emgs

LECTURE 5

Rune Mittet
Chief Scientist, EMGS
Adjunct Professor, NTNU

Spot the difference.

Uncertainty – Error propagation

Shallow water

Up-down decomposition

Anisotropy

Uncertainty – Error propagation

Propagation of uncertainty.
Error propagation

Example with two variables
Taylor to first order:

$$f(x,y) = f(x_0,y_0) + \frac{\partial f(x_0,y_0)}{\partial x}(x - x_0) + \frac{\partial f(x_0,y_0)}{\partial y}(y - y_0)$$

Or

$$f(x,y) - f(x_0,y_0) = \frac{\partial f(x_0,y_0)}{\partial x}(x - x_0) + \frac{\partial f(x_0,y_0)}{\partial y}(y - y_0)$$

Rewrite as:

$$\delta f(x,y) = \frac{\partial f(x_0,y_0)}{\partial x}\delta x + \frac{\partial f(x_0,y_0)}{\partial y}\delta y$$

Variation in function f as a function of variations in parameters x and y.

Have:

$$\delta f(x,y) = \frac{\partial f(x_0,y_0)}{\partial x}\delta x + \frac{\partial f(x_0,y_0)}{\partial y}\delta y$$

Suppose we do N measurements of $f(x,y)$. For the n'th measurement:

$$\delta f_n = \frac{\partial f(x_0,y_0)}{\partial x}\delta x_n + \frac{\partial f(x_0,y_0)}{\partial y}\delta y_n$$

$$\delta f_n^2 = \left(\frac{\partial f}{\partial x}\right)^2 \delta x_n^2 + \left(\frac{\partial f}{\partial y}\right)^2 \delta y_n^2 + 2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y} cov(x_n, y_n)$$

Have:

$$\delta f_n^2 = \left(\frac{\partial f}{\partial x}\right)^2 \delta x_n^2 + \left(\frac{\partial f}{\partial y}\right)^2 \delta y_n^2 + 2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y} cov(x_n, y_n)$$

If independent variables:

$$\frac{\sum_{n=1}^{N} \delta f_n^2}{N} = \left(\frac{\partial f}{\partial x}\right)^2 \frac{\sum_{n=1}^{N} \delta x_n^2}{N} + \left(\frac{\partial f}{\partial y}\right)^2 \frac{\sum_{n=1}^{N} \delta y_n^2}{N}$$

In terms of standard deviations:

$$s_f^2 = \left(\frac{\partial f}{\partial x}\right)^2 s_x^2 + \left(\frac{\partial f}{\partial y}\right)^2 s_y^2$$

Will use notation of the form: $$\delta f^2 = \left(\frac{\partial f}{\partial x}\right)^2 \delta x^2 + \left(\frac{\partial f}{\partial y}\right)^2 \delta y^2$$

Have

$$\delta f^2 = \left(\frac{\partial f}{\partial x}\right)^2 \delta x^2 + \left(\frac{\partial f}{\partial y}\right)^2 \delta y^2$$

The expected uncertainty/error in maesuring f due to uncertainty/error in x and y :

$$\delta f = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 \delta x^2 + \left(\frac{\partial f}{\partial y}\right)^2 \delta y^2}$$

Generalized:

$$\delta f(\boldsymbol{x}) = \sqrt{\sum_i |\frac{\partial f(\boldsymbol{x})}{\partial x_i}|^2 \ |\delta x_i \ |^2}$$

Example

Speed trap:
Measured fixed distance and interval time measurement

$$v = \frac{s_2 - s_1}{t_2 - t_1} = \frac{s}{t}$$

The distance from A to B is $s = 100$ m but there is an uncertainty related to measuring the distance s : δs

Likewise, there is an uncertainty related to measuring the time it takes to drive from A to B: δt

The velocity is: $v(s,t) = \frac{s}{t}$

The uncertainty in the velocity measurement is:

$$\delta v = \sqrt{\left(\frac{\partial v}{\partial s}\right)^2 \delta s^2 + \left(\frac{\partial v}{\partial t}\right)^2 \delta t^2}$$

Explicitly:

$$\delta v = \sqrt{\left(\frac{1}{t}\right)^2 \delta s^2 + \left(\frac{s}{t^2}\right)^2 \delta t^2}$$

Have

$$\delta v = \sqrt{\left(\frac{1}{t}\right)^2 \delta s^2 + \left(\frac{s}{t^2}\right)^2 \delta t^2}$$

Use $v = \frac{s}{t}$ to obtain:

$$\frac{\delta v}{v} = \sqrt{\left(\frac{\delta s}{s}\right)^2 + \left(\frac{\delta t}{t}\right)^2}$$

Suppose the task is to measure velocities up to 110 km/h with accuracy 1.1 km/h or better.

For the moment, assume perfect timing ($\delta t = 0$). Sufficient accuracy if:

$$\frac{\delta v}{v} \geq \frac{\delta s}{s}$$

Have for perfect time measurements:

$$\frac{\delta s}{s} \leq 0.01$$

If the distance $s = 100$ m : $\delta s \leq 1$ m

Assumption: $\delta s \approx 0.1$ m with laser

$$\frac{\delta v}{v} = \sqrt{\left(\frac{\delta s}{s}\right)^2 + \left(\frac{\delta t}{t}\right)^2} \longrightarrow \quad 0.01 \geq \sqrt{(0.001)^2 + \left(\frac{\delta t}{t}\right)^2}$$

Obtain

$$\frac{\delta t}{t} < 0.01$$

Good accuracy on distance measurement implies that allmost all potential uncertainty is related to time measurement

Have

$$\frac{\delta t}{t} < 0.01$$

Small time intervals from A to B will give largest uncertainty. This is for highest velocity.

Have 110 km/h ≈ 30 m/s. Expected shortest time is $t = 3.333$ s

Acceptable uncertainty in time measurement, δt, is 0.033 s or 33 msec

Manual timing with stopwatch or electronic timing?

Uncertainty in CSEM measurements

Assume observed inline electric field can be approximated by:

$$E_x(\boldsymbol{x}_r|\boldsymbol{x}_s) = G_{xn}^{EJ}(\boldsymbol{x}_r|\boldsymbol{x}_s)LJ_n\alpha + N$$

$G_{xn}^{EJ}(\boldsymbol{x}_r|\boldsymbol{x}_s)$: Electric field Green's function. Often named «The Earth's impulse response»
Here it serves the role as an ideal response without errors or uncertainty.

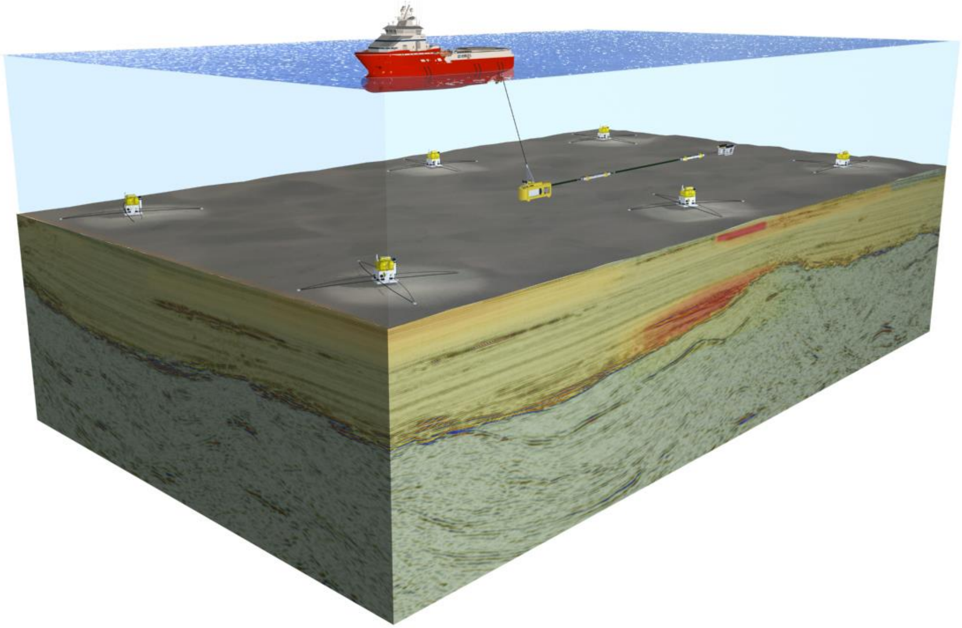
L: Length of electric dipole

J_n: Strength of transmitted current

α: Receiver calibration factor, nominal value is 1.0

N: Additive noise. Can be receiver self noise, MT noise, swell noise, motion noise, ...

Uncertainty



Receiver:
- Position
- Direction
- Calibration
- Timing
- Self noise
- Motion noise - turbulence
- Swell noise
- MT noise (Can be estimated/partly removed)

Transmitter:
- Front electrode position
- Aft electrode position
- Current measurement
- Timing

(Front electrode position, Aft electrode position) } Effective length, feathering, pitch

Have

$$E_x(\boldsymbol{x}_r|\boldsymbol{x}_s) = G_{xn}^{EJ}(\boldsymbol{x}_r|\boldsymbol{x}_s)LJ_n\alpha + N$$

For simplicity of derivation we assume a plane layer earth $\boldsymbol{x} = \boldsymbol{x}_r - \boldsymbol{x}_s$

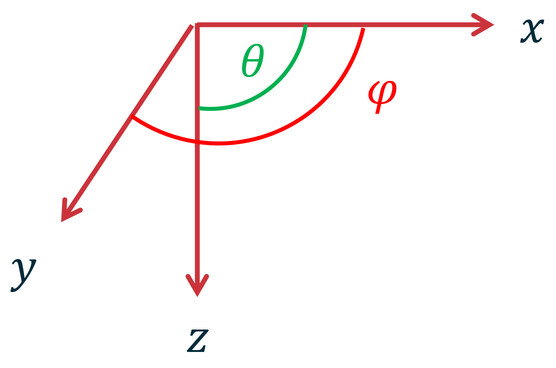
$$E_x(\boldsymbol{x}) = G_{xn}^{EJ}(\boldsymbol{x})LJ_n\alpha + N$$

For the source components

$$J_x = Jcos(\varphi)\cos(\theta)$$

$$J_y = Jsin(\varphi)\cos(\theta)$$

$$J_z = J\sin(\theta)$$



Nominal $\varphi = 0, \quad \theta = 0$

Perfect inline transmitter give: $J_x = J, \quad J_y = 0, \quad J_z = 0$

$$E_x(\boldsymbol{x}) \rightarrow E_x(\boldsymbol{p}) \qquad \boldsymbol{p} = [\boldsymbol{x}, \alpha, J, L, \theta, \varphi, N]^T$$

Have derived

$$\delta f(\boldsymbol{x}) = \sqrt{\sum_i \left|\frac{\partial f(\boldsymbol{x})}{\partial x_i}\right|^2 \ |\delta x_i\ |^2}$$

For inline electromagnetic field:

$$\delta E_x(\boldsymbol{p}) = \sqrt{\sum_i \left|\frac{\partial E_x(\boldsymbol{p})}{\partial p_i}\right|^2 \ |\delta p_i\ |^2}$$

$$E_x(\boldsymbol{p}) = G_{xn}^{EJ}(\boldsymbol{x}) L J_n \alpha + N$$

$$\boldsymbol{p} = [\boldsymbol{x}, \alpha, J, L, \theta, \varphi, N]^T$$

For simplicity of notation: $D = LJ\alpha$

Next step is to carry out calculation of partial derivatives with respect to parameter vector

$$E_x(\boldsymbol{p}) = G_{xn}^{EJ}(\boldsymbol{x})LJ_n\alpha + N \qquad \boldsymbol{p} = [\boldsymbol{x}, \alpha, J, L, \theta, \varphi, N]^T \qquad D = LJ\alpha$$

Spatial coordinates:

$$\partial_x E_x(\boldsymbol{p}) = \partial_x G_{xx}^{EJ}(\boldsymbol{x})D$$

$$\partial_y E_x(\boldsymbol{p}) = 0$$

$$\partial_z E_x(\boldsymbol{p}) = \partial_z G_{xx}^{EJ}(\boldsymbol{x})D$$

Directional coordinates:

$$J_x = Jcos(\varphi)\cos(\theta)$$

$$J_y = Jsin(\varphi)\cos(\theta) \qquad (G_{xy} = 0)$$

$$J_z = J\sin(\theta)$$

$$\partial_\varphi E_x(\boldsymbol{p}) = 0$$

$$\partial_\theta E_x(\boldsymbol{p}) = G_{xz}^{EJ}(\boldsymbol{x})D$$

Next step is to carry out calculation of partial derivatives with respect to parameter vector

$$E_x(\boldsymbol{p}) = G_{xn}^{EJ}(\boldsymbol{x}) L J_n \alpha + N \qquad \boldsymbol{p} = [\boldsymbol{x}, \alpha, J, L, \theta, \varphi, N]^T \qquad D = LJ\alpha$$

Dipole moment-receiver calibration:

$$\partial_L E_x(\boldsymbol{p}) = G_{xx}^{EJ}(\boldsymbol{x}) D \frac{1}{L}$$

$$\partial_J E_x(\boldsymbol{p}) = G_{xx}^{EJ}(\boldsymbol{x}) D \frac{1}{J}$$

$$\partial_\alpha E_x(\boldsymbol{p}) = G_{xx}^{EJ}(\boldsymbol{x}) D \frac{1}{\alpha}$$

Additive noise:

$$\partial_N E_x(\boldsymbol{p}) = 1$$

Partial uncertainties

$$\partial_x E_x(\boldsymbol{p}) = \partial_x G_{xx}^{EJ}(\boldsymbol{x})D \longrightarrow \delta E_x(\mathrm{X}) = |\partial_x G_{xx}^{EJ}(\boldsymbol{x})D\ \delta x|$$

$$\partial_z E_x(\boldsymbol{p}) = \partial_z G_{xx}^{EJ}(\boldsymbol{x})D \longrightarrow \delta E_x(\mathrm{Z}) = |\partial_z G_{xx}^{EJ}(\boldsymbol{x})D\ \delta z|$$

$$\left.\begin{aligned} \partial_L E_x(\boldsymbol{p}) &= G_{xx}^{EJ}(\boldsymbol{x})D\,\frac{1}{L} \\ \partial_J E_x(\boldsymbol{p}) &= G_{xx}^{EJ}(\boldsymbol{x})D\,\frac{1}{J} \\ \partial_\alpha E_x(\boldsymbol{p}) &= G_{xx}^{EJ}(\boldsymbol{x})D\,\frac{1}{\alpha} \end{aligned}\right\} \quad \delta E_x(\mathrm{C}) = |G_{xx}^{EJ}(\boldsymbol{x})D|\sqrt{\left(\frac{\delta L}{L}\right)^2 + \left(\frac{\delta J}{J}\right)^2 + \left(\frac{\delta \alpha}{\alpha}\right)^2}$$

$$\partial_\theta E_x(\boldsymbol{p}) = G_{xz}^{EJ}(\boldsymbol{x})D \longrightarrow \delta E_x(\theta) = |G_{xz}^{EJ}(\boldsymbol{x})D\ \delta\theta|$$

$$\partial_N E_x(\boldsymbol{p}) = 1 \longrightarrow \delta E_x(\mathrm{N}) = |\Delta N|$$

Have

$$\delta E_x(\mathrm{X}) = |\partial_x G_{xx}^{EJ}(\boldsymbol{x})D\ \delta x|$$
$$\delta E_x(\mathrm{Z}) = |\partial_z G_{xx}^{EJ}(\boldsymbol{x})D\ \delta z|$$
$$\delta E_x(\mathrm{C}) = |G_{xx}^{EJ}(\boldsymbol{x})D|\sqrt{\left(\frac{\delta L}{L}\right)^2 + \left(\frac{\delta J}{J}\right)^2 + \left(\frac{\delta \alpha}{\alpha}\right)^2}$$
$$\delta E_x(\theta) = |G_{xz}^{EJ}(\boldsymbol{x})D\ \delta\theta|$$

Collect all terms that scales with Green's functions and dipole moment:

$$|\delta E_x(\mathrm{M})|^2 = |\delta E_x(\mathrm{X})|^2 + |\delta E_x(\mathrm{Z})|^2 + |\delta E_x(\mathrm{C})|^2 + |\delta E_x(\theta)|^2$$

The additive term:

$$\delta E_x(\mathrm{N}) = |\Delta N|$$

The total uncertainty:

$$\delta E_x(\boldsymbol{p}) = \sqrt{\sum_i |\frac{\partial E_x(\boldsymbol{p})}{\partial p_i}|^2\ |\delta p_i\ |^2} \longrightarrow \delta E_x(\boldsymbol{p}) = \sqrt{|\delta E_x(\mathrm{M})|^2 + |\delta E_x(\mathrm{N})|^2}$$

Plot color coding

$$\delta E_x(\mathrm{X}) = |\partial_x G_{xx}^{EJ}(\boldsymbol{x}) D\ \delta x|$$

$$\delta E_x(\mathrm{Z}) = |\partial_z G_{xx}^{EJ}(\boldsymbol{x}) D\ \delta z|$$

$$\delta E_x(\mathrm{C}) = |G_{xx}^{EJ}(\boldsymbol{x}) D| \sqrt{\left(\frac{\delta L}{L}\right)^2 + \left(\frac{\delta J}{J}\right)^2 + \left(\frac{\delta \alpha}{\alpha}\right)^2}$$

$$\delta E_x(\theta) = |G_{xz}^{EJ}(\boldsymbol{x}) D\ \delta \theta|$$

$$\delta E_x(\mathrm{N}) = |\Delta N|$$

$$\delta E_x(\boldsymbol{p}) = \sqrt{|\delta E_x(\mathrm{M})|^2 + |\delta E_x(\mathrm{N})|^2}$$

Scattered fields

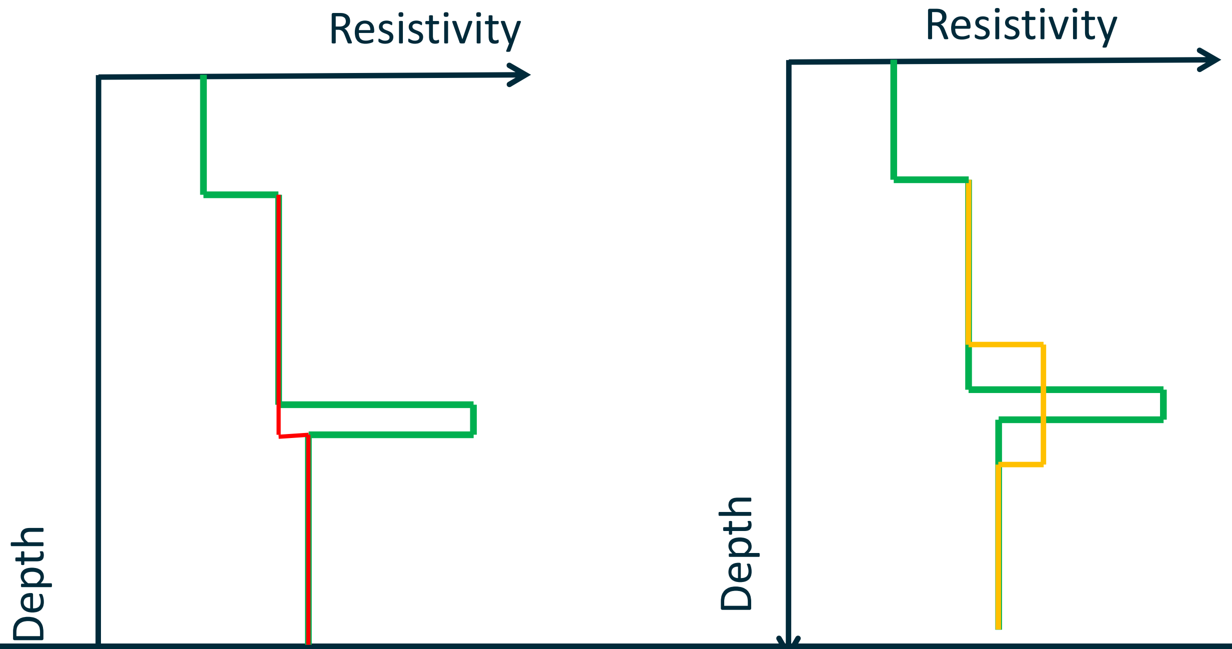
Misfit in first iteration:

$$\Delta E_x^0(\boldsymbol{p}) = |E_x^{Obs}(\boldsymbol{p}) - E_x^0(\boldsymbol{x}, \omega)|$$

Assume that 67 percent (2/3) of transverse resistance recovered at iteration n:

$$\Delta E_x^n(\boldsymbol{p}) = |E_x^{Obs}(\boldsymbol{p}) - E_x^n(\boldsymbol{x}, \omega)|$$

True model
Start model
Partially recovered model after n iterations



Inversion

L1 inversion data misfit kernel:

$$\Psi^n(\boldsymbol{p}) = \frac{\Delta E_x^n(\boldsymbol{p})}{\delta E_x(\boldsymbol{p})}$$

Inspect ratio of residual misfit field to uncertainty:
Hard to extract more resistivity information if residual data misfit is of same magnitude as uncertainty. Critical value for Ψ is 1. Further iterations make sense if Ψ larger than unity

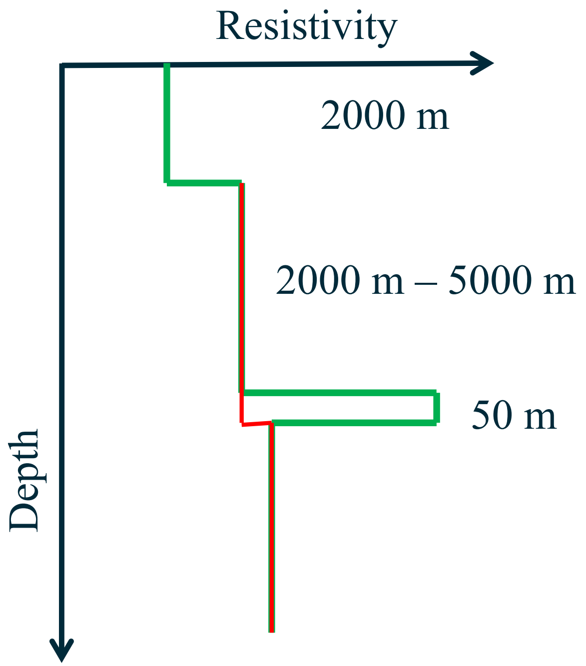
Usually L2 inversion data misfit kernels used:

$$\varepsilon = \sum_{Observations} \left(\frac{\Delta E_x^n}{\delta E_x} \right)^2$$

Resistivity model

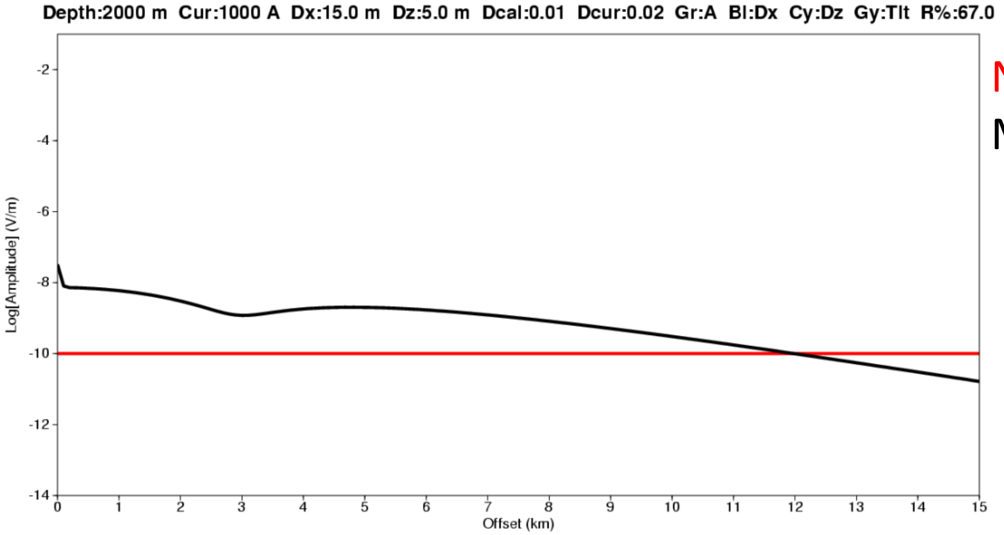
f=0.25 Hz

ρ_V	ρ_H
0.3125	0.3125
3.0	1.5
3.0	1.5
4.0	2.0



ρ_V	ρ_H
0.3125	0.3125
3.0	1.5
50.0	50.0
4.0	2.0

Examples



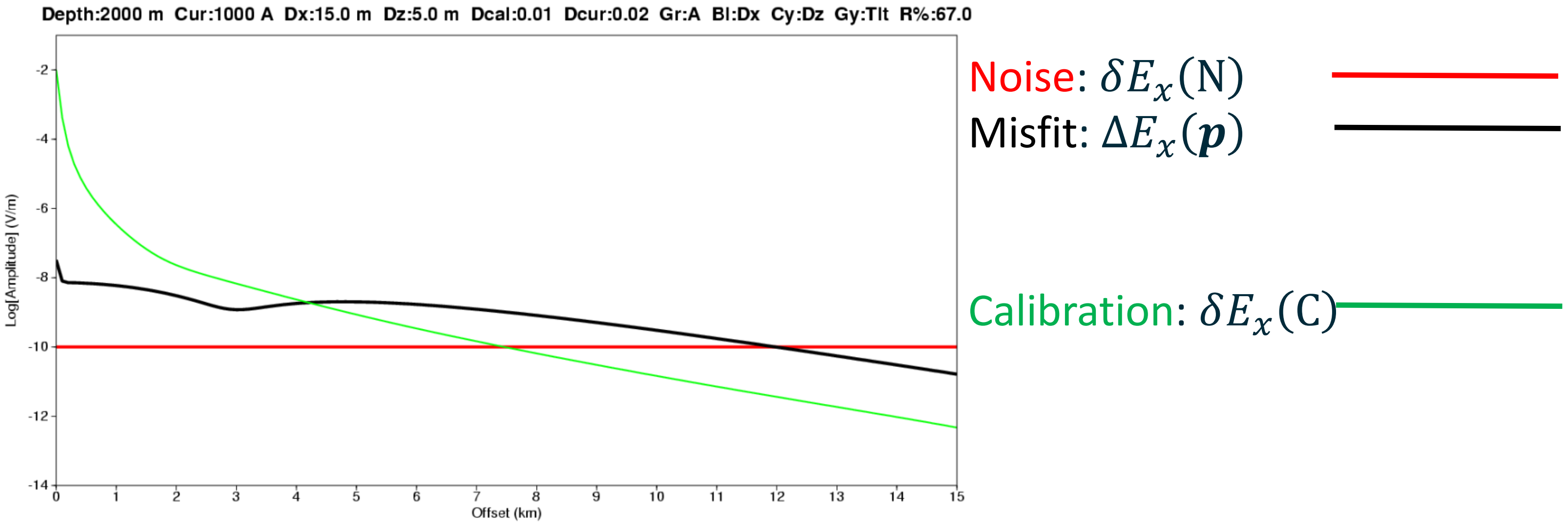
Noise: $\delta E_x(\mathrm{N})$

Misfit: $\Delta E_x(\boldsymbol{p})$

$$\Delta E_x^0(\boldsymbol{p}) = |E_x^{Obs}(\boldsymbol{p}) - E_x^0(\boldsymbol{x}, \omega)|$$

$$\delta E_x(\mathrm{N}) = |\Delta N|$$

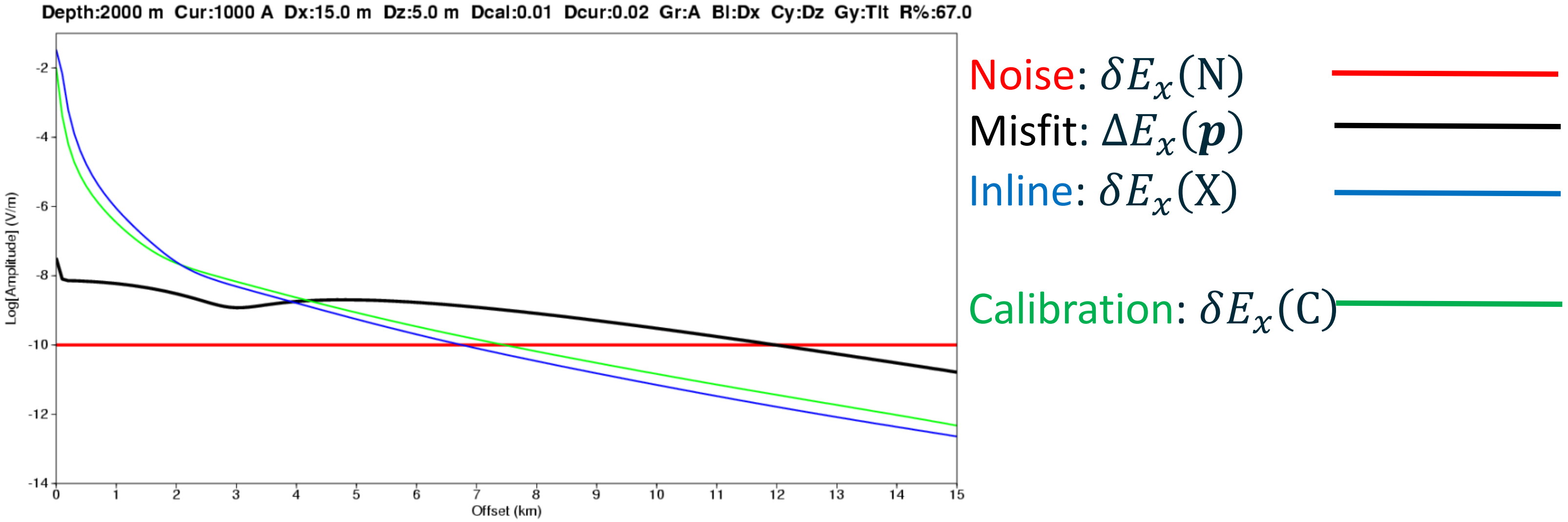
Examples



$$\Delta E_x^0(\boldsymbol{p}) = |E_x^{Obs}(\boldsymbol{p}) - E_x^0(\boldsymbol{x}, \omega)|$$

$$\delta E_x(\mathrm{C}) = |G_{xx}^{EJ}(\boldsymbol{x})D| \sqrt{\left(\frac{\delta L}{L}\right)^2 + \left(\frac{\delta J}{J}\right)^2 + \left(\frac{\delta \alpha}{\alpha}\right)^2}$$

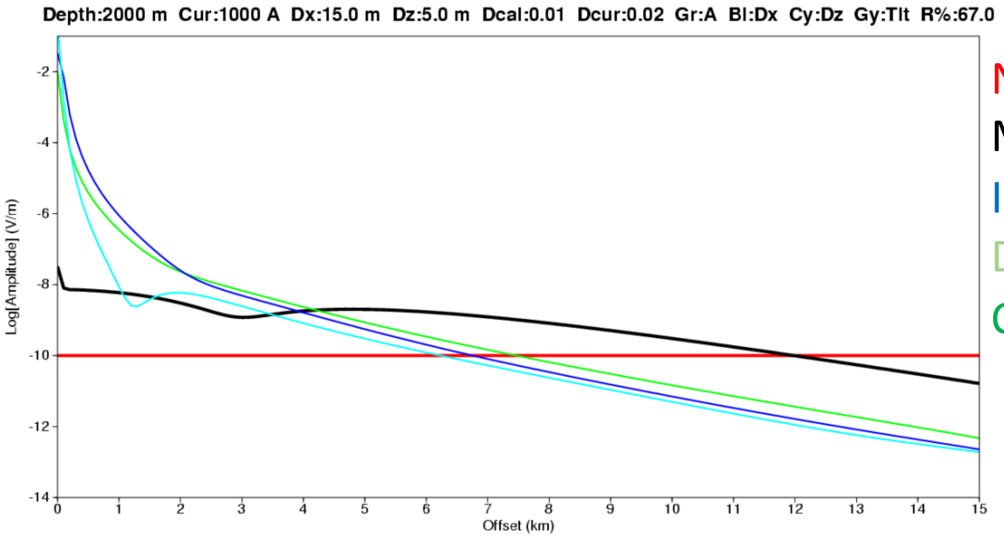
Examples



$$\Delta E_x^0(\boldsymbol{p}) = |E_x^{Obs}(\boldsymbol{p}) - E_x^0(\boldsymbol{x}, \omega)|$$

$$\delta E_x(\mathrm{X}) = |\partial_x G_{xx}^{EJ}(\boldsymbol{x}) D\, \delta x|$$

Examples



Noise: $\delta E_x(\mathrm{N})$

Misfit: $\Delta E_x(\boldsymbol{p})$

Inline: $\delta E_x(\mathrm{X})$

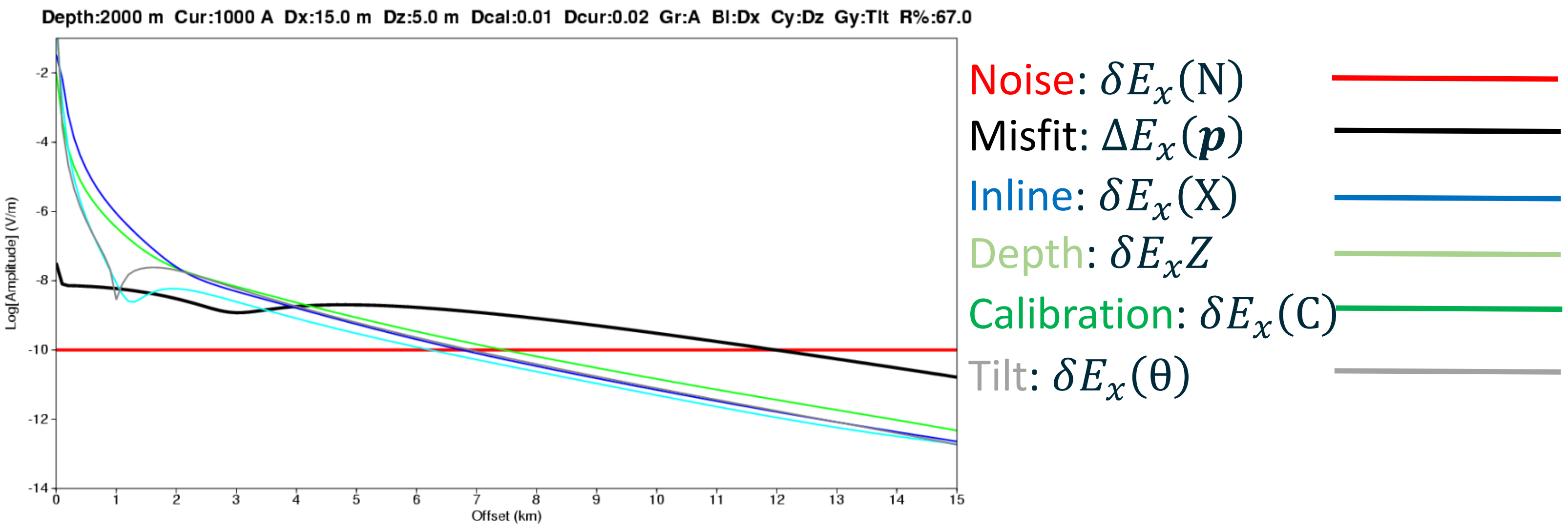
Depth: $\delta E_x(\mathrm{Z})$

Calibration: $\delta E_x(\mathrm{C})$

$$\Delta E_x^0(\boldsymbol{p}) = |E_x^{Obs}(\boldsymbol{p}) - E_x^0(\boldsymbol{x}, \omega)|$$

$$\delta E_x(\mathrm{Z}) = |\partial_z G_{xx}^{EJ}(\boldsymbol{x}) D\ \delta z|$$

Examples



$$\Delta E_x^0(\boldsymbol{p}) = |E_x^{Obs}(\boldsymbol{p}) - E_x^0(\boldsymbol{x}, \omega)|$$

$$\delta E_x(\theta) = |G_{xz}^{EJ}(\boldsymbol{x}) D \; \delta\theta|$$

For

$$\delta E_x(\mathrm{C}) = |G_{xx}^{EJ}(\boldsymbol{x})D| \sqrt{\left(\frac{\delta L}{L}\right)^2 + \left(\frac{\delta J}{J}\right)^2 + \left(\frac{\delta \alpha}{\alpha}\right)^2}$$

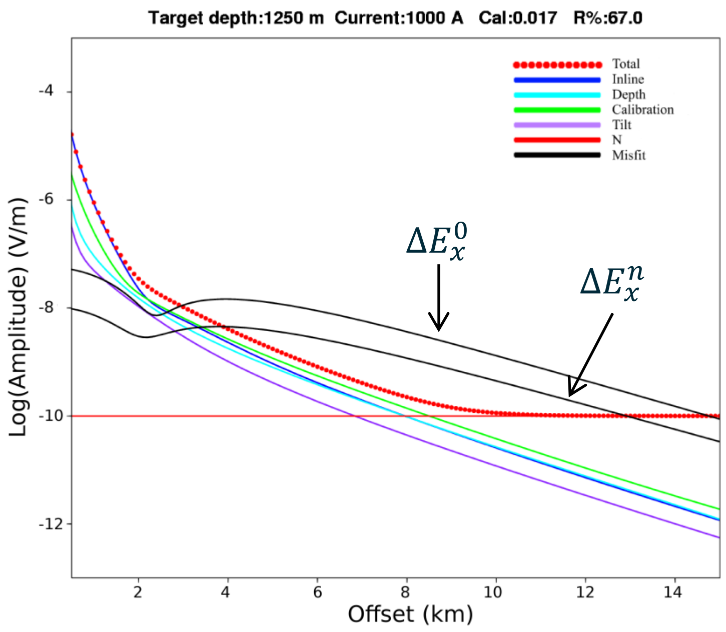
Assume in the following:

$$\frac{\delta L}{L} = \frac{\delta J}{J} = \frac{\delta \alpha}{\alpha}$$

$$\delta E_x(\mathrm{C}) = |G_{xx}^{EJ}(\boldsymbol{x})D| \frac{\delta A}{A}$$

For example

$$\frac{\delta A}{A} = \sqrt{3}\frac{\delta L}{L} \approx 1.7\frac{\delta L}{L}$$



Noise: $\delta E_x(\mathrm{N})$

Misfit: $\Delta E_x(\boldsymbol{p})$

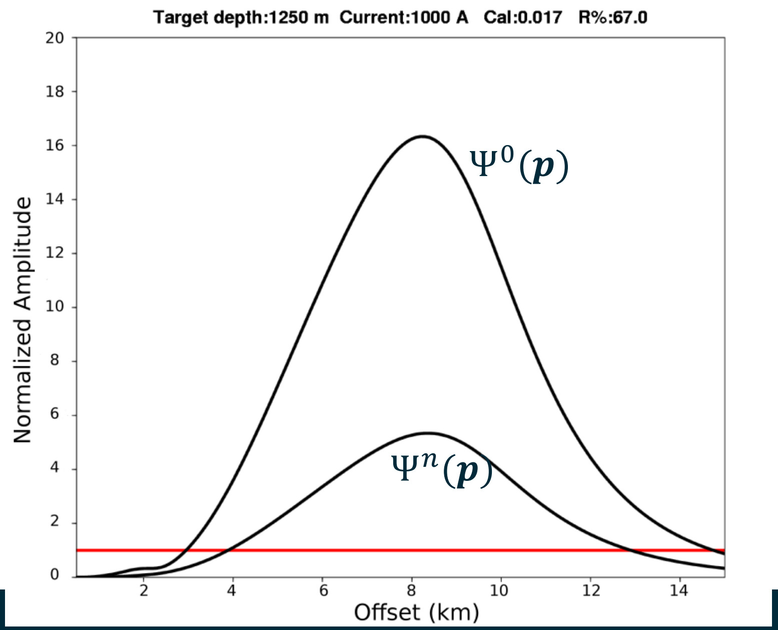
Inline: $\delta E_x(\mathrm{X})$

Depth: $\delta E_x(\mathrm{Z})$

Calibration: $\delta E_x(\mathrm{C})$

Tilt: $\delta E_x(\theta)$

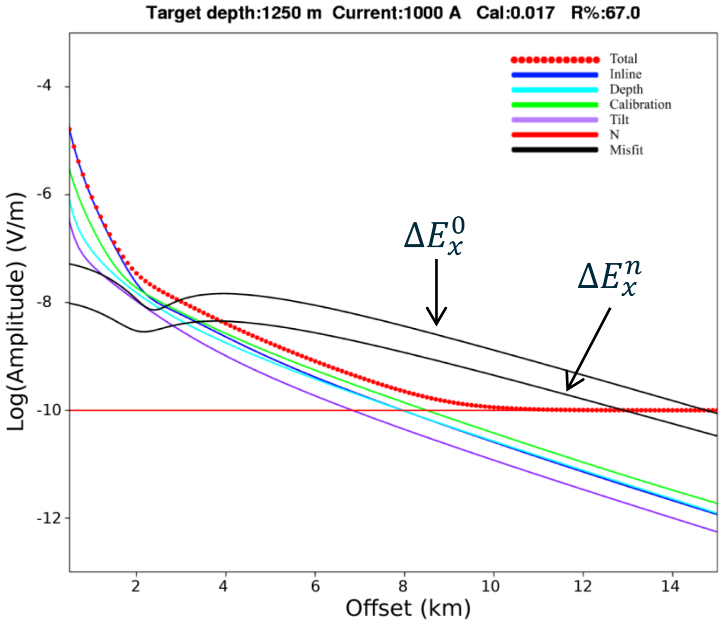
Total: $\delta E_x(\boldsymbol{p})$

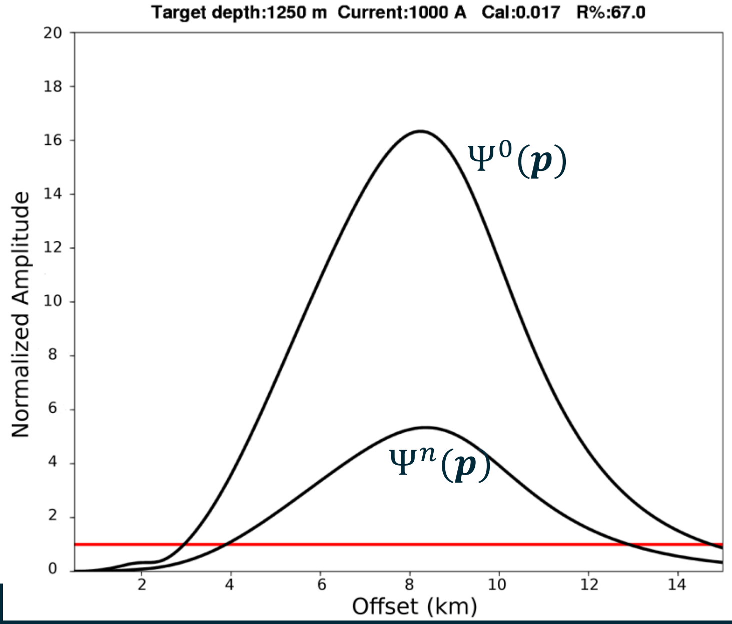


$$\Delta E_x^0(\boldsymbol{p}) = |E_x^{Obs}(\boldsymbol{p}) - E_x^0(\boldsymbol{x}, \omega)|$$

$$\Delta E_x^n(\boldsymbol{p}) = |E_x^{Obs}(\boldsymbol{p}) - E_x^n(\boldsymbol{x}, \omega)|$$

$$\delta E_x(\boldsymbol{p}) = \sqrt{|\delta E_x(\mathrm{M})|^2 + |\delta E_x(\mathrm{N})|^2}$$



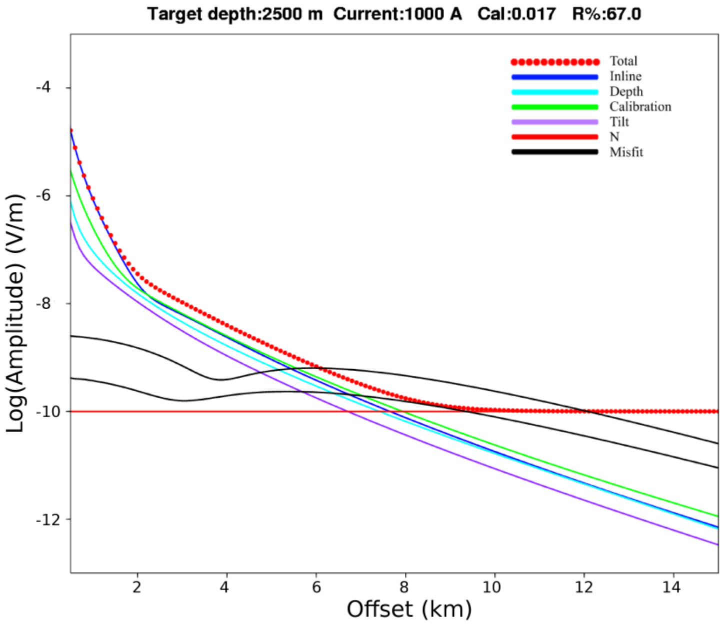


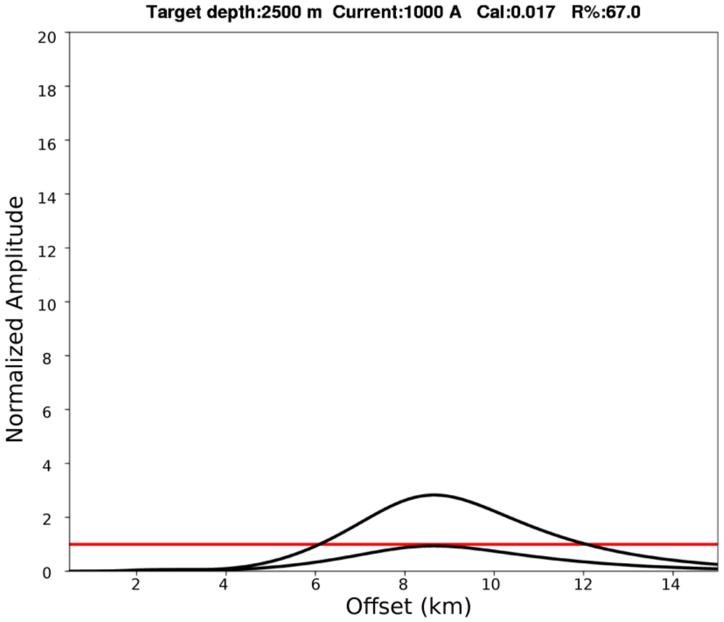
Typical accuracy as of 2010

$$\delta x = 15 \text{ m}$$

$$\delta z = 5 \text{ m}$$

$$\delta \theta = 1^\circ$$





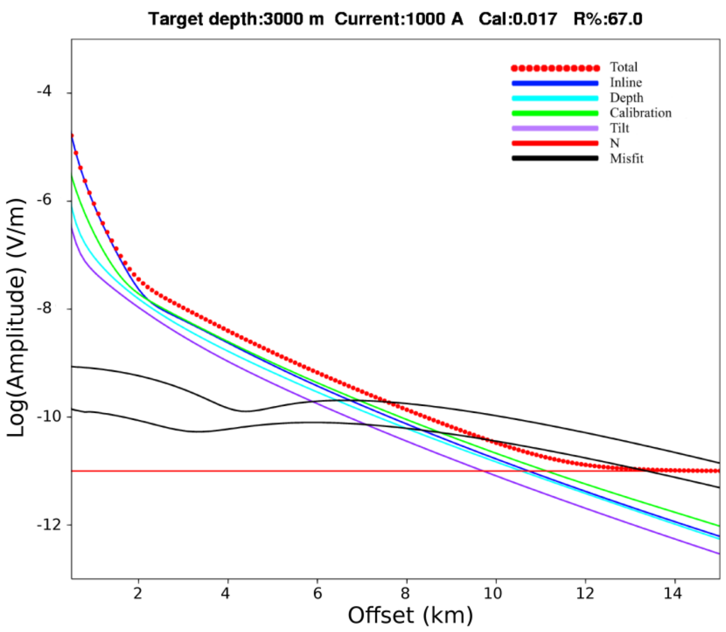
Target down

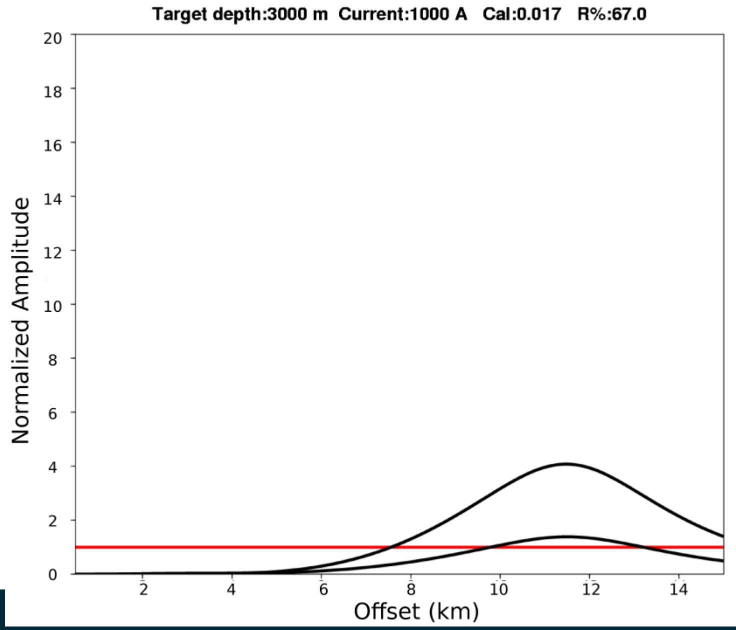
Typical accuracy as of 2010

$$\delta x = 15 \text{ m}$$

$$\delta z = 5 \text{ m}$$

$$\delta \theta = 1°$$





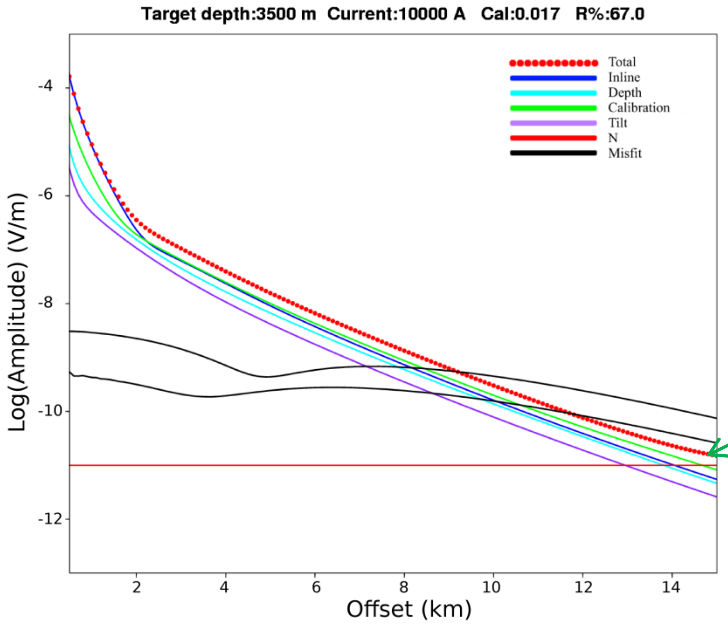
Target down

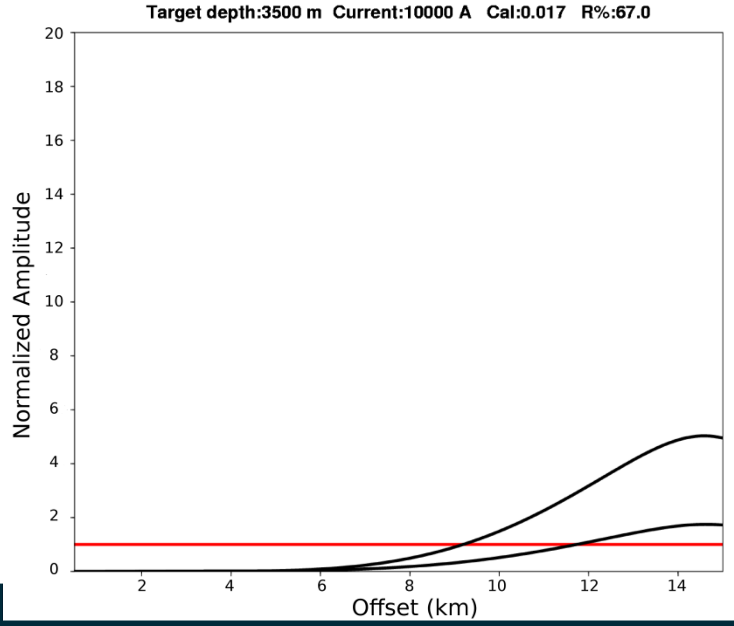
$\delta x = 15$ m

$\delta z = 5$ m

$\delta \theta = 1°$

Receiver noise 10^{-11} V/m





Target down

Largest contribution

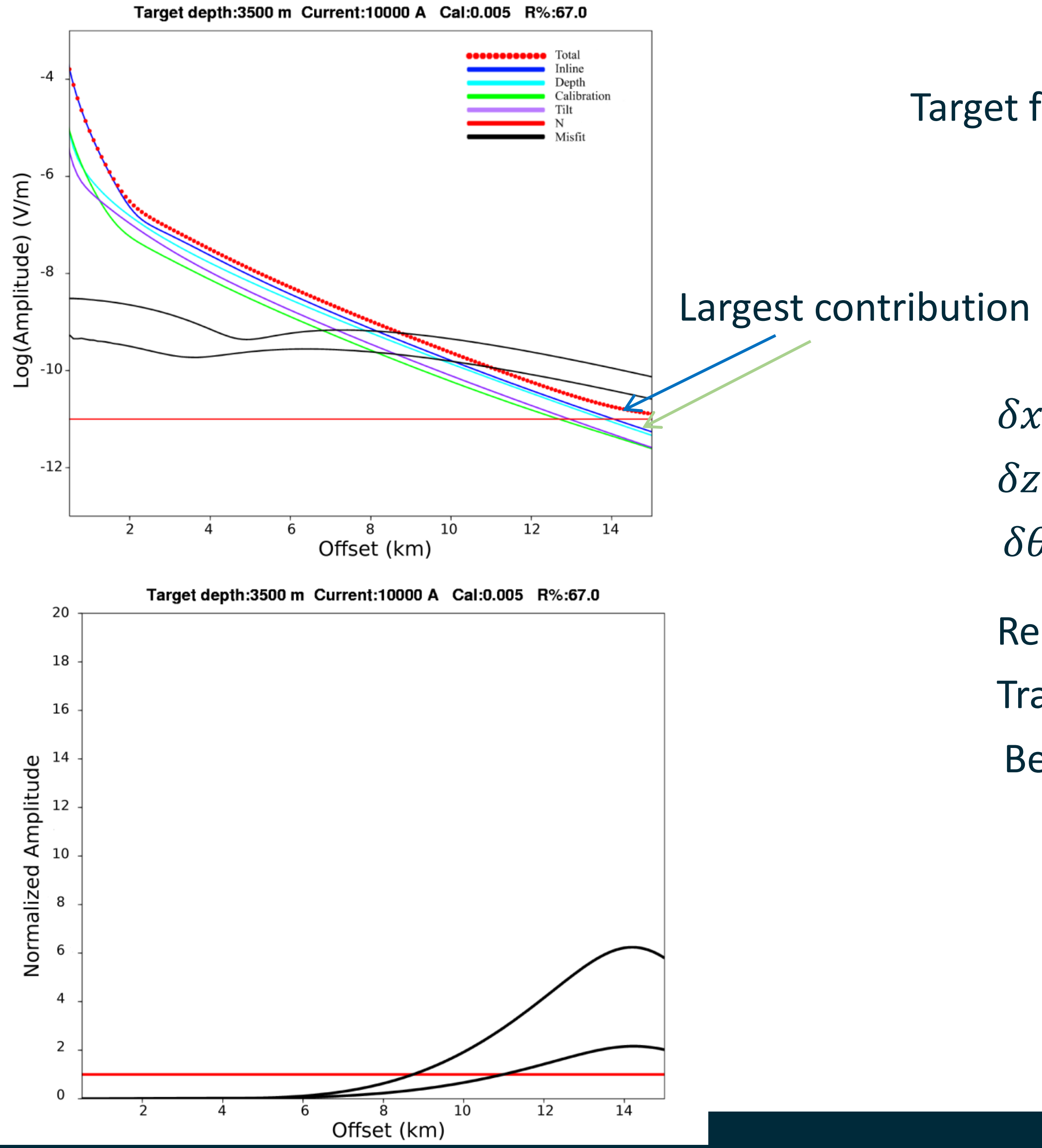
$\delta x = 15$ m

$\delta z = 5$ m

$\delta\theta = 1°$

Receiver noise 10^{-11} V/m

Transmitter current 10 kA



Target fixed

$\delta x = 15$ m

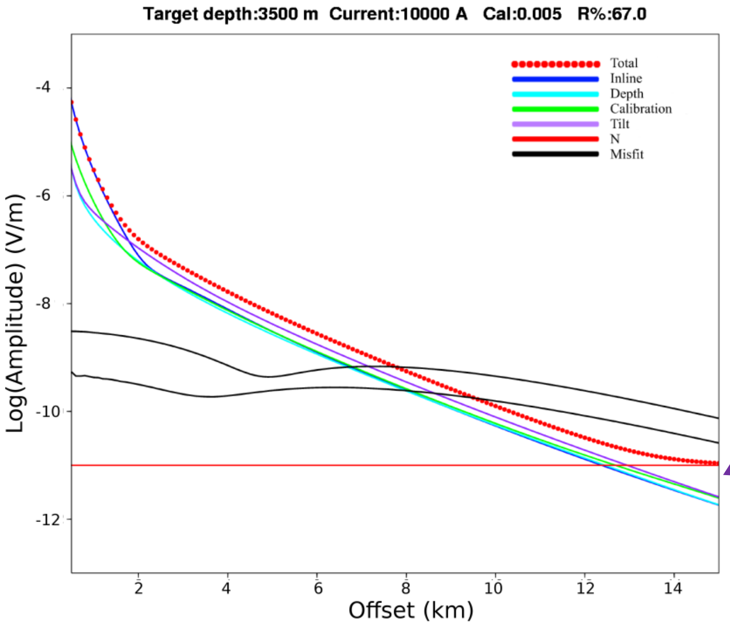
$\delta z = 5$ m

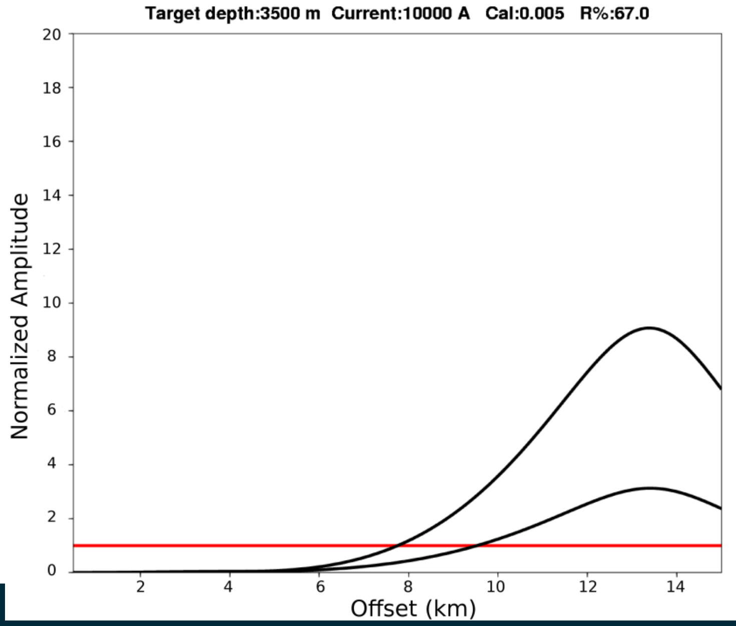
$\delta\theta = 1°$

Receiver noise 10^{-11} V/m

Transmitter current 10 kA

Better calibration and δL





Target fixed

Largest contribution

$\delta x = 5$ m

$\delta z = 2$ m

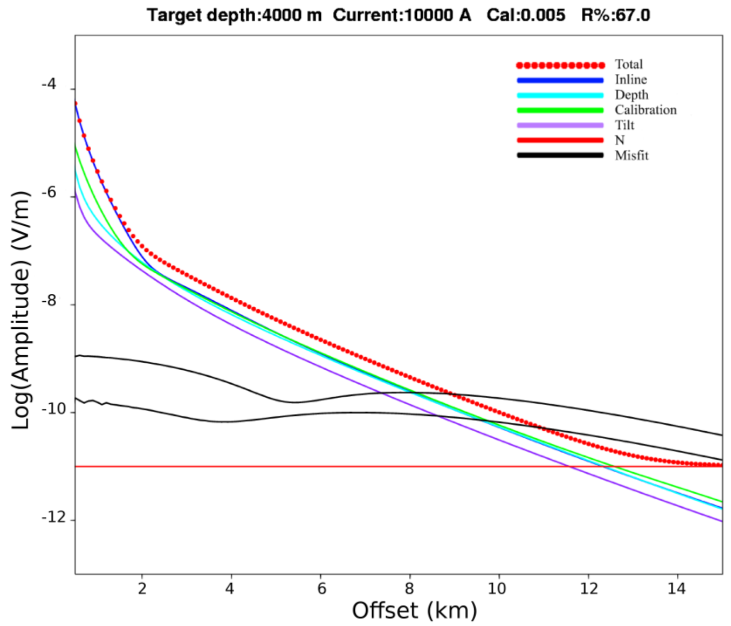
$\delta \theta = 1°$

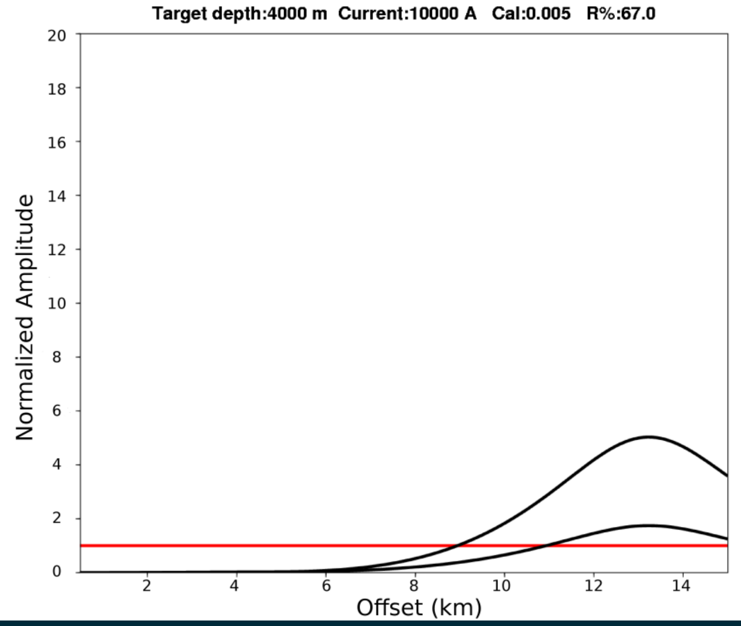
Receiver noise 10^{-11} V/m

Transmitter current 10 kA

Better calibration and δL

Better navigation x & z





Target down

$\delta x = 5$ m

$\delta z = 2$ m

$\delta \theta = 0.4°$

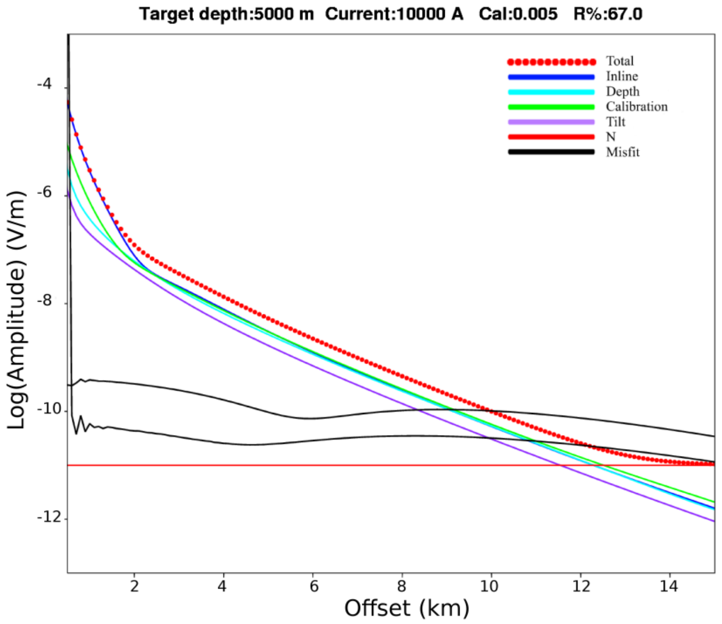
Receiver noise 10^{-11} V/m

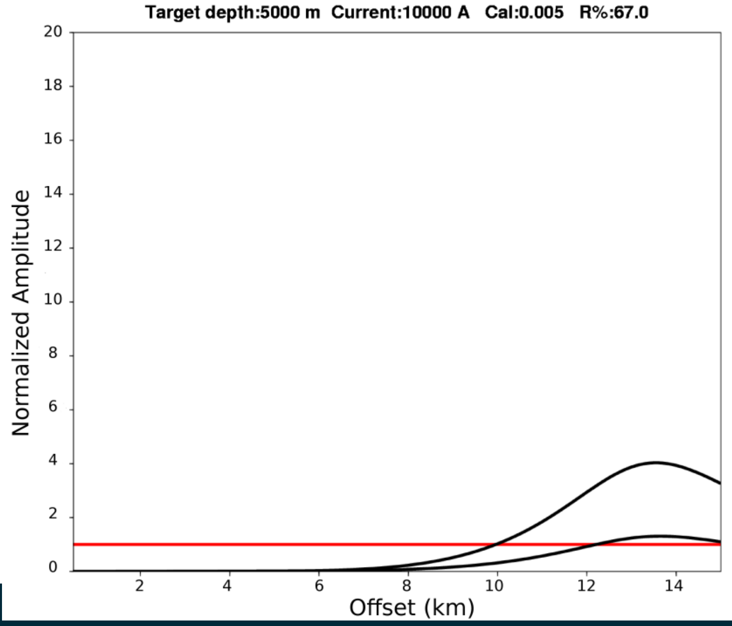
Transmitter current 10 kA

Better calibration and δL

Better navigation x & z

Better navigation θ





Increased transverse resistance

Target down

$\delta x = 5$ m

$\delta z = 2$ m

$\delta \theta = 0.4°$

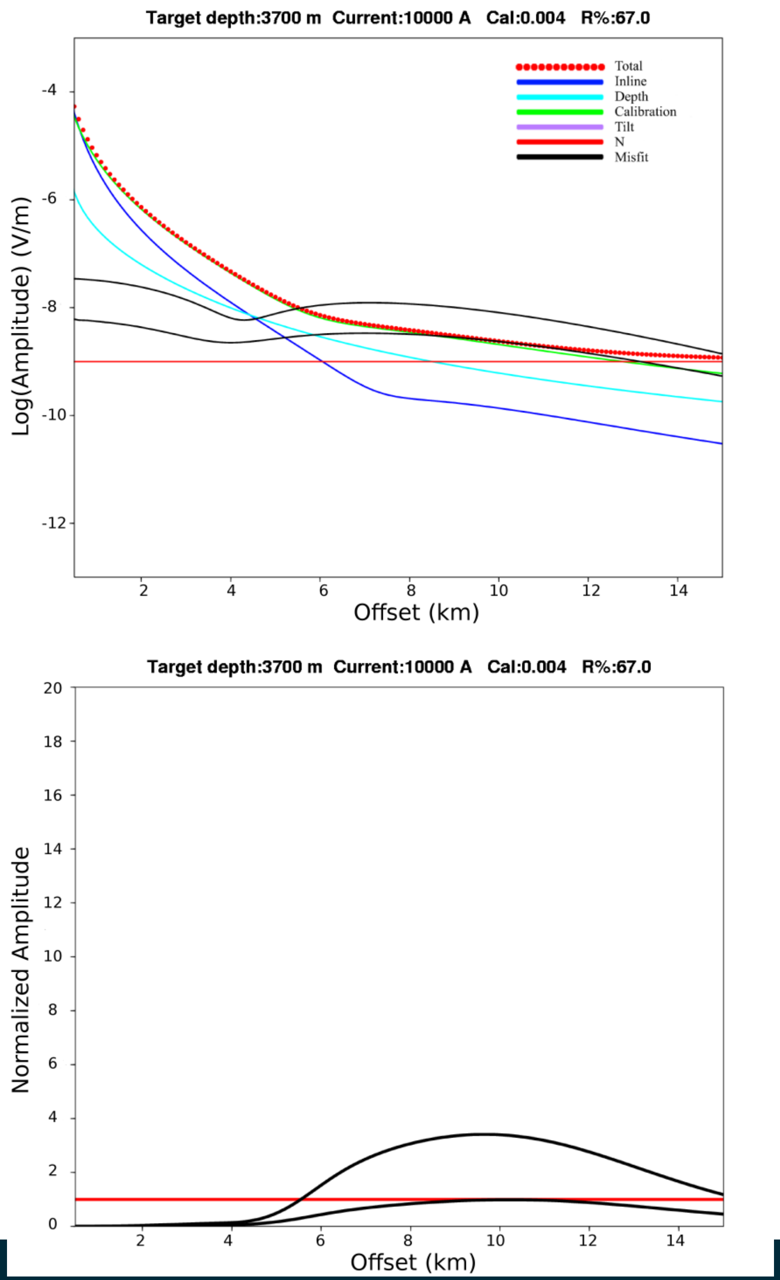
Receiver noise 10^{-11} V/m

Transmitter current 10 kA

Better calibration and δL

Better navigation x & z

Better navigation θ



Shallow water – 40 m

Problem is MT – swell – motion noise

$\delta x = 1$ m

$\delta z = 1$ m

$\delta \theta = 0°$

Receiver noise 10^{-9} V/m

Have

$$\delta E_x(\boldsymbol{p}) = \sqrt{|\delta E_x(\mathrm{M})|^2 + |\delta E_x(\mathrm{N})|^2}$$

$$|\delta E_x(\mathrm{M})|^2 = |\delta E_x(\mathrm{X})|^2 + |\delta E_x(\mathrm{Z})|^2 + |\delta E_x(\mathrm{C})|^2 + |\delta E_x(\theta)|^2$$

$$\Delta E_x^0(\boldsymbol{p}) = |E_x^{Obs}(\boldsymbol{p}) - E_x^0(\boldsymbol{x}, \omega)|$$

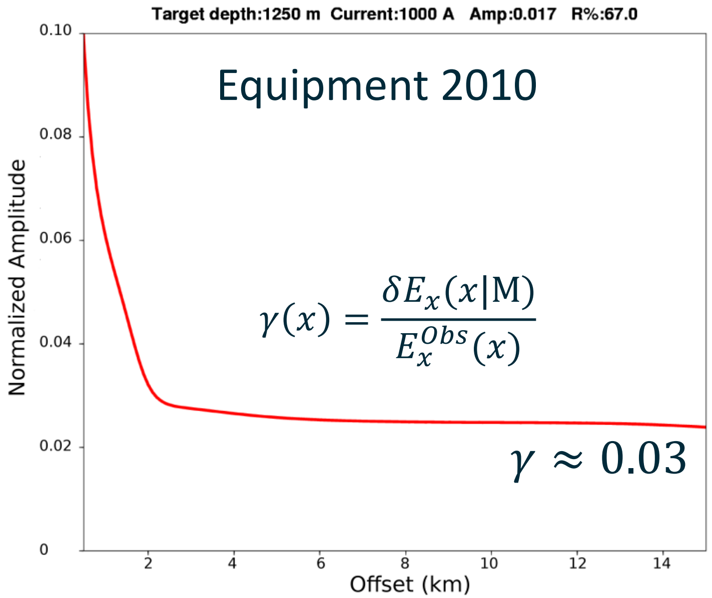
Let x here mean source-receiver offset

$$\gamma(x) = \frac{\delta E_x(x|\mathrm{M})}{E_x^{Obs}(x)}$$

Can write:

$$\delta E_x(x) = \sqrt{|\gamma(x) E_x^{Obs}(x)|^2 + \Delta N^2}$$

How does γ behave as a function of source-receiver offset?

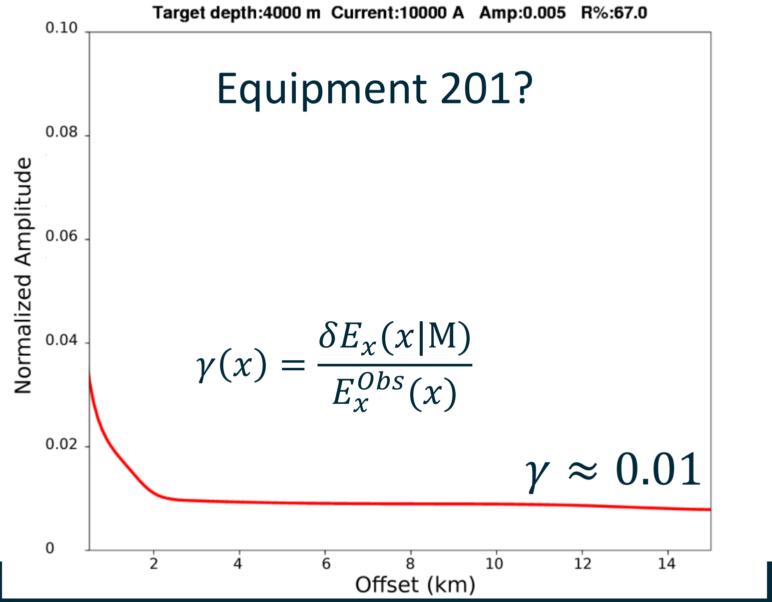


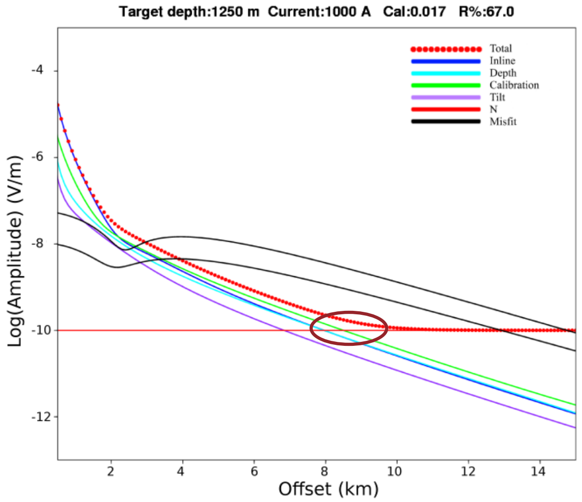
Quick and dirty estimate of uncertainty:

$$\delta E_x(x) \approx \sqrt{\gamma^2 |E_x^{Obs}(x)|^2 + \Delta N^2}$$

Even dirtier estimate of uncertainty:

$$\delta E_x(x) \approx \gamma |E_x^{Obs}(x)| + \Delta N$$

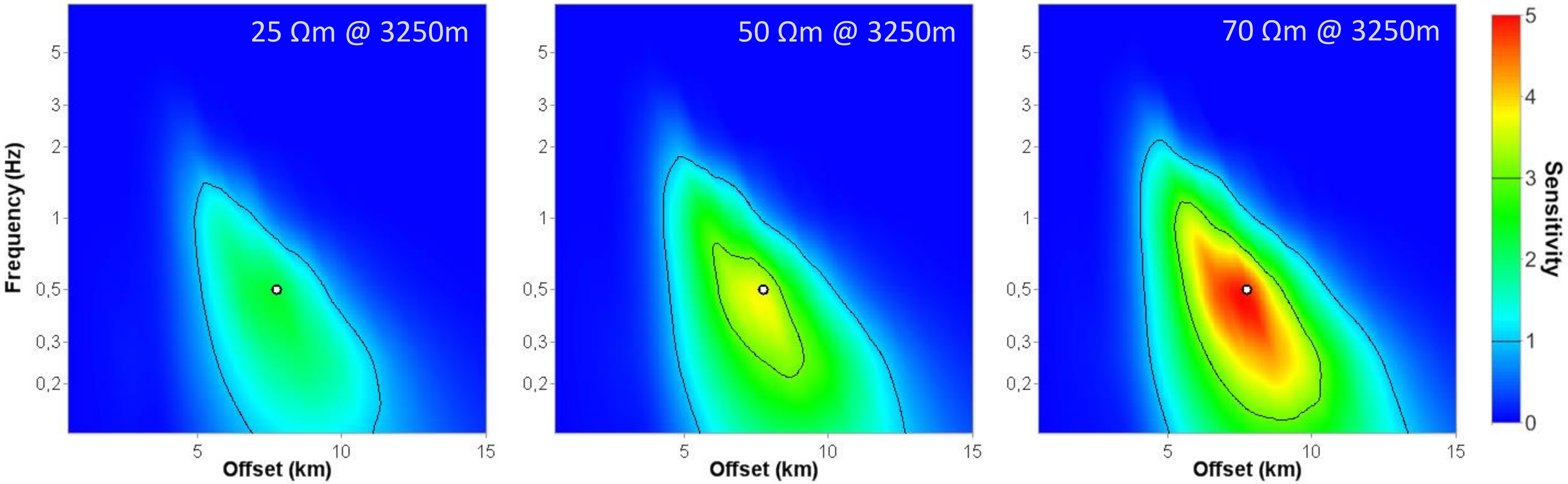




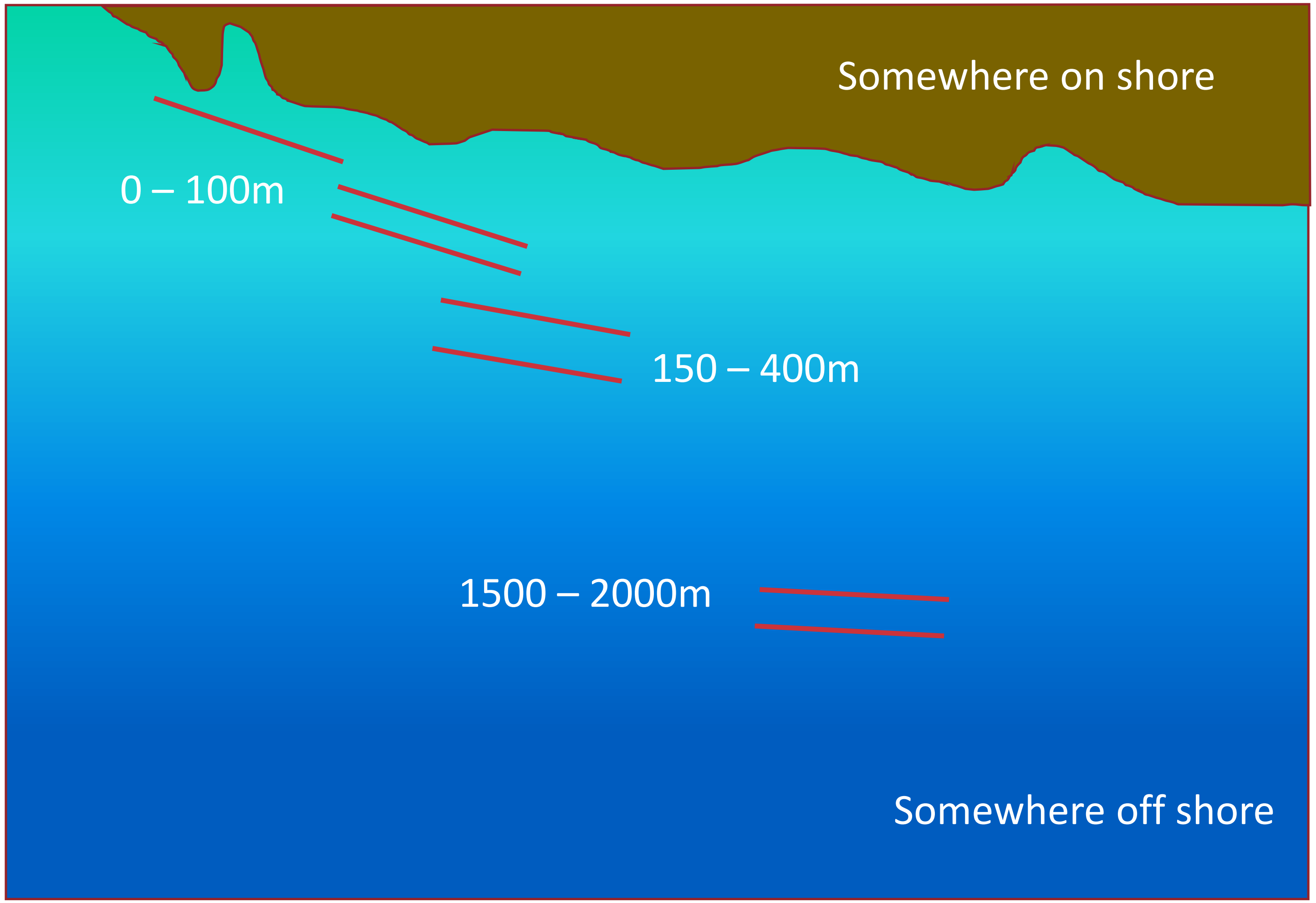
Frequency vs. Offset

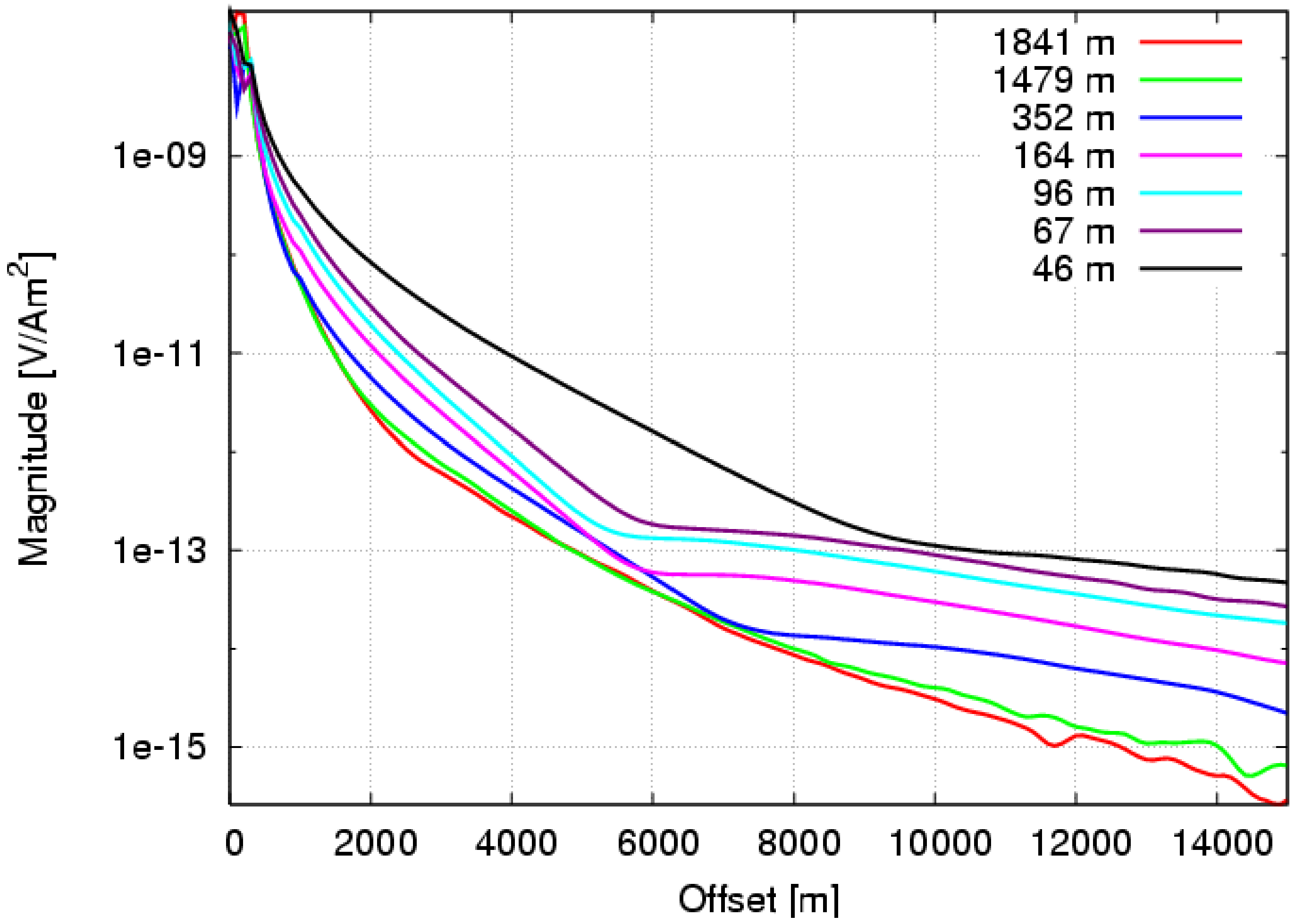
In order to find the best fitting range of frequencies for a survey it is important to find a waveform with frequencies at and around the peak sensitivity for a given target.

At the same time one should keep in mind that different frequencies have different penetration and resolution.

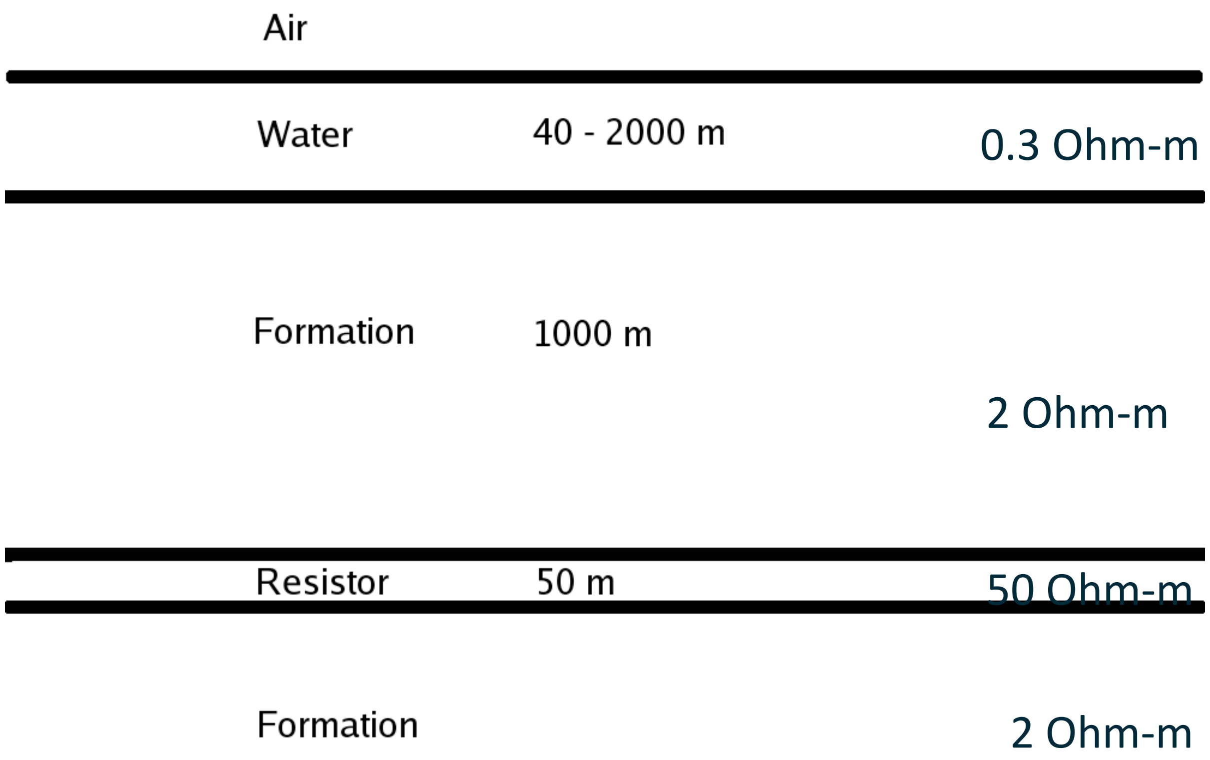


Shallow water

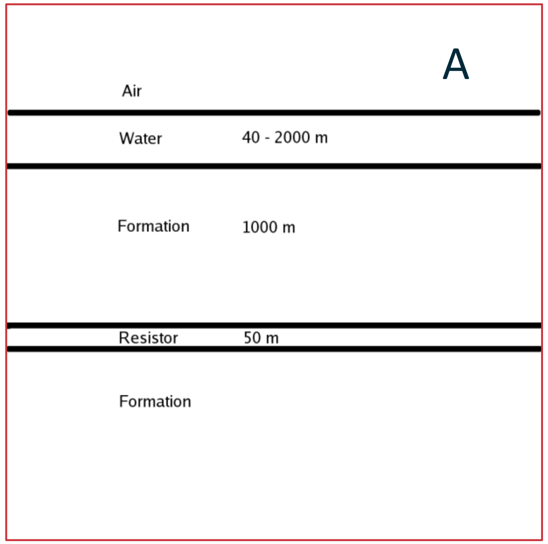

Somewhere on shore
0 – 100m
150 – 400m
1500 – 2000m
Somewhere off shore

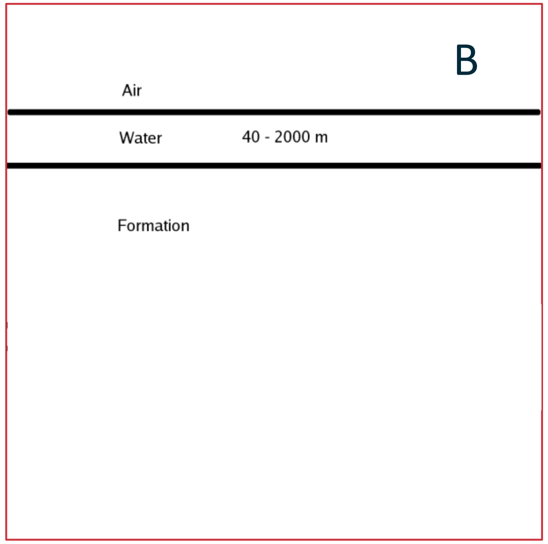


1841 m
1479 m
352 m
164 m
96 m
67 m
46 m
1e-09
1e-11
1e-13
1e-15
0
2000
4000
6000
8000
10000
12000
14000
Offset [m]
Magnitude [V/Am^2]



All examples are for 0.25 Hz
Results not particular for that frequency

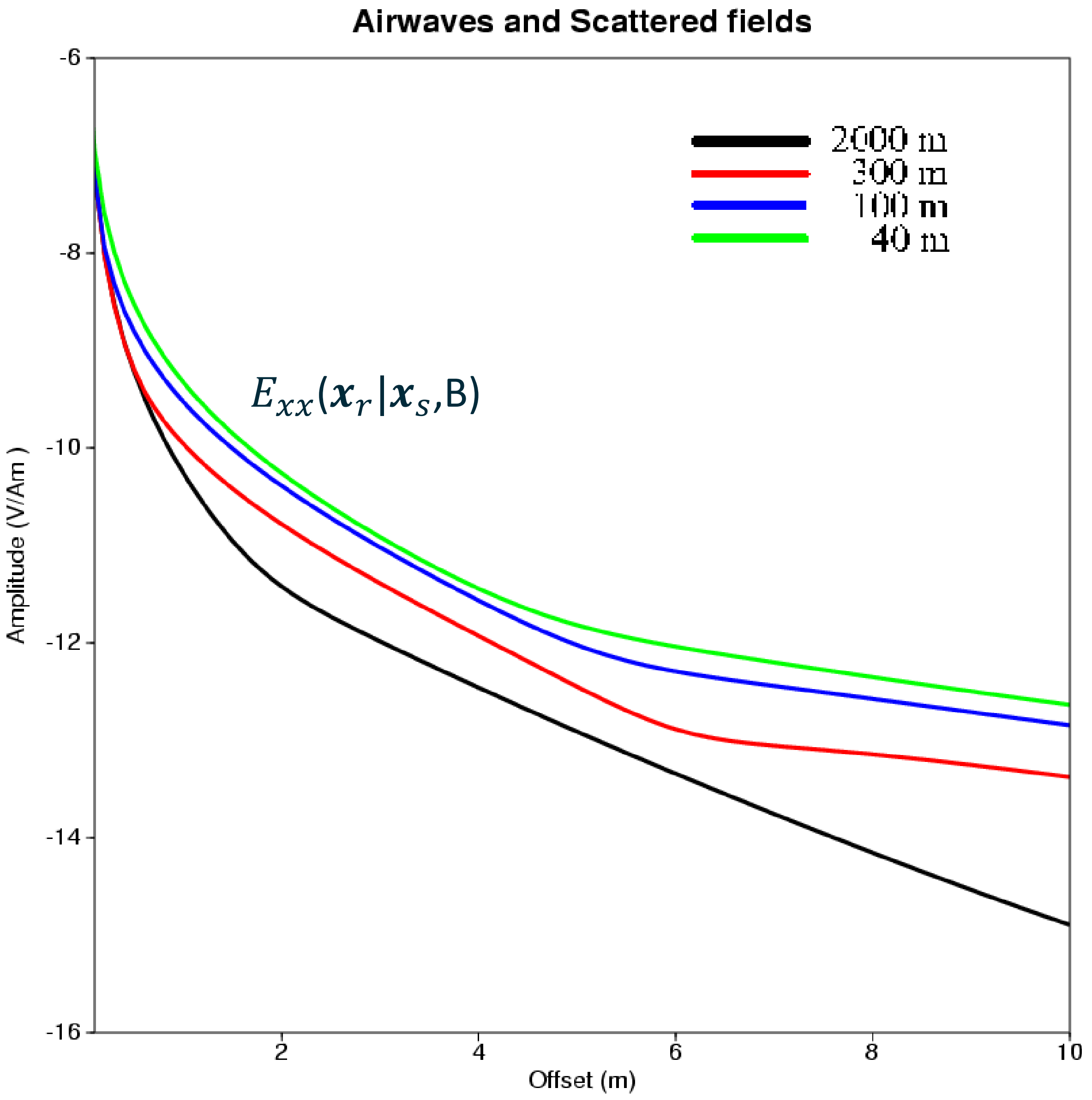


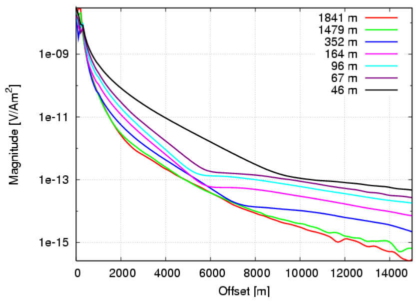


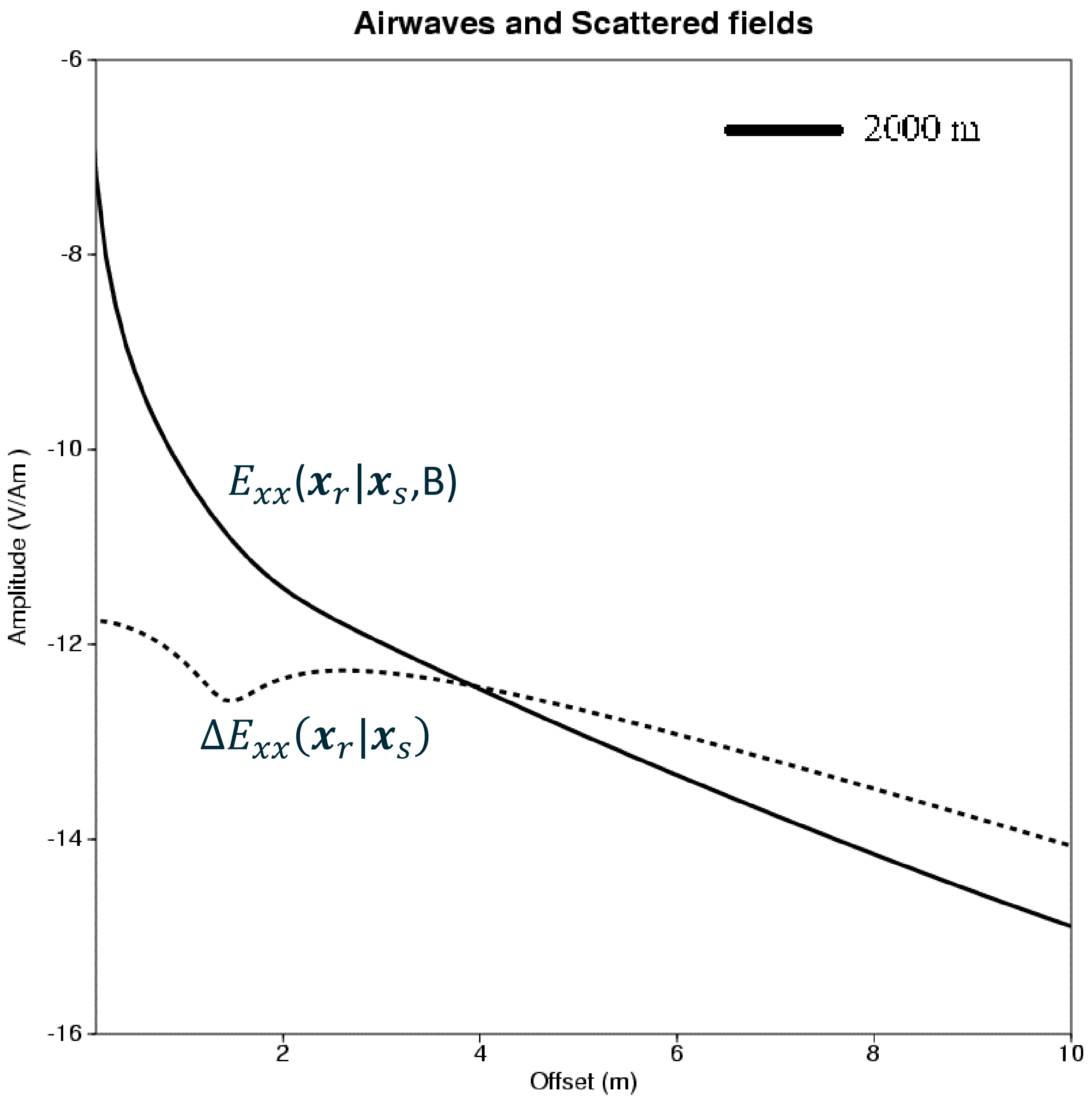
Full waveform modeling of the scattered field from a thin resistor

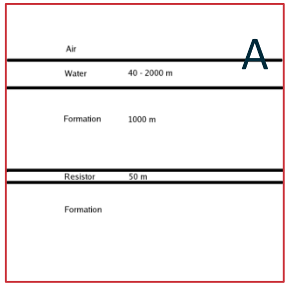
$$\Delta E_{xx}(\boldsymbol{x}_r|\boldsymbol{x}_s) = E_{xx}(\boldsymbol{x}_r|\boldsymbol{x}_s,\mathrm{A}) - E_{xx}(\boldsymbol{x}_r|\boldsymbol{x}_s,\mathrm{B})$$

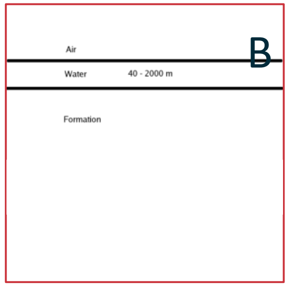
$$E_{xx}(\boldsymbol{x}_r|\boldsymbol{x}_s,\mathrm{A}) = E_{xx}(\boldsymbol{x}_r|\boldsymbol{x}_s,\mathrm{B}) + \Delta E_{xx}(\boldsymbol{x}_r|\boldsymbol{x}_s)$$

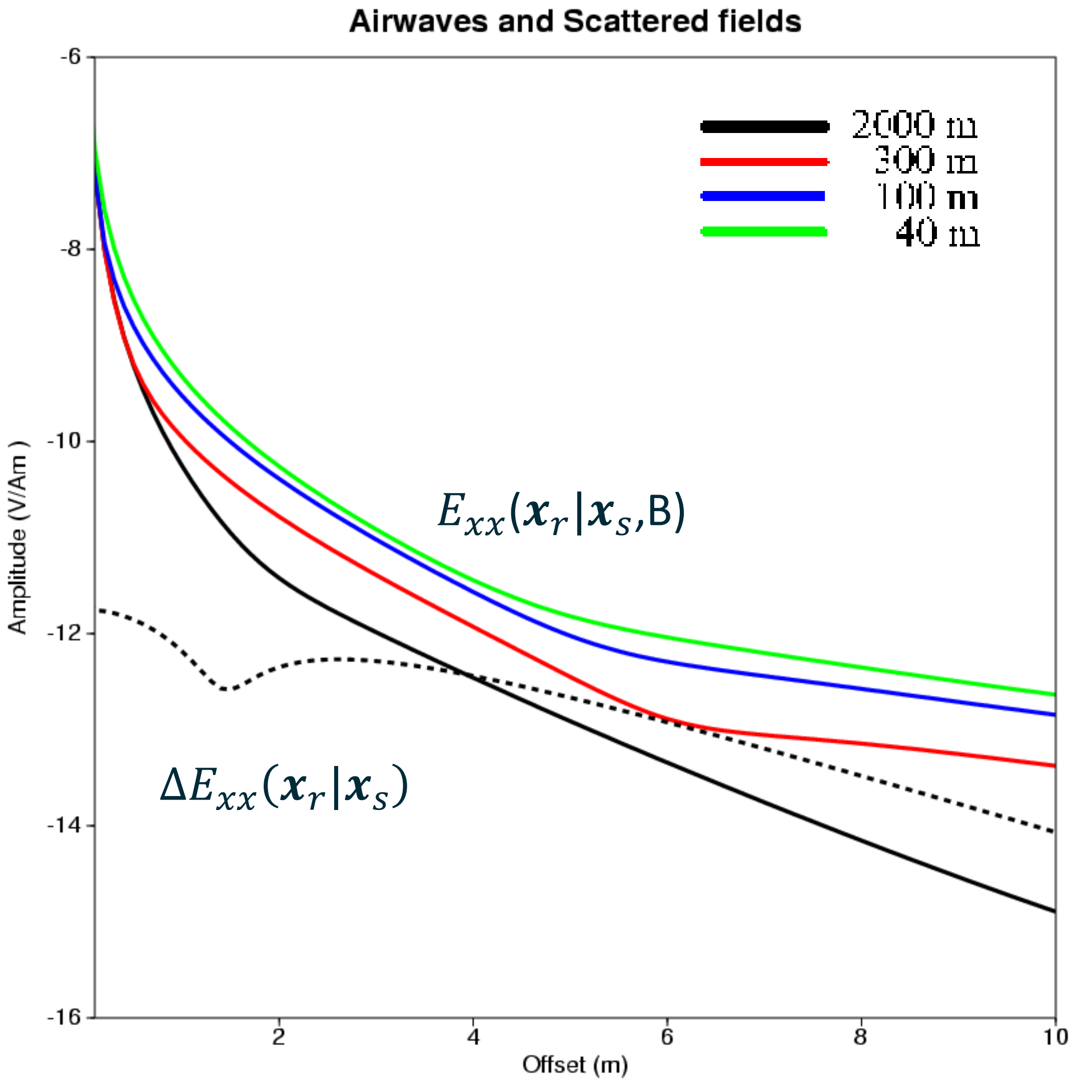

Airwaves and Scattered fields
2000 m
300 m
100 m
40 m
$E_{xx}(\boldsymbol{x}_r|\boldsymbol{x}_s,\mathrm{B})$
Amplitude (V/Am)
-6
-8
-10
-12
-14
-16
2
4
6
8
10
Offset (m)


1841 m
1479 m
352 m
164 m
96 m
67 m
46 m
1e-09
1e-11
1e-13
1e-15
Magnitude [V/Am^2]
0
2000
4000
6000
8000
10000
12000
14000
Offset [m]

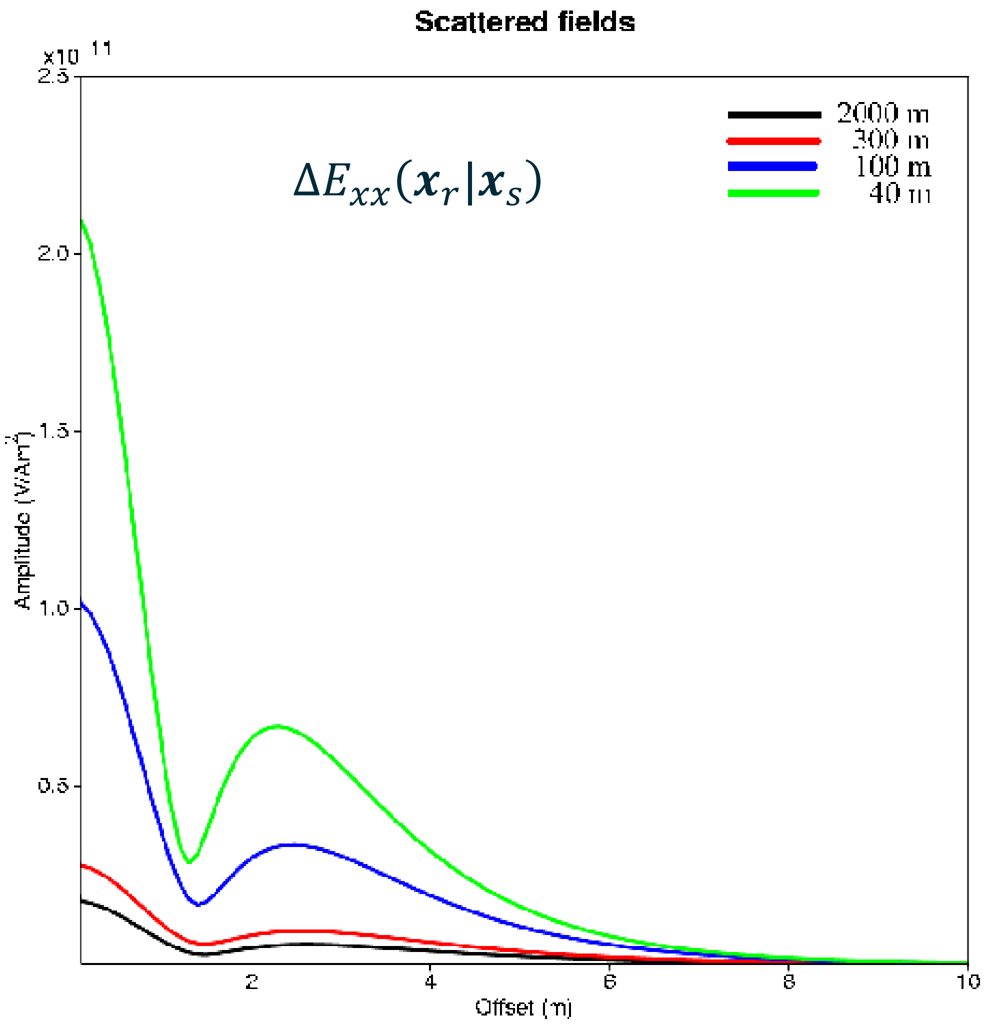

Airwaves and Scattered fields
2000 m
$E_{xx}(\boldsymbol{x}_r|\boldsymbol{x}_s, B)$
$\Delta E_{xx}(\boldsymbol{x}_r|\boldsymbol{x}_s)$
Amplitude (V/Am)
Offset (m)
-6
-8
-10
-12
-14
-16
2
4
6
8
10


A
Air
Water
40 - 2000 m
Formation
1000 m
Resistor
50 m
Formation

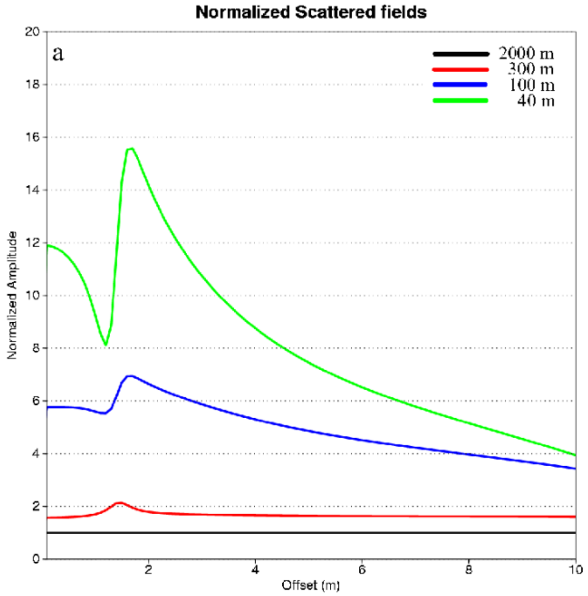

B
Air
Water
40 - 2000 m
Formation



Shallow water CSEM very difficult if scattered field same amplitude for all waterdepths

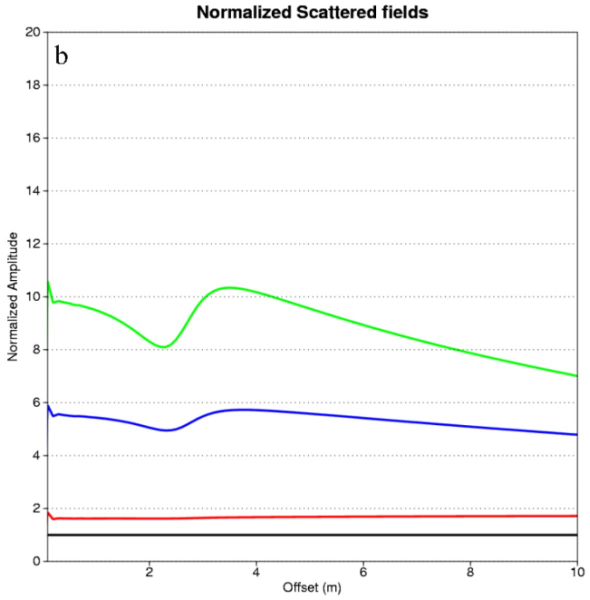


The amplitude of the scattered field increase significantly in waterdepths less than 300 m



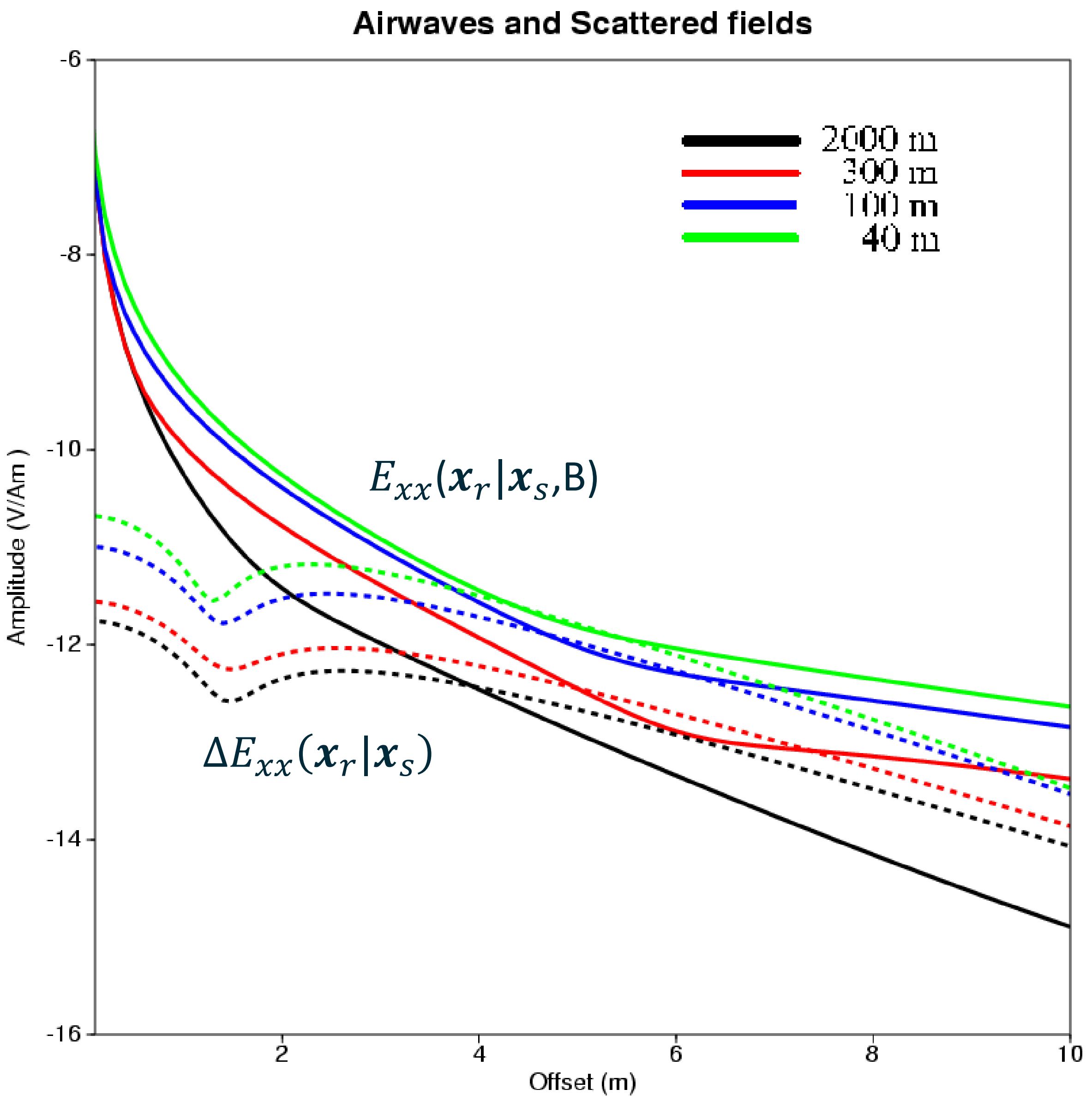
Scattered fields normalized on the 2000 m waterdepth case

Resistor burial depth 1000 m



Resistor burial depth 3000 m

Enhanced scattered-field effect is not restricted to a particular burial depth or frequency



Scattered field of same magnitude as background field for a fairly large offset interval.

Marine CSEM in shallow water feasible.

Magnitude of airwave increase as waterdepth is reduced

The response from a thin resistive layer increase as waterdepth is reduced

The increase in the response from a thin resistive layer is sufficiently strong to make marine CSEM in shallow water feasible

Mittet and Morten 2012:
Error propagation analysis to estimate uncertainty in observation

$$E_{xx}^{Obs}(\boldsymbol{x}_r|\boldsymbol{x}_s;\mathrm{L},\mathrm{J},\beta,N,\ldots.) \approx \; G_{xx}(\boldsymbol{x}_r|\boldsymbol{x}_s)\ \mathrm{LJ}\ \beta + \mathrm{N}$$

$$\delta E_{xx}^{Obs}(\boldsymbol{p}) = \sqrt{\sum_i \left|\frac{\partial E_{xx}^{Obs}(\boldsymbol{p})}{\partial p_i}\right|^2 |\Delta p_i|^2}$$

Contributions to uncertainty are both multiplicative and additive

For model used here we find that multplicative contributions approximately constant with offset (Offset > 2 km)

Simplified model for the uncertainty in the observed data:

$$\delta E_{xx}(\boldsymbol{x}_r|\boldsymbol{x}_s) = \sqrt{\alpha^2 |E_{xx}^{Obs}(\boldsymbol{x}_r|\boldsymbol{x}_s)|^2 + \eta^2}$$

Scattered (≈misfit) field from full waveform modeling:

$$\Delta E_{xx}(\boldsymbol{x}_r|\boldsymbol{x}_s) = E_{xx}(\boldsymbol{x}_r|\boldsymbol{x}_s,\text{A}) - E_{xx}(\boldsymbol{x}_r|\boldsymbol{x}_s,\text{B})$$

Uncertainty in the observed field:

$$\delta E_{xx}(\boldsymbol{x}_r|\boldsymbol{x}_s) = \sqrt{\alpha^2 |E_{xx}^{Obs}(\boldsymbol{x}_r|\boldsymbol{x}_s)|^2 + \eta^{\,2}}$$

L1 inversion kernel at first iteration:

$$\Psi(\boldsymbol{x}_r|\boldsymbol{x}_s) = \frac{|\Delta E_{xx}(\boldsymbol{x}_r|\boldsymbol{x}_s)|}{|\delta E_{xx}(\boldsymbol{x}_r|\boldsymbol{x}_s)|}$$

L2 inversion kernel at first iteration:

$$\Psi(\boldsymbol{x}_r|\boldsymbol{x}_s) = \frac{|\Delta E_{xx}(\boldsymbol{x}_r|\boldsymbol{x}_s)|^2}{|\delta E_{xx}(\boldsymbol{x}_r|\boldsymbol{x}_s)|^2}$$

(Tarantola, 1984)

Noise models

$$\delta E_{xx}(\boldsymbol{x}_r|\boldsymbol{x}_s) = \sqrt{\alpha^2|E_{xx}^{Obs}(\boldsymbol{x}_r|\boldsymbol{x}_s)|^2|) + \eta^{\,2}}$$

Based on error propagation analysis

$$\alpha = 0.03$$

Based on real data

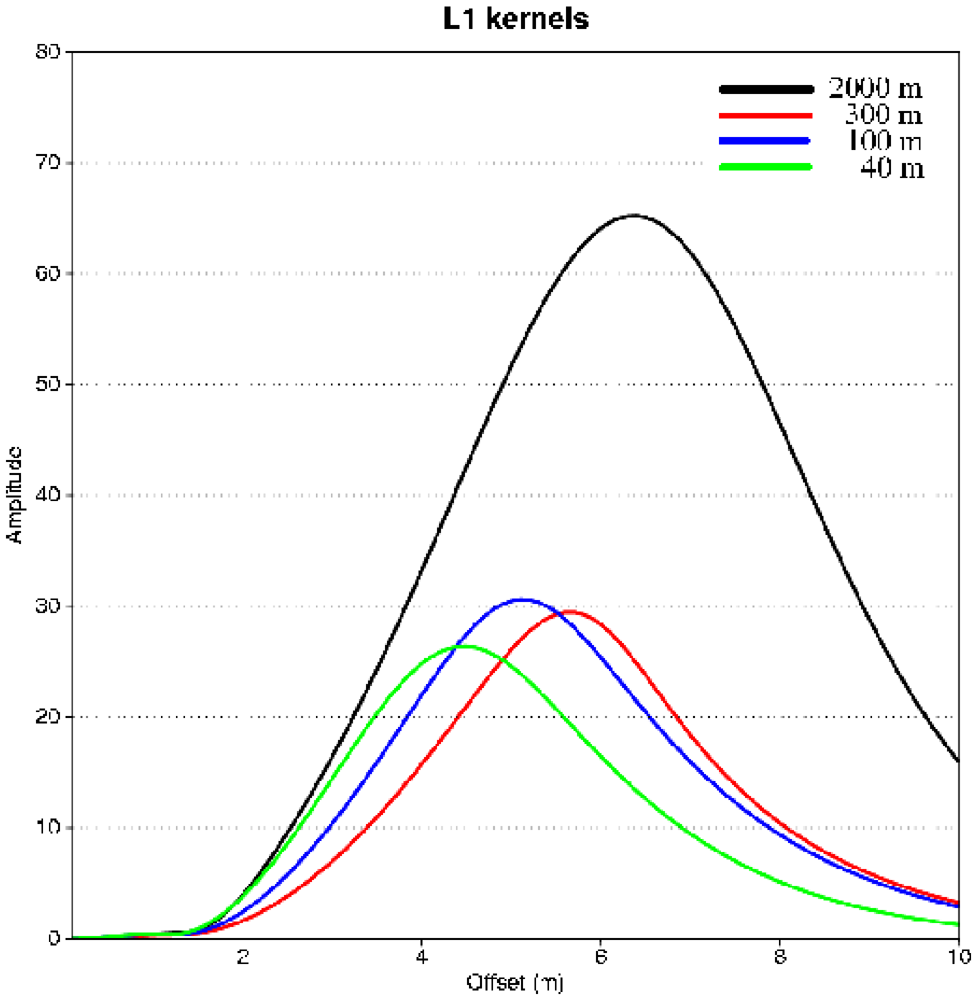
$$\eta(2000\ \mathrm{m}) = 5\ \times 10^{-16}\,\frac{\mathrm{V}}{\mathrm{Am}^2}$$

$$\eta(300\ \mathrm{m}) = 3\ \times 10^{-15}\,\frac{\mathrm{V}}{\mathrm{Am}^2}$$

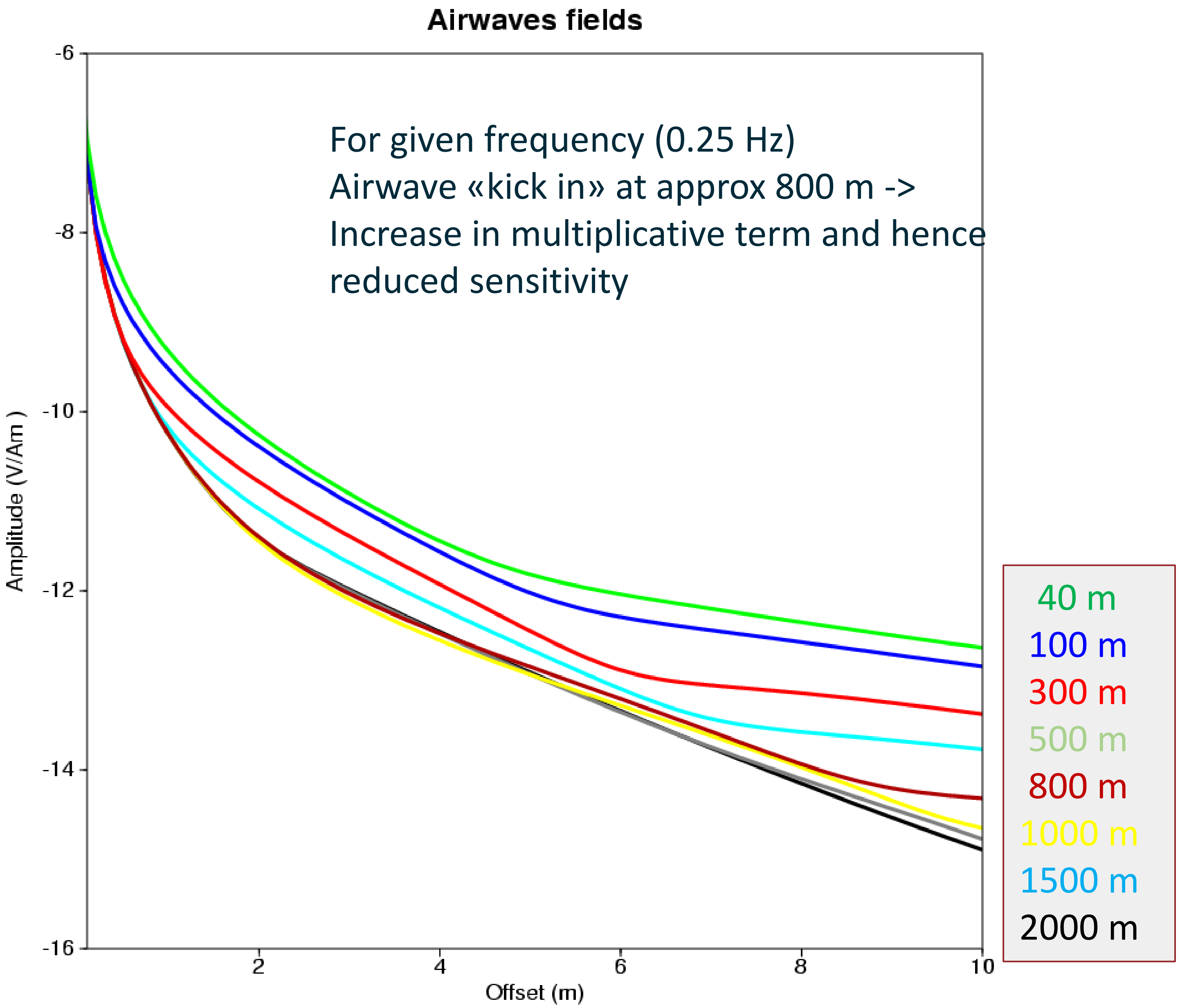
$$\eta(100\ \mathrm{m}) = 6\ \times 10^{-15}\,\frac{\mathrm{V}}{\mathrm{Am}^2}$$

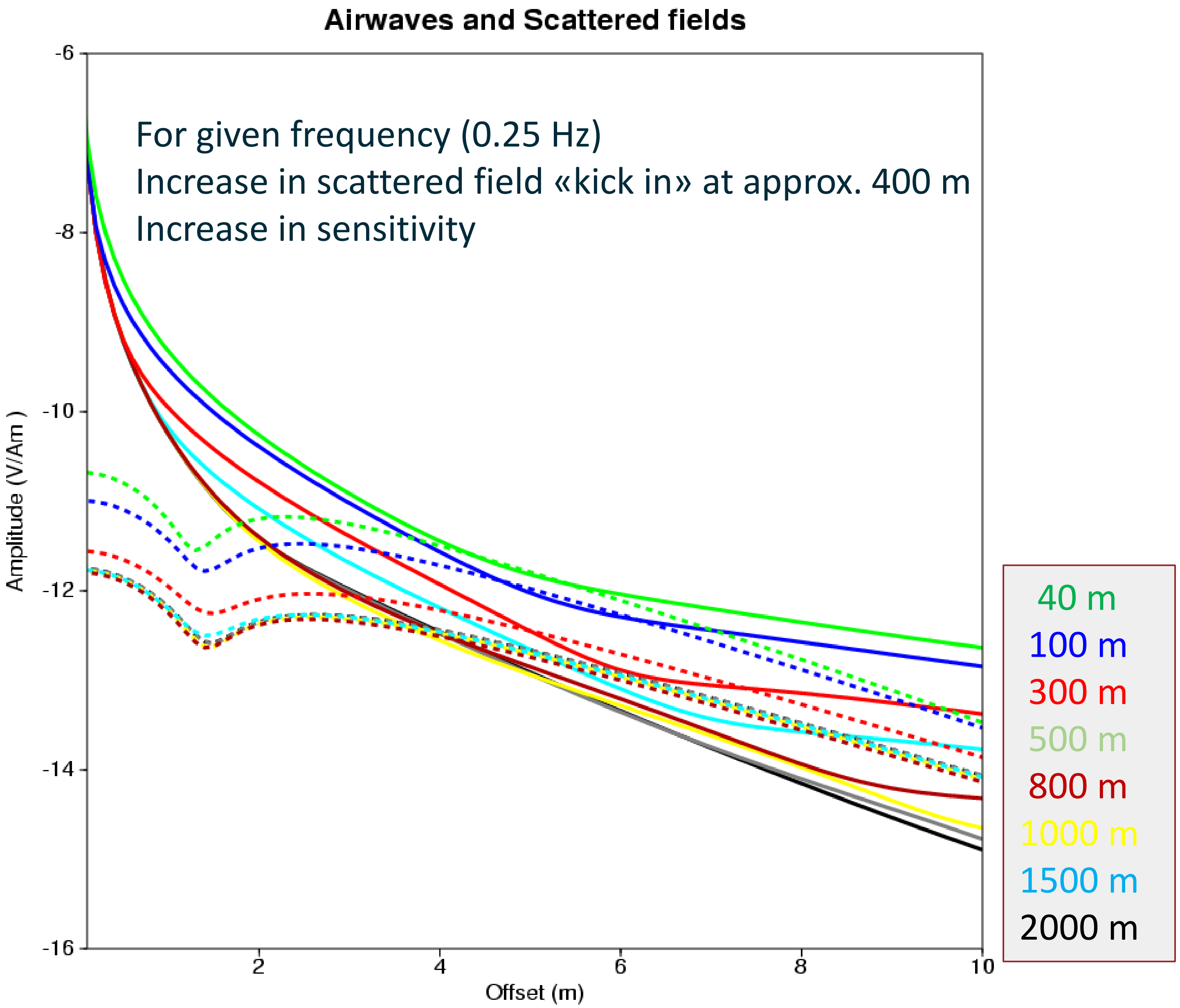
$$\eta(40\ \mathrm{m}) = 1.5\ \times 10^{-14}\,\frac{\mathrm{V}}{\mathrm{Am}^2}$$

L=270 m
I=1250 A

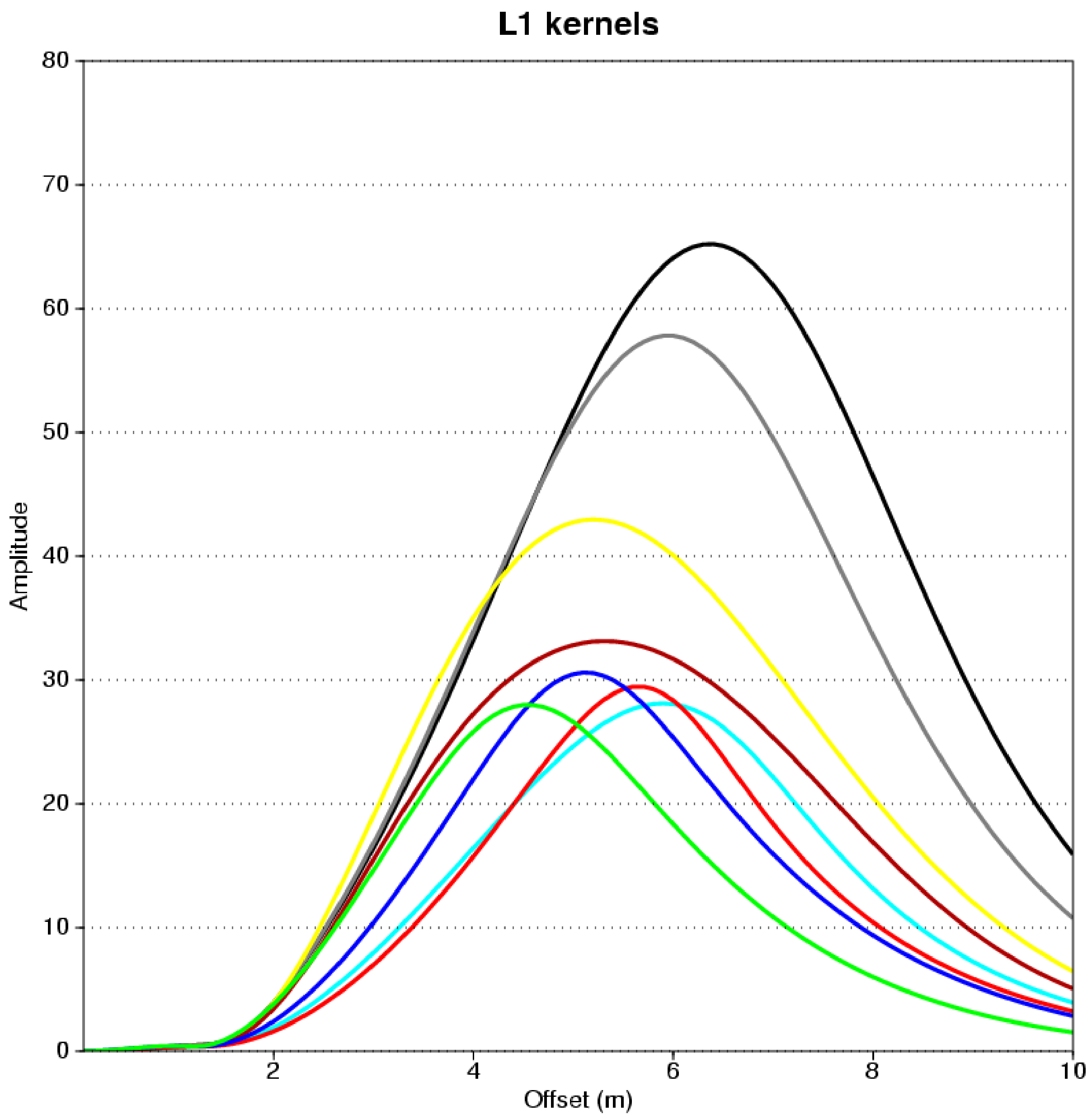


$$\Psi(\boldsymbol{x}_r|\boldsymbol{x}_s) = \frac{|\Delta E_{xx}(\boldsymbol{x}_r|\boldsymbol{x}_s)|}{|\delta E_{xx}(\boldsymbol{x}_r|\boldsymbol{x}_s)|}$$


Airwaves fields
-6
-8
-10
-12
-14
-16
Amplitude (V/Am)
2
4
6
8
10
Offset (m)
For given frequency (0.25 Hz)
Airwave «kick in» at approx 800 m ->
Increase in multiplicative term and hence
reduced sensitivity
40 m
100 m
300 m
500 m
800 m
1000 m
1500 m
2000 m



Airwaves and Scattered fields
For given frequency (0.25 Hz)
Increase in scattered field «kick in» at approx. 400 m
Increase in sensitivity
Amplitude (V/Am)
Offset (m)
-6
-8
-10
-12
-14
-16
2
4
6
8
10
40 m
100 m
300 m
500 m
800 m
1000 m
1500 m
2000 m



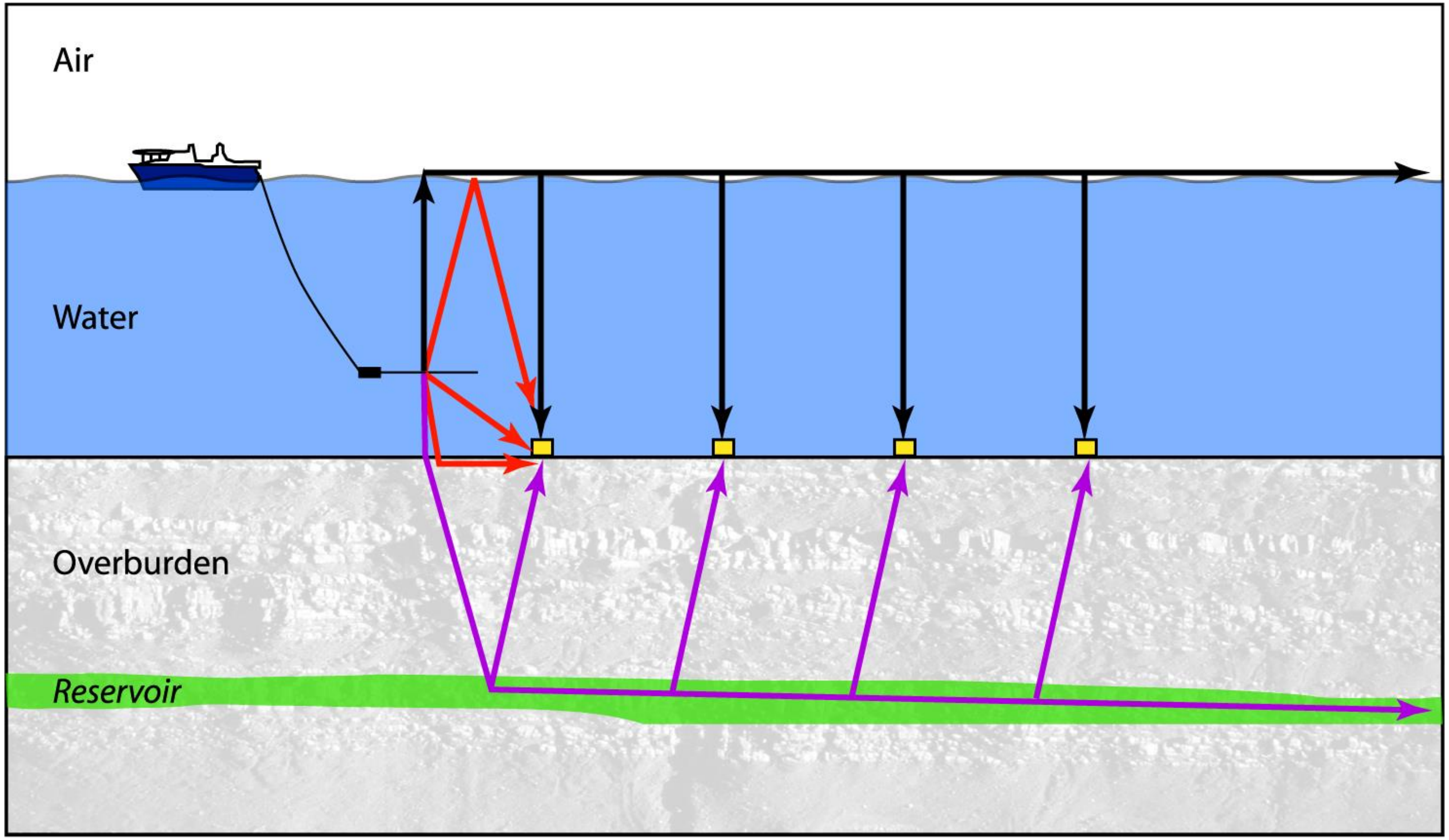
From 2000 m – 400 m:
Increase in airwave
and additive noise give
reduced sensitivity

From 400 m – 40 m :
Increase in scattered
field balance the airwave
effect

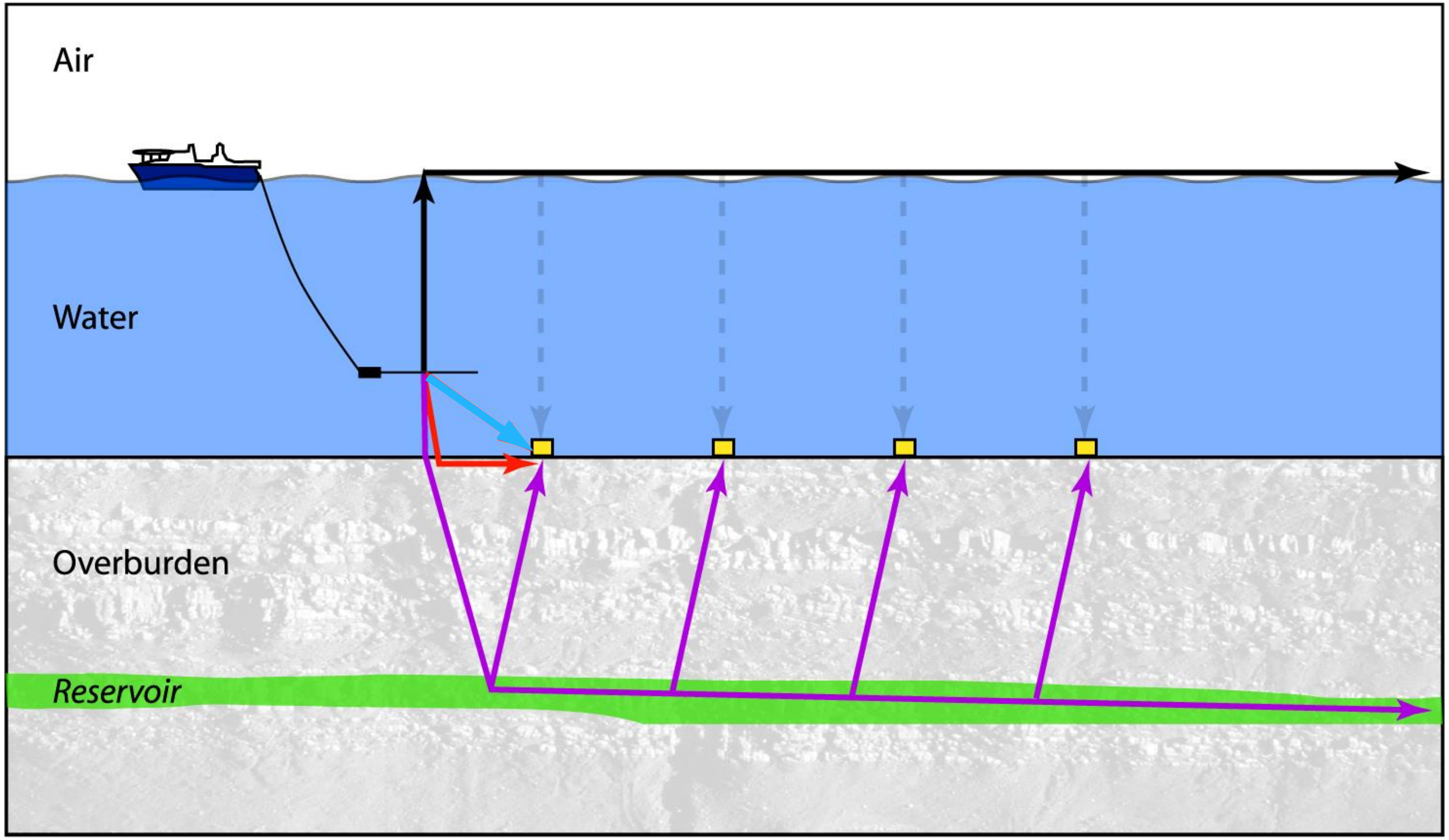
40 m
100 m
300 m
500 m
800 m
1000 m
1500 m
2000 m

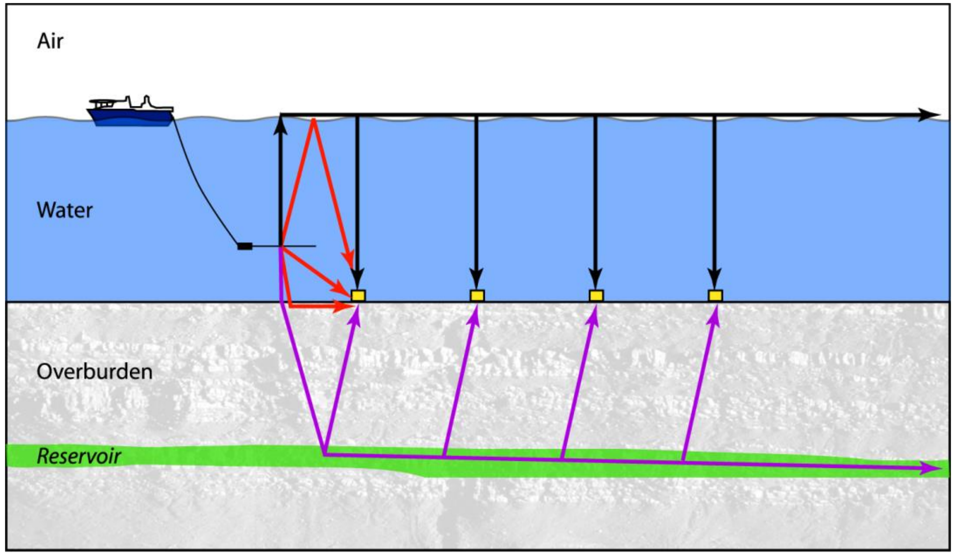
Up-down decomposition

Before up-down decomposition



After up-down decomposition





The purpose of U-D decomposition is to reduce the contribution from «large amplitude» downgoing field components like the airwave and MT fields

After decomposition further processing is performed on the upgoing field that has interacted with the subsurface

Maxwell equations for 1D MT

$$\begin{bmatrix} J_x^s + \sigma E_x \\ J_y^s + \sigma E_y \\ 0 + \sigma E_z \end{bmatrix} = \begin{bmatrix} -\partial_z H_y \\ \partial_z H_x \\ 0 \end{bmatrix} \qquad \begin{bmatrix} i\omega\mu_0 H_x \\ i\omega\mu_0 H_y \\ i\omega\mu_0 H_z \end{bmatrix} = \begin{bmatrix} -\partial_z E_y \\ \partial_z E_x \\ 0 \end{bmatrix}$$

Obtain two sets of equations that describe two different polarizations:

$$\partial_z H_y + \sigma E_x = -J_x^s \qquad \partial_z H_x + \sigma E_y = -J_y^s$$

$$\boxed{\partial_z E_x - i\omega\mu_0 H_y = 0} \qquad \partial_z E_y + i\omega\mu_0 H_x = 0$$

Equations for both polarizations :

$$\partial_z^2 E_x + \mathrm{i}\omega\mu_0\sigma E_x = -\,\mathrm{i}\omega\mu_0\, J_x^s \qquad \partial_z^2 E_y + \mathrm{i}\omega\mu_0\sigma E_y = -\,\mathrm{i}\omega\mu_0\, J_y^s$$

Sufficient to concentrate on x-polarization to understand the physics.

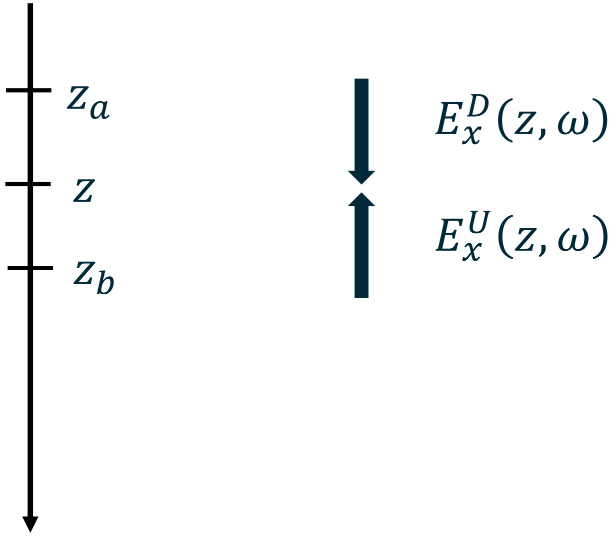
$$\partial_z E_x - i\omega\mu_0 H_y = 0$$

$$k_\omega^2 = i\omega\mu_0\sigma$$

$$E_x^D(z,\omega) = E_x(z_a,\omega)e^{ik_\omega\,(z-z_a)}$$

$$E_x^U(z,\omega) = E_x(z_b,\omega)e^{i\,k_\omega(z_b-z)}$$

$$k_\omega = \sqrt{i\omega\mu_0\sigma}$$



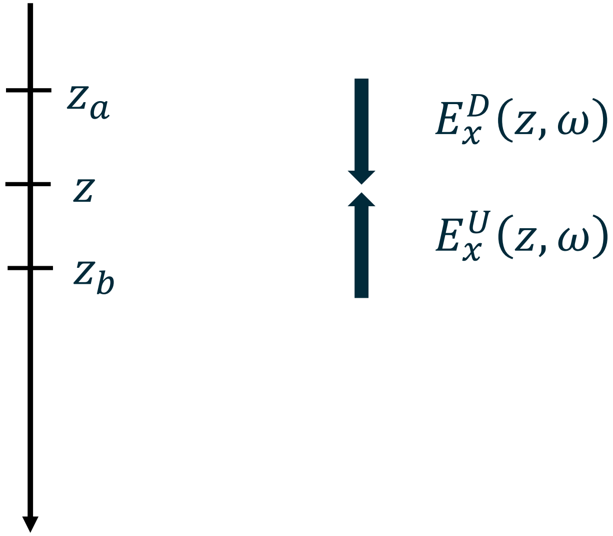
$$\partial_z E_x - i\omega\mu_0 H_y = 0$$

$$k_\omega^2 = i\omega\mu_0\sigma$$

$$E_x^D(z,\omega) = E_x(z_a,\omega)e^{ik_\omega(z-z_a)}$$

$$E_x^U(z,\omega) = E_x(z_b,\omega)e^{i\,k_\omega(z_b-z)}$$

$$k_\omega = \sqrt{i\omega\mu_0\sigma}$$



Have in general: $\partial_z E_x = i\omega\mu_0 H_y$

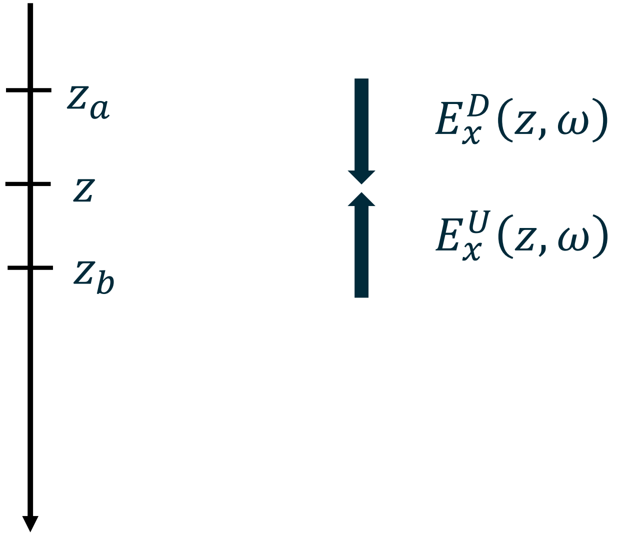
$$\partial_z E_x \text{-} i\omega\mu_0 H_y = 0$$

$$k_\omega^2 = i\omega\mu_0\sigma$$

$$E_x^D(z,\omega) = E_x(z_a,\omega)e^{ik_\omega\,(z-z_a)}$$

$$E_x^U(z,\omega) = E_x(z_b,\omega)e^{i\,k_\omega(z_b-z)}$$

$$k_\omega = \sqrt{i\omega\mu_0\sigma}$$



Have in general: $\partial_z E_x = i\omega\mu_0 H_y$

Assume downgoing field only at z:

$$\partial_z E_x^D(z,\omega) = i\omega\mu_0 H_y^D(z,\omega)$$

$$E_x^D(z,\omega) = \frac{\omega\mu_0}{k_\omega} H_y^D(z,\omega)$$

$$E_x^D(z,\omega) = \frac{\omega\mu_0}{\sqrt{i\omega\mu_0\sigma}} H_y^D(z,\omega)$$

$$E_x^D(z,\omega) = Z H_y^D(z,\omega)$$

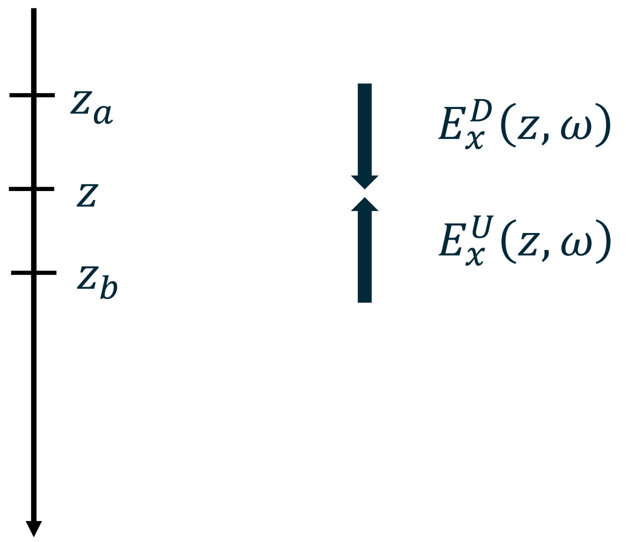
$$\partial_z E_x \text{-} i\omega\mu_0 H_y = 0$$

$$k_\omega^2 = i\omega\mu_0\sigma$$

$$E_x^D(z,\omega) = E_x(z_a,\omega)e^{ik_\omega(z-z_a)}$$

$$E_x^U(z,\omega) = E_x(z_b,\omega)e^{i\,k_\omega(z_b-z)}$$

$$k_\omega = \sqrt{i\omega\mu_0\sigma}$$



Have in general: $\partial_z E_x = i\omega\mu_0 H_y$

Assume upgoing field only at z:

$$\partial_z E_x^U(z,\omega) = i\omega\mu_0 H_y^U(z,\omega)$$

$$E_x^U(z,\omega) = -\frac{\omega\mu_0}{k_\omega} H_y^U(z,\omega)$$

$$E_x^U(z,\omega) = -\frac{\omega\mu_0}{\sqrt{i\omega\mu_0\sigma}} H_y^U(z,\omega)$$

$$E_x^U(z,\omega) = -Z H_y^U(z,\omega)$$

Characteristic impedance:

$$Z = \frac{\omega\mu_0}{\sqrt{i\omega\mu_0\sigma}}$$

$$Z = \sqrt{-i\omega\mu_0\rho}$$

Characteristic impedance is a medium property and is completely determined by the medium resistivity (or conductivity).

Must not be confused by the expression for field impedance Z_{xy} used in MT processing:

$$Z_{xy} = \frac{E_x}{H_y}$$

We measure the total fields:

$$E_x(z,\omega) = E_x^D(z,\omega) + E_x^U(z,\omega)$$

$$H_y(z,\omega) = H_y^D(z,\omega) + H_y^U(z,\omega)$$

$$E_x^D(z,\omega) = ZH_y^D(z,\omega)$$

$$E_x^U(z,\omega) = -ZH_y^U(z,\omega)$$

$$E_x^D(z,\omega) = ZH_y^D(z,\omega) \longrightarrow E_x(z,\omega) - E_x^U(z,\omega) = ZH_y(z,\omega) - ZH_y^U(z,\omega)$$

$$E_x(z,\omega) - E_x^U(z,\omega) = ZH_y(z,\omega) + E_x^U(z,\omega)$$

Upgoing and downgoing fields are calculated from the measured fields:

$$E_x^U(z,\omega) = \frac{1}{2}[E_x(z,\omega) - ZH_y(z,\omega)]$$

$$E_x^D(z,\omega) = \frac{1}{2}[E_x(z,\omega) + ZH_y(z,\omega)]$$

Up/Down separation 3D

For vertically traveling field:

$$E_x^U(z,\omega) = \frac{1}{2}[E_x(z,\omega) - ZH_y(z,\omega)]$$

General solution:

$$E_x^U(z,\omega) = \frac{1}{2}[E_x(z,\omega) - Z\frac{(\ k_x k_y H_x(z,\omega) + (k_\omega^2 - k_x^2) H_y(z,\omega)\ )}{k_\omega \sqrt{k_\omega^2 - k_x^2 - k_y^2}}]$$

Vertical propagation $k_x = k_y = 0$

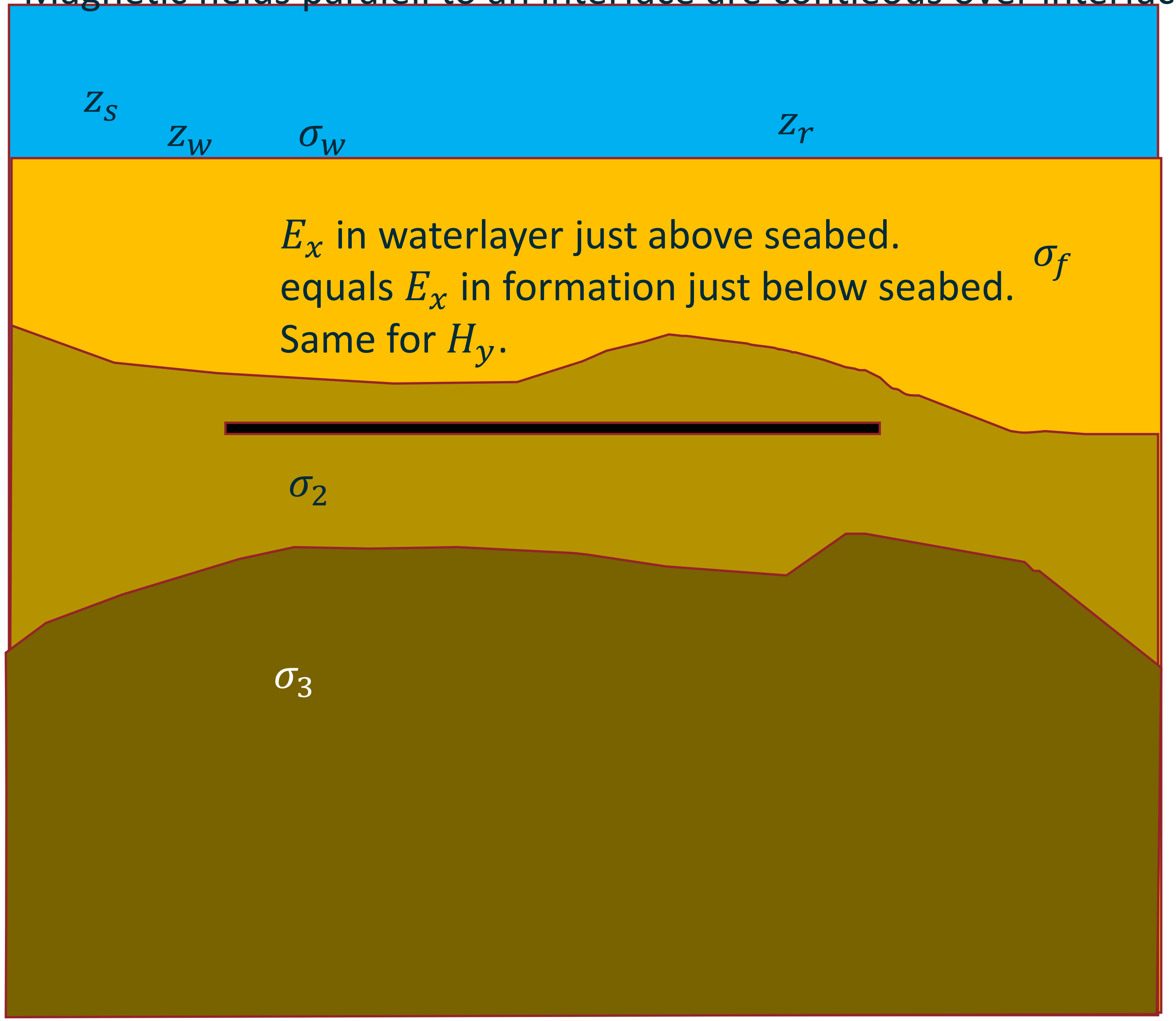
Practical Up-Down decomposition is performed with: $E_x^U(z,\omega) = \frac{1}{2}[E_x(z,\omega) - ZH_y(z,\omega)]$

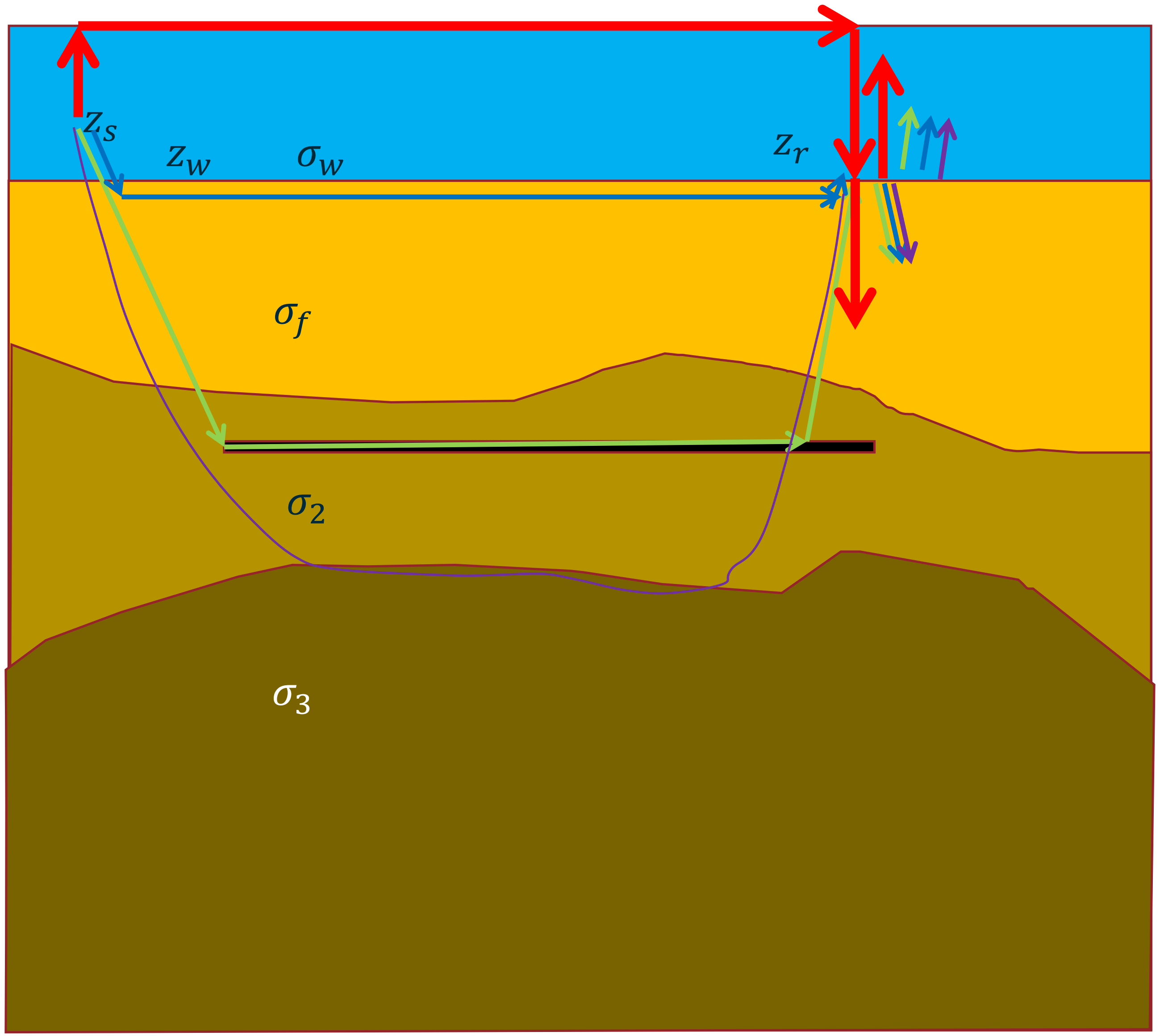
Electric fields paralell to an interface are contineous over interface.

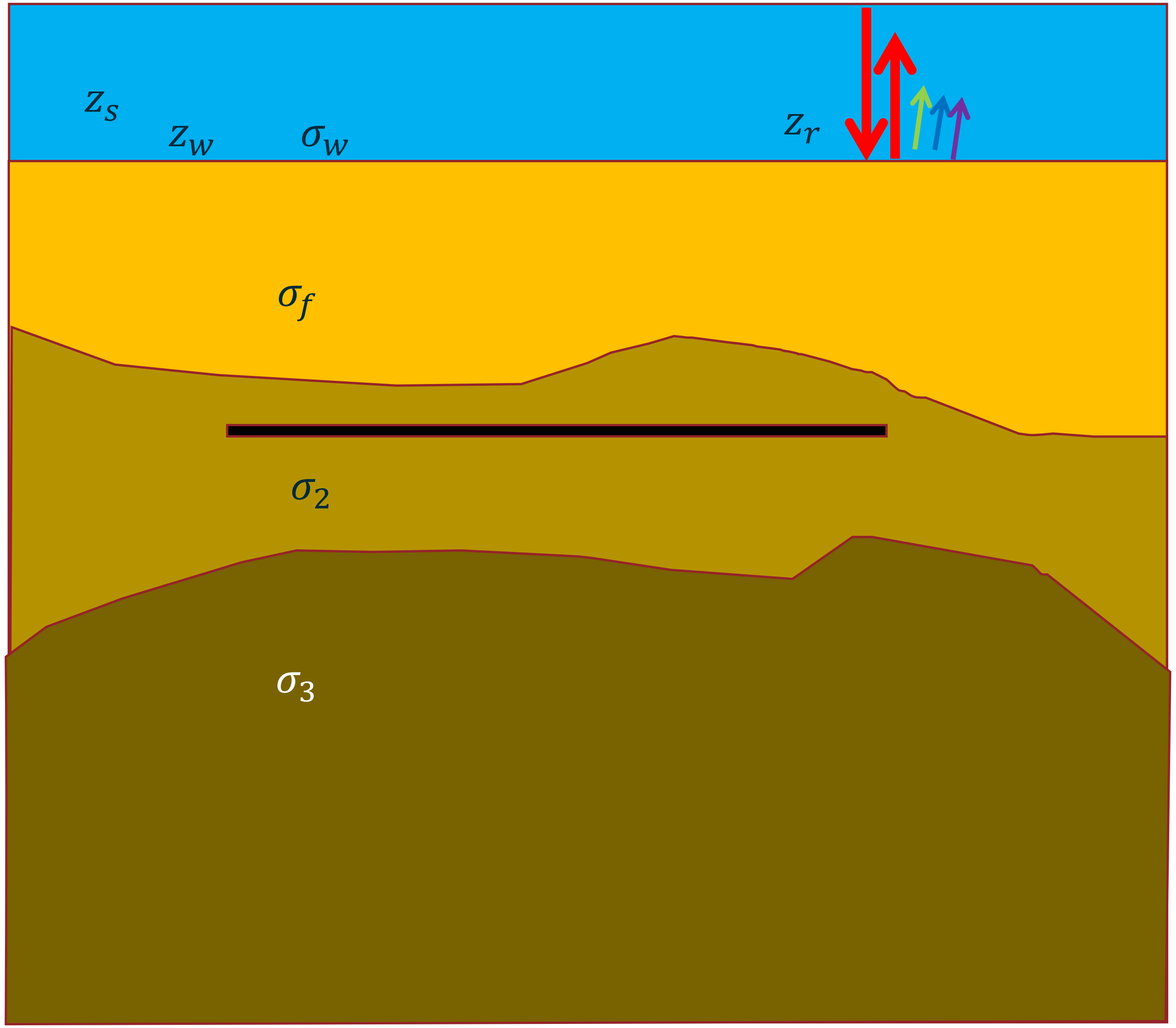
Current normal to interfaces is contineous

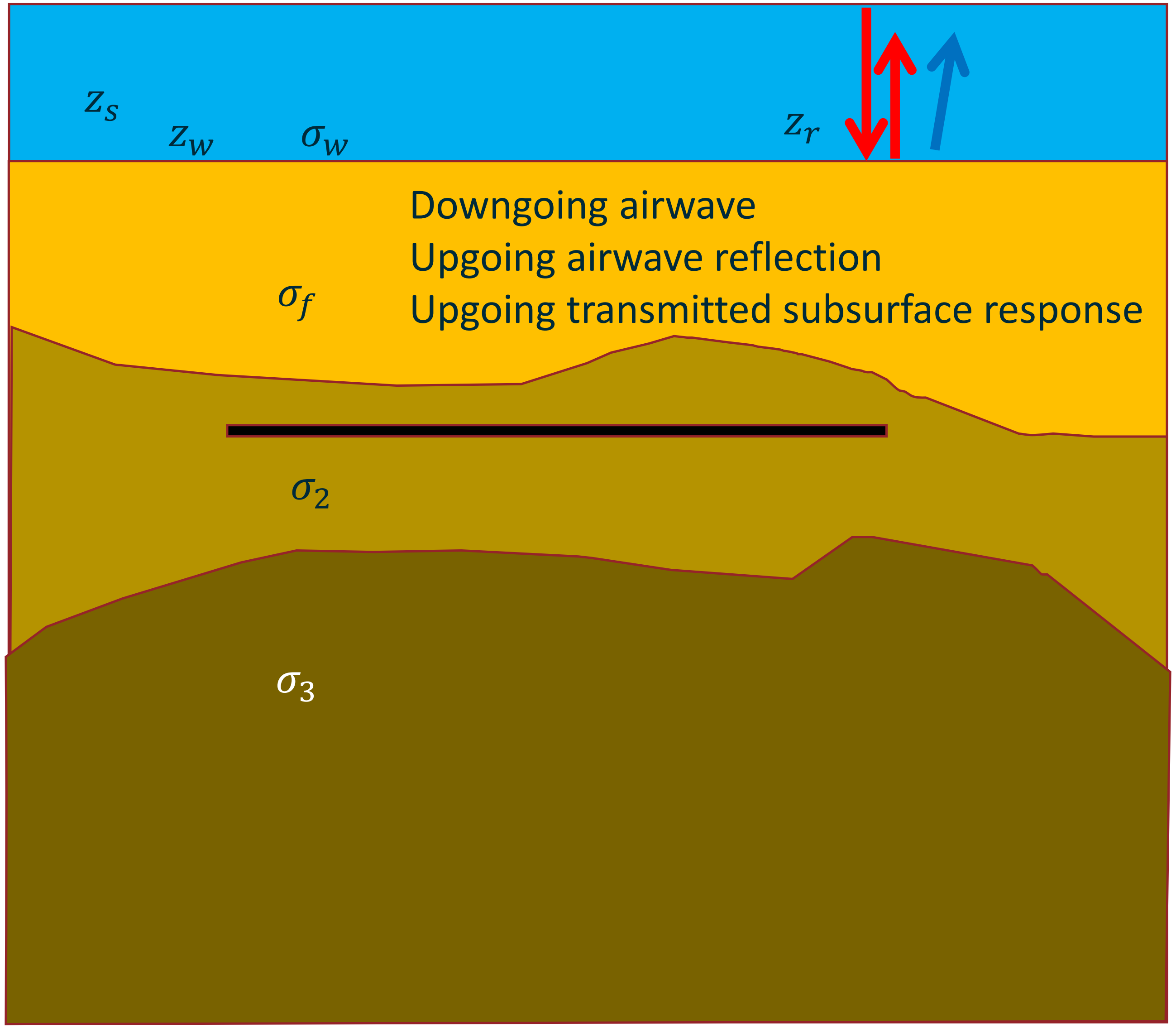
Magnetic fields continous if non-magnetic material

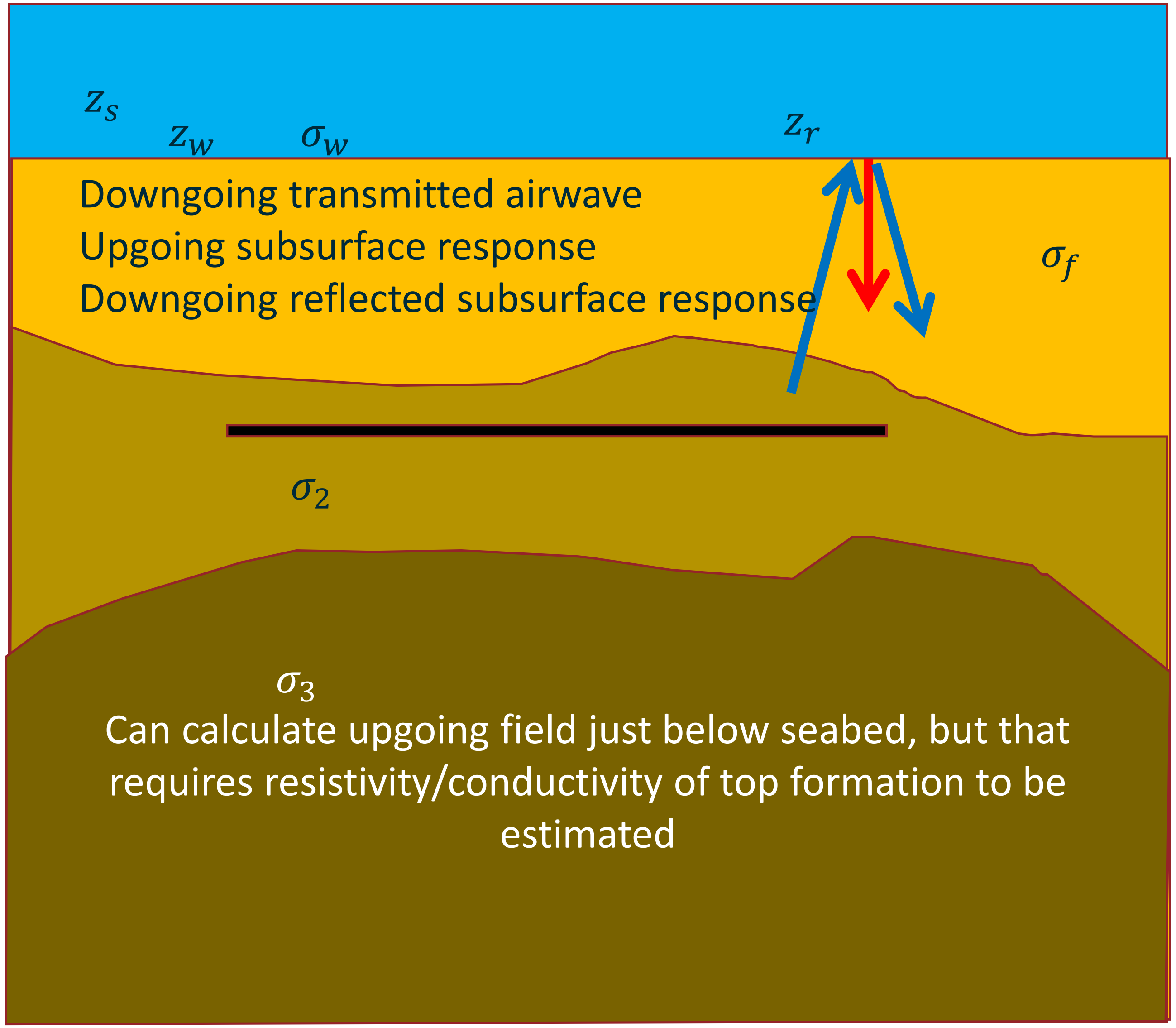
Electric fields paralell to an interface are contineous over interface.
Magnetic fields paralell to an interface are contieous over interface.




z_s
z_w
σ_w
z_r
σ_f
σ_2
σ_3

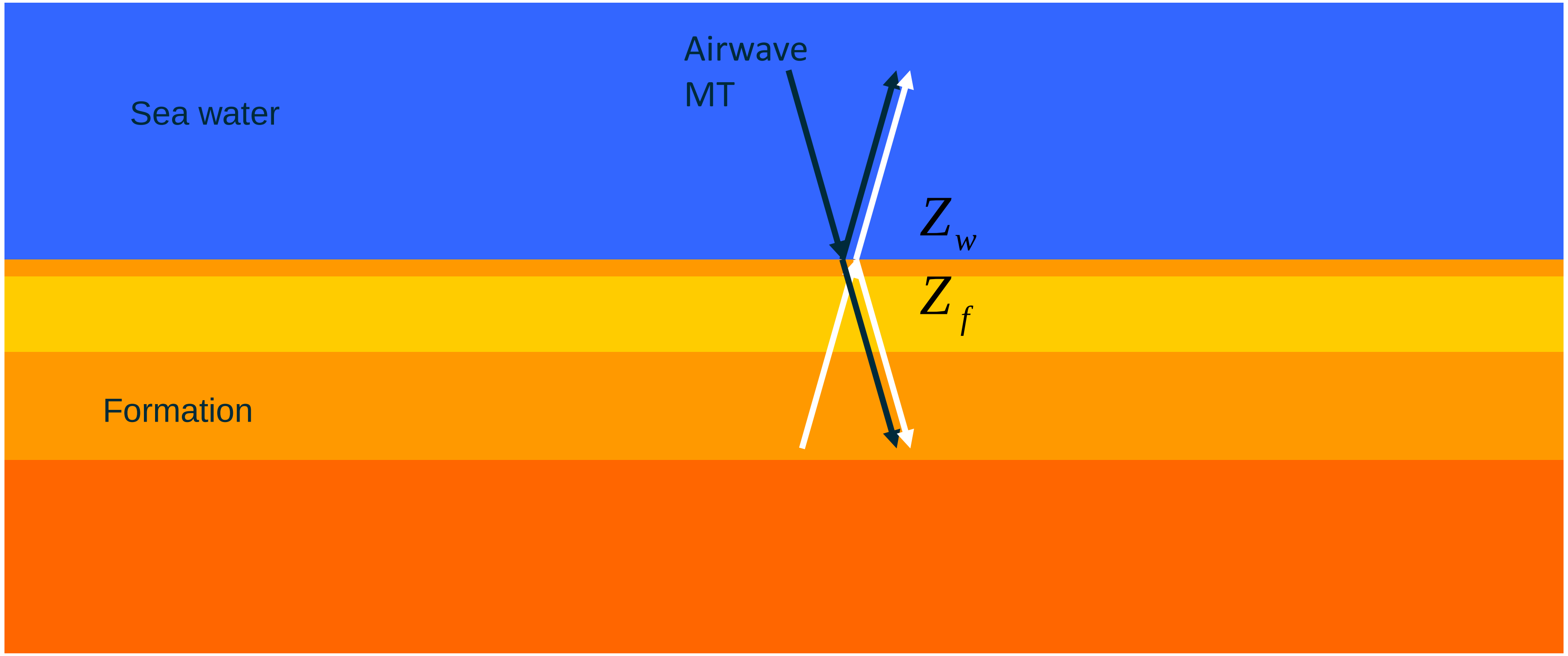

z_s
z_w
σ_w
z_r
σ_f
σ_2
σ_3

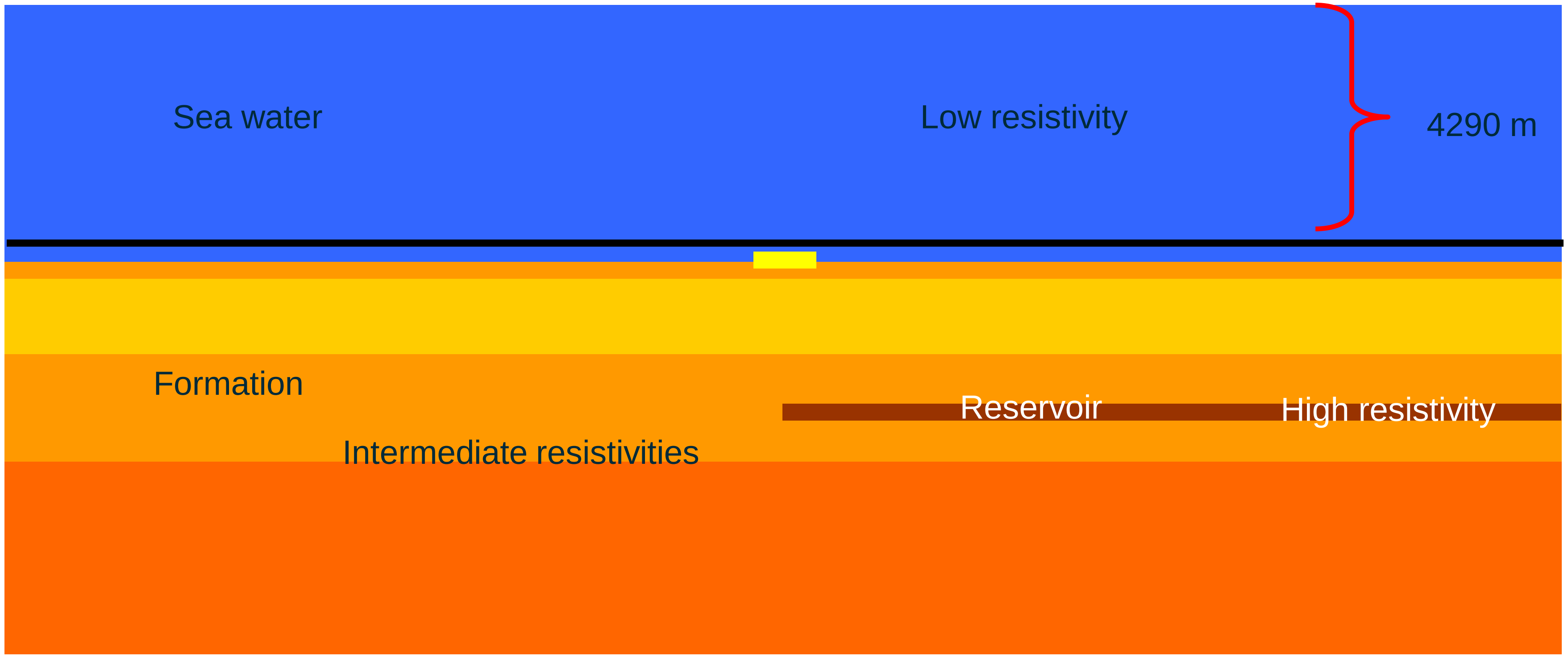

z_s
z_w
σ_w
z_r
Downgoing airwave
Upgoing airwave reflection
Upgoing transmitted subsurface response
σ_f
σ_2
σ_3

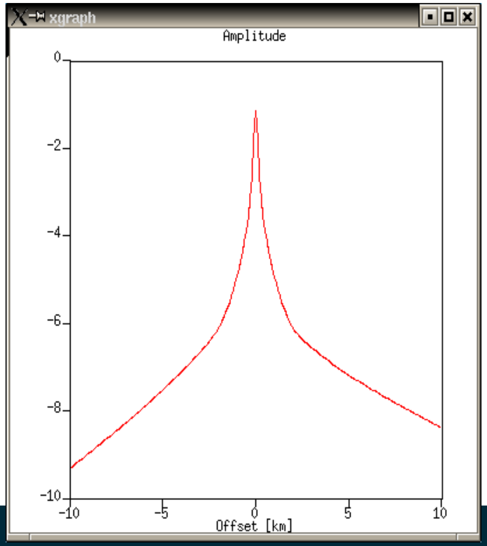

z_s
z_w
σ_w
z_r
Downgoing transmitted airwave
Upgoing subsurface response
Downgoing reflected subsurface response
σ_f
σ_2
σ_3
Can calculate upgoing field just below seabed, but that requires resistivity/conductivity of top formation to be estimated

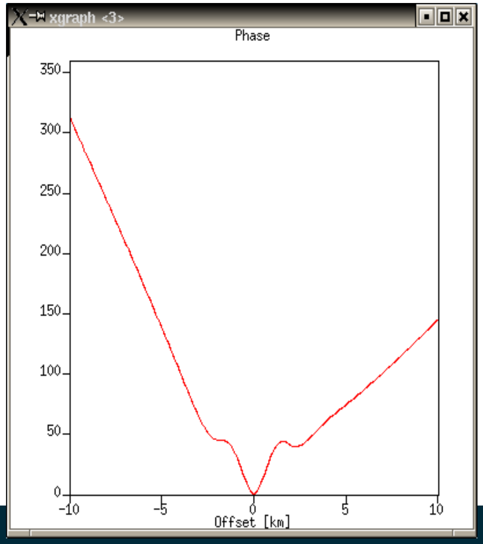
Up/Down separation

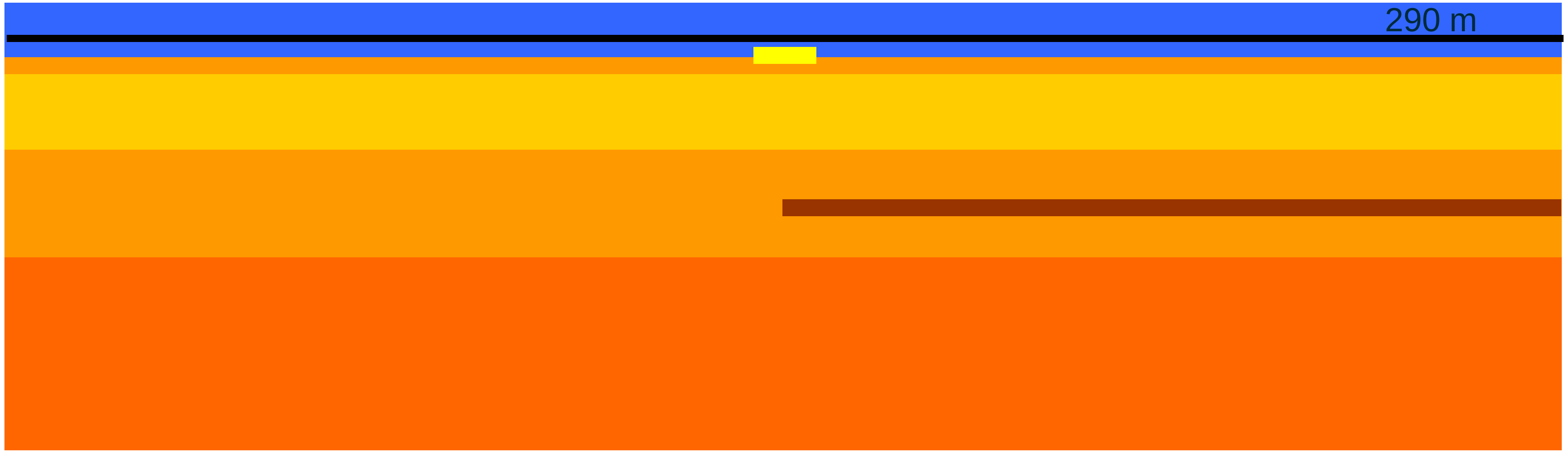
Separation above and below seabed

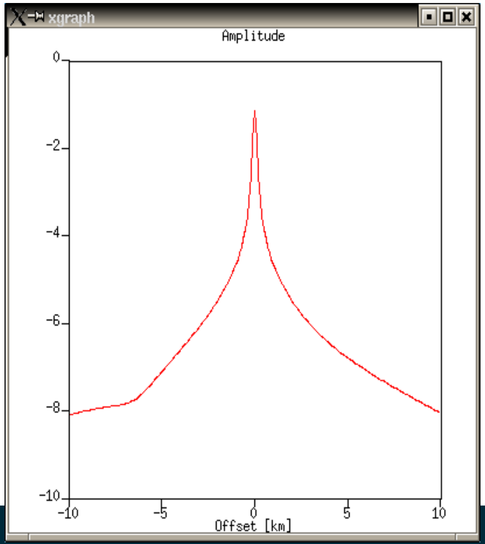


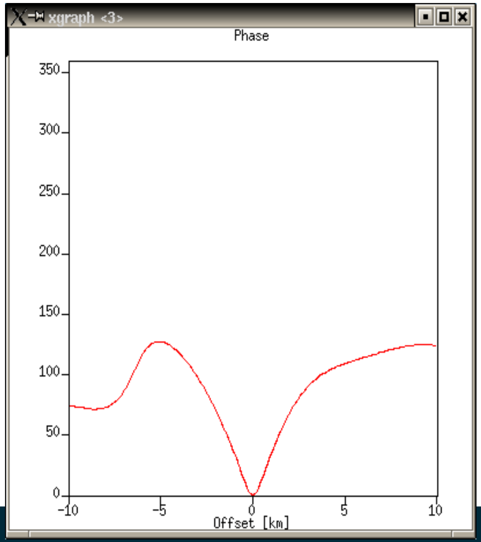

Sea water
Low resistivity
4290 m
Formation
Reservoir
High resistivity
Intermediate resistivities


xgraph
Amplitude
0
-2
-4
-6
-8
-10
-10
-5
0
5
10
Offset [km]


xgraph <3>
Phase
350
300
250
200
150
100
50
0
-10
-5
0
5
10
Offset [km]

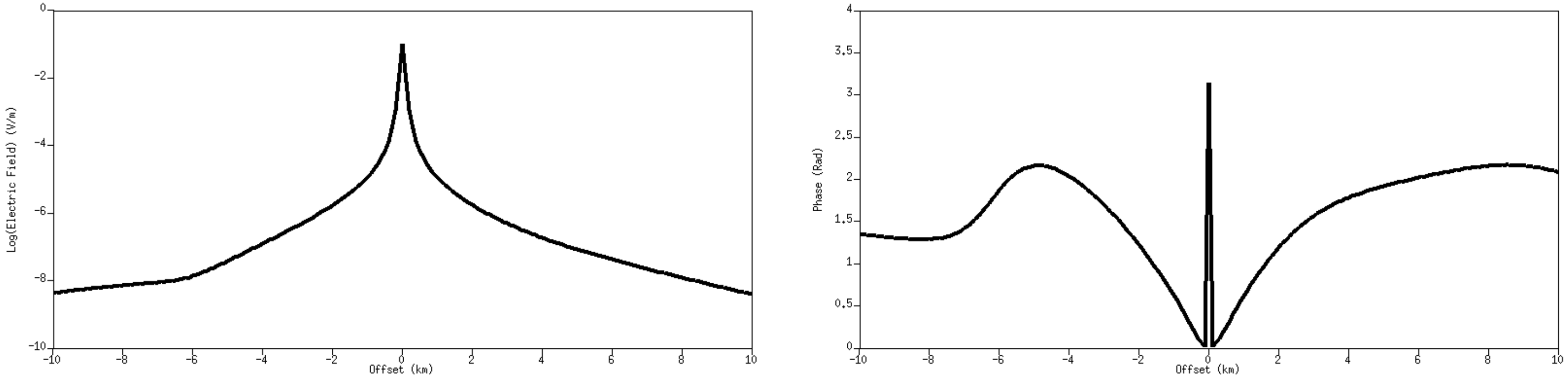

290 m


xgraph
Amplitude
0
-2
-4
-6
-8
-10
-10
-5
0
5
10
Offset [km]


xgraph <3>
Phase
350
300
250
200
150
100
50
0
-10
-5
0
5
10
Offset [km]

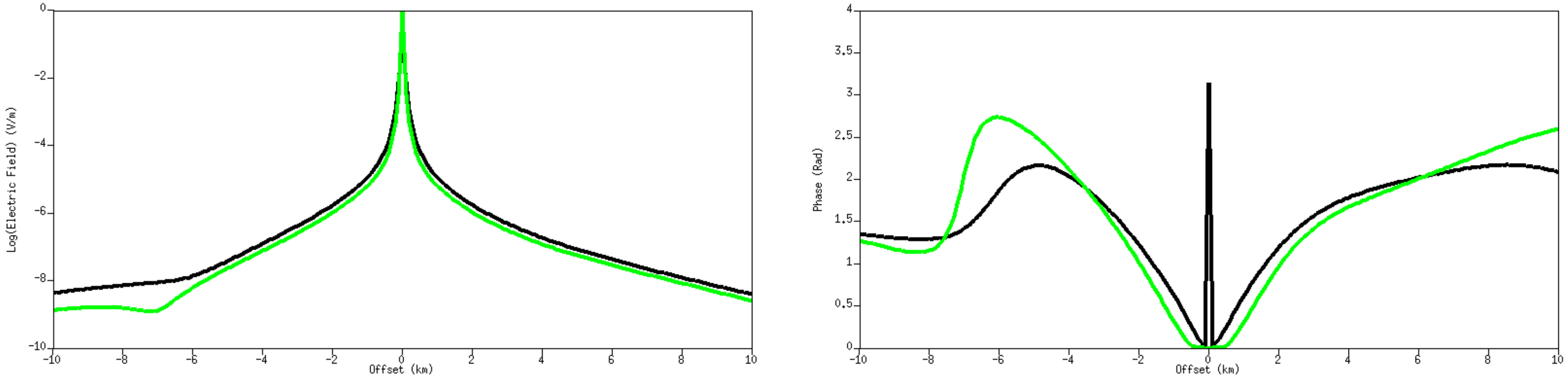
Up/Down separation

Measured field
Modeled upgoing
Calculated from measured field 3D
Calculated from measured field 1D above
Calculated from measured field 1D below



Up/Down separation

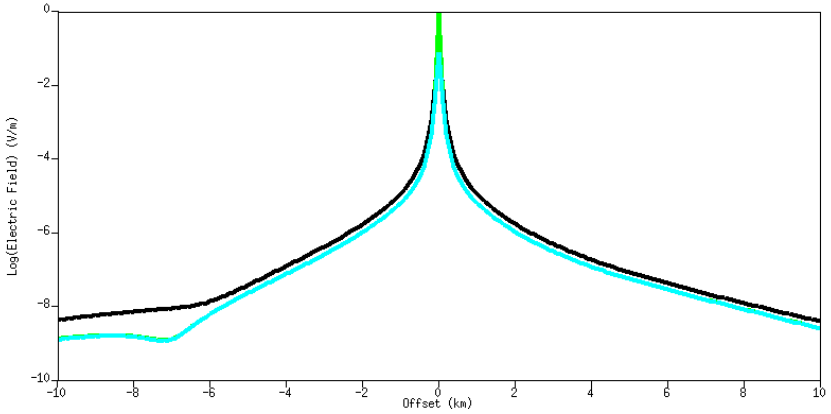
Measured field
Modeled upgoing
Calculated from measured field 3D
Calculated from measured field 1D above
Calculated from measured field 1D below

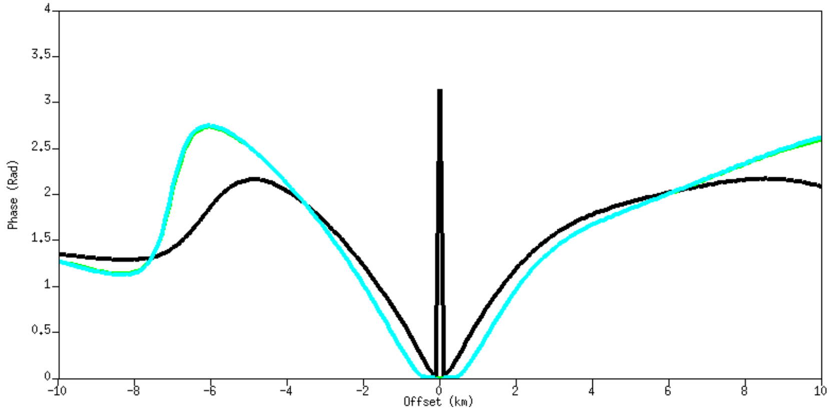


Up/Down separation

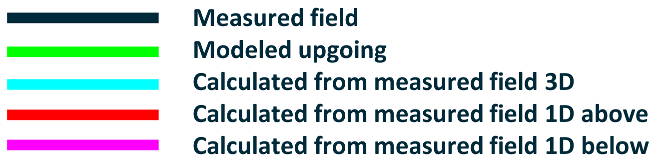
- Measured field
- Modeled upgoing
- Calculated from measured field 3D
- Calculated from measured field 1D above
- Calculated from measured field 1D below

$$E_x^U(z,\omega) = \frac{1}{2}\left[E_x(z,\omega) - Z\,\frac{\left(k_x k_y H_x(z,\omega) + \left(k_\omega^2 - k_x^2\right) H_y(z,\omega)\right)}{k_\omega \sqrt{k_\omega^2 - k_x^2 - k_y^2}}\right]$$

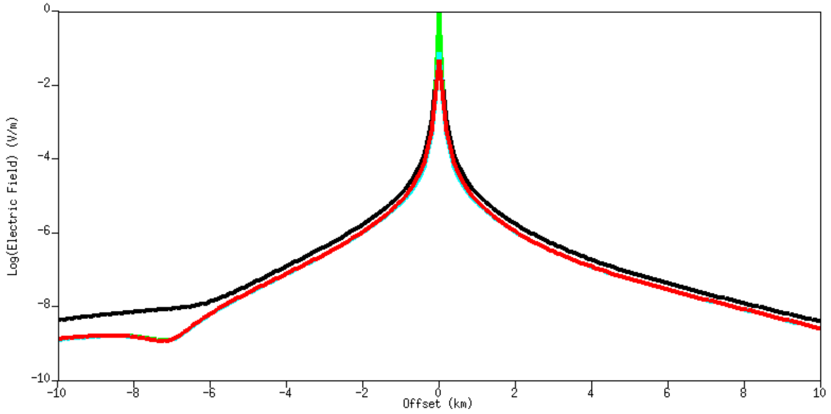


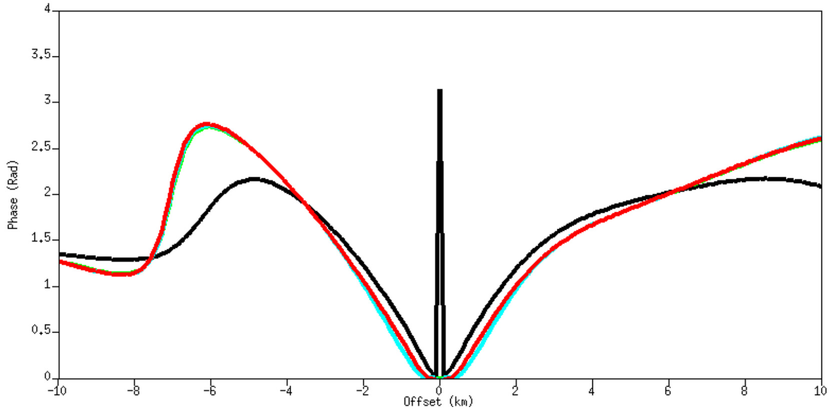


Up/Down separation

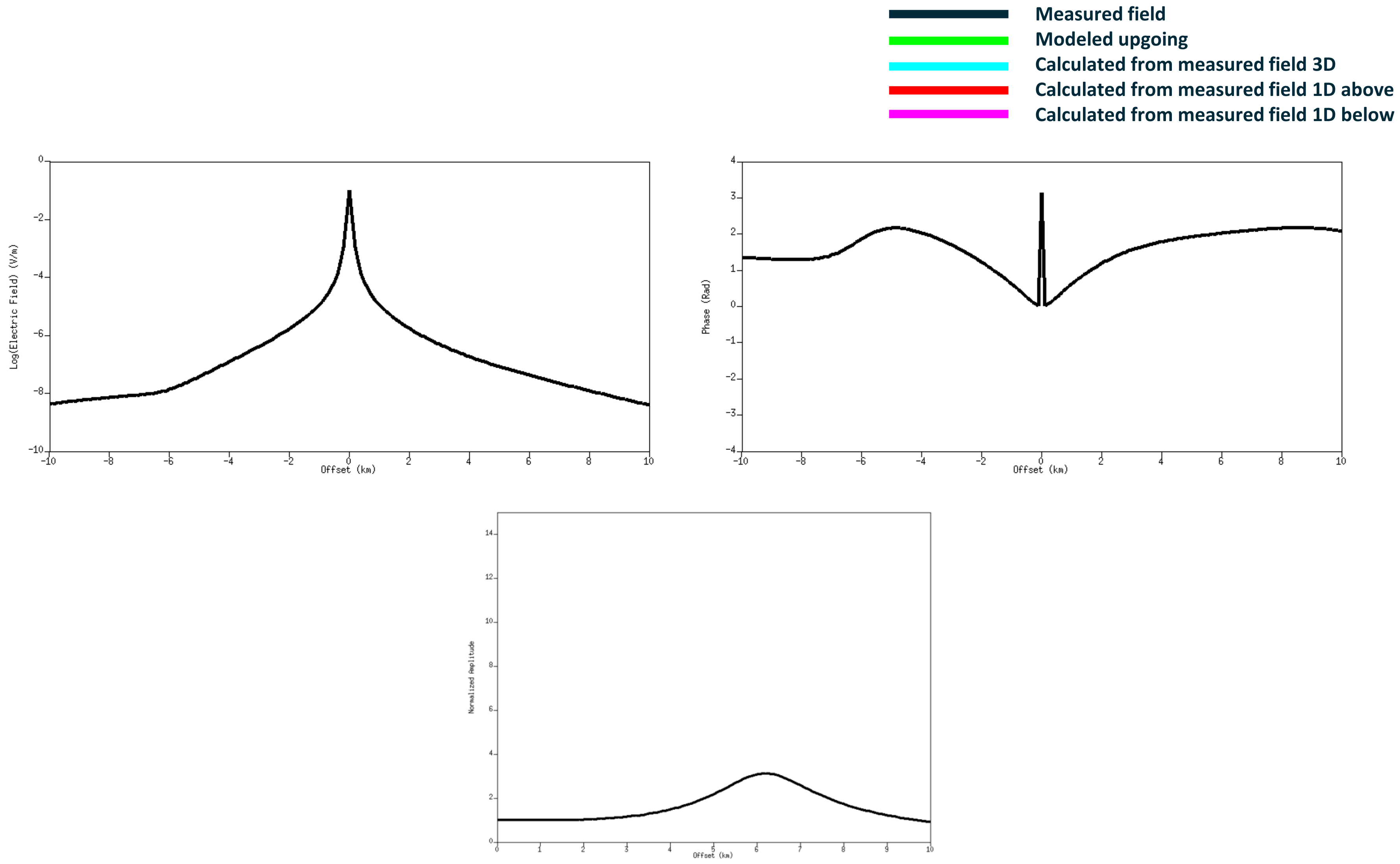


$$E_x^U(z,\omega) = \frac{1}{2}\left[E_x(z,\omega) - Z_w H_y(z,\omega)\right]$$



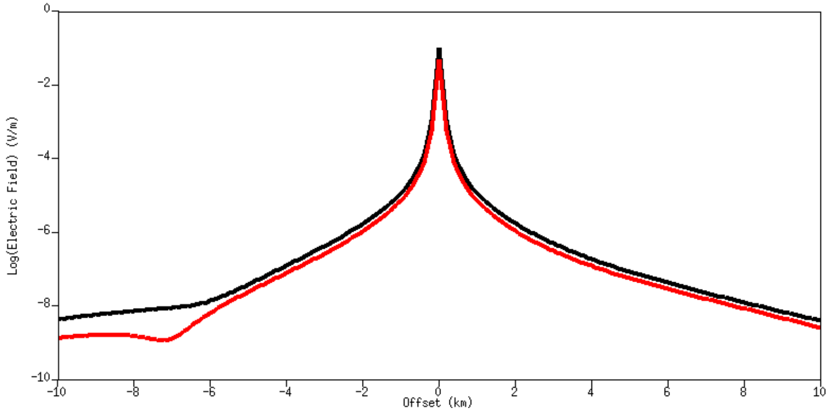


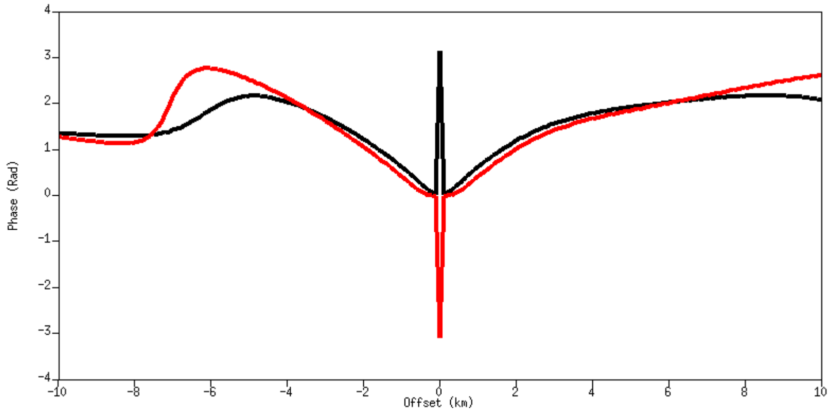
Up/Down separation

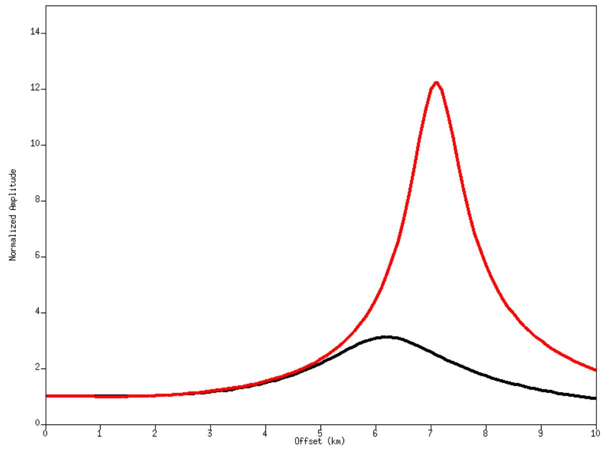


Up/Down separation

Measured field
Modeled upgoing
Calculated from measured field 3D
Calculated from measured field 1D above
Calculated from measured field 1D below





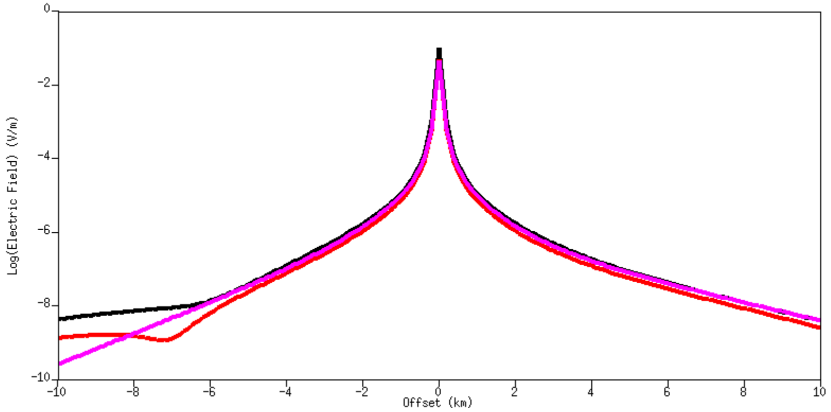


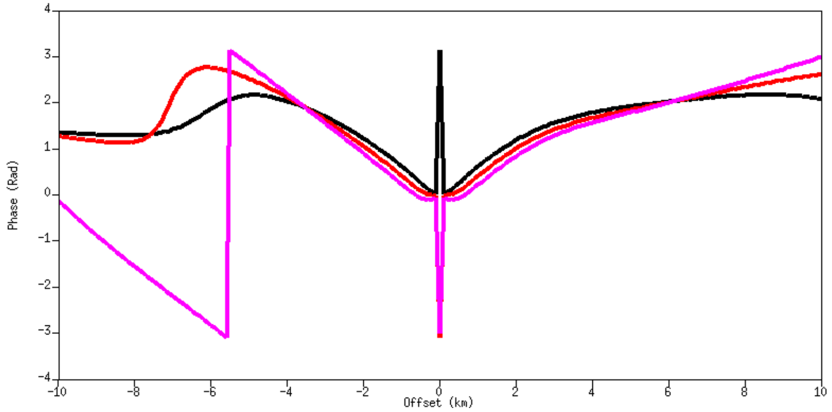
Up/Down separation

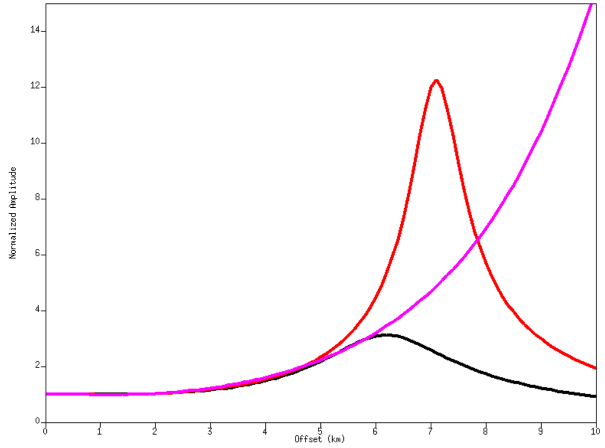
$$E_x^U(z,\omega) = \frac{1}{2}[E_x(z,\omega) - Z_w H_y(z,\omega)]$$

$$E_x^U(z,\omega) = \frac{1}{2}[E_x(z,\omega) - Z_f H_y(z,\omega)]$$

Measured field
Modeled upgoing
Calculated from measured field 3D
Calculated from measured field 1D above
Calculated from measured field 1D below

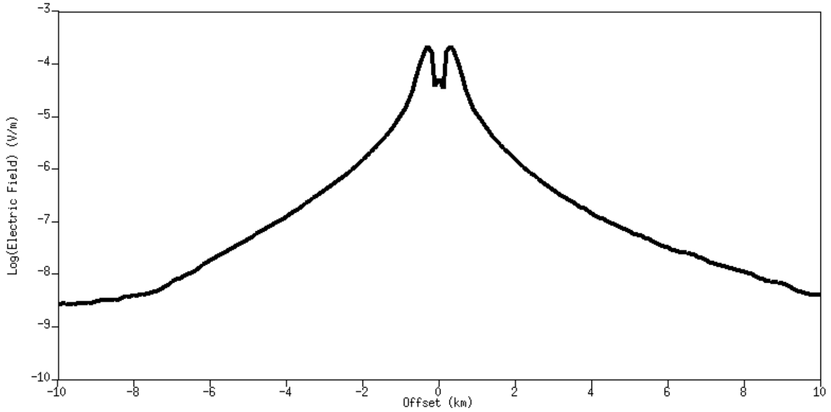


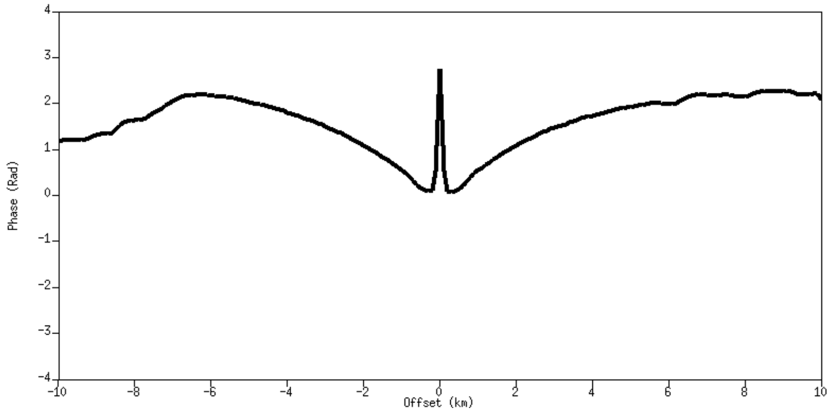


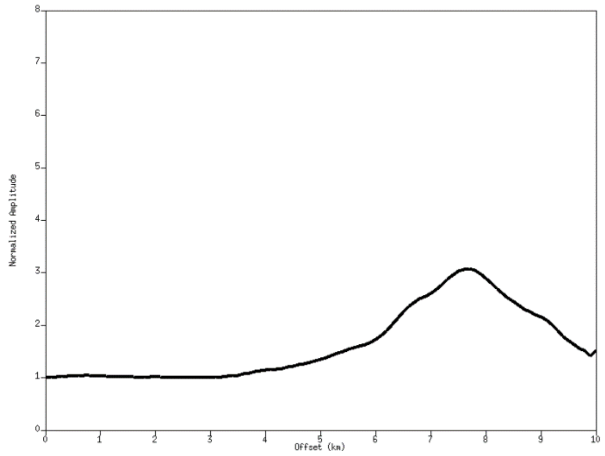


Up/Down separation

- Recorded field
- Modeled upgoing
- Calculated from measured field 3D
- Calculated from measured field 1D above
- Calculated from recorded field 1D below

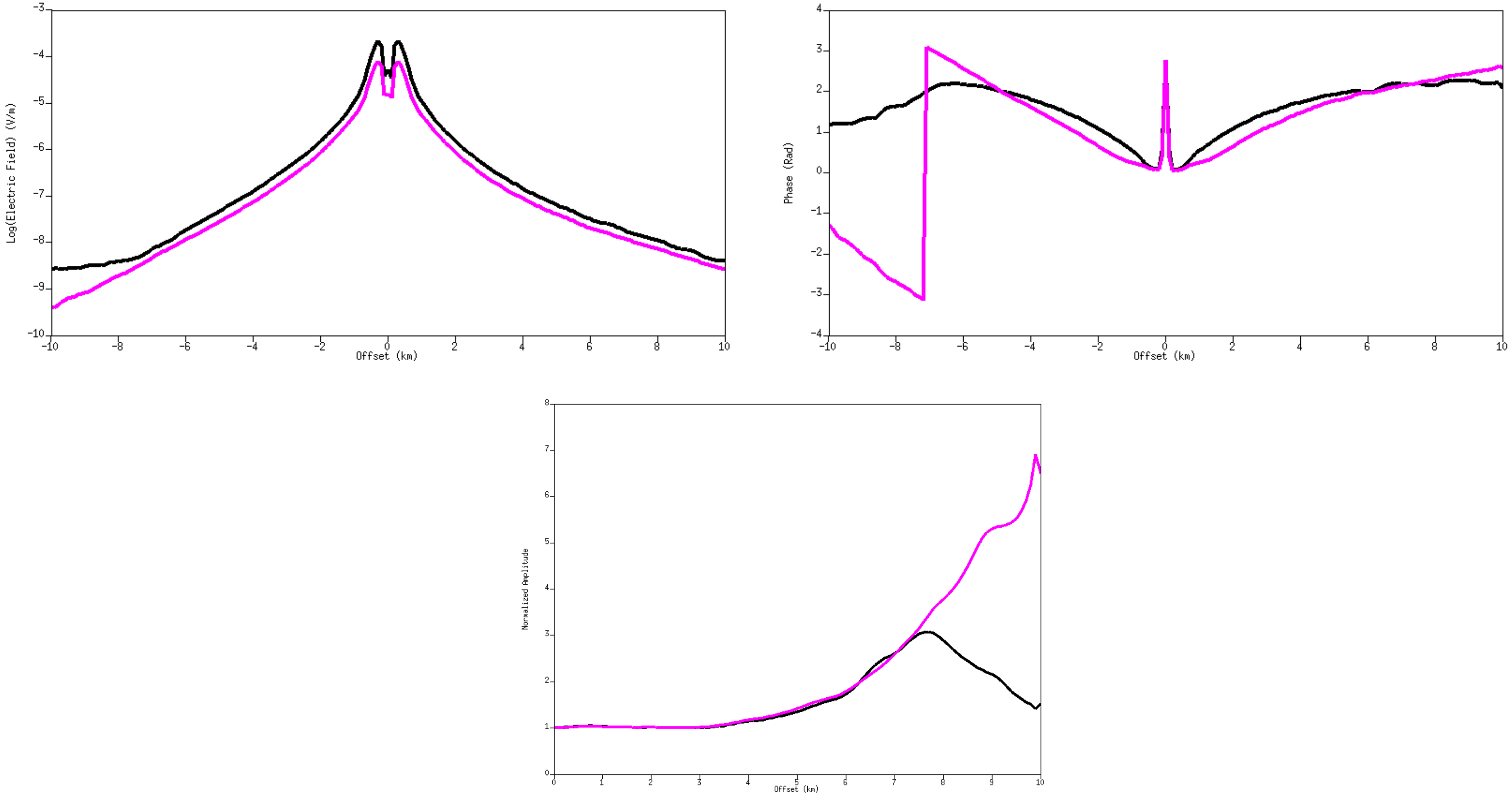




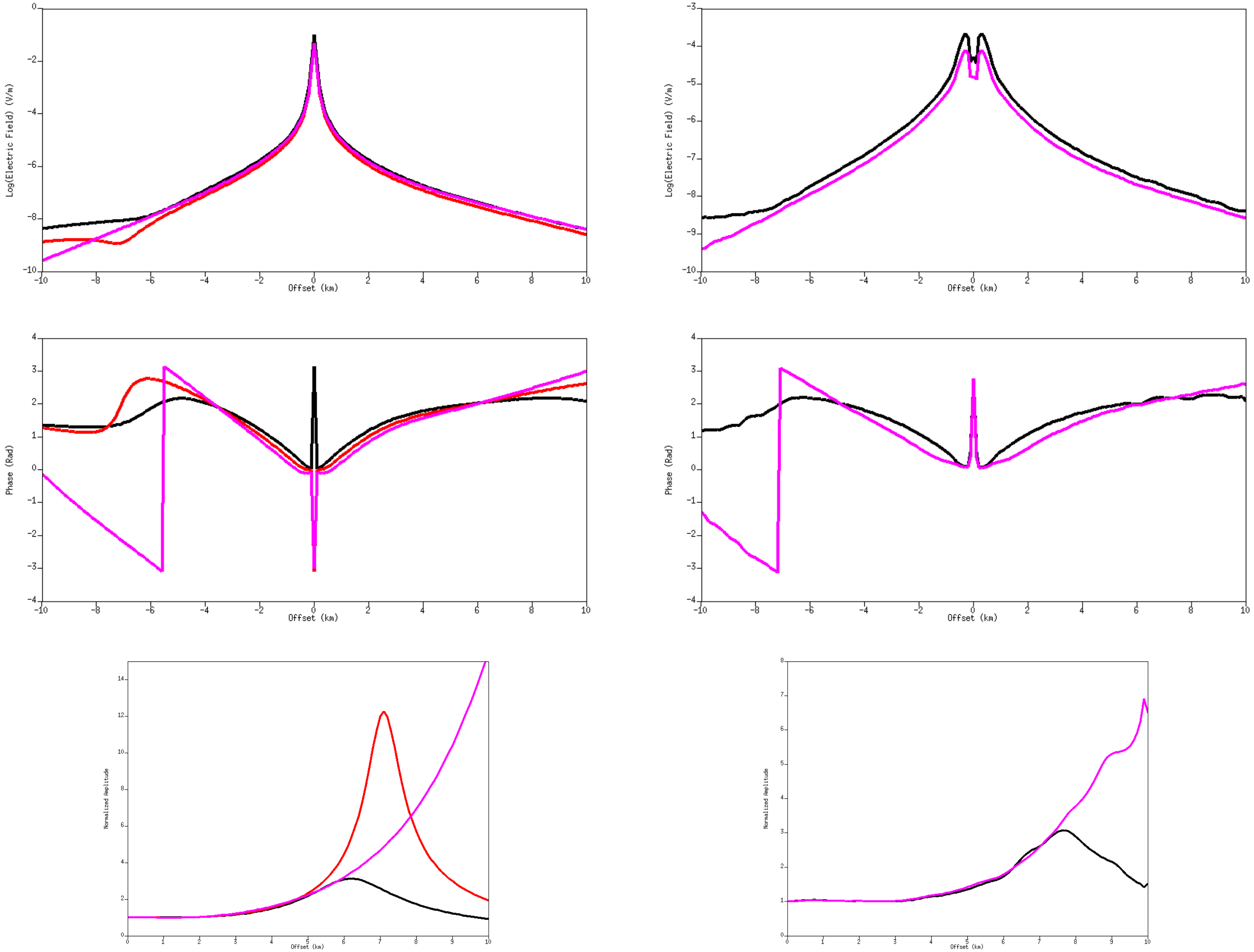


Up/Down separation

- Recorded field
- Modeled upgoing
- Calculated from measured field 3D
- Calculated from measured field 1D above
- Calculated from recorded field 1D below

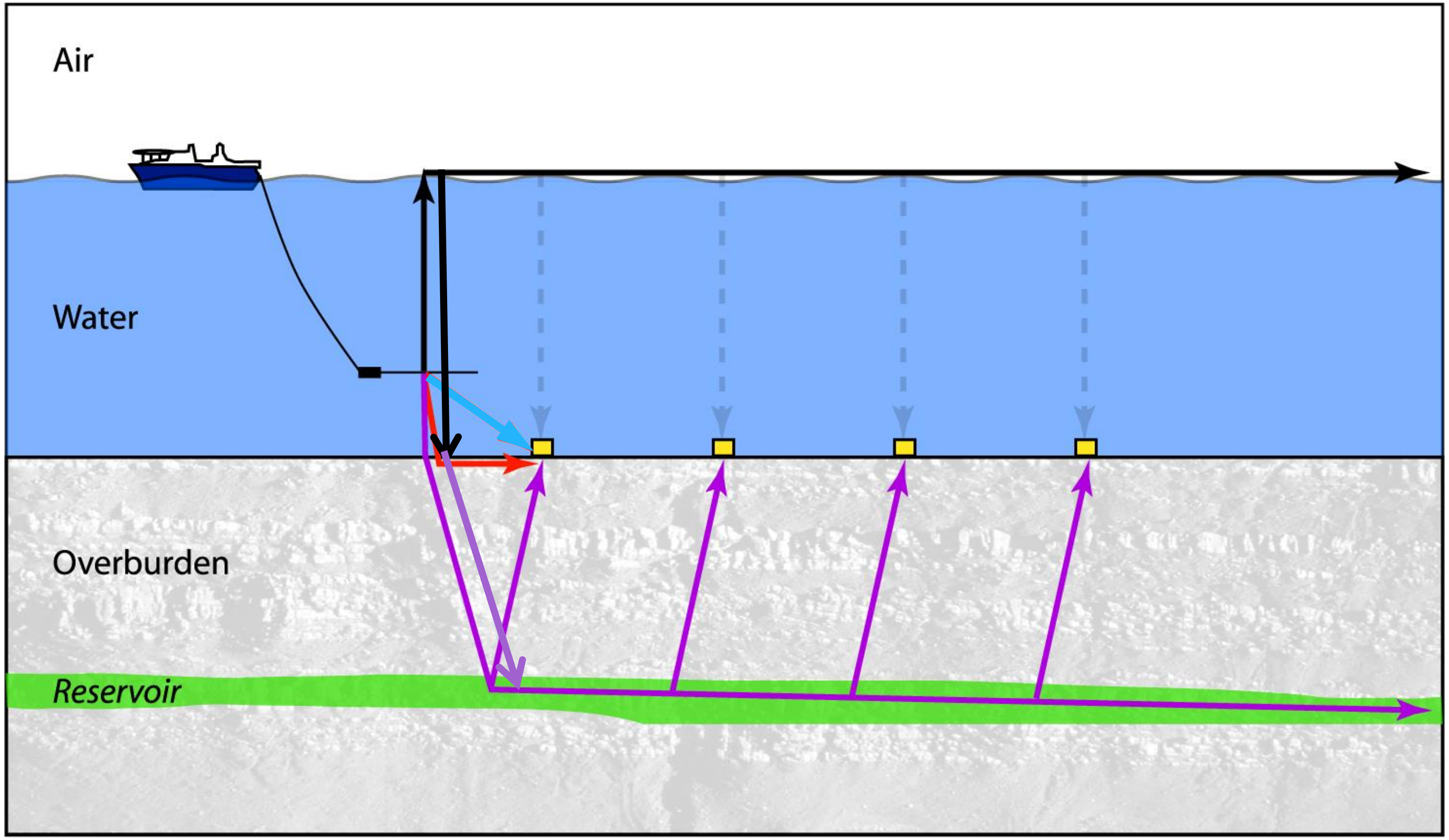


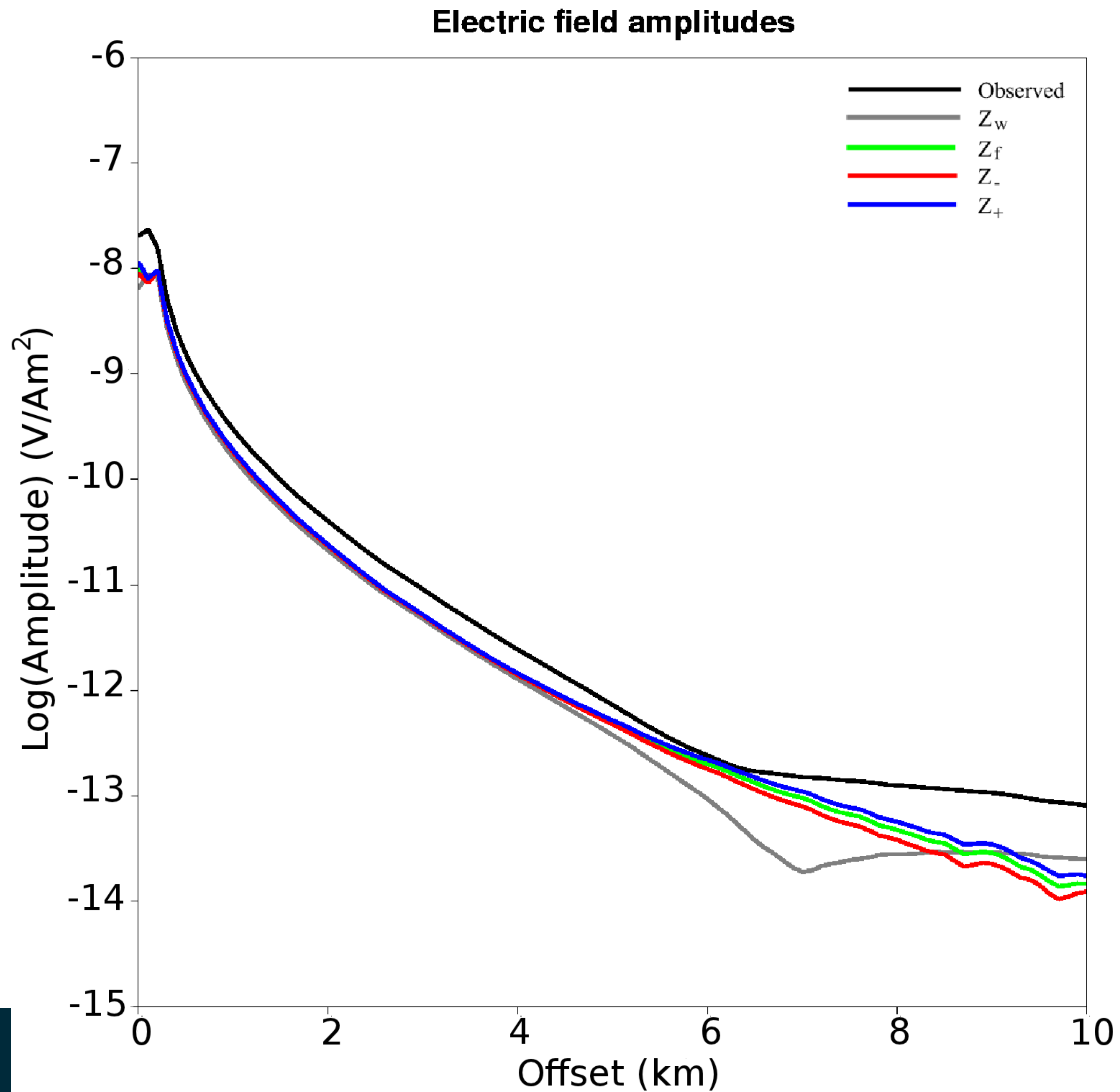
Up/Down separation



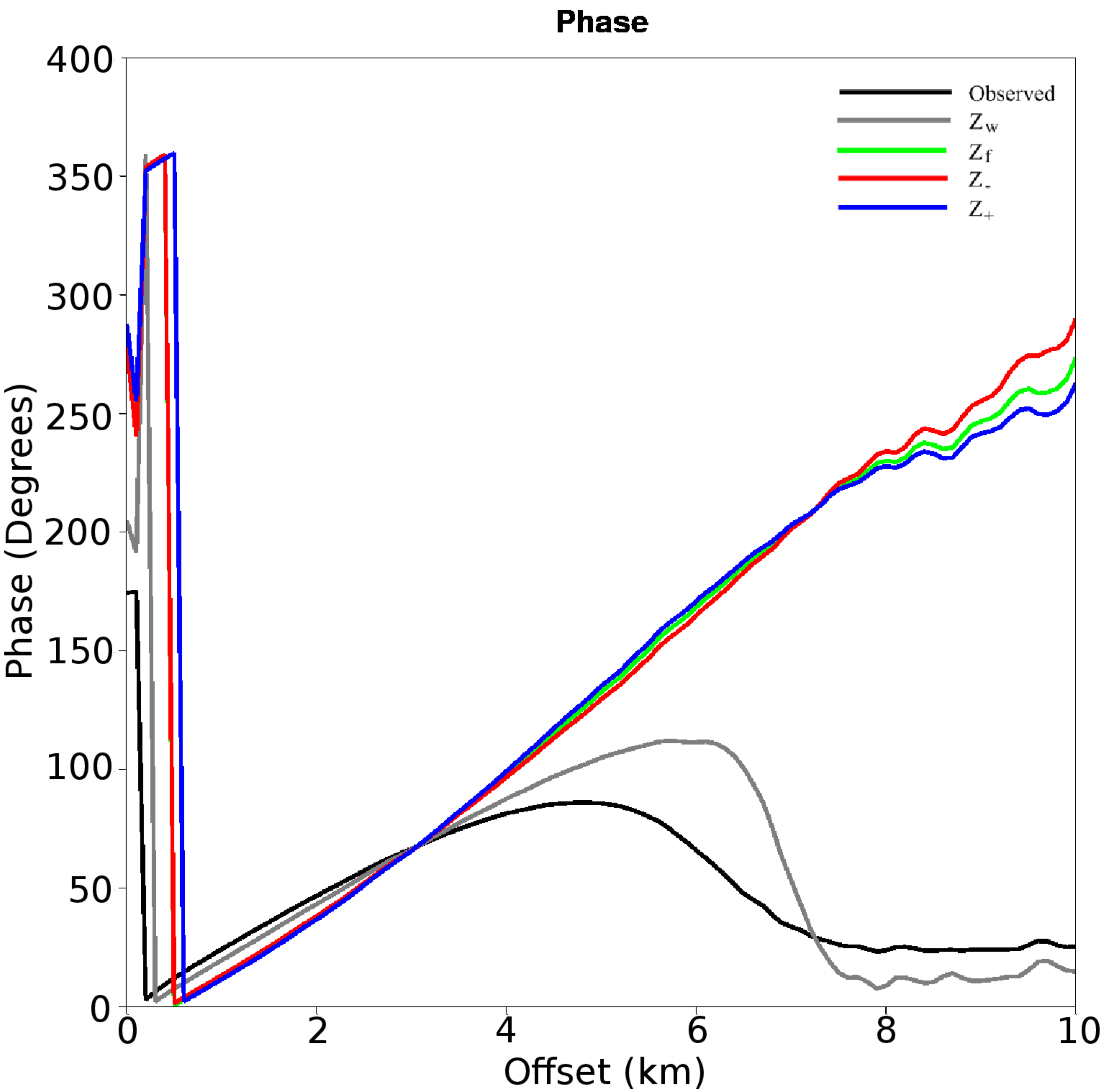
Doing Up-Down decomposition is not the same as doing a deep water experiment

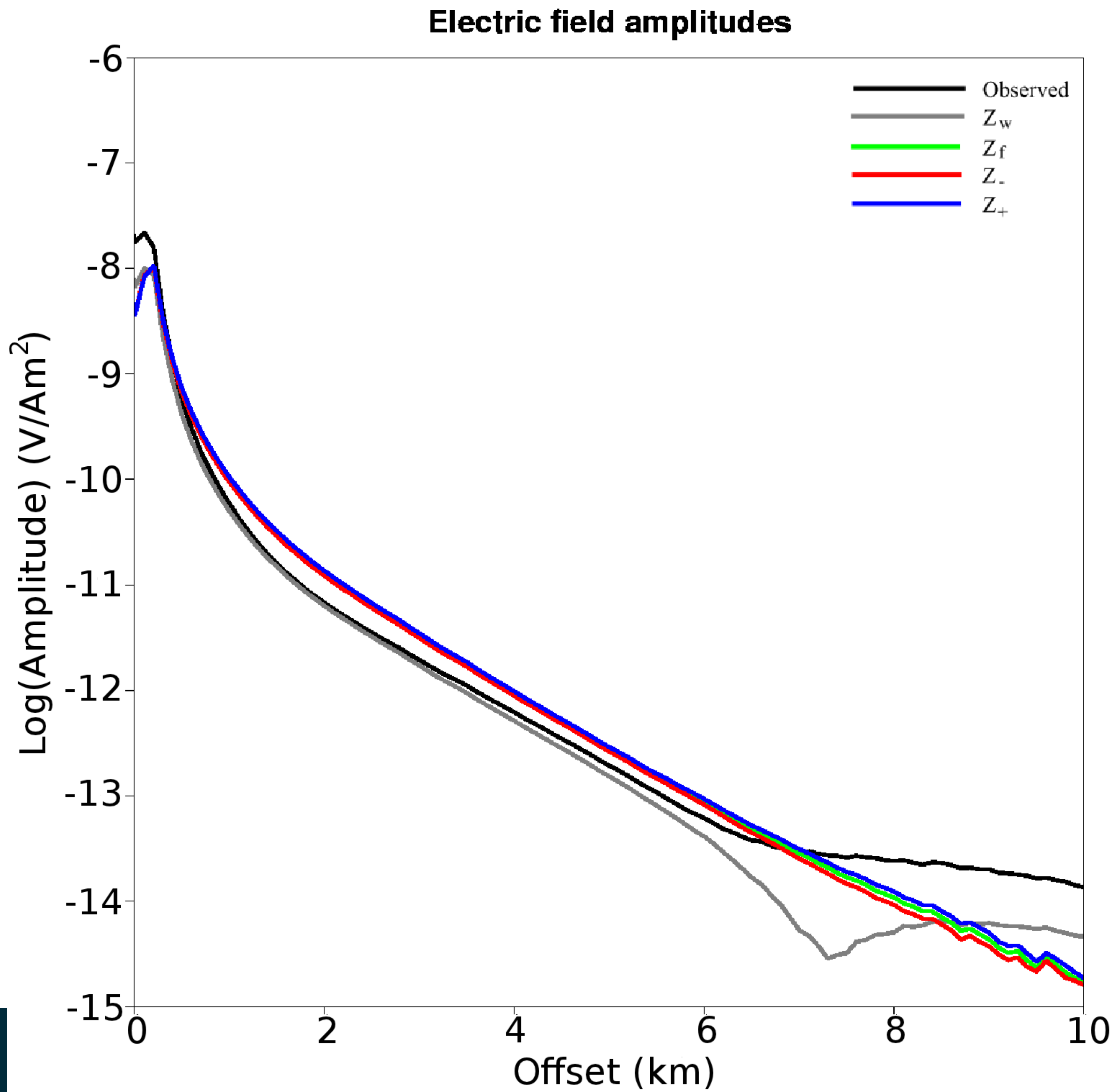
Internal multiples in waterlayer is not removed.

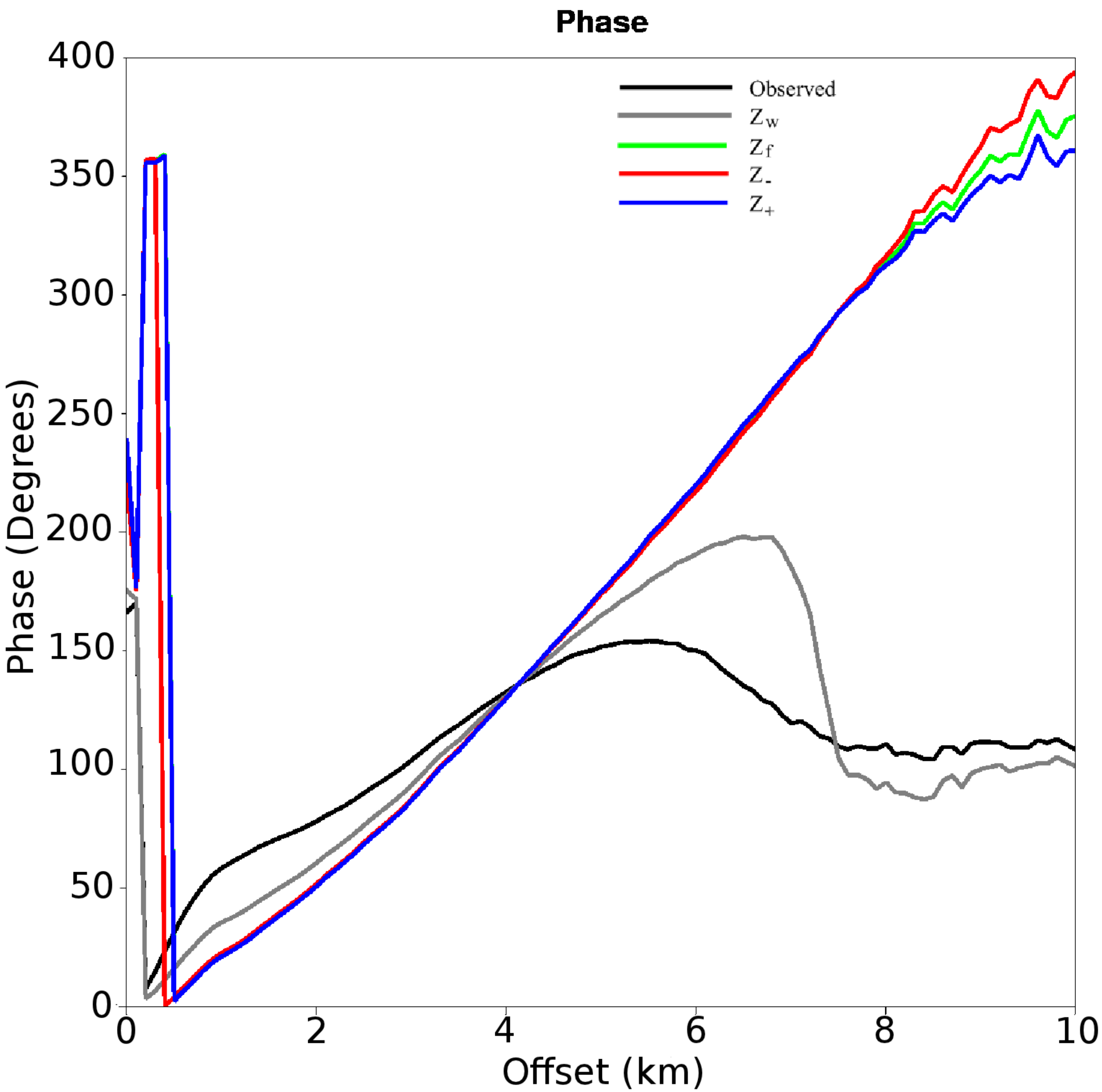


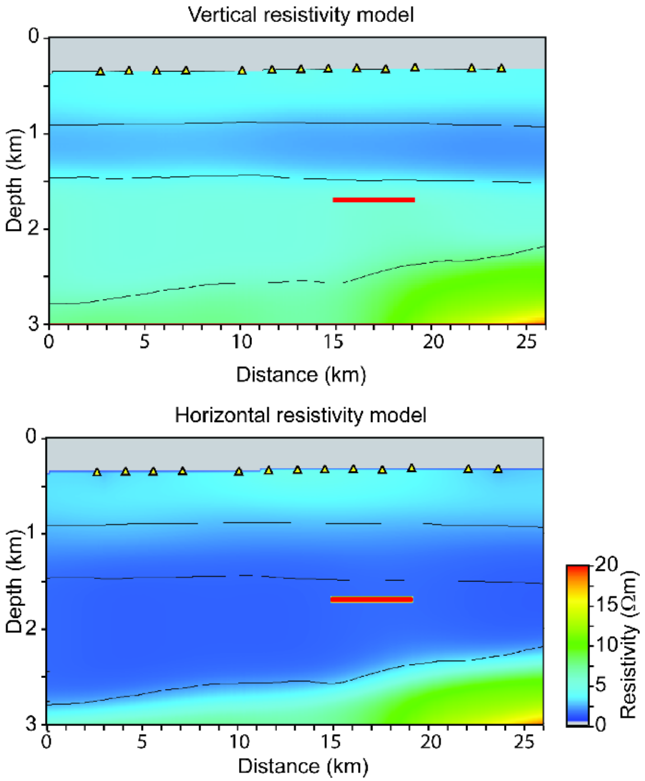


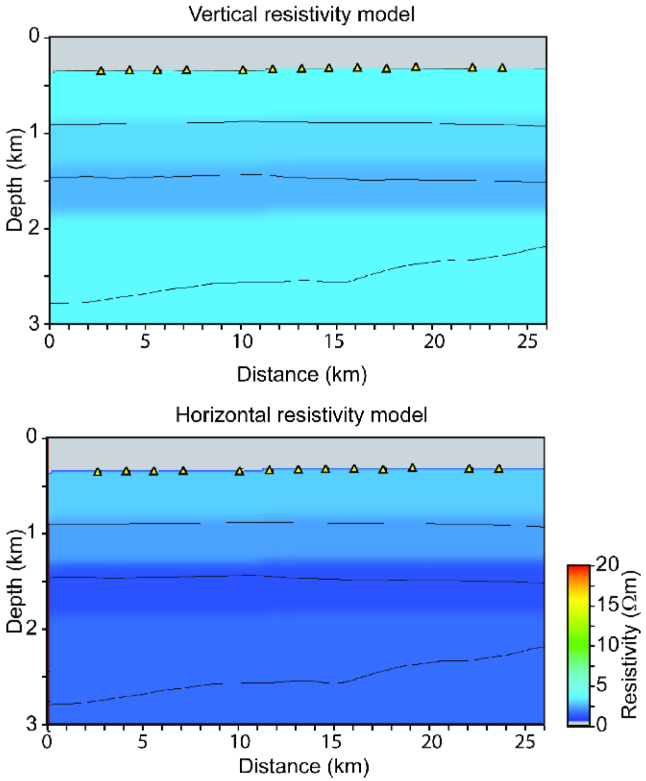
Electric field amplitudes
Observed
Zw
Zf
Z-
Z+
Log(Amplitude) (V/Am2)
Offset (km)
-6
-7
-8
-9
-10
-11
-12
-13
-14
-15
0
2
4
6
8
10

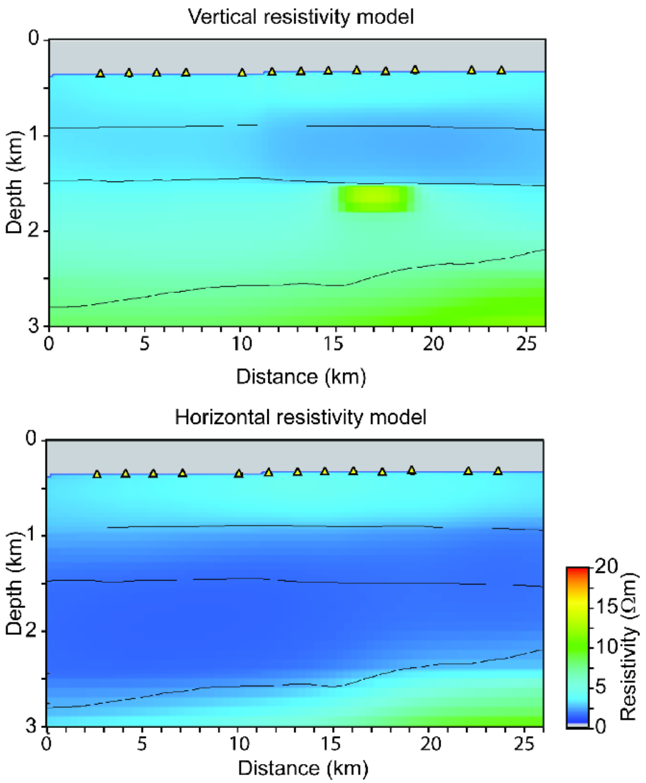

Phase
Observed
Zw
Zf
Z-
Z+
Phase (Degrees)
Offset (km)
0
50
100
150
200
250
300
350
400
0
2
4
6
8
10

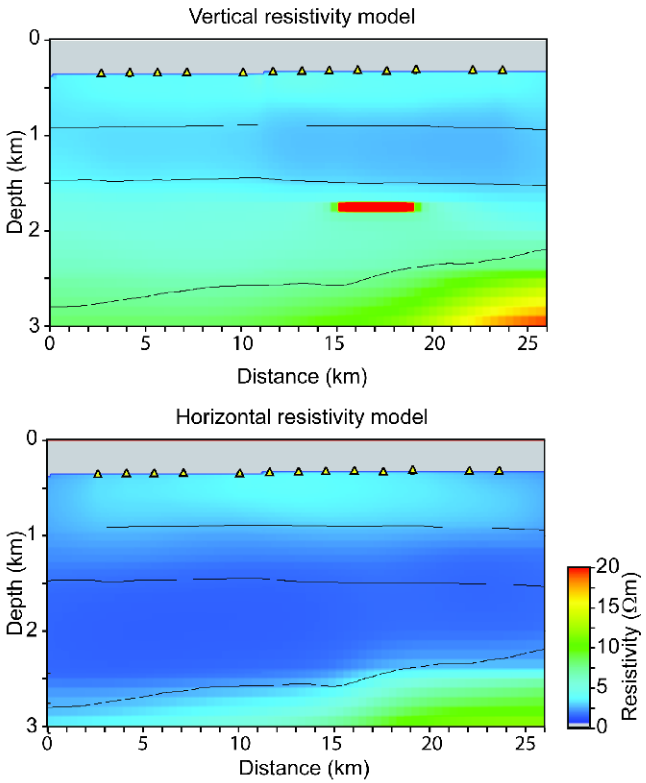

Electric field amplitudes
Observed
Z_w
Z_f
Z_-
Z_+
Log(Amplitude) (V/Am^2)
-6
-7
-8
-9
-10
-11
-12
-13
-14
-15
0
2
4
6
8
10
Offset (km)

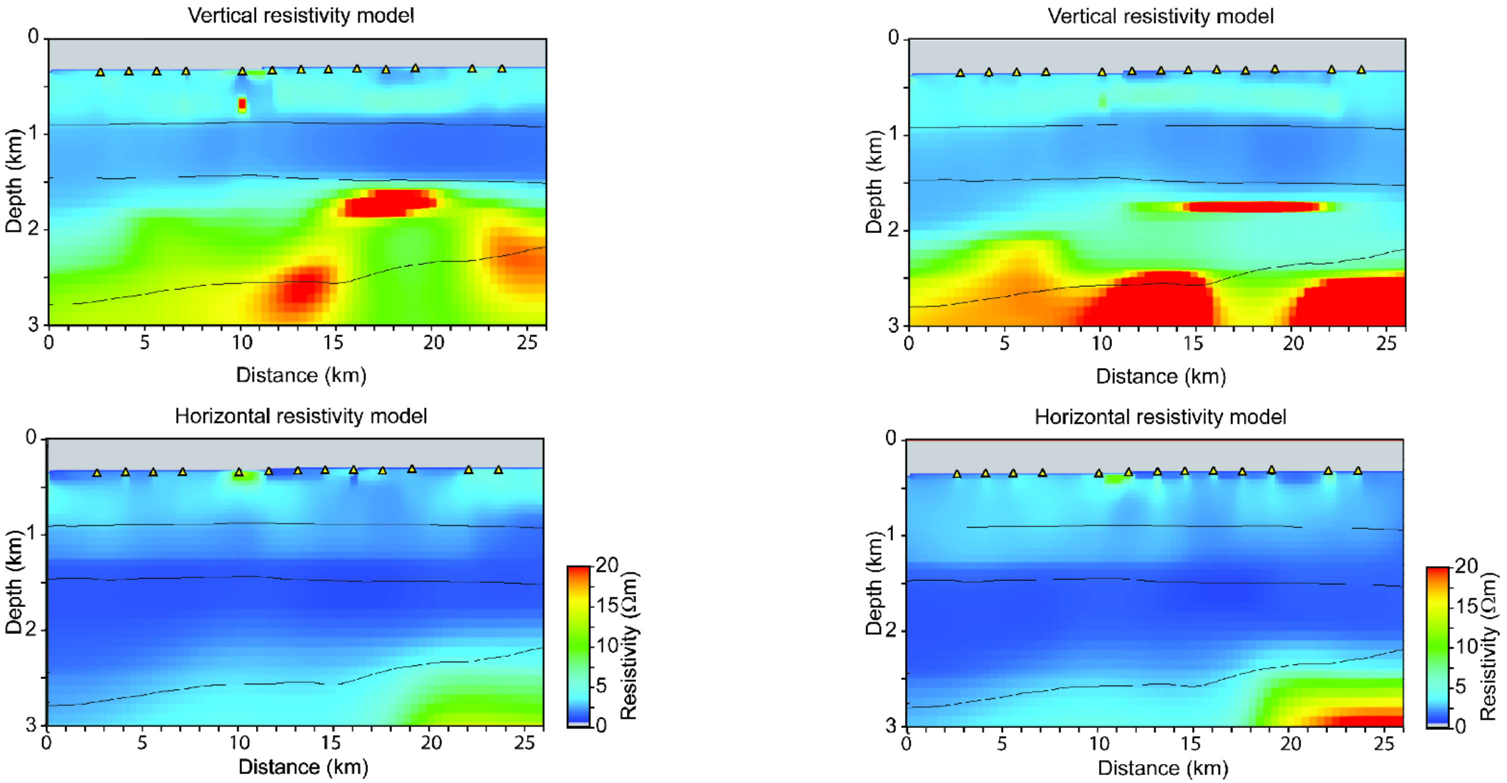

Phase
Observed
Z_w
Z_f
Z_-
Z_+
Phase (Degrees)
Offset (km)
0
50
100
150
200
250
300
350
400
0
2
4
6
8
10


Vertical resistivity model
Depth (km)
0
1
2
3
0
5
10
15
20
25
Distance (km)
Horizontal resistivity model
Depth (km)
0
1
2
3
0
5
10
15
20
25
Distance (km)
20
15
10
5
0
Resistivity (Ωm)


Vertical resistivity model
Depth (km)
0
1
2
3
0
5
10
15
20
25
Distance (km)
Horizontal resistivity model
Depth (km)
0
1
2
3
0
5
10
15
20
25
Distance (km)
20
15
10
5
0
Resistivity (Ωm)


Vertical resistivity model
Depth (km)
Distance (km)
Horizontal resistivity model
Depth (km)
Distance (km)
Resistivity (Ωm)


Vertical resistivity model
Depth (km)
Distance (km)
Horizontal resistivity model
Depth (km)
Distance (km)
Resistivity (Ωm)


Vertical resistivity model
Vertical resistivity model
Horizontal resistivity model
Horizontal resistivity model
Depth (km)
Distance (km)
Resistivity (Ωm)
0
1
2
3
5
10
15
20
25

Anisotropy

Electrical anisotropy

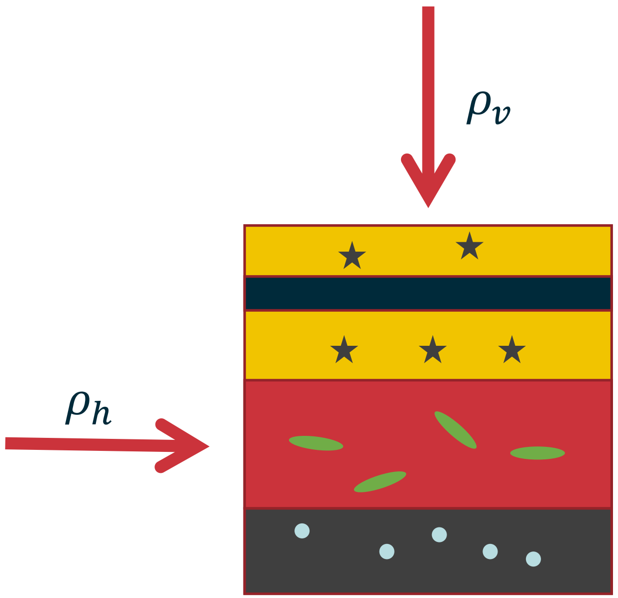
Resistivity within a formation is different in the vertical and horizontal directions.

Reasons for this:

Lithology, layering, grain orientation

Fractures

Diagenesis



Anisotropy Factor= ρ_v / ρ_h
Values range from basin to basin and stratigraphic intervals.

Electrical anisotropy

A formation is said to be electrically anisotropic if its conductivity is direction dependendent.

Ohm's law: $\boldsymbol{J} = \sigma \boldsymbol{E}$

Isotropy

$$\sigma = \begin{bmatrix} \sigma & & \\ & \sigma & \\ & & \sigma \end{bmatrix}$$

1 independent value

General anisotropy

$$\sigma = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix}$$

6 independent values
(symmetry property of tensor)

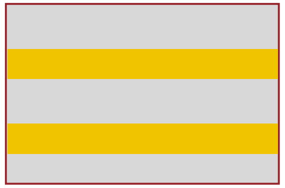
TIV

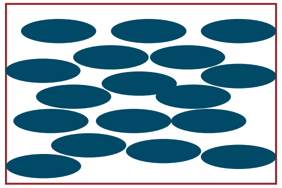
$$\sigma = \begin{bmatrix} \sigma_h & & \\ & \sigma_h & \\ & & \sigma_v \end{bmatrix}$$

2 independent values

Principle causes of anisotropy are: Lamination and bedding, grain shape and alignment, and fracturing.

TIV is typical for a formation with horizontal bedding and grain alignment.





General anisotropy is typical for a dipping formation.

In CSEM, it is most common to work with a TIV model.

Good to know

- TIV stands for ”transverse isotropy with respect to a vertical axis of rotational symmetry”.
- In vertical wells, resistivity lo

Electrical ansiotropy and resolution

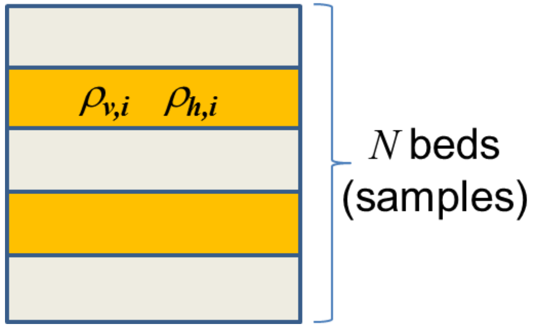
- CSEM is a low-frequency technique, so we cannot hope to resolve conductivity variations on a scale similar to well log resistivity measurements.

- All we can expect is to measure a bulk conductivity of a rock slab with dimensions on the order of several meters.

- The bulk conductivity is, however, determined by the fine-scale structure and constituents of the slab.

- Material averaging laws dictate that the bulk conductivity is anisotropic even if the constituents are isotropic.

Material averaging for a formation with horizontal bedding

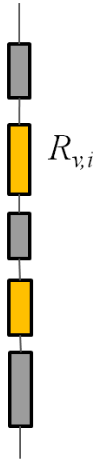


<u>Good to know</u>

The effective vertical resistivity is typically higher than the effective horizontal resistivity.

The ratio $\lambda=\rho_v/\rho_h$ is called anisotropy factor.

Vertical current flow
→Equivalent circuit:
Resistors in series

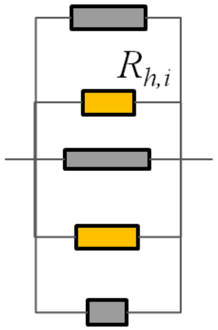


$$\rho_v = \frac{1}{N}\sum_i \rho_{v,i}$$

"arithmetic" average of vertical resistivity

Emphasis on beds with relatively high resistivity

Horizontal current flow
→Equivalent circuit:
Resistors in parallel



$$\frac{1}{\rho_h} = \frac{1}{N}\sum_i \frac{1}{\rho_{h,i}}$$

"harmonic" average of horizontal resistivity

Emphasis on beds with relatively high conductivity

SPOT THE DIFFERENCE.

Thank you