LECTURE 4

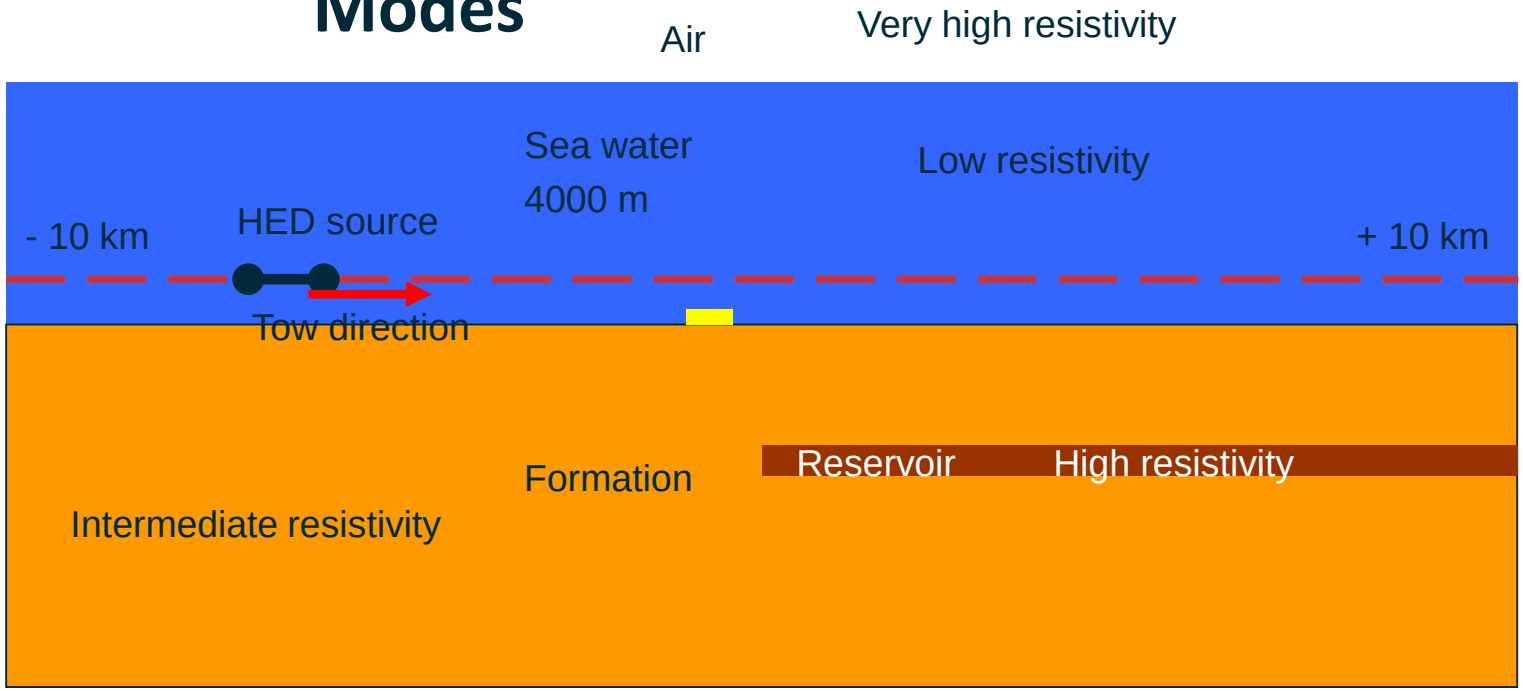
Rune Mittet
Chief Scientist, EMGS
Adjunct Professor, NTNU

Spot the difference.

Simulation examples - Amplitude and phase
Normalized plots
Non-uniqueness
Transmitter waveform

Simulation examples
Amplitude and phase

Modes



$c \approx 3160\sqrt{\rho\, f}$

$f = 0.25$ Hz

$c \approx 1580\sqrt{\rho}$

$c_{air} \approx 3 \times 10^8$ m/s

$c_{water} \approx 880$ m/s

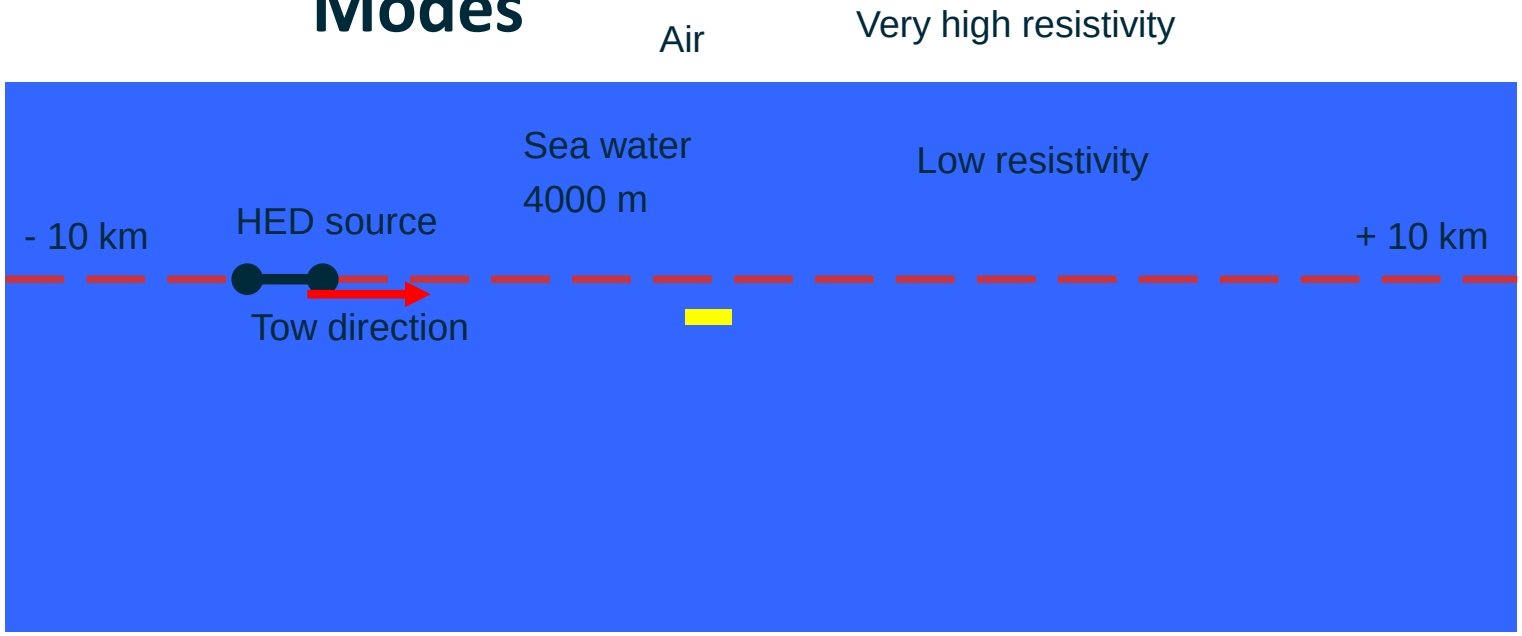
$c_{formation} \approx 2200$ m/s

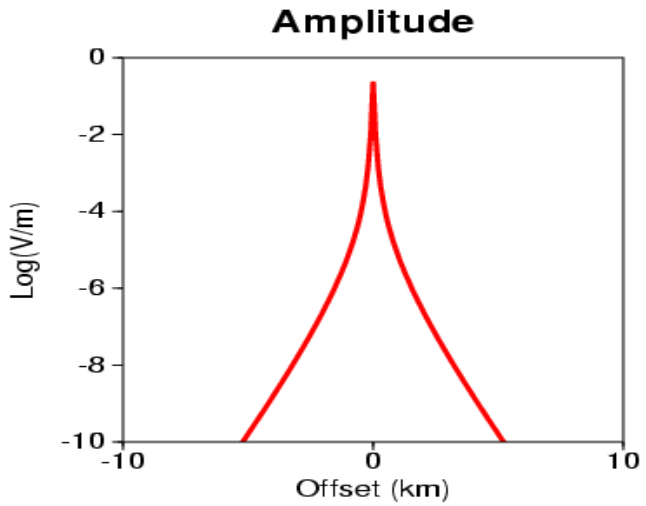
$c_{reservoir} \approx 14000$ m/s

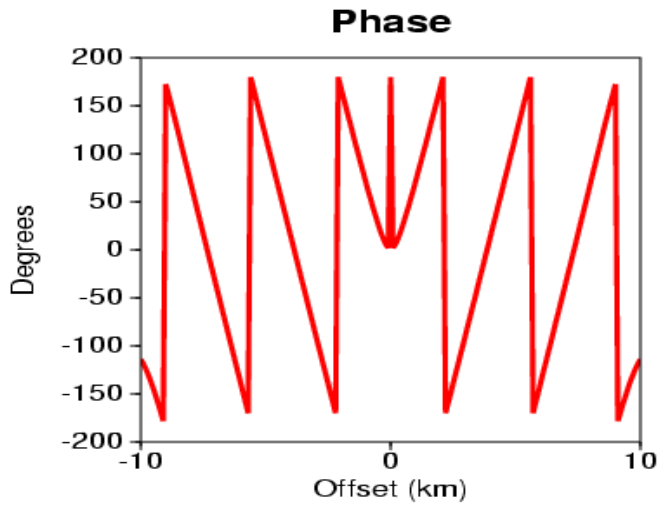
Simplified picture for EM data:
Phase depends on propagation velocity

$$\varphi \sim \frac{\omega}{c} x_{\text{offset}}$$

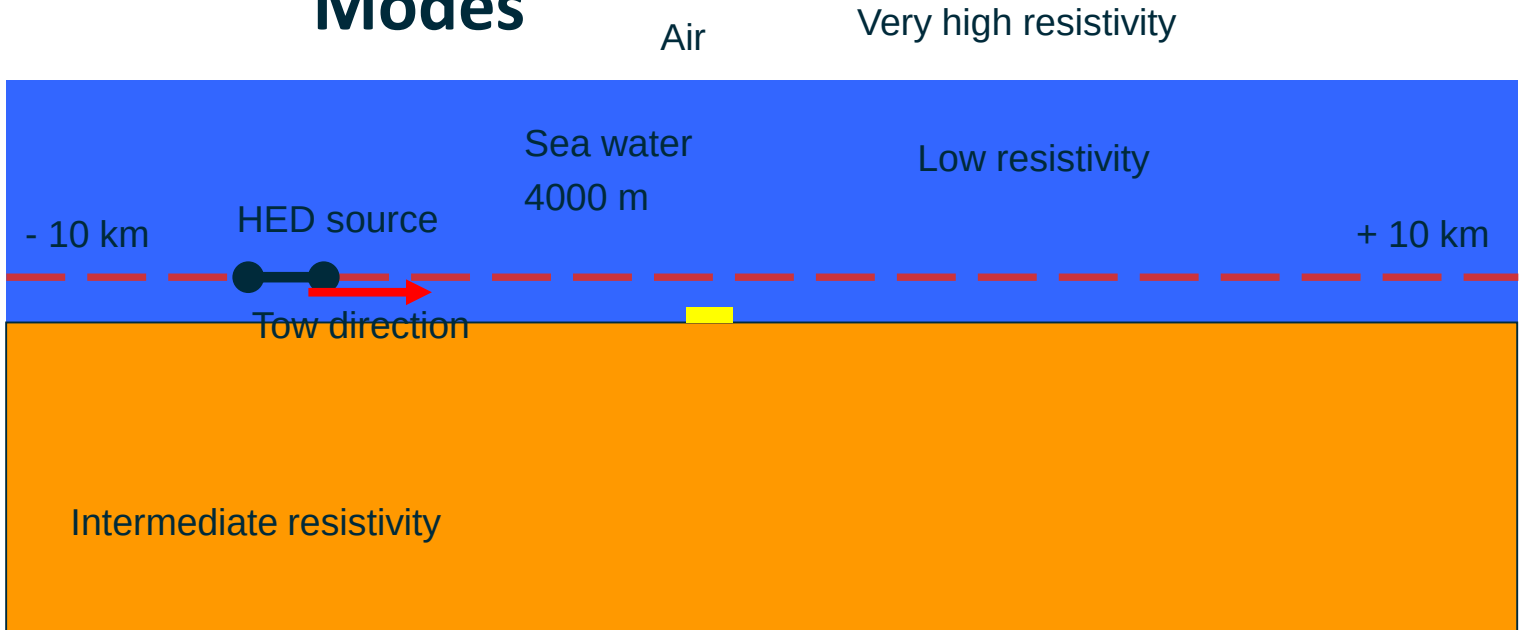
Modes

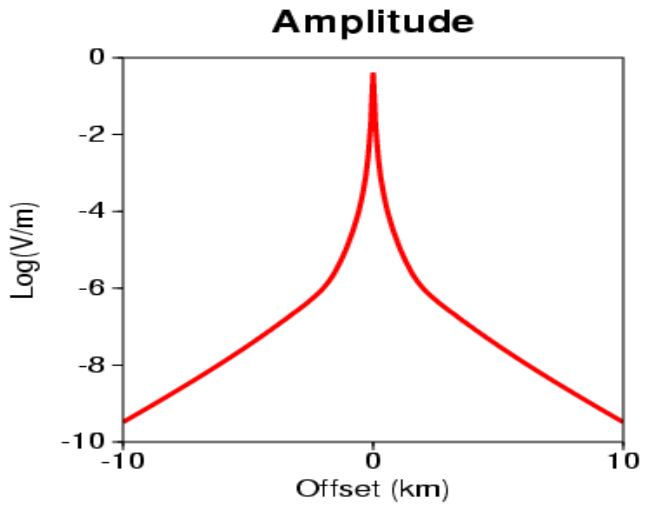

Air
Very high resistivity
Sea water
4000 m
Low resistivity
- 10 km
HED source
+ 10 km
Tow direction

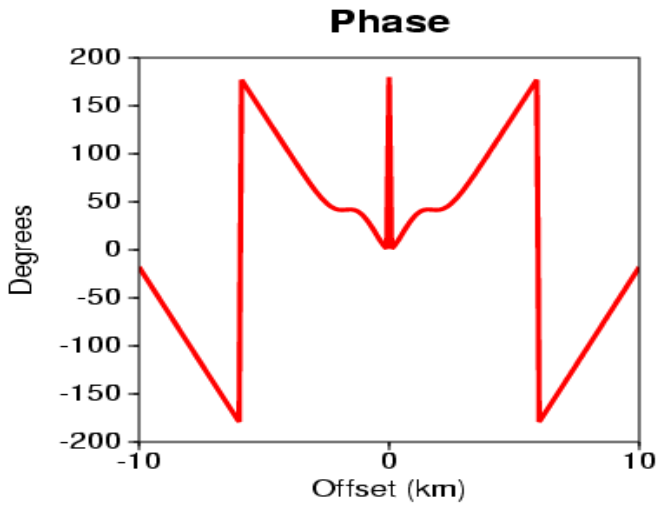

Amplitude
0
-2
-4
-6
-8
-10
Log(V/m)
-10
0
10
Offset (km)


Phase
200
150
100
50
0
-50
-100
-150
-200
Degrees
-10
0
10
Offset (km)

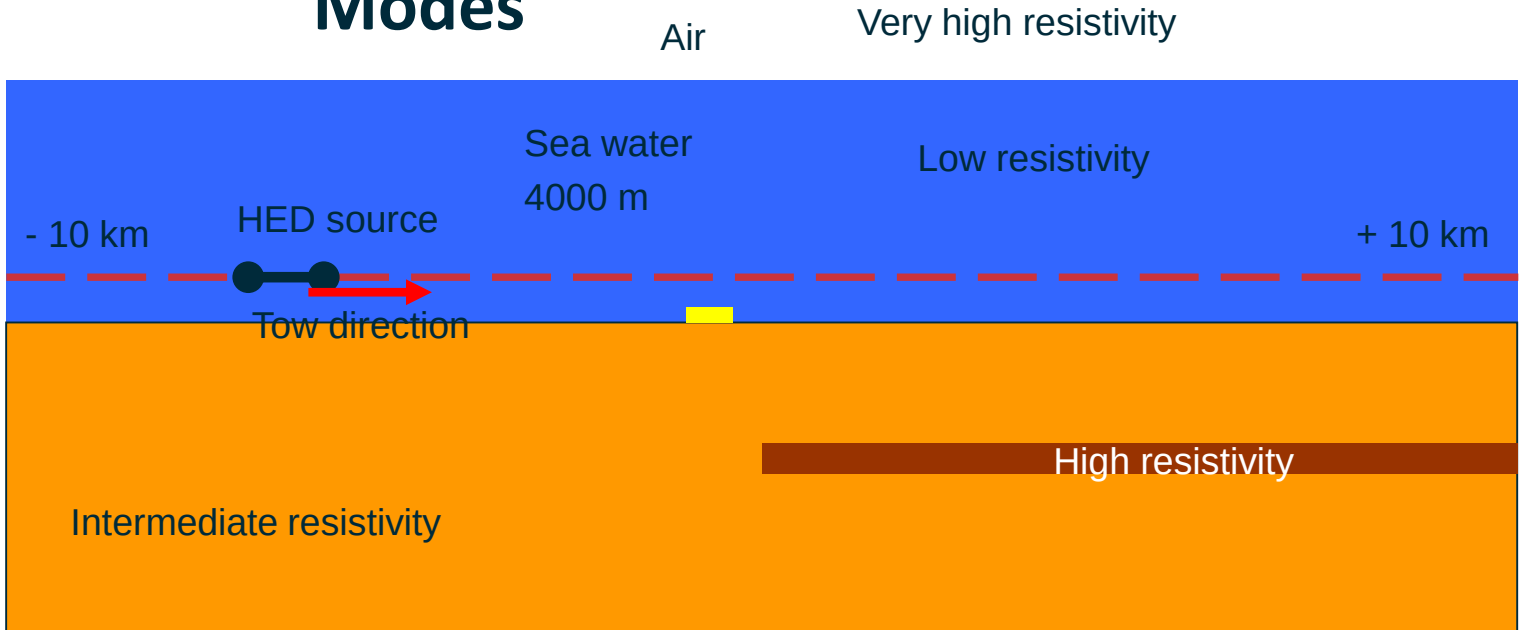
Modes

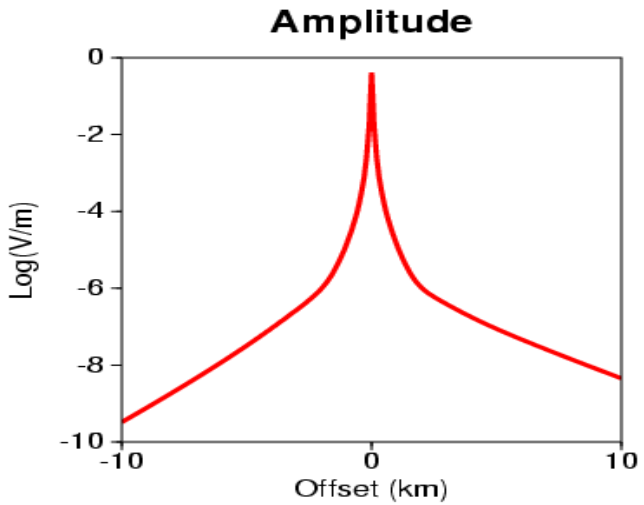

Air
Very high resistivity
Sea water
4000 m
Low resistivity
- 10 km
HED source
+ 10 km
Tow direction
Intermediate resistivity

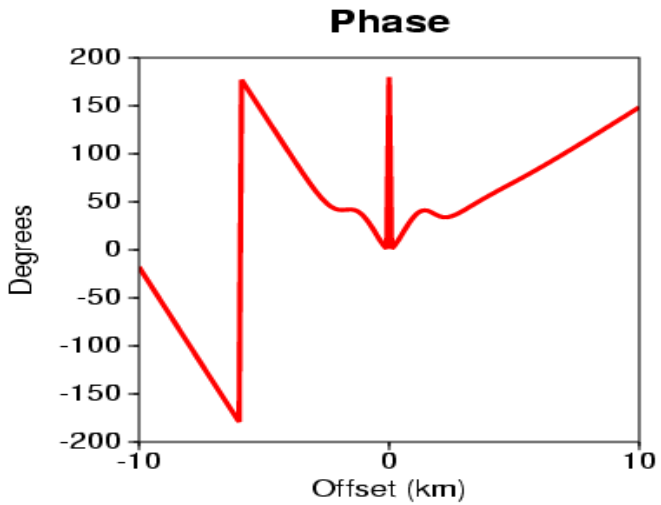

Amplitude
0
-2
-4
-6
-8
-10
Log(V/m)
-10
0
10
Offset (km)


Phase
200
150
100
50
0
-50
-100
-150
-200
Degrees
-10
0
10
Offset (km)

Modes


Air
Very high resistivity
Sea water
4000 m
Low resistivity
- 10 km
HED source
+ 10 km
Tow direction
High resistivity
Intermediate resistivity

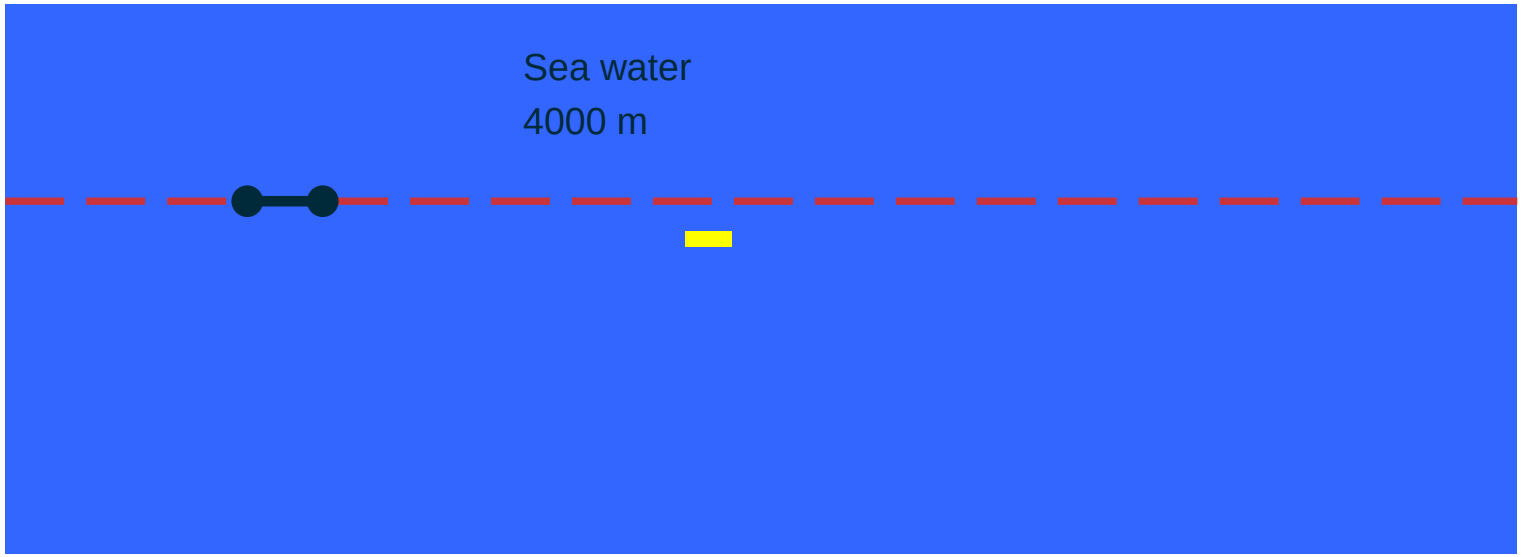

Amplitude
Log(V/m)
0
-2
-4
-6
-8
-10
-10
0
10
Offset (km)

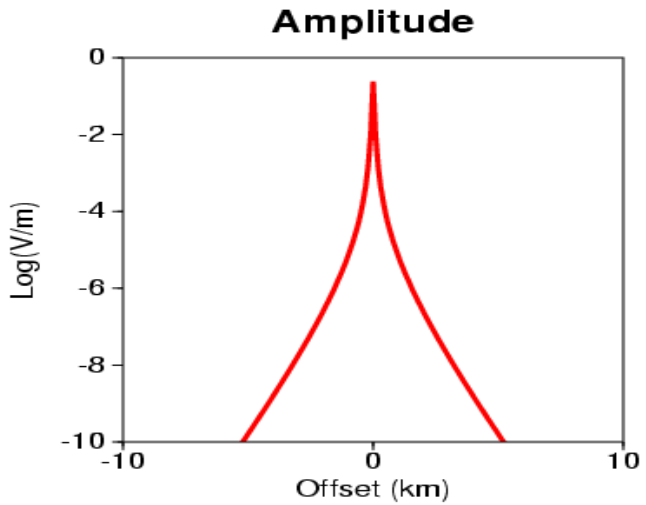

Phase
Degrees
200
150
100
50
0
-50
-100
-150
-200
-10
0
10
Offset (km)

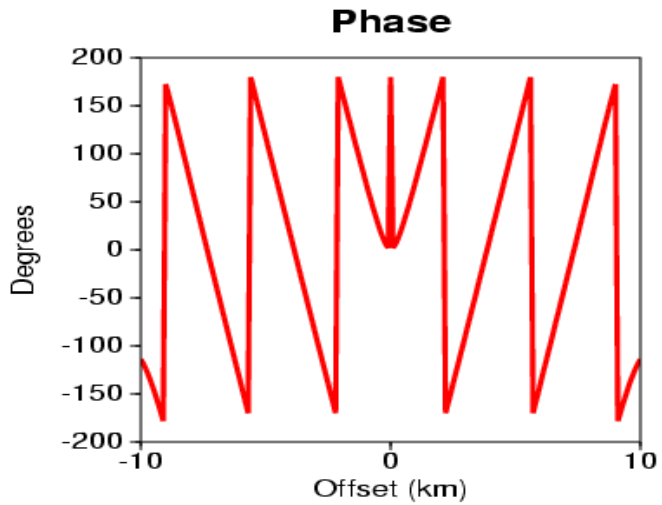
Modes

Air

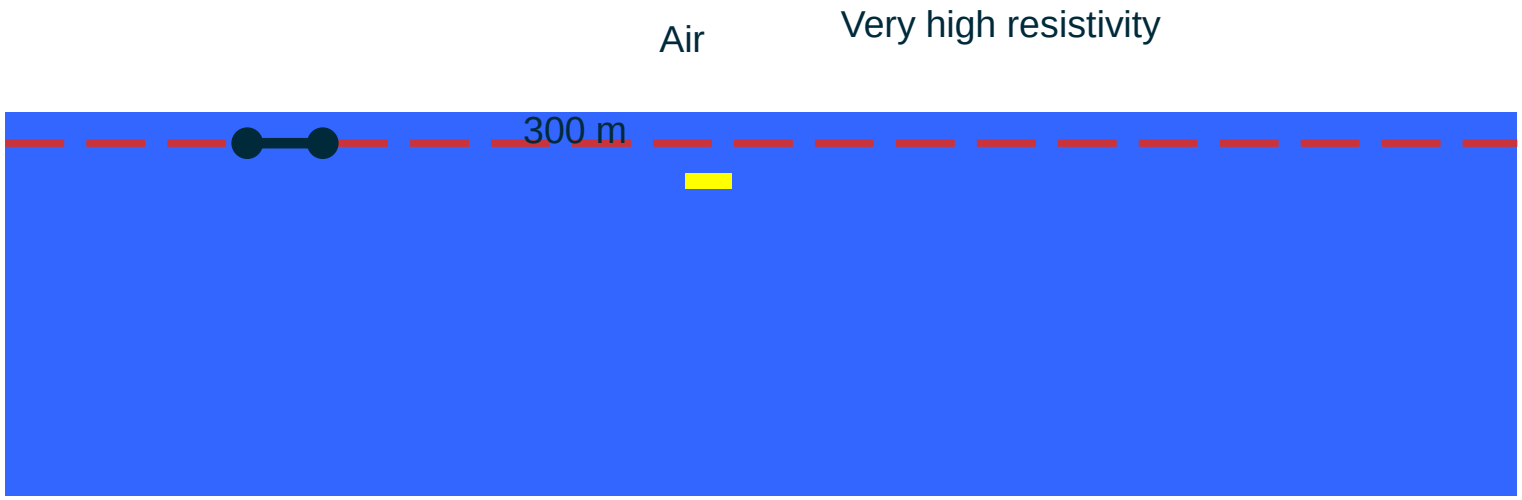
Very high resistivity

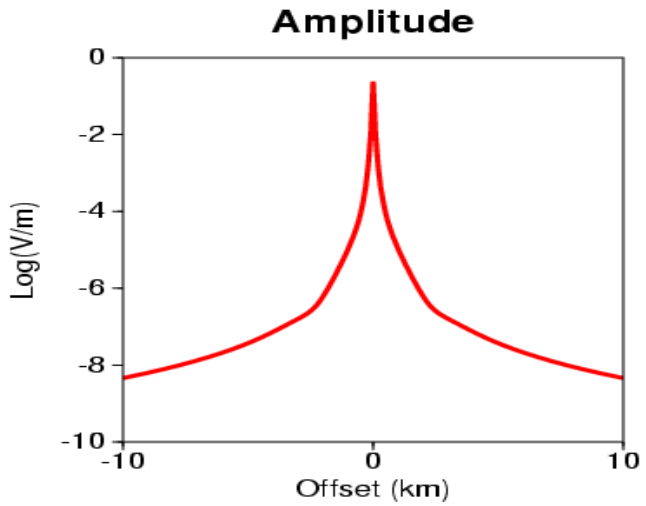


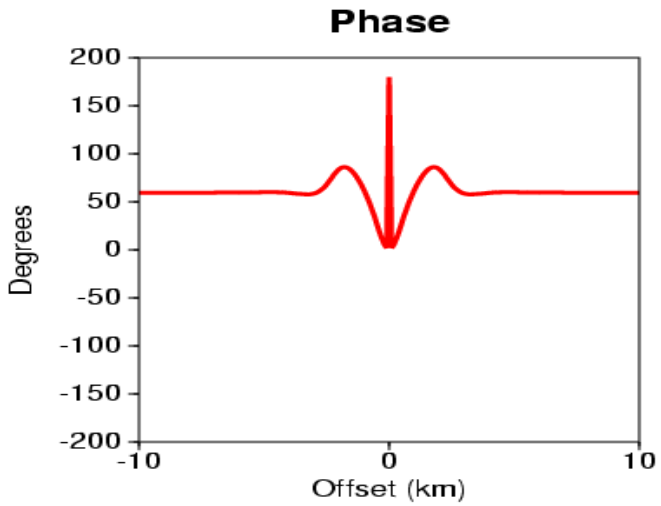




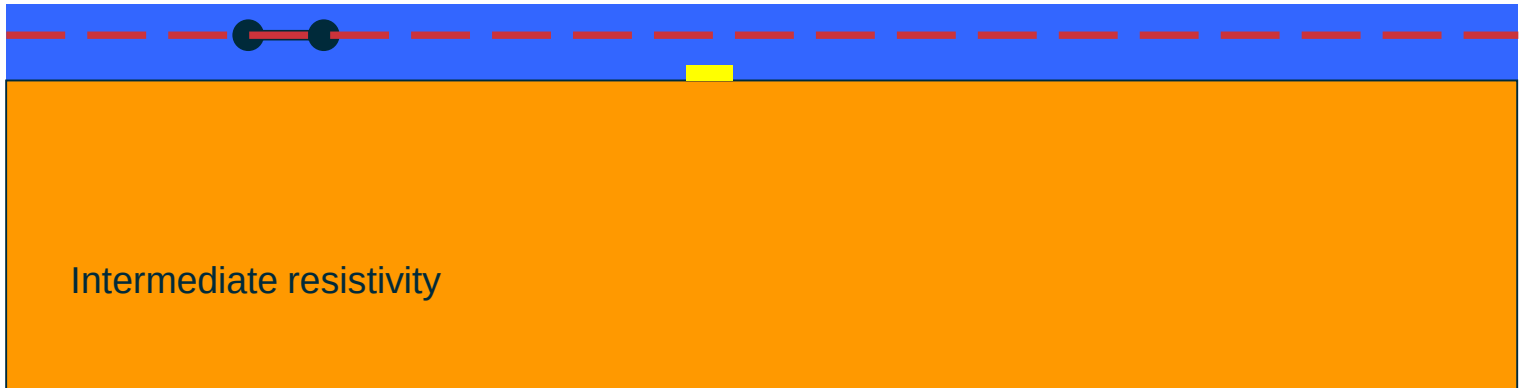
Modes

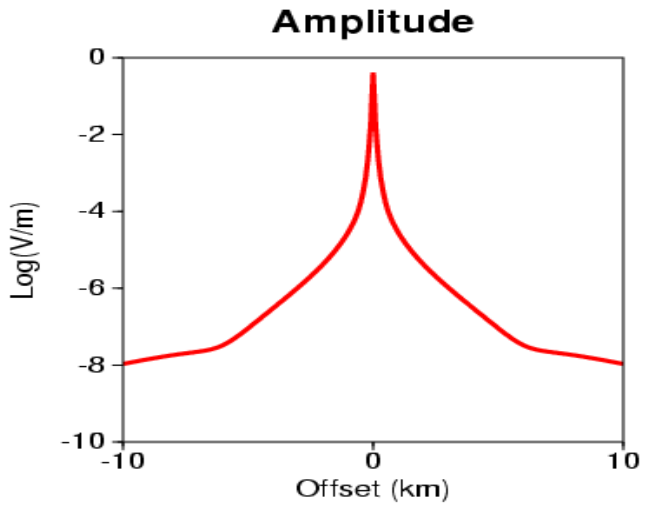

Air
Very high resistivity
300 m

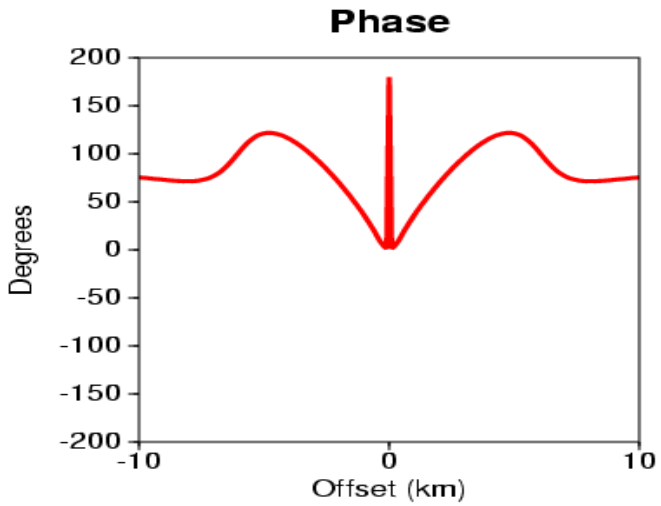

Amplitude
0
-2
-4
-6
-8
-10
Log(V/m)
-10
0
10
Offset (km)


Phase
200
150
100
50
0
-50
-100
-150
-200
Degrees
-10
0
10
Offset (km)

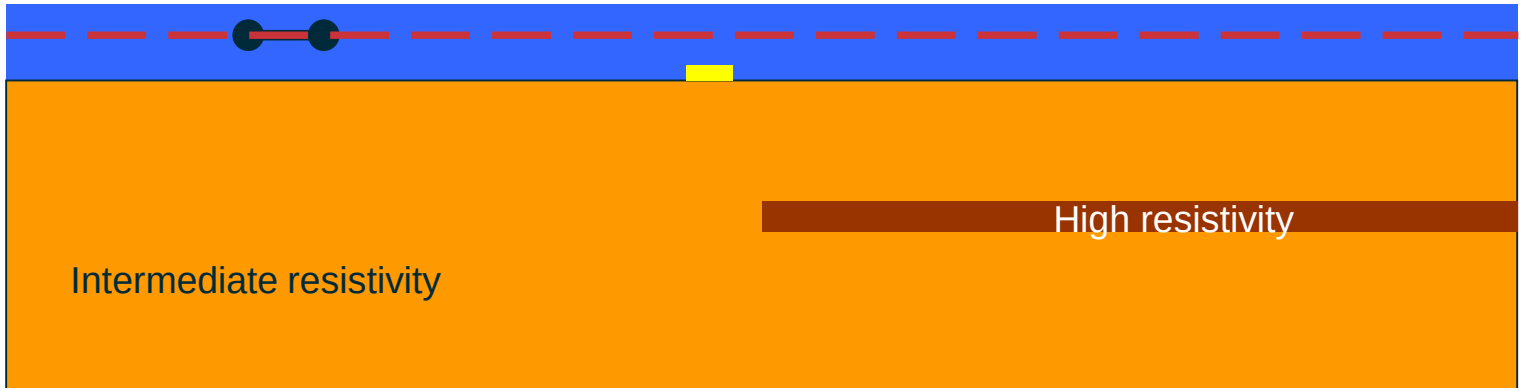
Modes

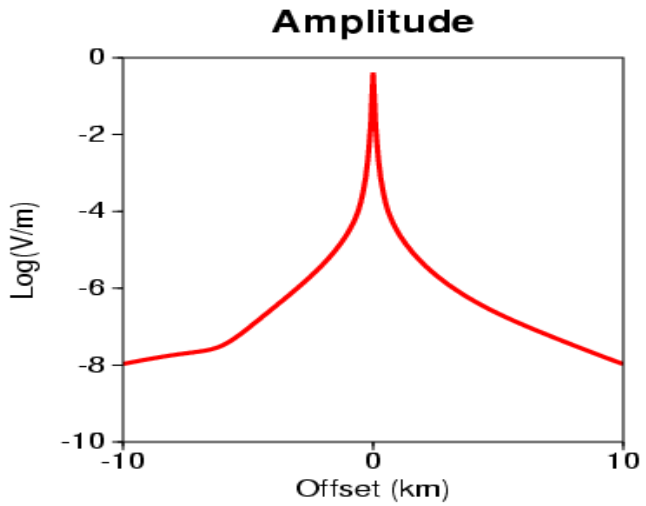

Intermediate resistivity

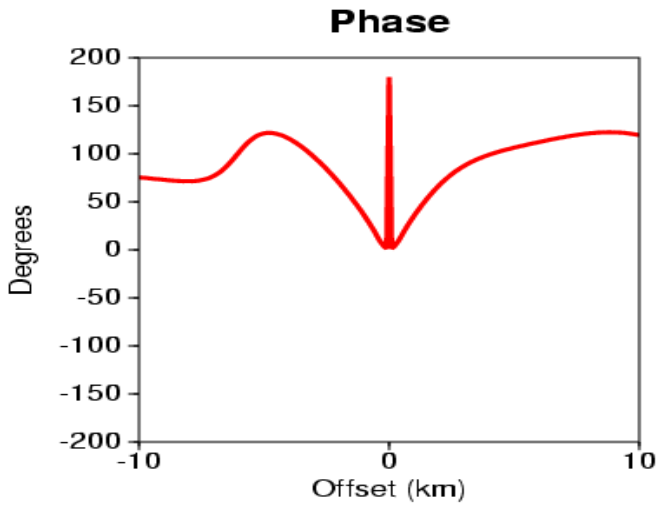

Amplitude
0
-2
-4
-6
-8
-10
Log(V/m)
-10
0
10
Offset (km)


Phase
200
150
100
50
0
-50
-100
-150
-200
Degrees
-10
0
10
Offset (km)

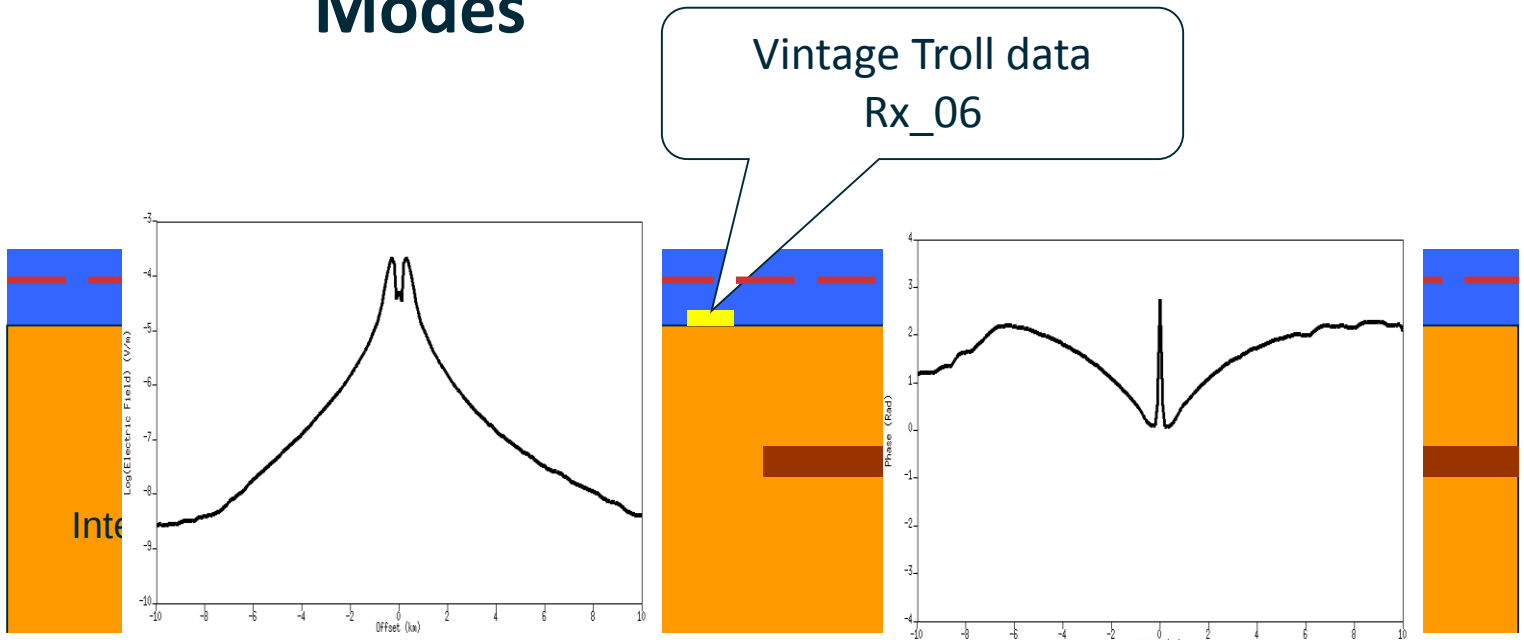
Modes

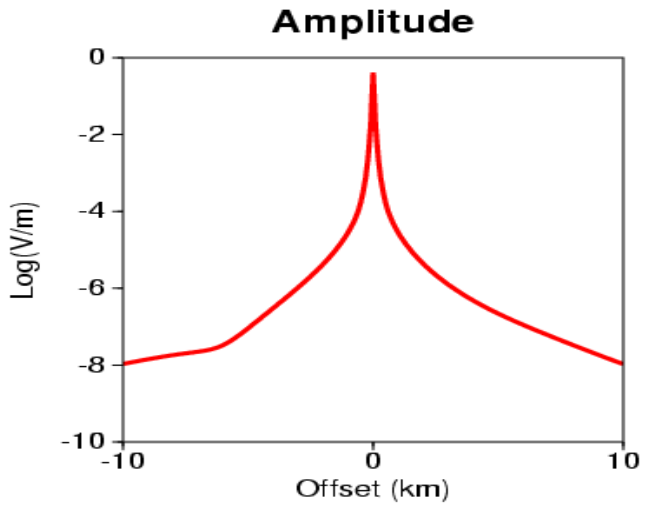

High resistivity
Intermediate resistivity

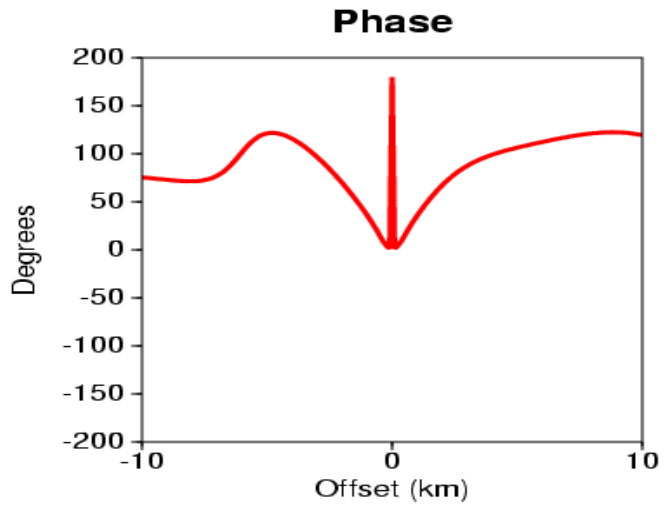

Amplitude
0
-2
-4
-6
-8
-10
Log(V/m)
-10
0
10
Offset (km)


Phase
200
150
100
50
0
-50
-100
-150
-200
Degrees
-10
0
10
Offset (km)

Modes


Vintage Troll data
Rx_06
Inte
Log(Electric Field) (V/m)
Offset (km)
Phase (Rad)
Offset (km)


Amplitude
Log(V/m)
0
-2
-4
-6
-8
-10
-10
0
10
Offset (km)

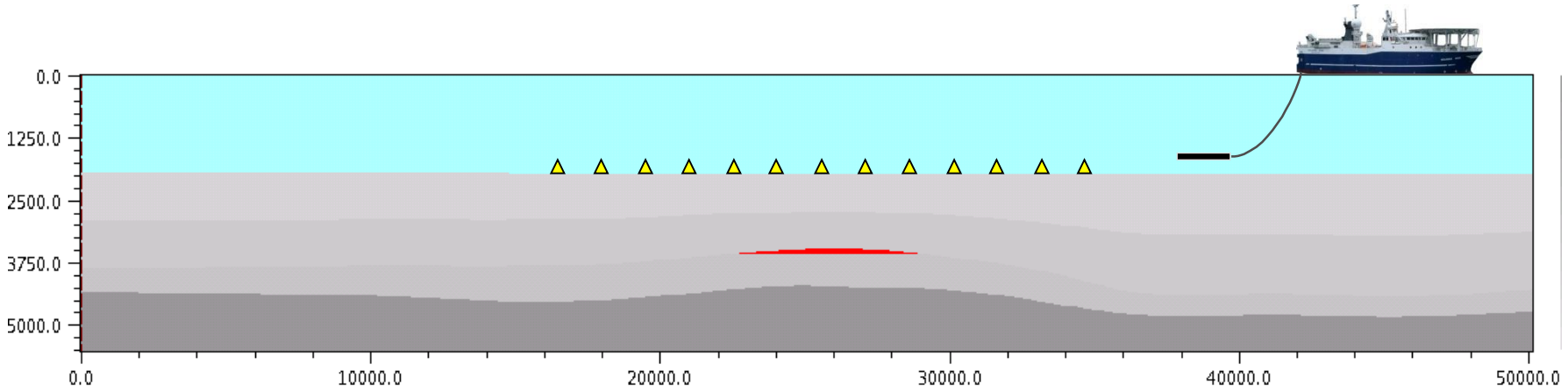

Phase
Degrees
200
150
100
50
0
-50
-100
-150
-200
-10
0
10
Offset (km)

Indications of high resistivity:

- Increased amplitude
- Reduced phase gradient as function of source-receiver offset

Normalized plots

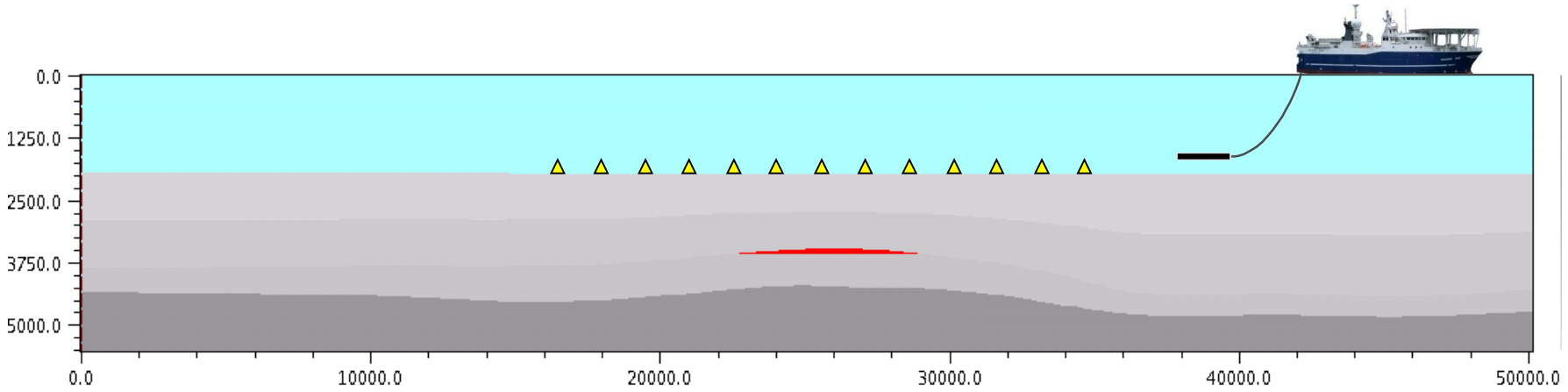
EM attributes



Normalized Magnitude (NM)

Phase difference (PD)

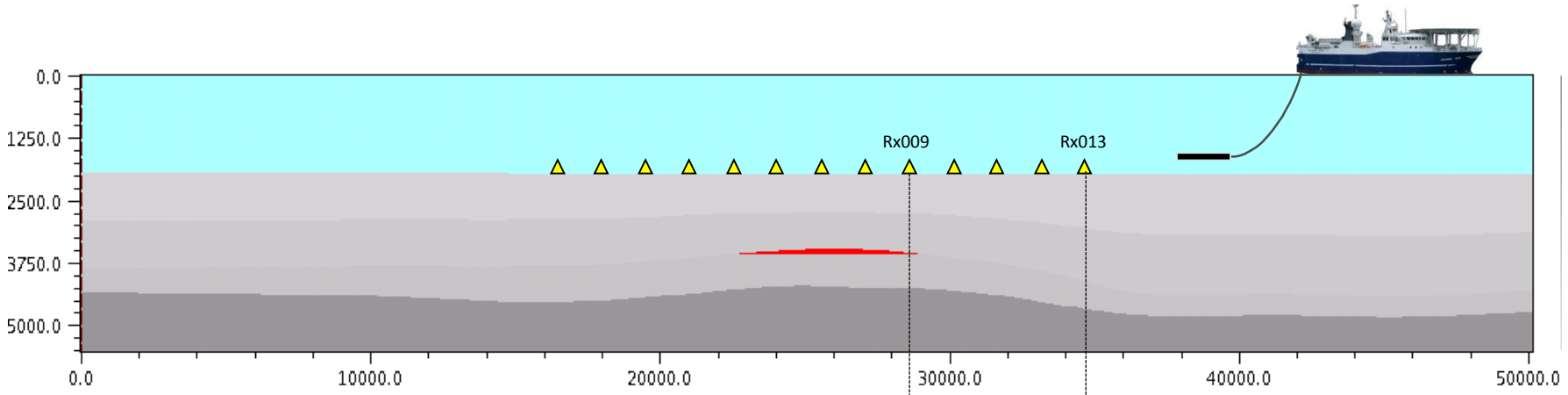
EM attributes

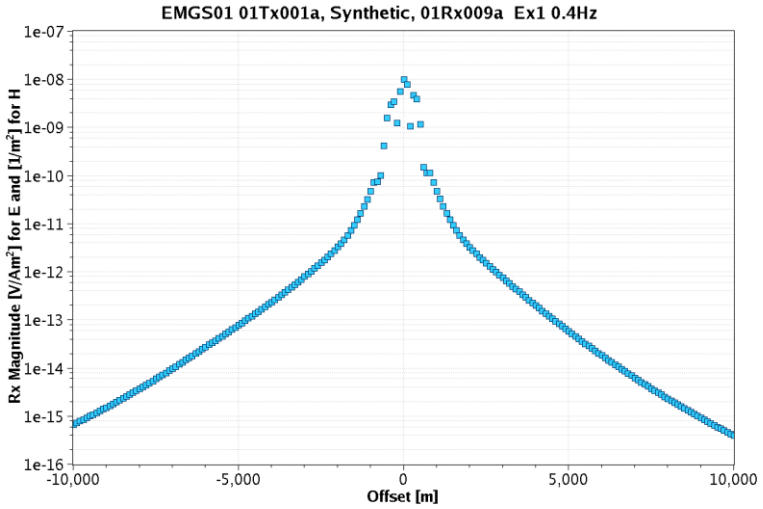


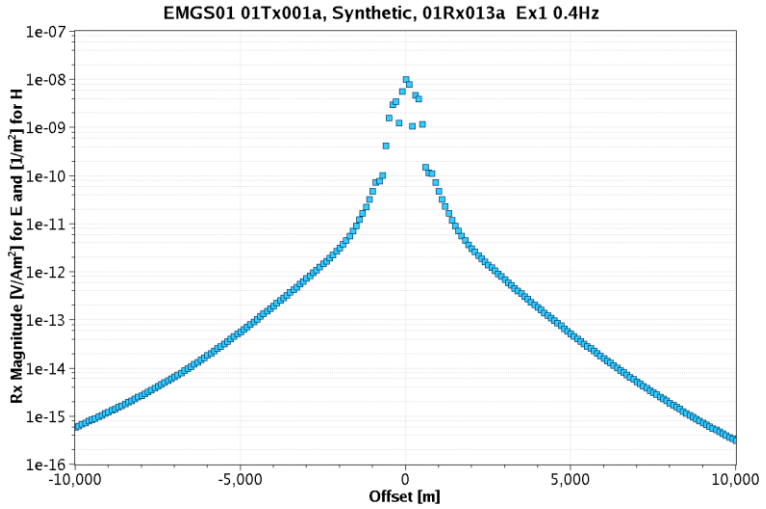
Normalized Magnitude (NM)

Phase difference (PD)

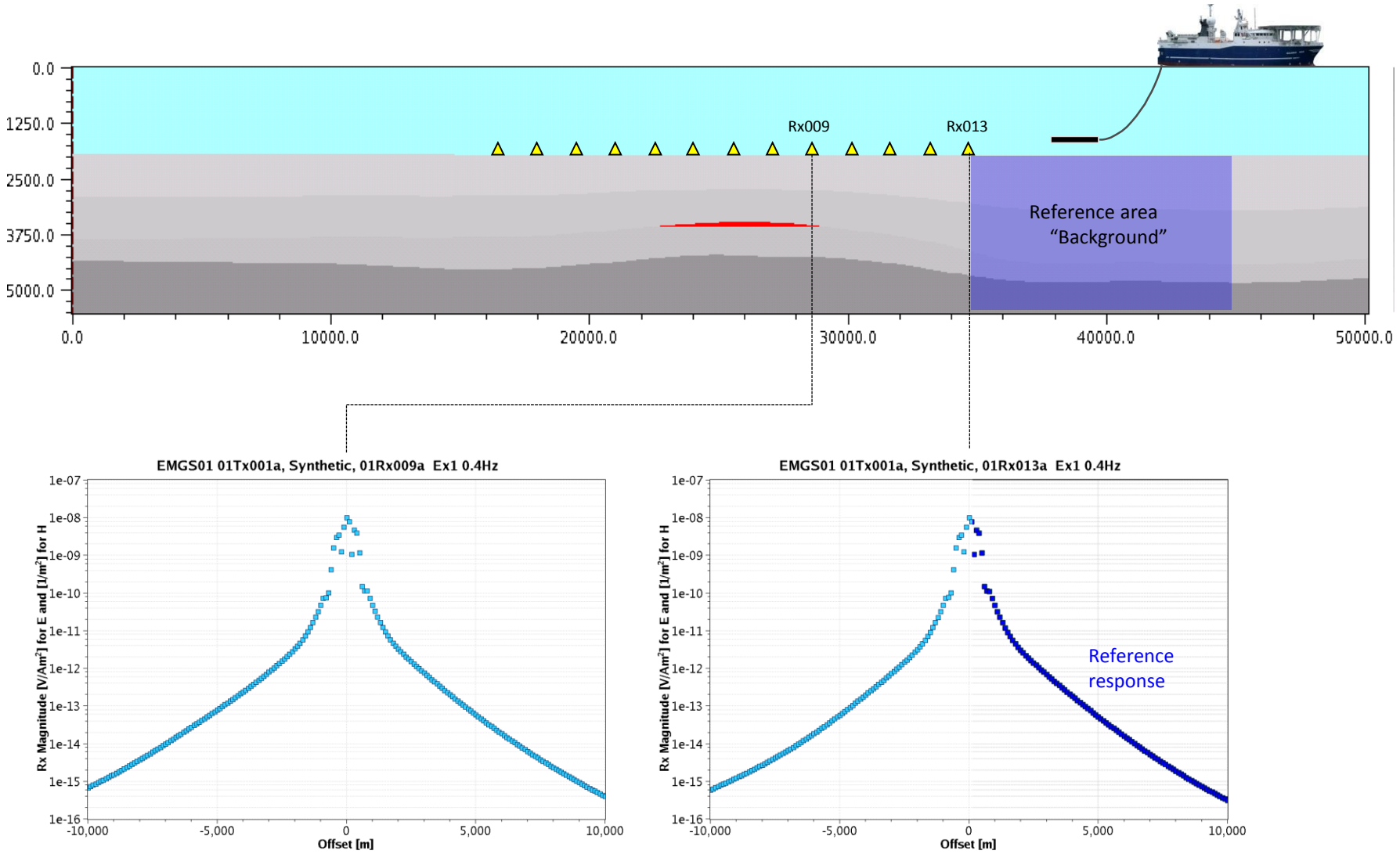
Magnitude versus Offset



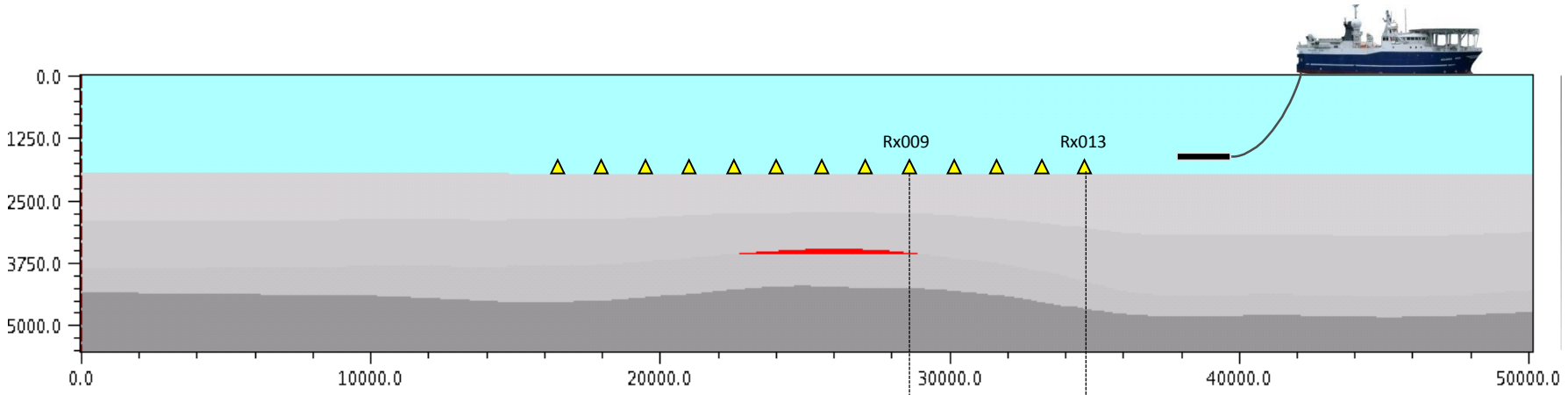


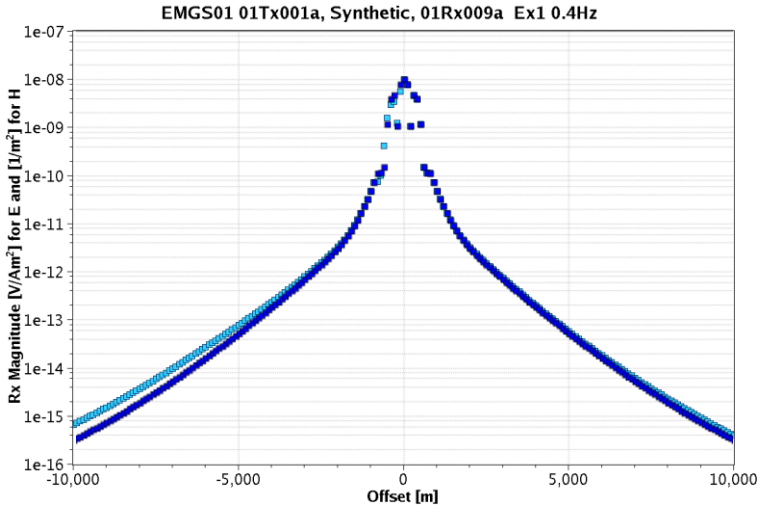


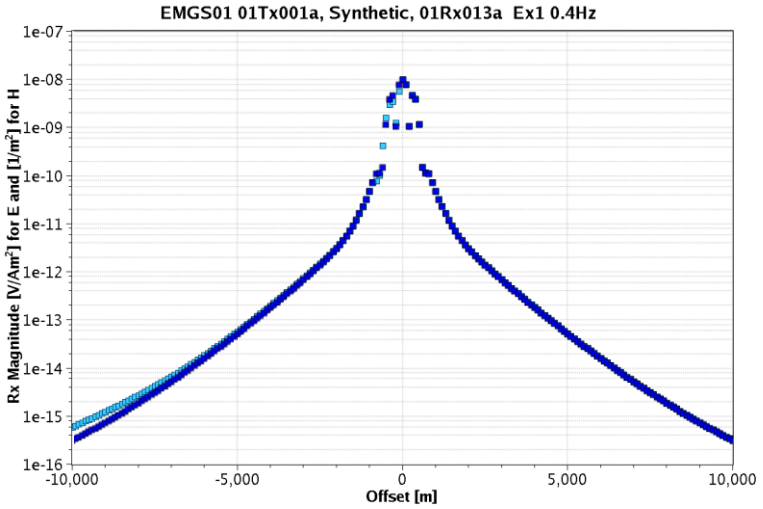
Selecting a reference response



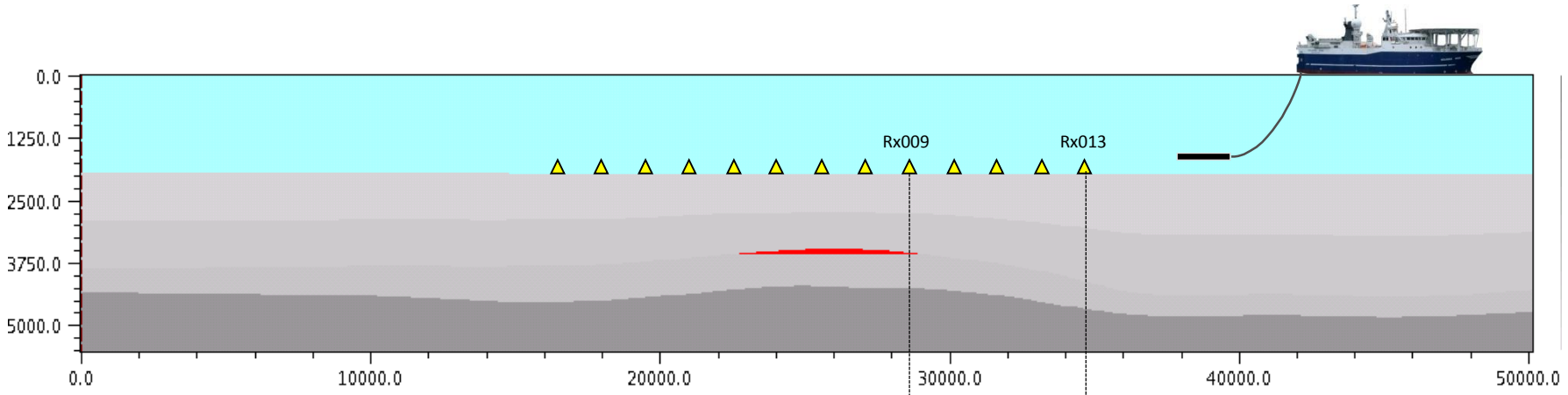
MvO comparison

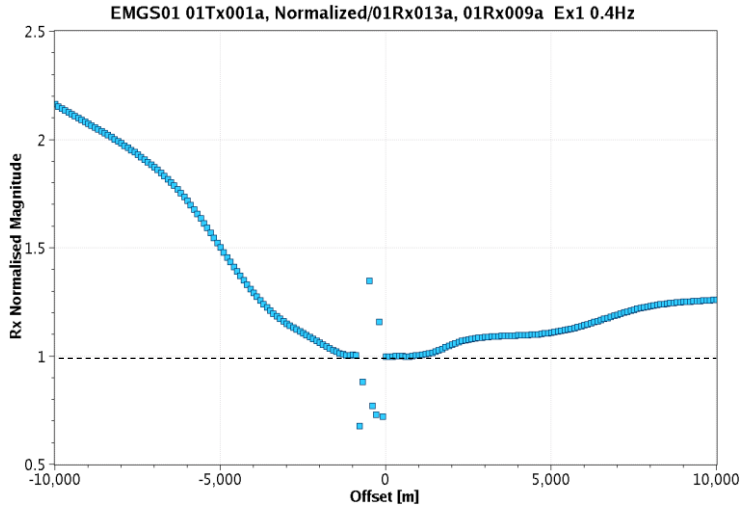


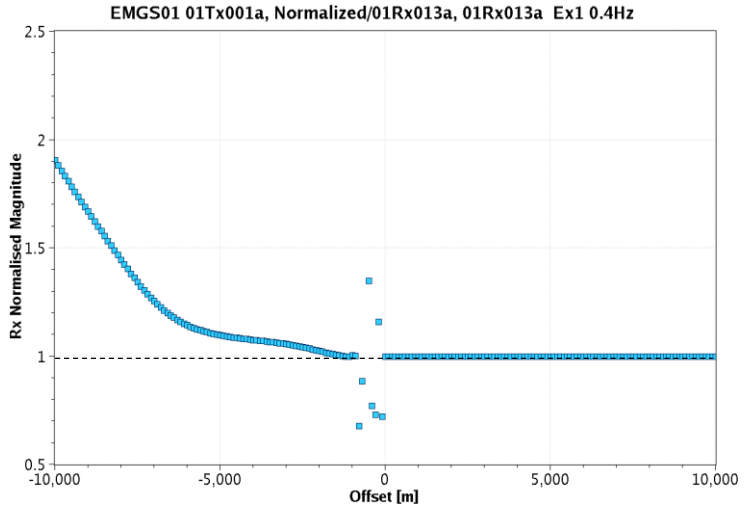




Normalized Magnitude versus Offset (NMvO)



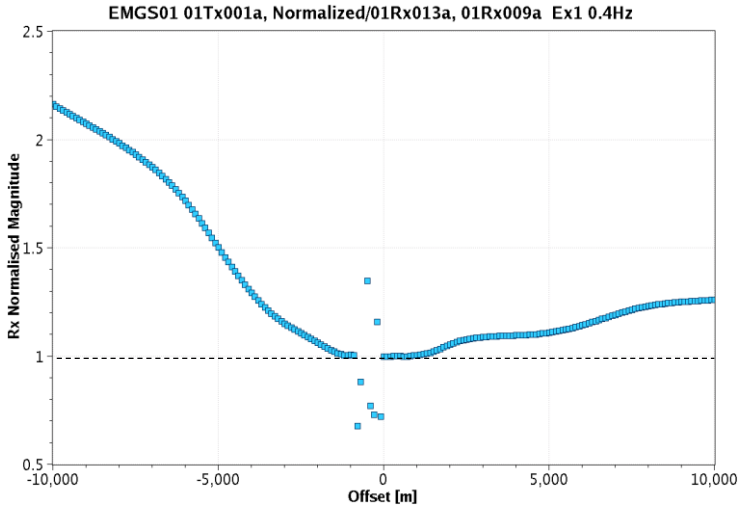


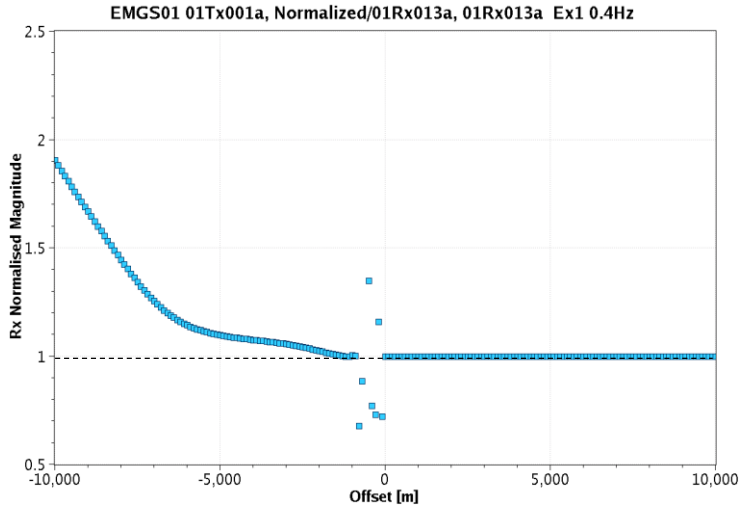


Normalized Magnitude versus Offset (NMvO)

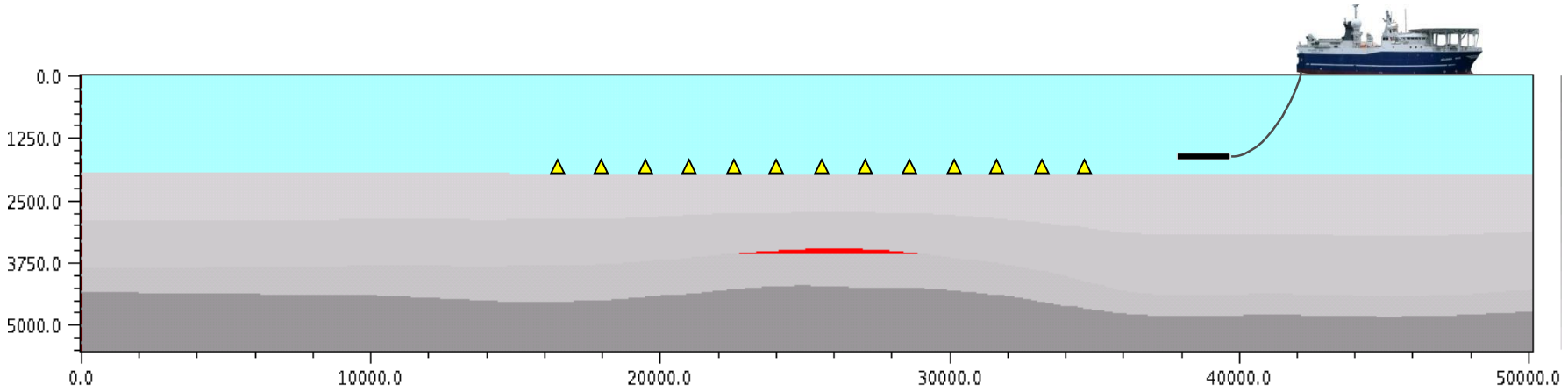
Modulus (magnitude) of the normalized response.

$$\mathrm{NM} = \left| \frac{E_i}{E_i^{\mathrm{reference}}} \right| = \frac{|E_i|}{\left|E_i^{\mathrm{reference}}\right|}$$





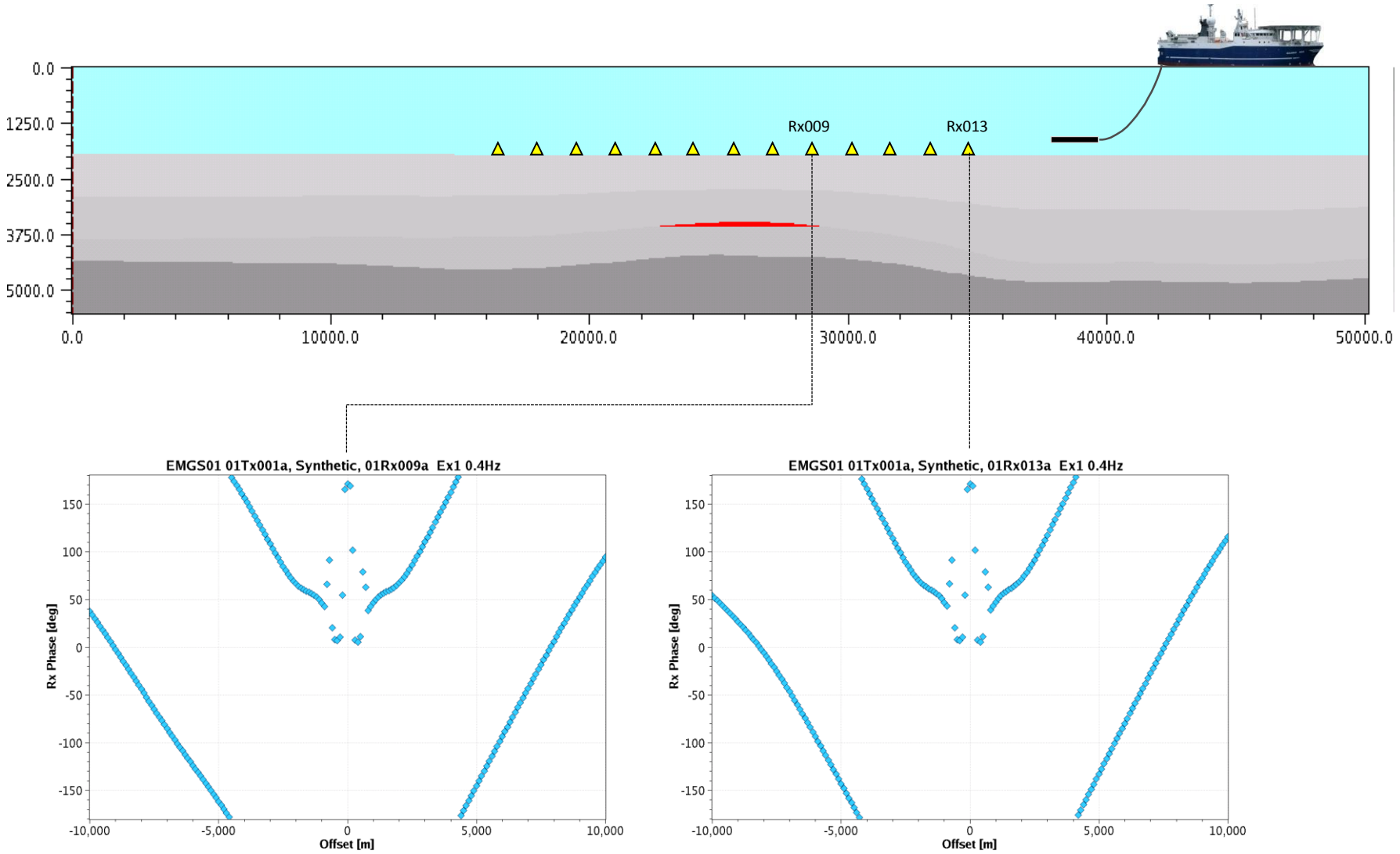
EM attributes



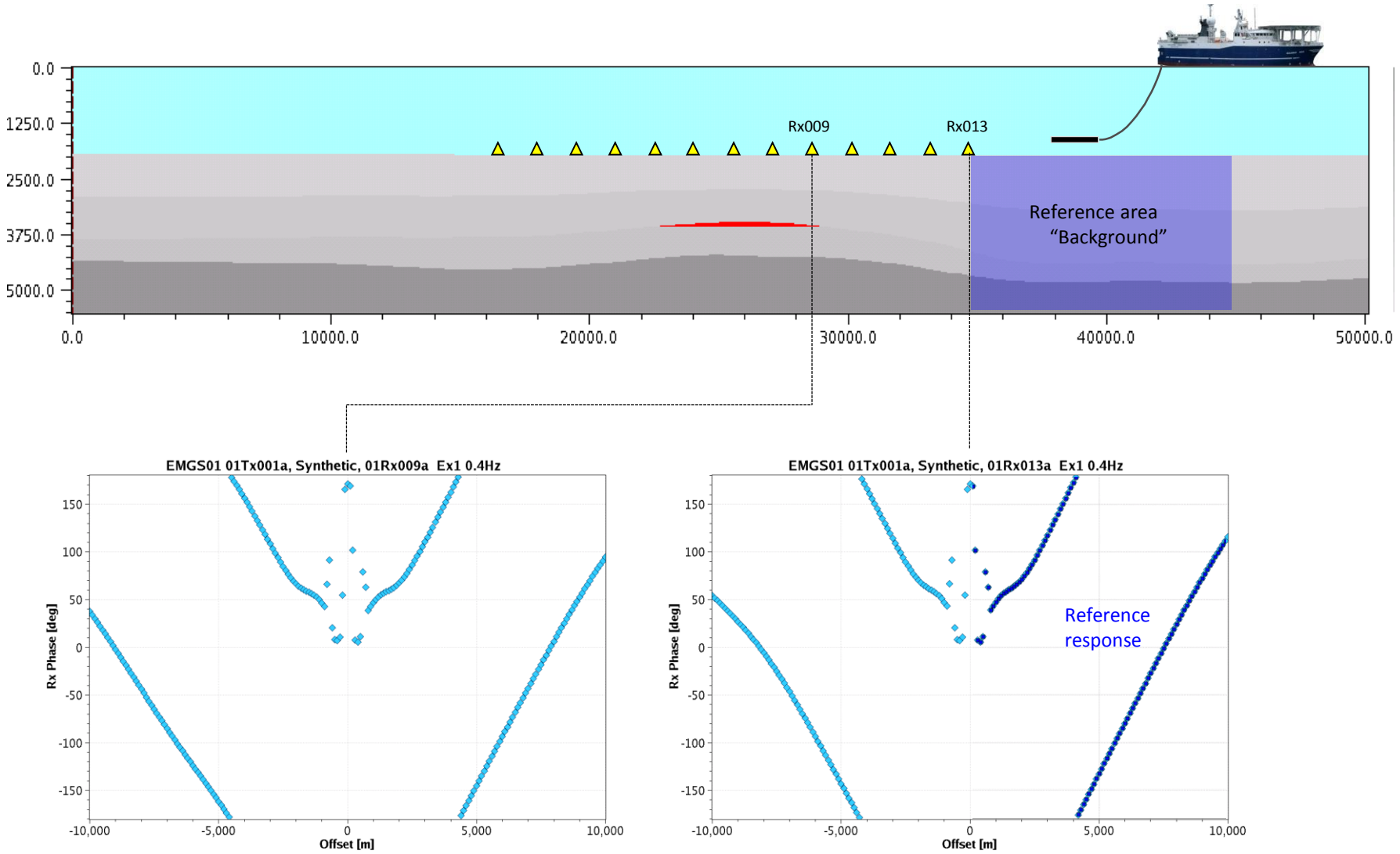
Normalized Magnitude (NM)

Phase difference (PD)

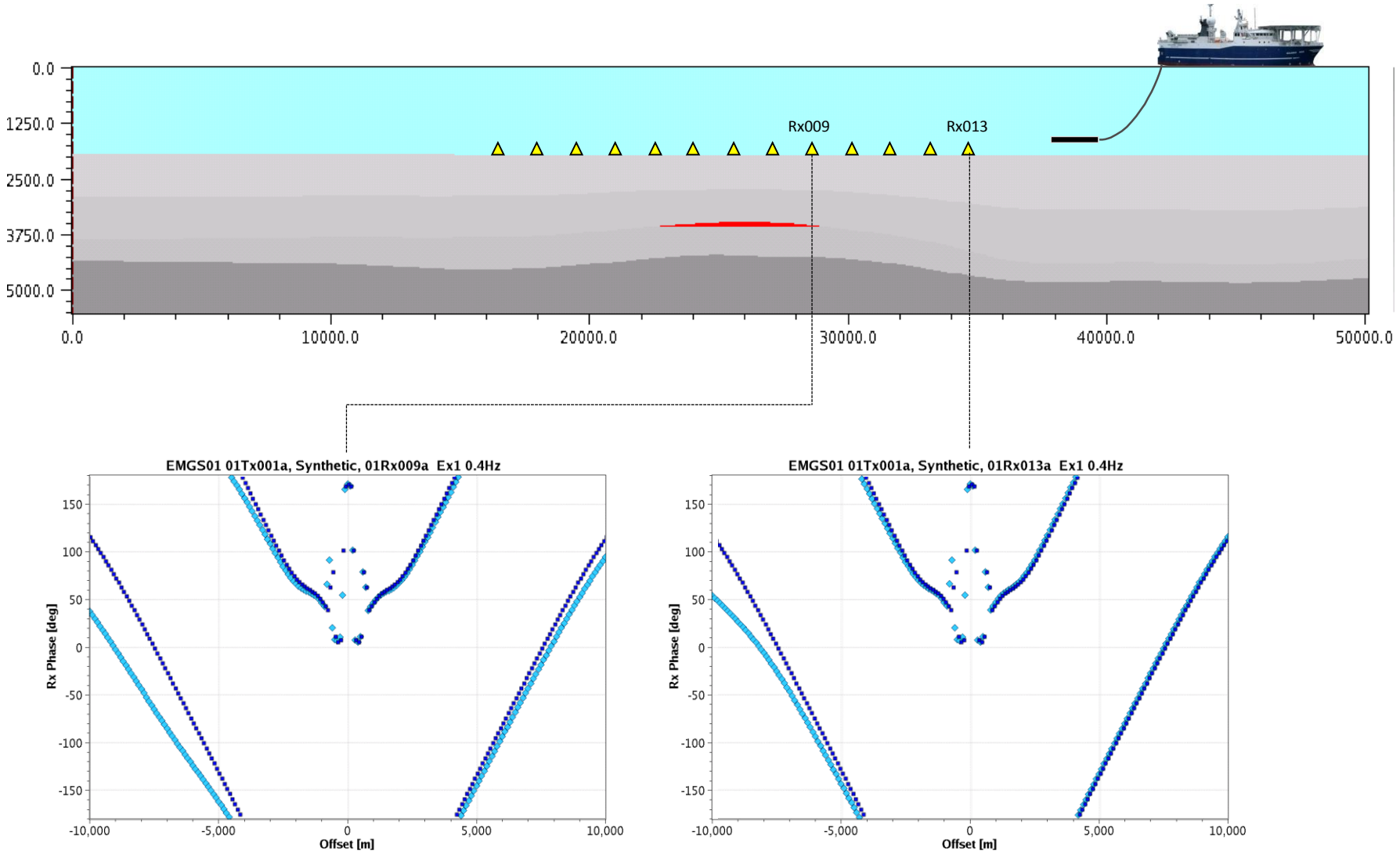
Phase versus Offset



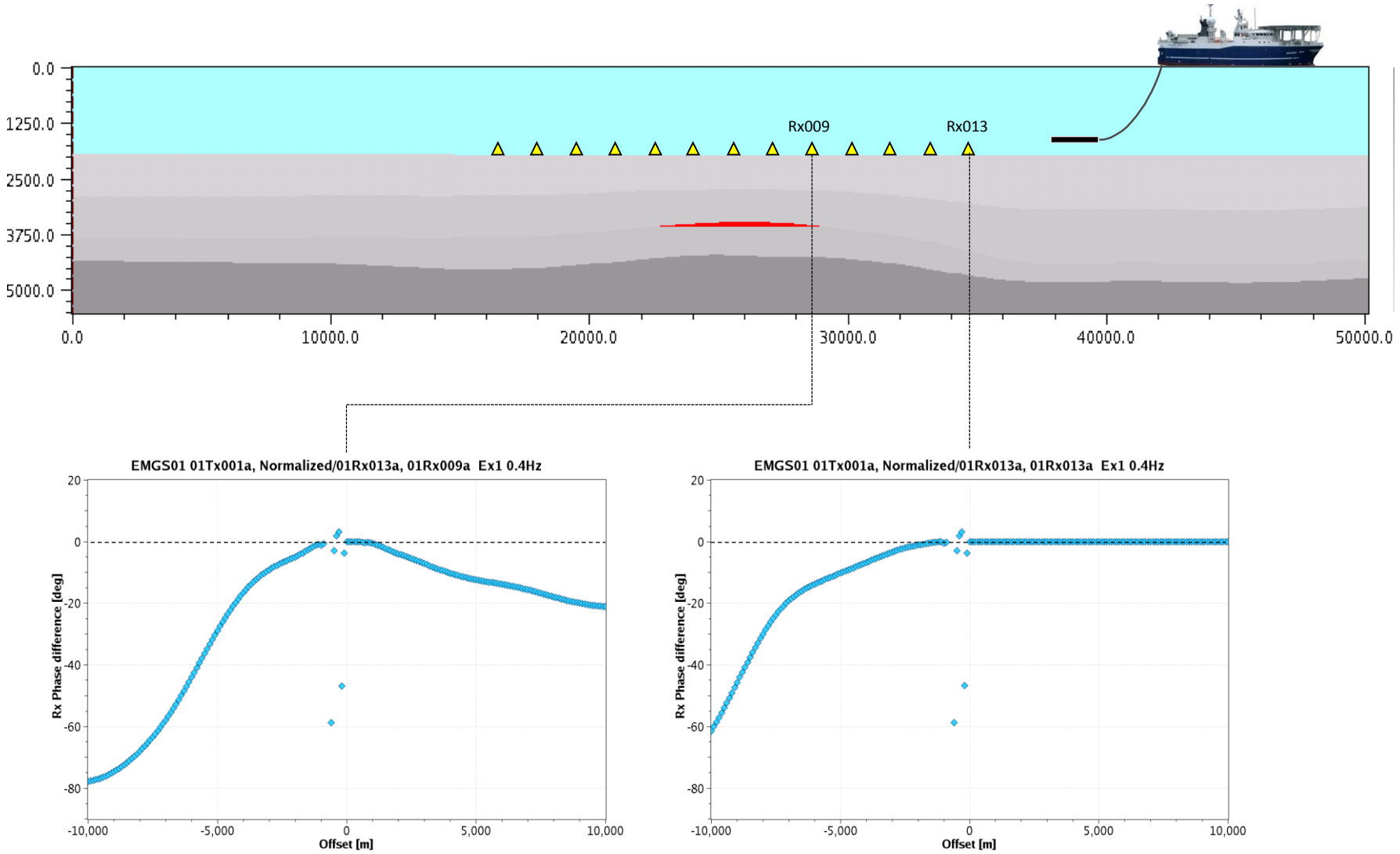
Selecting a reference response



PvO comparison



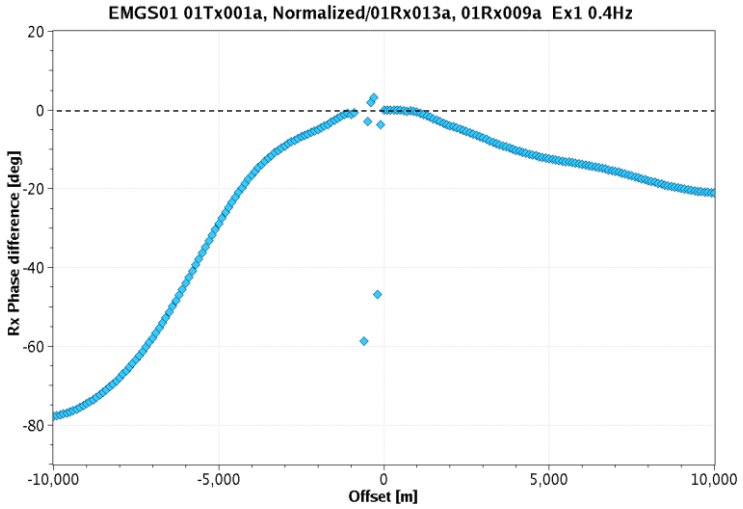
Phase Difference versus Offset (PDVO)

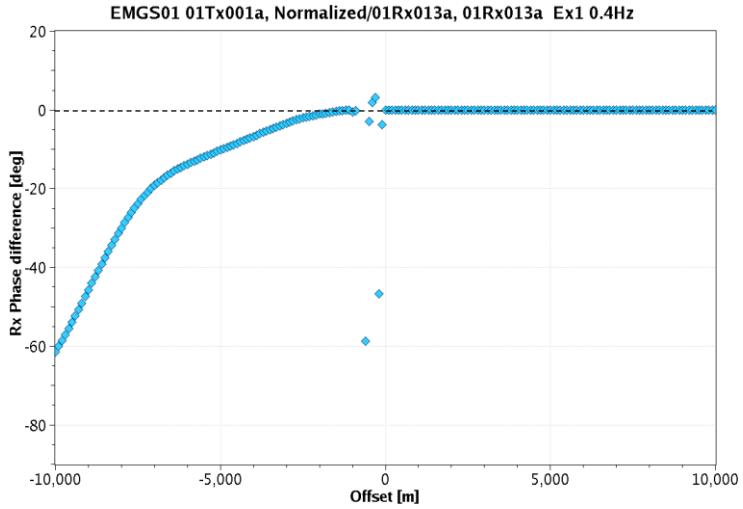


Phase Difference versus Offset (PDVO)

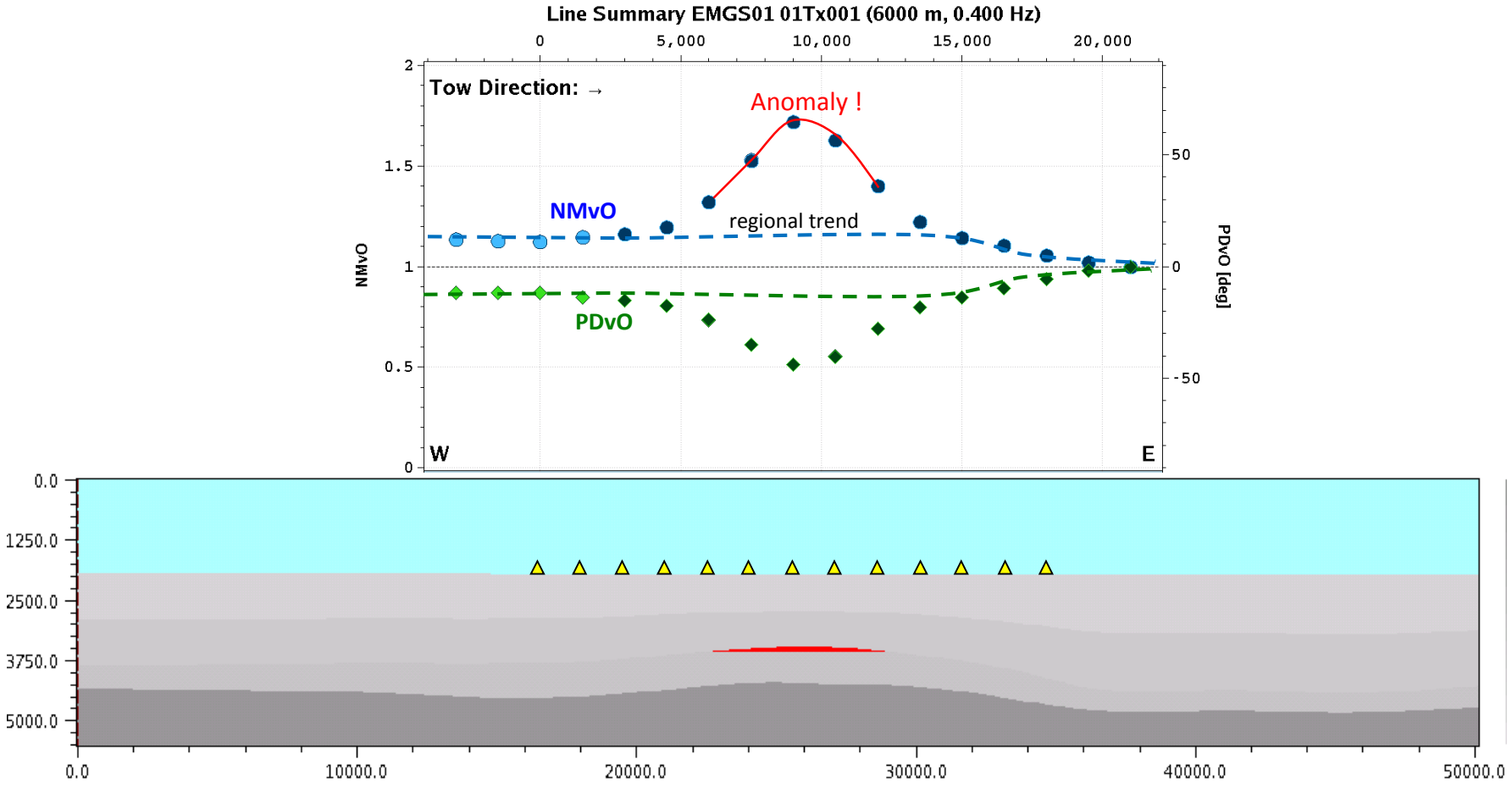
Argument (phase) of the normalized response.

$$\mathrm{PD} = \arg\left(\frac{E_{\mathrm{i}}}{E_{\mathrm{i}}^{\mathrm{reference}}}\right) = \arg(E_{\mathrm{i}}) - \arg(E_{\mathrm{i}}^{\mathrm{reference}})$$





Line summary



Data example Troll Western Gas Province

The Troll field is the largest gas discovery on the Norwegian shelf.

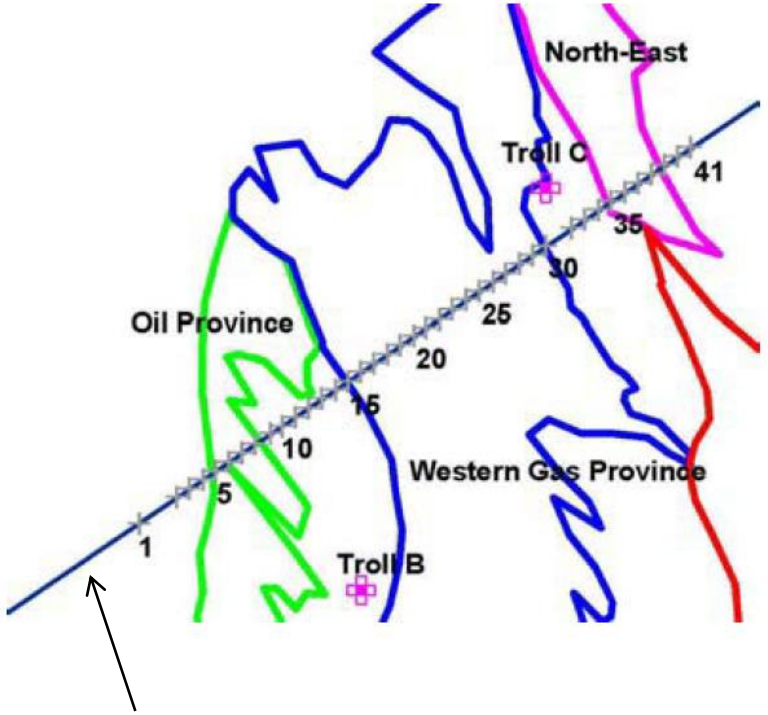
- Gas column thickness: ~300 m
- Burial depth: ~1000 m below seabed
- Resistivity of gas filled reservoir: up to 250 Ωm

→ Excellent target for CSEM

Calibration survey acquired in 2003

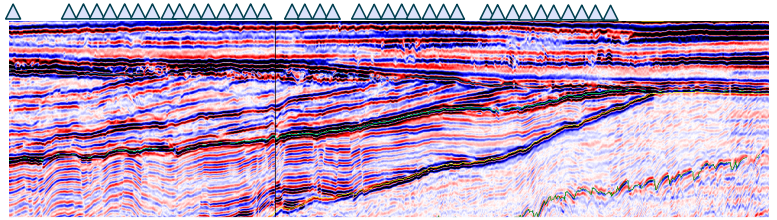
- 41 receivers
- Square wave source current:
  - 0.25 Hz (base frequency)
  - 0.75 Hz, 1.25 Hz, … (odd harmonics)

In the following slides, the 0.25Hz electric inline response is analyzed.



Data example Troll Western Gas Province

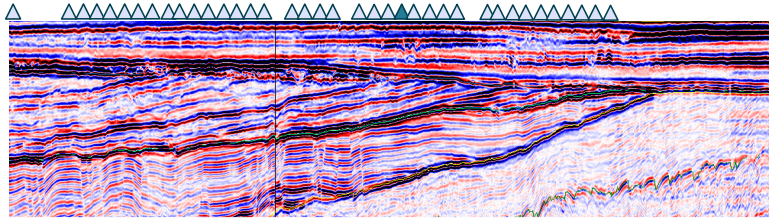
Midpoint-offset domain sorting



With courtesy of Statoil

Data example Troll Western Gas Province

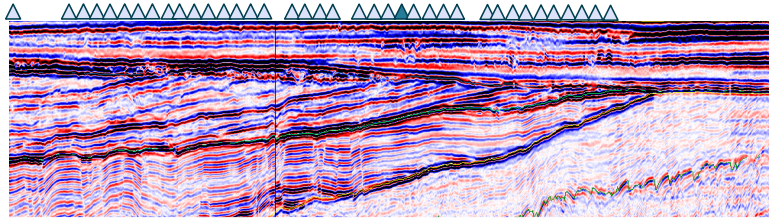
Midpoint-offset domain sorting



With courtesy of Statoil

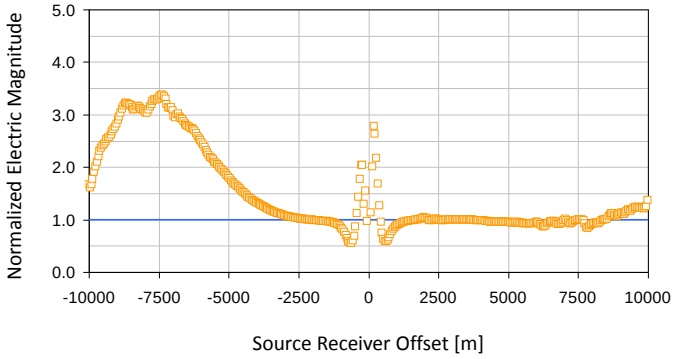
Data example Troll Western Gas Province

Midpoint-offset domain sorting



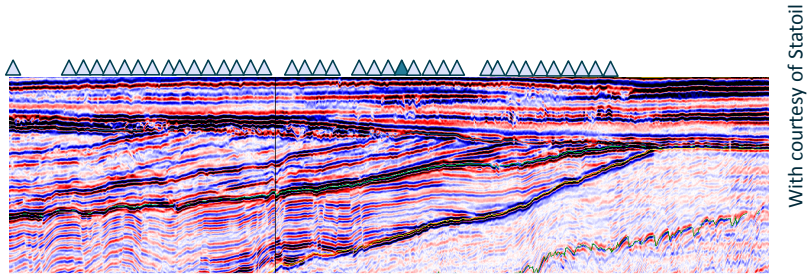
With courtesy of Statoil

Receiver amplitudes normalized by reference amplitudes

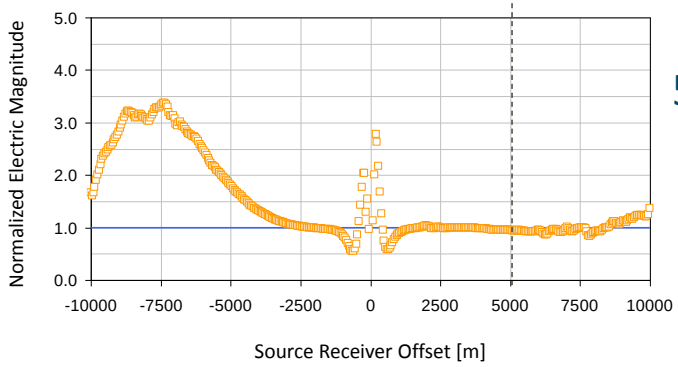


Data example Troll Western Gas Province

Midpoint-offset domain sorting



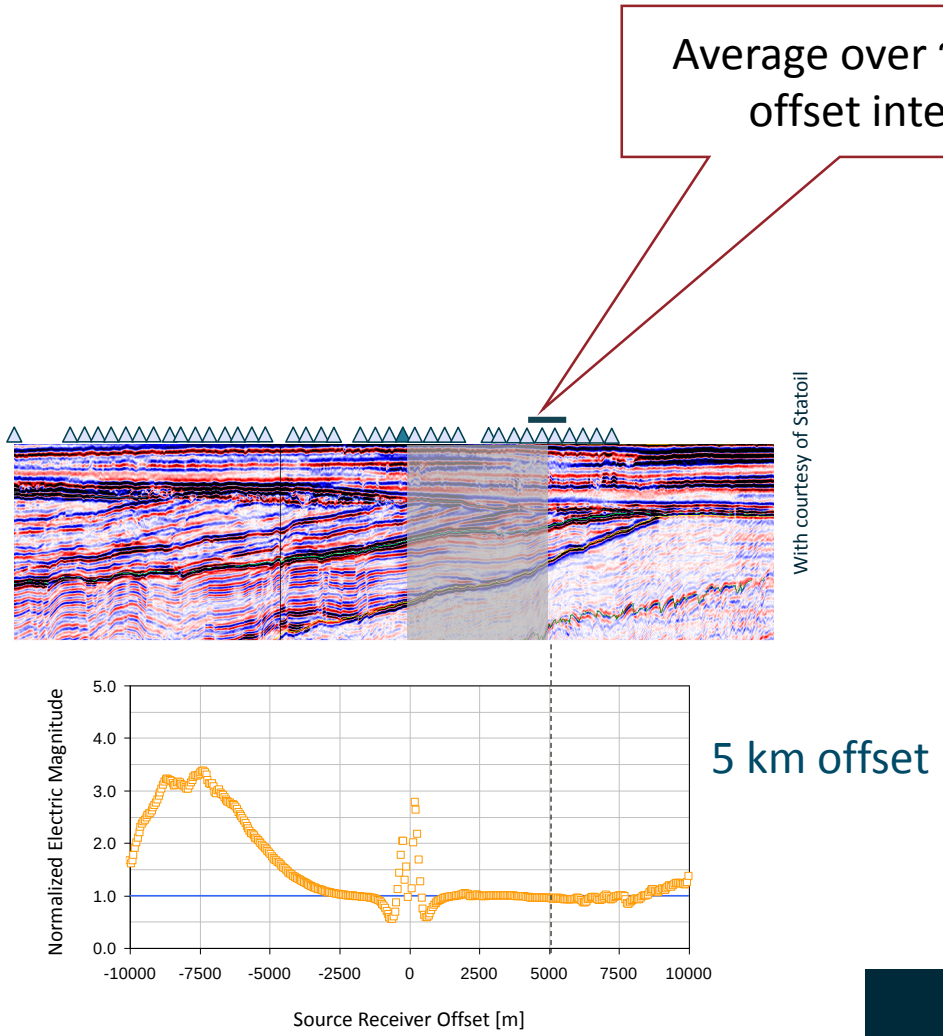
With courtesy of Statoil



5 km offset

Data example Troll Western Gas Province

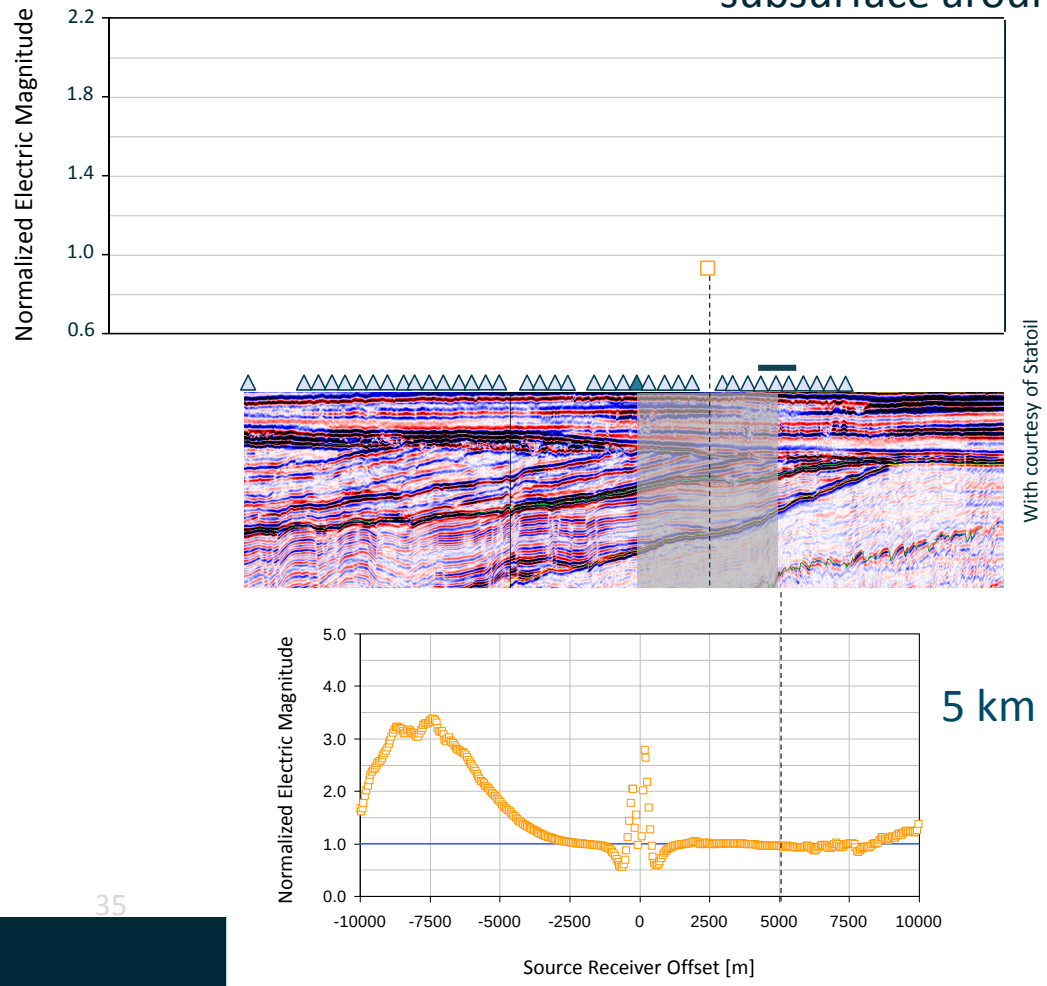
Midpoint-offset domain sorting



Data example Troll Western Gas Province

Midpoint-offset domain sorting

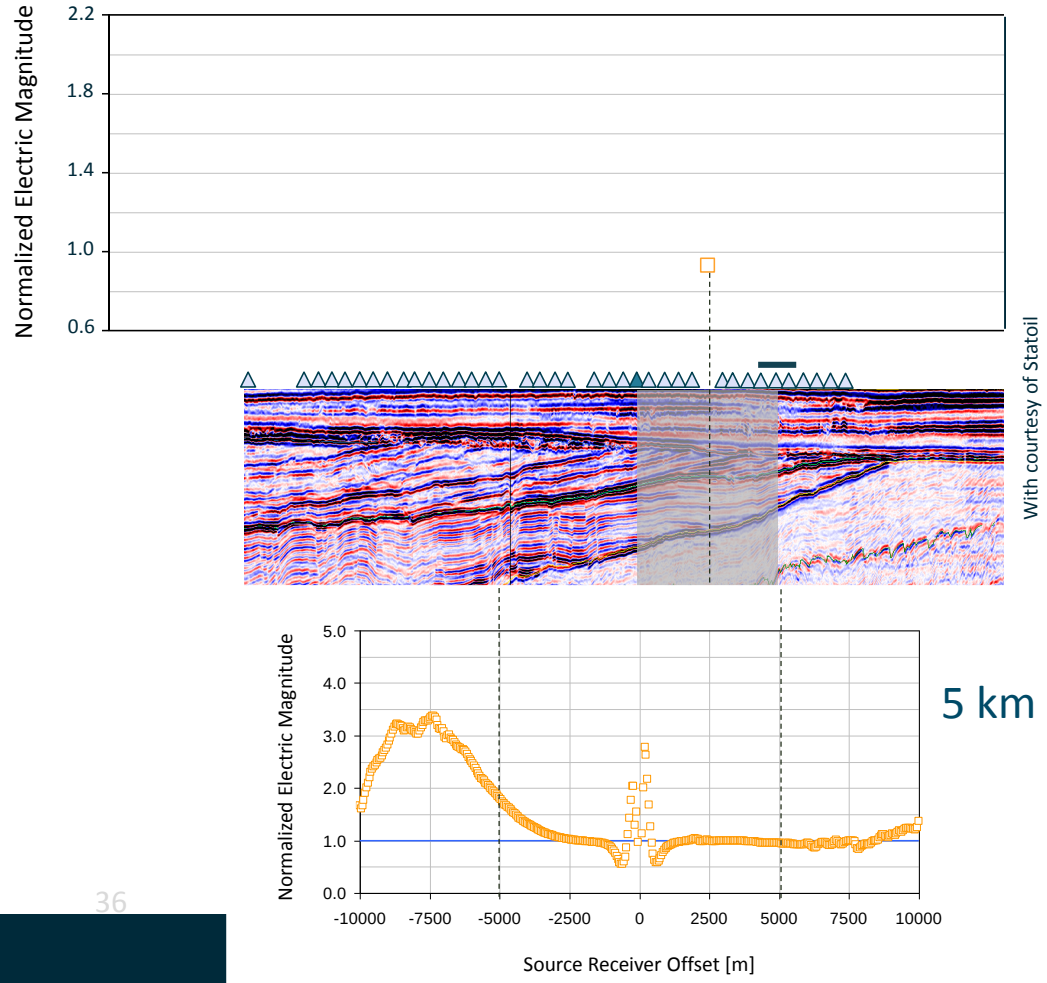
A rough basic approximation:
The response at 5 km offset is most sensitive to subsurface around 2.5 km.



5 km offset

Data example Troll Western Gas Province

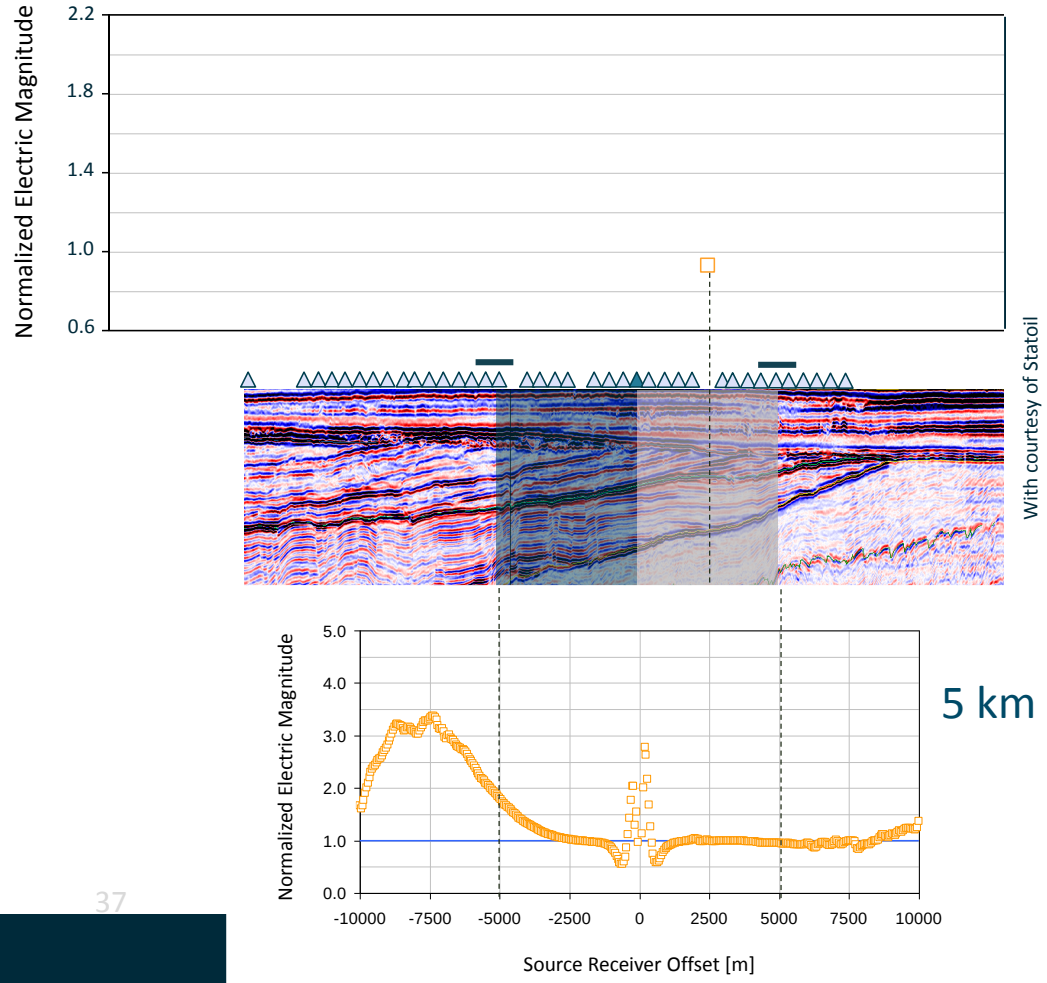
Midpoint-offset domain sorting



5 km offset

Data example Troll Western Gas Province

Midpoint-offset domain sorting

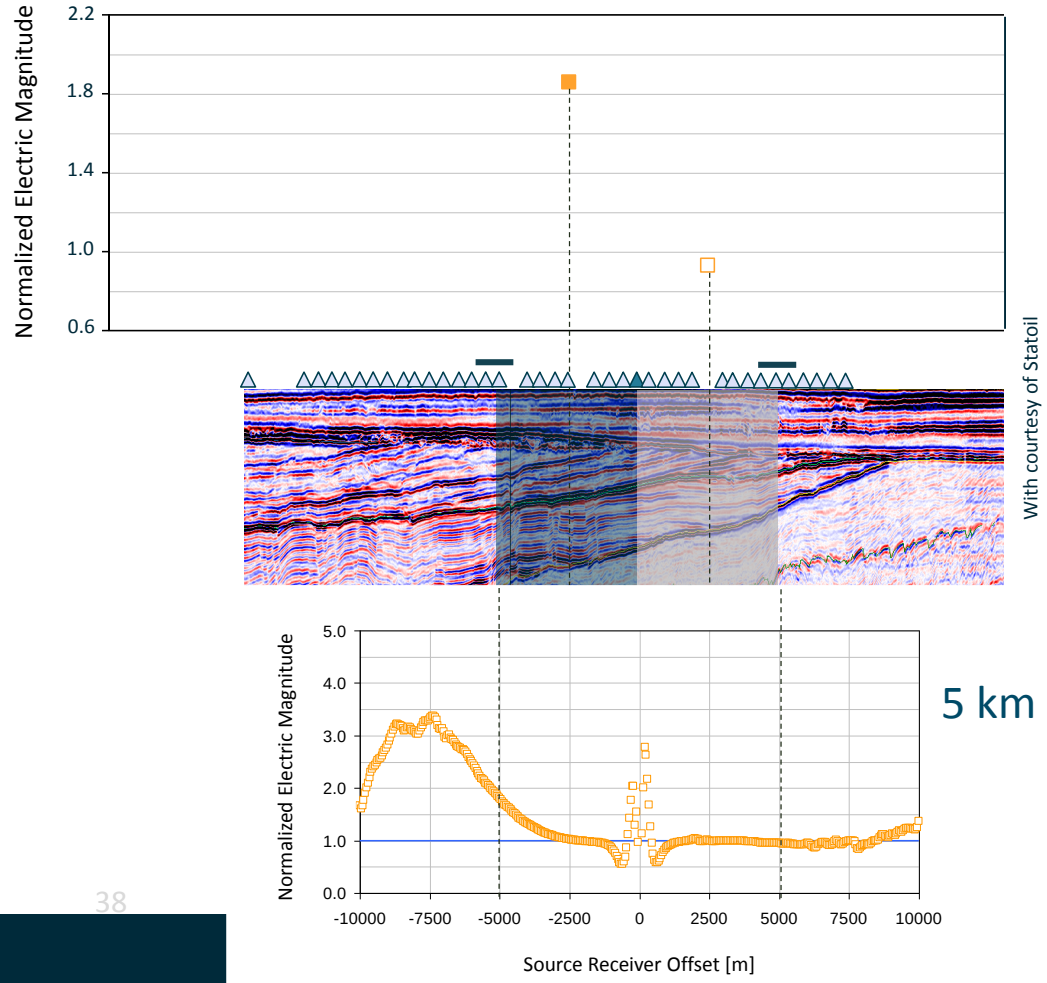


5 km offset

Data example Troll Western Gas Province

Midpoint-offset domain sorting

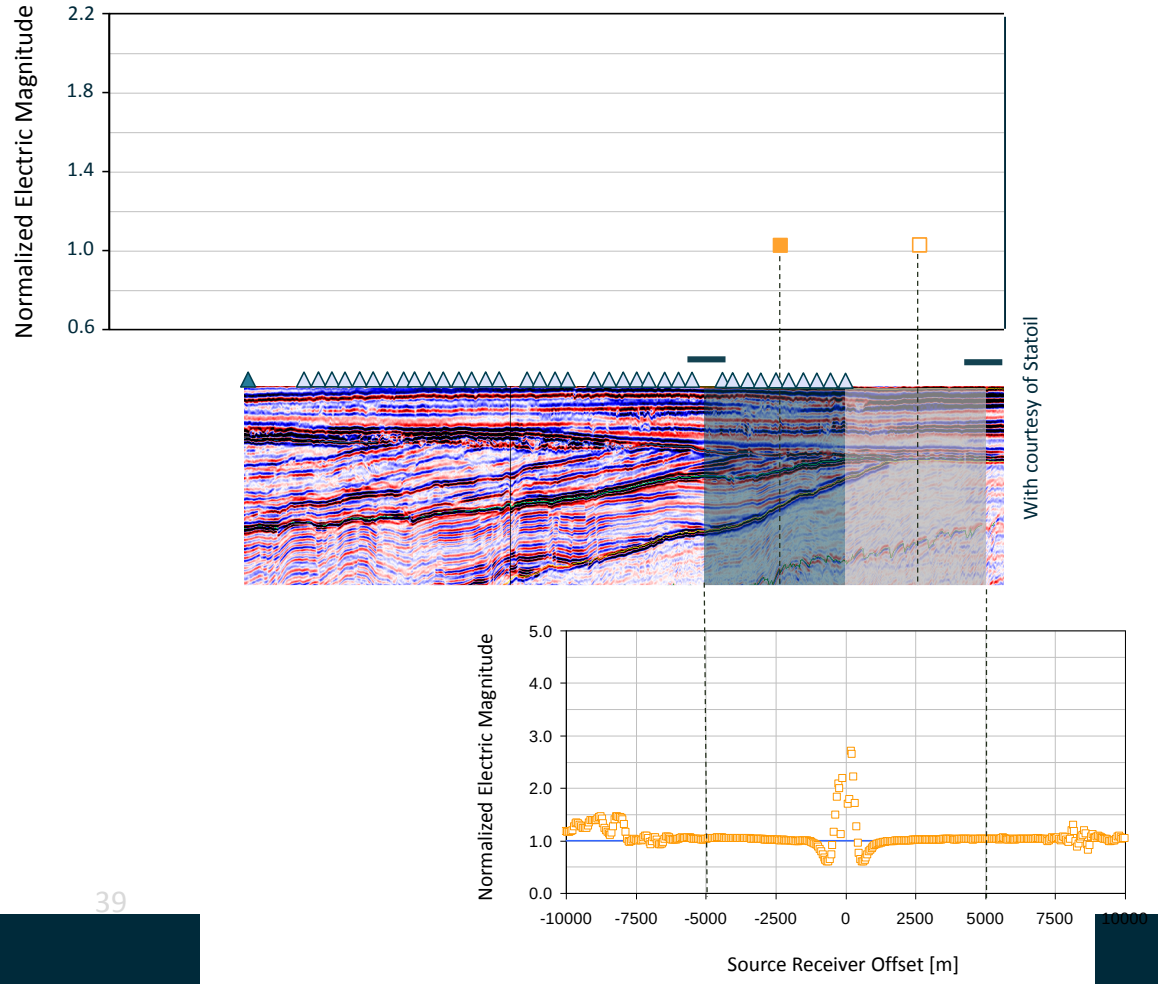
Each receiver give one CMP value for intow and one CMP value for outtow



5 km offset

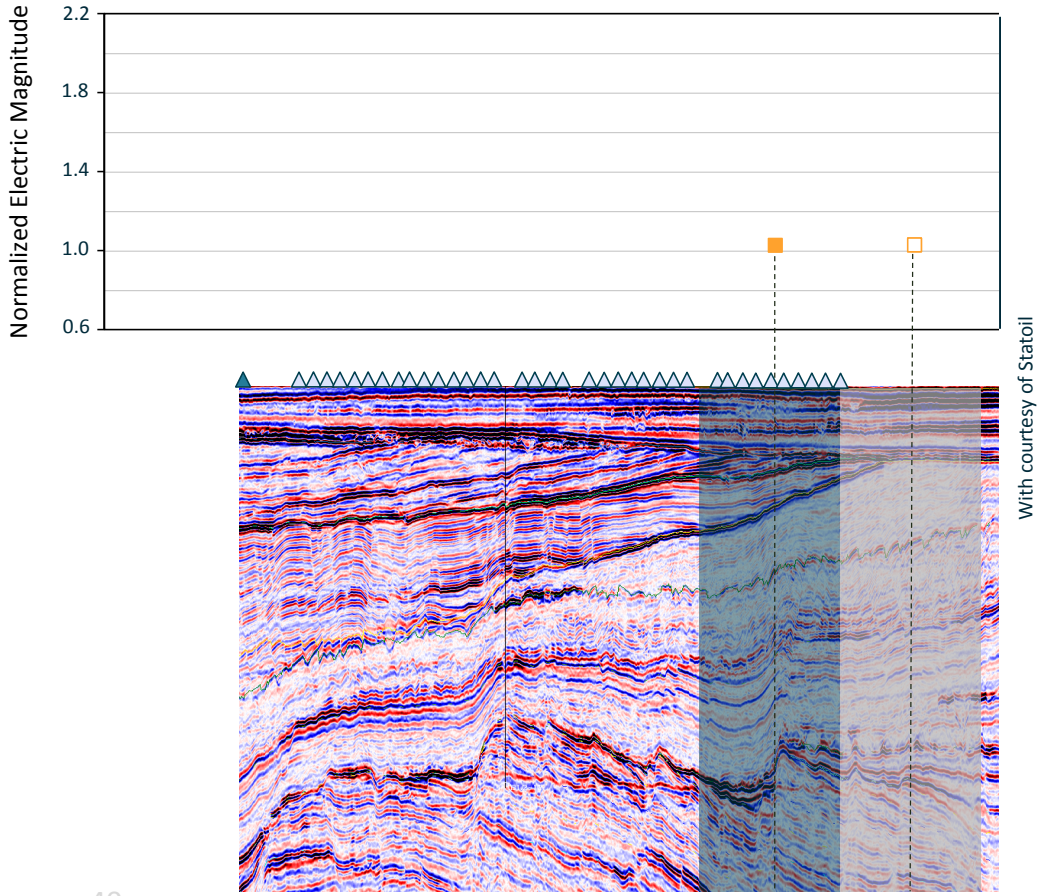
Data example Troll Western Gas Province

Midpoint-offset domain sorting



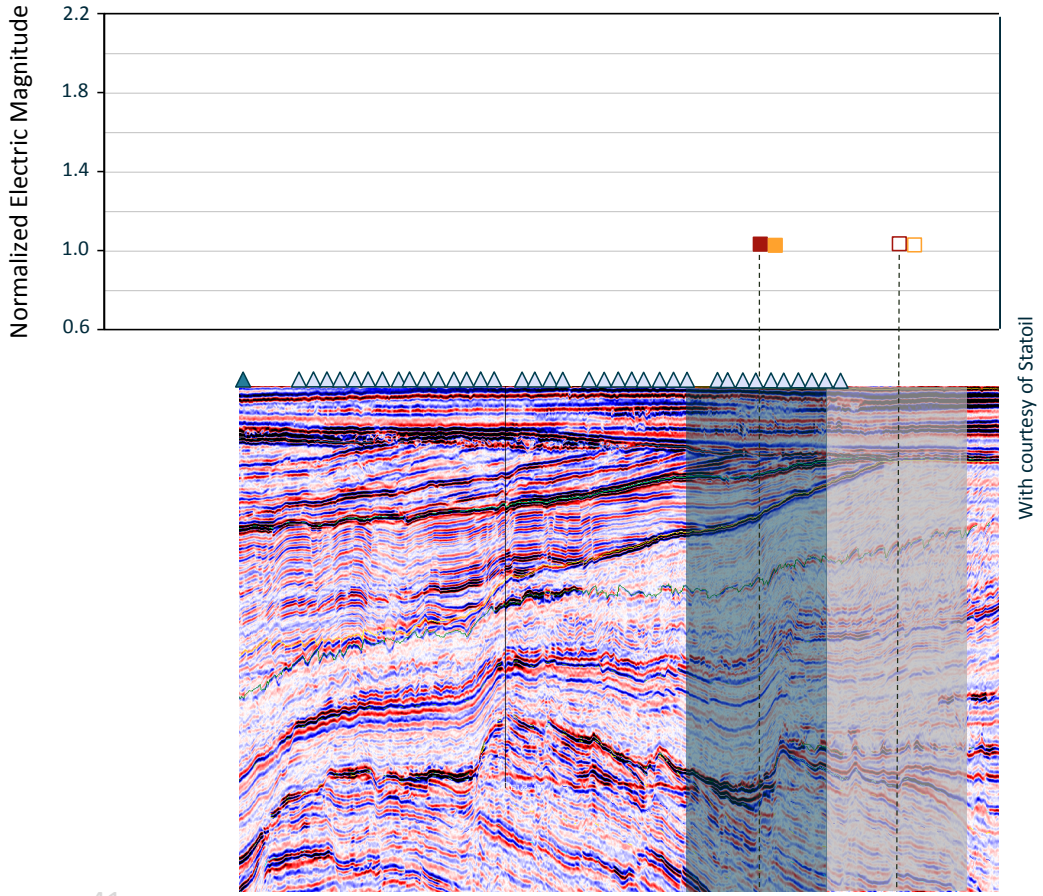
Data example Troll Western Gas Province

Midpoint-offset domain sorting



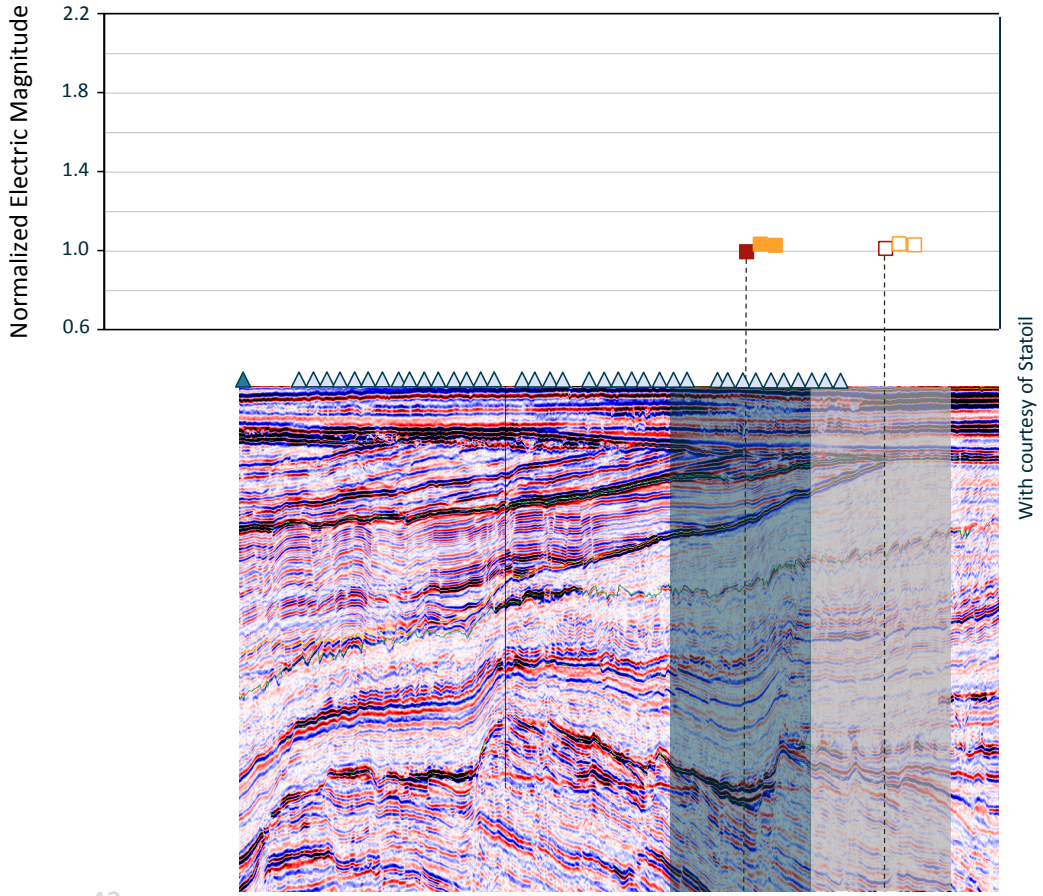
Data example Troll Western Gas Province

Midpoint-offset domain sorting



Data example Troll Western Gas Province

Midpoint-offset domain sorting



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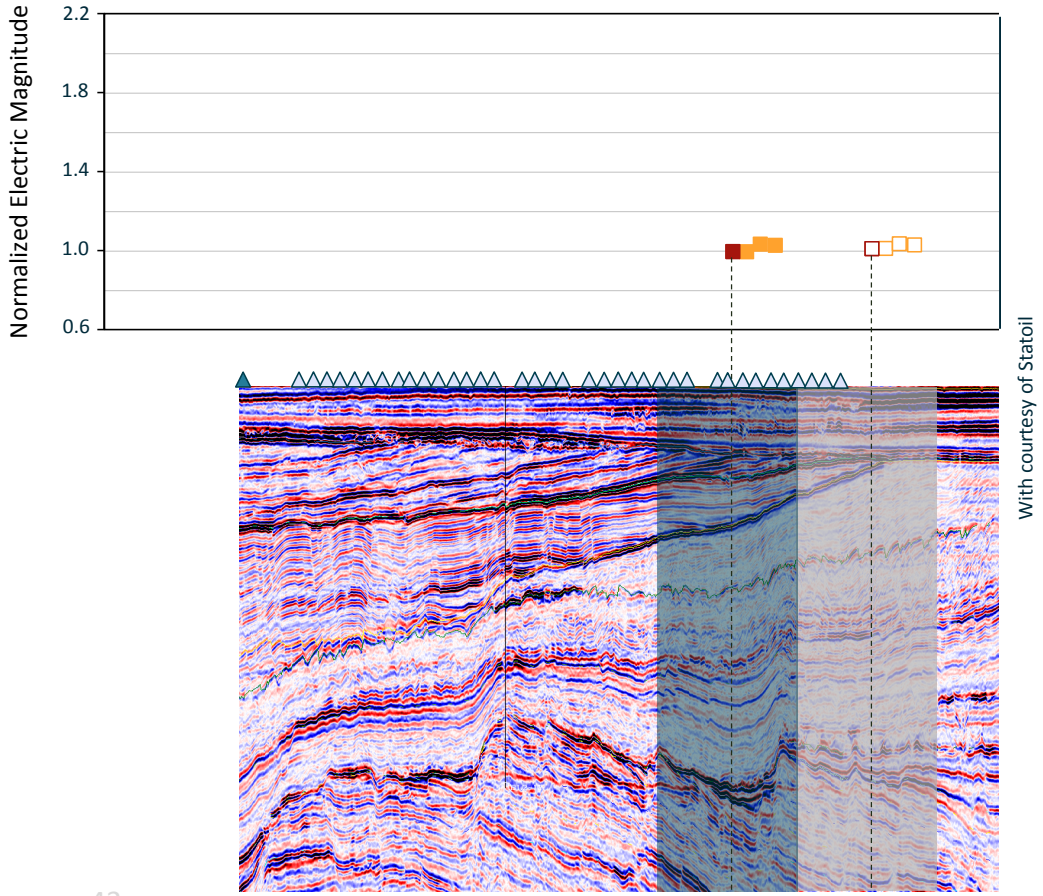
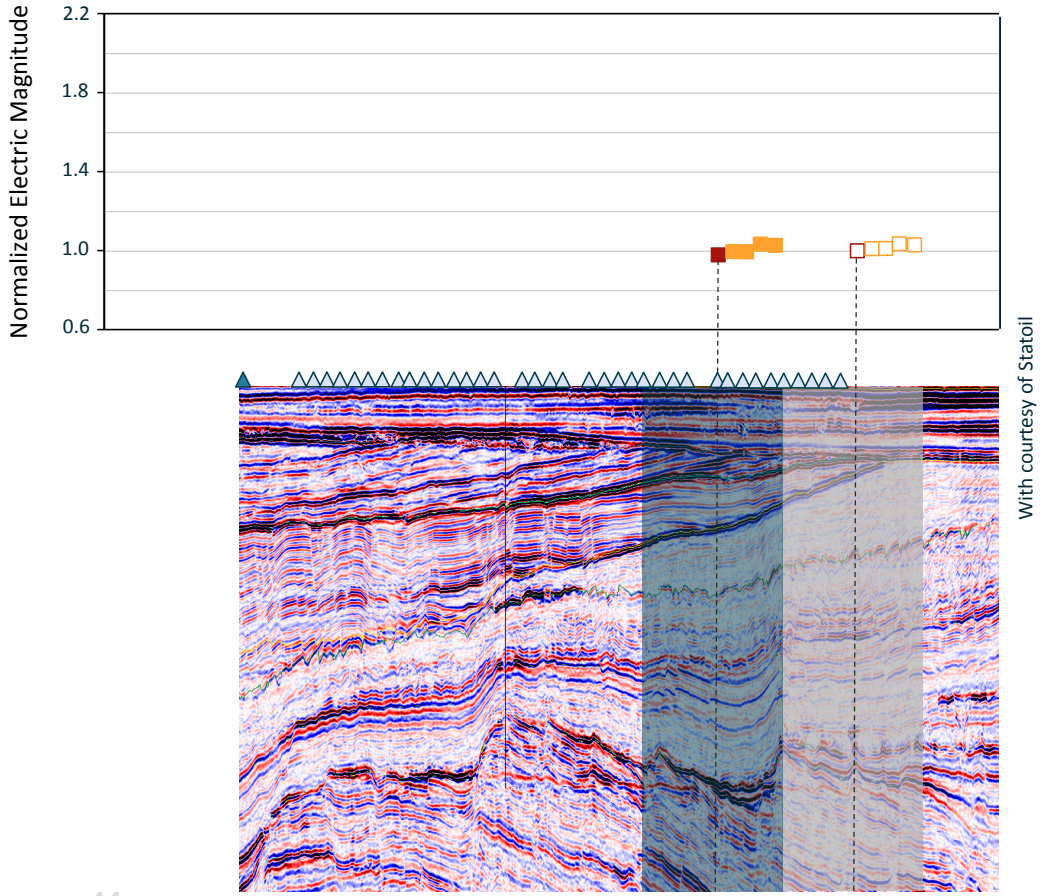


Data example Troll Western Gas Province

Midpoint-offset domain sorting



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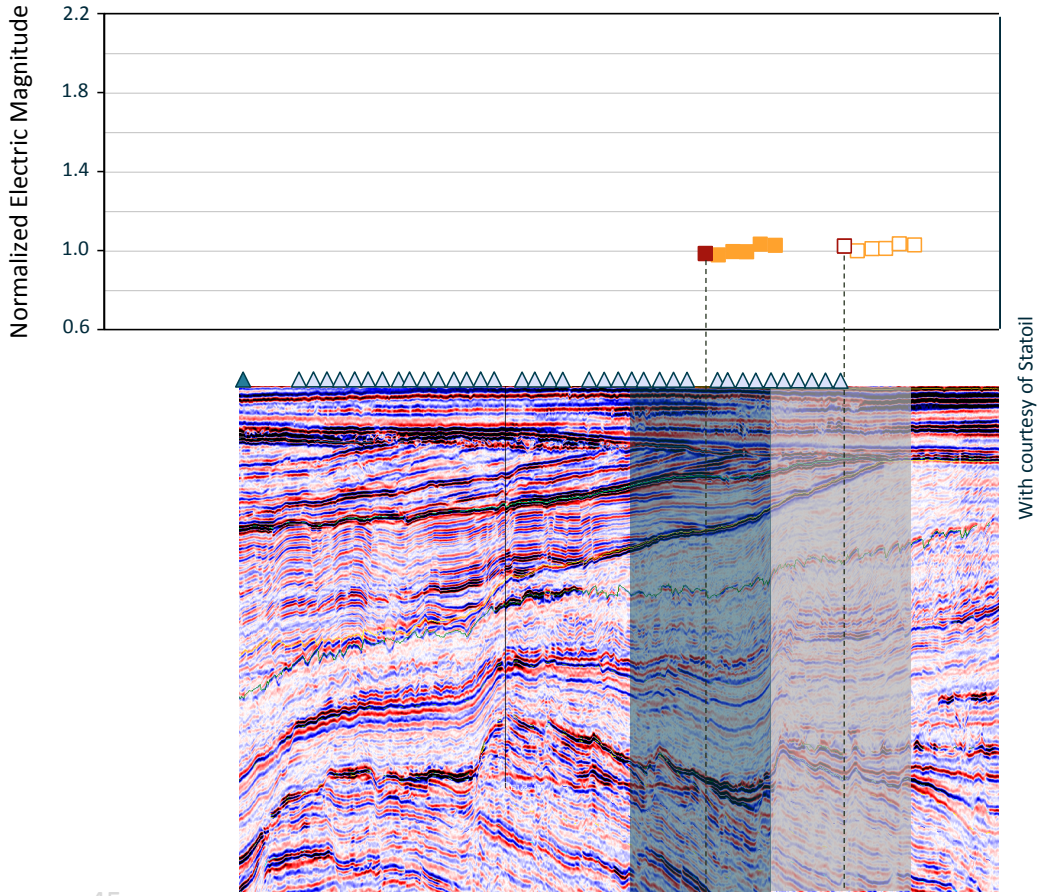
Midpoint-offset domain sorting



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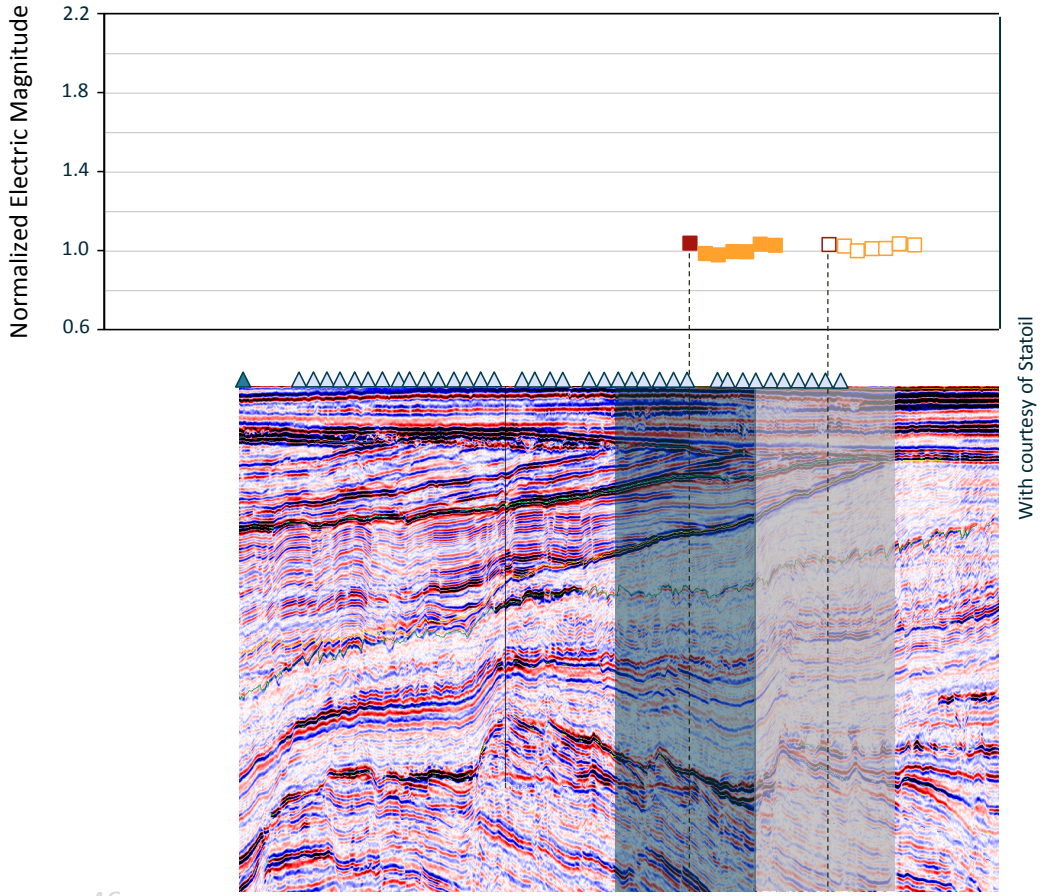
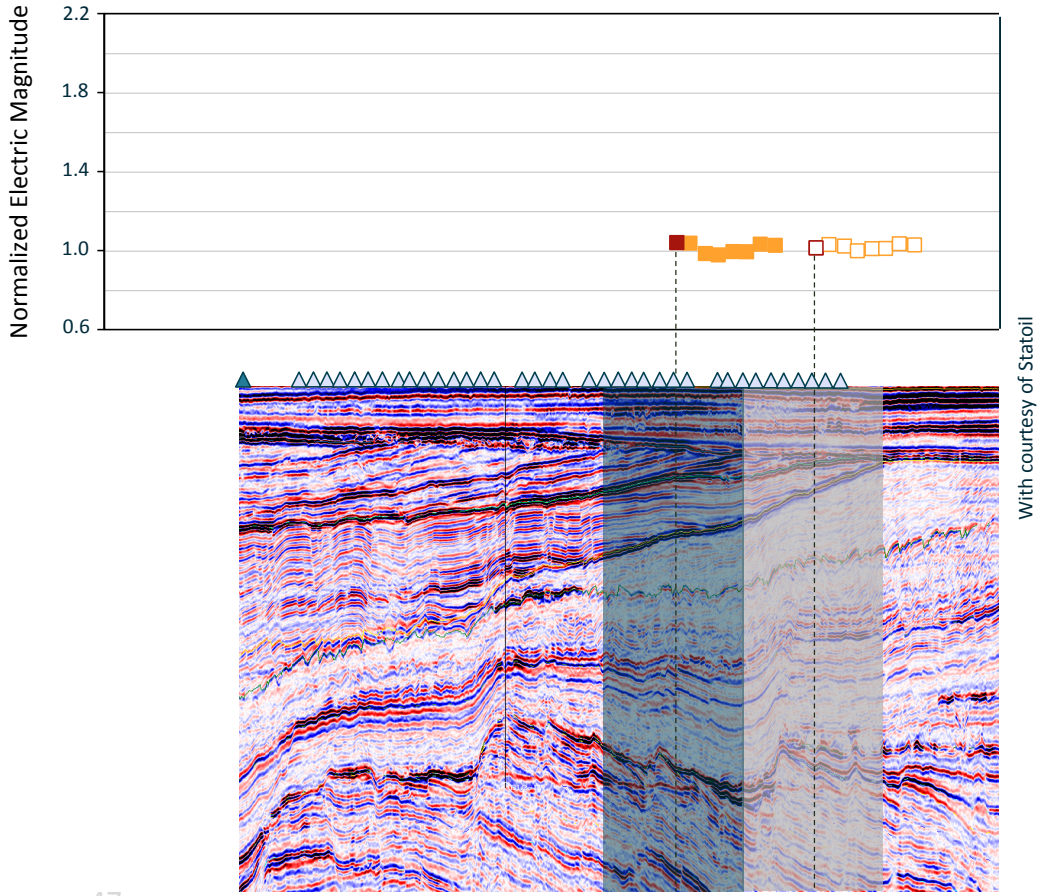


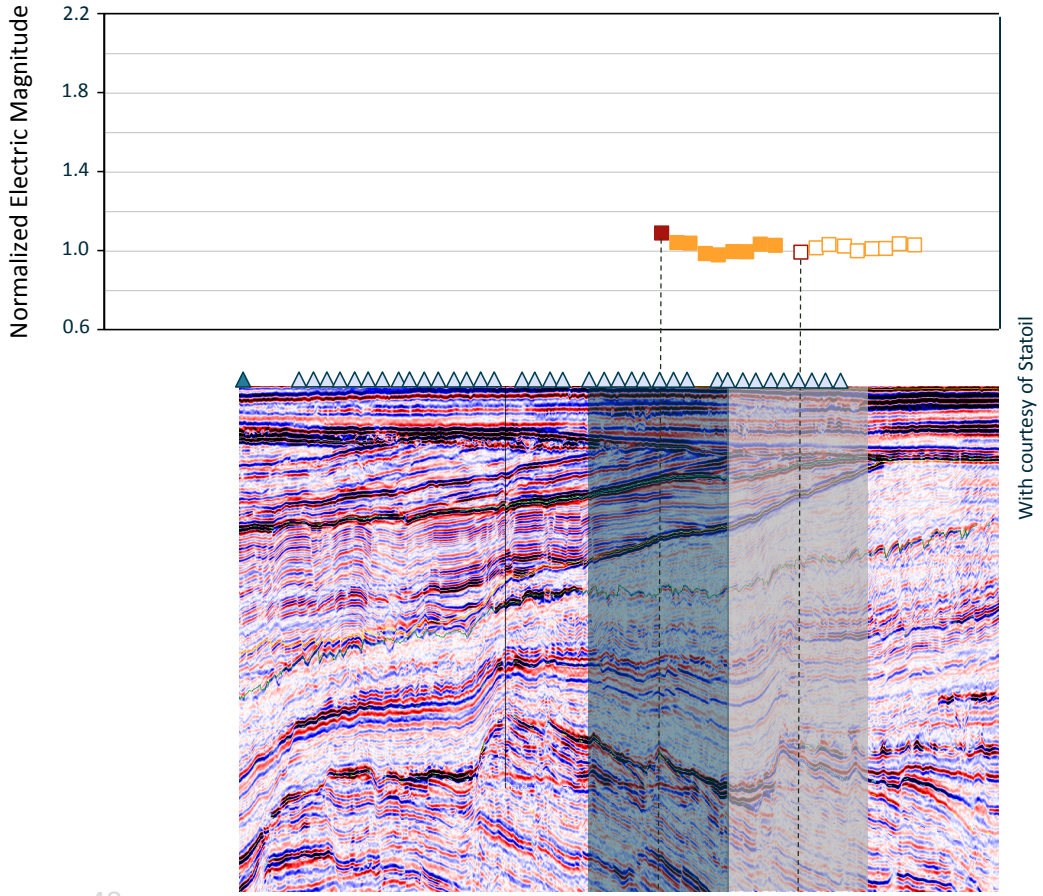
Data example Troll Western Gas Province

Midpoint-offset domain sorting



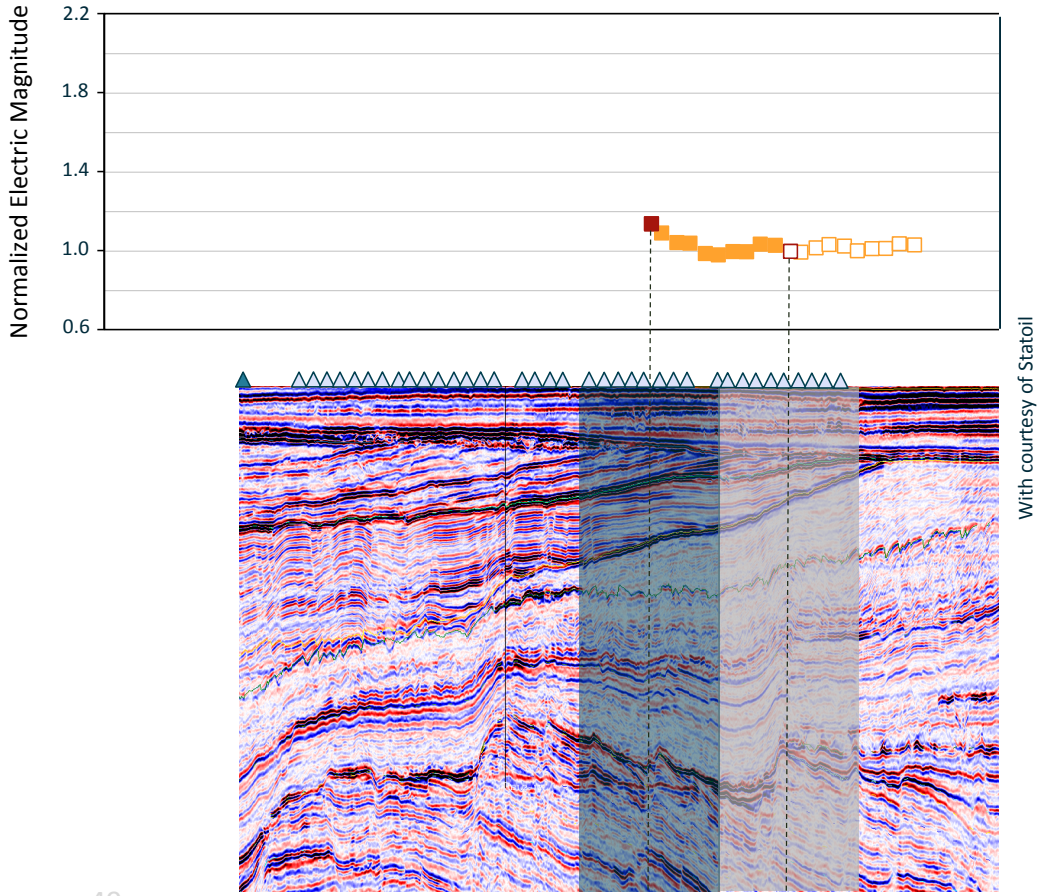
Data example Troll Western Gas Province

Midpoint-offset domain sorting



Data example Troll Western Gas Province

Midpoint-offset domain sorting



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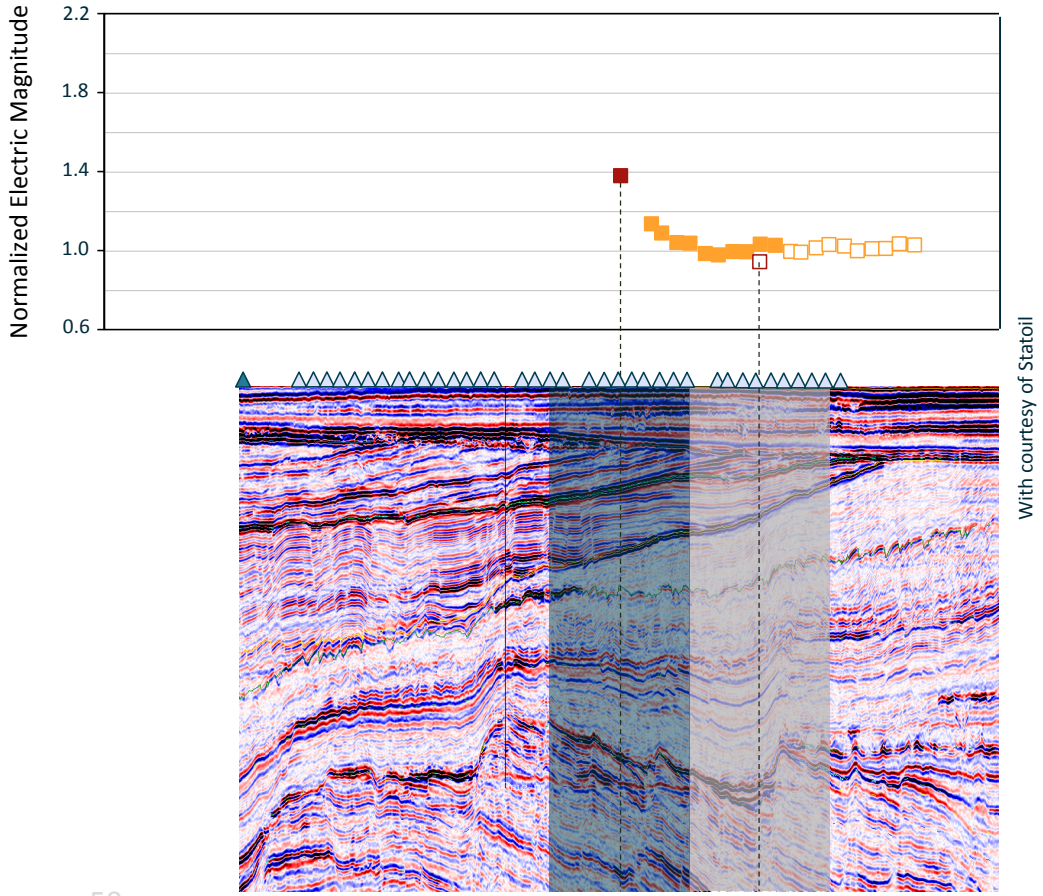

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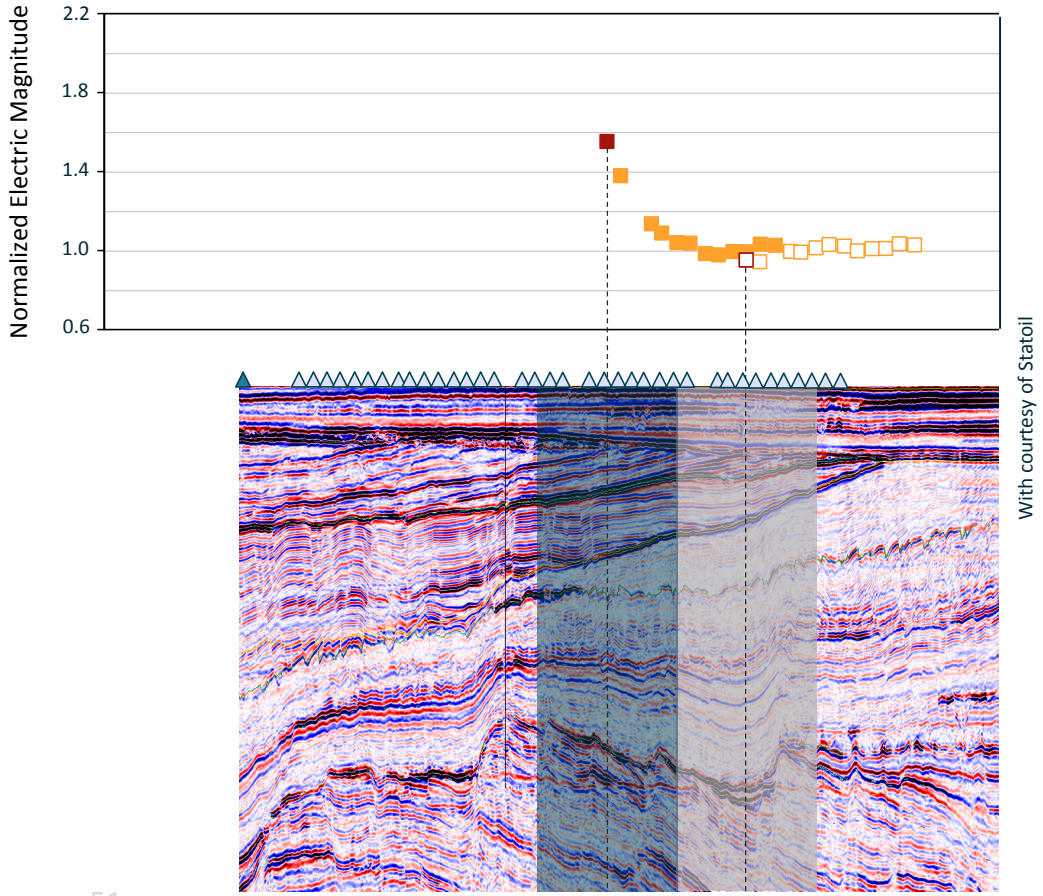
Midpoint-offset domain sorting



Data example Troll Western Gas Province

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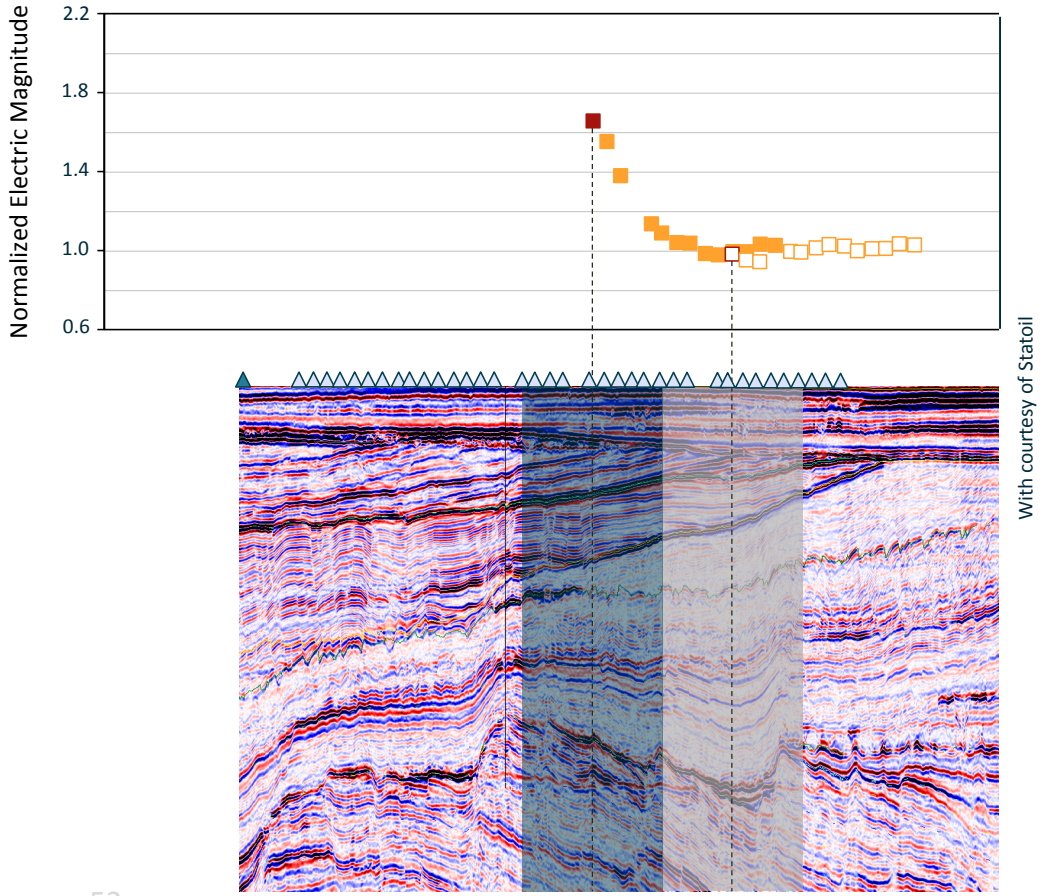

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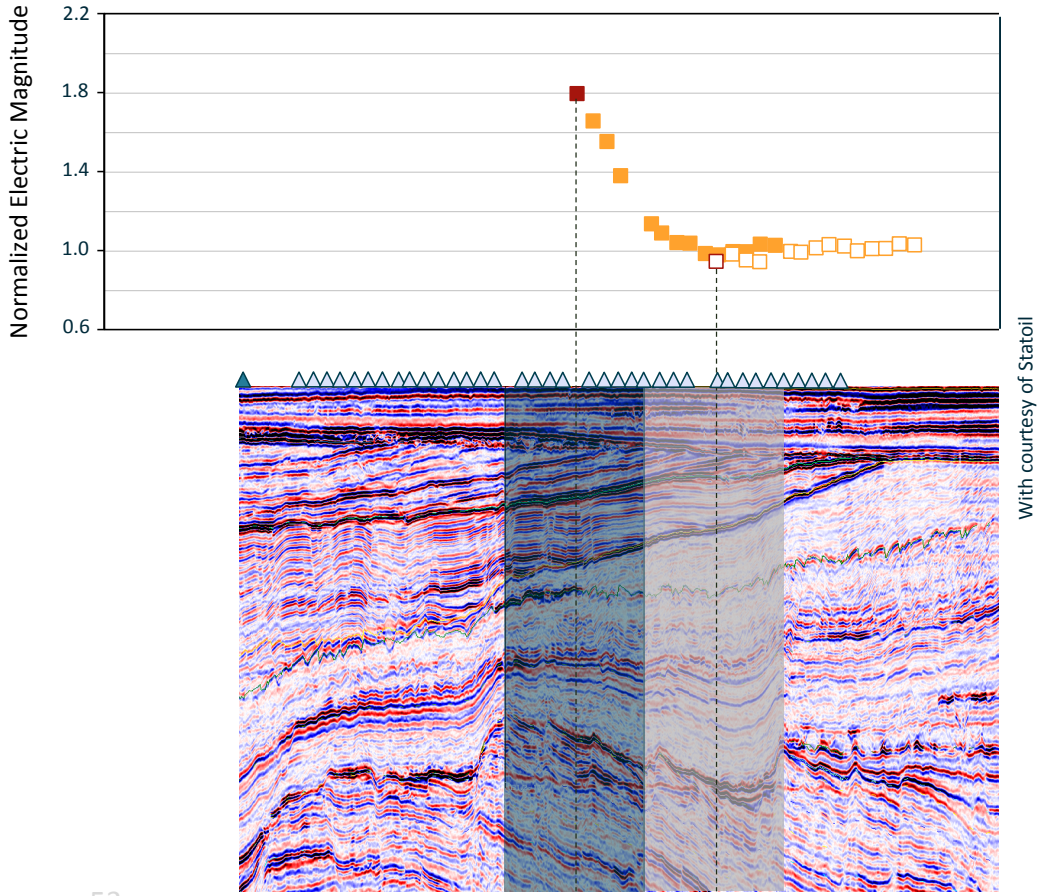
Midpoint-offset domain sorting



Data example Troll Western Gas Province

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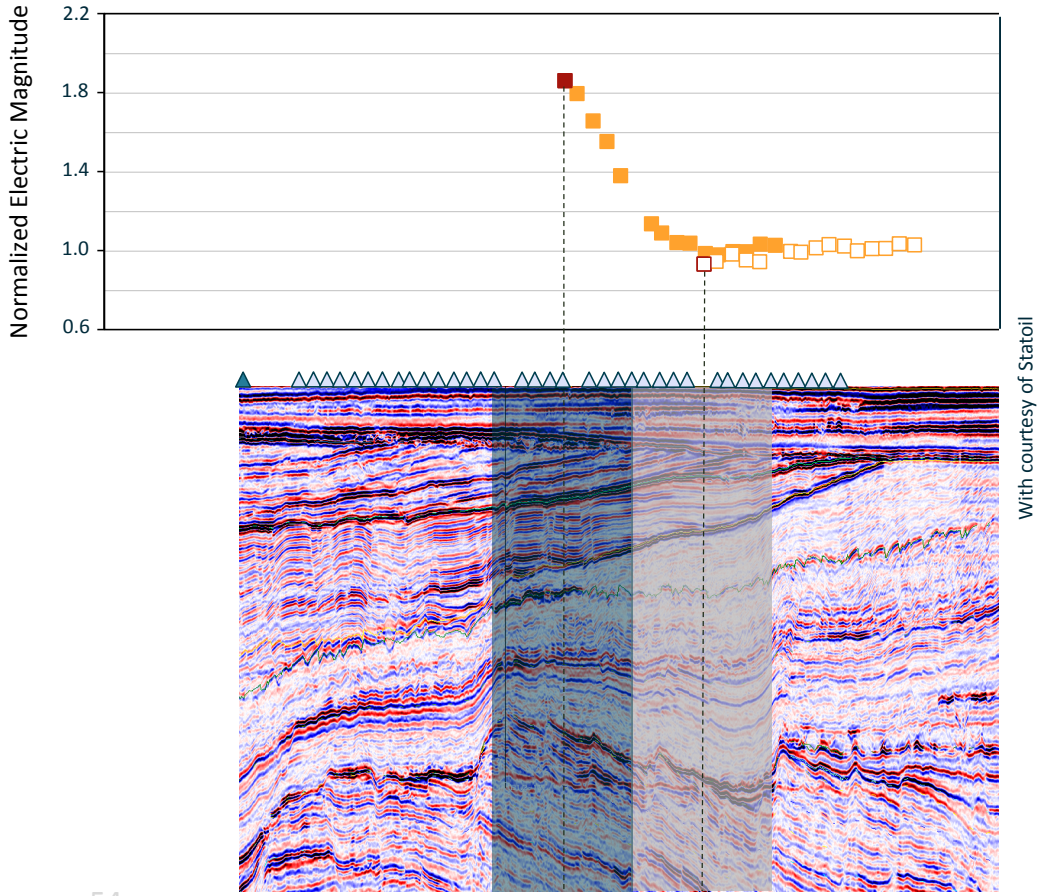
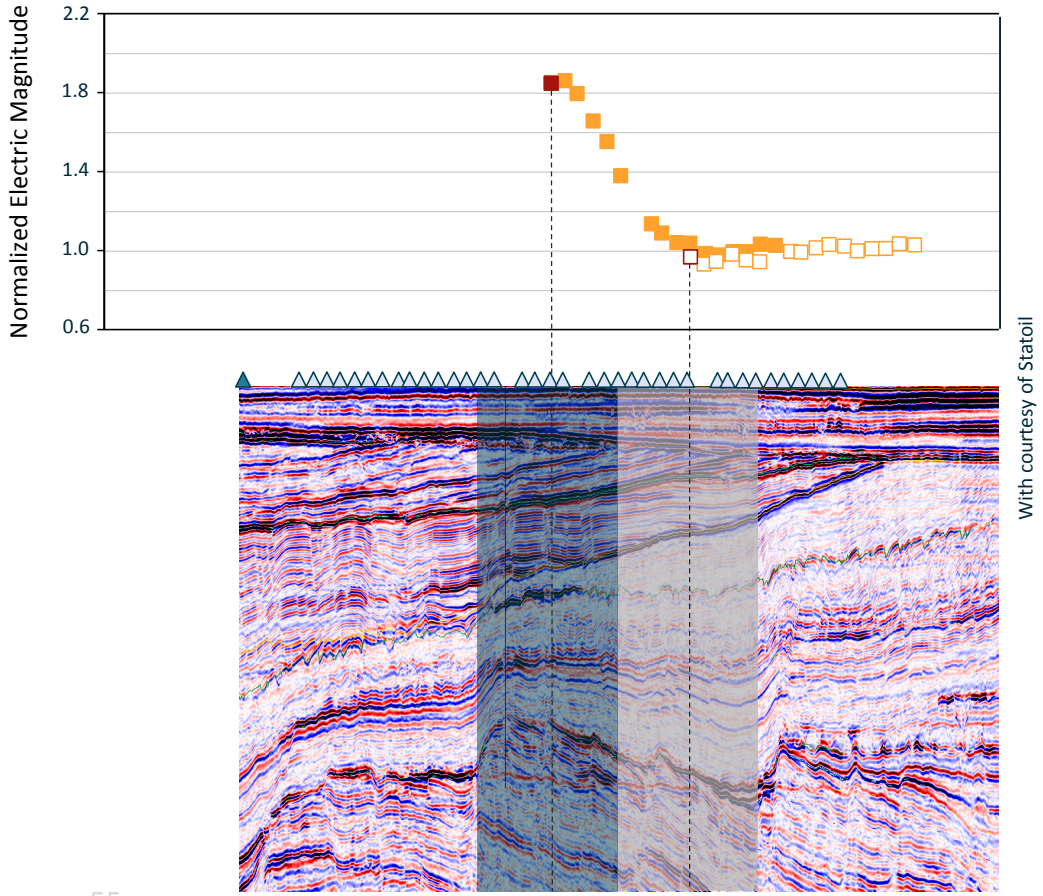


Data example Troll Western Gas Province

Midpoint-offset domain sorting



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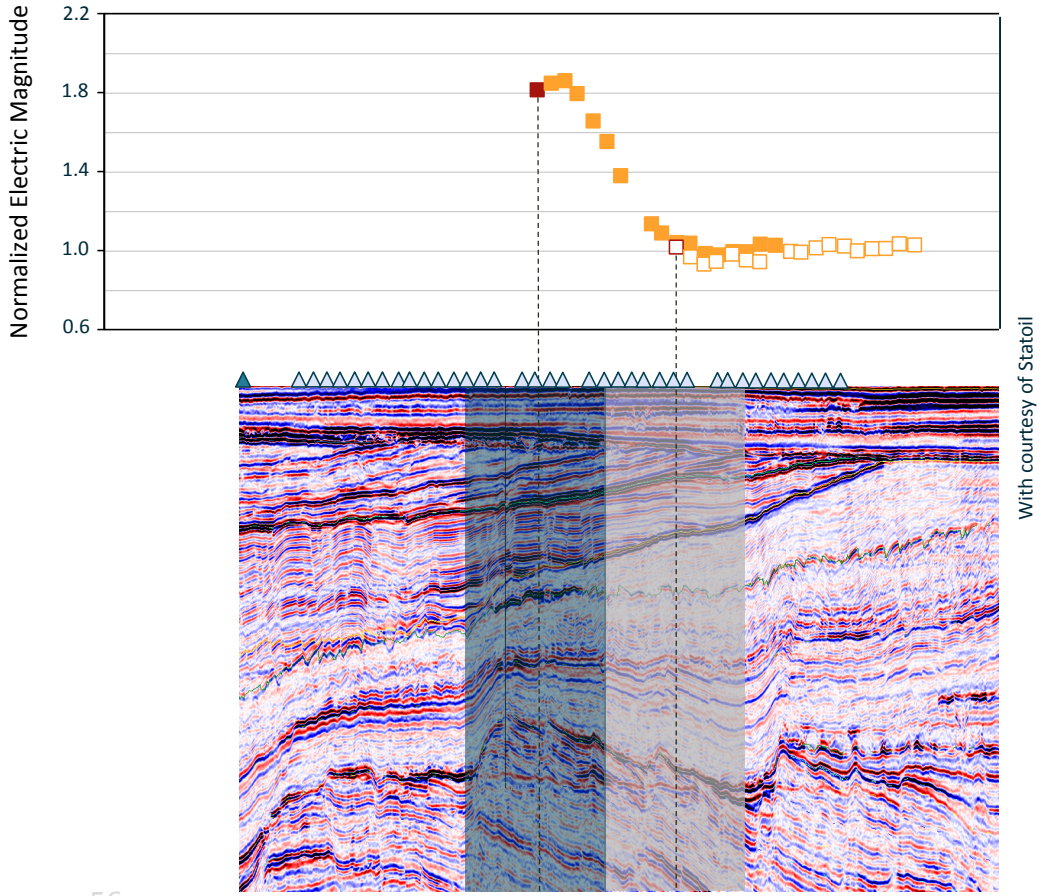

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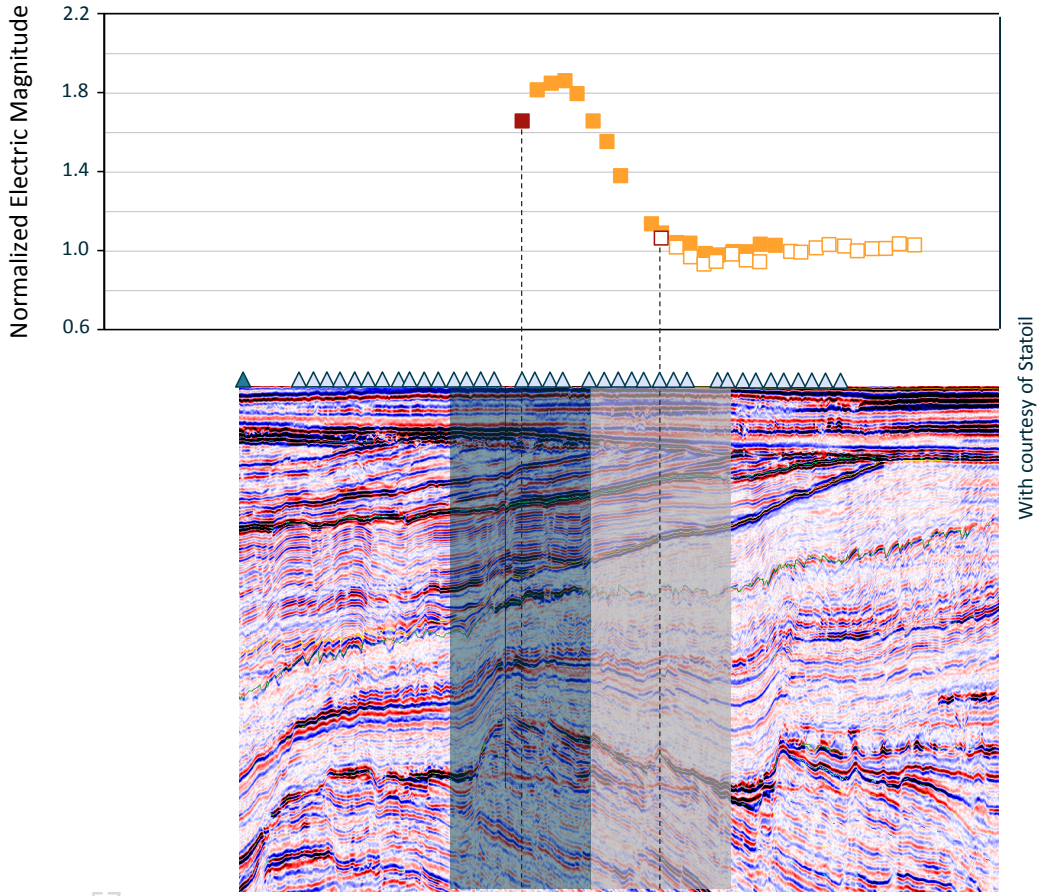

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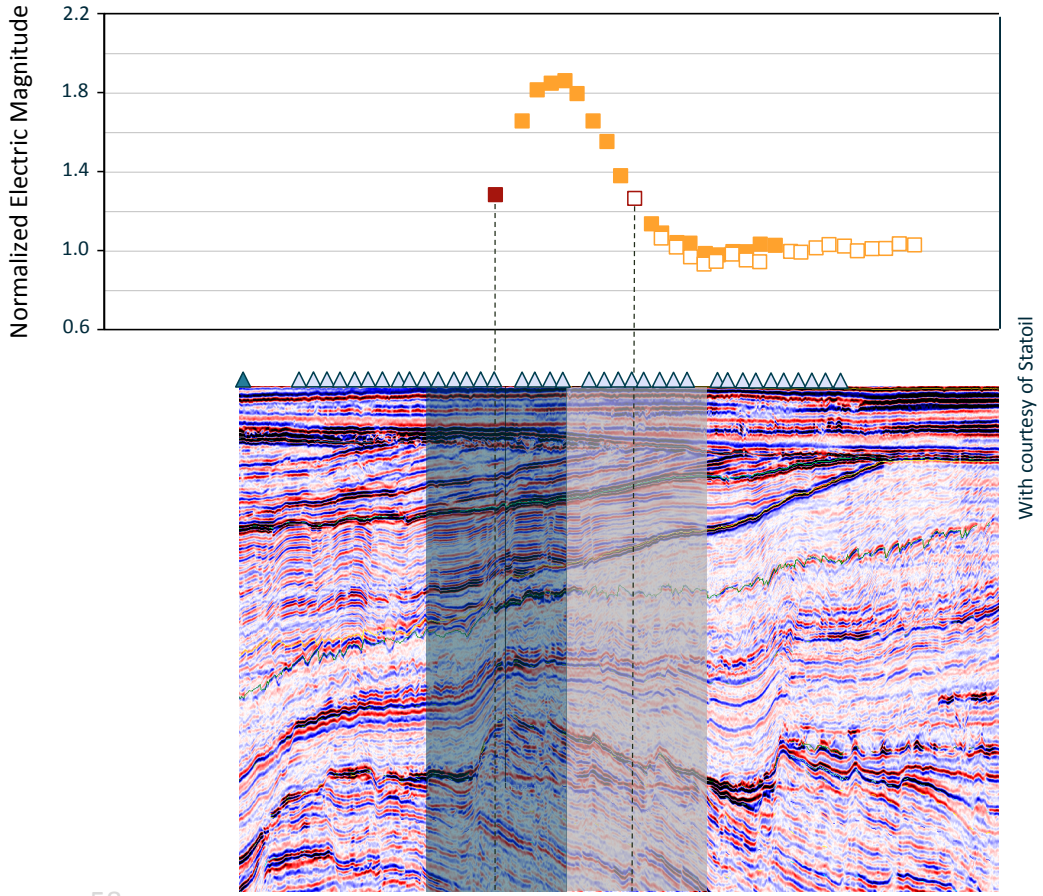
Midpoint-offset domain sorting



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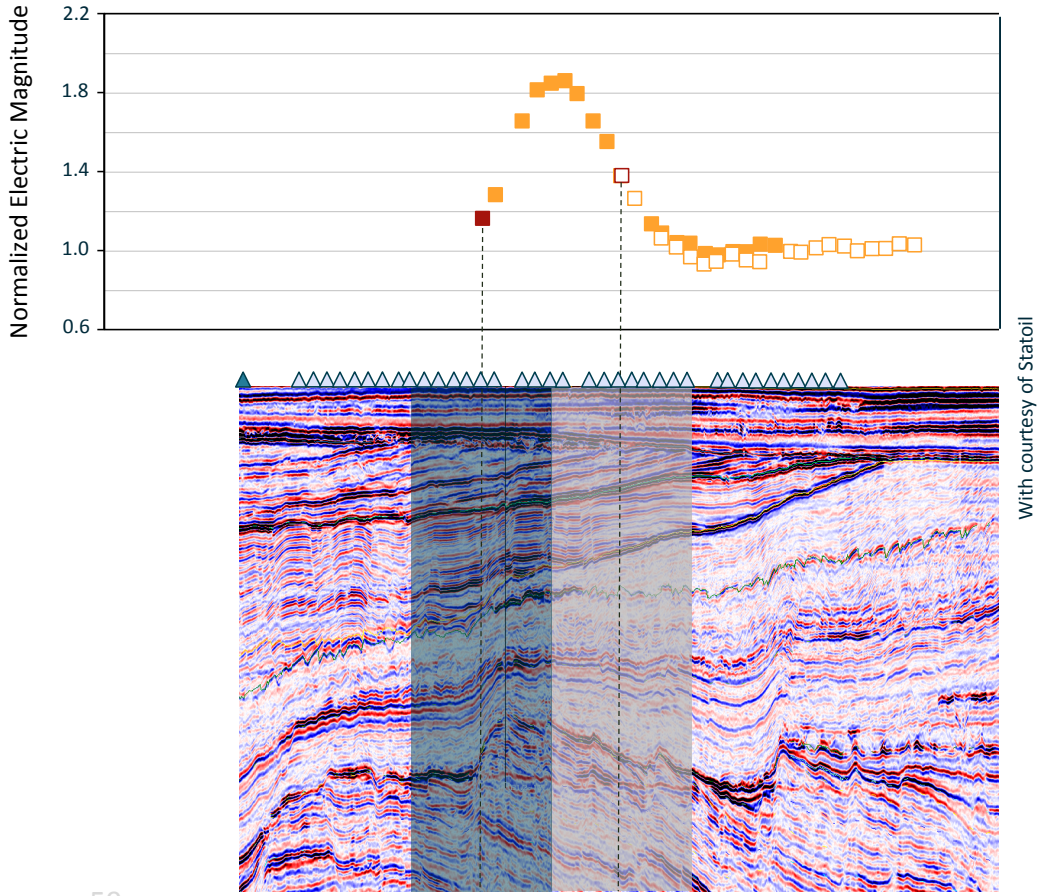

Data example Troll Western Gas Province

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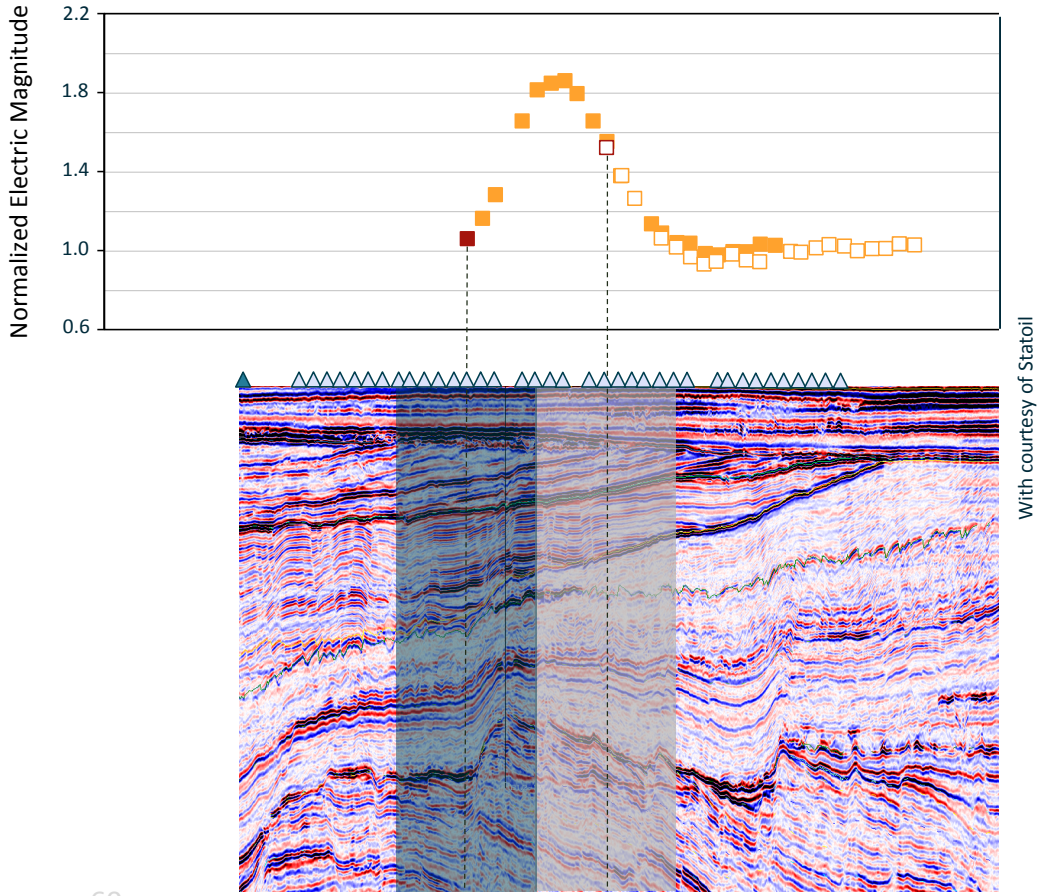

Data example Troll Western Gas Province

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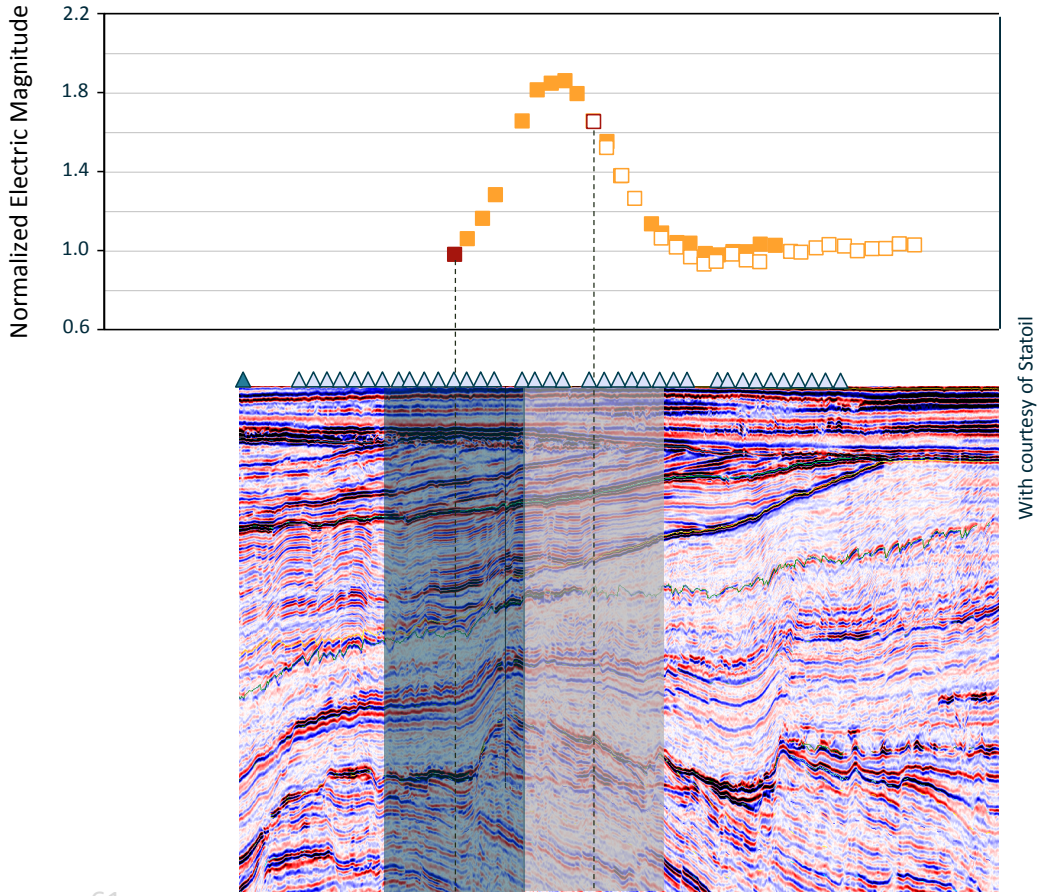
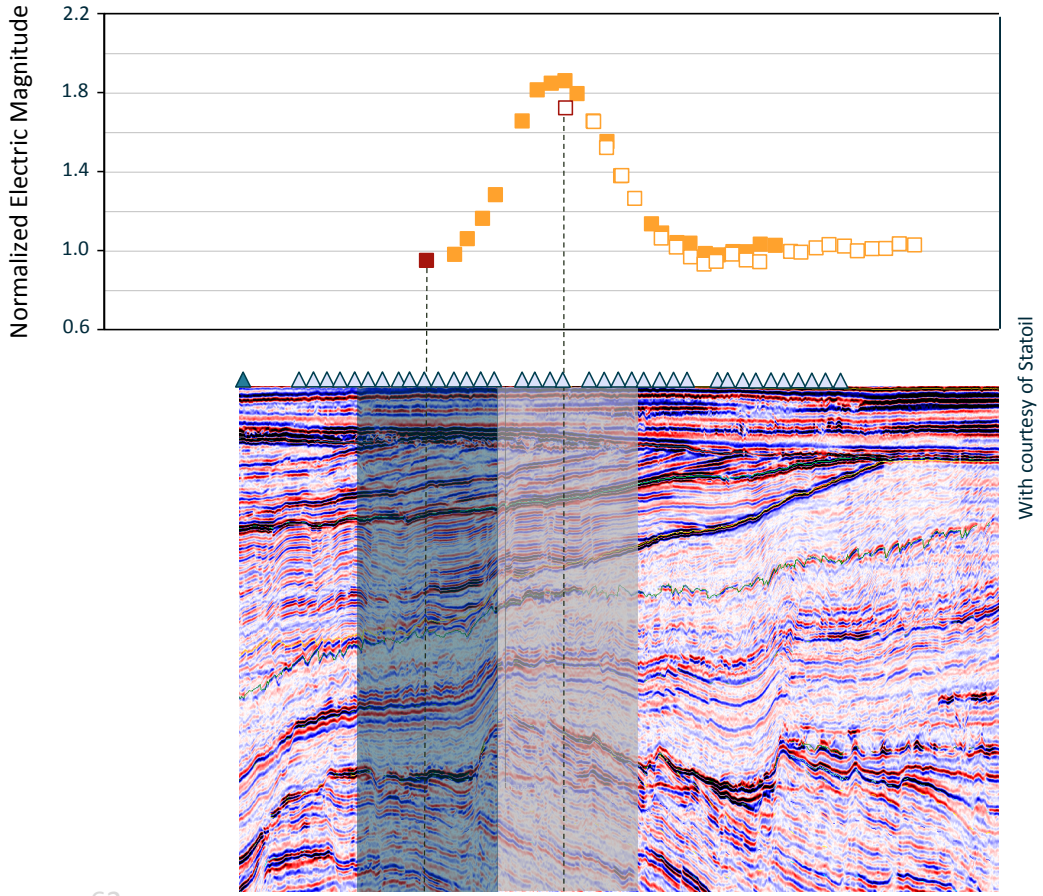


61

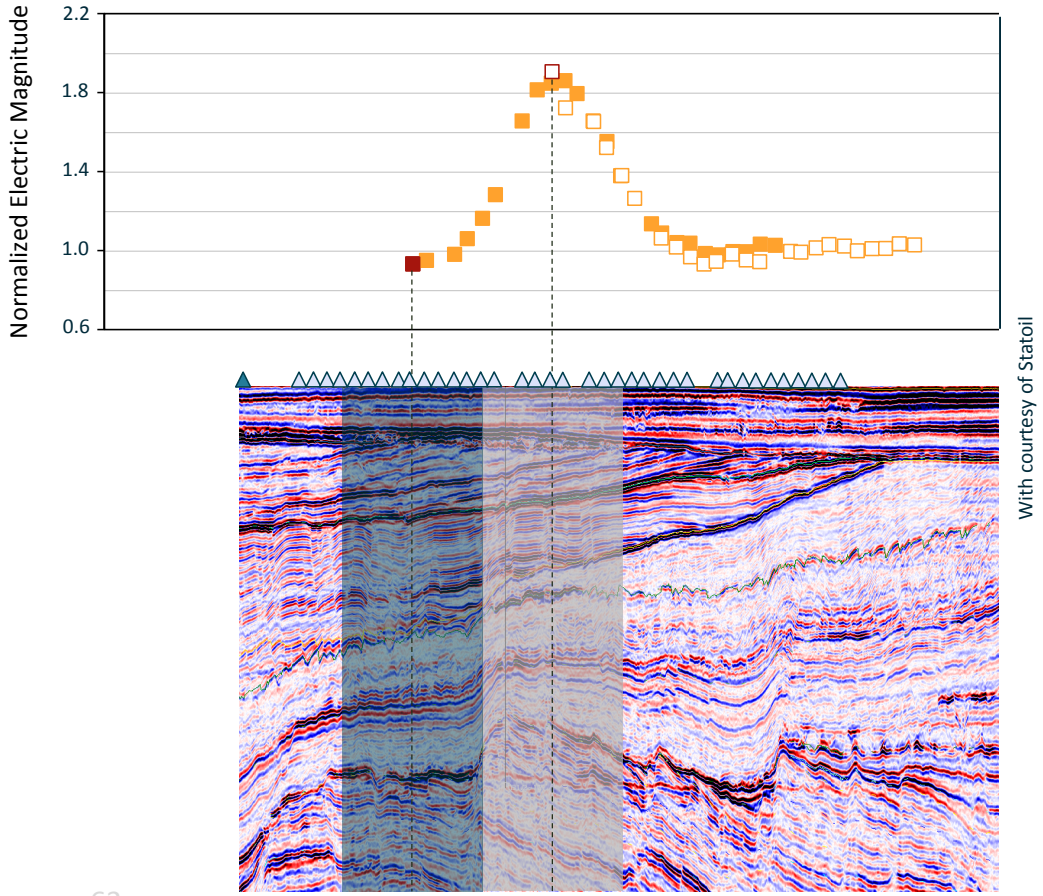
Data example Troll Western Gas Province

Midpoint-offset domain sorting



Data example Troll Western Gas Province

Midpoint-offset domain sorting



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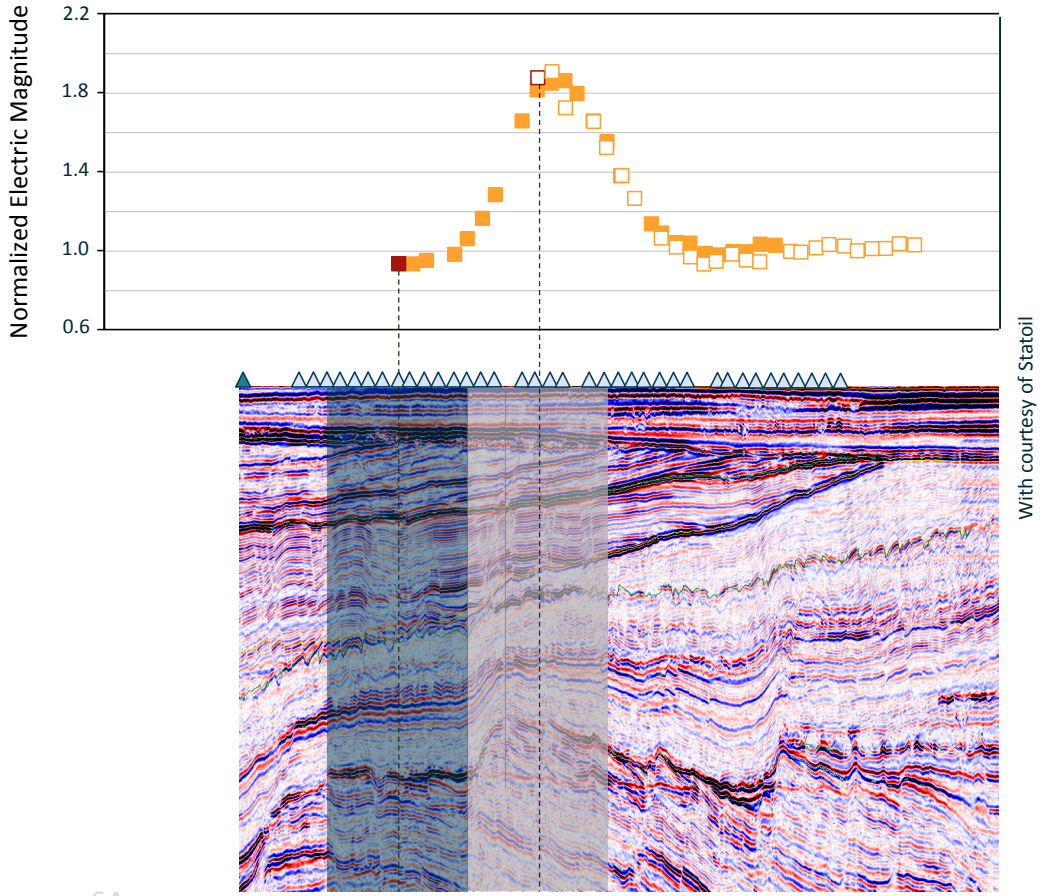

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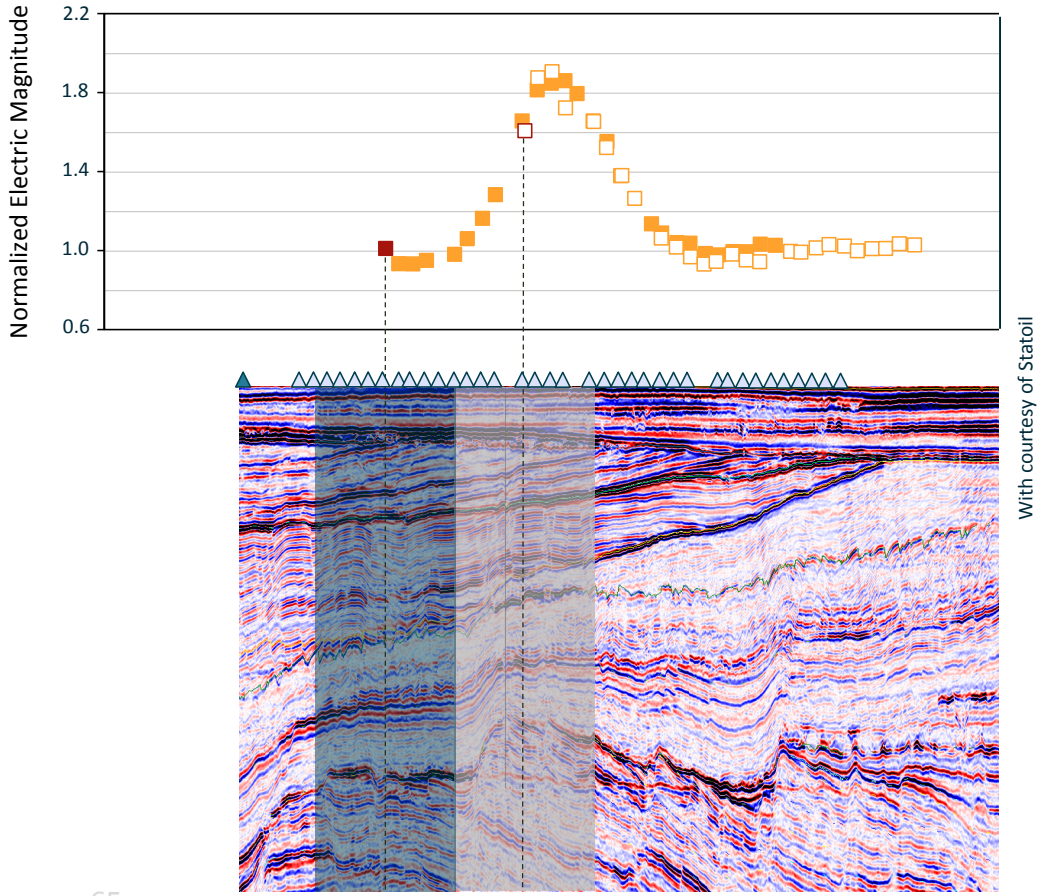

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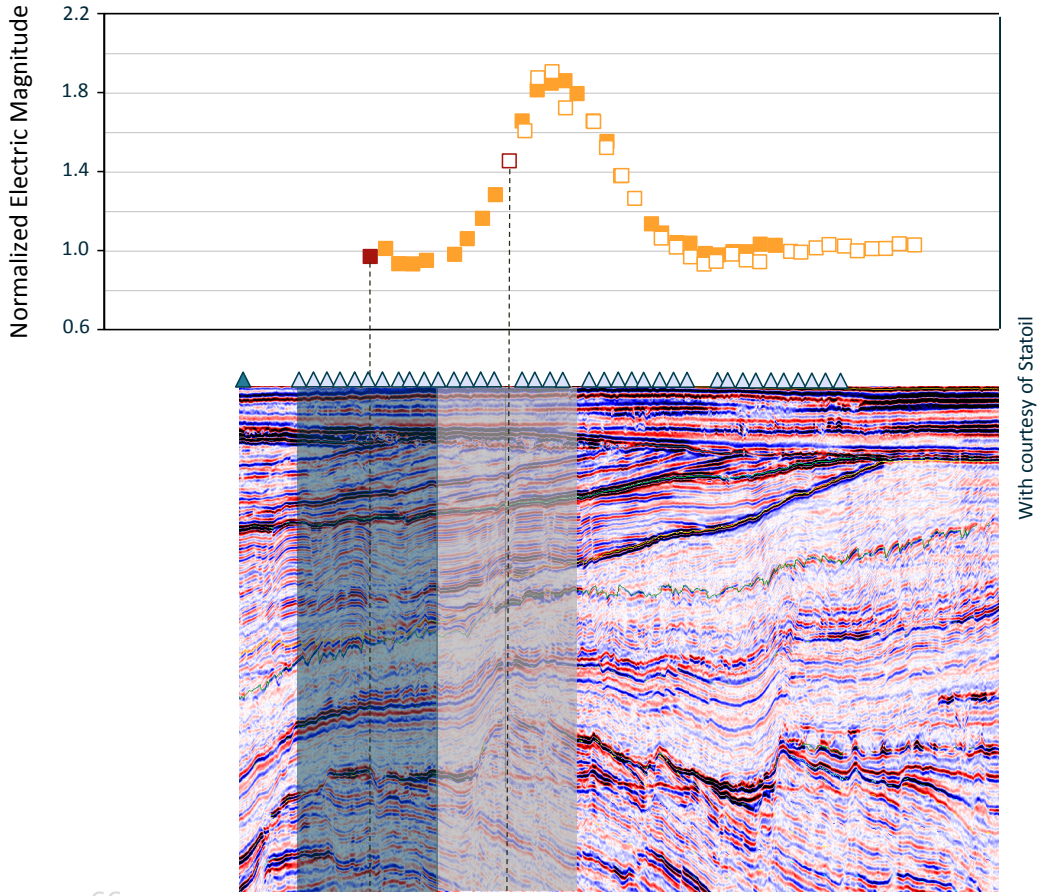

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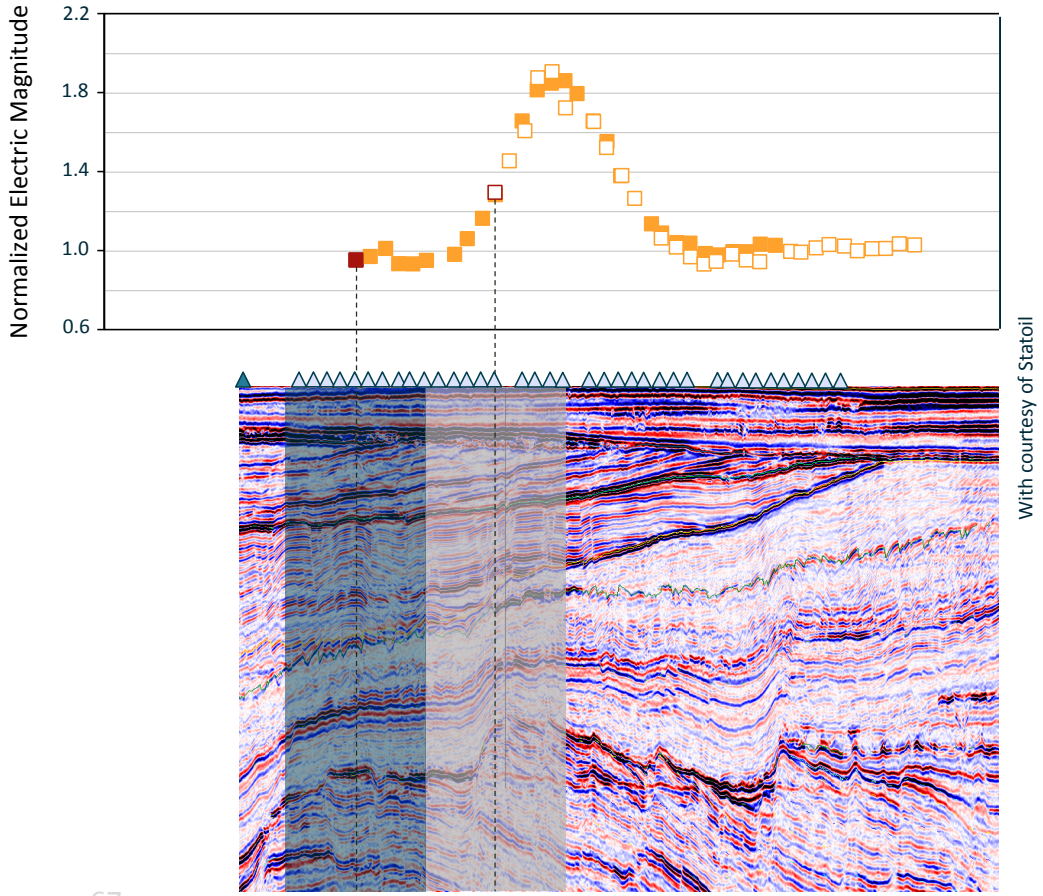

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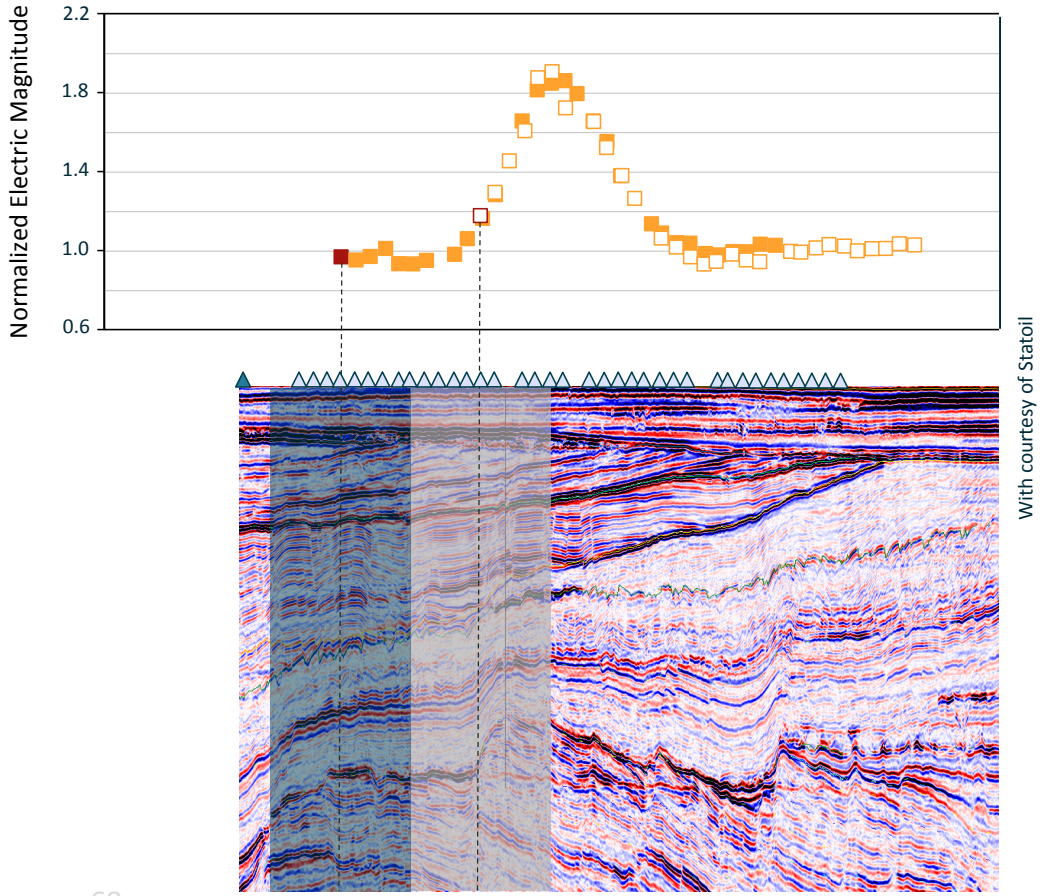

Data example Troll Western Gas Province

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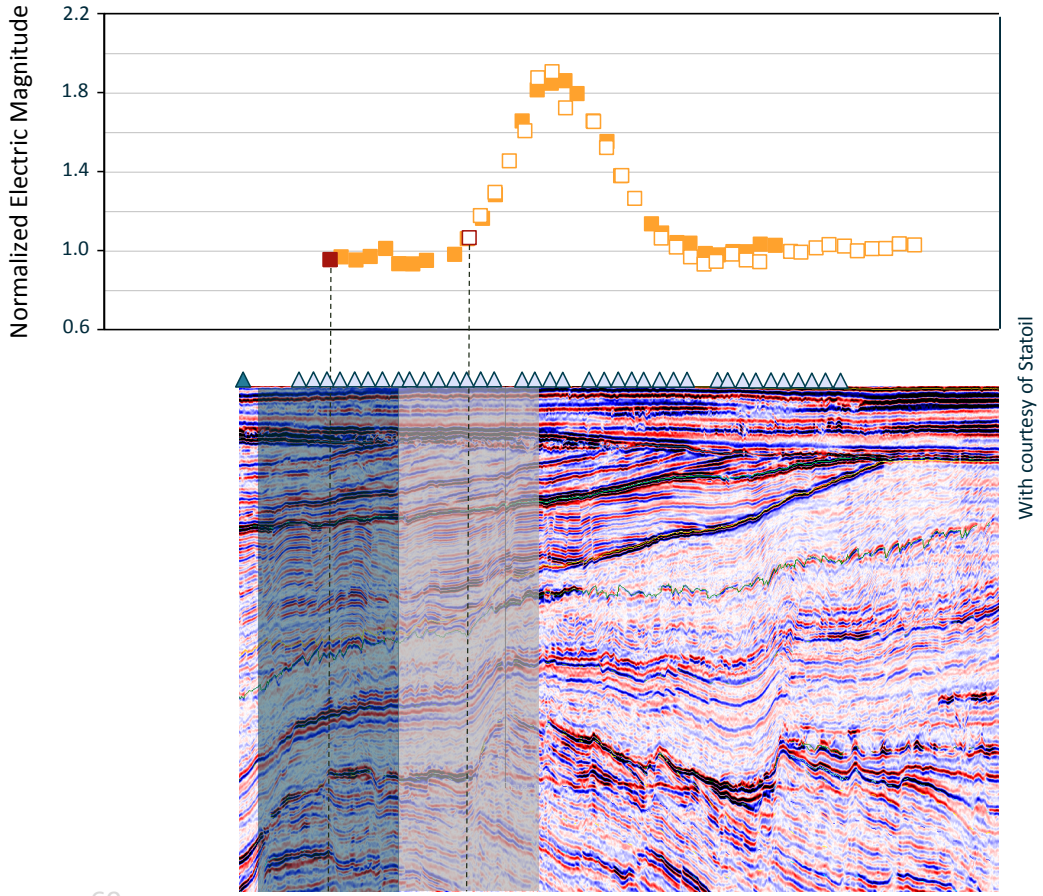
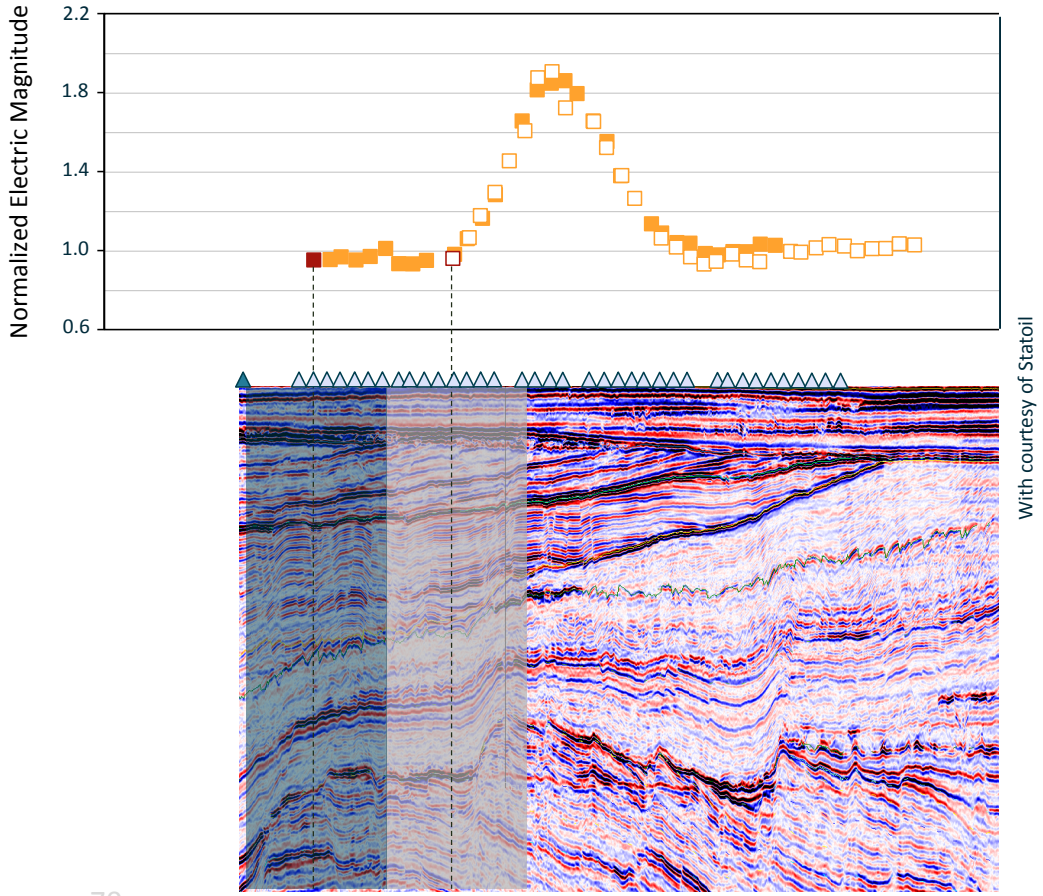


Data example Troll Western Gas Province

Midpoint-offset domain sorting



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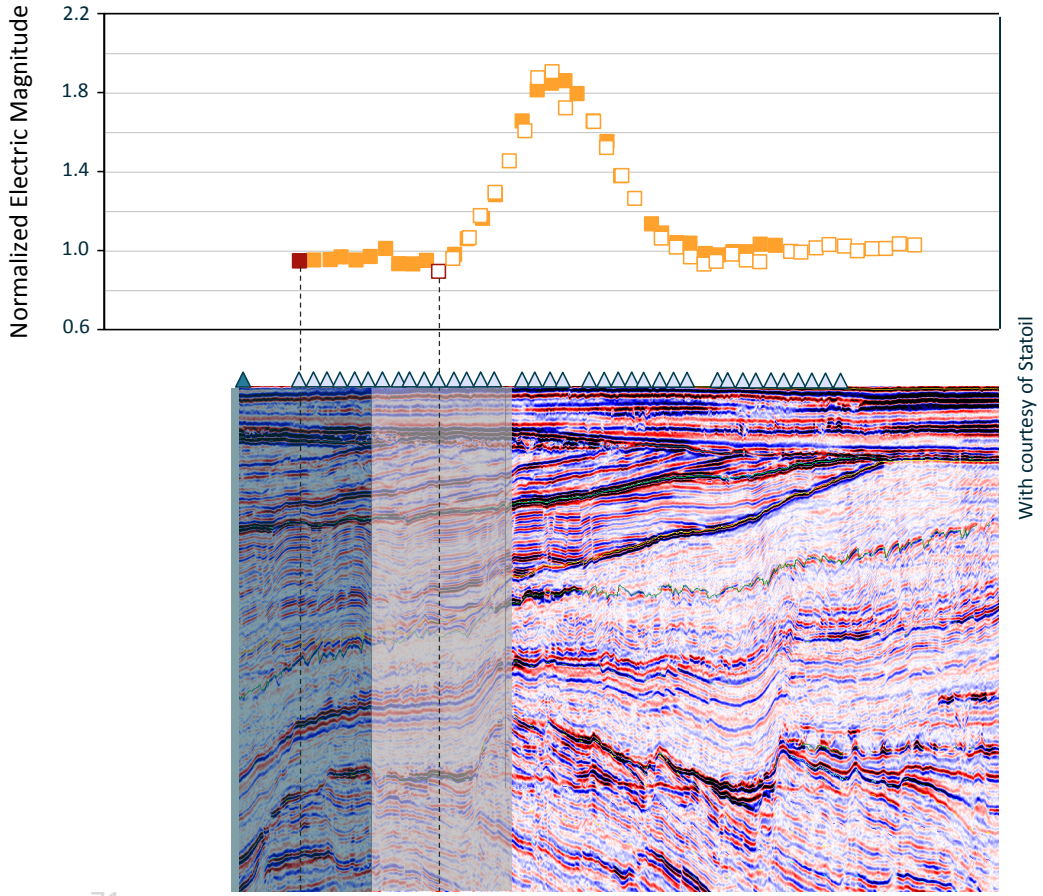

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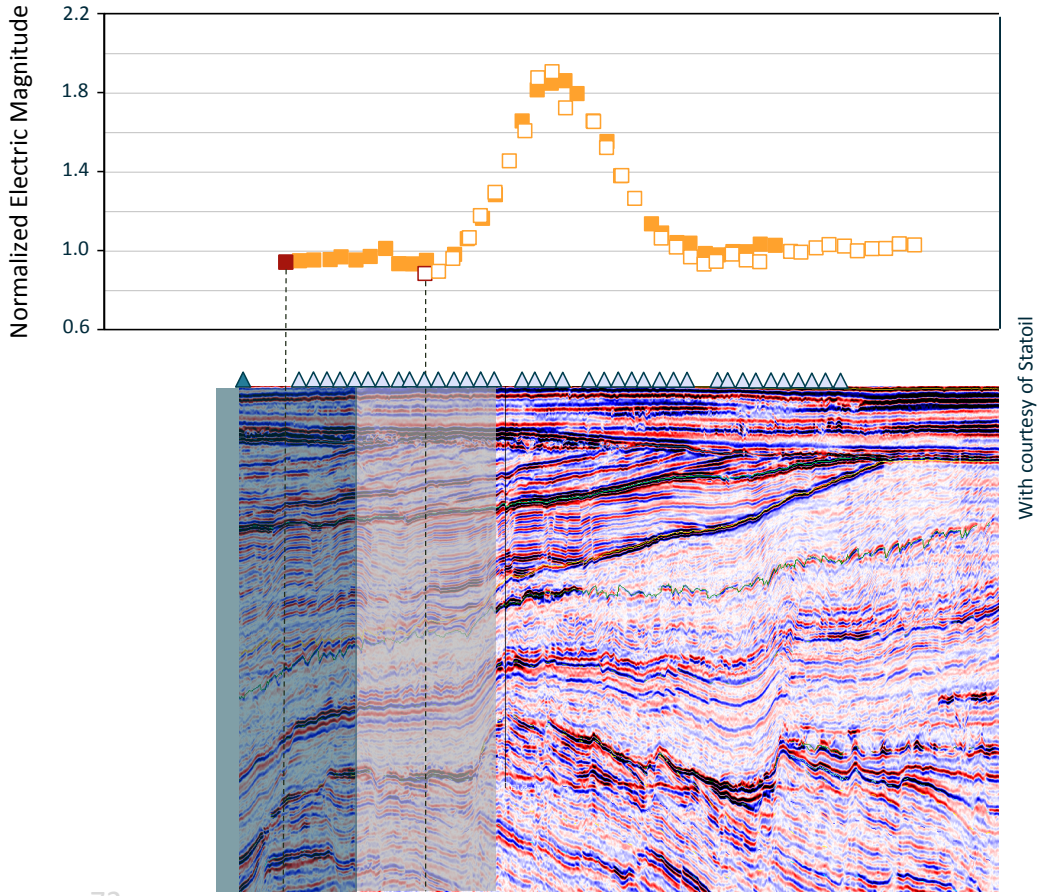

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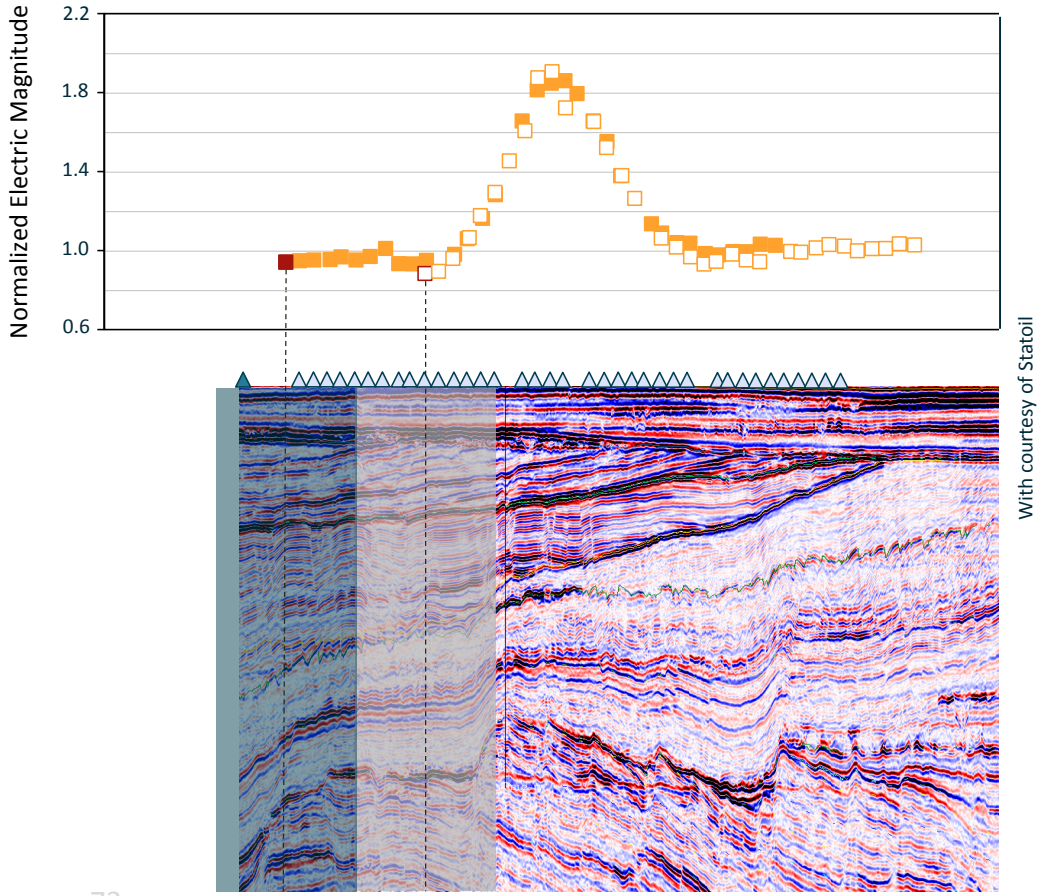

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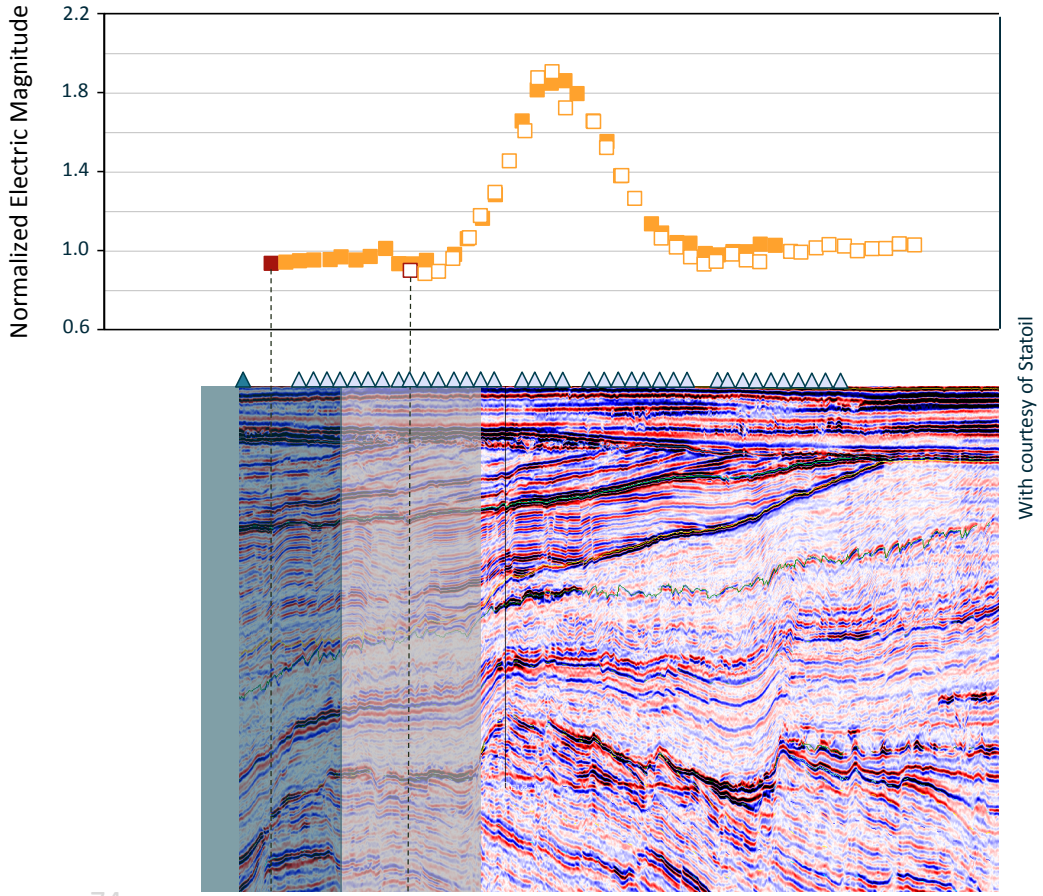
Midpoint-offset domain sorting



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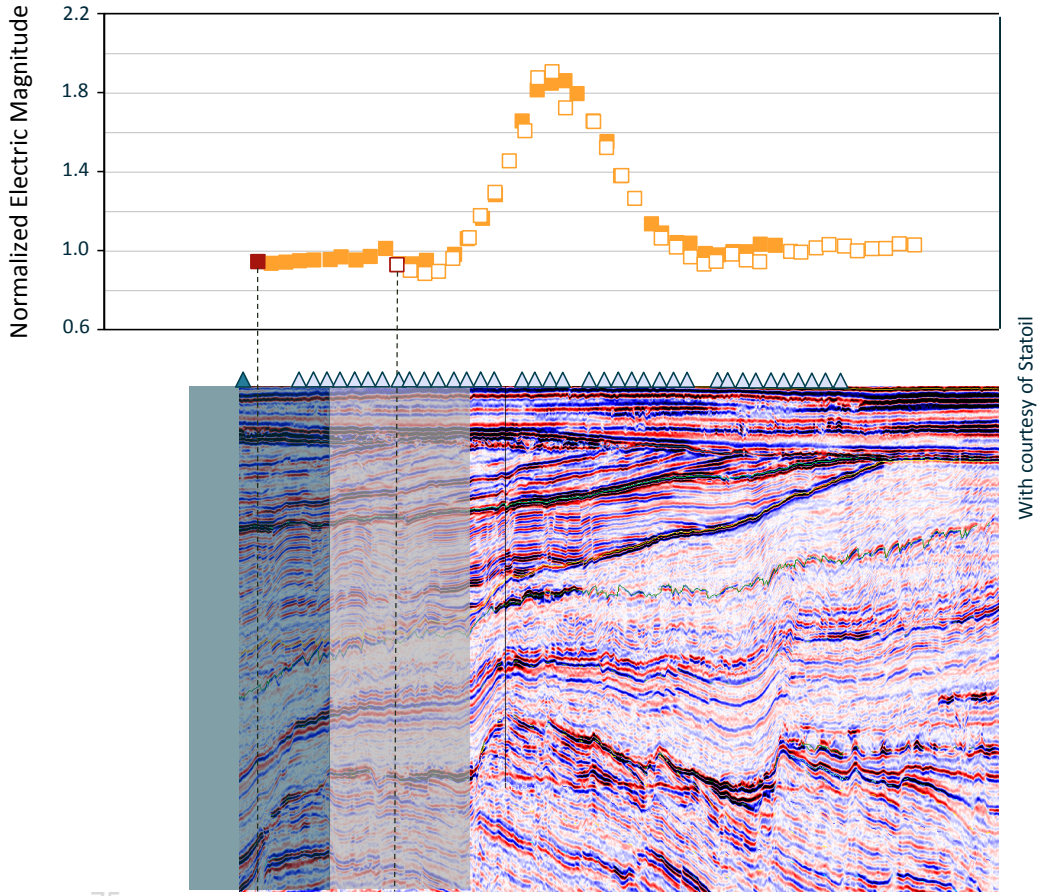

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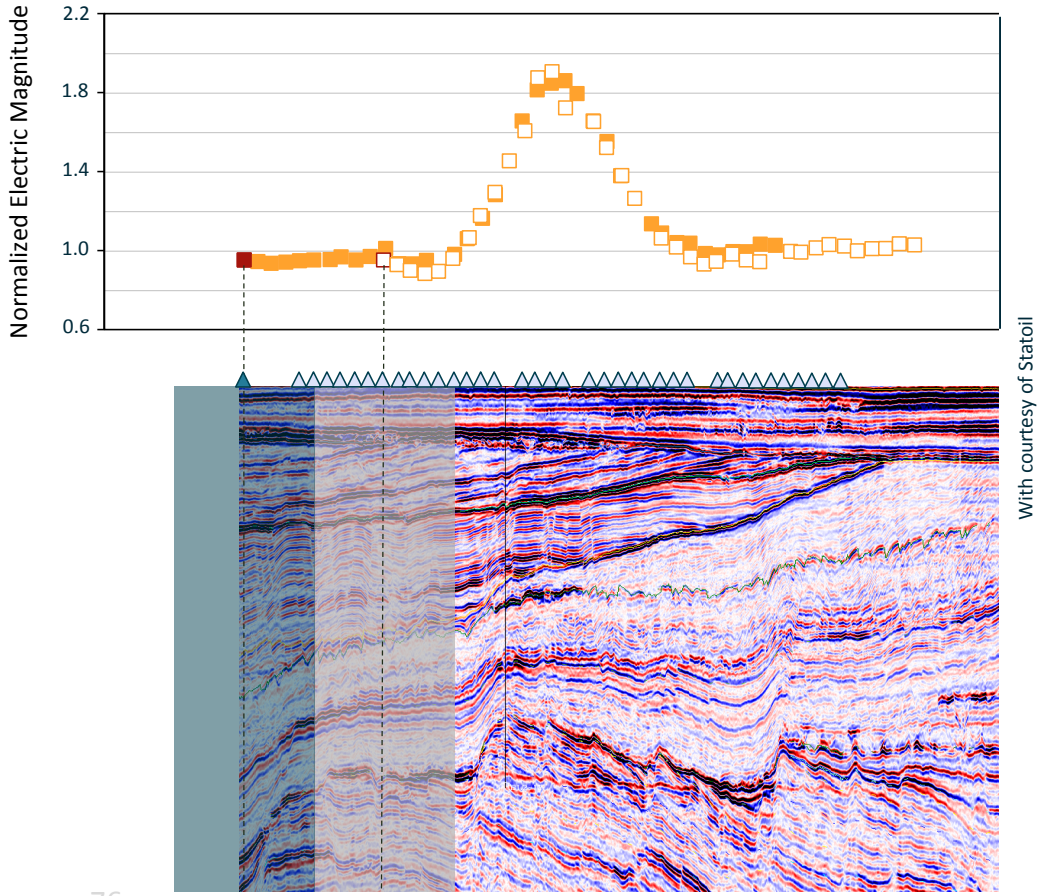

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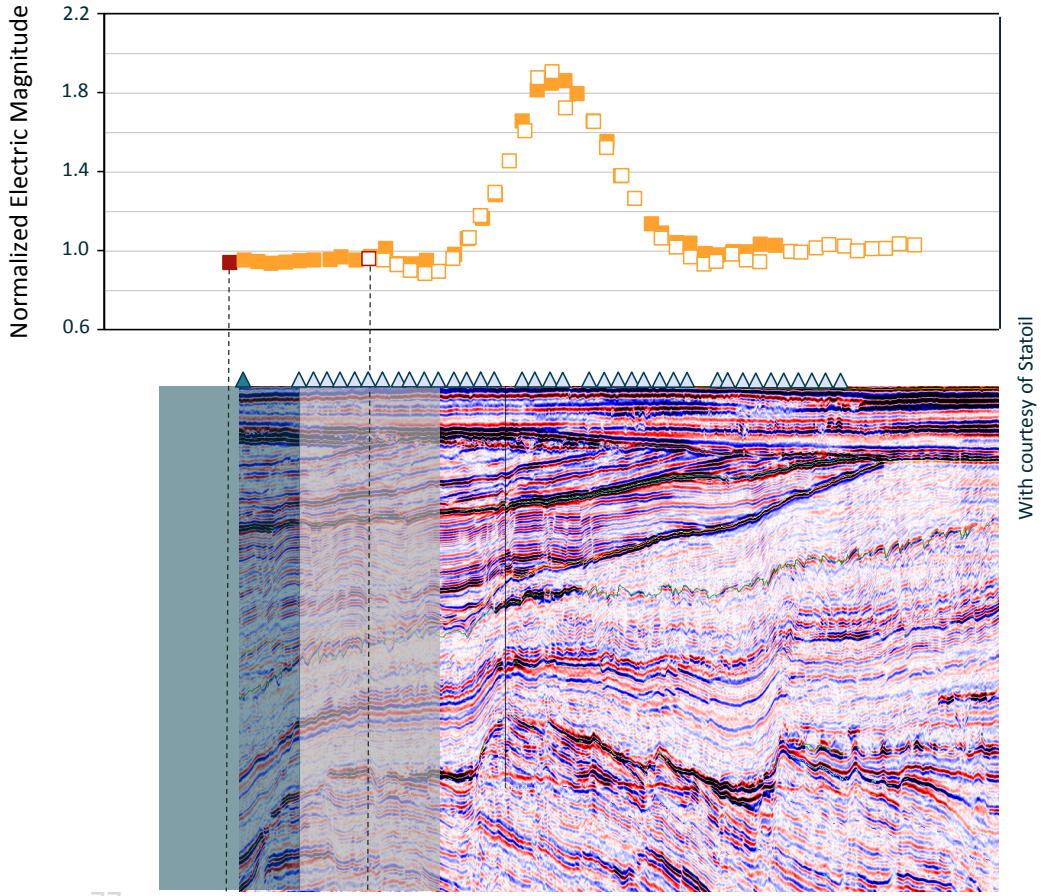

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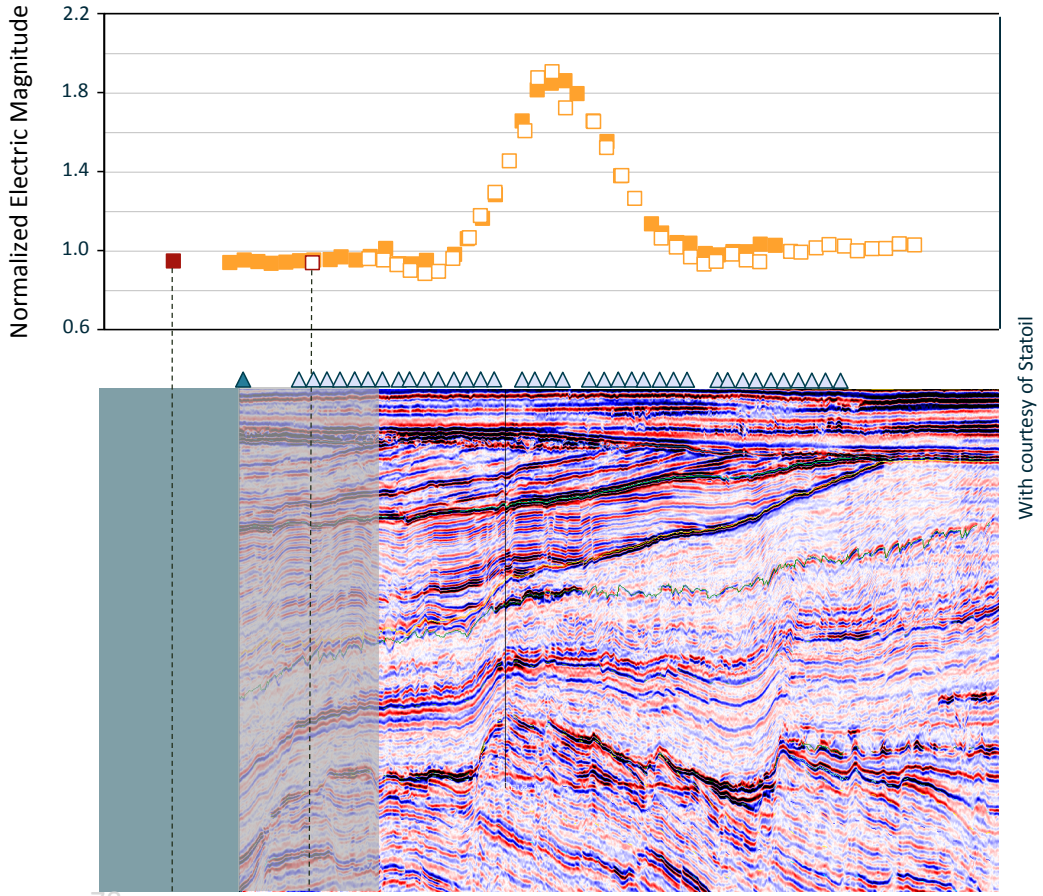
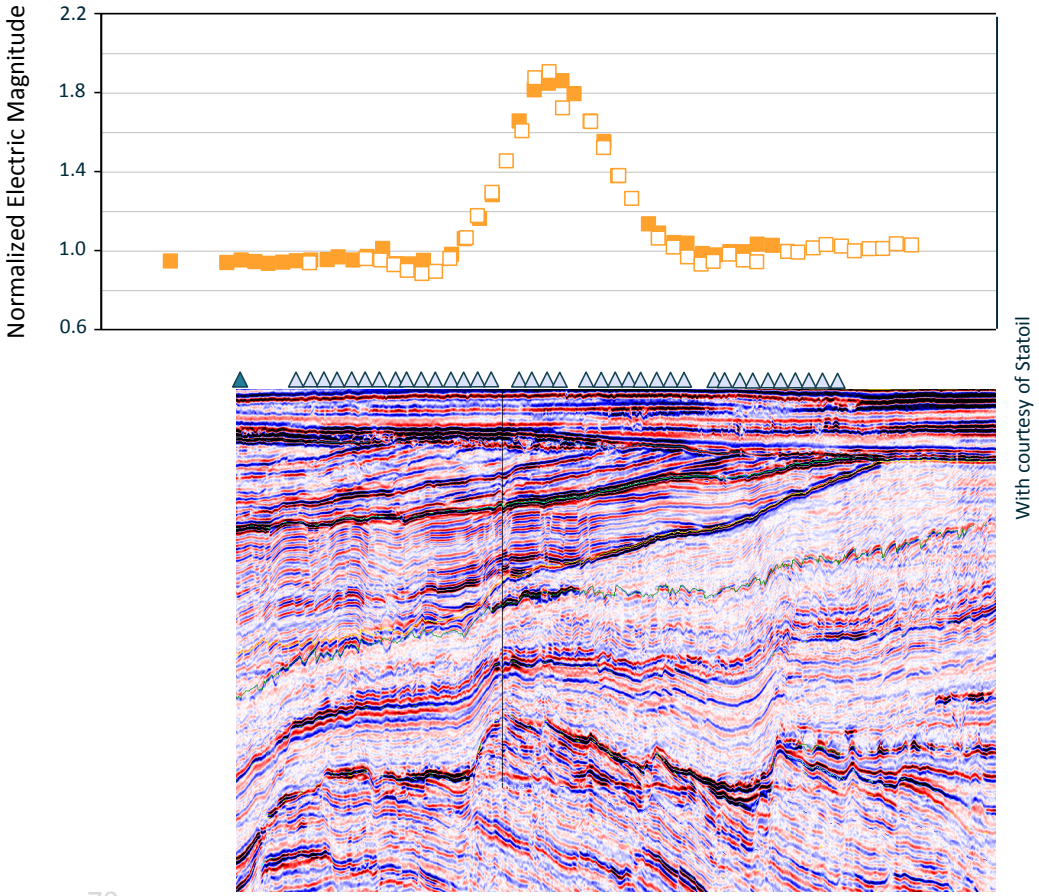


Data example Troll Western Gas Province

Line summary of the electric inline response at 5 km offset



Data example Troll Western Gas Province

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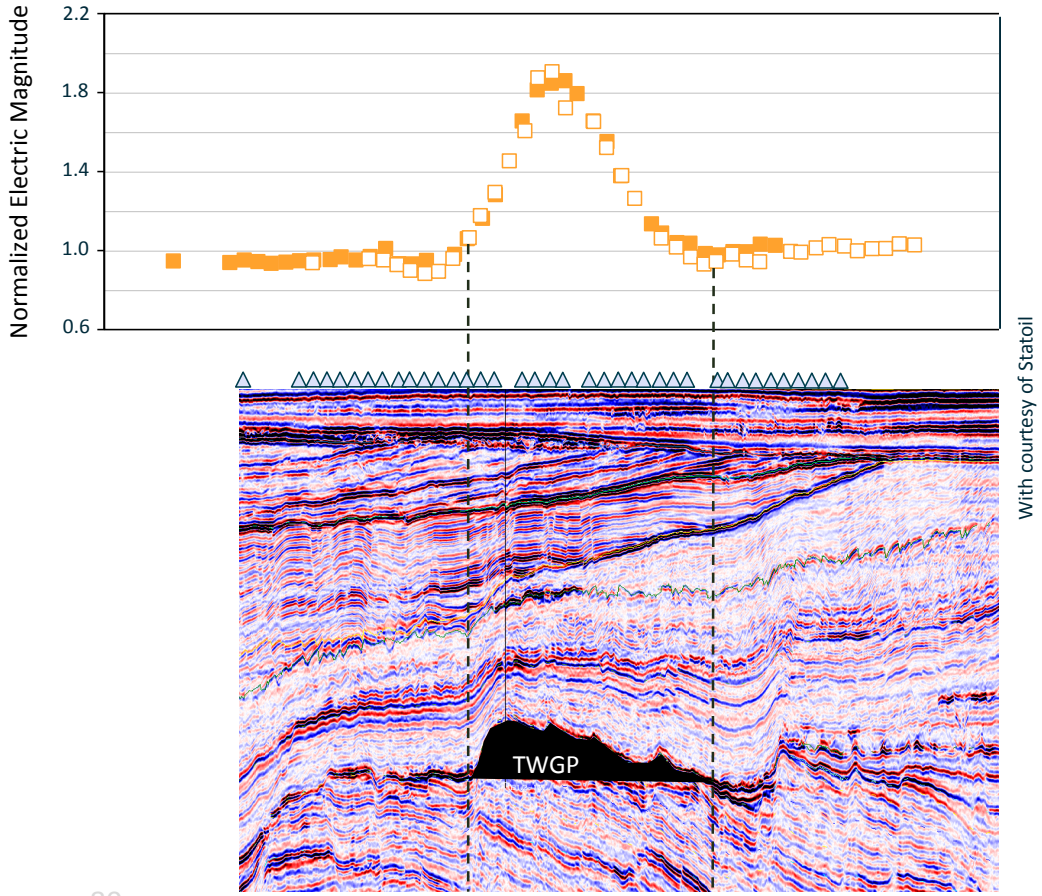
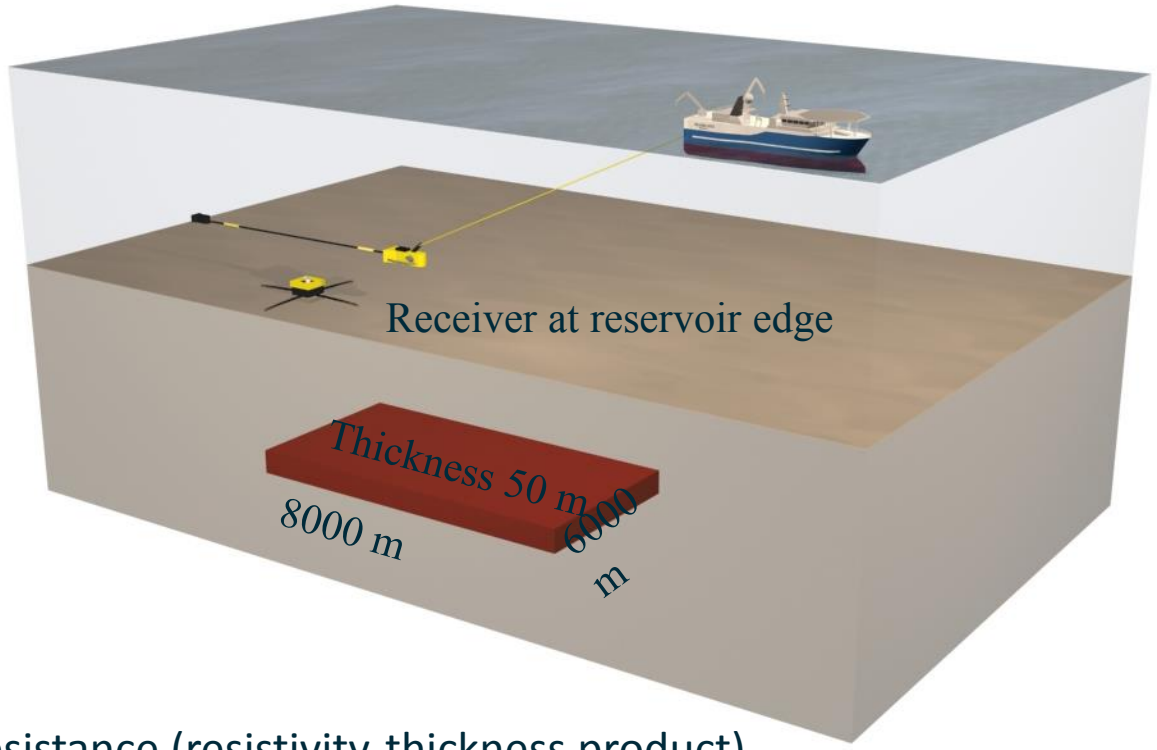


Non-uniqueness

Non-uniqueness



Transvers resistance (resistivity-thickness product)

$$T_r = (\rho_{anomaly} - \rho_{background})(z_{bottom} - z_{top})$$

$$T_r = \Delta\rho\Delta z$$

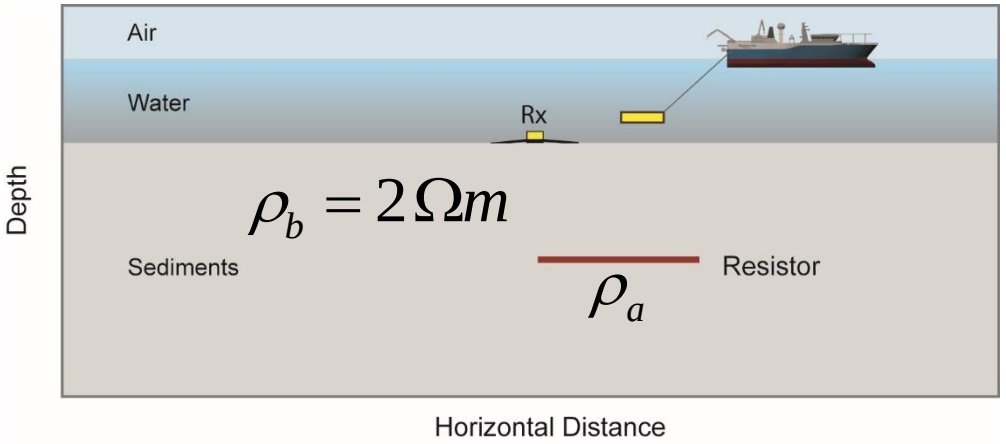
$$T_r \approx \rho_{anomaly}\Delta z$$

Non-uniqueness

$$T_R = \Delta\rho\,\Delta z = 5000\,\Omega\mathrm{m}^2 \qquad \Delta\rho = \rho_a - \rho_b$$

$$\Delta z = 50\,\mathrm{m}$$

$$\rho_a = 102\,\Omega\mathrm{m}$$

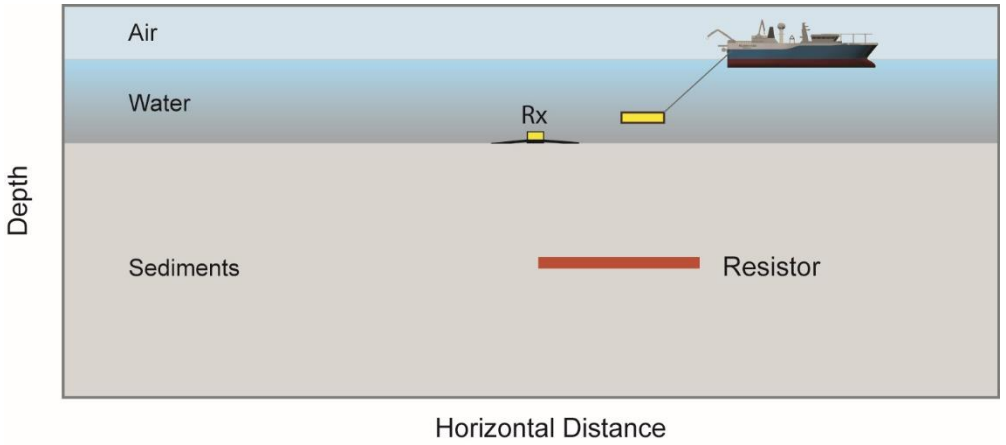


Non-uniqueness

$$T_R = \Delta\rho\,\Delta z = 5000\,\Omega\mathrm{m}^2$$

$$\Delta z = 100\,\mathrm{m}$$

$$\rho_a = 52\,\Omega\mathrm{m}$$

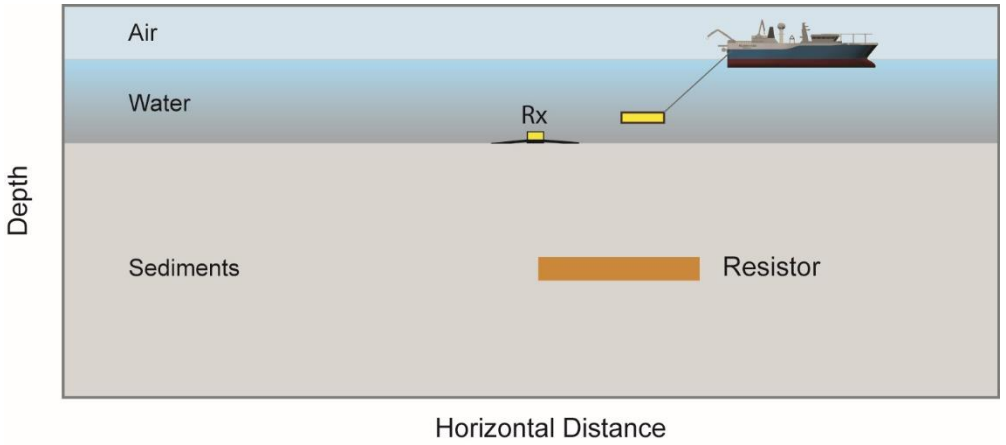


Non-uniqueness

$$T_R = \Delta\rho\,\Delta z = 5000\,\Omega\mathrm{m}^2$$

$$\Delta z = 200\,\mathrm{m}$$

$$\rho_a = 27\,\Omega\mathrm{m}$$

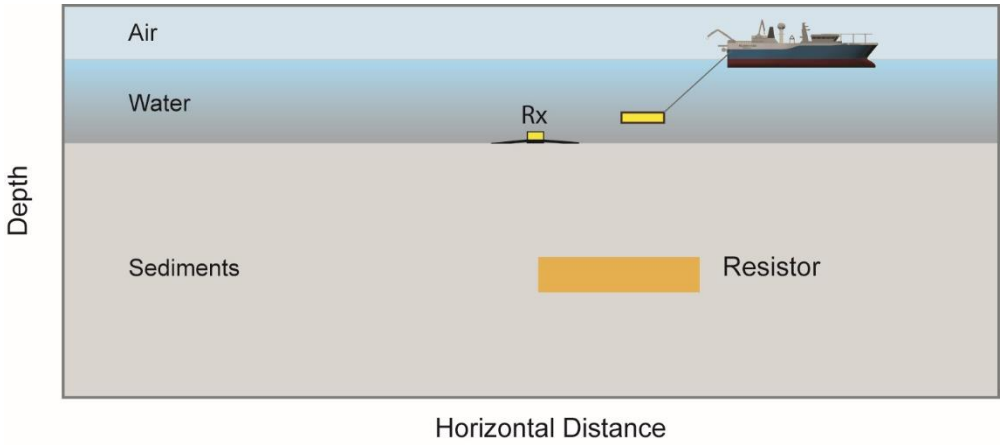


Non-uniqueness

$$T_R = \Delta\rho\,\Delta z = 5000\,\Omega\mathrm{m}^2$$

$$\Delta z = 300\,\mathrm{m}$$

$$\rho_a = 18.7\,\Omega\mathrm{m}$$

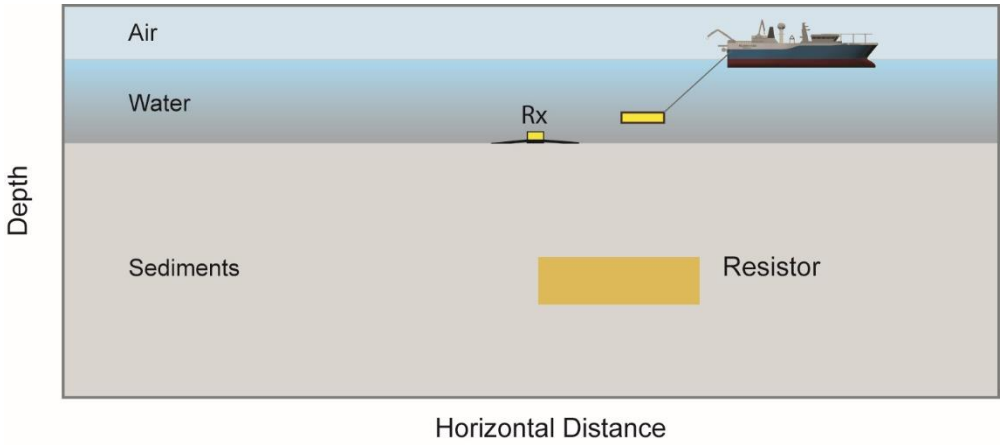


Non-uniqueness

$$T_R = \Delta\rho\,\Delta z = 5000\,\Omega\mathrm{m}^2$$

$$\Delta z = 400\,\mathrm{m}$$

$$\rho_a = 14.5\,\Omega\mathrm{m}$$

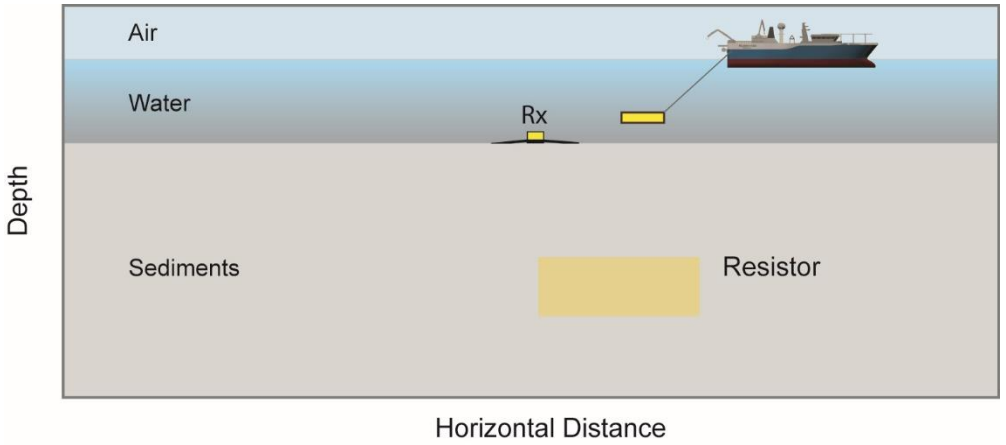


Non-uniqueness

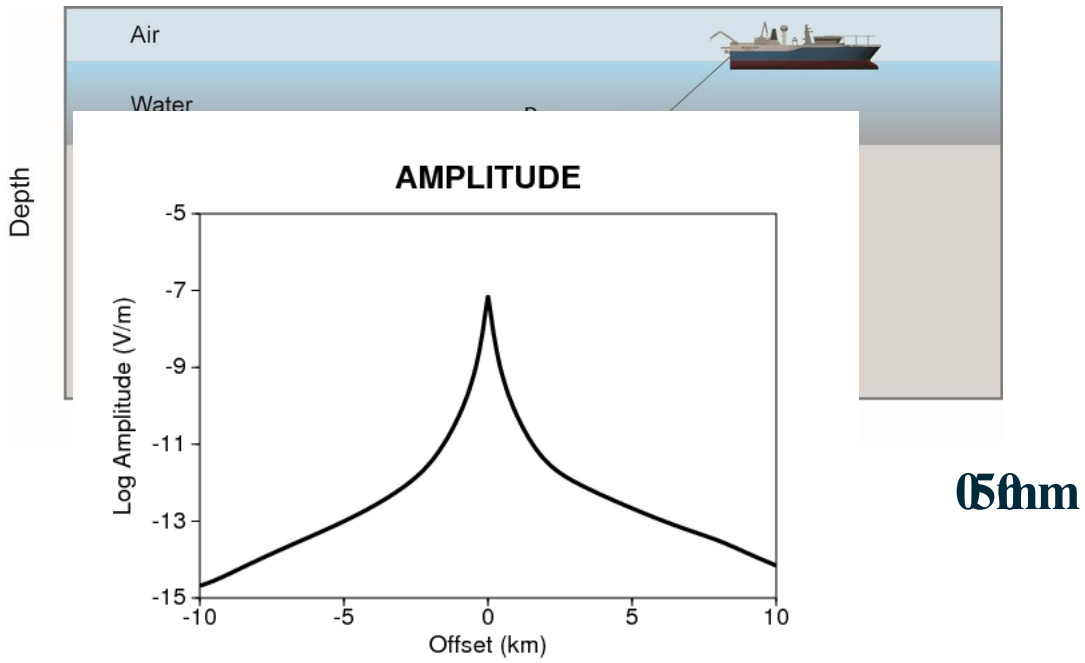
$$T_R = \Delta\rho\,\Delta z = 5000\,\Omega\mathrm{m}^2$$

$$\Delta z = 500\,\mathrm{m}$$

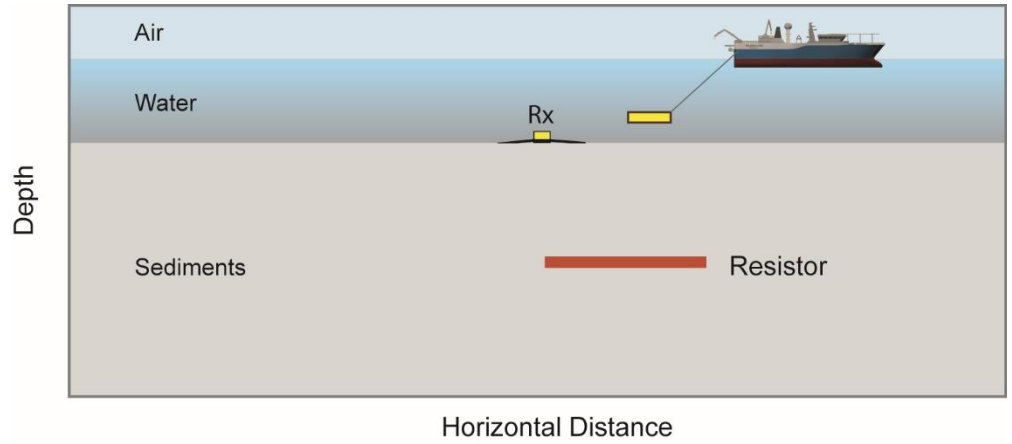
$$\rho_a = 12\,\Omega\mathrm{m}$$

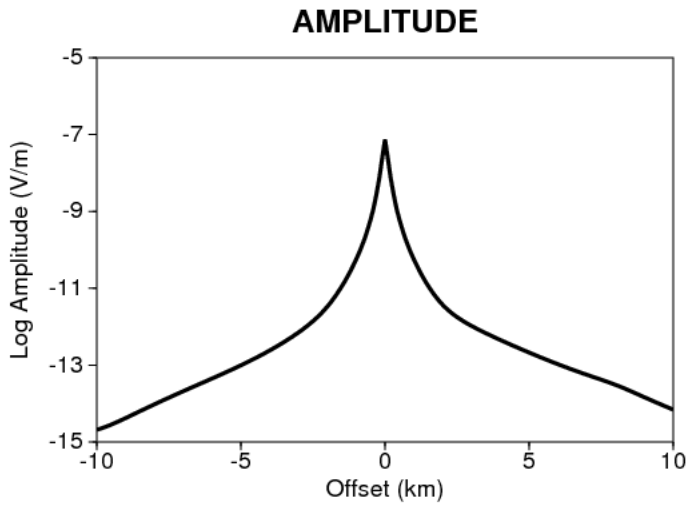


Non-uniqueness

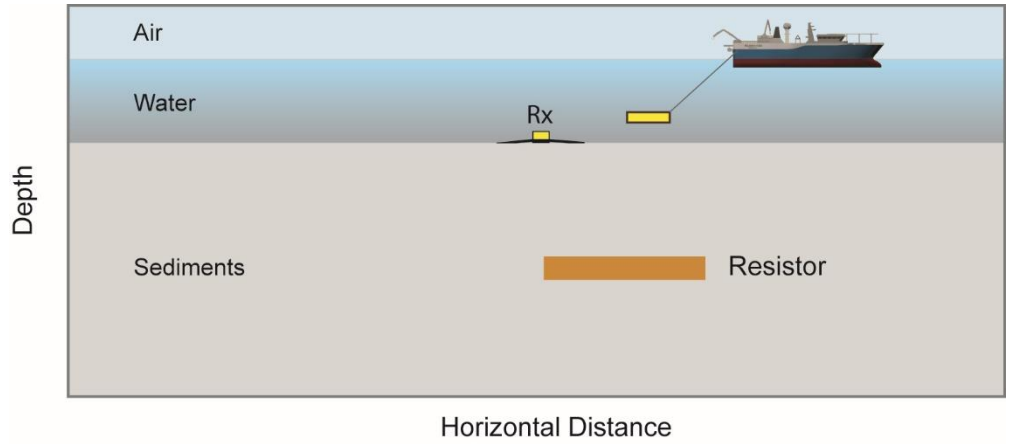


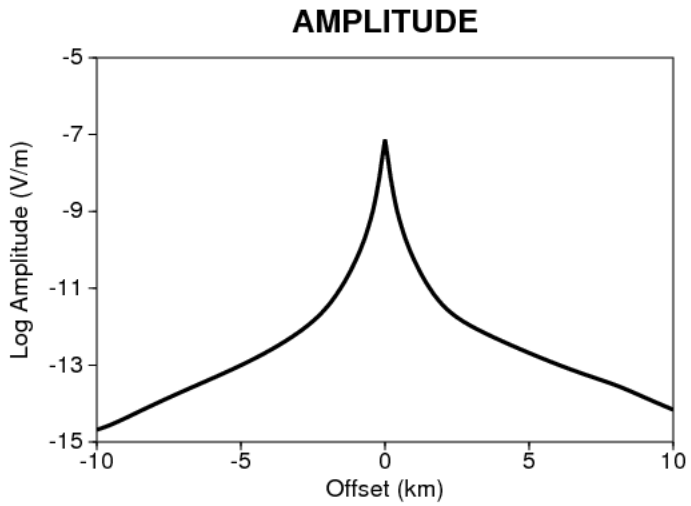
50 Ohm


Air
Water
Rx
Depth
Sediments
Resistor
Horizontal Distance

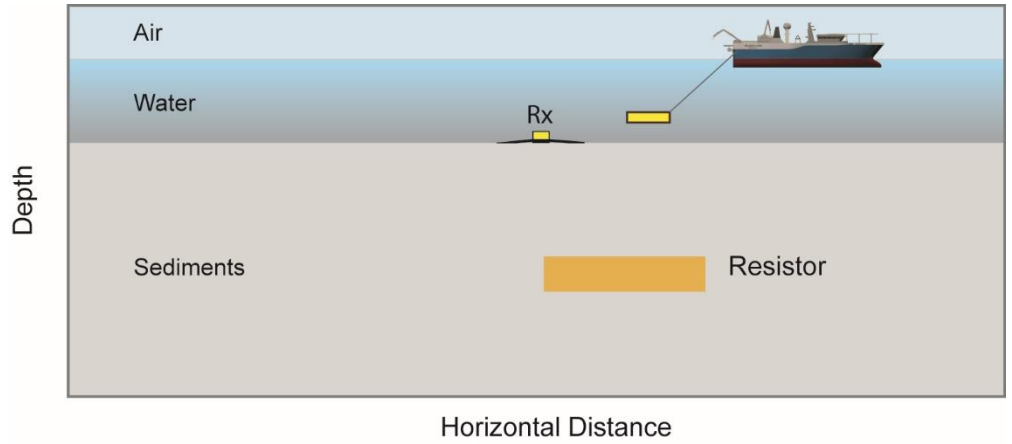

AMPLITUDE
Log Amplitude (V/m)
-5
-7
-9
-11
-13
-15
-10
-5
0
5
10
Offset (km)

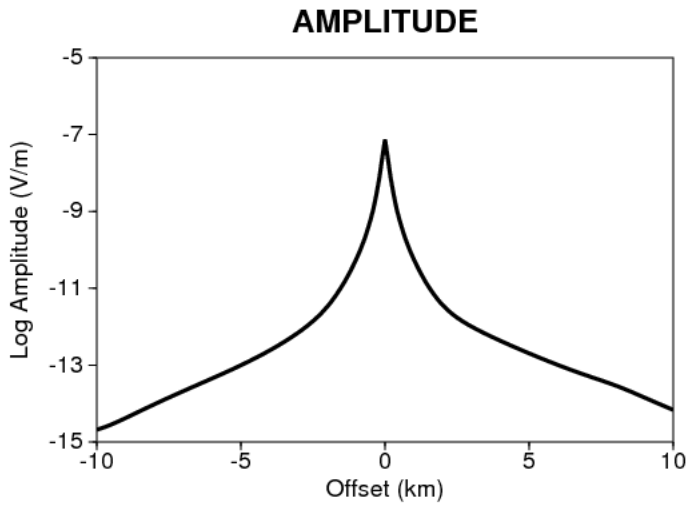
100 m


Air
Water
Rx
Depth
Sediments
Resistor
Horizontal Distance

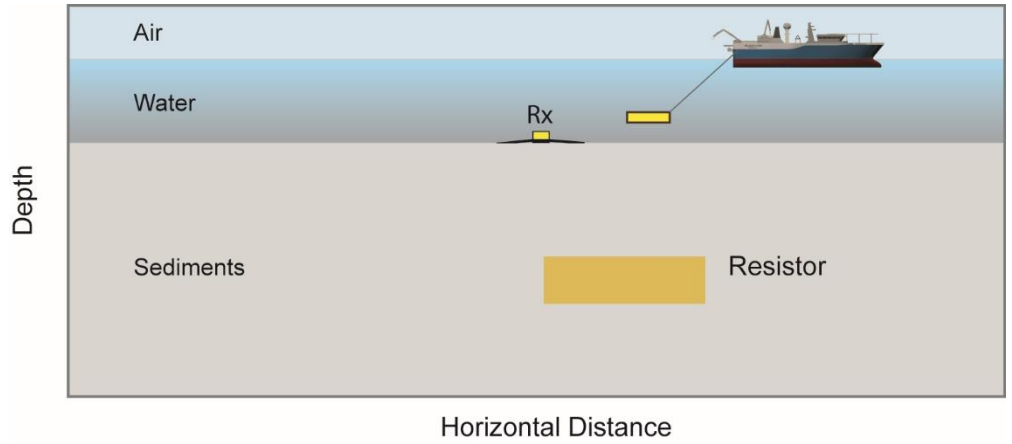

AMPLITUDE
Log Amplitude (V/m)
-5
-7
-9
-11
-13
-15
-10
-5
0
5
10
Offset (km)

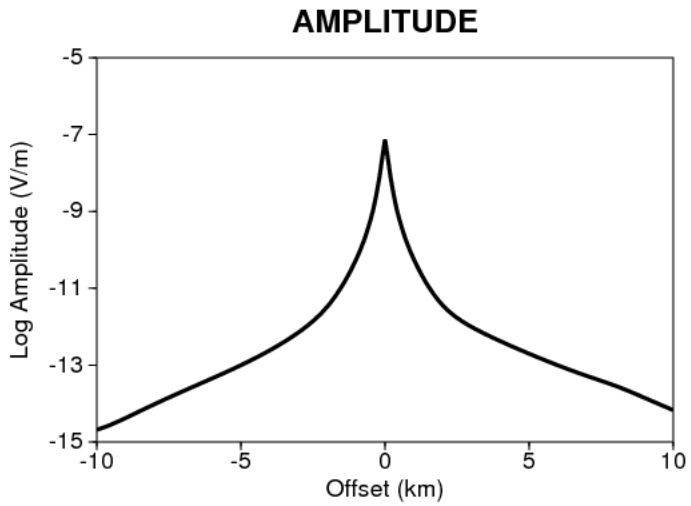
200 m


Air
Water
Rx
Depth
Sediments
Resistor
Horizontal Distance

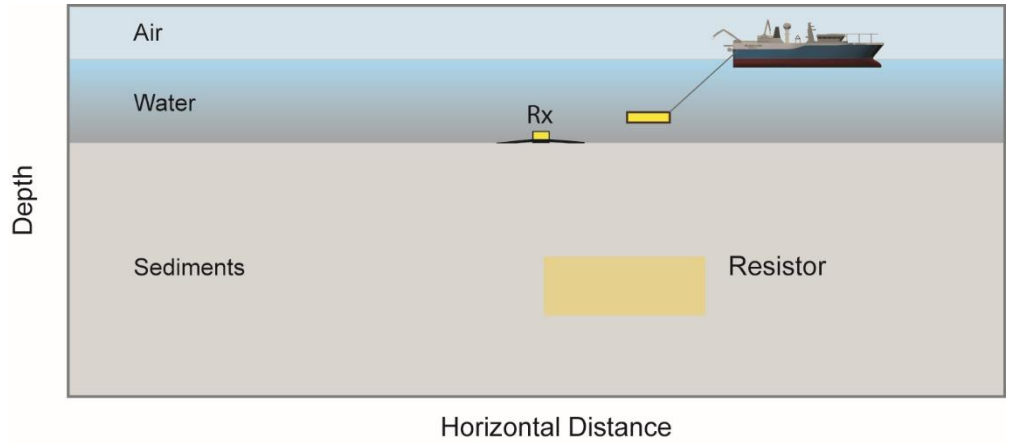

AMPLITUDE
Log Amplitude (V/m)
-5
-7
-9
-11
-13
-15
-10
-5
0
5
10
Offset (km)

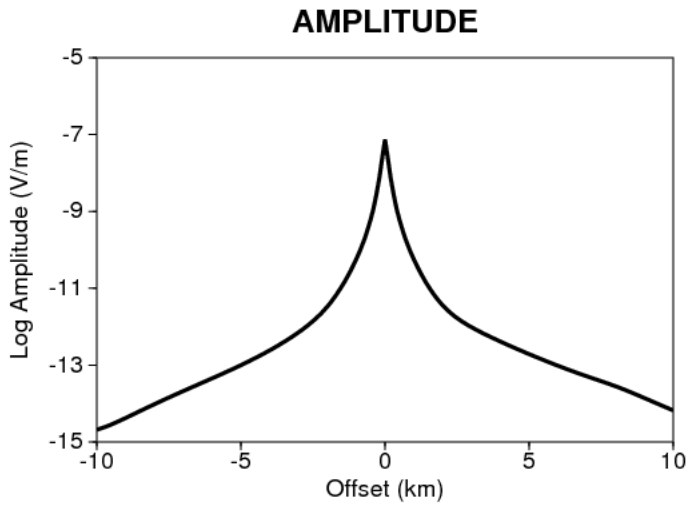
300 m


Air
Water
Rx
Depth
Sediments
Resistor
Horizontal Distance

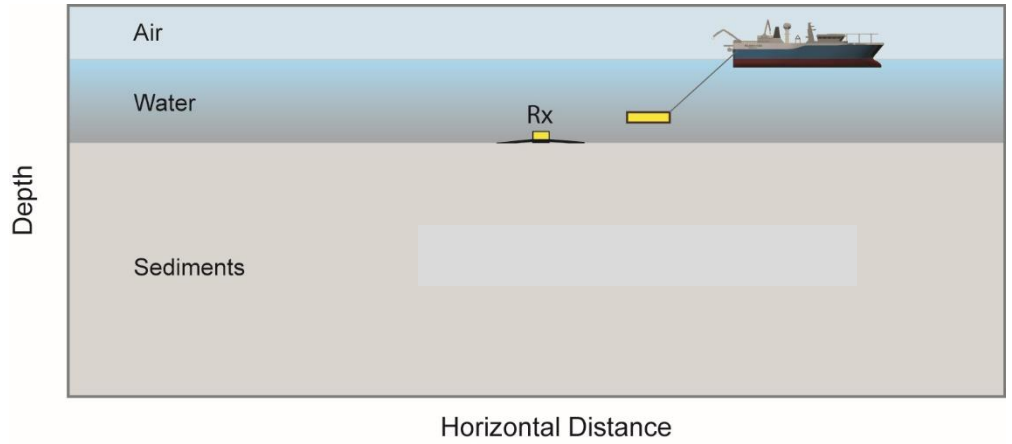

AMPLITUDE
Log Amplitude (V/m)
-5
-7
-9
-11
-13
-15
-10
-5
0
5
10
Offset (km)

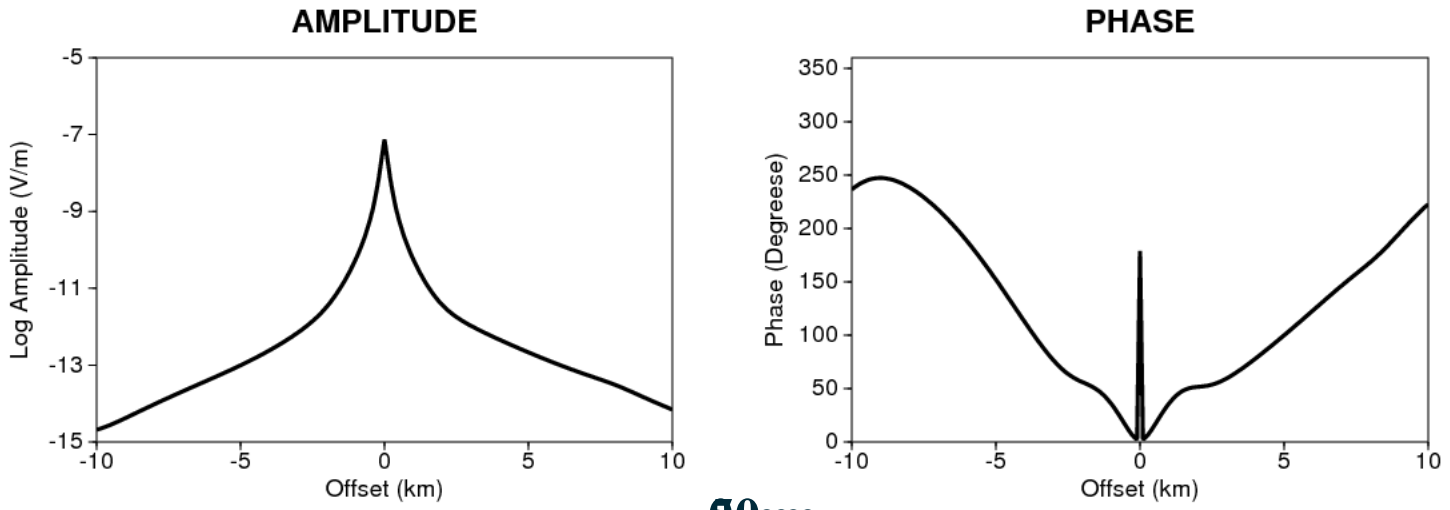
400 m


Air
Water
Rx
Depth
Sediments
Resistor
Horizontal Distance

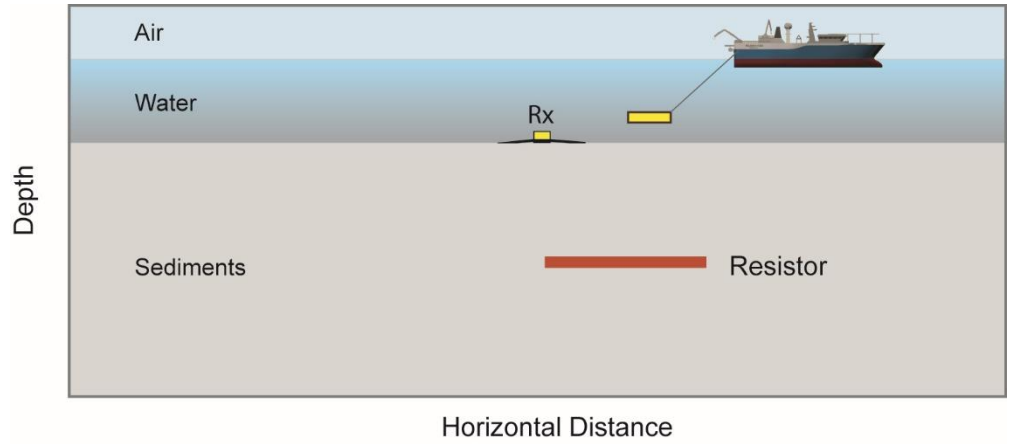

AMPLITUDE
Log Amplitude (V/m)
-5
-7
-9
-11
-13
-15
-10
-5
0
5
10
Offset (km)

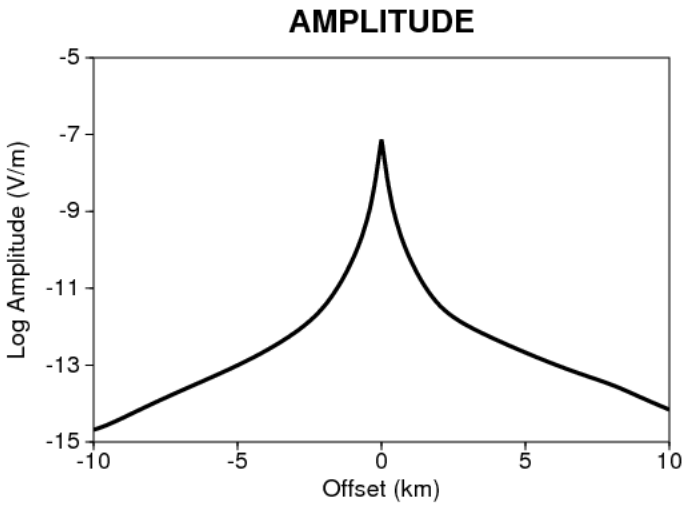
500 m

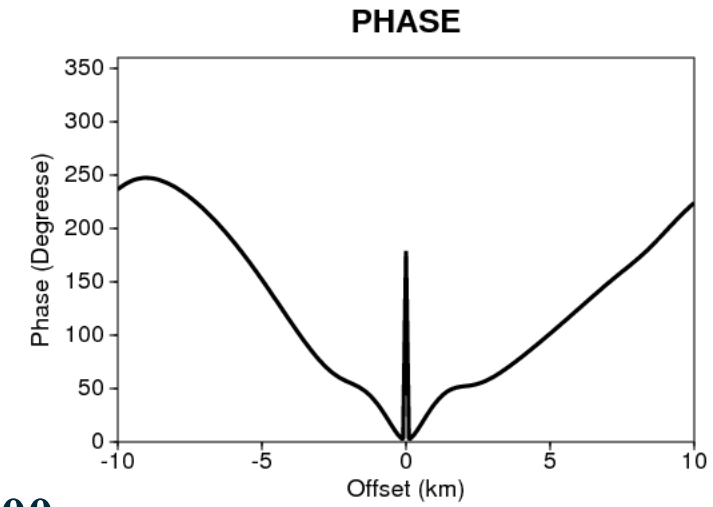

Air
Water
Rx
Sediments
Depth
Horizontal Distance


AMPLITUDE
Log Amplitude (V/m)
-5
-7
-9
-11
-13
-15
-10
-5
0
5
10
Offset (km)
PHASE
Phase (Degreese)
350
300
250
200
150
100
50
0
-10
-5
0
5
10
Offset (km)

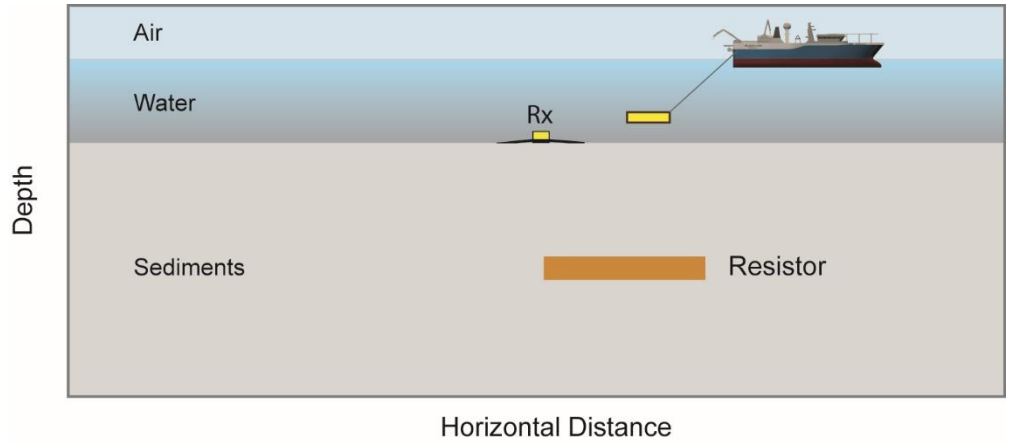
60m

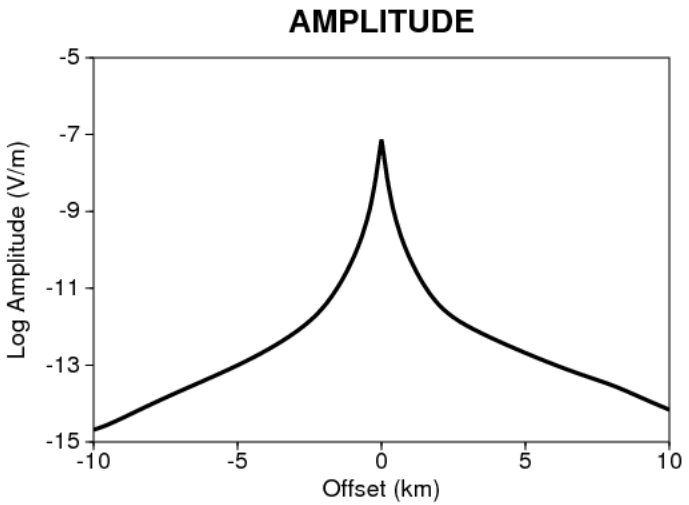

Air
Water
Rx
Depth
Sediments
Resistor
Horizontal Distance

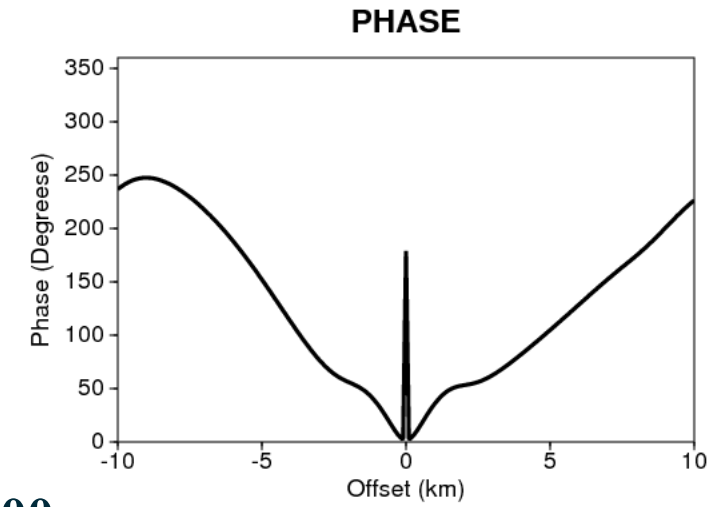

AMPLITUDE
Log Amplitude (V/m)
-5
-7
-9
-11
-13
-15
-10
-5
0
5
10
Offset (km)


PHASE
Phase (Degreese)
350
300
250
200
150
100
50
0
-10
-5
0
5
10
Offset (km)

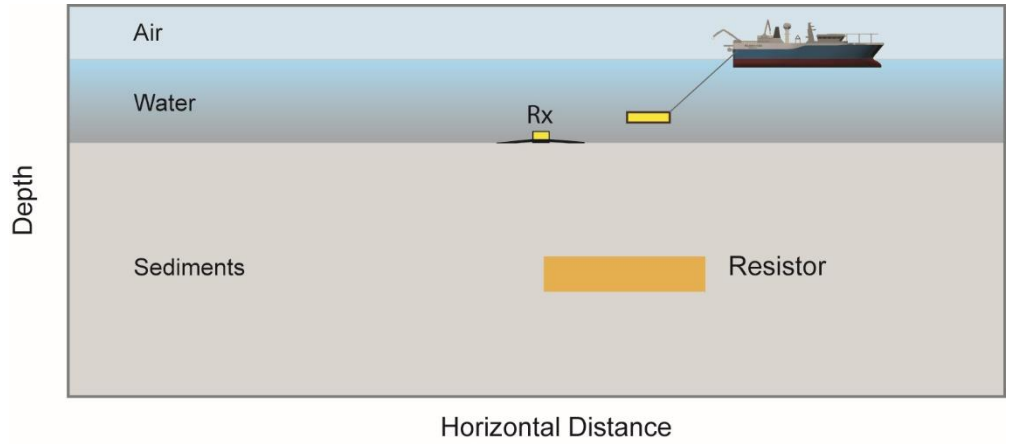
100 m

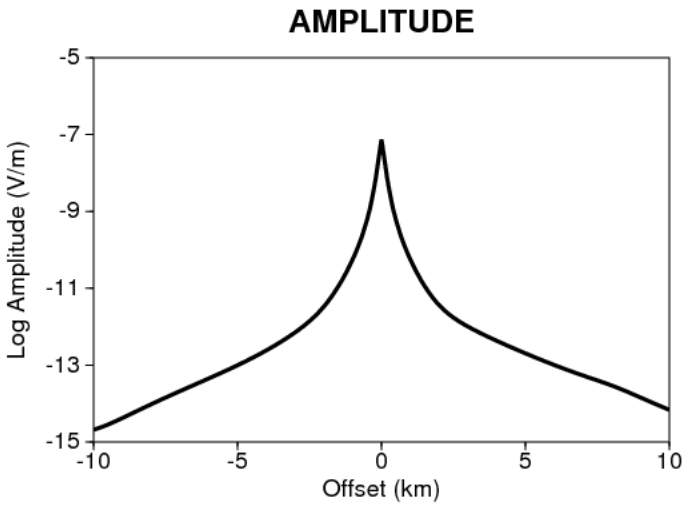

Air
Water
Rx
Depth
Sediments
Resistor
Horizontal Distance

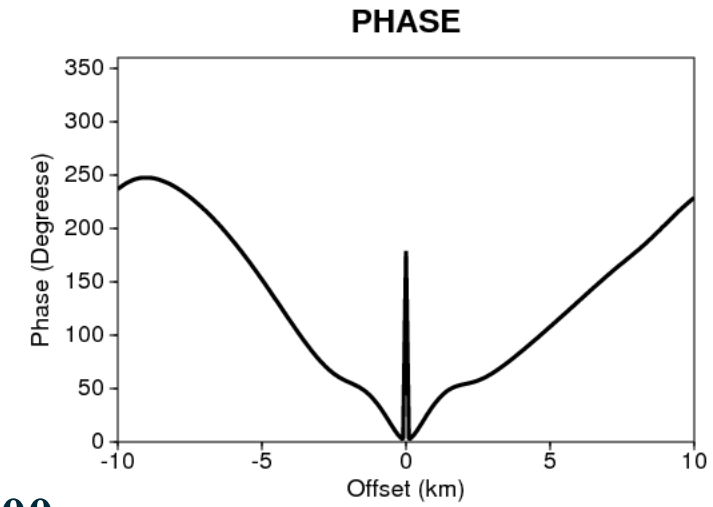

AMPLITUDE
Log Amplitude (V/m)
-5
-7
-9
-11
-13
-15
-10
-5
0
5
10
Offset (km)


PHASE
Phase (Degreese)
350
300
250
200
150
100
50
0
-10
-5
0
5
10
Offset (km)

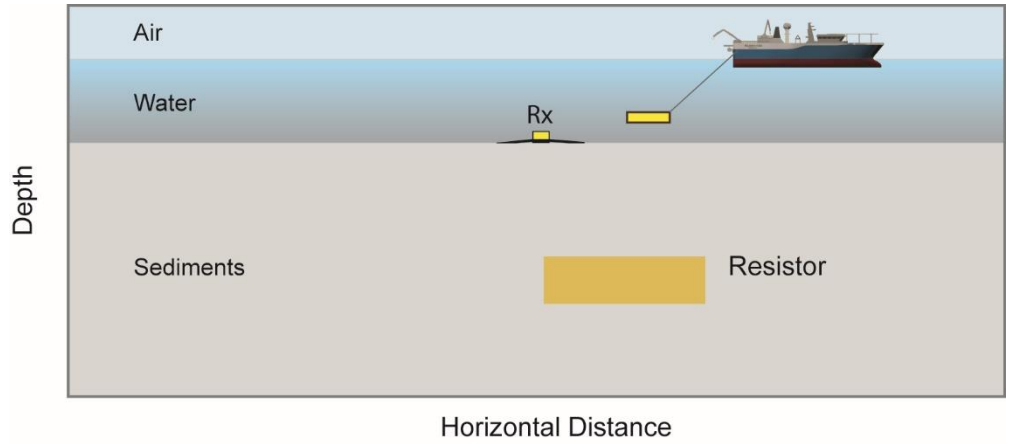
200 m

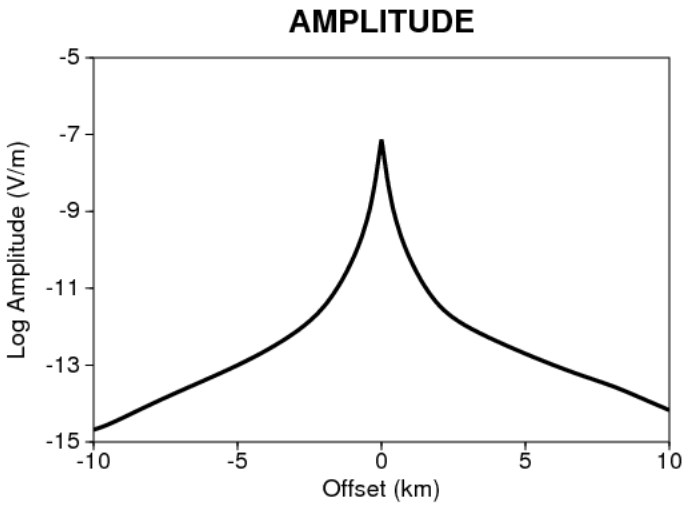

Air
Water
Rx
Depth
Sediments
Resistor
Horizontal Distance

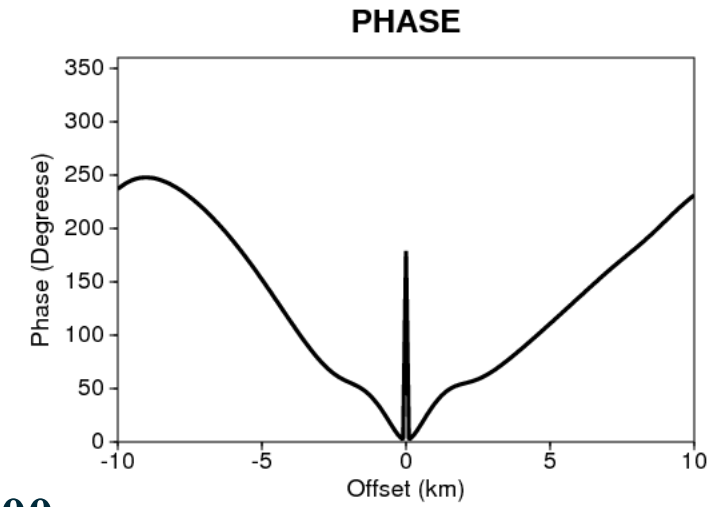

AMPLITUDE
Log Amplitude (V/m)
-5
-7
-9
-11
-13
-15
-10
-5
0
5
10
Offset (km)


PHASE
Phase (Degreese)
350
300
250
200
150
100
50
0
-10
-5
0
5
10
Offset (km)

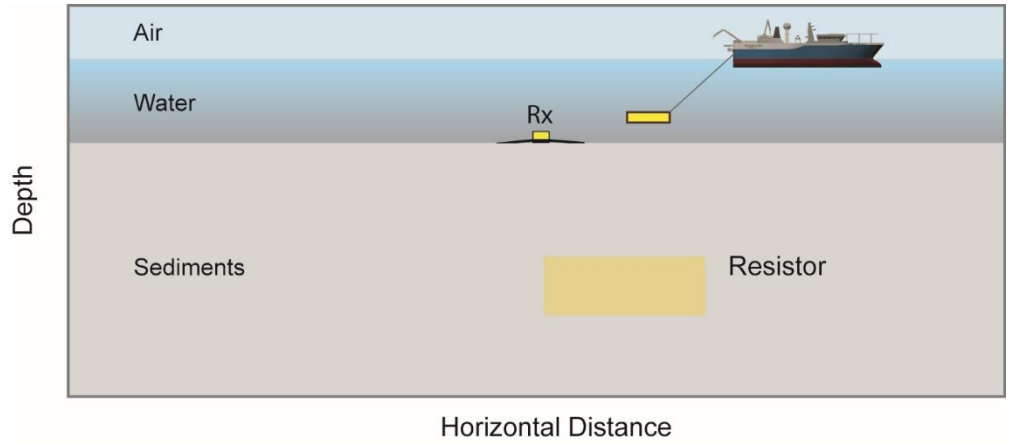
300 m

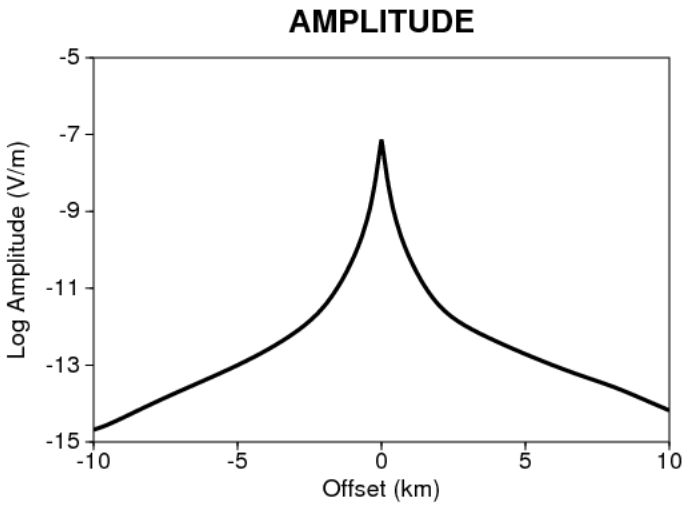

Air
Water
Rx
Depth
Sediments
Resistor
Horizontal Distance

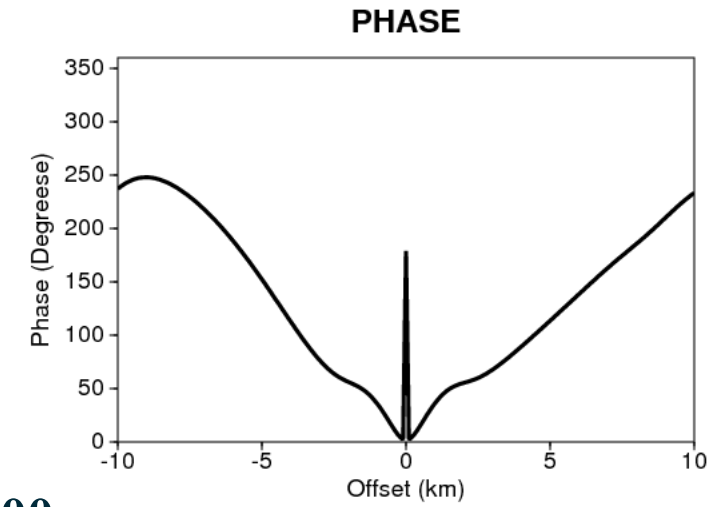

AMPLITUDE
Log Amplitude (V/m)
-5
-7
-9
-11
-13
-15
-10
-5
0
5
10
Offset (km)


PHASE
Phase (Degreese)
350
300
250
200
150
100
50
0
-10
-5
0
5
10
Offset (km)

400 m


Air
Water
Rx
Depth
Sediments
Resistor
Horizontal Distance


AMPLITUDE
Log Amplitude (V/m)
-5
-7
-9
-11
-13
-15
-10
-5
0
5
10
Offset (km)


PHASE
Phase (Degreese)
350
300
250
200
150
100
50
0
-10
-5
0
5
10
Offset (km)

500 m

There are other forms of non-uniqueness in marine CSEM data

Burial depth versus thin layer resistivity is another example

Inversion of marine CSEM data

•Formulate the inversion as a regularized non-linear least-squares problem.

- The cost function consist of a data misfit function and a regularization term:

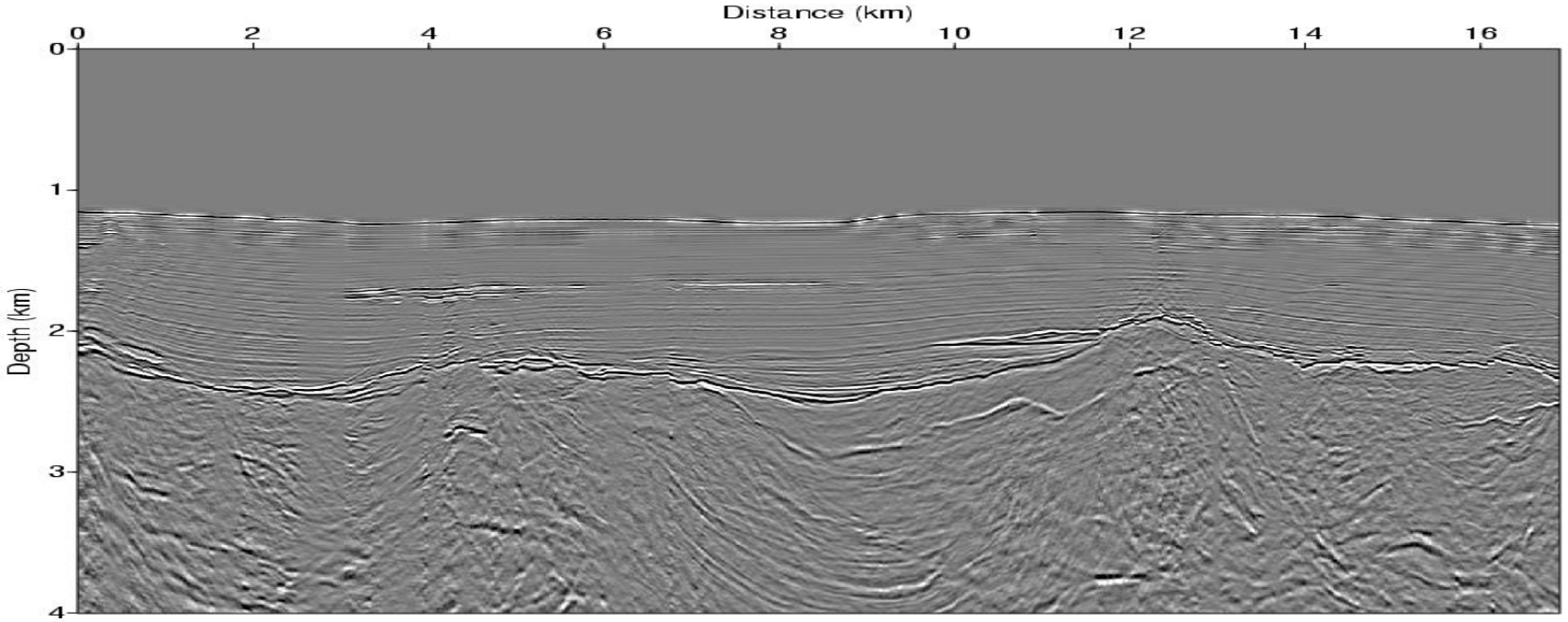
$$\varepsilon(\rho^n, \rho_{ref}) = \sum_j W_j \left| \left(E_j^{obs} - E_j^{pred}(\rho^n) \right) \right|^2 + \lambda_n R(\rho^n, \rho_{ref})$$

- The data weights W provide a measure of the data uncertainty:

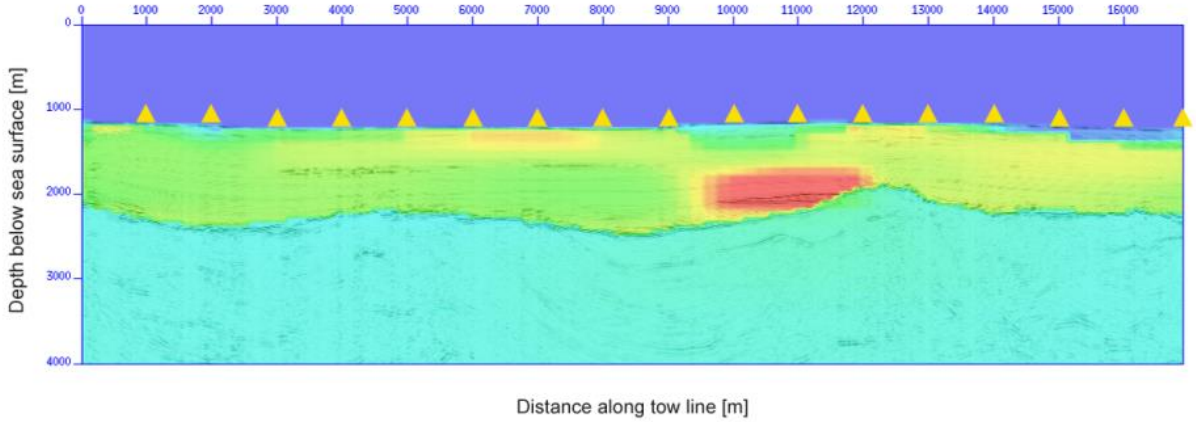
$$W_j = \frac{1}{\alpha^2 \left| E_j^{obs} \right|^2 + \eta^2}$$

- We assume that the data uncertainty can be described as the sum of a multiplicative uncertainty, proportional to the measured field amplitude, and a noise floor.

$$R(\rho^n, \rho_{ref}) = \int \left| W_x (\partial_x \rho^n - \partial_x \rho_{ref}) \right|^2 dxdz + \int \left| W_z (\partial_z \rho^n - \partial_z \rho_{ref}) \right|^2 dxdz$$



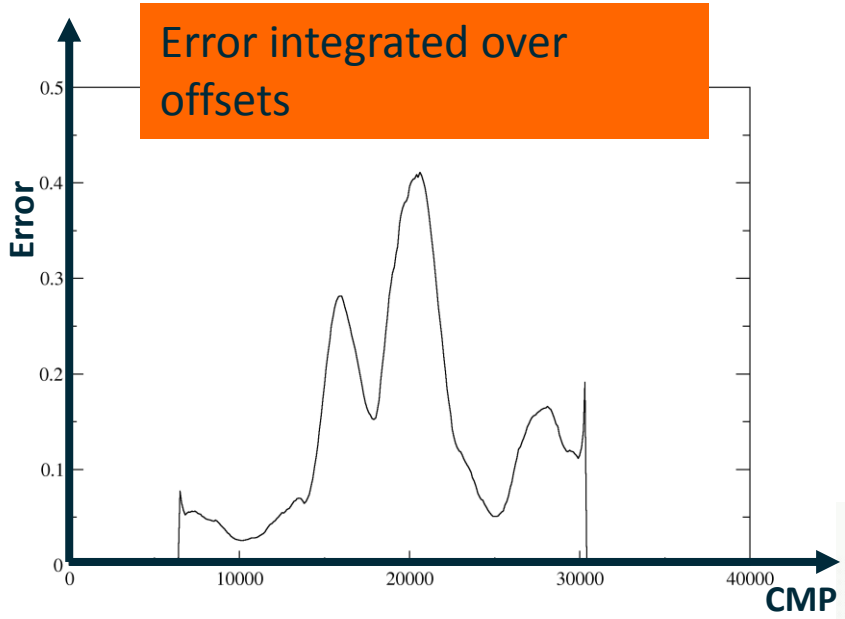
2.5D CSEM inversion with soft seismic constraints

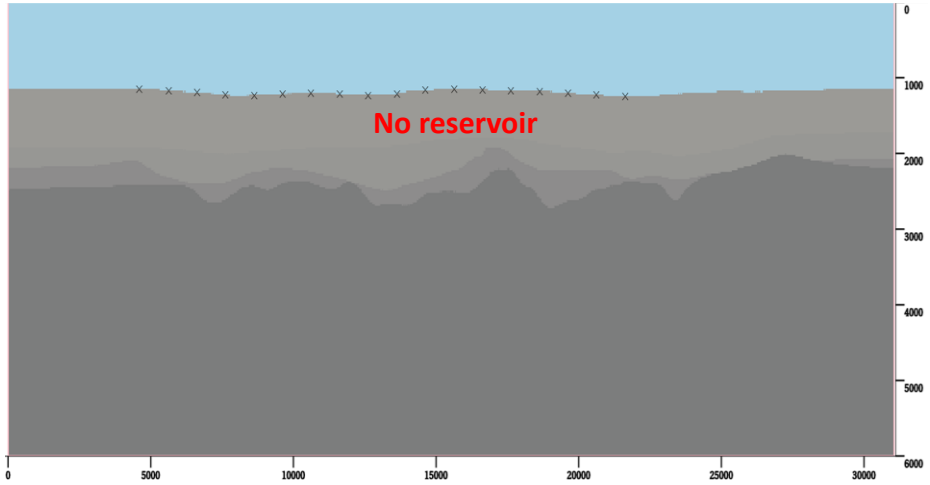


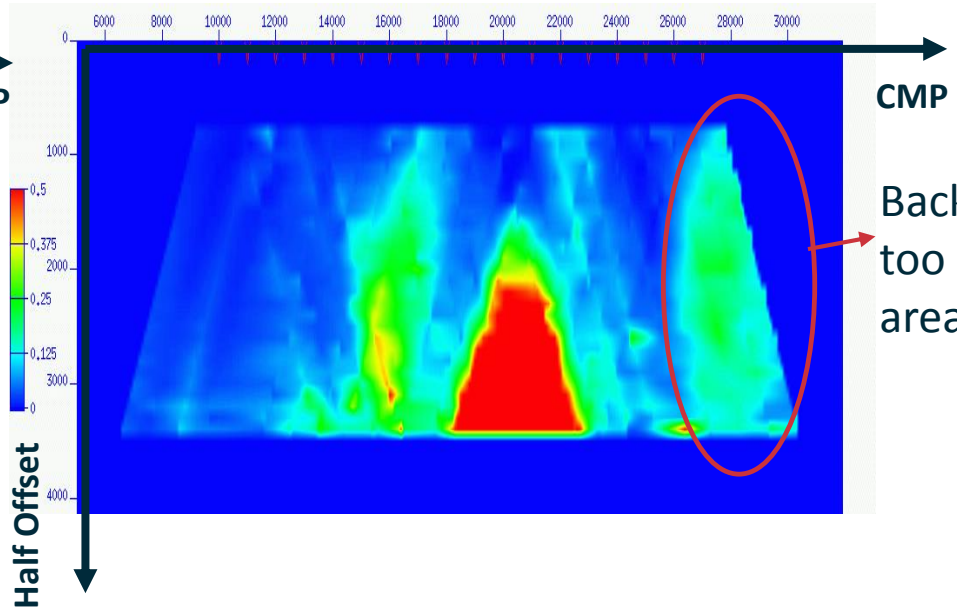
2.5D: 2D «geology» and 3D field

3D modeling real data

Error integrated over offsets



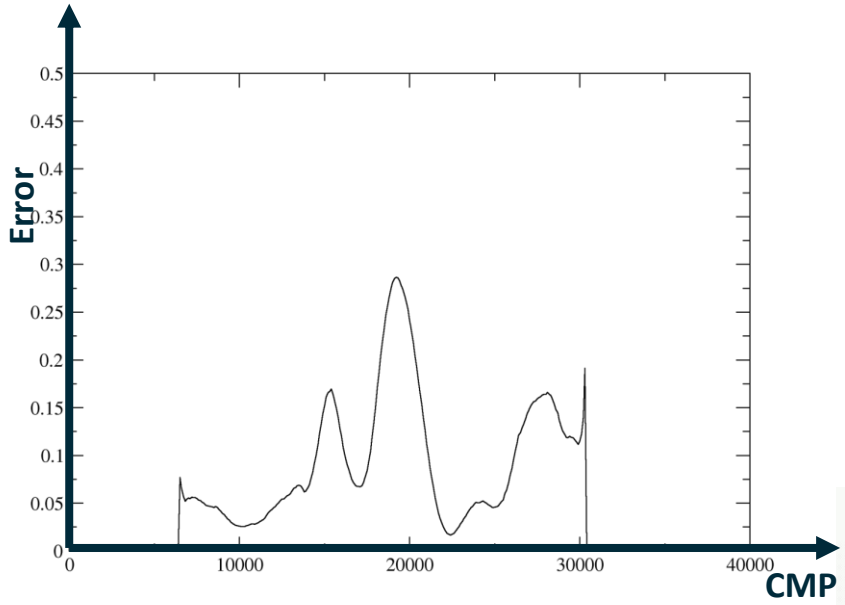


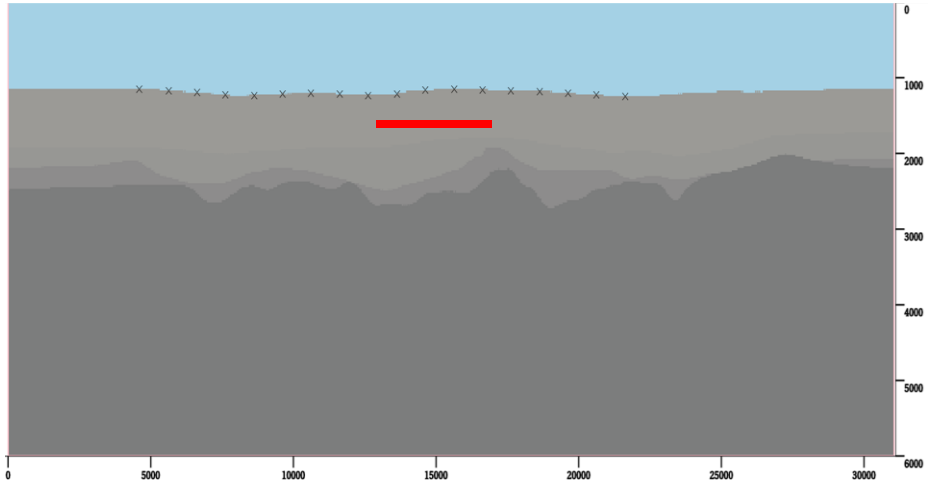


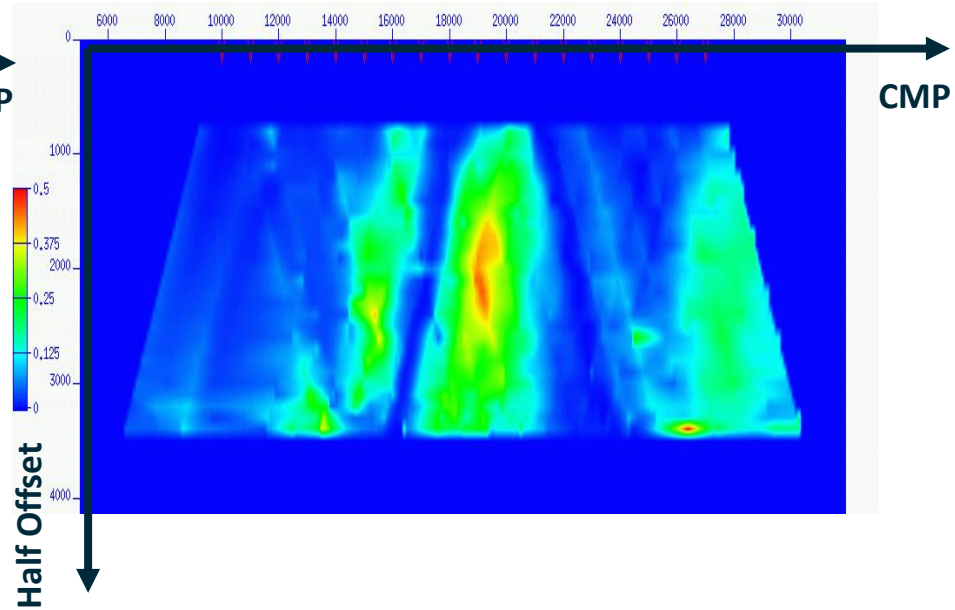
Blue : Small error

Background model too resistive in this area

3D modeling real data

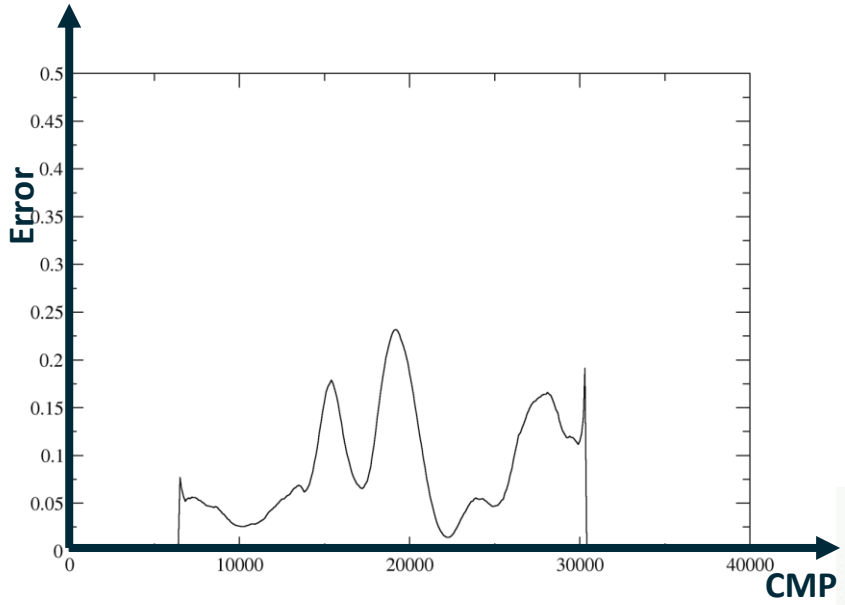


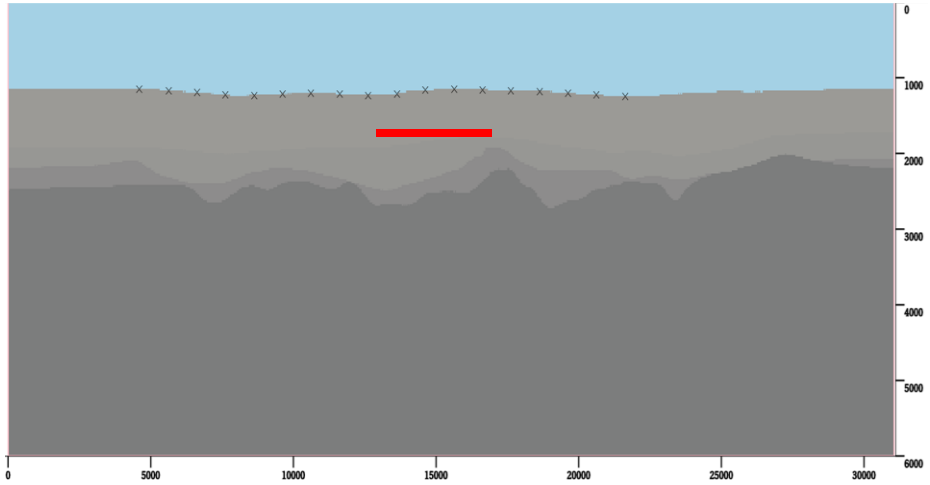


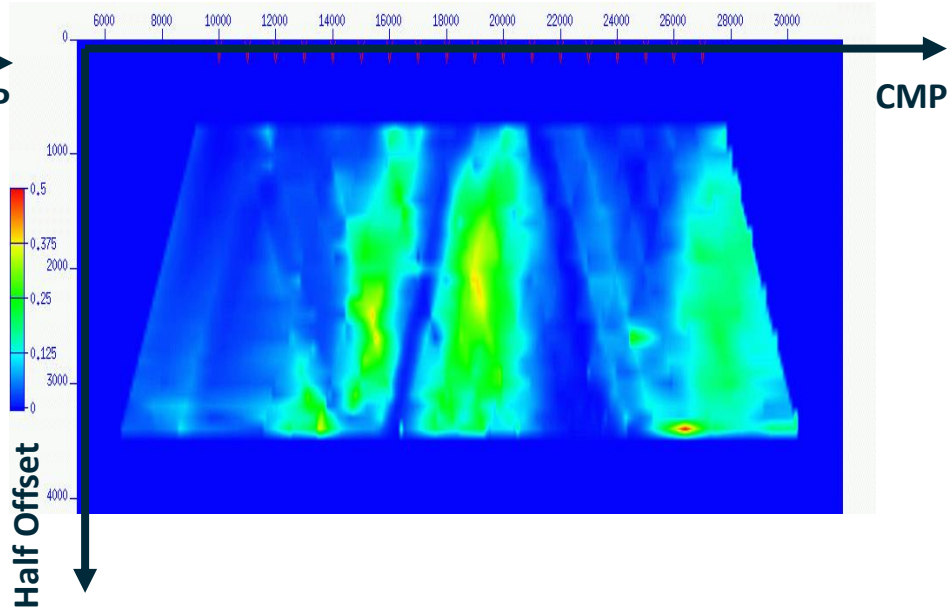


Blue : Small error

3D modeling real data

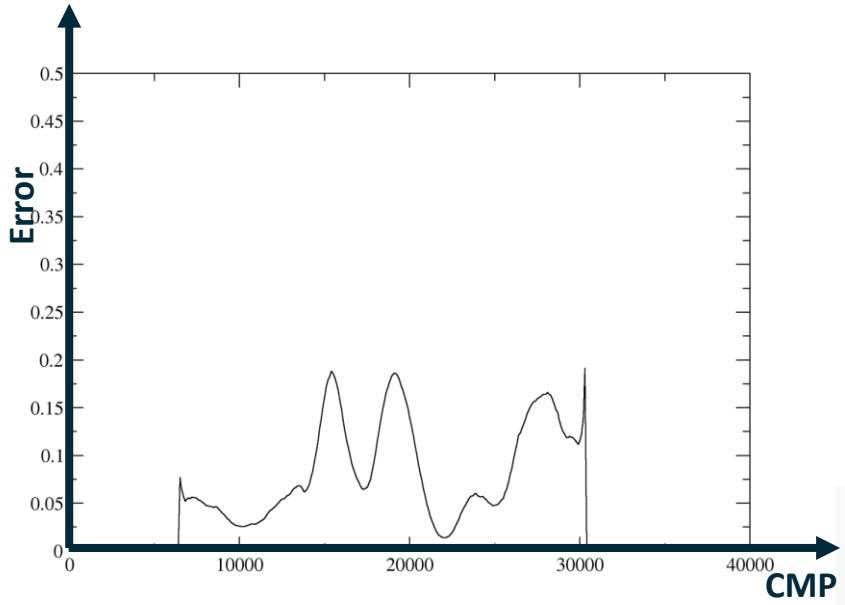


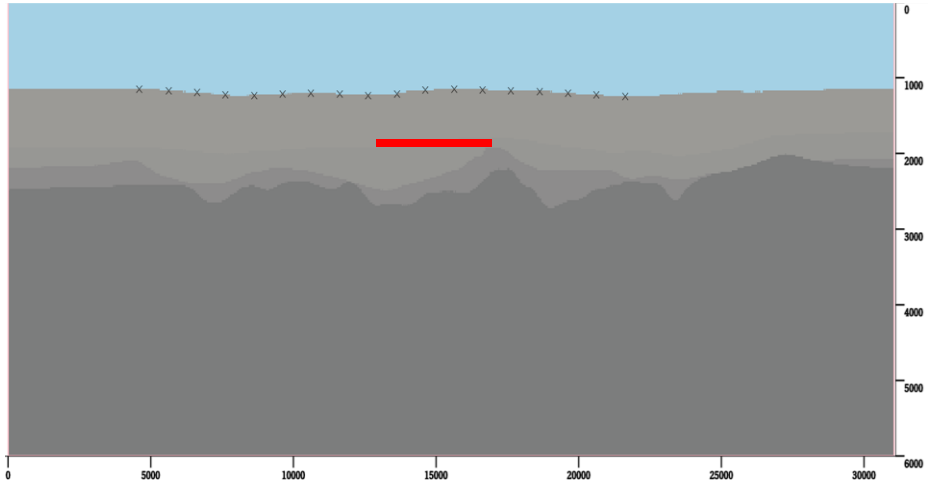


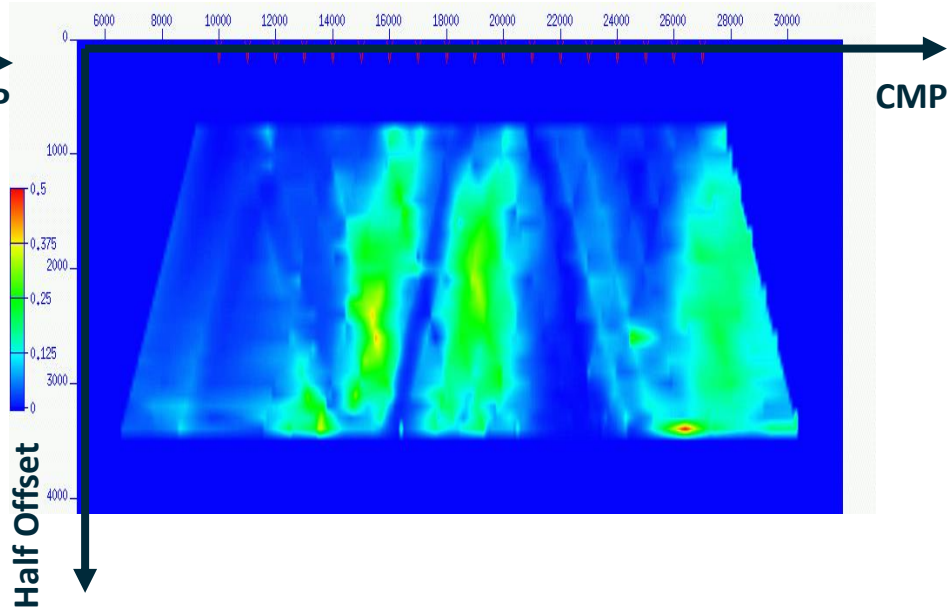


Blue : Small error

3D modeling real data

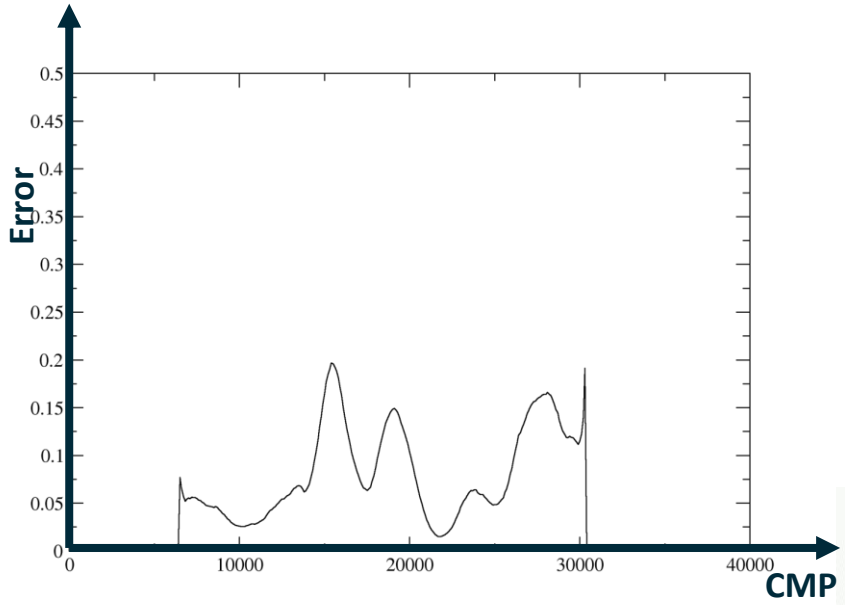


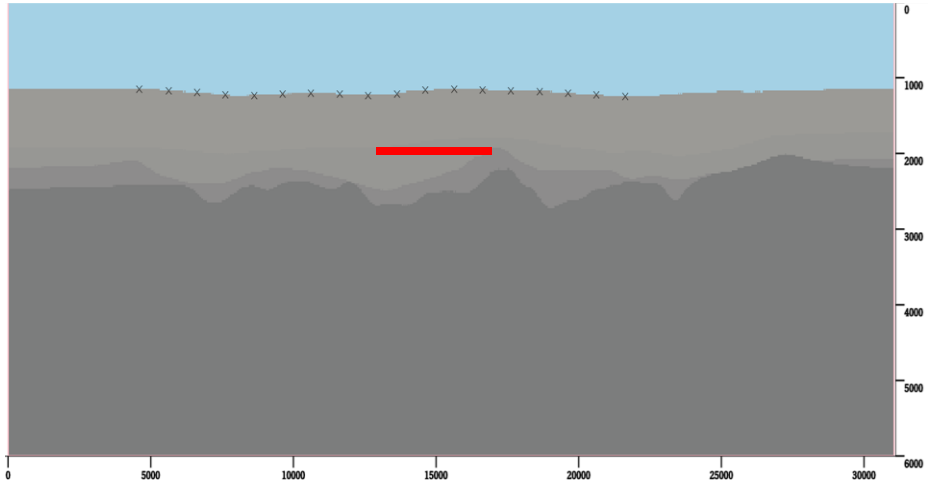


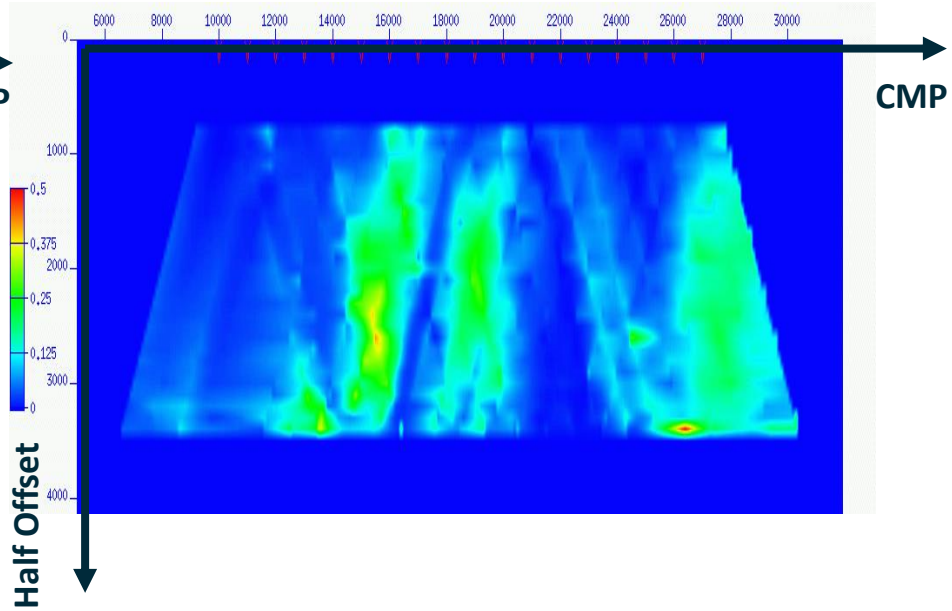


Blue : Small error

3D modeling real data

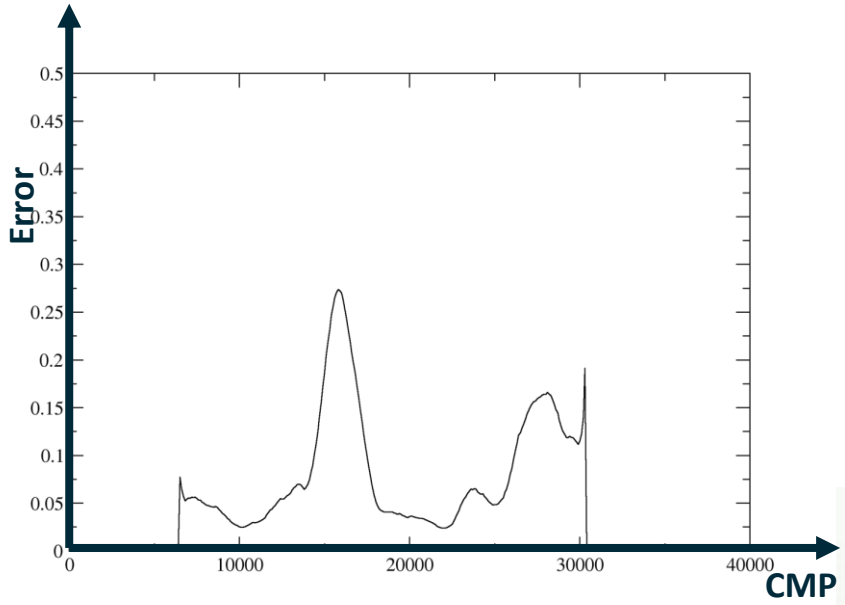


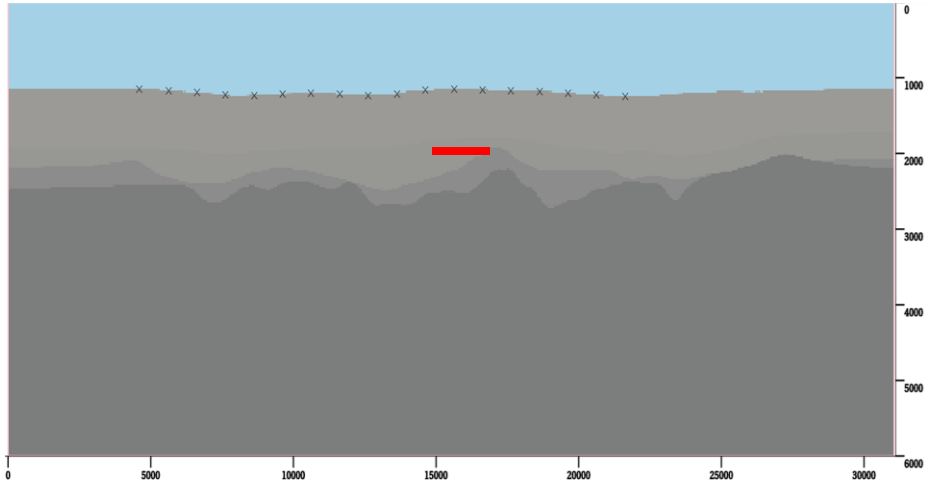


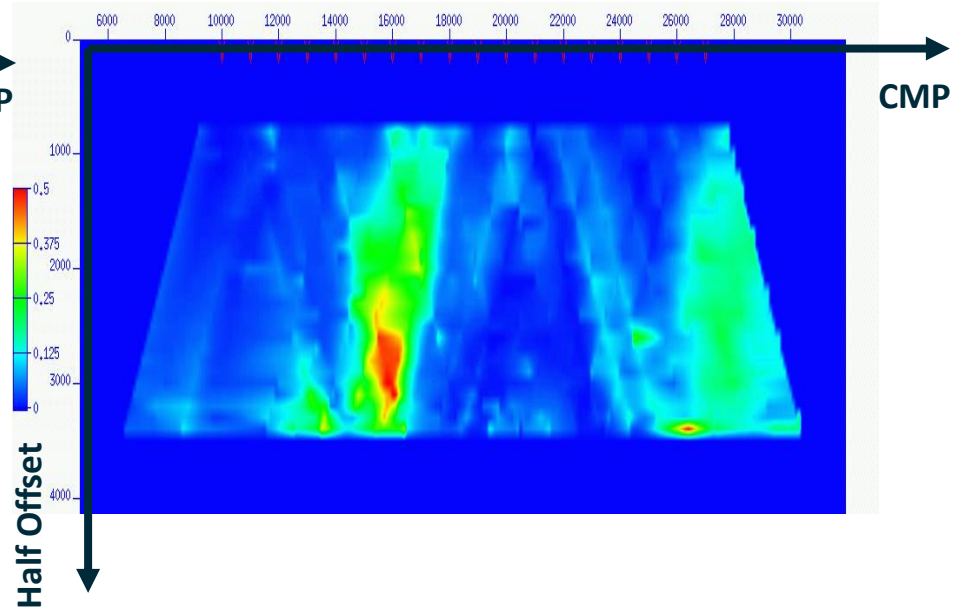


Blue : Small error

3D modeling real data

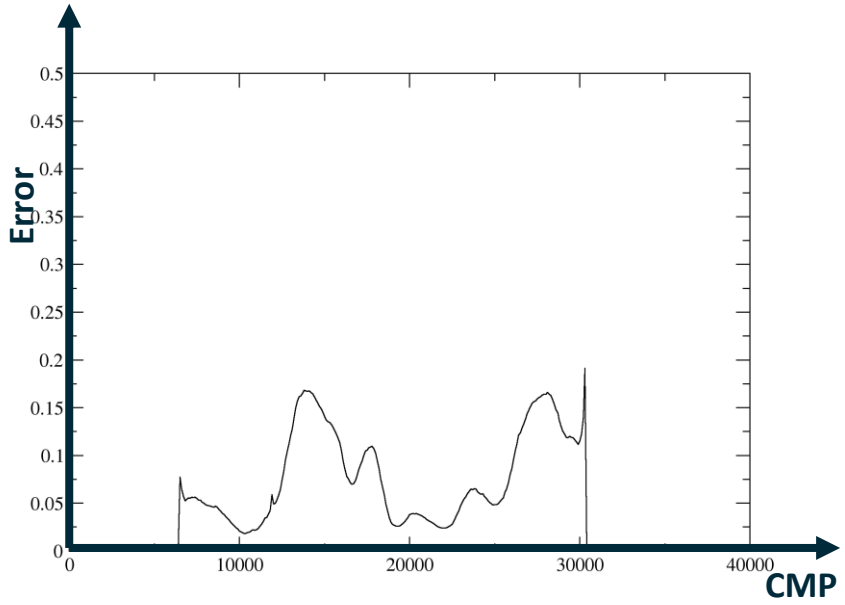


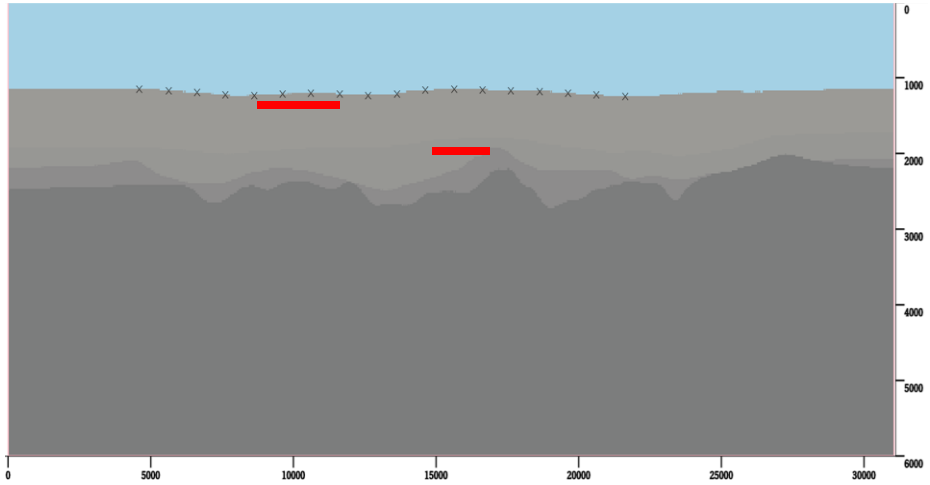


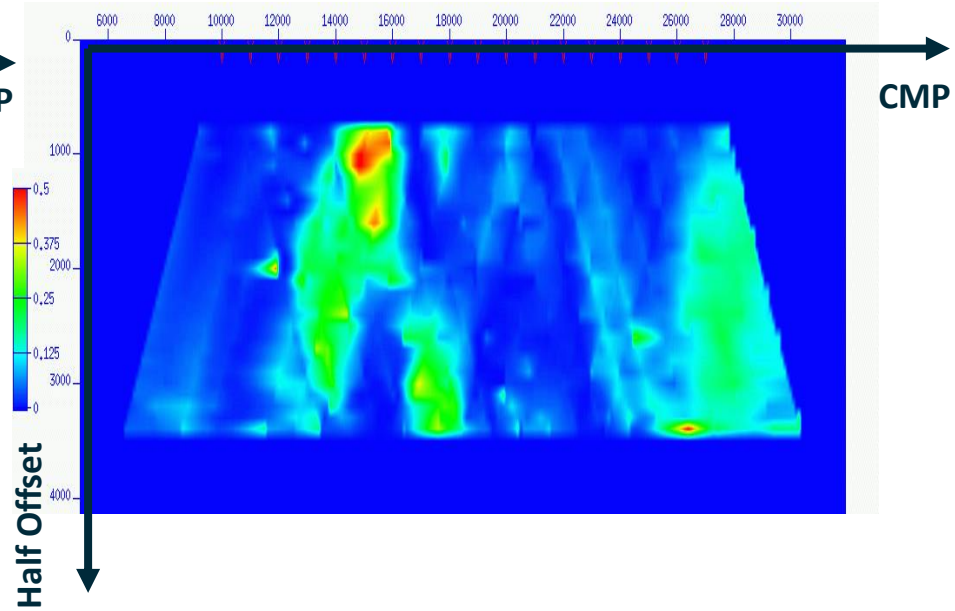


Blue : Small error

3D modeling real data

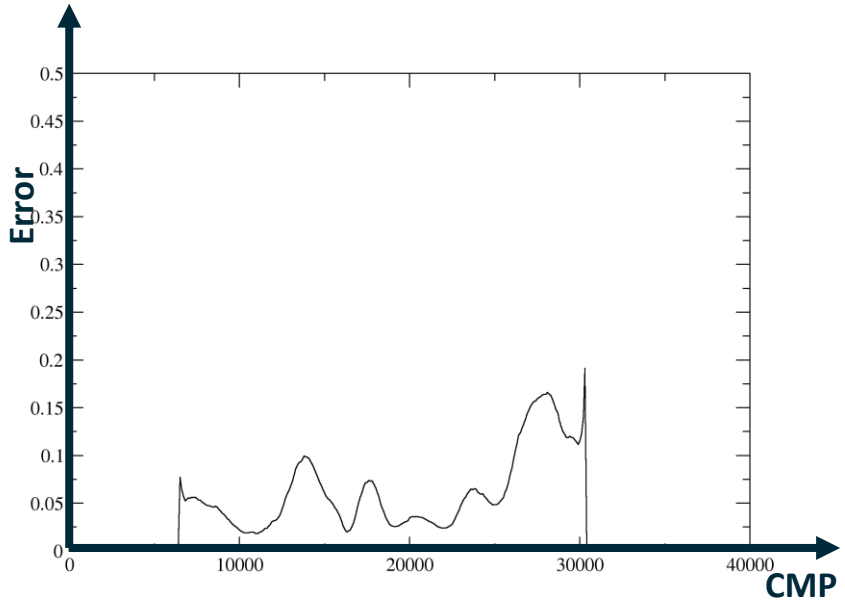


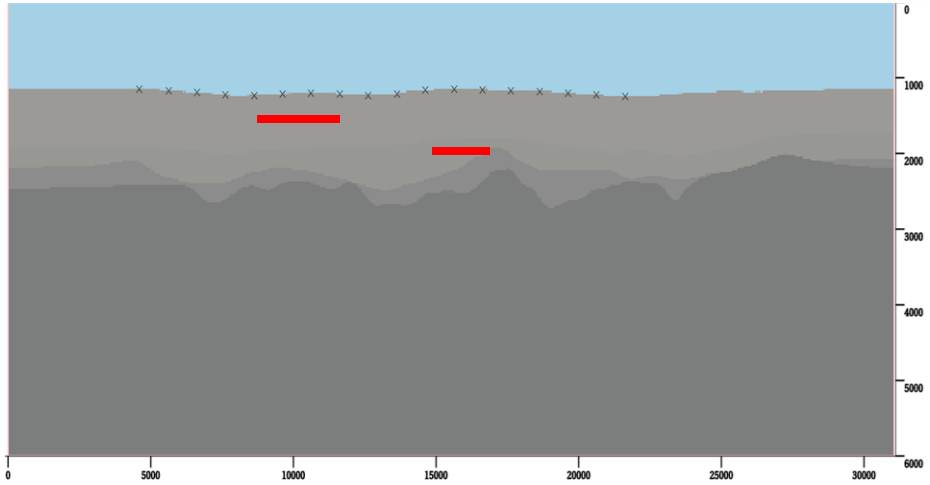


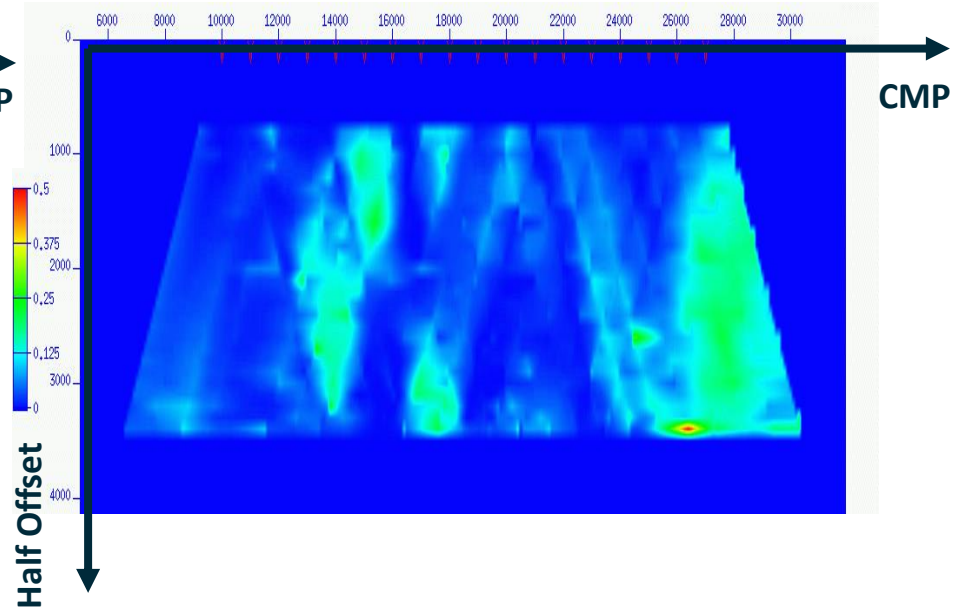


Blue : Small error

3D modeling real data

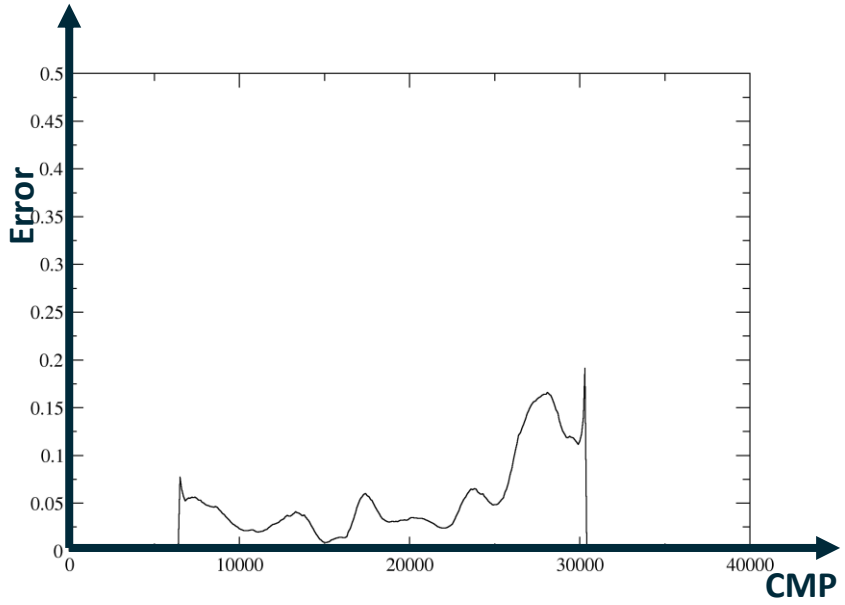


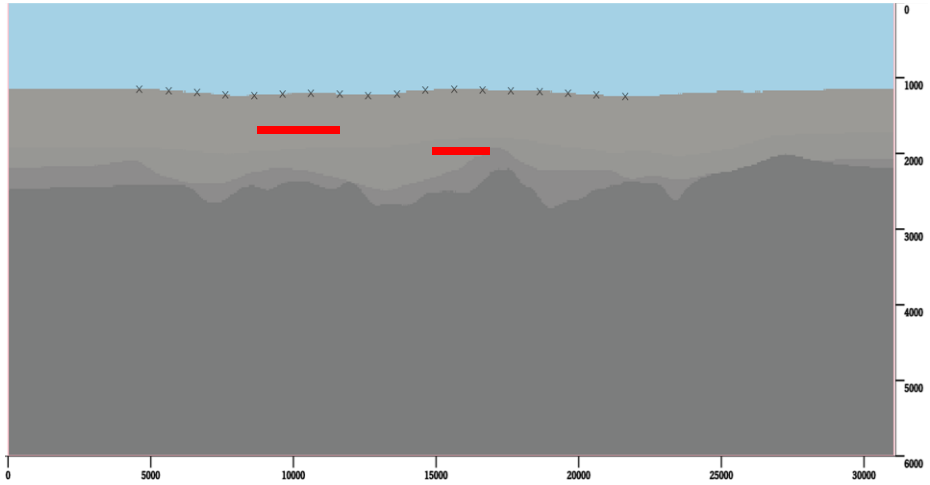


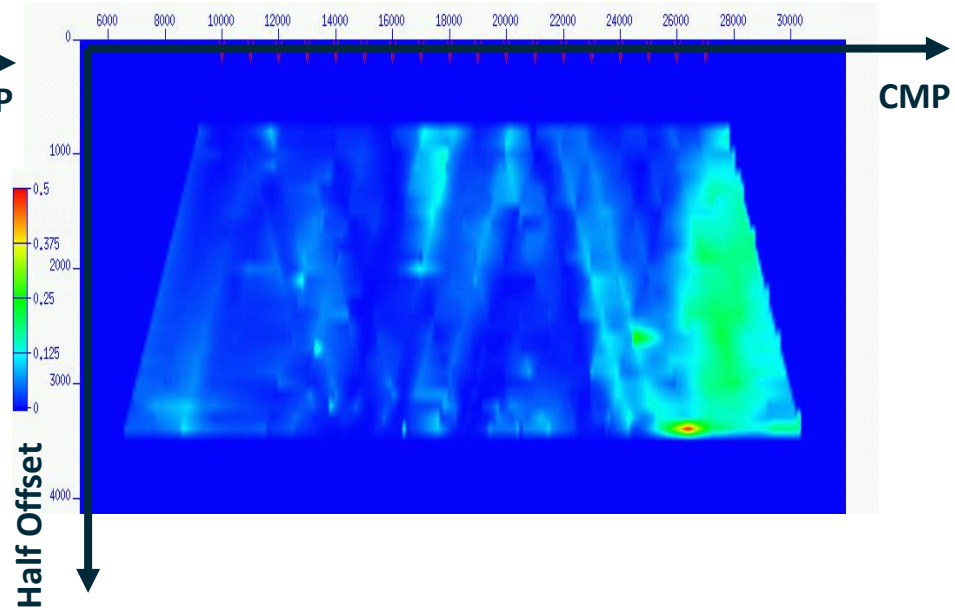


Blue : Small error

3D modeling real data

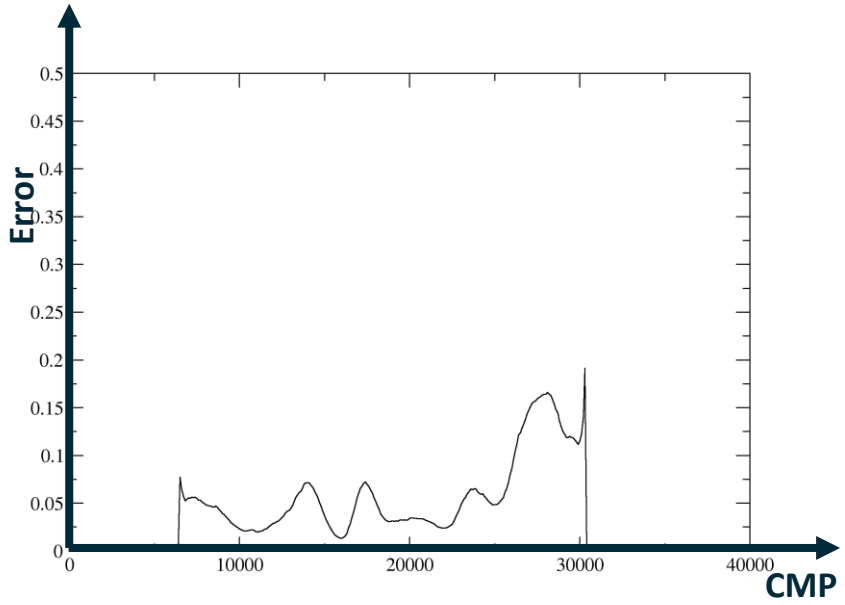


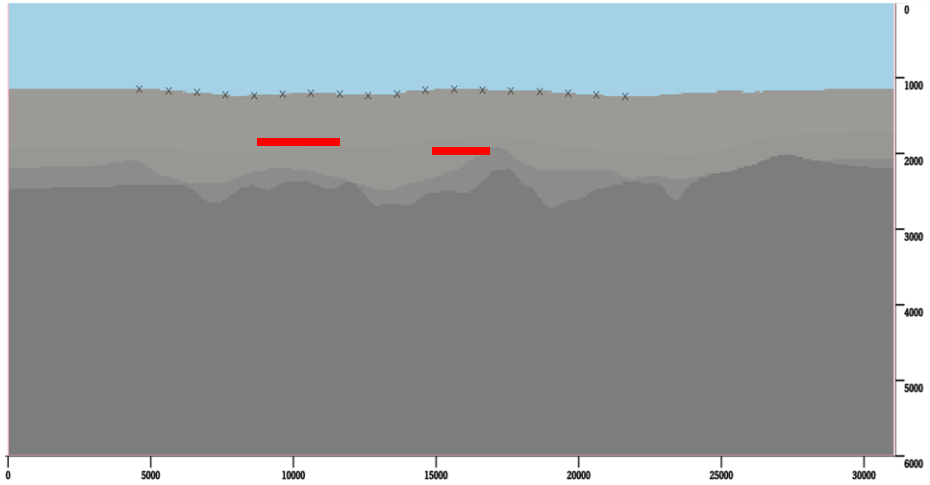


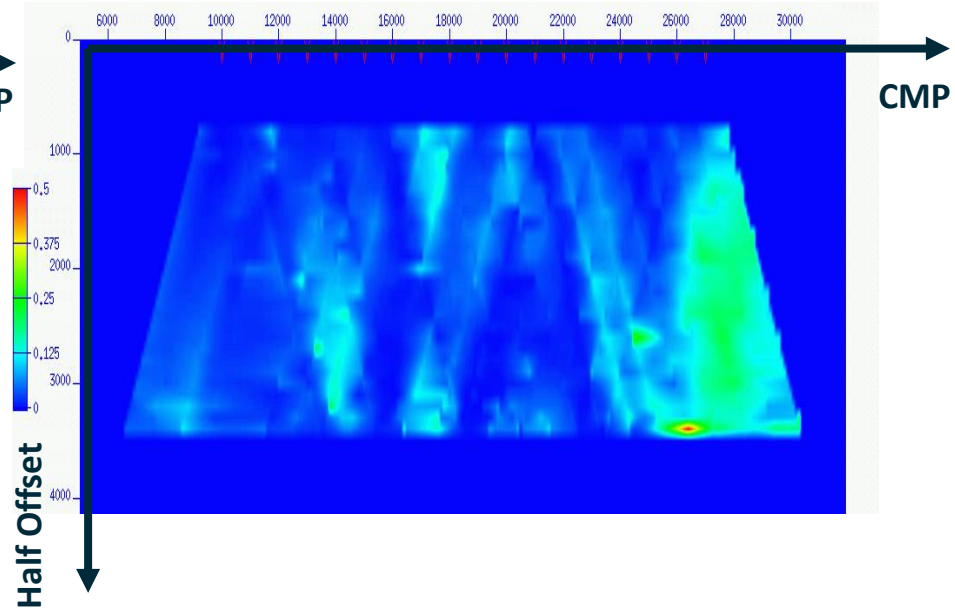


Blue : Small error

3D modeling real data

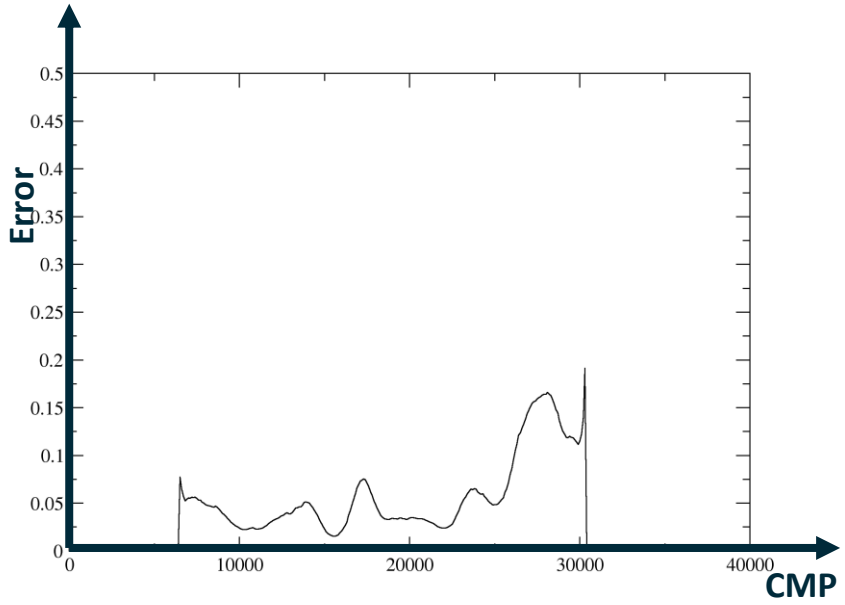


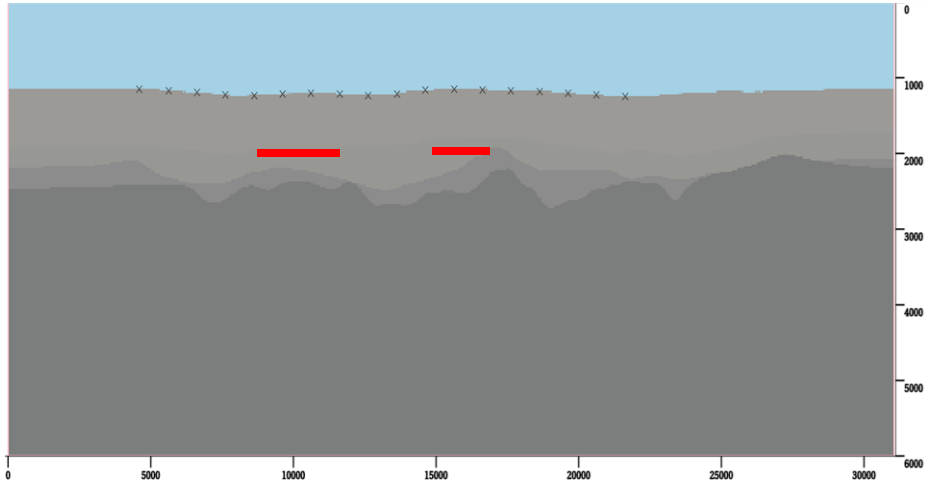


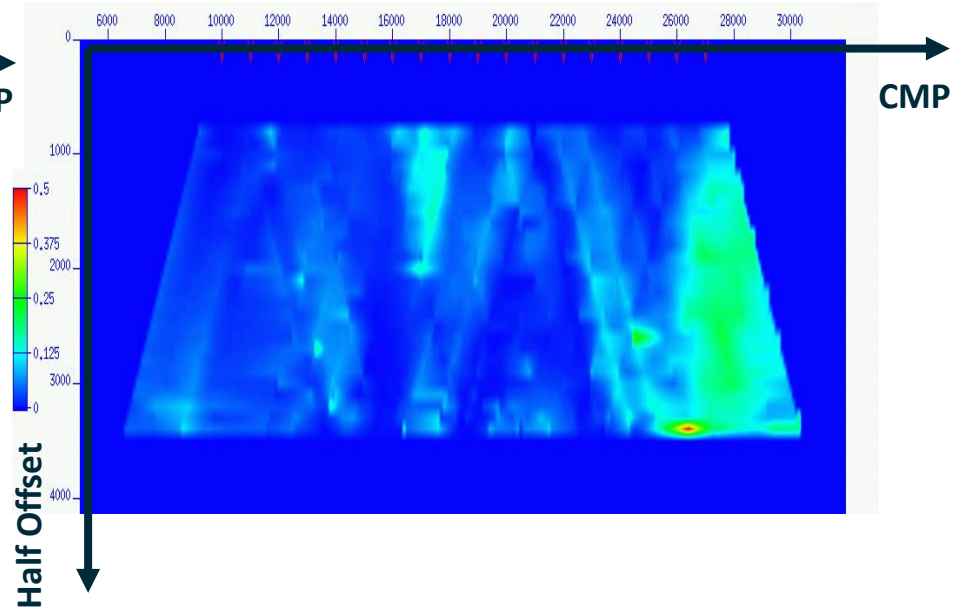


Blue : Small error

3D modeling real data

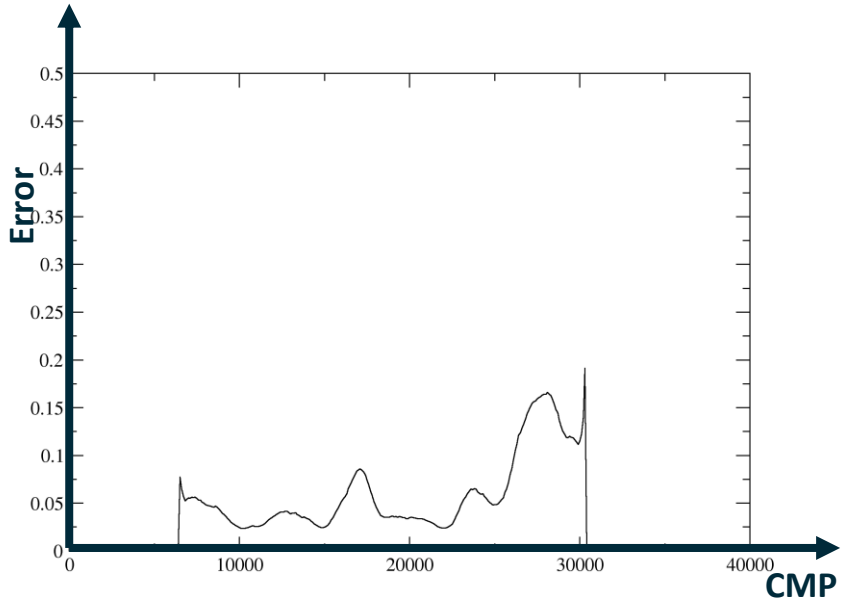


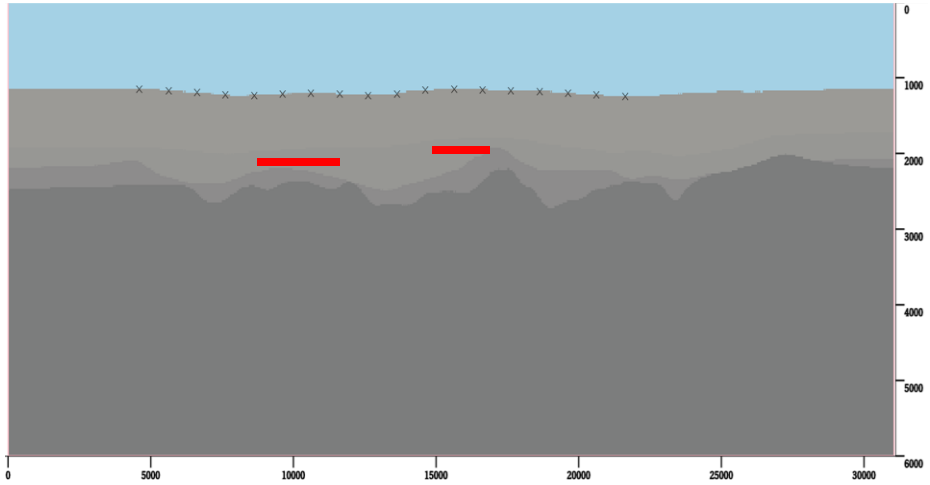


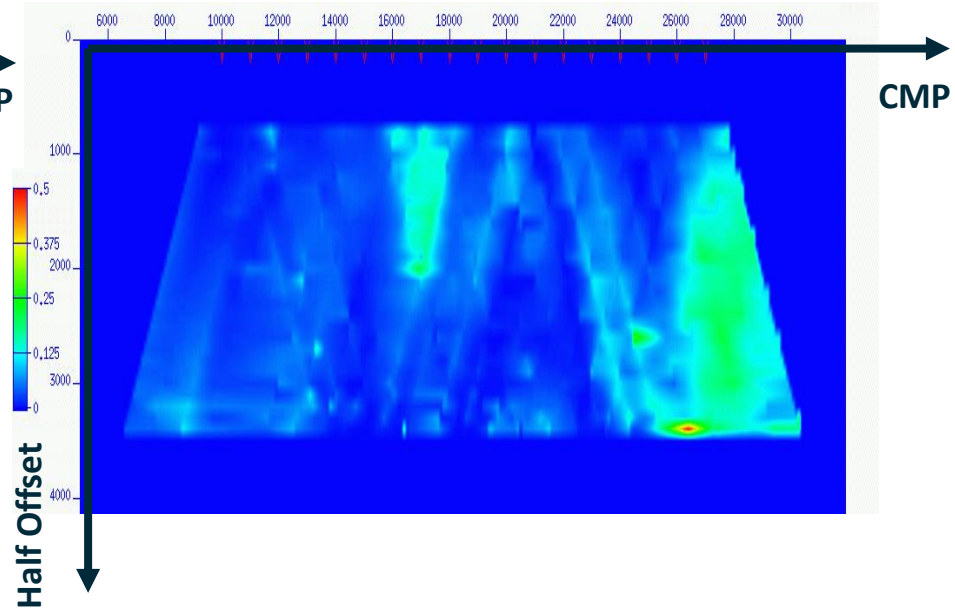


Blue : Small error

3D modeling real data



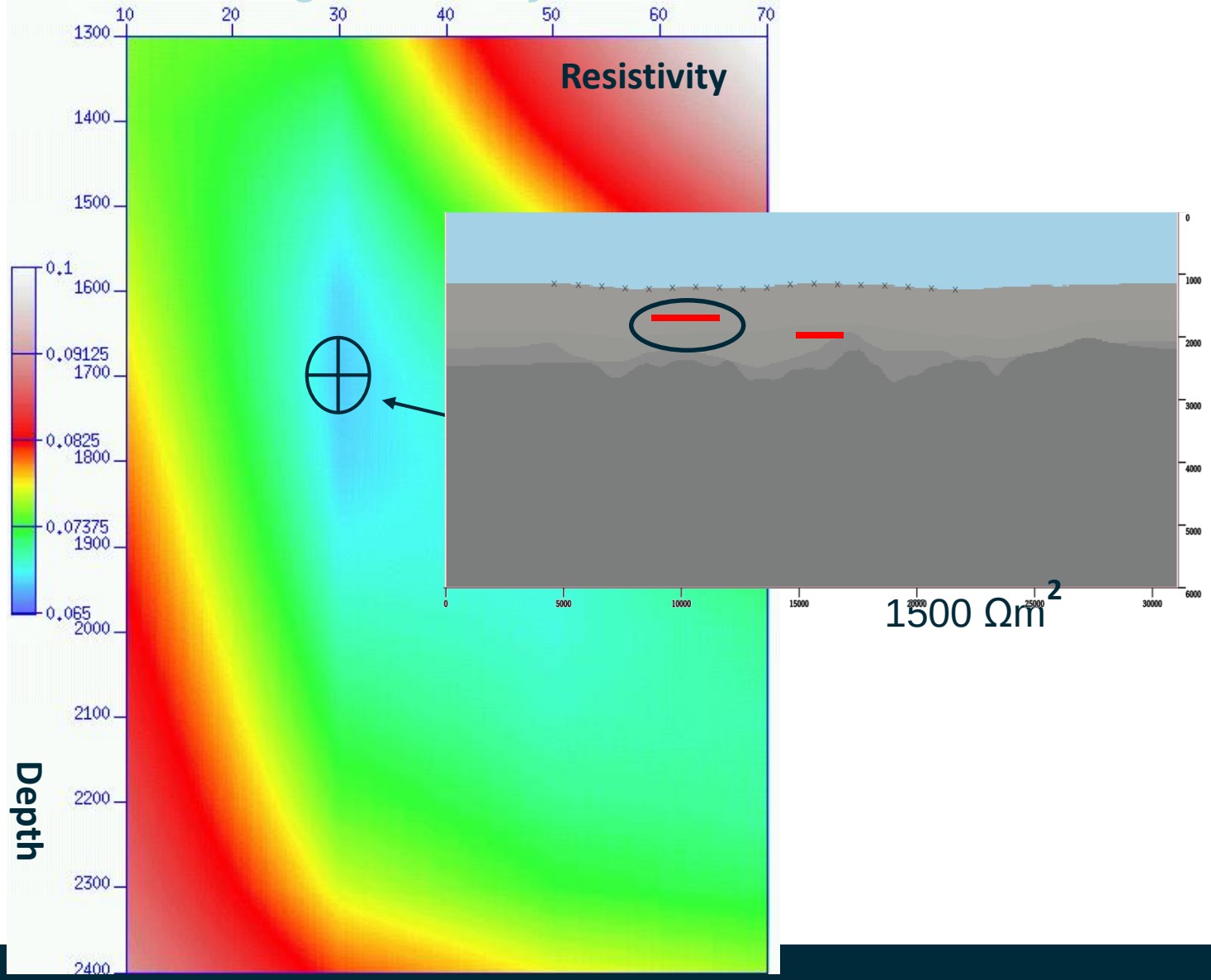




Blue : Small error

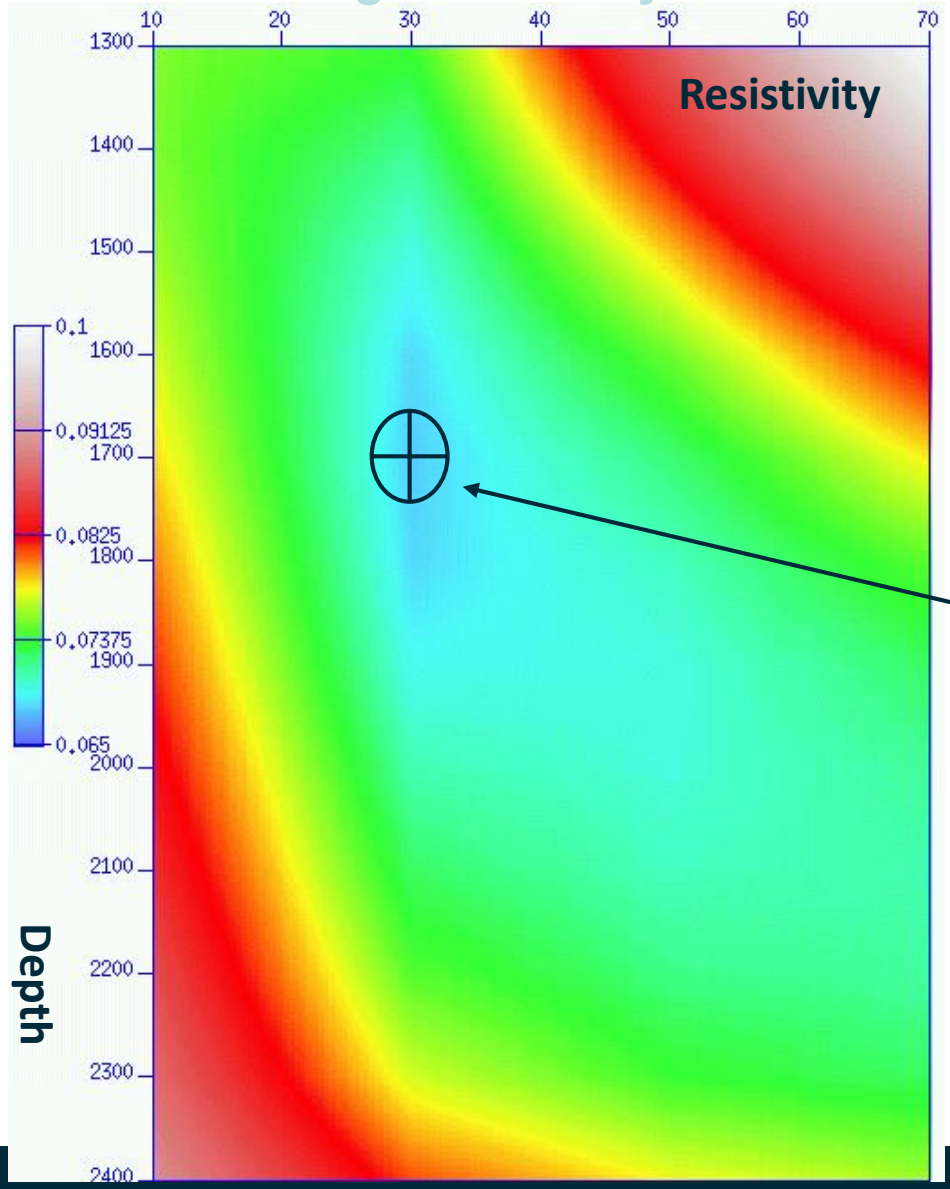
Weighted L2 error as a function of burial depth and resistivity

Rigth anomaly fixed



Weighted L2 error as a function of burial depth and resistivity

Rigth anomaly fixed



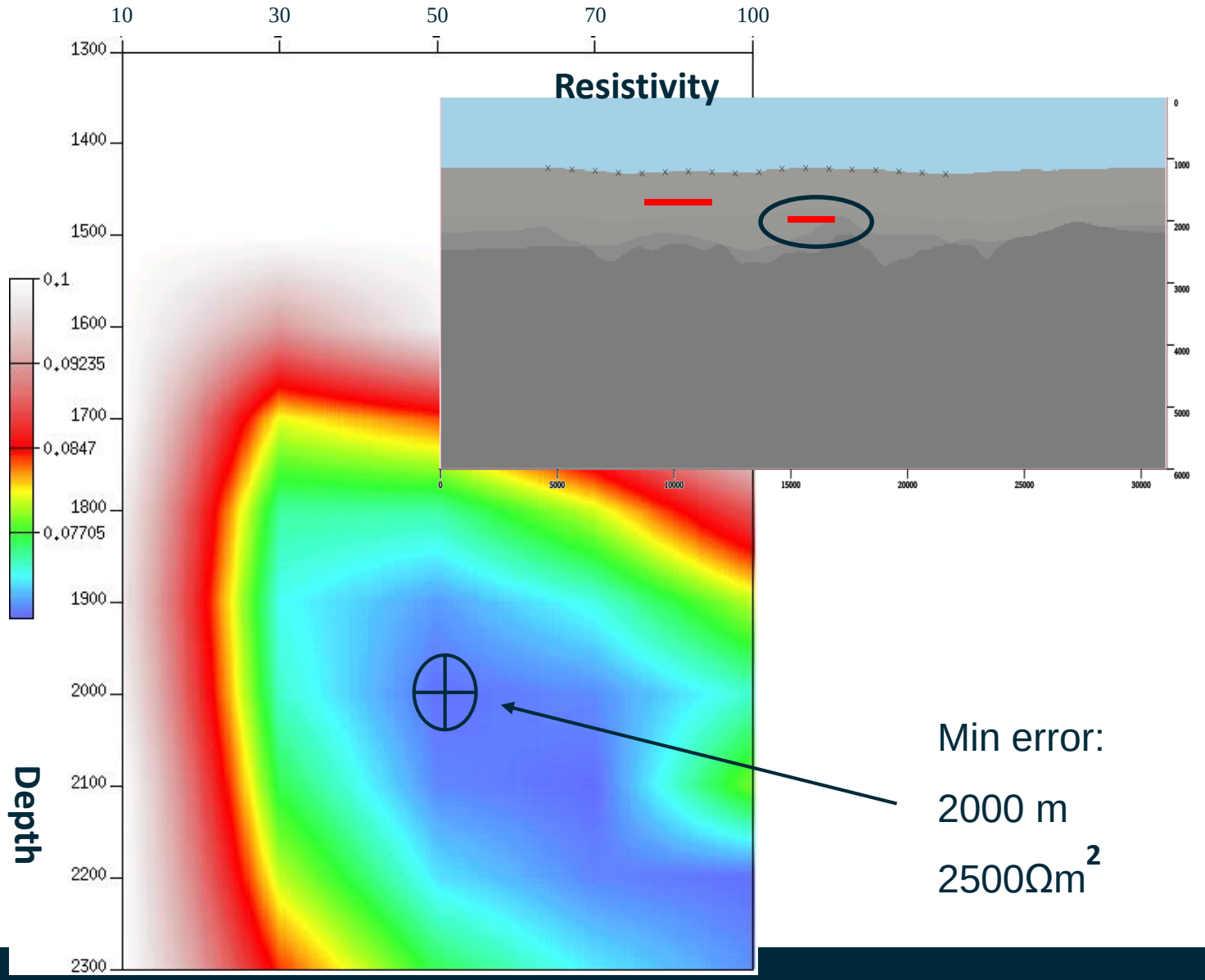
Min error:

1700 m

1500 Ωm^2

Weighted L2 error as a function of burial depth and resistivity

Left anomaly fixed



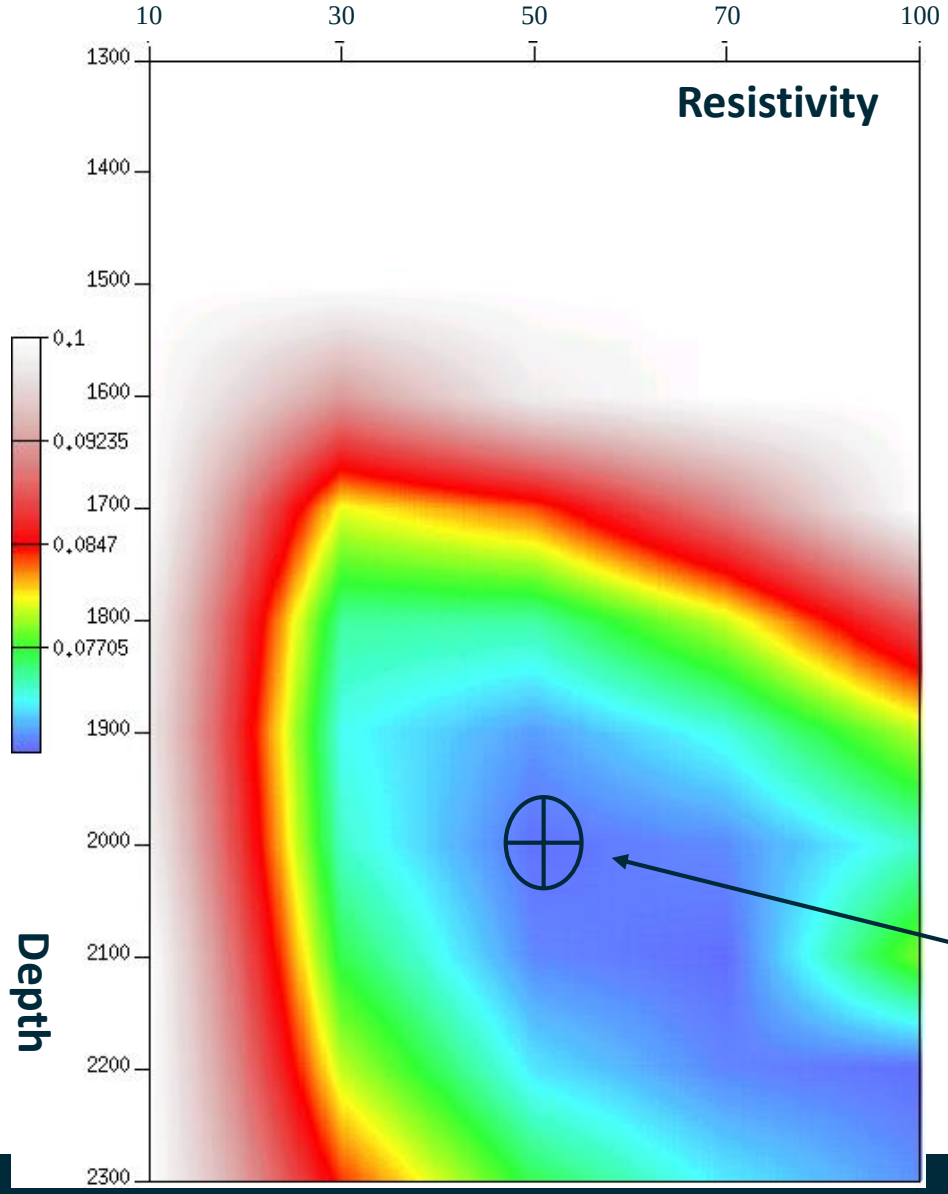
Min error:

2000 m

2500Ωm^2

Weighted L2 error as a function of burial depth and resistivity

Left anomaly fixed

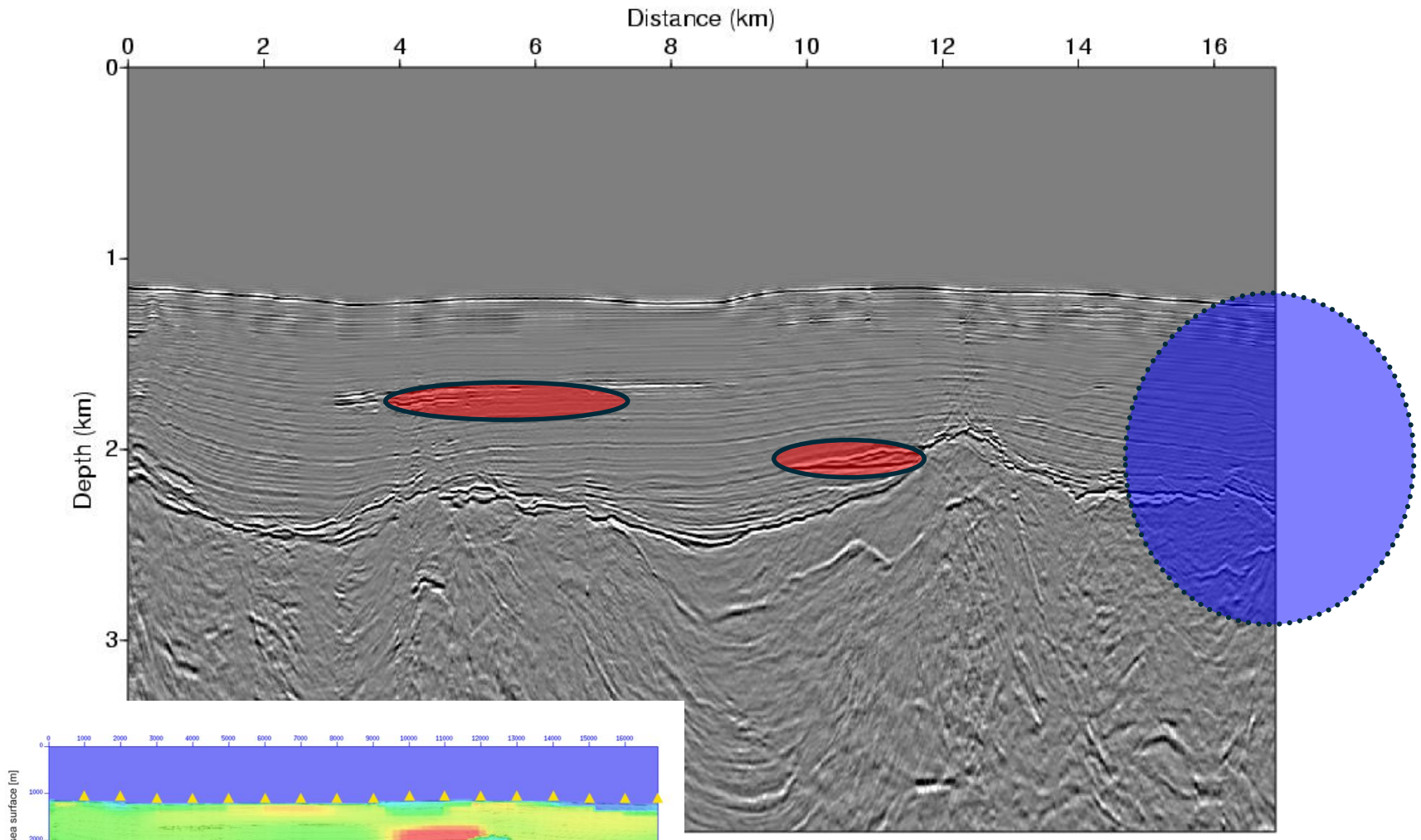


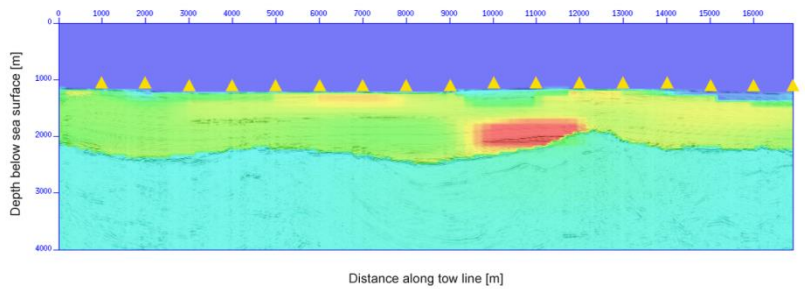
Min error:

2000 m

2500Ωm^2

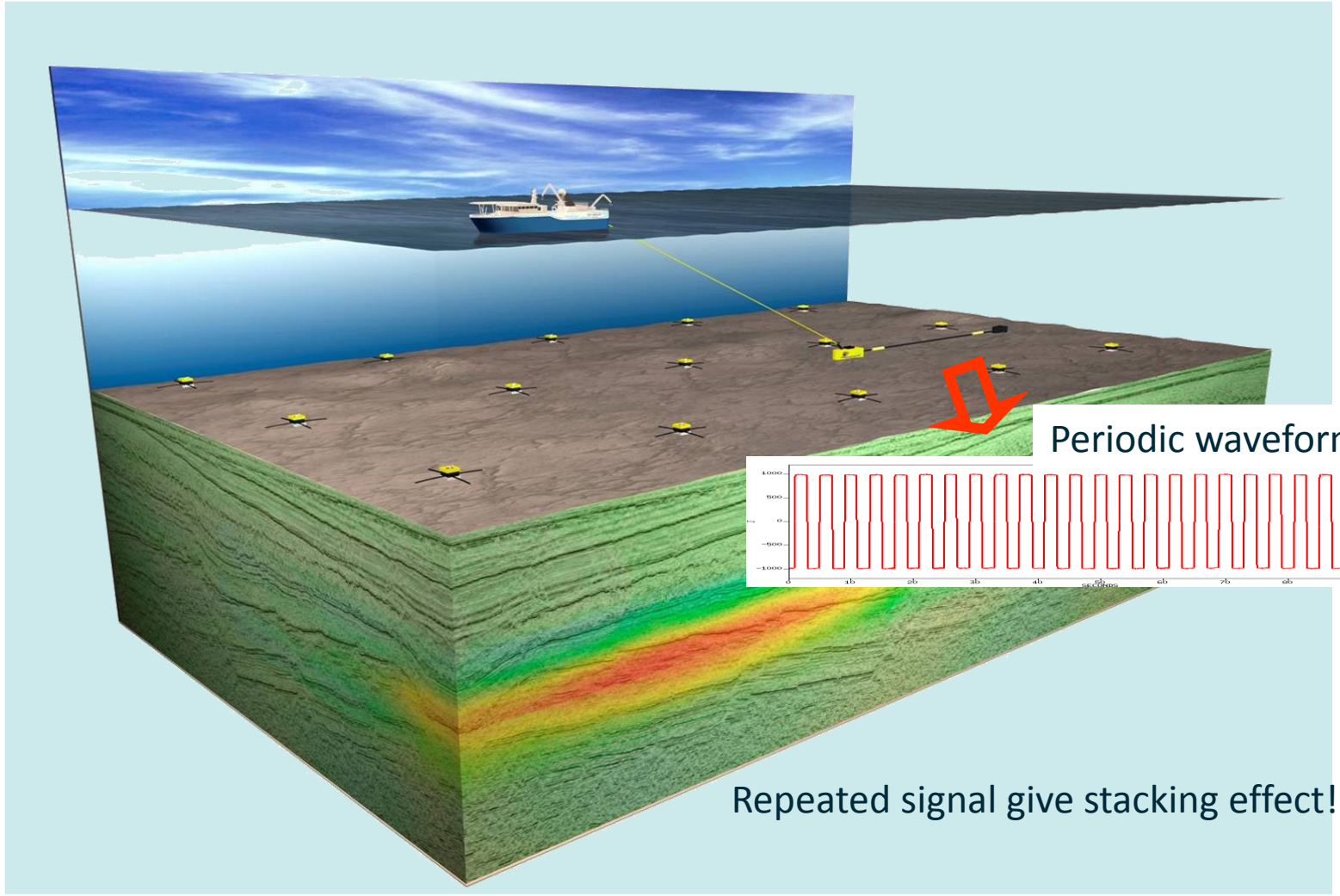
Test on real data





Transmitter waveform

Introduction



The CSEM dilemma:

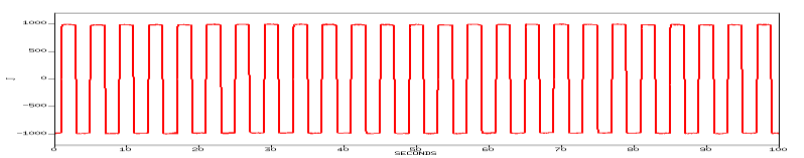
We want to transmit high frequencies for good resolution

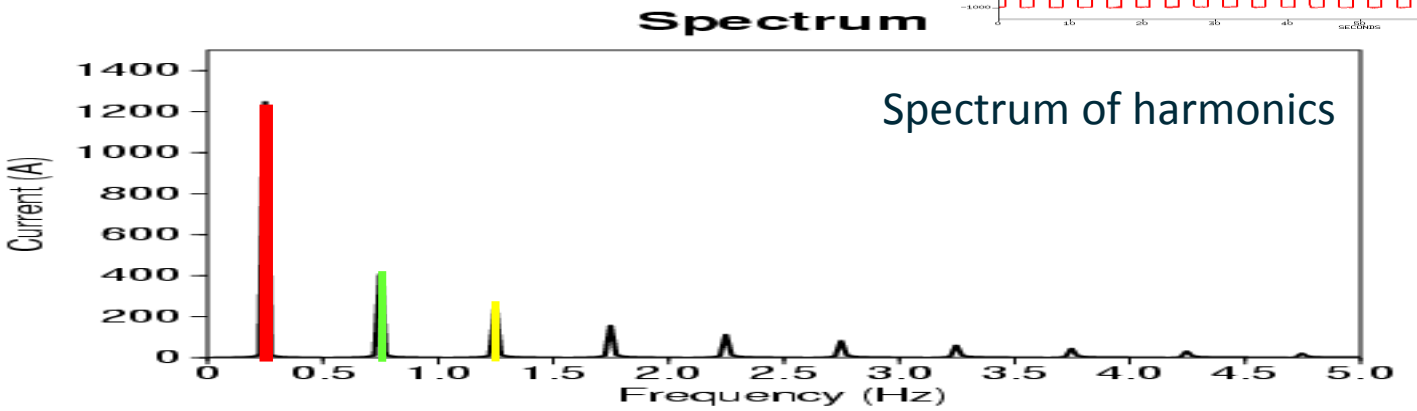
We want the electric field to be above the receiver noise floor for large source-receiver distances in order to investigate the subsurface to large depths

The skin-depth effect implies that high frequency signals have smaller penetration depth than low frequency signals

The electromagnetic field amplitude is proportional to the transmitted current amplitude: $E_x(\omega) = G_{xx}\ (\omega) J_x(\omega)\ L$

Source radiation

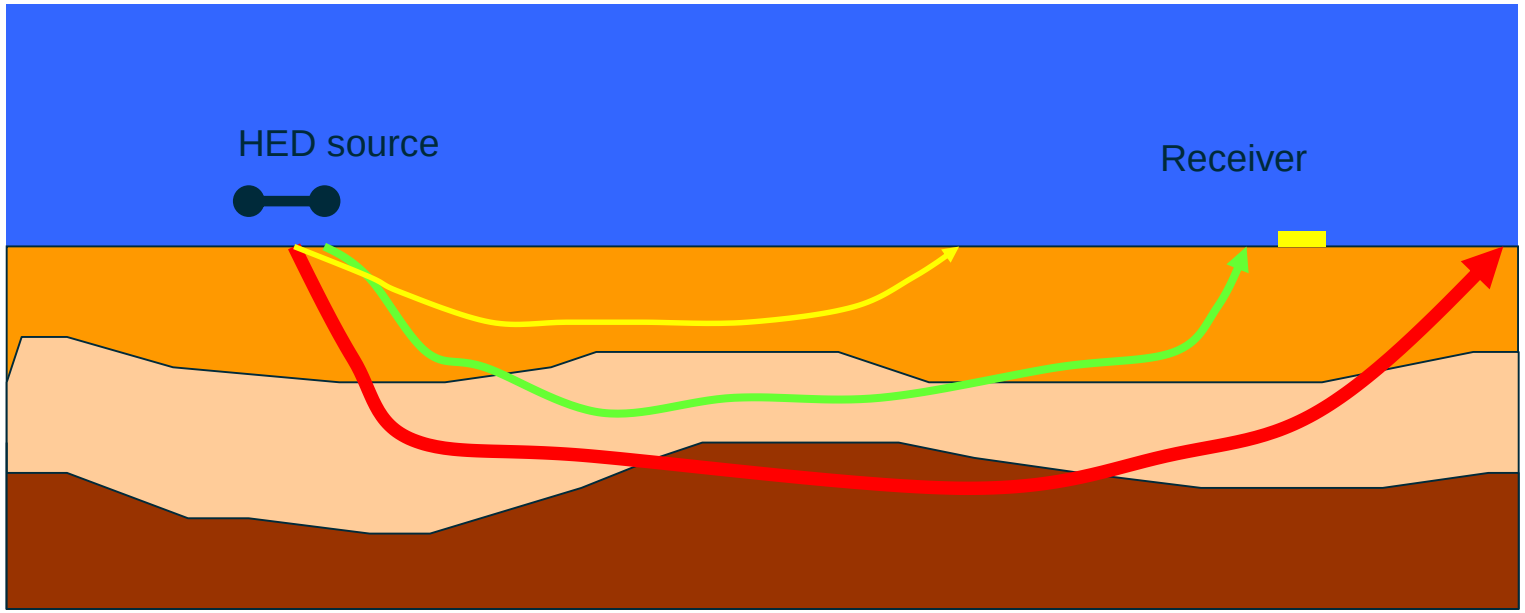




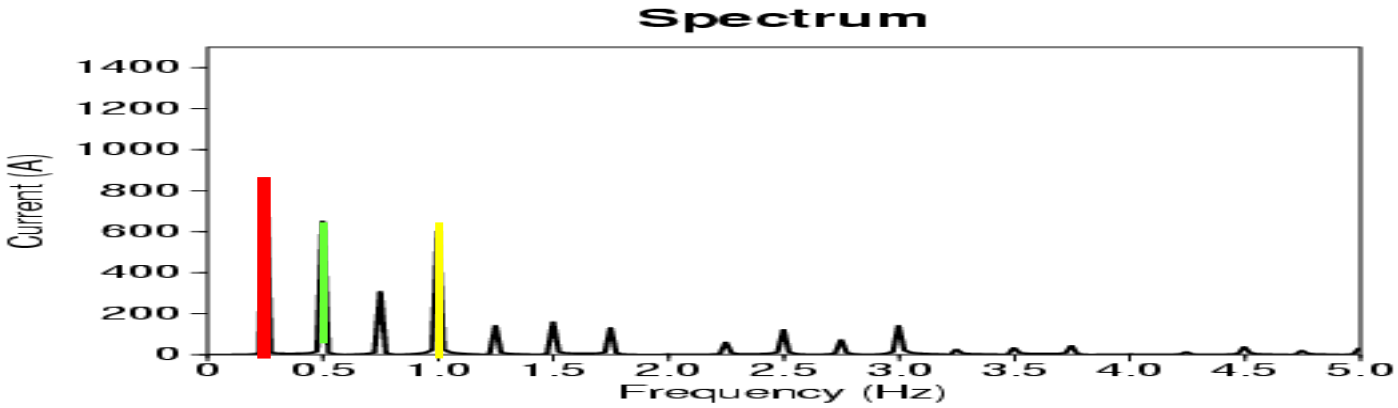
$$T_0 = 4\ \text{s}$$

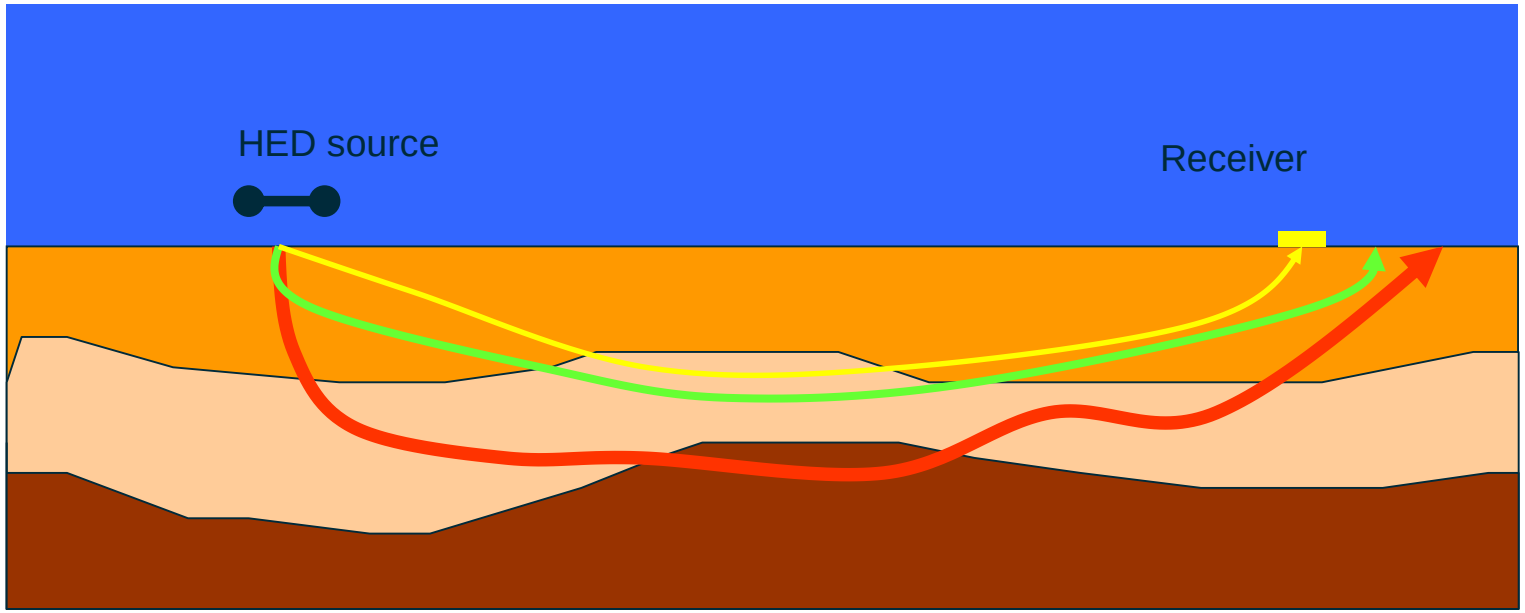
$$f_0 = \frac{1}{T_0}$$

$$f_0 = 0.25\ \text{Hz}$$

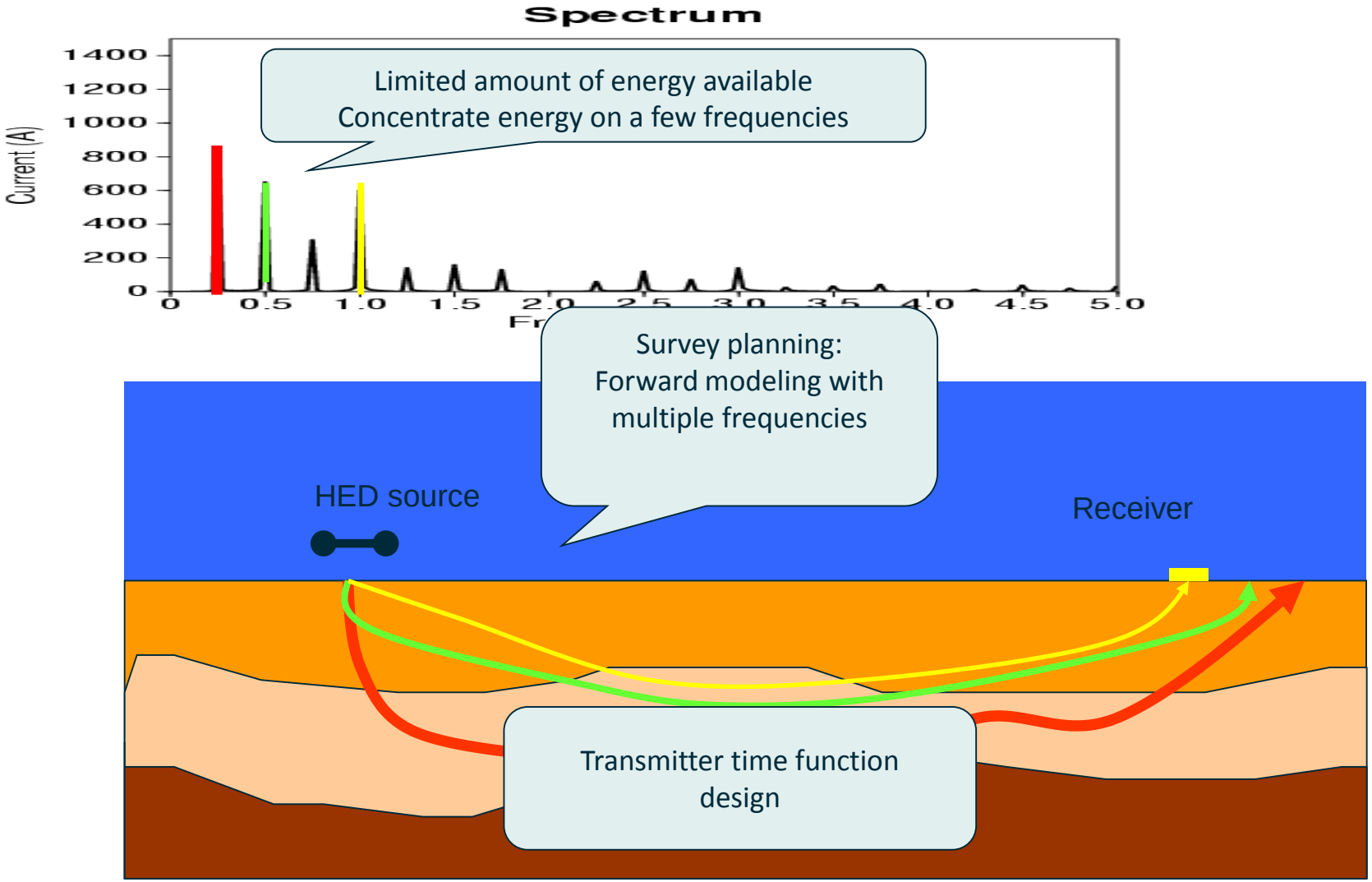


Source radiation



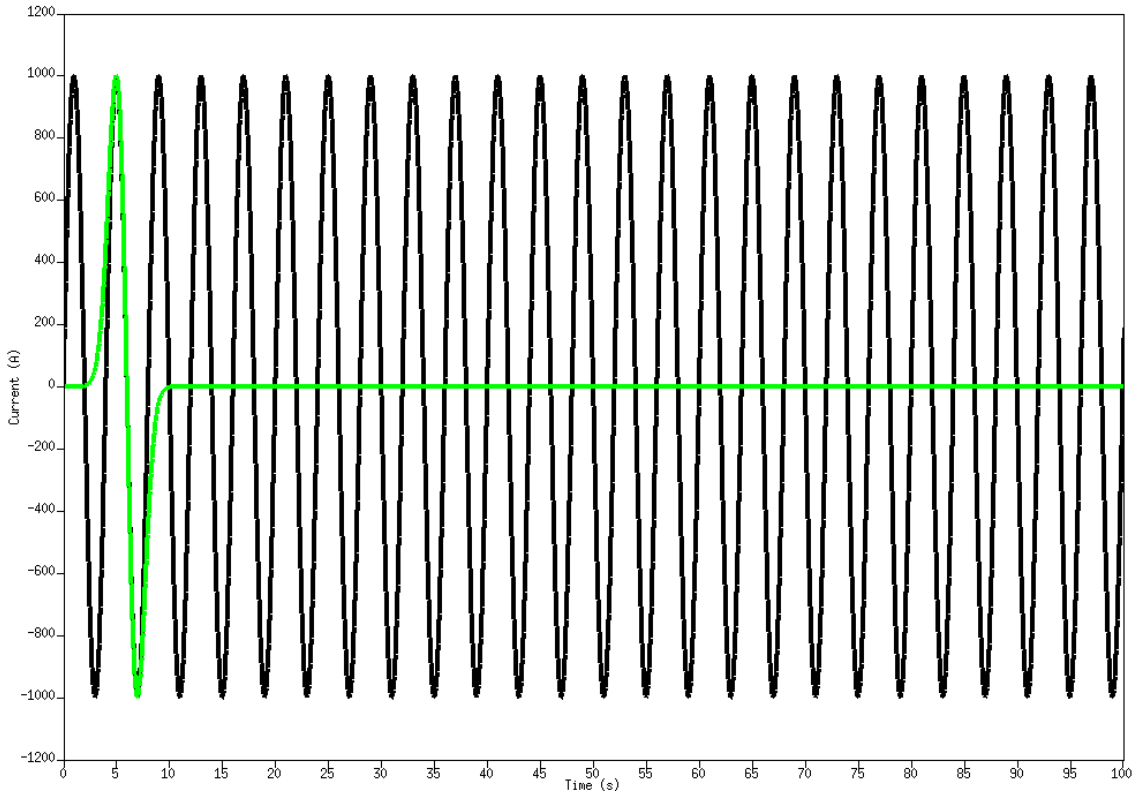


Source radiation



Time domain

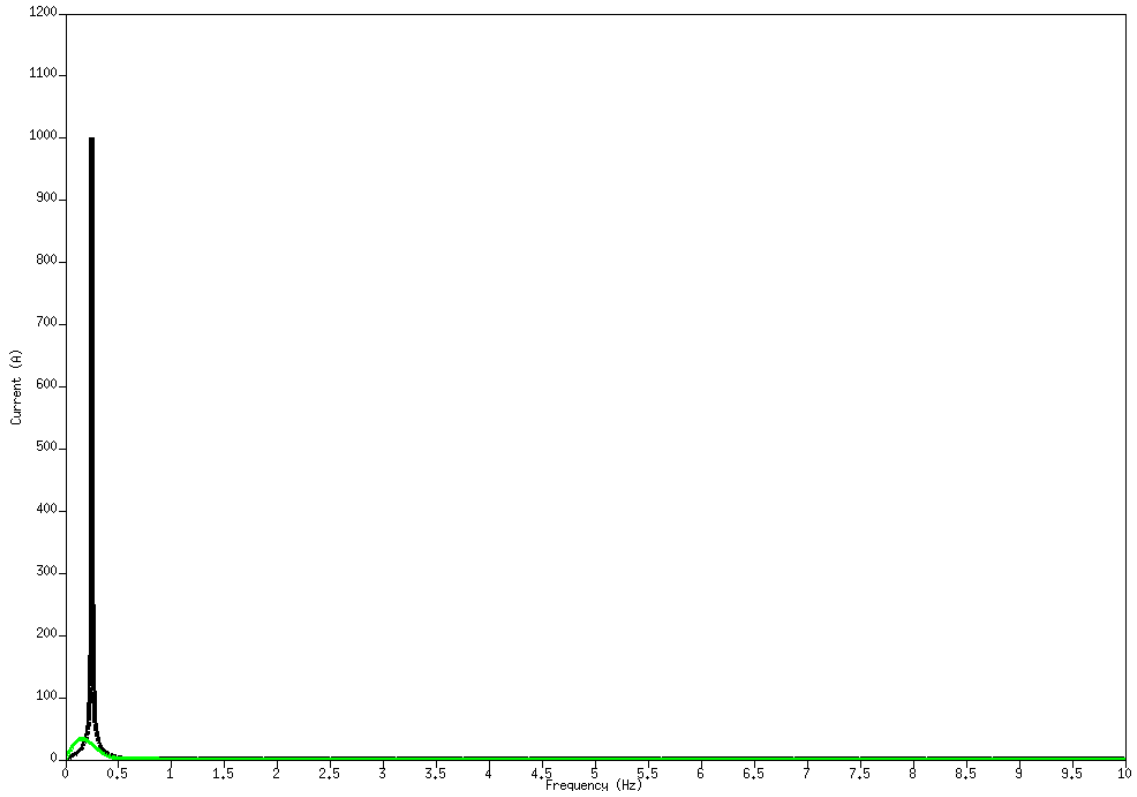
Black: Harmonic signal at 0.25 Hz with peak amplitude 1000 A
Green: Transient with peak amplitude 1000 A



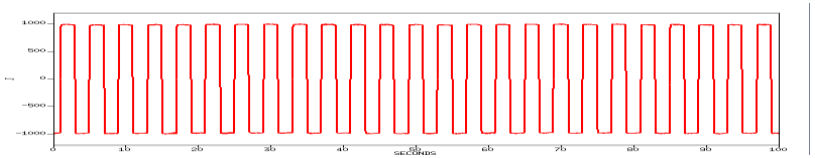
First harmonic for 0.25 Hz signal -> $T_0 = 4$ s

Frequency domain:

Current amplitudes for the transient are small
Better to have the source active at all times
However, mono-frequency signal not optimal



Transmit harmonic square wave signal



Hardware limitation: Limited maximum current amplitude available

Transmit most energy into subsurface if source is running with maximum current at all times

Switch polarity of signal with peak amplitude.
Called square wave if time interval between each switch is constant.
Called generalized square wave if time intervals can vary within one period.

The amplitude of the transmitted and hence measured electric field is proportional to the current amplitude.

The transmitted current signal can be transformed to the Fourier domain. The result is transmitted current as a function of frequency.

Strategy:

Design transmitter signal $J_x(t)$ that give optimal $J_x(\omega)$

Fourier transform

$$J_x(\omega_n) = \frac{2}{T_0}\int_0^{T_0} J_x(t)e^{-i\omega_n t}$$

$$\omega_n = n\frac{2\pi}{T_0} = n2\pi f_0$$

$$\omega_n = n\frac{2\pi}{T_0} = n2\pi f_0$$

$$f_n = nf_0 = \frac{n}{T_0}$$

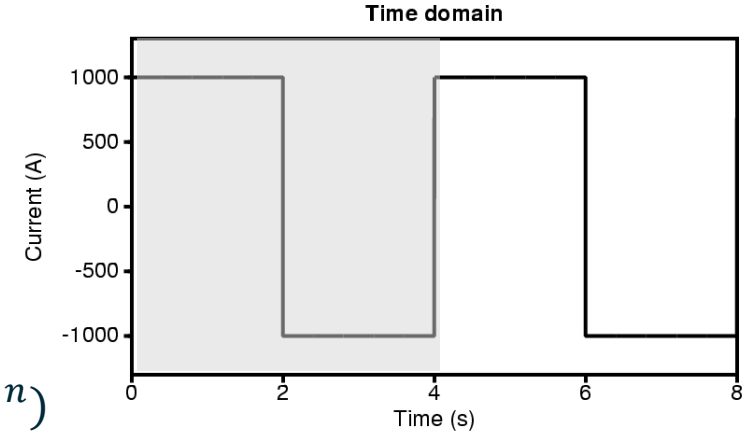
Assume $T_0 = 4$ s $\quad \rightarrow \quad f_0 = 0.25$ Hz

Harmonic number		Frequency
1	$f_1 = 1f_0$	0.25 Hz
2	$f_2 = 2f_0$	0.5 Hz
3	$f_3 = 3f_0$	0.75 Hz
4	$f_4 = 4f_0$	1.0 Hz
5	$f_5 = 5f_0$	1.25 Hz

Introduction

Standard square wave

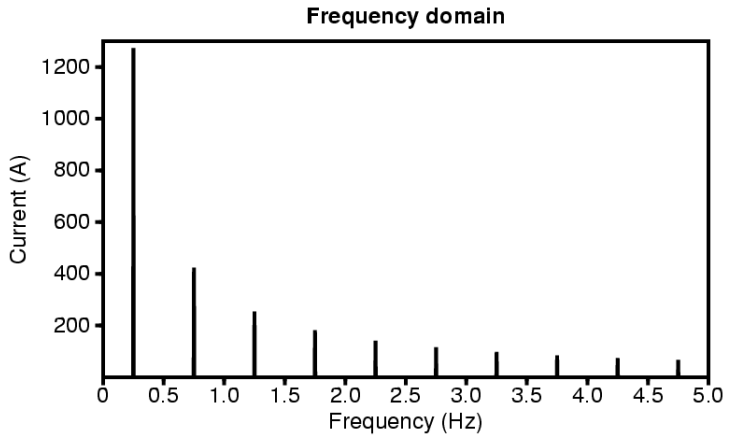
Period 4 s (f_0 =0.25 Hz)



$$J_n = J_{Max}\frac{4}{n\pi}\frac{(1-(-1)^n)}{2}$$

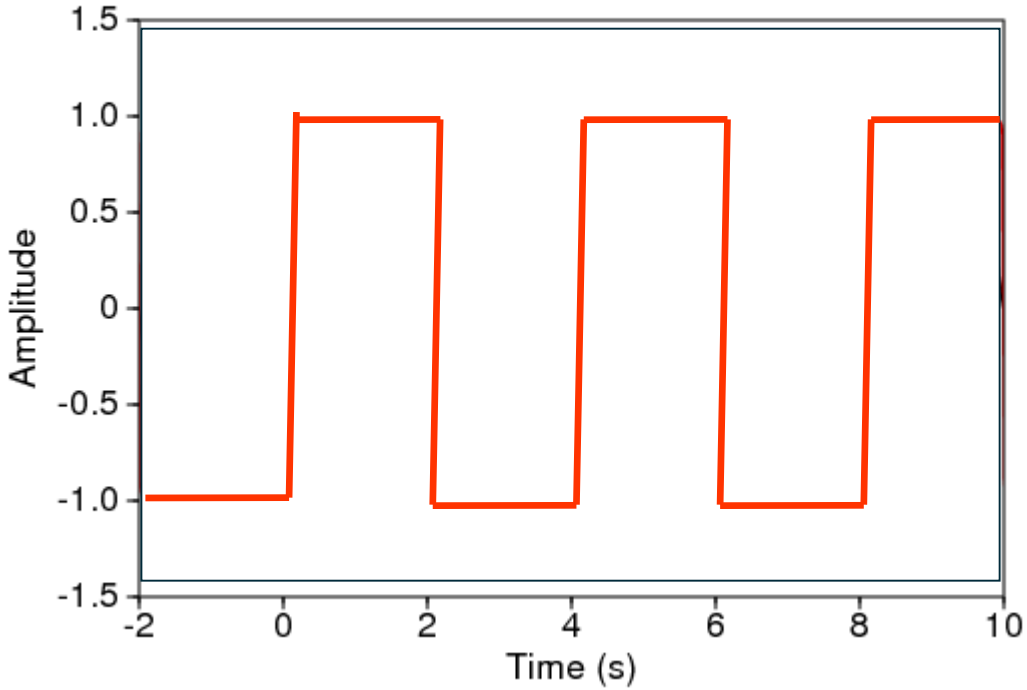
$$J_{Max} = 1000\ \text{A}$$

1: 1273 A
3: 424 A
5: 255 A
7: 182 A
9: 141 A

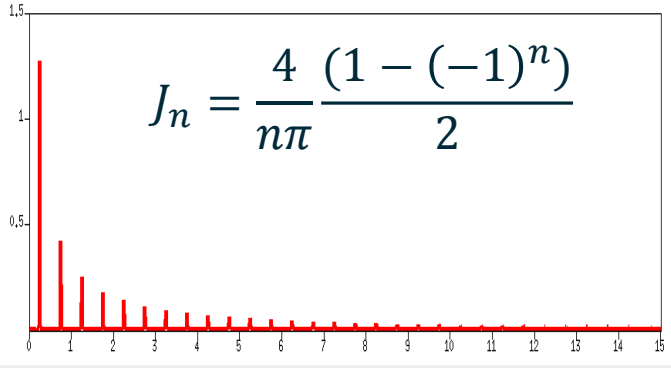


High current amplitudes on more than one frequency may be useful

Square wave



$$J_n = \frac{4}{n\pi}\frac{(1-(-1)^n)}{2}$$



Square wave can be represented by a sine transform

$$J(t) = \sum_{n=0}^{\infty} J_n \sin(\omega_n t)$$

$$J_n = \frac{2}{T_0} \int_{-T_0/2}^{T_0/2} dt \; J(t) \sin(\omega_n t)$$

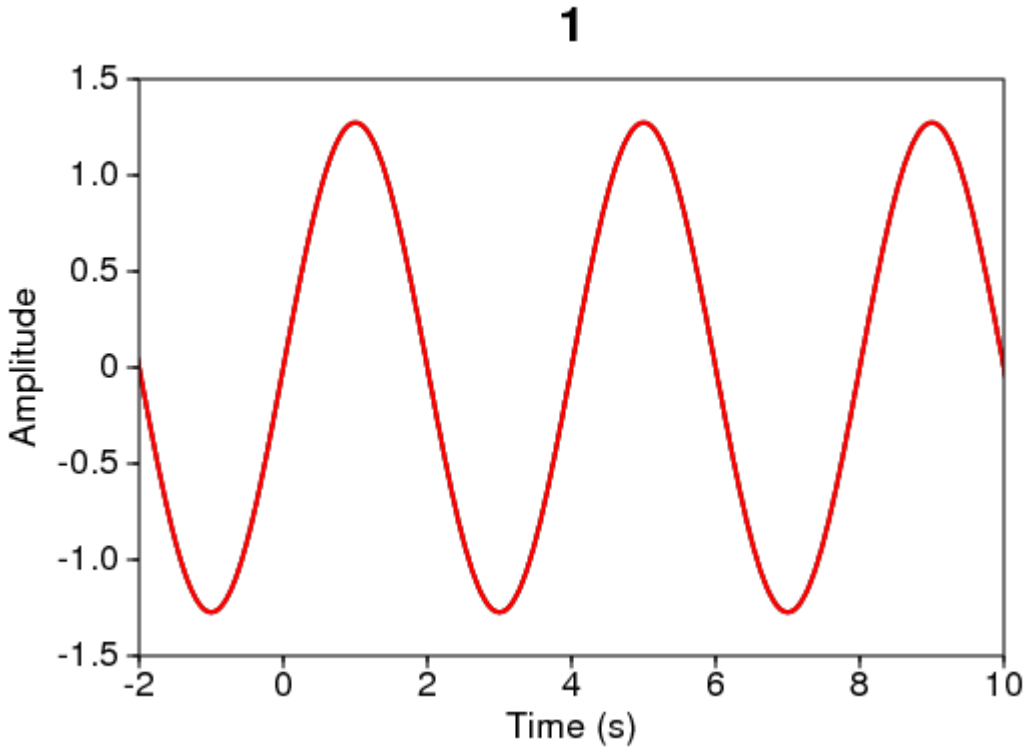
$$\omega_n = n\omega_0 = n2\pi f_0$$

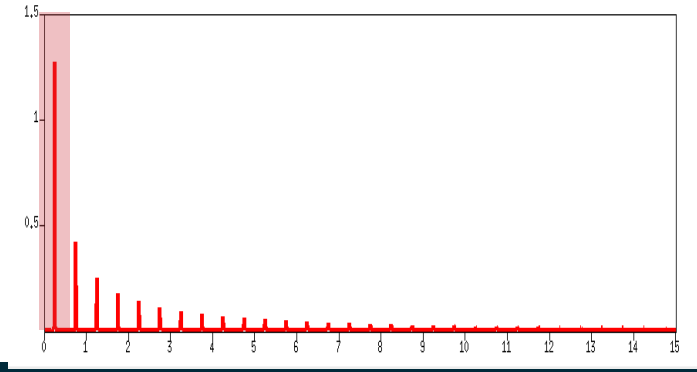
$$f_0 = \frac{1}{T_0}$$

$$J(t) = \sum_{n=0}^{\infty} J_n \sin(n2\pi \frac{t}{T_0})$$

$$J(t) = \sum_{n=0}^{\infty} J_n \sin(\omega_n t)$$

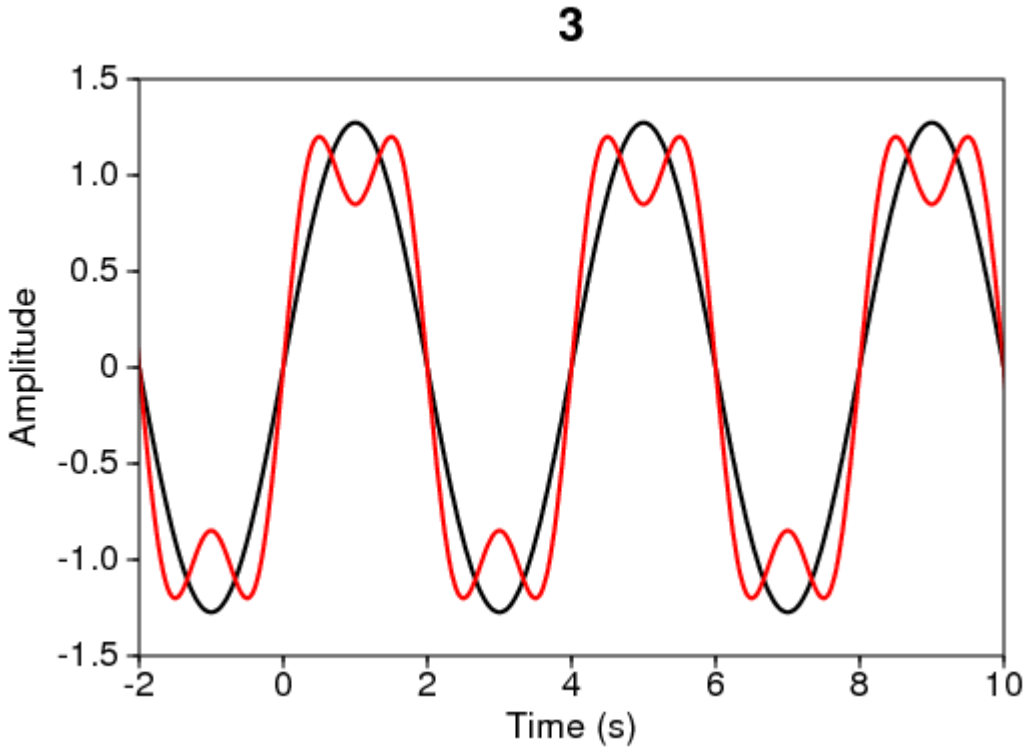
$$J(t) = J_1 \sin(\omega_1 t)$$

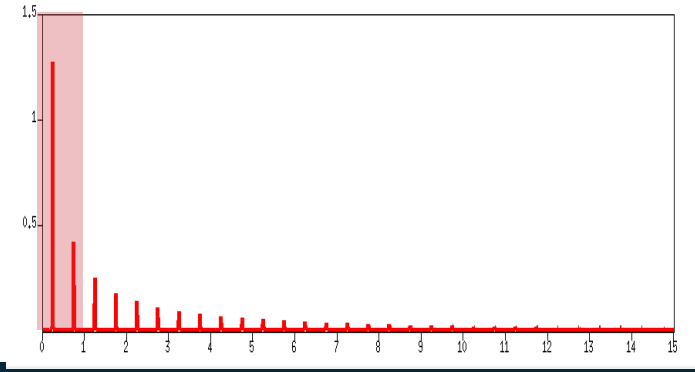




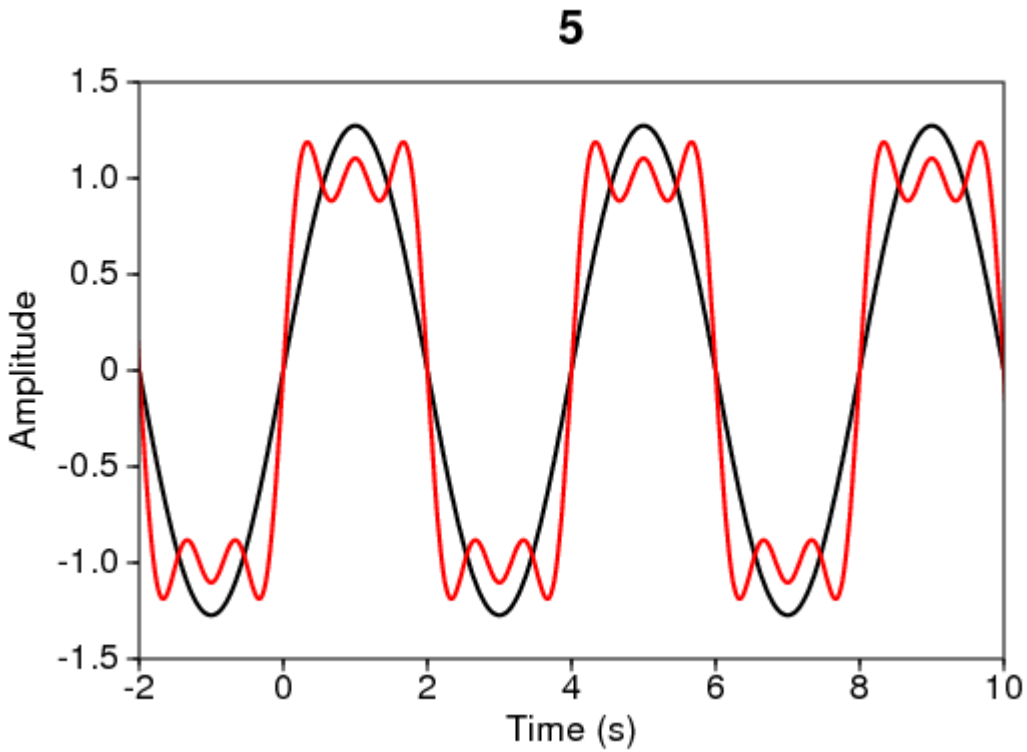
$$J(t) = \sum_{n=0}^{\infty} J_n \sin(\omega_n t)$$

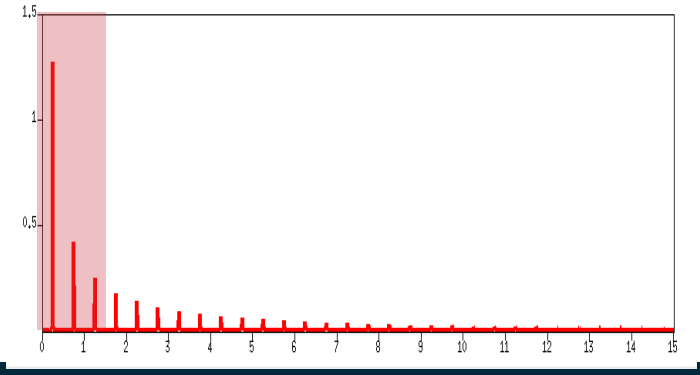
$$J(t) = J_1 \sin(\omega_1 t) + J_3 \sin(\omega_3 t)$$





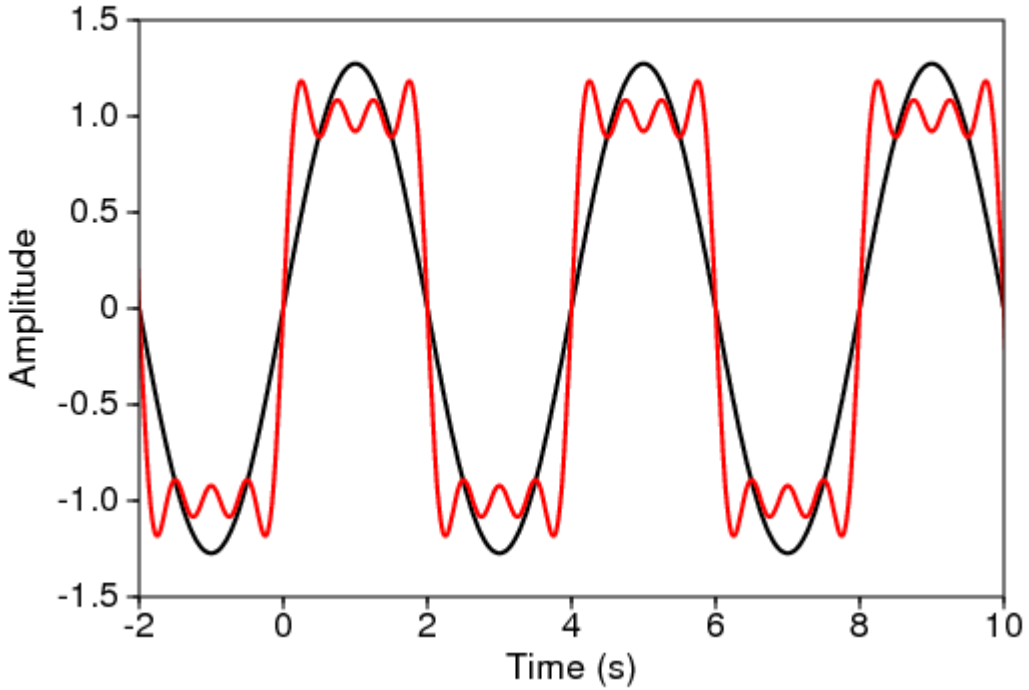
$$J(t) = J_1 \sin(\omega_1 t) + J_3 \sin(\omega_3 t) + J_5 \sin(\omega_5 t)$$

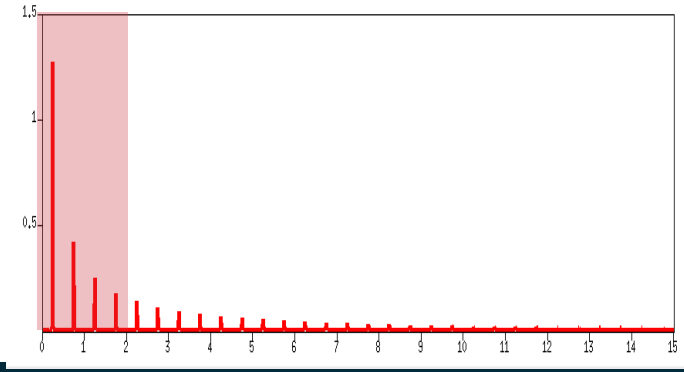




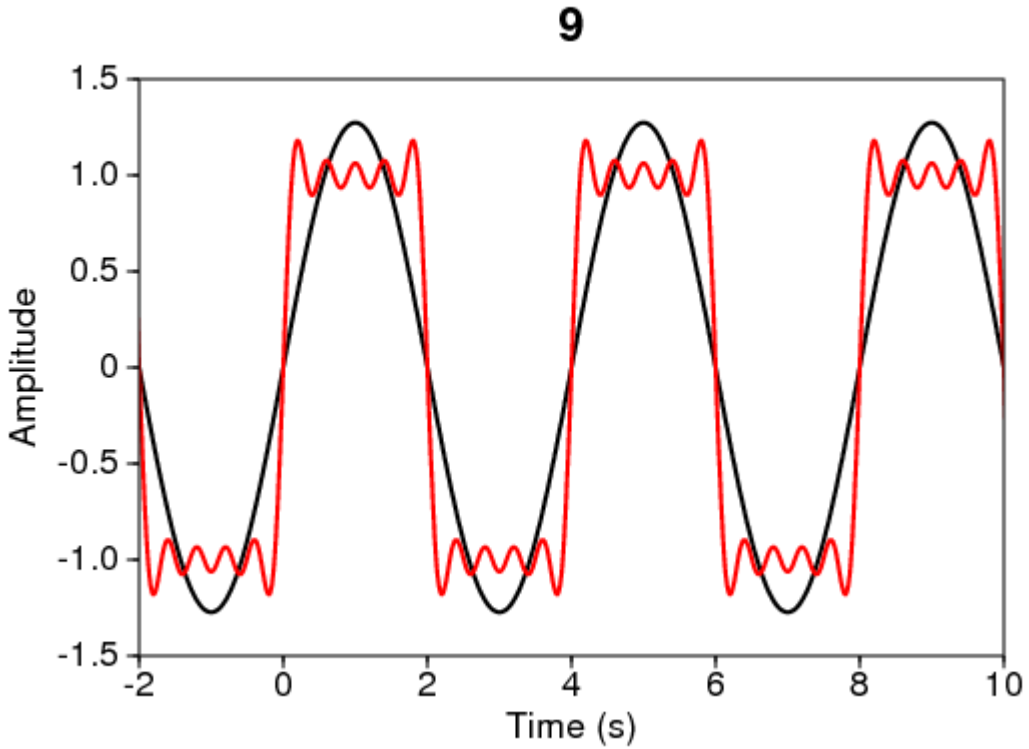
$$J(t) = J_1 \sin(\omega_1 t) + J_3 \sin(\omega_3 t) + J_5 \sin(\omega_5 t) + J_7 \sin(\omega_7 t)$$

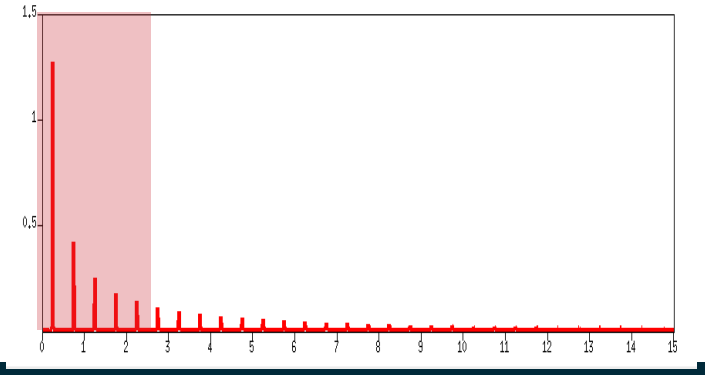
7

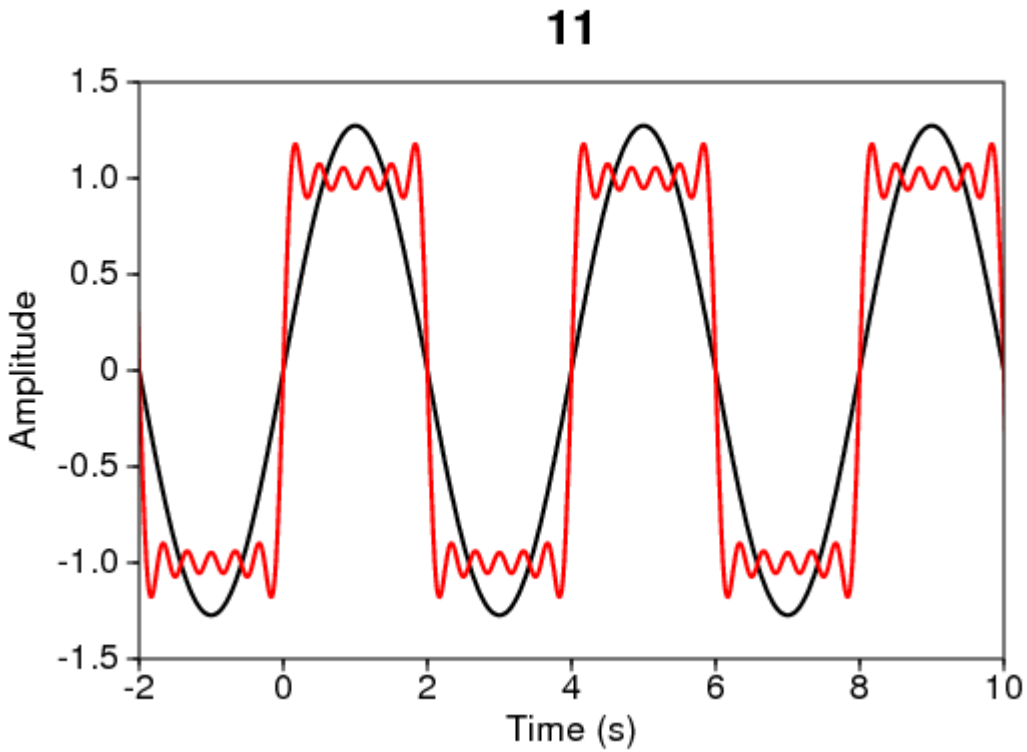




$$J(t) = J_1 \sin(\omega_1 t) + J_3 \sin(\omega_3 t) + J_5 \sin(\omega_5 t)$$
$$+ J_7 \sin(\omega_7 t) + J_9 \sin(\omega_9 t)$$






11
1.5
1.0
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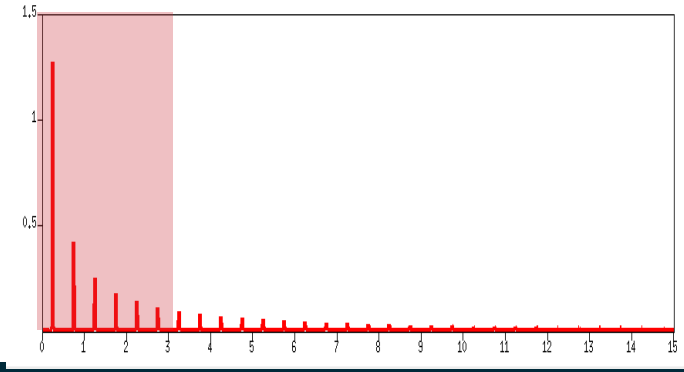
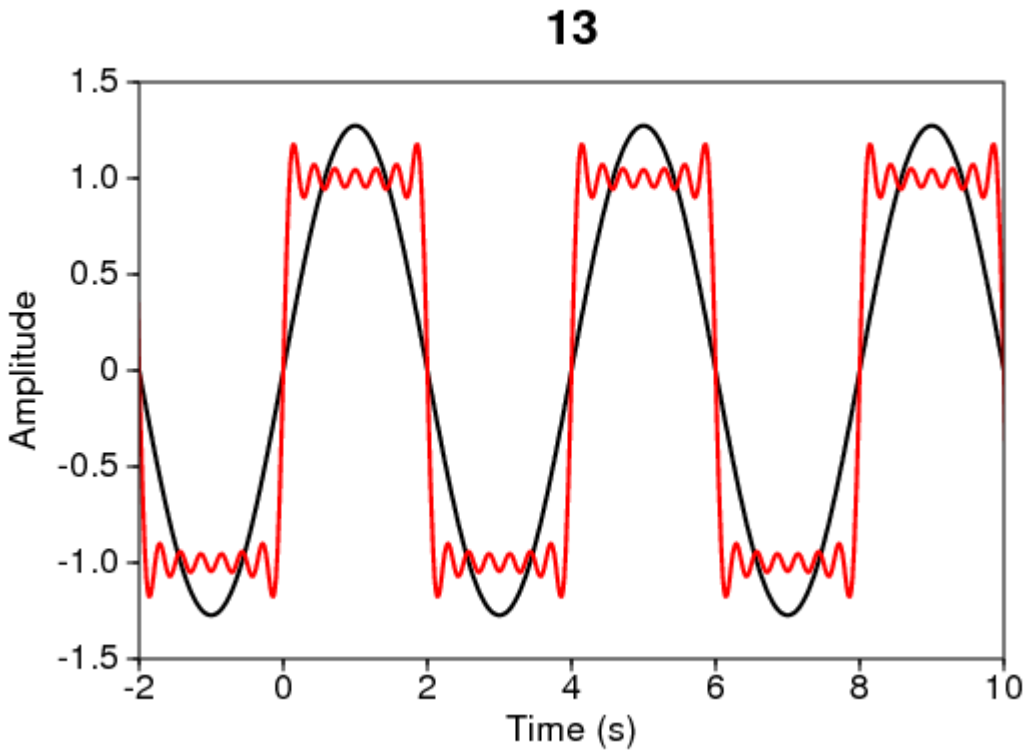
0
-0.5
-1.0
-1.5
Amplitude
-2
0
2
4
6
8
10
Time (s)

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1.0
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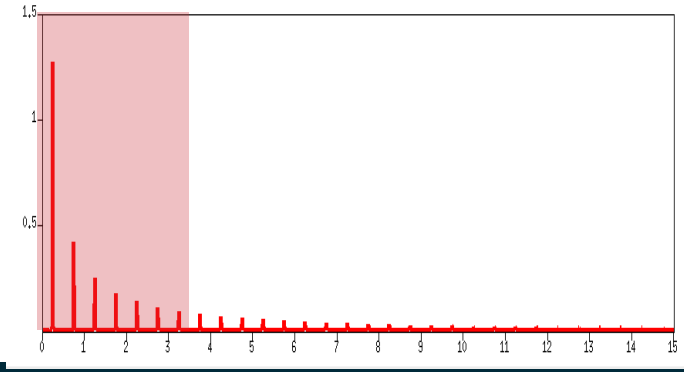
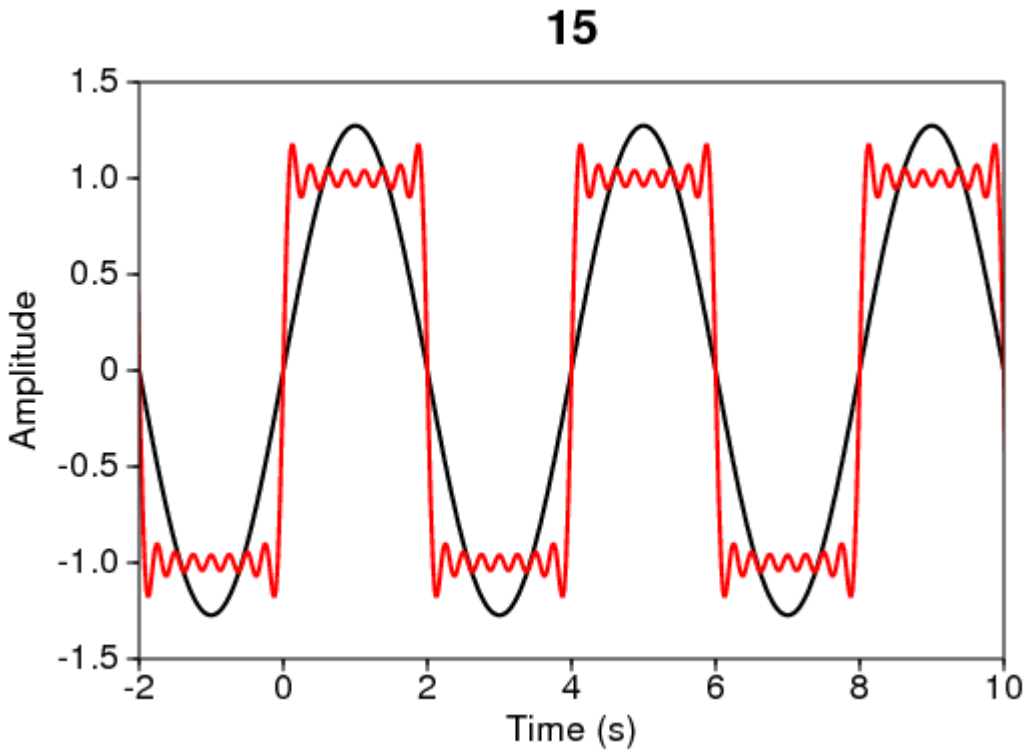
-1.0
-1.5
Amplitude
-2
0
2
4
6
8
10
Time (s)

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1.5
1.0
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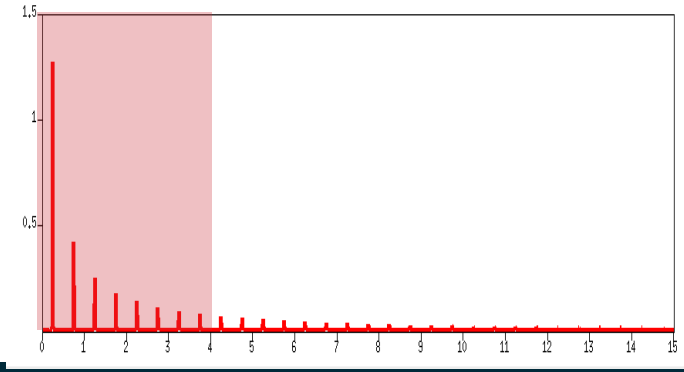
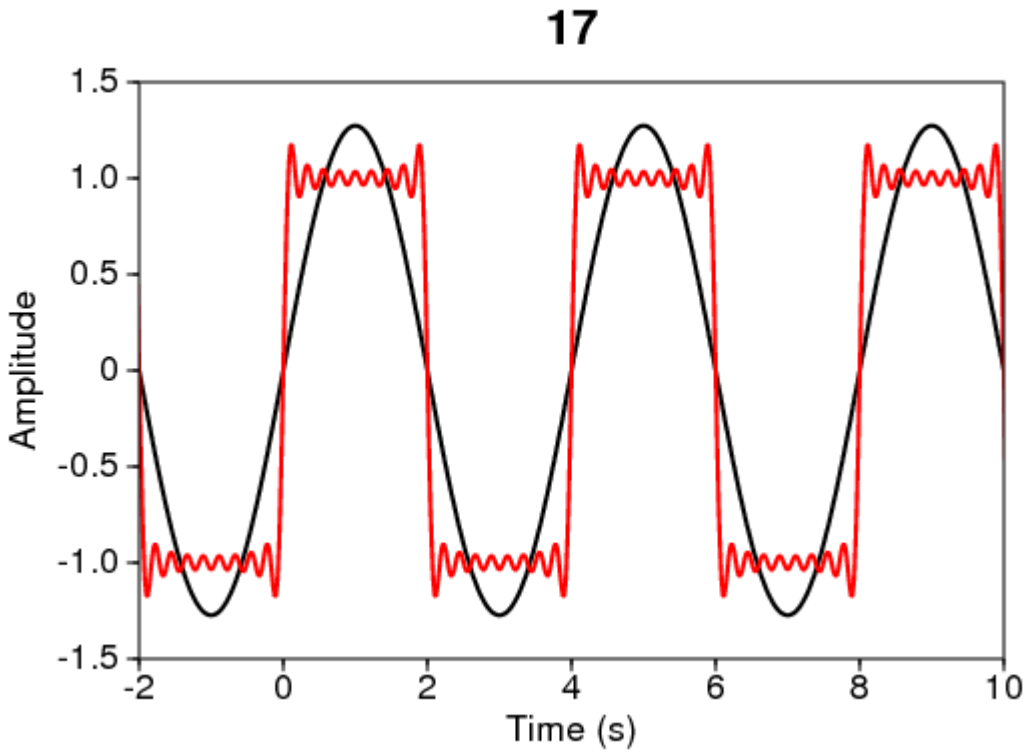
-1.0
-1.5
Amplitude
-2
0
2
4
6
8
10
Time (s)

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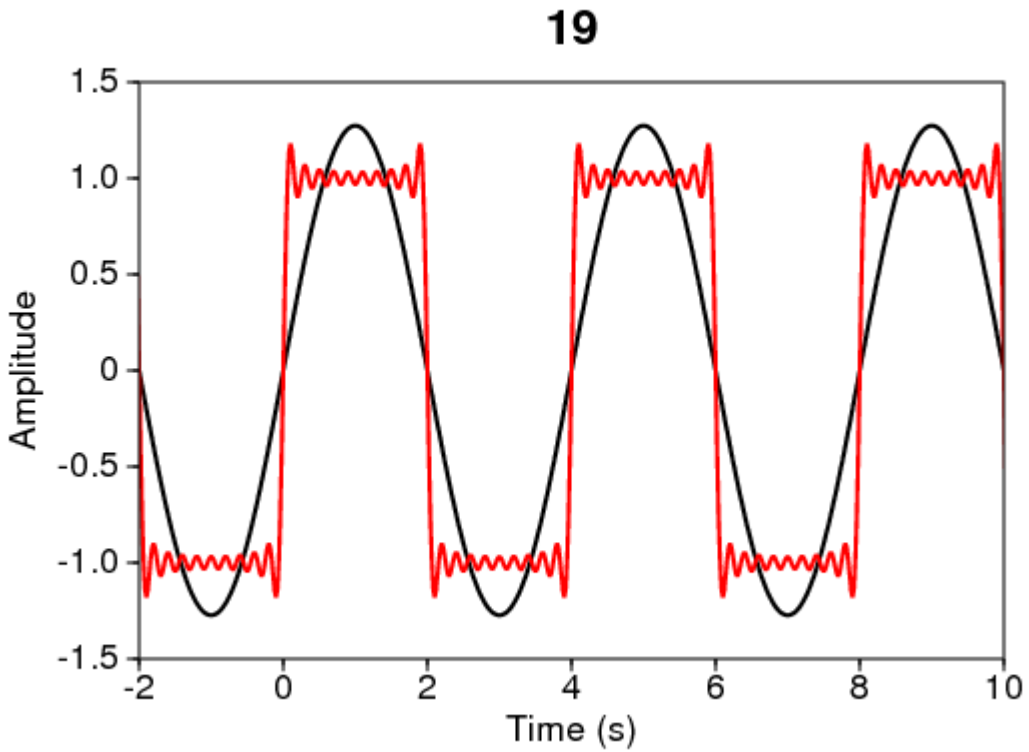
-1.0
-1.5
Amplitude
-2
0
2
4
6
8
10
Time (s)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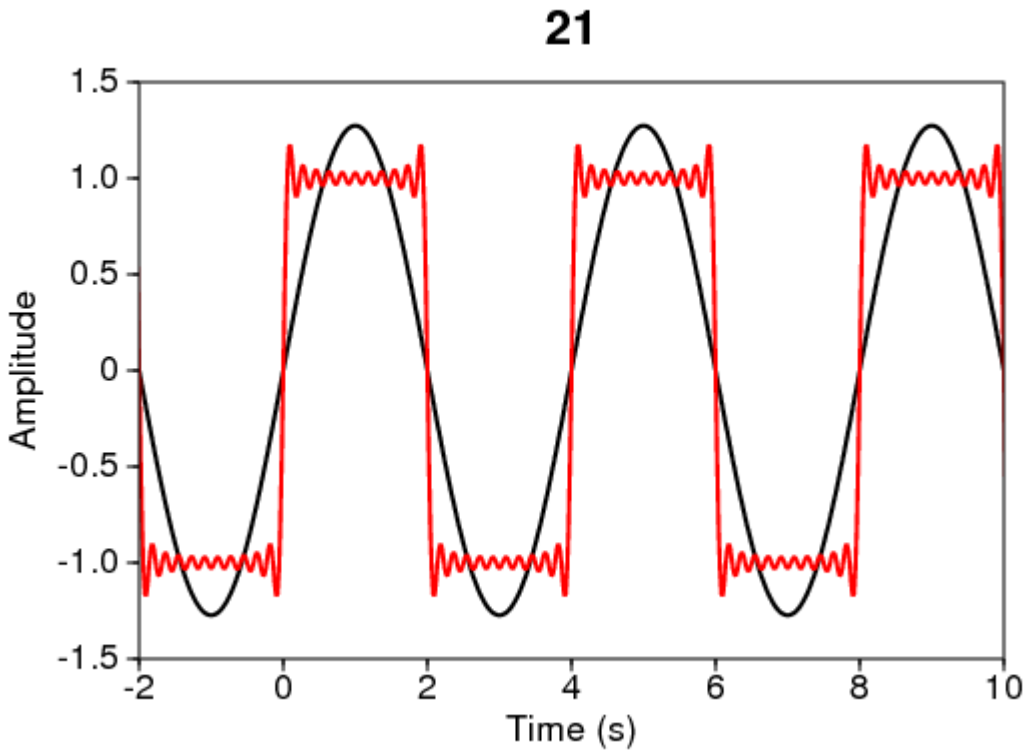
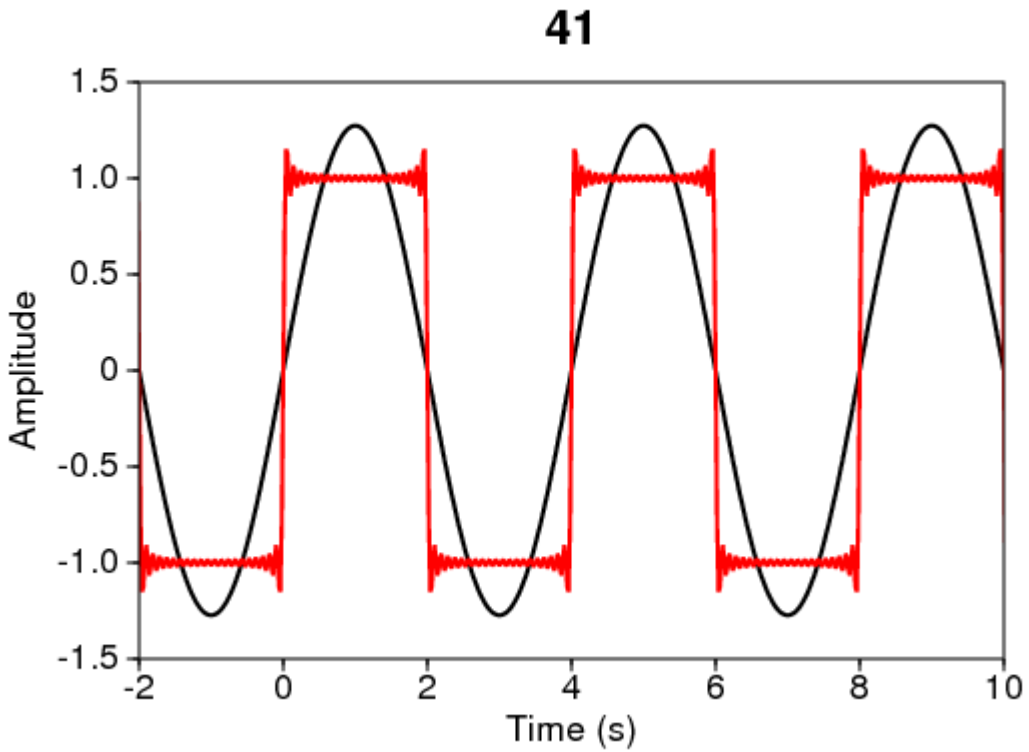
-1.0
-1.5
Amplitude
-2
0
2
4
6
8
10
Time (s)


21
1.5
1.0
0.5
0
-0.5
-1.0
-1.5
Amplitude
-2
0
2
4
6
8
10
Time (s)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1.5
1.0
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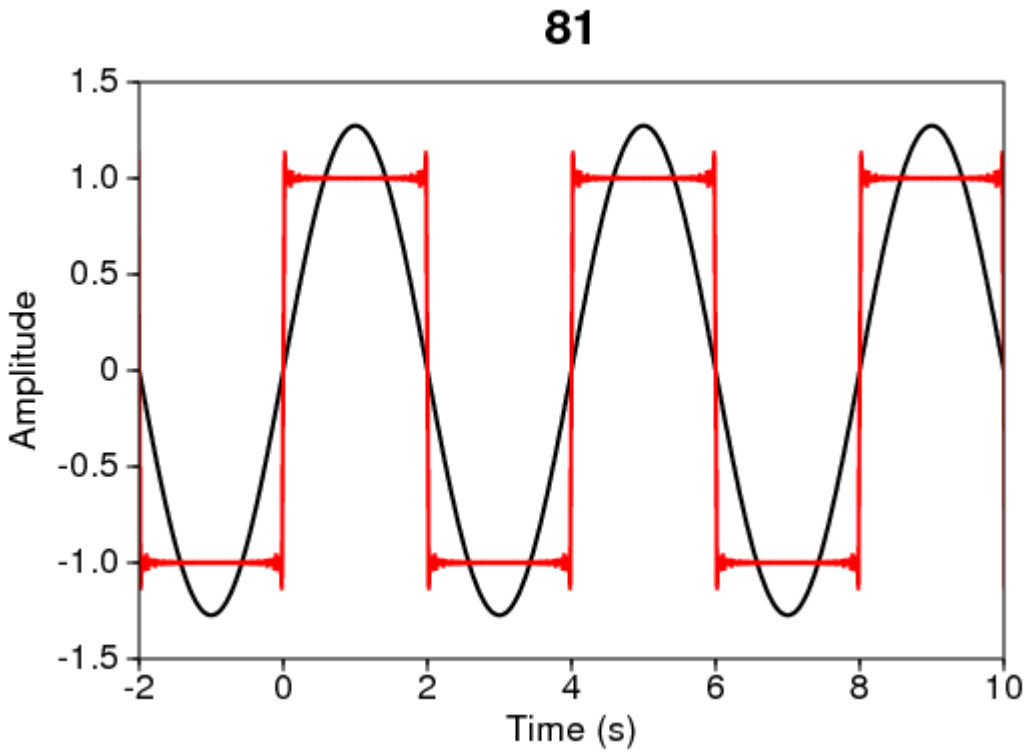
-1.0
-1.5
Amplitude
-2
0
2
4
6
8
10
Time (s)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1.5
1.0
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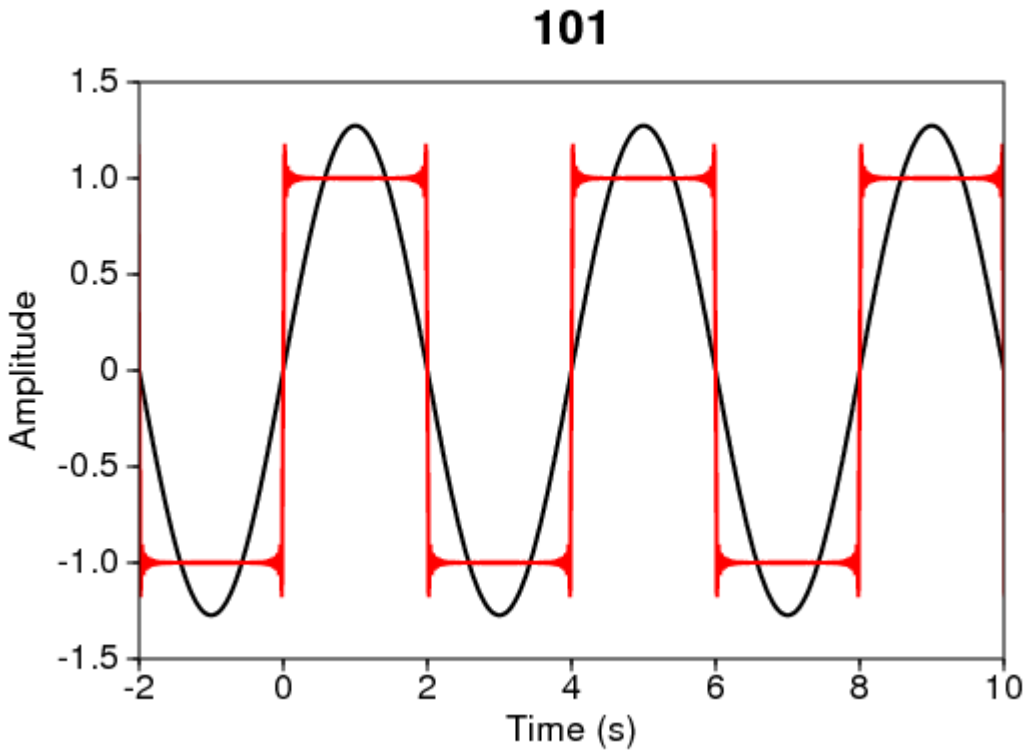
-0.5
-1.0
-1.5
Amplitude
-2
0
2
4
6
8
10
Time (s)


101
1.5
1.0
0.5
0
-0.5
-1.0
-1.5
Amplitude
-2
0
2
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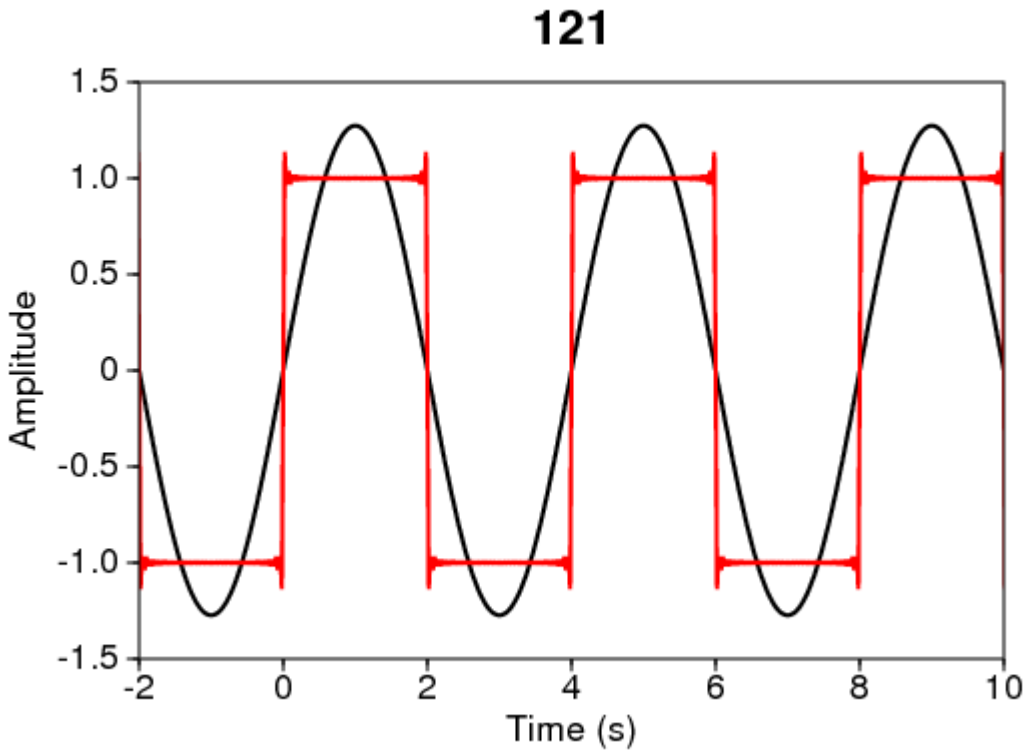
6
8
10
Time (s)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1.5
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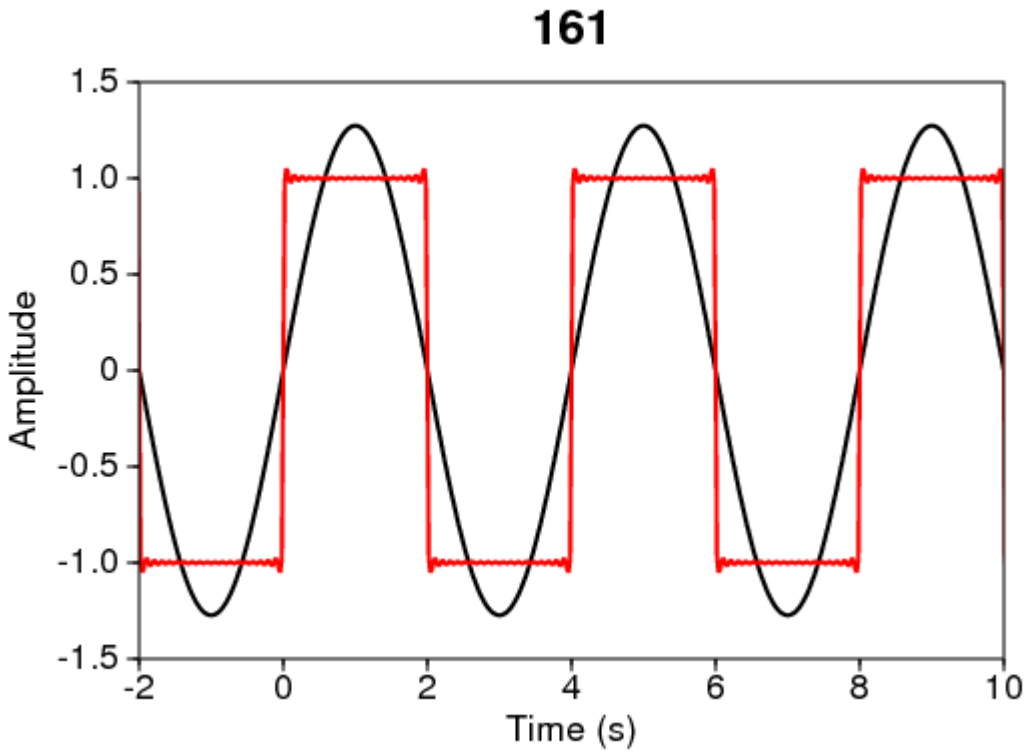
-1.0
-1.5
Amplitude
-2
0
2
4
6
8
10
Time (s)


161
1.5
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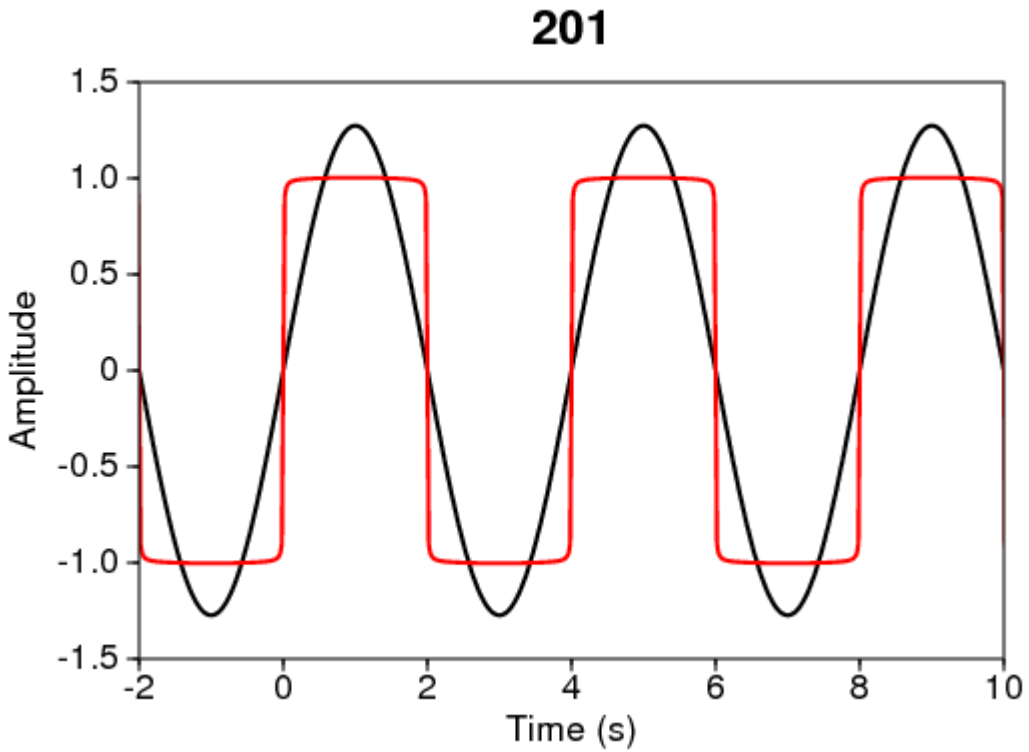
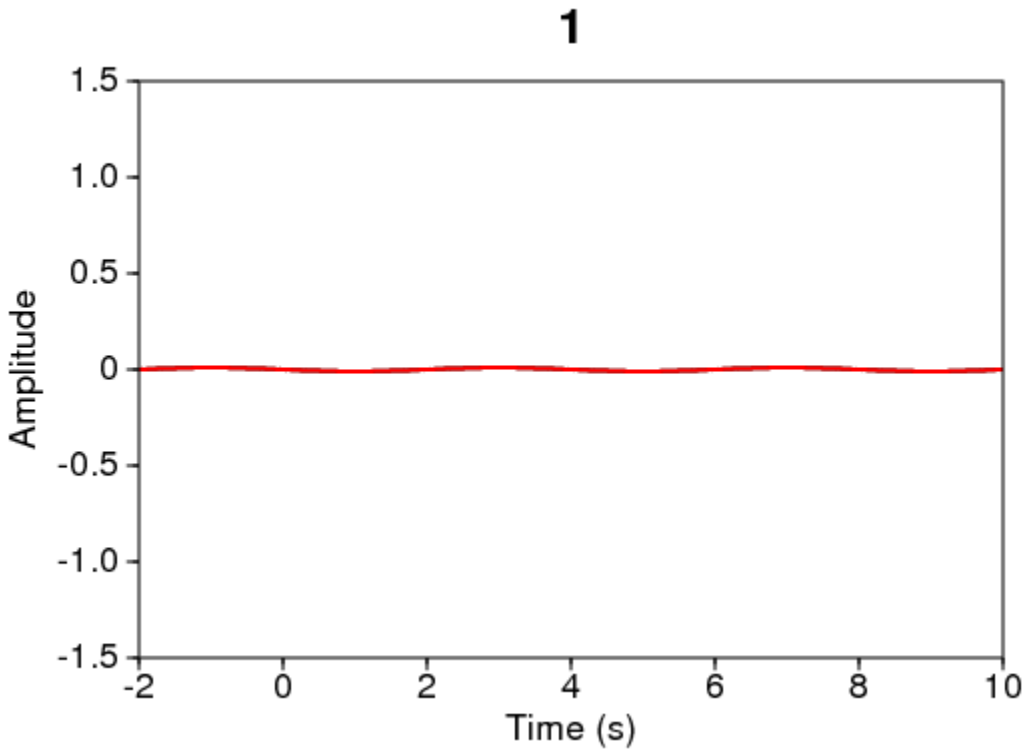
-0.5
-1.0
-1.5
Amplitude
-2
0
2
4
6
8
10
Time (s)

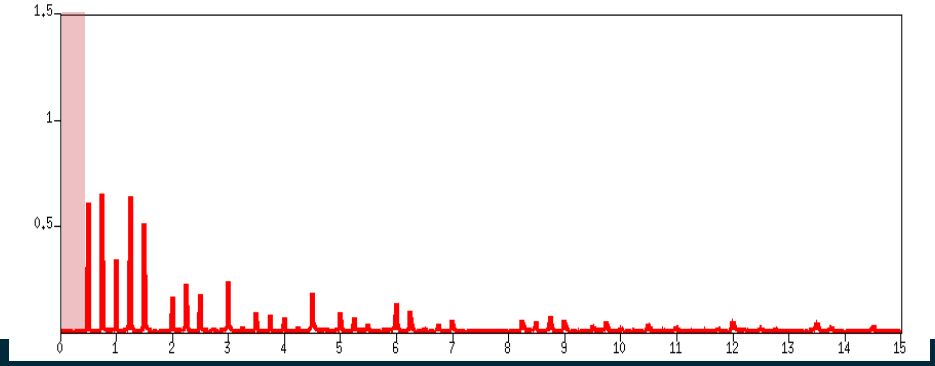

201
1.5
1.0
0.5
0
-0.5
-1.0
-1.5
Amplitude
-2
0
2
4
6
8
10
Time (s)

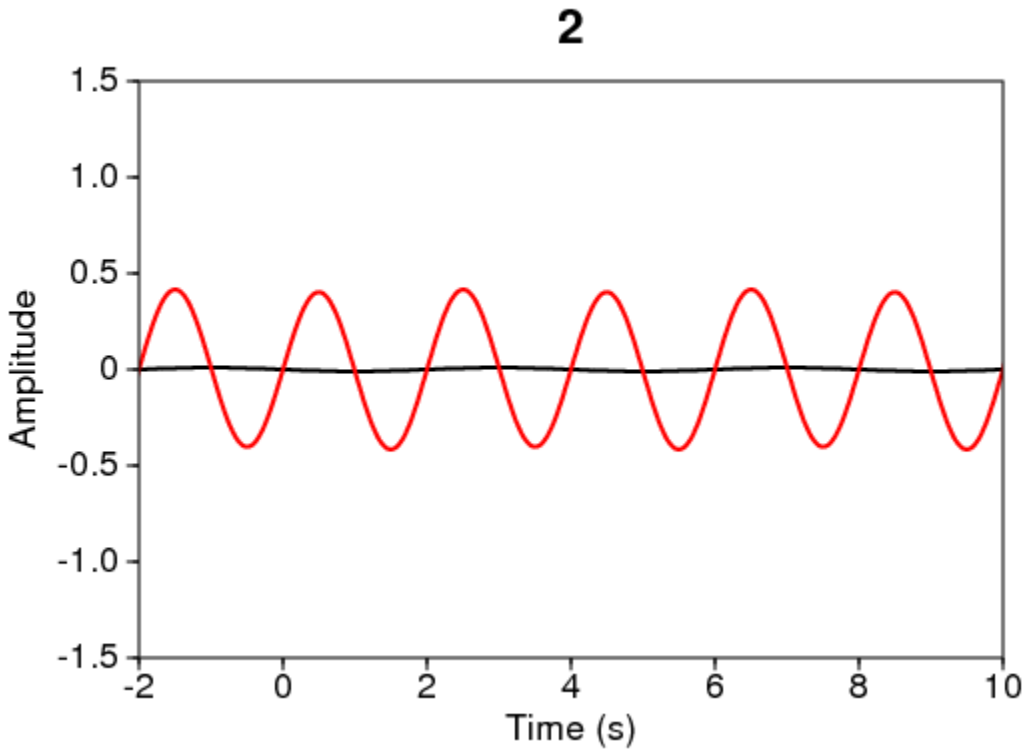
Assume a different source current spectrum:

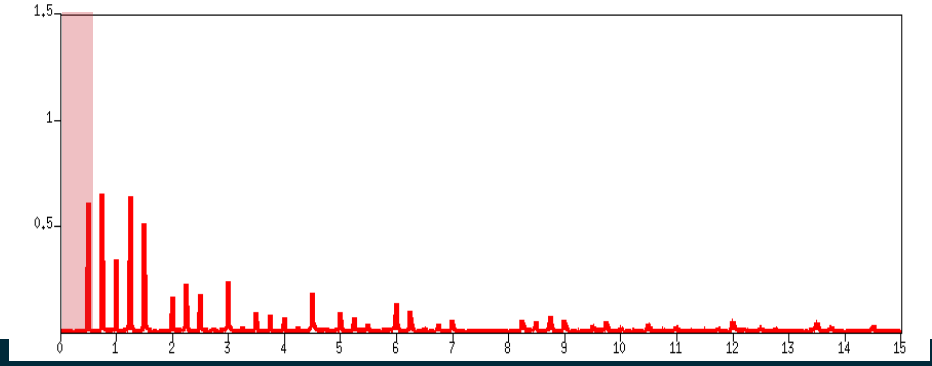
What will the resulting time signal look like?



The first harmonic is almost negligible




2
1.5
1.0
0.5
0
-0.5
-1.0
-1.5
Amplitude
-2
0
2
4
6
8
10
Time (s)


1.5
1
0.5
0
1
2
3
4
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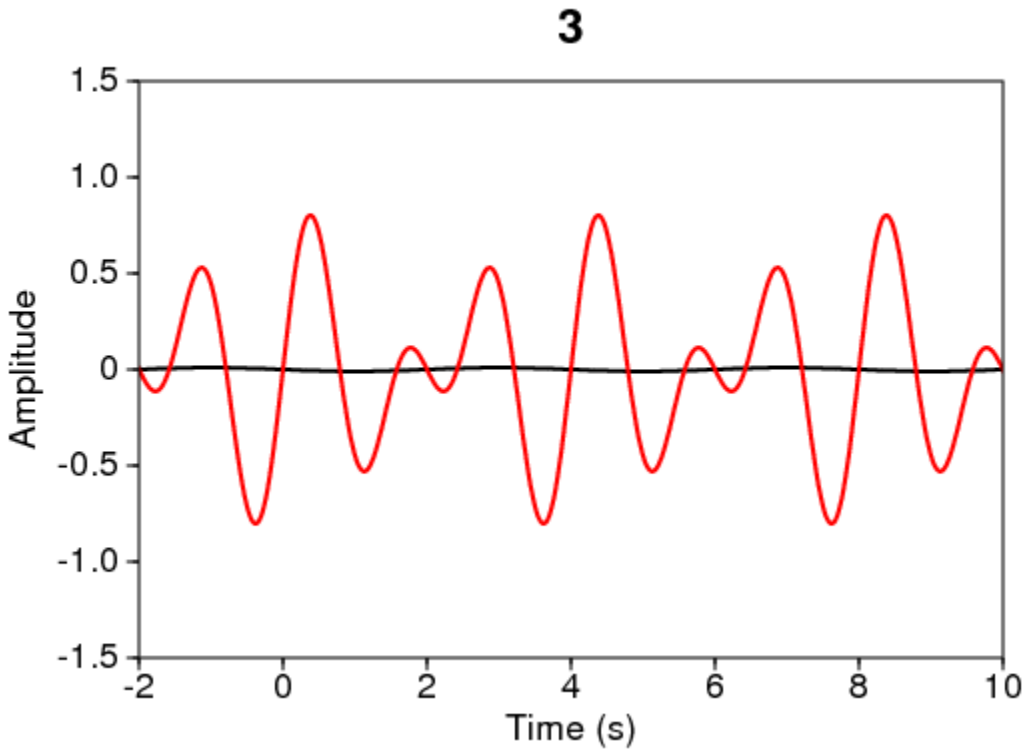
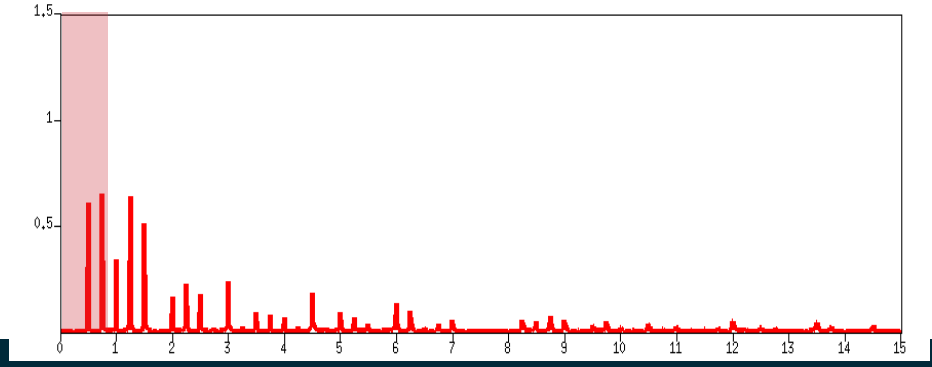
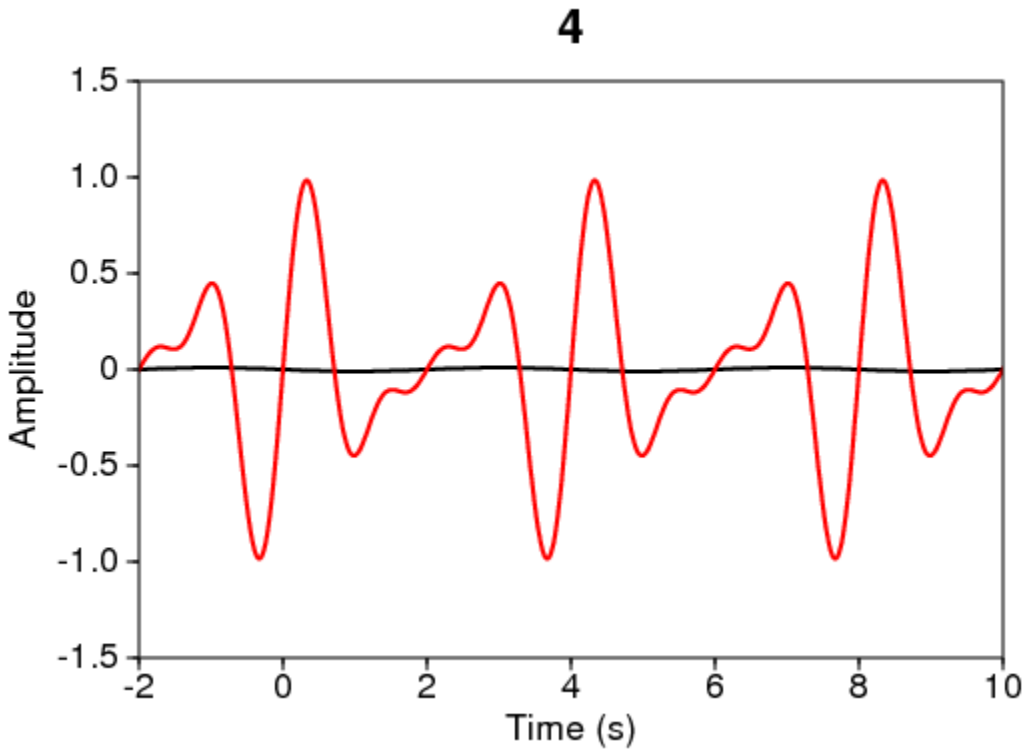
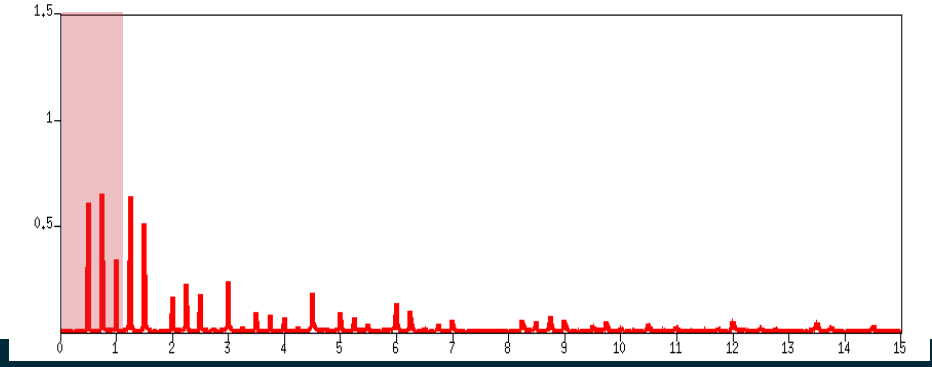
6
7
8
9
10
11
12
13
14
15


3
1.5
1.0
0.5
0
-0.5
-1.0
-1.5
Amplitude
-2
0
2
4
6
8
10
Time (s)


1.5
1
0.5
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15


4
1.5
1.0
0.5
0
-0.5
-1.0
-1.5
Amplitude
-2
0
2
4
6
8
10
Time (s)


1.5
1
0.5
0
1
2
3
4
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8
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12
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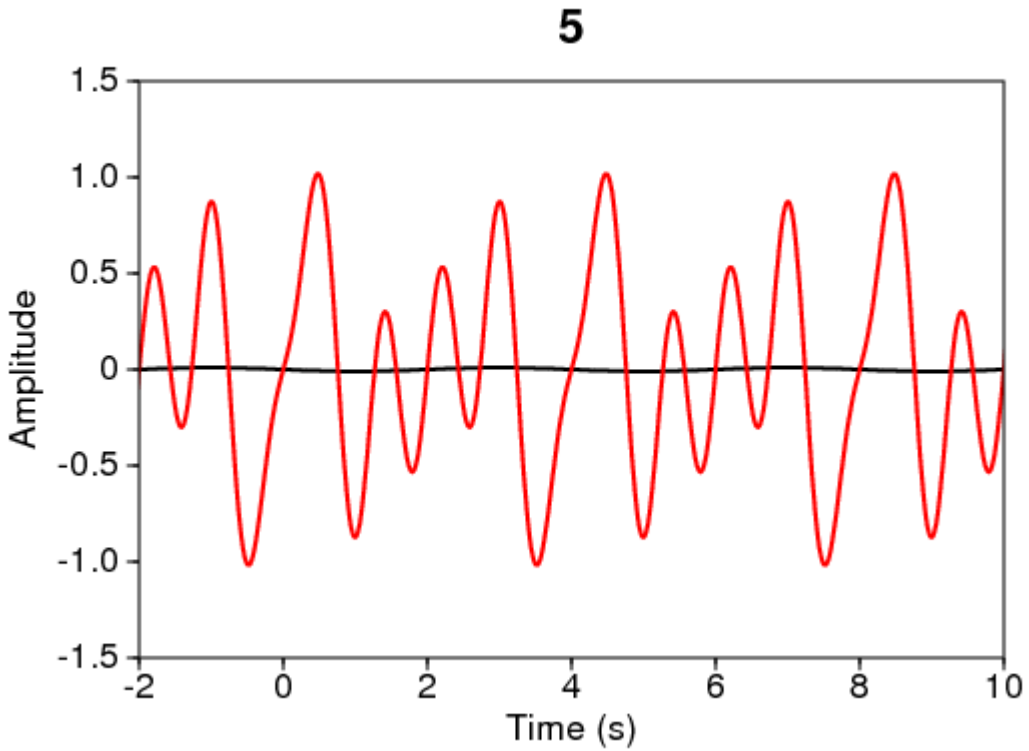
14
15


5
1.5
1.0
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0
-0.5
-1.0
-1.5
Amplitude
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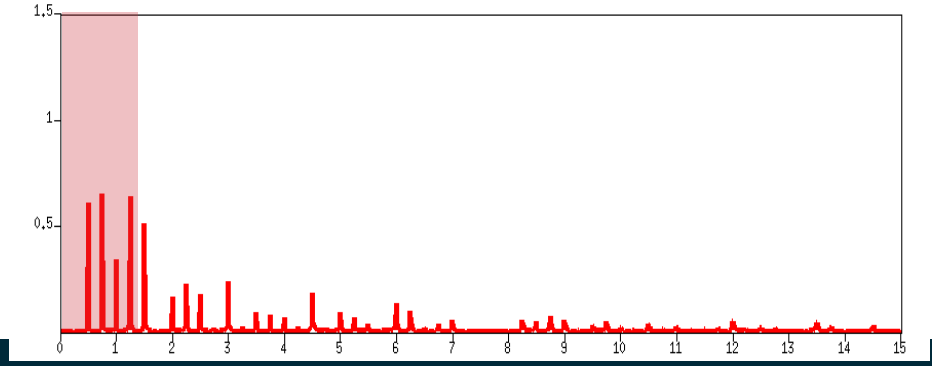
0
2
4
6
8
10
Time (s)


1.5
1
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0
1
2
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4
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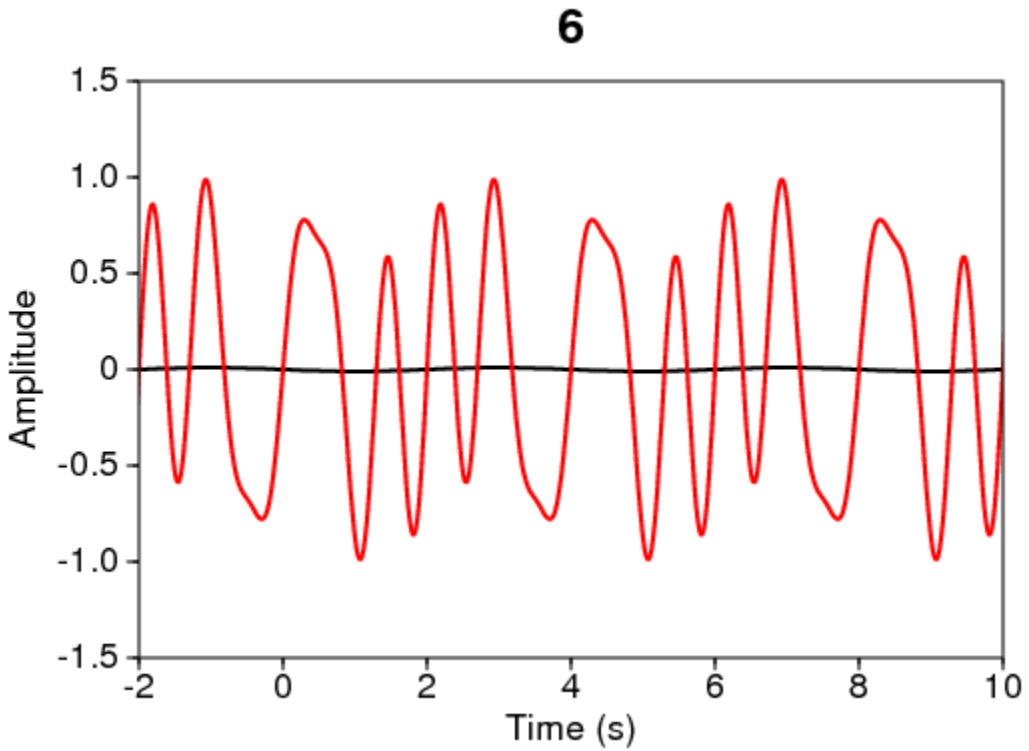
14
15


6
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-1.5
Amplitude
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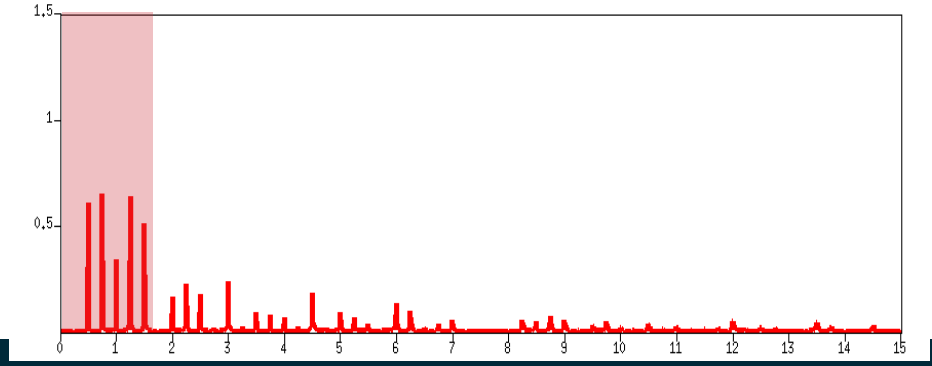
0
2
4
6
8
10
Time (s)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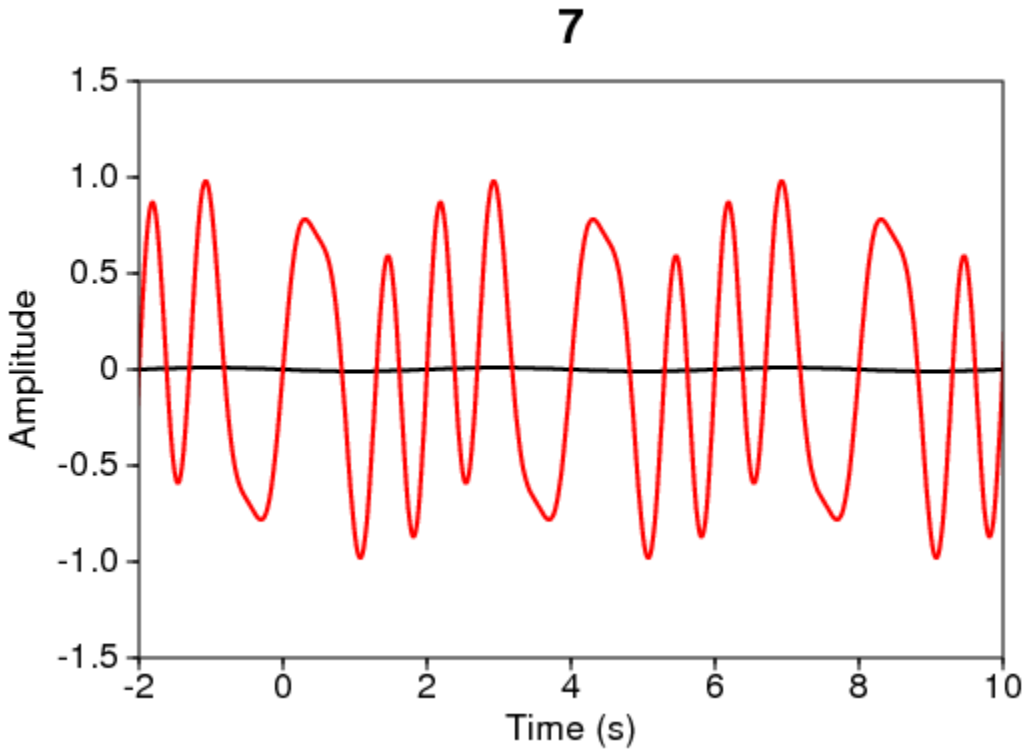
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Amplitude
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8
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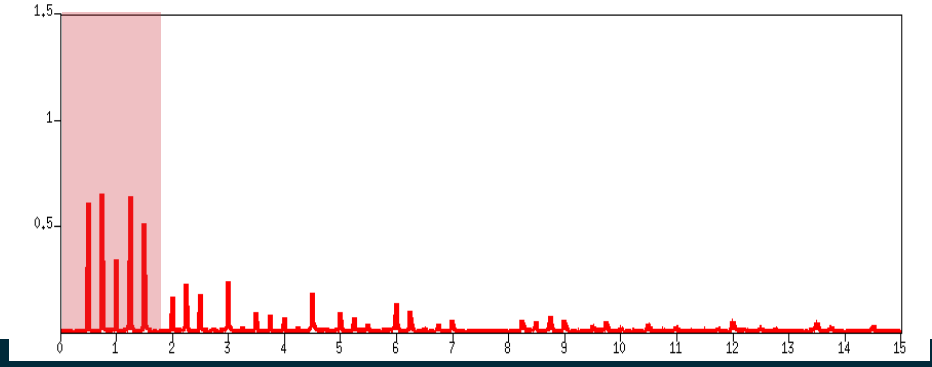
Time (s)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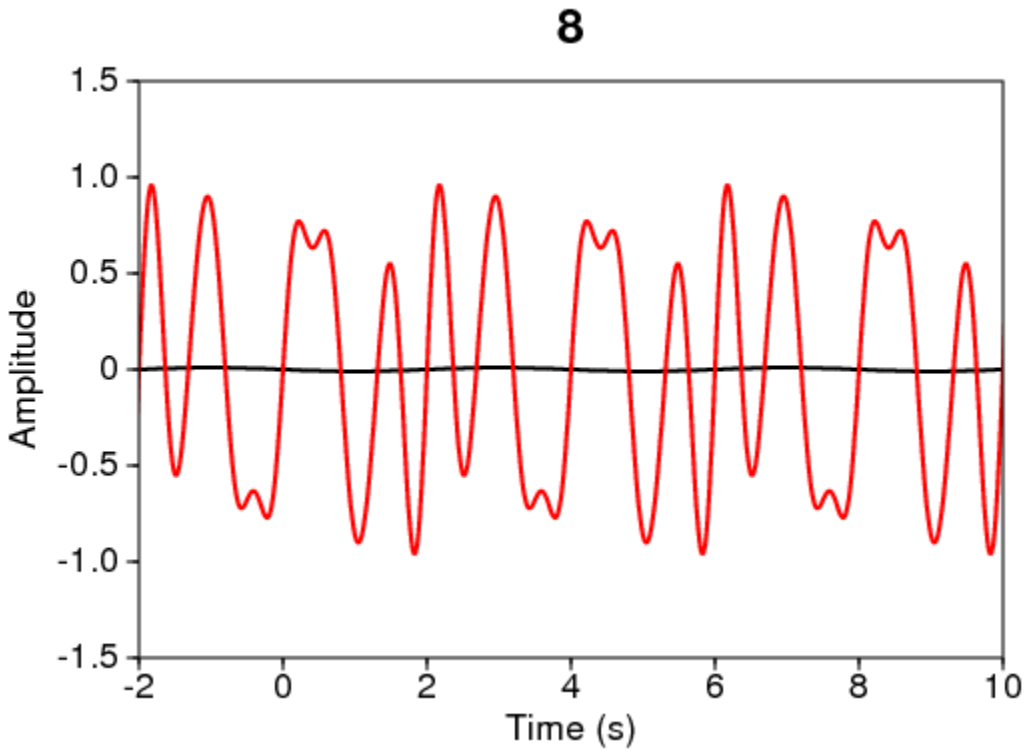
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Amplitude
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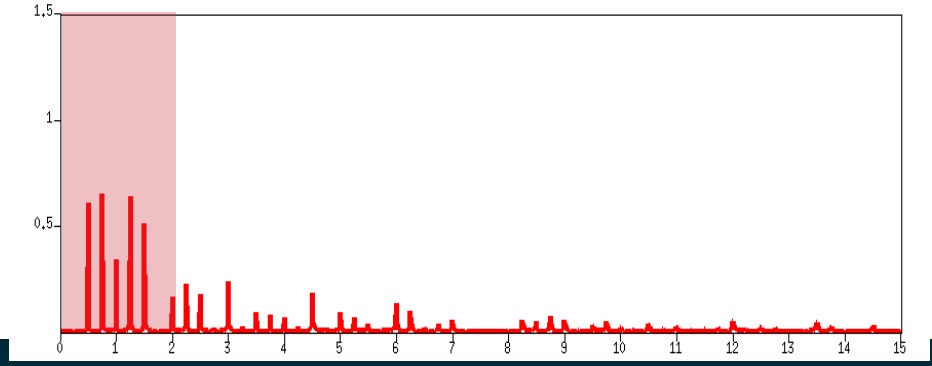
8
10
Time (s)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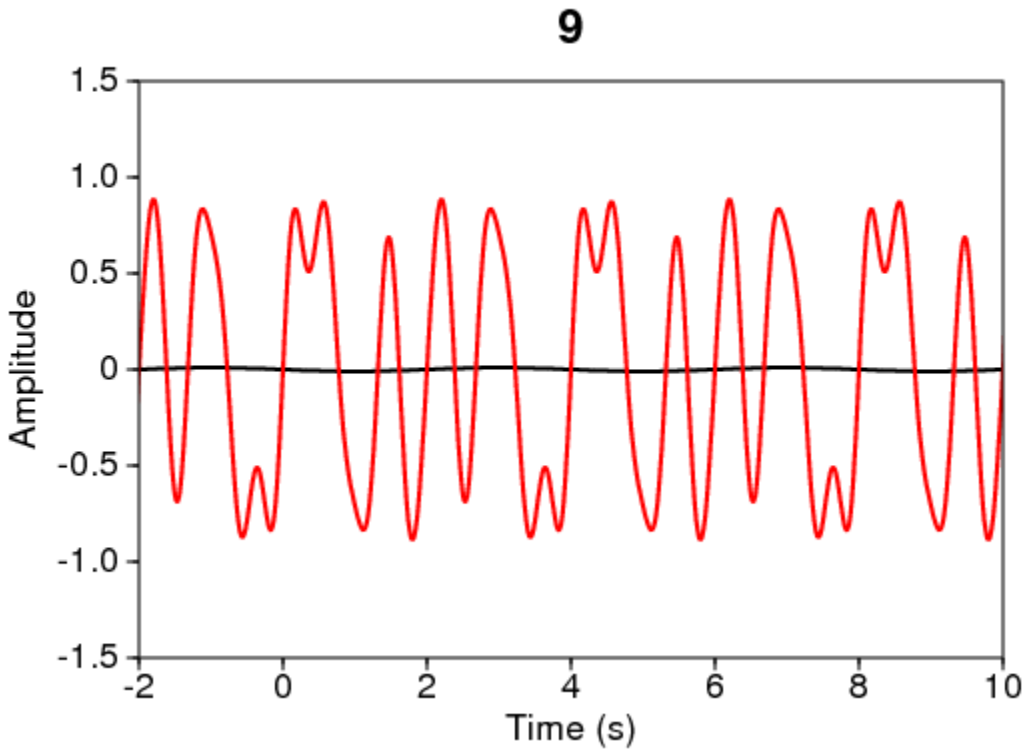
9
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-1.0
-1.5
Amplitude
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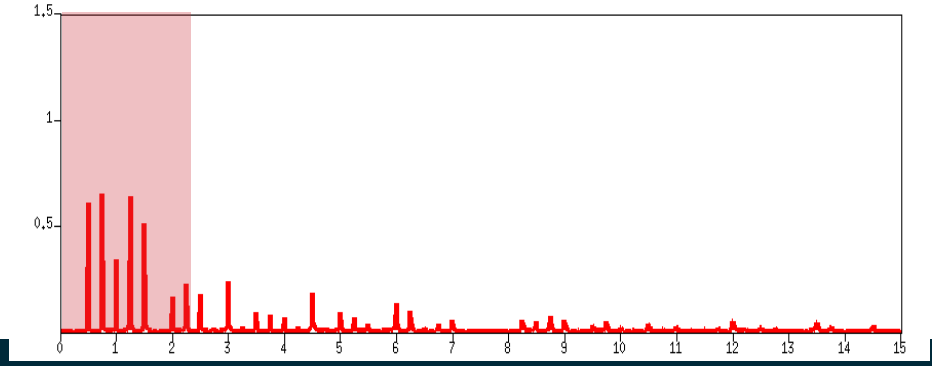
0
2
4
6
8
10
Time (s)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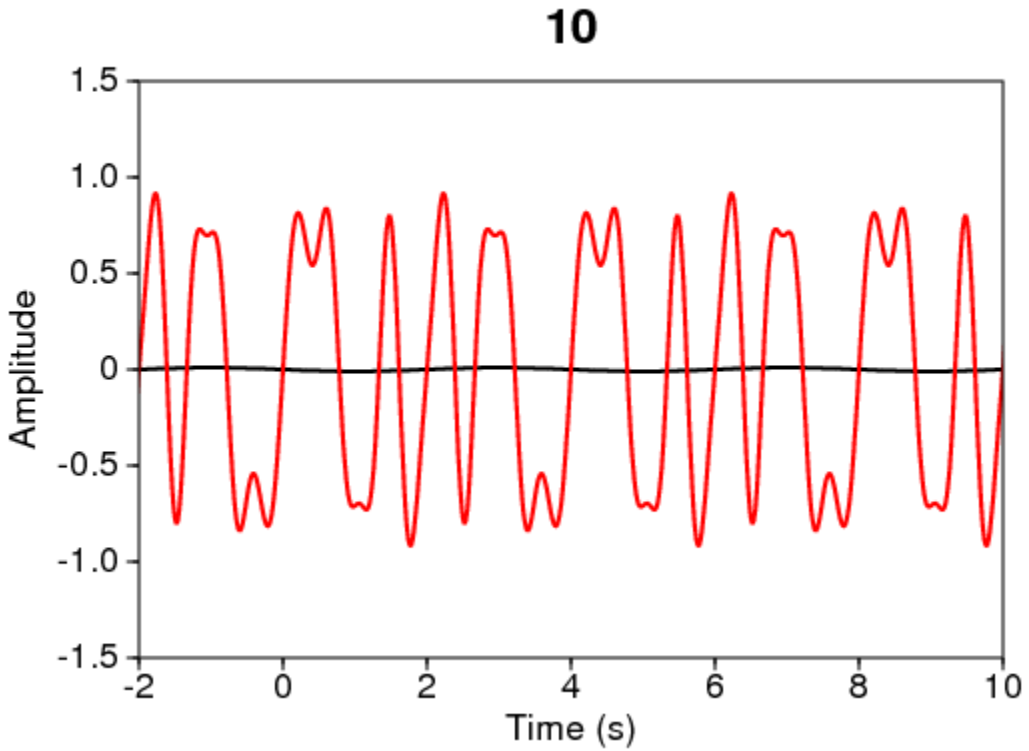
10
Amplitude
Time (s)
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-2
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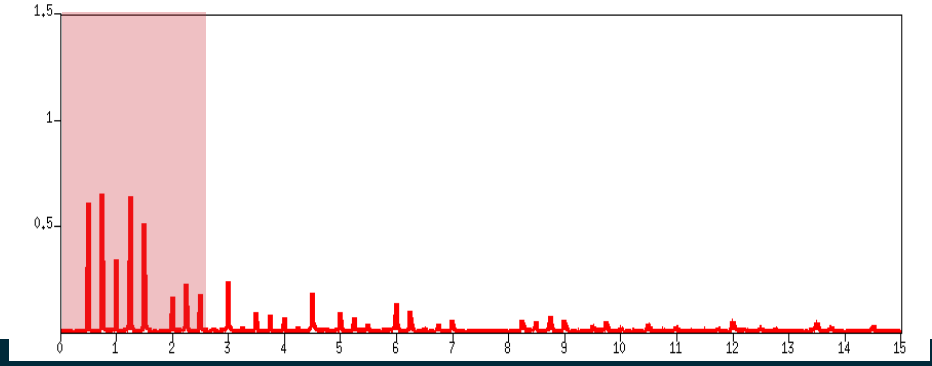
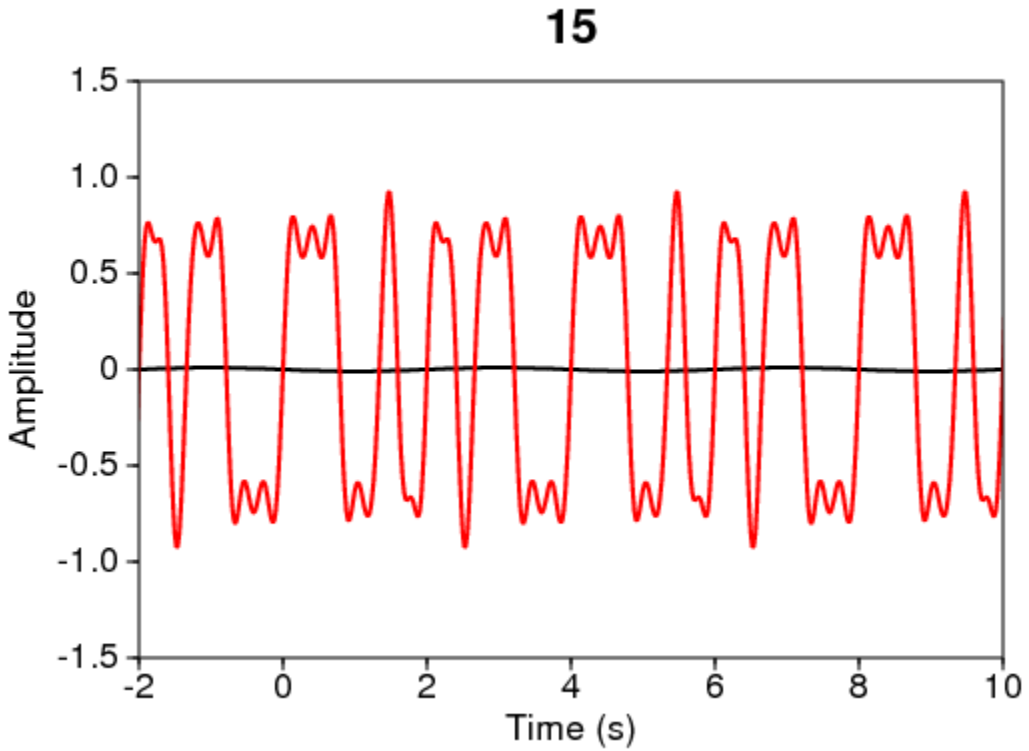
4
6
8
10




15
Amplitude
Time (s)
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-2
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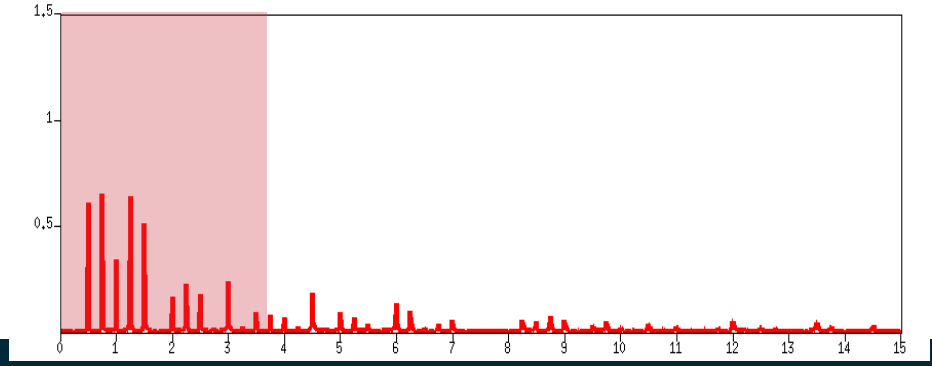
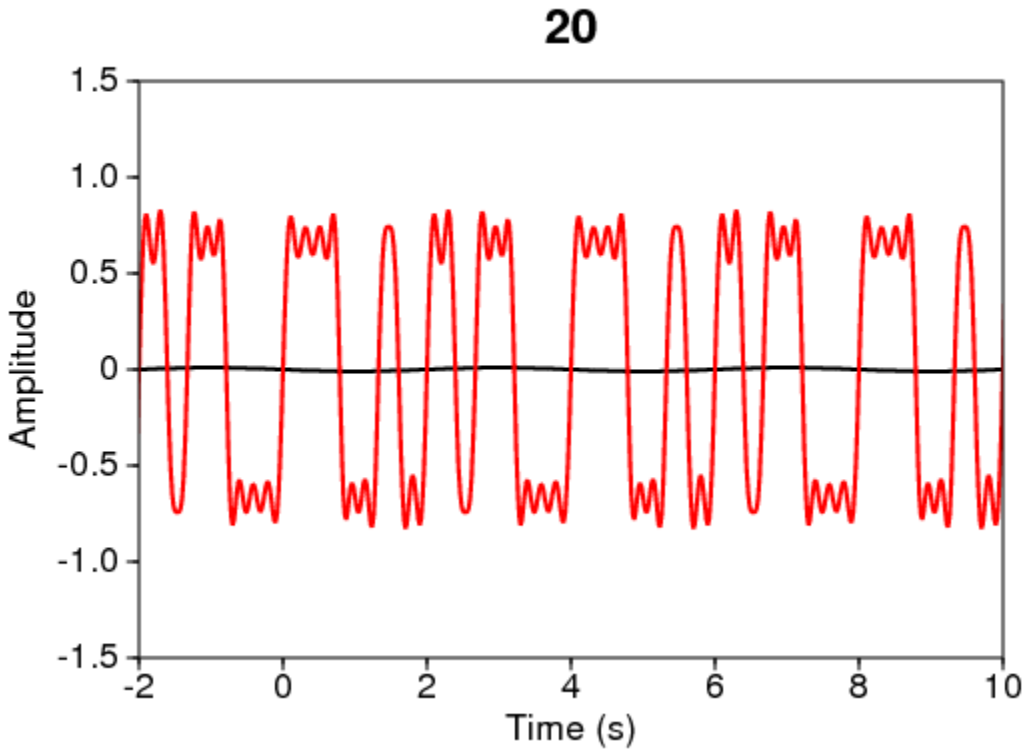
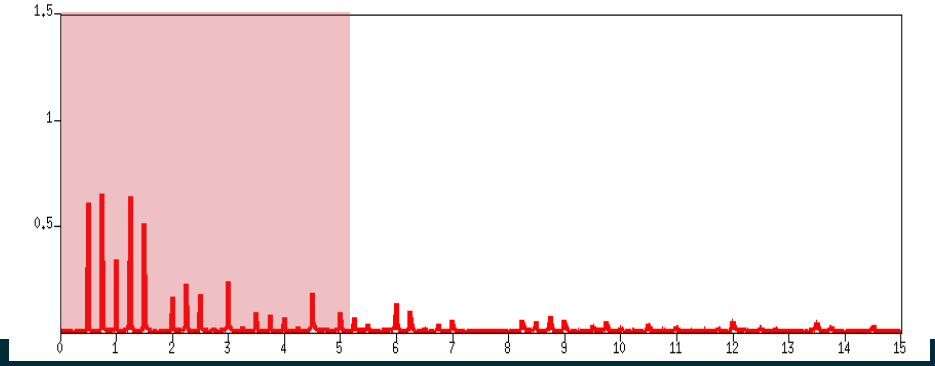
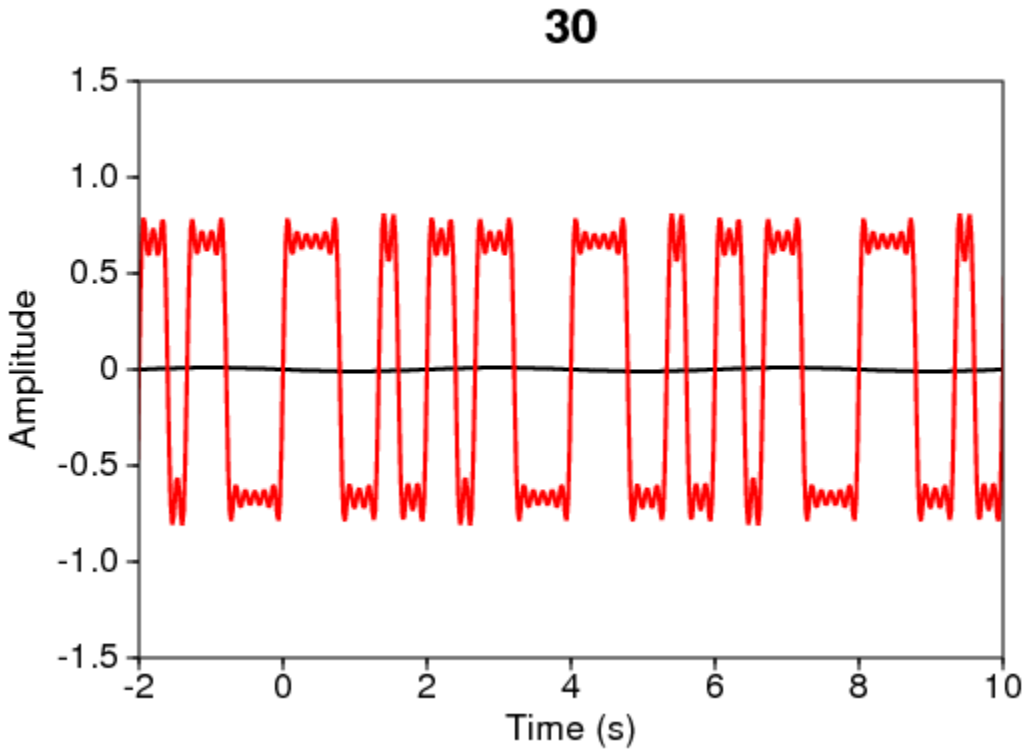
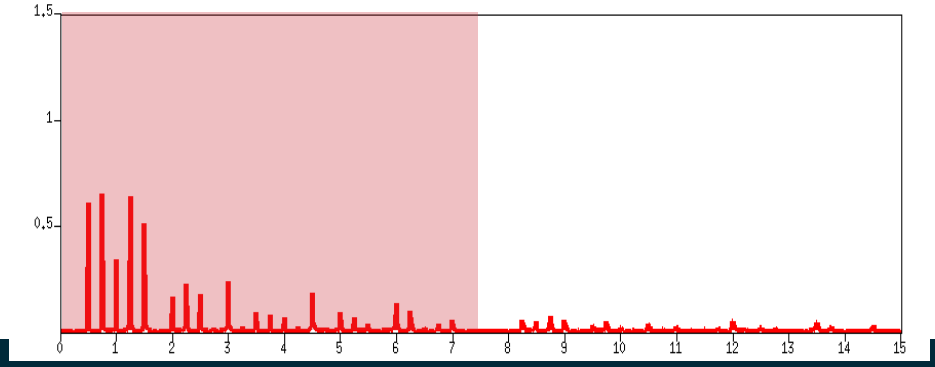
4
6
8
10



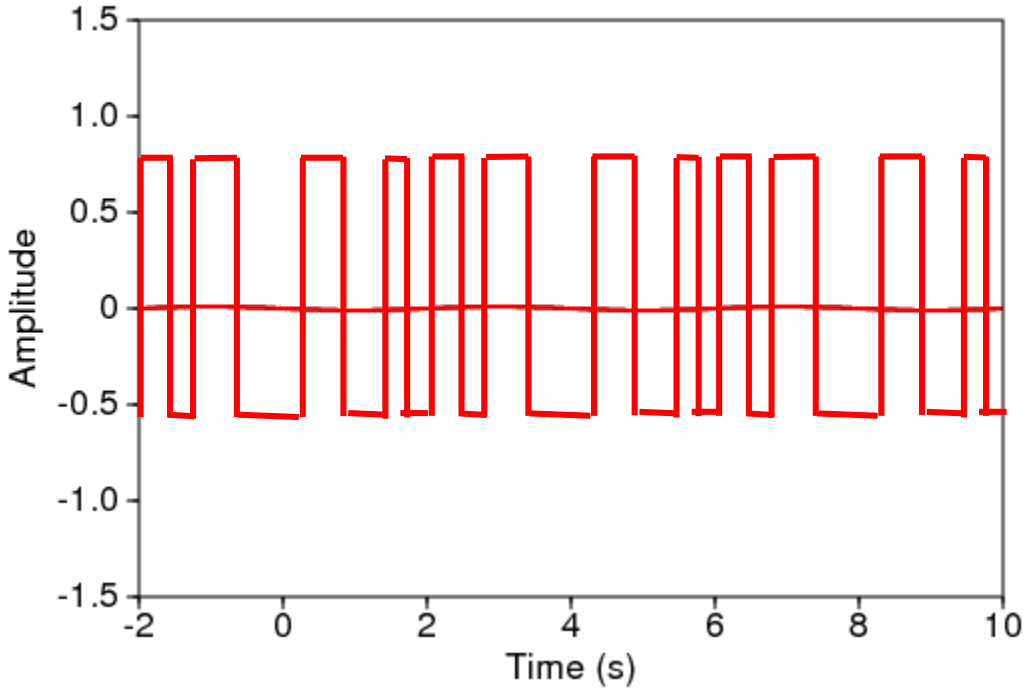

20
Amplitude
Time (s)
1.5
1.0
0.5
0
-0.5
-1.0
-1.5
-2
0
2
4
6
8
10

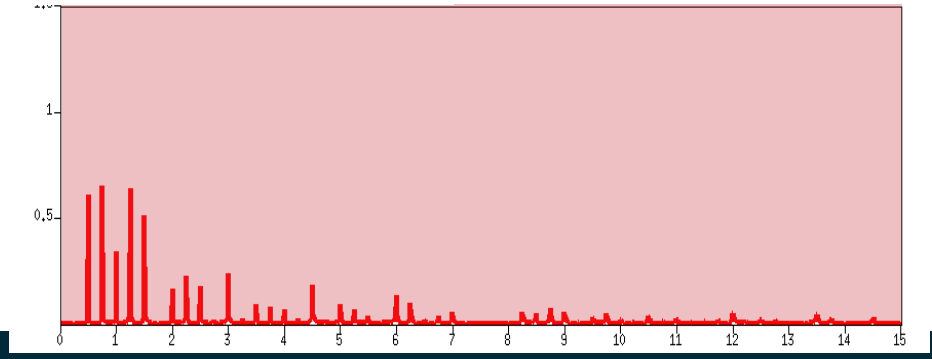


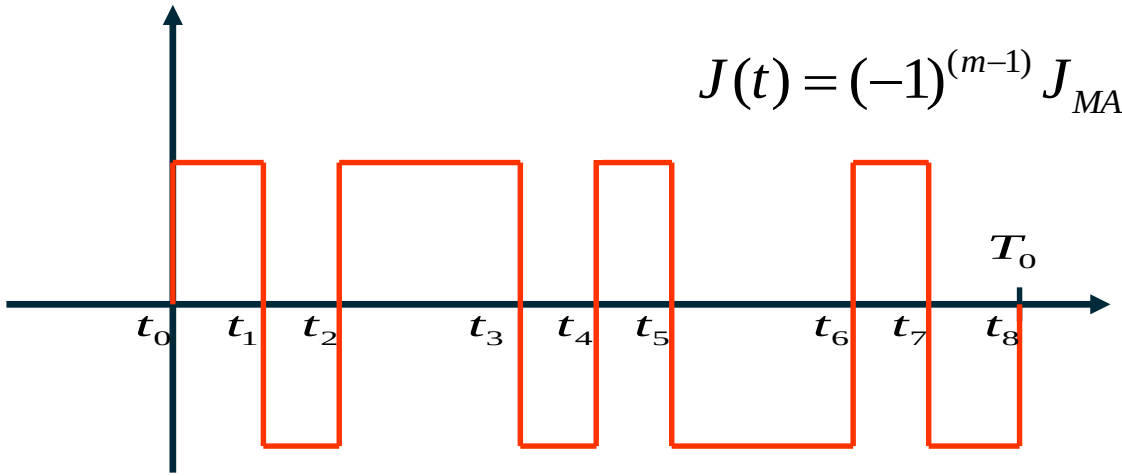

30
Amplitude
Time (s)
1.5
1.0
0.5
0
-0.5
-1.0
-1.5
-2
0
2
4
6
8
10


1.5
1
0.5
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15

200


1.5
1.0
0.5
0
-0.5
-1.0
-1.5
Amplitude
-2
0
2
4
6
8
10
Time (s)


1
0.5
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15

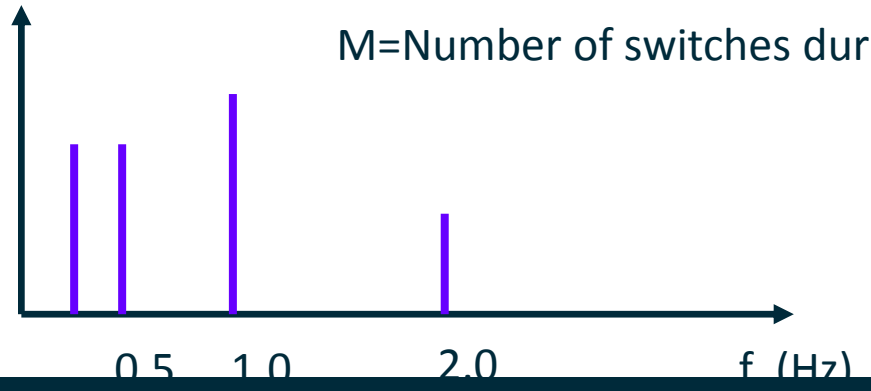


$$J(t) = (-1)^{(m-1)} J_{MAX} \qquad t_{m-1} < t \le t_m$$

Problem: Find time ordered sequence that give approximately desired current spectrum!

$$S_M = \{t_0, ..., t_{m-1}, t_m, t_{m+1}, ..., t_M\}$$

M=Number of switches during one period



GEOPHYSICS, VOL. 73, NO. 3 (MAY-JUNE 2008); P. F97–F104, 6 FIGS.
10.1190/1.2898410

Shaping optimal transmitter waveforms for marine CSEM surveys

Rune Mittet[1] and Tor Schaug-Pettersen[1]

$$I_5 = 0.7. \tag{27}$$

All other values of I_n are set to zero. The spectrum $|J_n|$ will depend on the switching time sequence S_M. The problem at hand is to find S_M such that $|J_n|$ has a distribution of amplitudes as a function of harmonics that is as close to I_n as possible. One possible criterion is to minimize the least-squares error ε,

$$\varepsilon = \sum_{n=0}^{N} (|J_n| - I_n)^2. \tag{28}$$

The maximum frequency f_{max} used in the optimization determines the highest harmonic N to use in equation 28. We normally use a maximum frequency of 15 Hz because higher frequencies are hard to use in an SBL survey; accordingly,

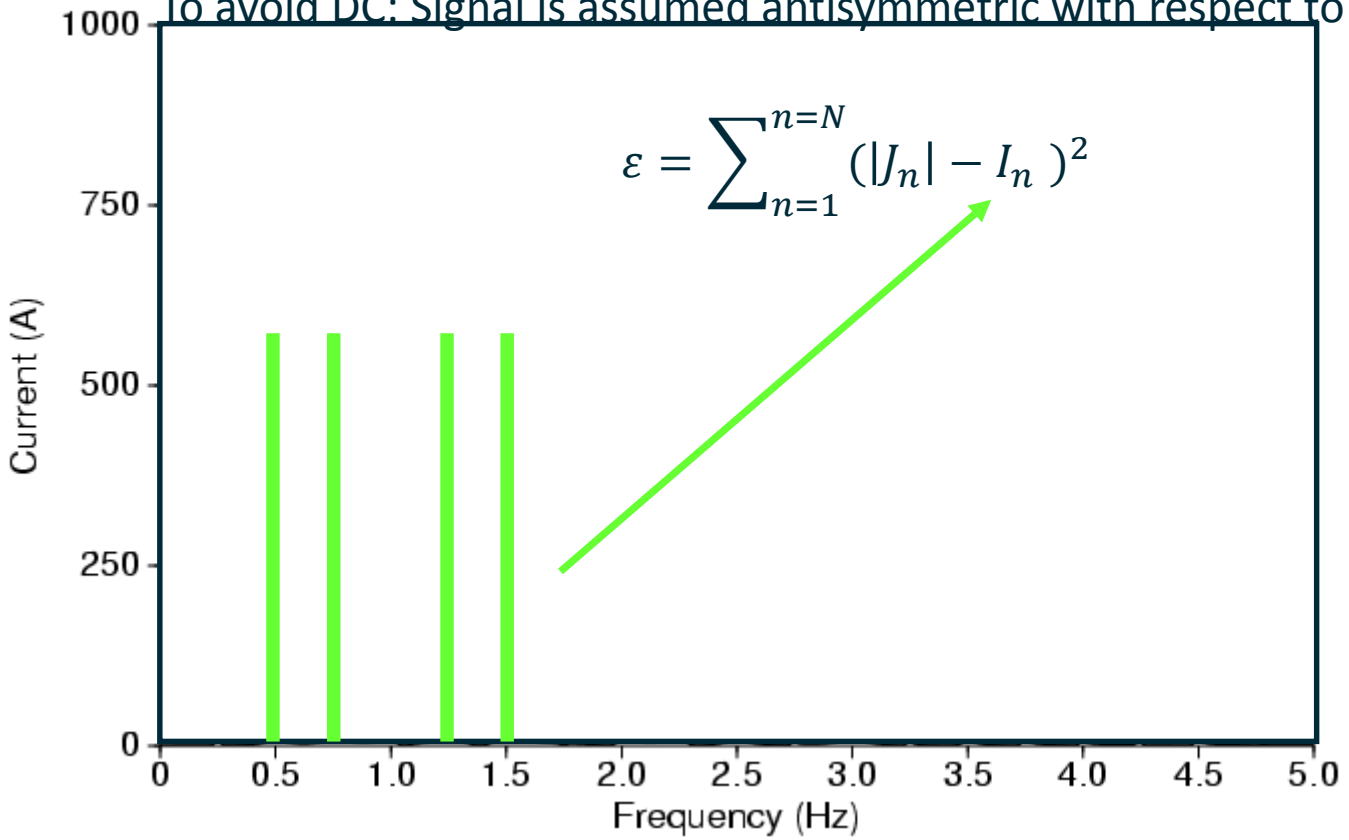
Desired current distribution

Problem: Find optimal M and corresponding switching times.

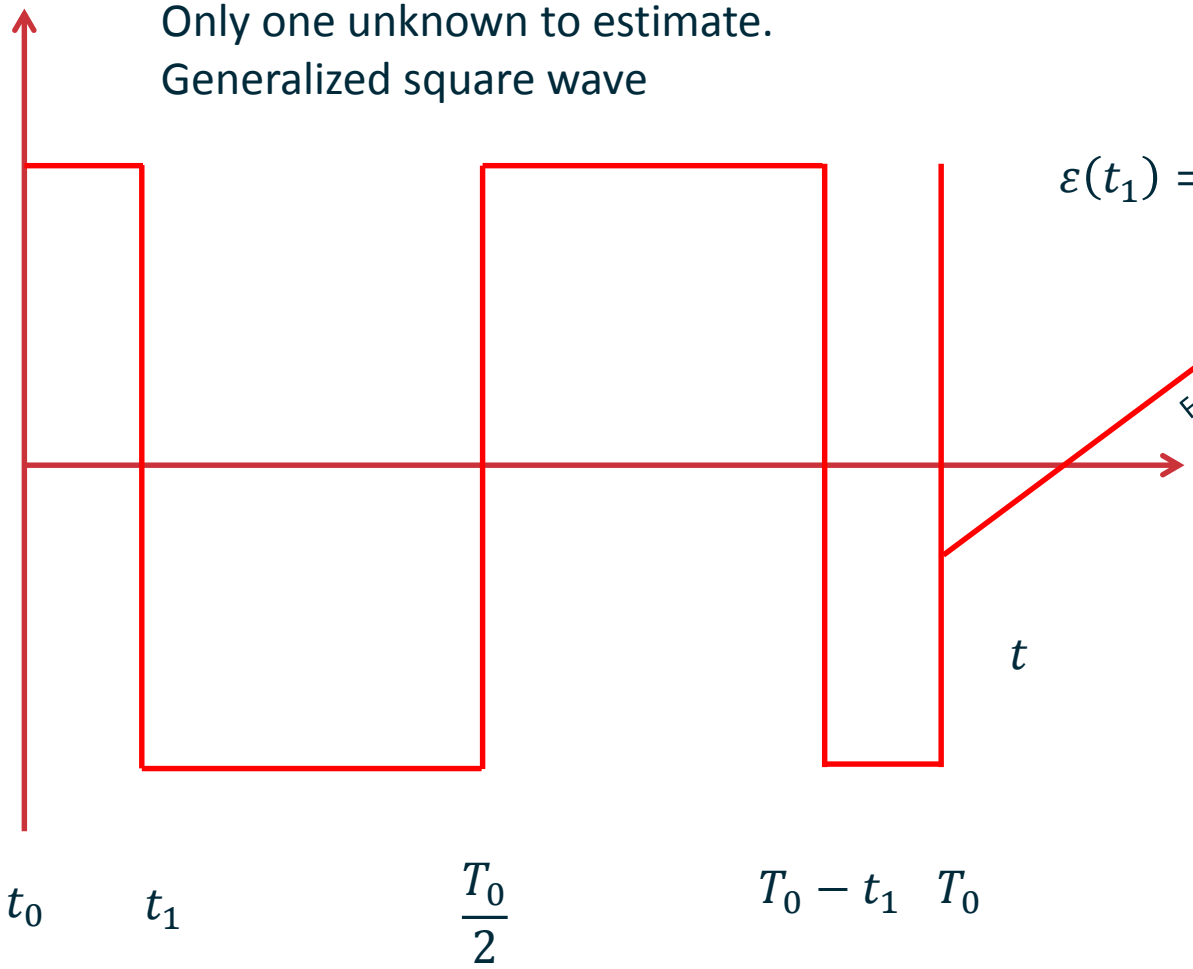
First harmonic is 0.25 Hz

Period is 4 s

To avoid DC: Signal is assumed antisymmetric with respect to $T_0/2$



4 switches within period T_0
Signal is DC free since antisymmetric with respect to half period.
DC free -> avoid electrode corrosion
Only one unknown to estimate.
Generalized square wave



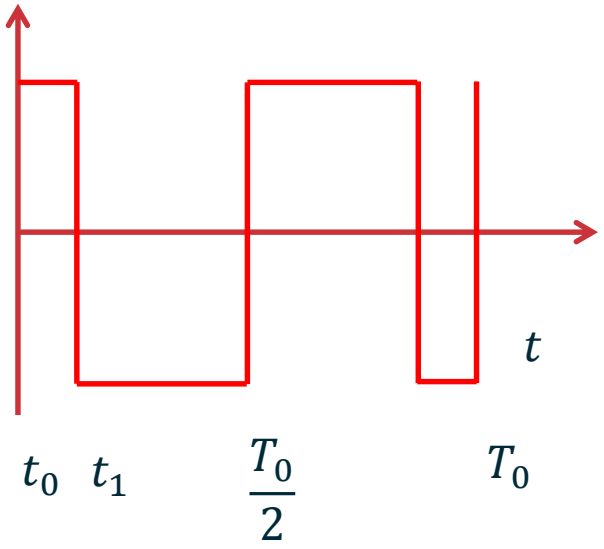
$$\varepsilon(t_1) = \sum_{n=1}^{n=N} (|J_n(t_1)| - I_n)^2$$

Fourier transform

$$t_0 = 0.0\ \text{s}$$

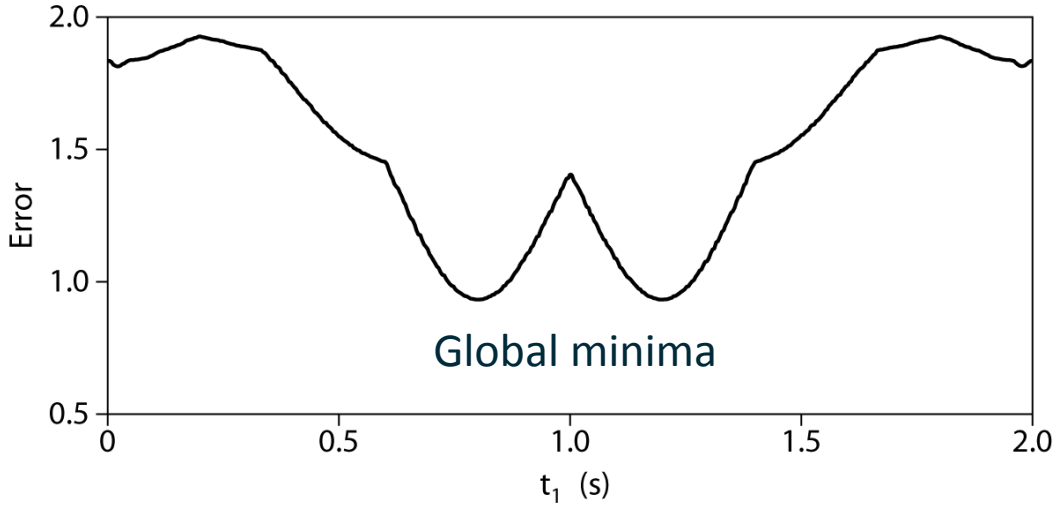
$$0.0\ \text{s} < t_1 < 2.0\ \text{s}$$

$$t_M = 4.0\ \text{s}$$



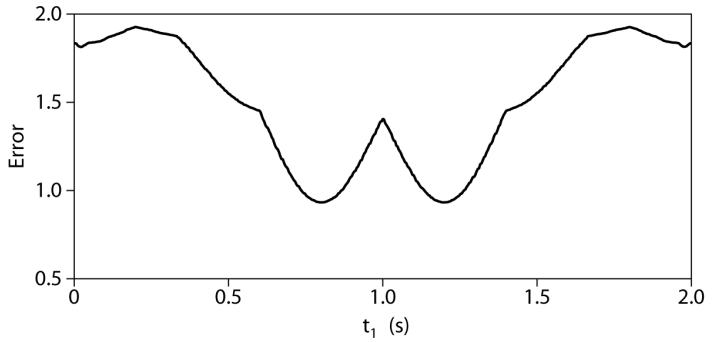
$$\varepsilon(t_1) = \sum_{n=1}^{n=N} (|J_n(t_1)| - I_n)^2$$

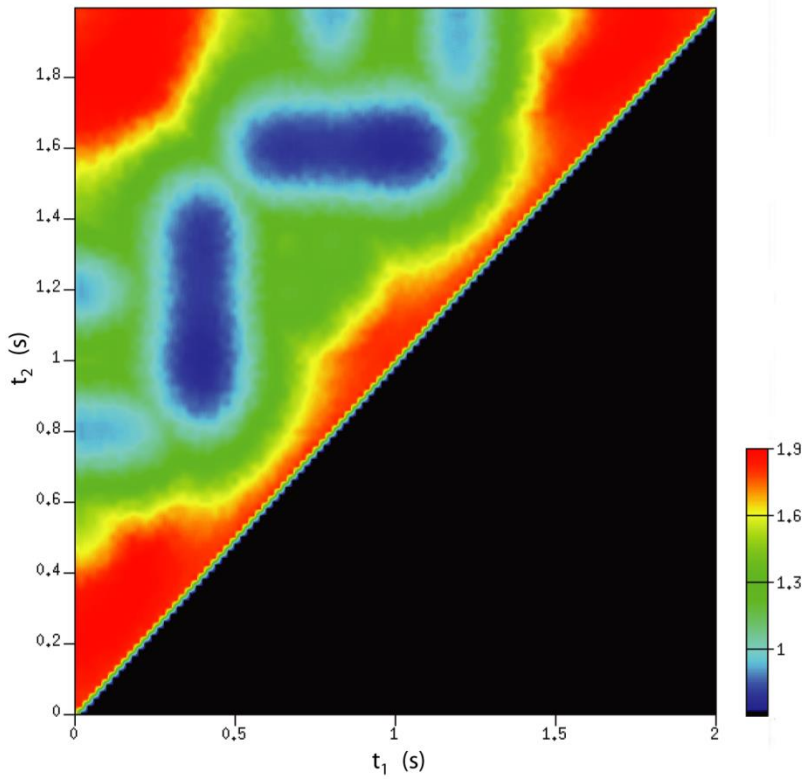
Local minimum



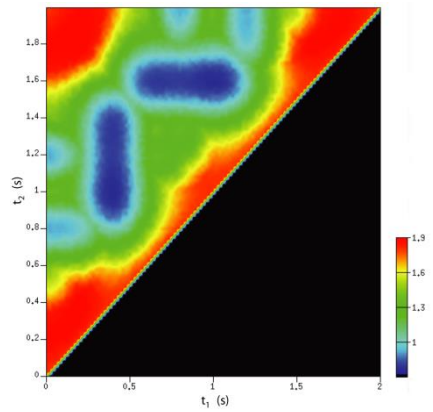
Local minimum

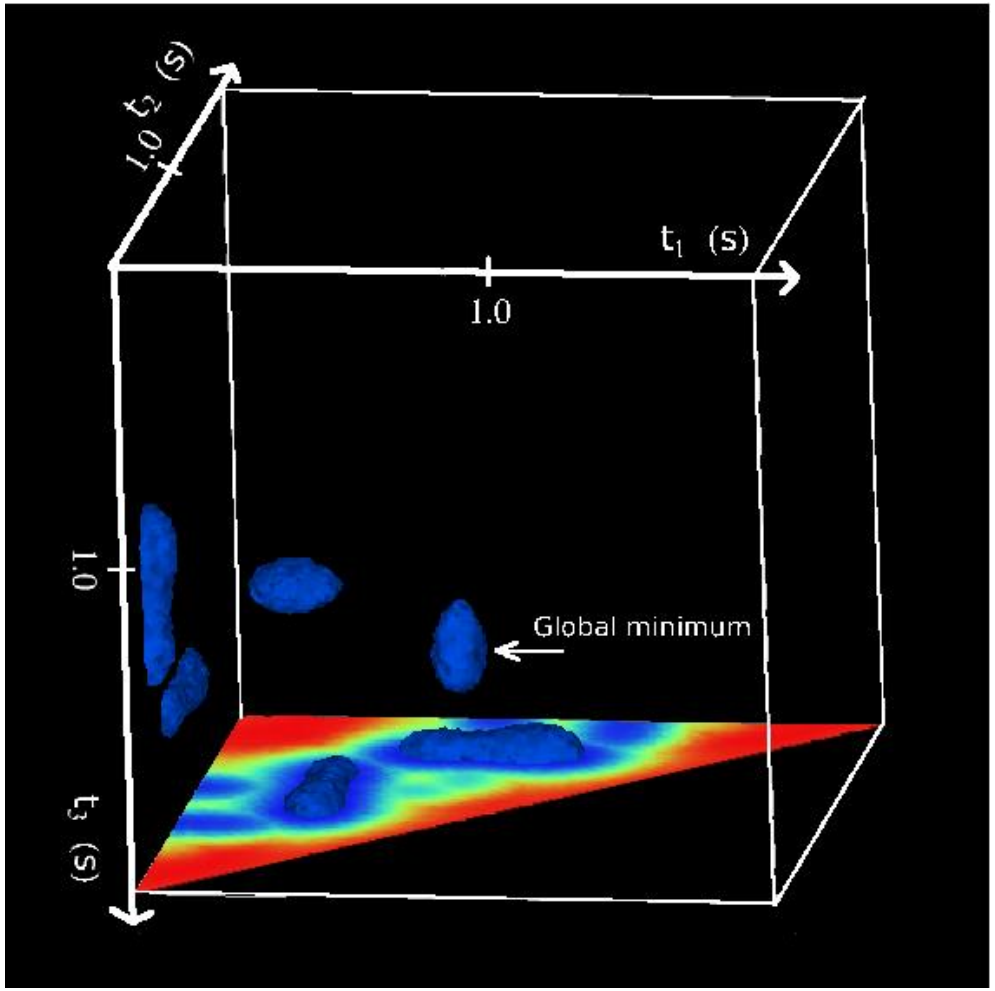
$$\varepsilon(t_1, t_2) = \sum_{n=1}^{n=N} (|J_n(t_1, t_2)| - I_n)^2$$

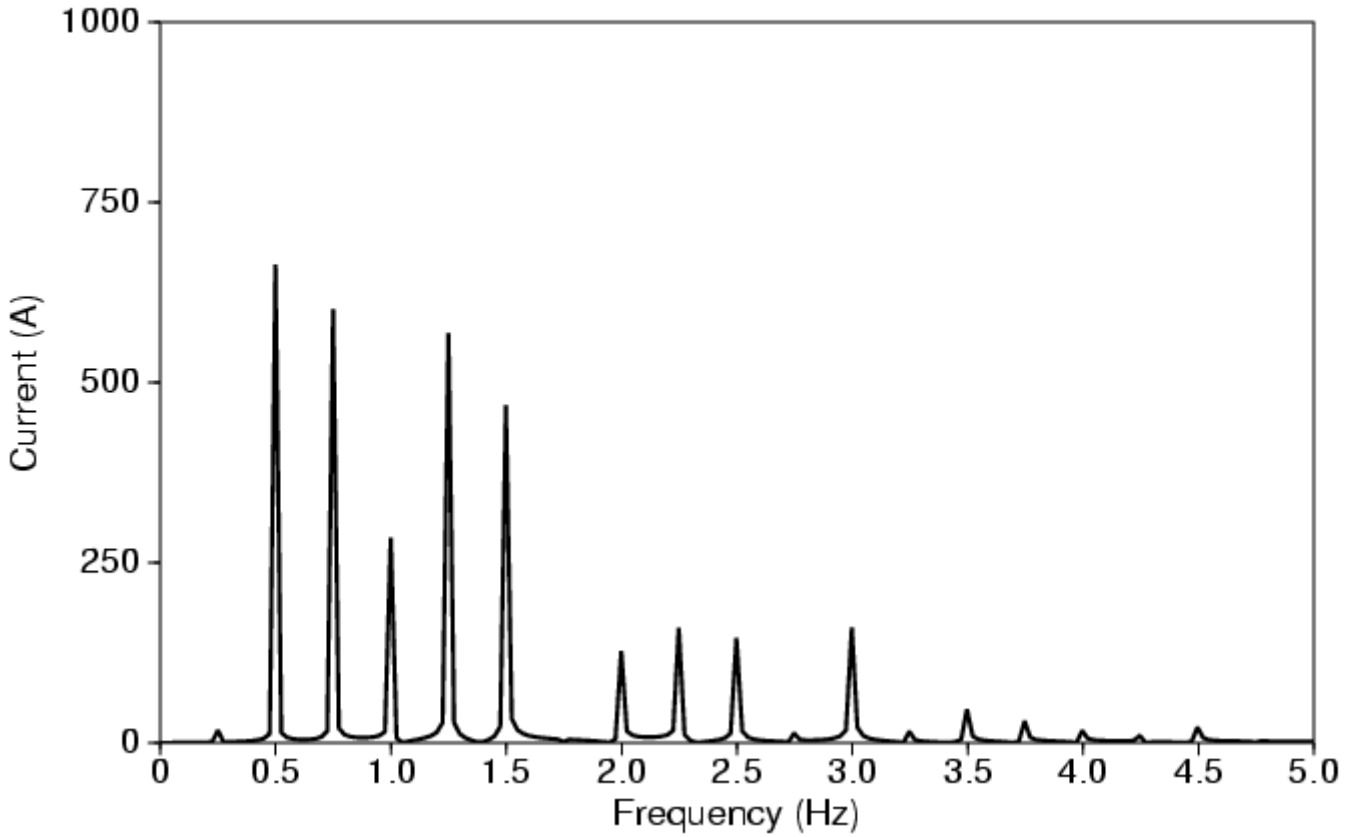




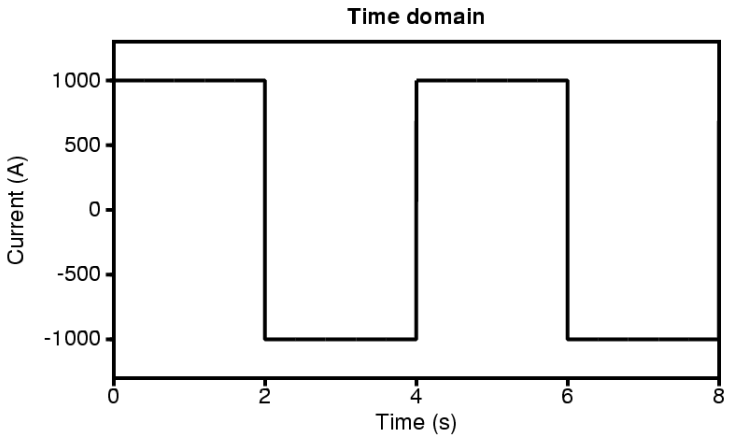
$$\varepsilon(t_1, t_2, t_3) = \sum_{n=1}^{n=N} (|J_n(t_1, t_2, t_3)| - I_n)^2$$

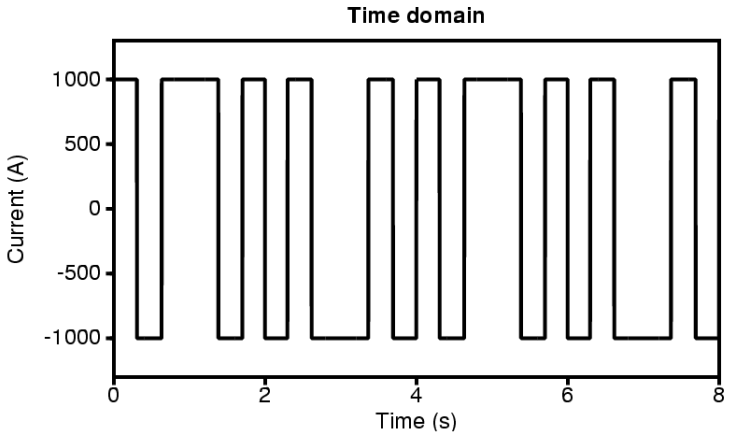


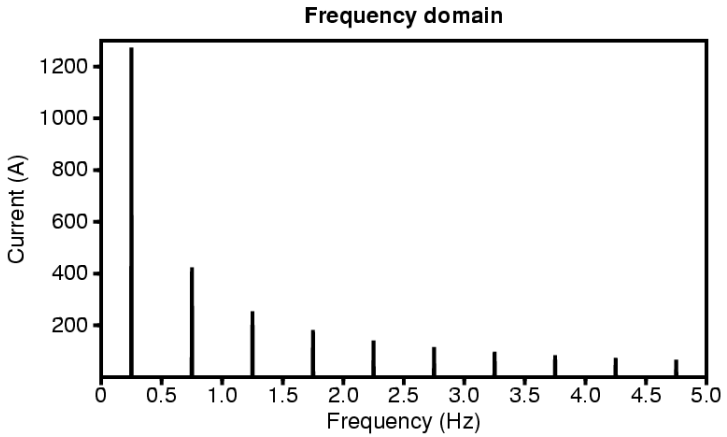


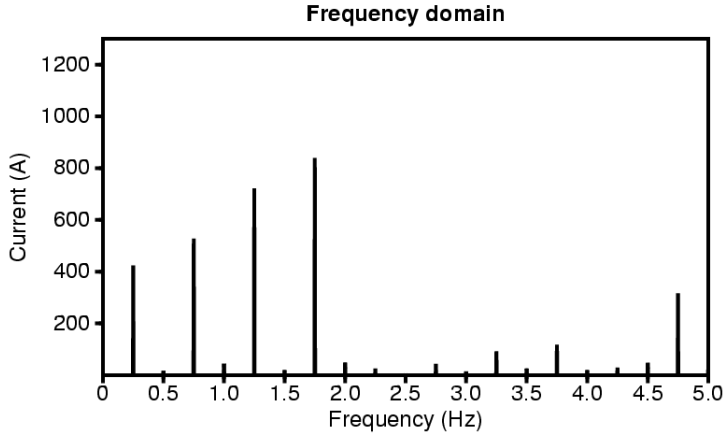

1000
750
500
250
0
Current (A)
0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
Frequency (Hz)

Examples


Time domain
1000
500
0
-500
-1000
Current (A)
0
2
4
6
8
Time (s)


Time domain
1000
500
0
-500
-1000
Current (A)
0
2
4
6
8
Time (s)

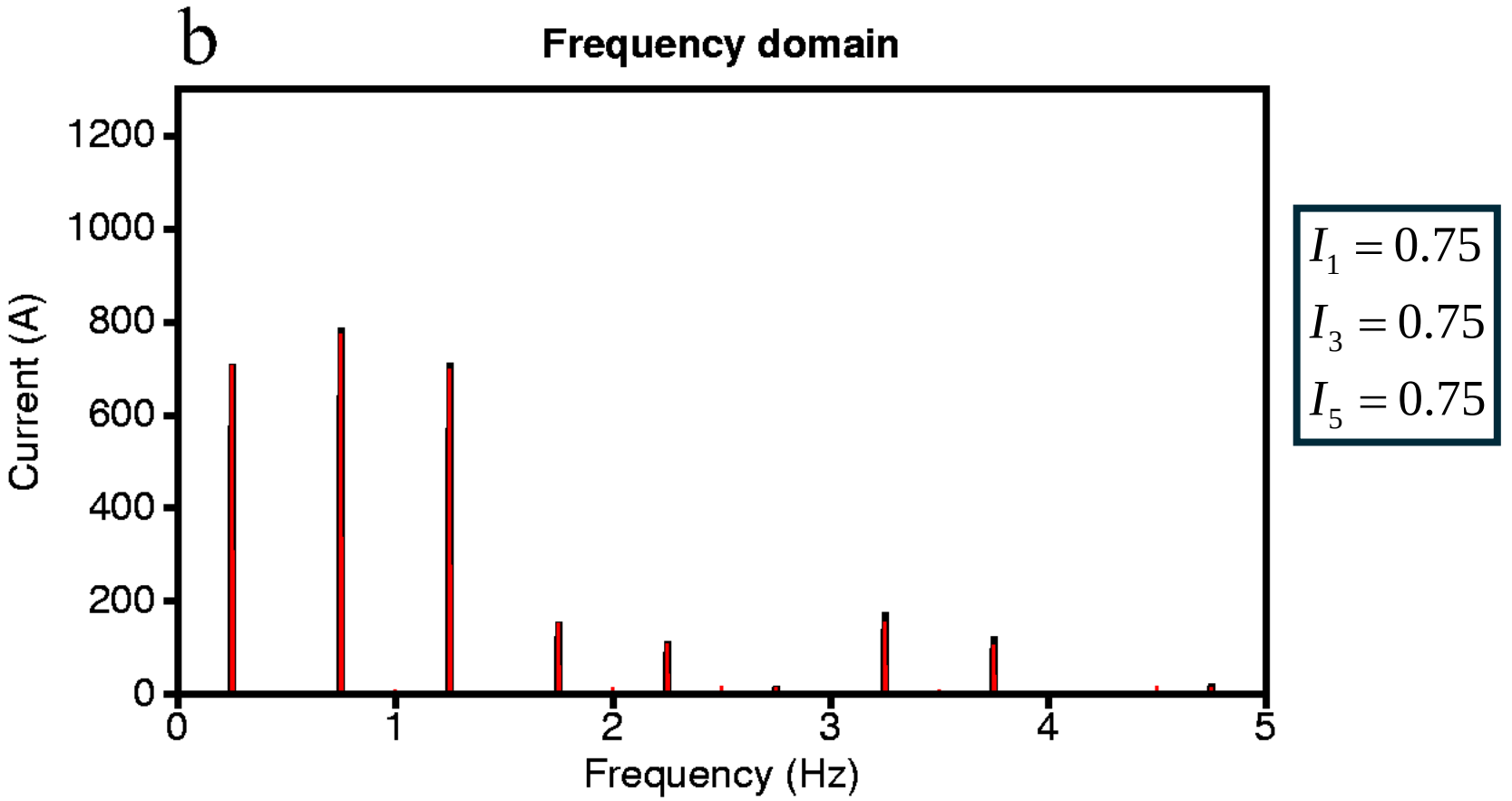

Frequency domain
1200
1000
800
600
400
200
Current (A)
0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
Frequency (Hz)


Frequency domain
1200
1000
800
600
400
200
Current (A)
0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
Frequency (Hz)

Real Data

Spectrum from designed waveform

Spectrum from waveform measured on transmitter

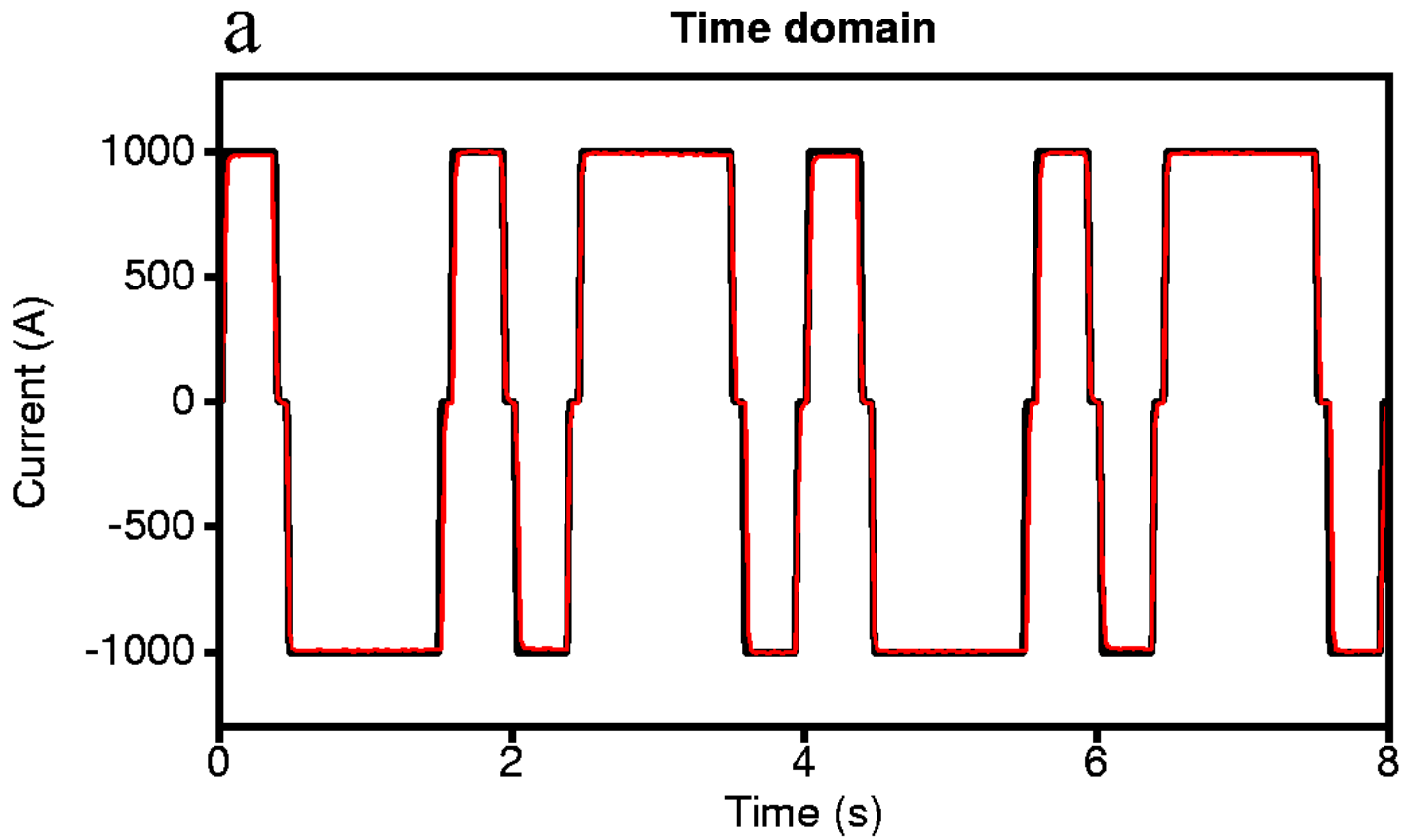


$$I_1 = 0.75$$
$$I_3 = 0.75$$
$$I_5 = 0.75$$

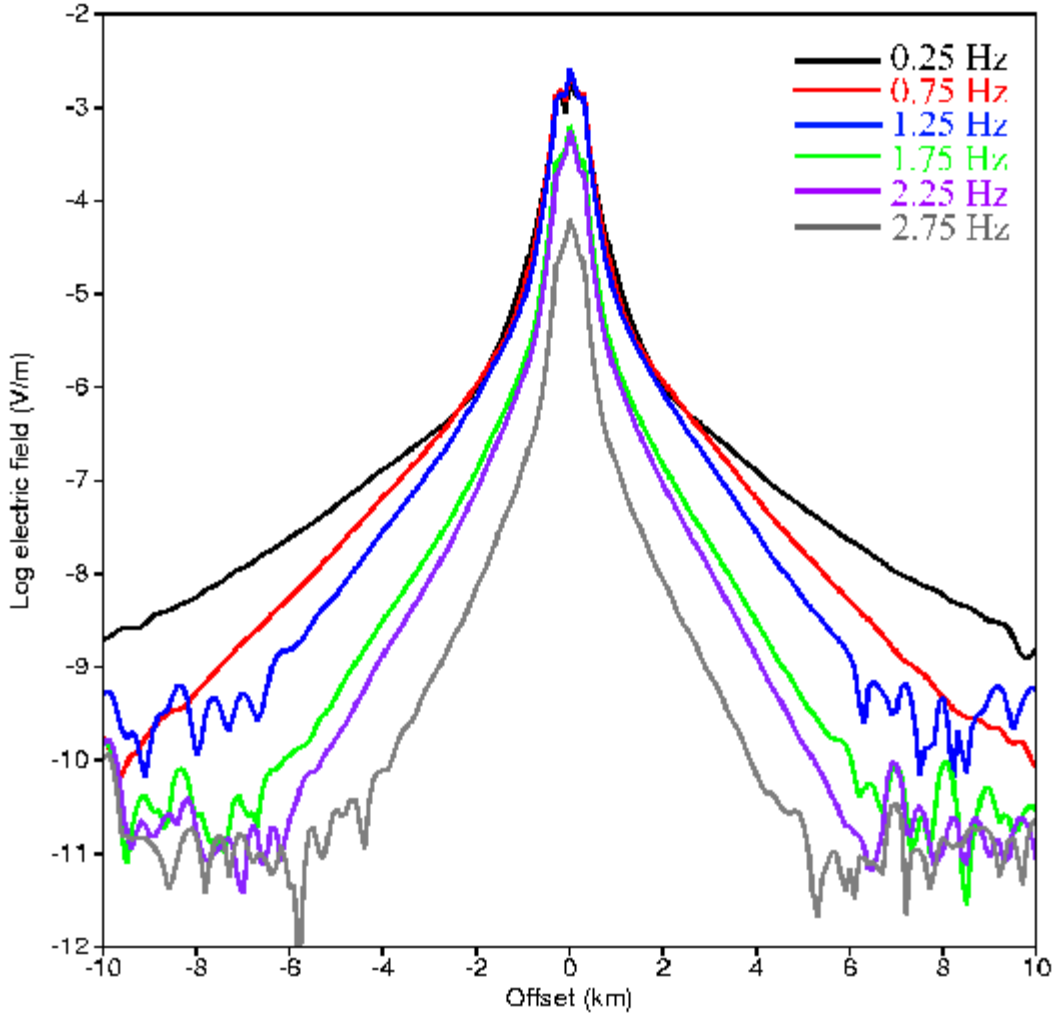
Real Data

Designed waveform

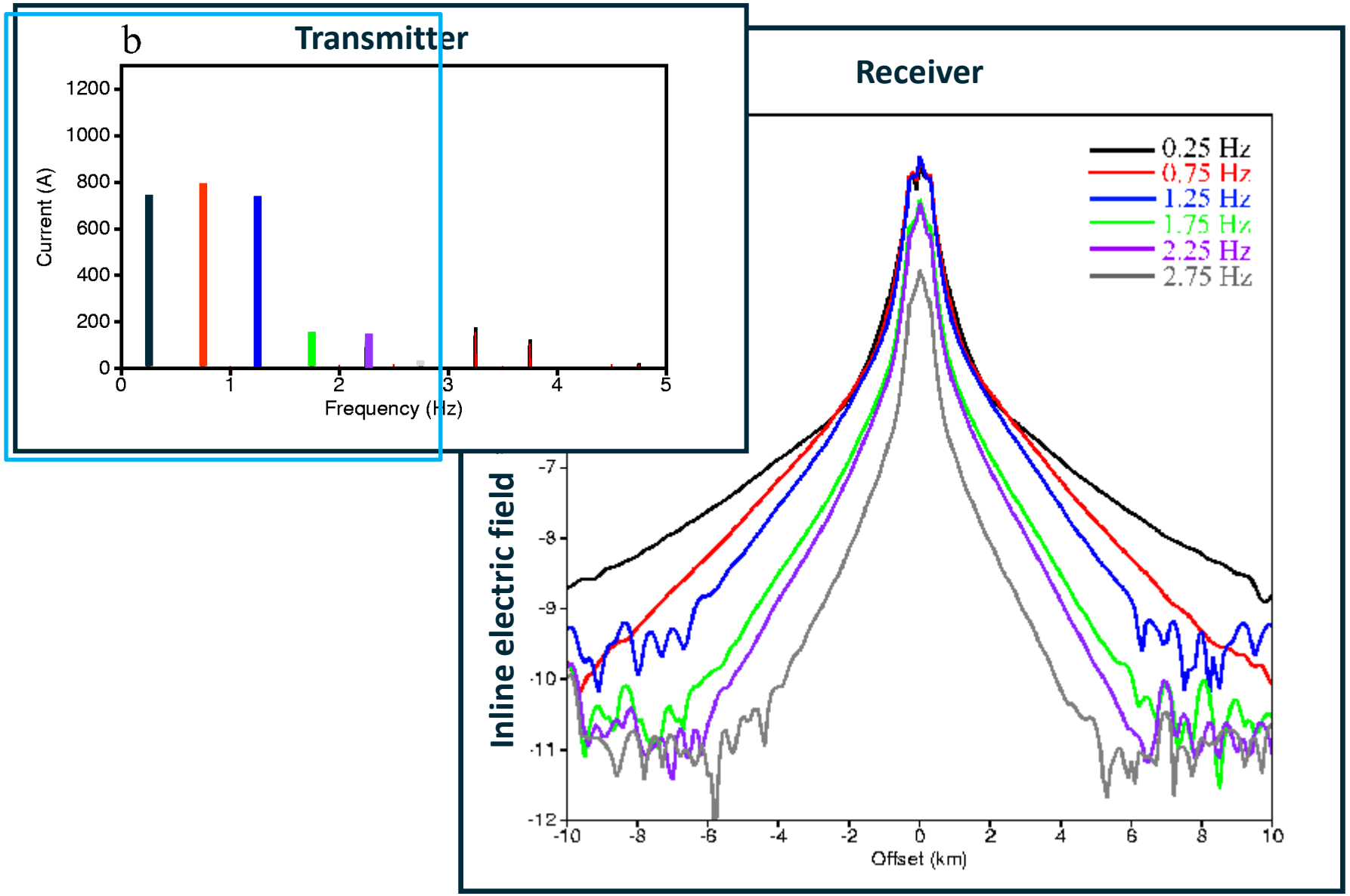
Waveform measured on transmitter



Real Data


0.25 Hz
0.75 Hz
1.25 Hz
1.75 Hz
2.25 Hz
2.75 Hz
Log electric field (V/m)
-2
-3
-4
-5
-6
-7
-8
-9
-10
-11
-12
-10
-8
-6
-4
-2
0
2
4
6
8
10
Offset (km)

Real Data


b
Transmitter
Current (A)
Frequency (Hz)
Receiver
0.25 Hz
0.75 Hz
1.25 Hz
1.75 Hz
2.25 Hz
2.75 Hz
Inline electric field
Offset (km)

emgs

SPOT THE DIFFERENCE.

Thank you