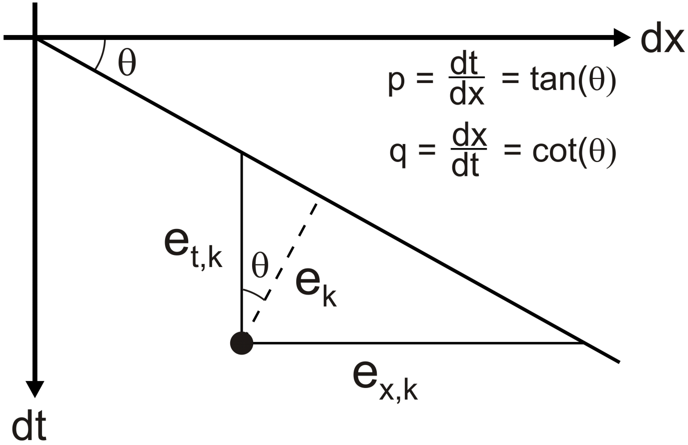
$$\phi_x = ||e_x||^2 = \sum_k |qDx_k + Dt_k|^2 = bq^2 + 2cq + a$$

The result is $$\hat{q} = -\frac{c}{b}$$

Note that $$\hat{p}\hat{q} = \frac{c^2}{ab}$$

Should have $\dfrac{dt}{dx}\dfrac{dx}{dt} = 1.$

GEOMETRY OF TOTAL LEAST SQUARES



TOTAL LEAST SQUARES I

Minimize $$\phi_p = ||e||^2 = ||e_t||^2 cos^2\theta = \frac{||e_t||^2}{1+p^2}$$

Solution $$p = -\frac{1}{2c}\left[(b-a) + \sqrt{(b-a)^2 + 4c^2}\right]$$

Alternatively, minimize

$$\phi_q = ||e||^2 = ||e_x||^2 sin^2\theta = \frac{||e_x||^2}{1+q^2}$$

with solution $$q = -\frac{1}{2c}\left[(a-b) + \sqrt{(a-b)^2 + 4c^2}\right]$$

Note $pq = 1$.

TOTAL LEAST SQUARES II

Van Huffel and Vandervalle (1991) define total least squares via SVD analysing the problem

$$\mathbf{C}\begin{bmatrix}p\\1\end{bmatrix} = \begin{bmatrix}Dt_k & Dx_k\end{bmatrix}\begin{bmatrix}p\\1\end{bmatrix} \approx 0$$

We can solve the eigenvalue problem (Porsani et al, 2013)

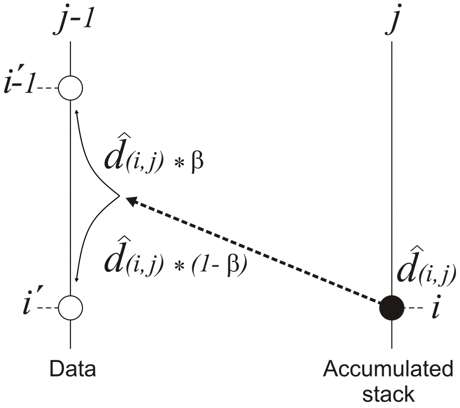
$$\mathbf{C}^T\mathbf{C}\begin{bmatrix}p\\1\end{bmatrix} = \begin{bmatrix}a & c\\c & b\end{bmatrix}\begin{bmatrix}p\\1\end{bmatrix} = \sigma^2\begin{bmatrix}p\\1\end{bmatrix}$$

The solution in p (and q) as before, or in a different form

$$p = \frac{-2c}{a-b+\sqrt{(a-b)^2+4c^2}} \quad , \quad q = \frac{-2c}{b-a+\sqrt{(b-a)^2+4c^2}}$$

STACKING ALONG LOCAL SLOPE

Started at far offset, hyperbolic extrapolation from near offset to zero offset.



STACKING ALONG THE NMO VELOCITY SLOPE

Standard traveltime approximation

$$T(x) = \sqrt{T(0)^2 + \frac{x^2}{v_{NMO}^2}}$$

$v_{NMO}(T(0))$ given. For (t,x) solve

$$t^2 = T(0)^2 + \frac{x^2}{T(0)\,v_{NMO}^2}$$

for $v_{NMO}(t,x)$. Local slope

$$p(t,x) = \frac{x}{t\,v_{NMO}^2(t,x)}$$

AUTOMATIC SLOPE STACK

Estimate local slope using total least squares. Check

$$\frac{x}{t\, v^2_{NMO,max}} < p(t,x) < \frac{x}{t\, v^2_{NMO,min}}$$

If $p(t,x)$ is outside range, the stack is restarted.

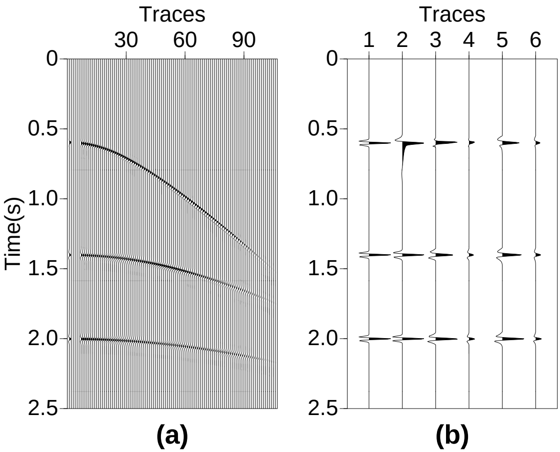
NUMERICAL EXAMPLES

1 - Ideal synthetic data

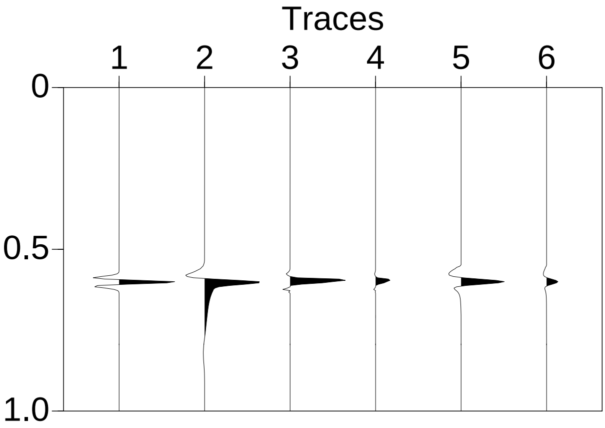
2 - Hess 2D VTI data

3 - Deepwater Gulf of Mexico data

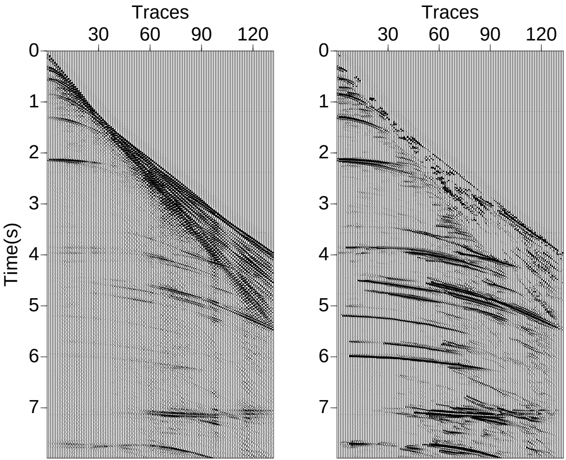
IDEAL SYNTHETIC DATA



IDEAL SYNTHETIC DATA

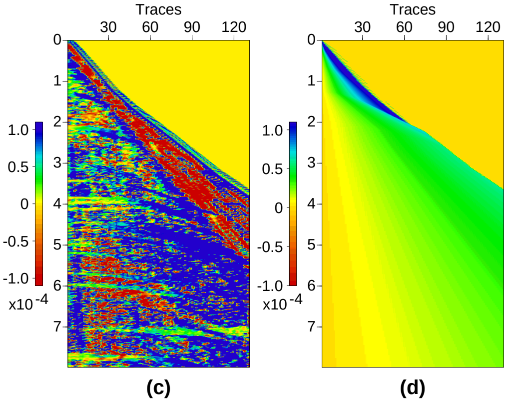


HESS 2D VTI DATA

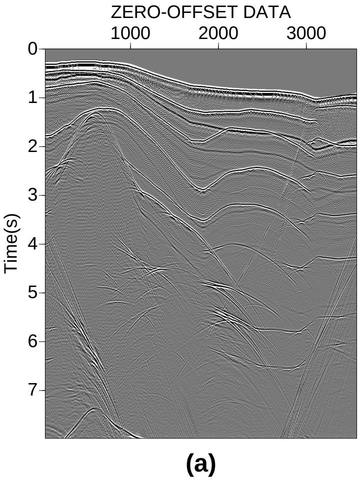


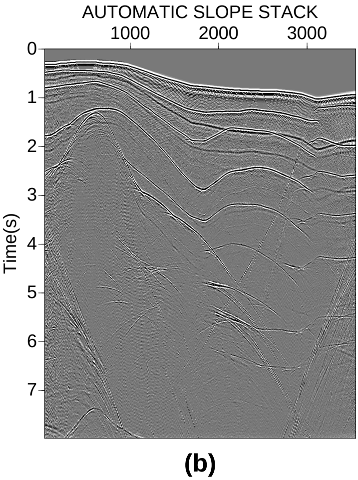
(a) (b)

HESS 2D VTI DATA

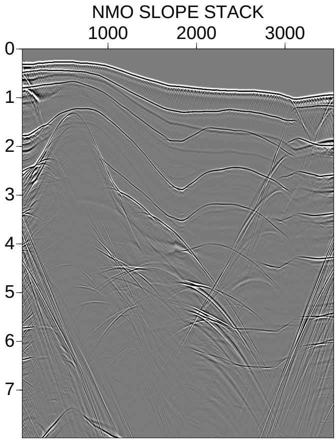


HESS 2D VTI DATA

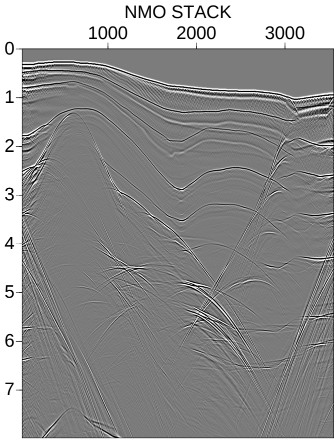




HESS 2D VTI DATA

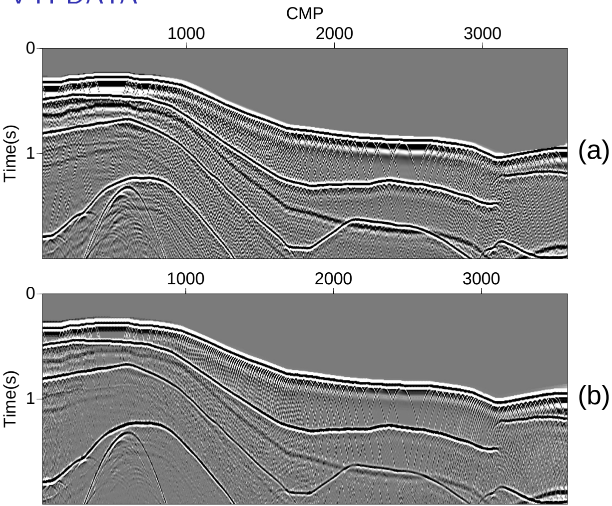


(c)

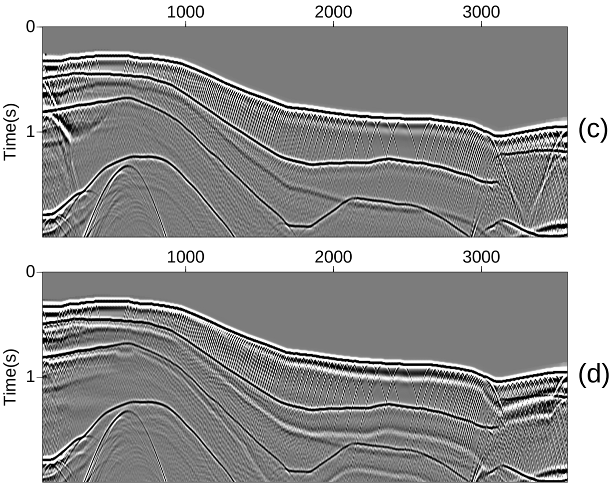


(d)

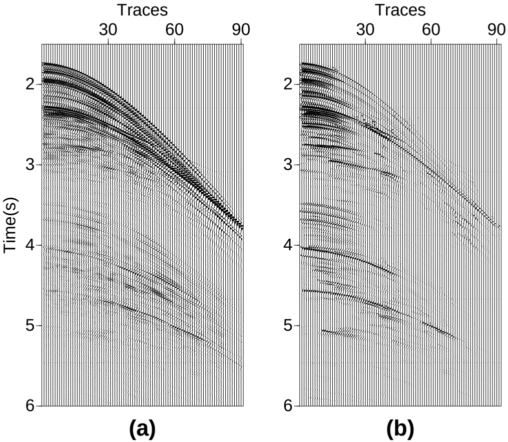
HESS 2D VTI DATA



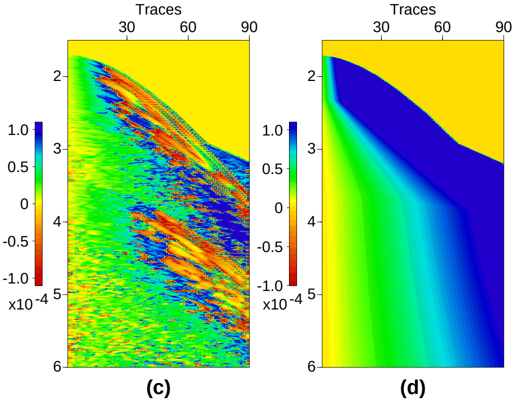
HESS 2D VTI DATA



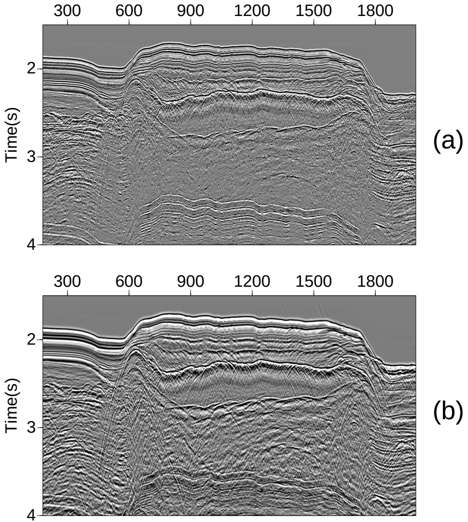
DEEPWATER GULF OF MEXICO DATA



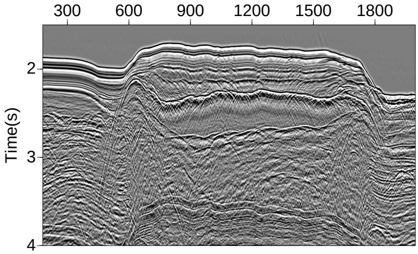
DEEPWATER GULF OF MEXICO DATA



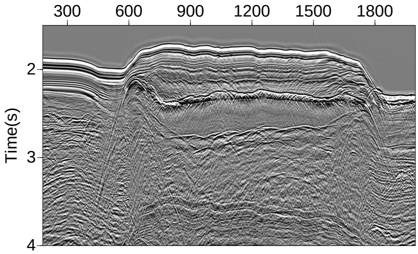
DEEPWATER GULF OF MEXICO DATA



DEEPWATER GULF OF MEXICO DATA



(b)



(c)

CONCLUSIONS

- **New estimate of local slope, applications:**
  - Stereotomography
  - Pre-stack time migration
  - Dip filtering
- **New method for slope stack**
  - Fully automatic, no mute
  - Can also use NMO velocities
- **No stretch effect, but only relative amplitudes**

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- **License:** Landmark
- **Financial support:** INCT-GP/CNPq/MCT, Petrobras, ANP, FINEP, FAPESB, Statoil NFR, Rose Project.

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LINEAR PROBLEM (Markovsky and Van Huffel, 2007)

Find $\mathbf{X}$, such that

$$\mathbf{AX} \approx \mathbf{B}$$

when $\mathbf{A}$ and $\mathbf{B}$ are given.

LEAST SQUARES

$$[\hat{\mathbf{X}}_{LS}, \Delta\mathbf{B}] = argmin_{\mathbf{X},\Delta\mathbf{B}} ||\Delta\mathbf{B}||_F$$

subject to $\mathbf{AX} = \mathbf{B} + \Delta\mathbf{B}$. The Frobenius matrix norm is

$$||\Delta\mathbf{B}||_F = \left\{ \sum_i \sum_j |\Delta\mathbf{B}_{i,j}|^2 \right\}^{1/2}$$

Solution $$\hat{\mathbf{x}}_{LS} = (\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\mathbf{B}$$

TOTAL LEAST SQUARES

Consider

$$[\mathbf{A} \quad \mathbf{B}] \begin{bmatrix} \mathbf{X} \\ -\mathbf{I} \end{bmatrix} \approx \mathbf{0}$$

and let $\mathbf{A} = \hat{\mathbf{A}} + \Delta\mathbf{A}$ and $\mathbf{B} = \hat{\mathbf{B}} + \Delta\mathbf{B}$. The total least squares solution

$$\left[\hat{\mathbf{X}}_{TLS} \quad \Delta\mathbf{A} \quad \Delta\mathbf{B}\right] = argmin_{\mathbf{X},\Delta\mathbf{A},\Delta\mathbf{B}} \,||\Delta\mathbf{A}|\Delta\mathbf{B}||_F$$

subject to $\hat{\mathbf{A}}\hat{\mathbf{X}}_{TLS} = \hat{\mathbf{B}}$.

SVD

$$\mathbf{C} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \end{bmatrix} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^T$$
$$= \mathbf{U} \begin{bmatrix} \mathbf{\Sigma}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{\Sigma}_2 \end{bmatrix} \begin{bmatrix} \mathbf{V}_{11}^T & \mathbf{V}_{21}^T \\ \mathbf{V}_{12}^T & \mathbf{V}_{22}^T \end{bmatrix}$$

Solution

$$\Delta\mathbf{C} = \begin{bmatrix} \Delta\mathbf{A} & \Delta\mathbf{B} \end{bmatrix} = \mathbf{U} \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{\Sigma_2} \end{bmatrix} \mathbf{V}^T$$

and

$$\hat{\mathbf{X}}_{TLS} = -\mathbf{V}_{12}\mathbf{V}_{22}^{-1}$$