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Bounds on Anisotropic Moduli

Constraints on C_{13} and their Consequences

by

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This is a summary of the presentation topic, not containing details. The details will be found in an upcoming publication.

Focus:
- VTI symmetry
- Shales

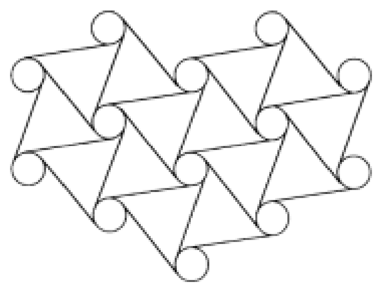
Isotropic Bounds on Elastic Moduli

Bulk modulus K
Shearmodulus G
Young's modulus E
Plane Wave Modulus H

} ³ 0

Poisson's ratio n

$- 1 £ n £ \frac{1}{2}$



G¹ 0; K® 0

?

K¹ 0; G® 0

Theoretical Basis: Positive Elastic Energy

Anisotropy

VTI (Vertical Transverse Isotropy):

$$\vec{C} = \begin{pmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{11} & C_{13} & 0 & 0 & 0 \\ C_{13} & C_{13} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{pmatrix}$$

$$\vec{s} = \vec{C}^{-1} = \begin{pmatrix} \frac{1}{E_H} & -\frac{\nu_{HH}}{E_H} & -\frac{\nu_{VH}}{E_V} & 0 & 0 & 0 \\ -\frac{\nu_{HH}}{E_H} & \frac{1}{E_H} & -\frac{\nu_{VH}}{E_V} & 0 & 0 & 0 \\ -\frac{\nu_{HV}}{E_H} & -\frac{\nu_{HV}}{E_H} & \frac{1}{E_V} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2G_{HV}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2G_{HV}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2G_{HH}} \end{pmatrix}$$

- Theoretical Basis: Positive Elastic Energy;
  Stiffness & Compliance Matrices Positive Definite

We want to link Static ("engineering") parameters (E, n)
to Dynamic (seismic / sonic / ultrasonic) parameters

Anisotropic Wave velocities:

$$r\,v_{PH}^2 = C_{11};\quad r\,v_{PV}^2 = C_{33};\quad r\,v_{SV}^2 = C_{44},\quad r\,v_{SH}^2 = C_{66}$$

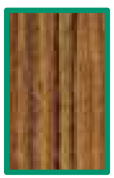
$$r\,v_{qP}^2(q) = f\,(C_{11}, C_{33}, C_{44}\ \&\ C_{13})$$

$$e = \frac{C_{11} - C_{33}}{2C_{33}};\quad g = \frac{C_{66} - C_{44}}{2C_{44}}$$

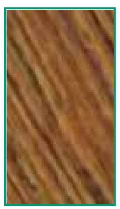
$$d = \frac{(C_{13} + C_{44})^2 - (C_{33} - C_{44})^2}{2C_{33}(C_{33} - C_{44})}$$

Û Anisotropic E-moduli & Poisson's ratios:

$$E_V = C_{33} - \frac{C_{13}^2}{(C_{11} - C_{66})} = \frac{(C_{11} - C_{66})C_{33} - C_{13}^{\ 2}}{(C_{11} - C_{66})};\quad n_{VH} = \frac{C_{13}}{2(C_{11} - C_{66})}$$



$$E_H = \frac{4C_{66}[(C_{11} - C_{66})C_{33} - C_{13}^2]}{C_{11}C_{33} - C_{13}^2};\quad n_{HV} = \frac{2C_{66}C_{13}}{C_{11}C_{33} - C_{13}^2};\quad n_{HH} = \frac{(C_{11} - 2C_{66})C_{33} - C_{13}^2}{C_{11}C_{33} - C_{13}^2}$$



Angular dependence of Young's modulus (core axis at angle q with z) :

$$\frac{1}{E(q)} = \frac{\cos^4 q}{E_z} + \frac{\sin^4 q}{E_r} + \frac{\sin^2 q \cos^2 q}{B};\quad \frac{1}{B} = \frac{1}{C_{44}} - \frac{C_{13}}{(C_{11} - C_{66})C_{33} - C_{13}^2}$$

Linking static and dynamic parameters

- **In the LAB:**
- **Dynamic (ultrasonic) measurements:**
  - We can easily obtain C_{33} & C_{44} + C_{11} & C_{66} from one core plug
  - C_{13}: Yes, but with more uncertainty

C_{13} ® d; "n" & "E"

- **Static (or quasistatic) measurements:**
  - We can obtain C_{33} , C_{13} & C_{66} from one core plug (at least 2 stress paths required!)
  - C_{11} from a second core plug drilled at 90 °
  - C_{44} from a third core plug drilled at oblique angle

Bounds may help to provide a best estimate of "uncertain" parameters + QC of measured data

Anisotropic Bounds for VTI Media

- Theoretical Basis: Positive Elastic Energy;
  Stiffness & Compliance Matrices Positive Definite

$$C_{44} > 0; \qquad C_{66} > 0$$

$$C_{11} > C_{66} > 0; \qquad C_{33} > 0$$

$$(C_{11} - C_{66})C_{33} - C_{13}^2 > 0$$

Note: No requirement that $C_{33} > C_{66}$

- Fundamentally; bounds may in general not always be found.
- Here we assume that C_{11} & C_{66} are known (C_{33} & C_{44} may be found if P- and S-wave anisotropies also are known):
- Question: What bounds do then exist for Poisson's ratios, Young's modulus anisotropy, and Thomsen's d?

Conclusions

- Anisotropic elastic mofuli are bounded by the requirement of positive elastic energy
  - Rigid bounds do not always exist

- Constraints on C_{13} lead to bounds on Poisson's ratios, E-modulus anisotropy and Thomsen's d.
  - "n" > 0.5 is accepted and expected
  - Do not use "n" from v_P/v_S in anisotropic media
  - d < 0 is expected, in particular for soft shales

- Laboratory observations show no values outside bounds
  - Trends within bounds may reveal fundamental insight

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SINTEF

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