LECTURE 2

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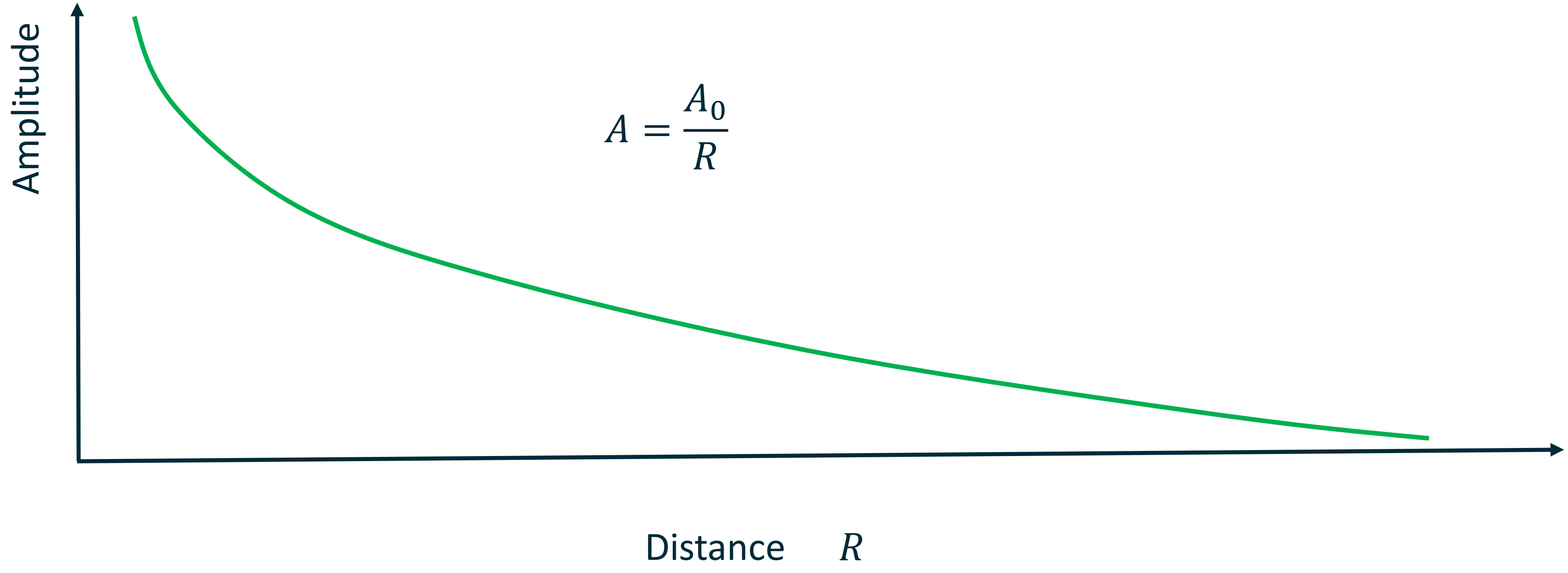
Spot the difference.

Data representation - Amplitude and phase
Preprocessing

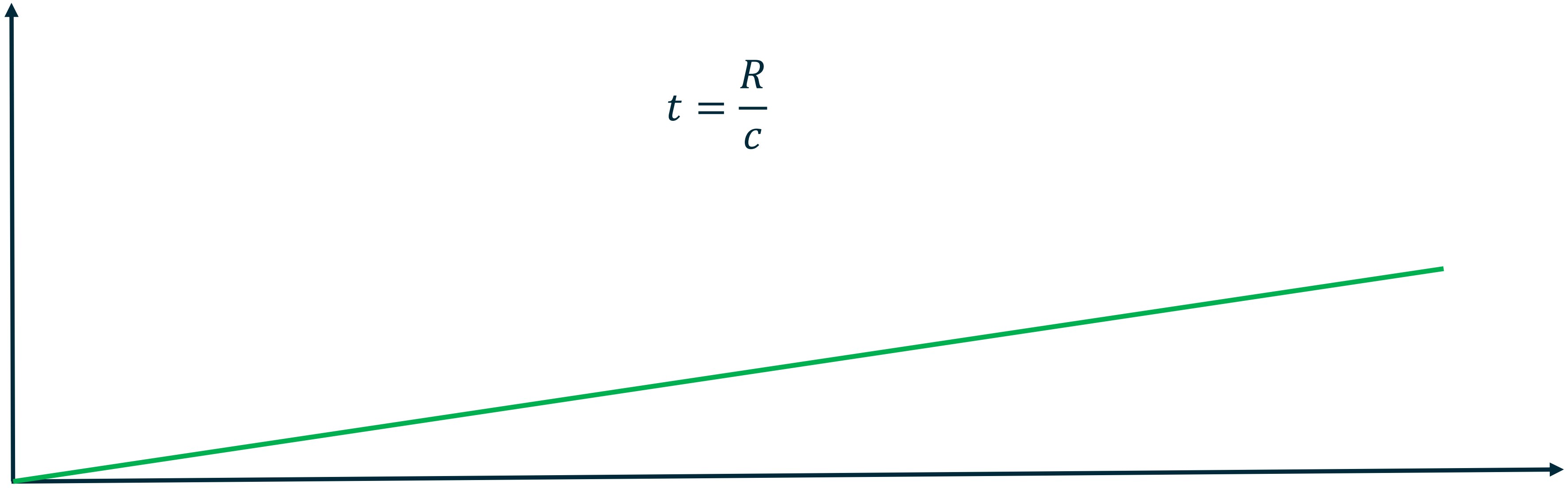
Data representation
Amplitude and phase

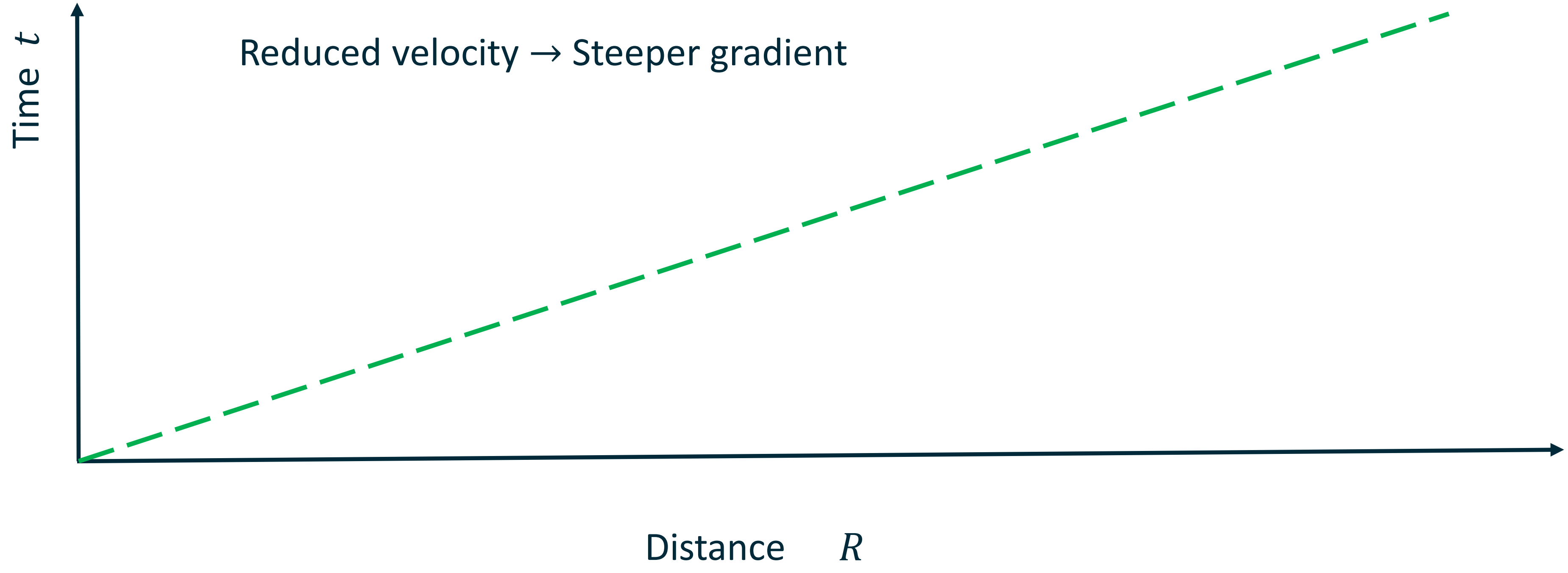


Amplitude
$A = \frac{A_0}{R}$
Distance R

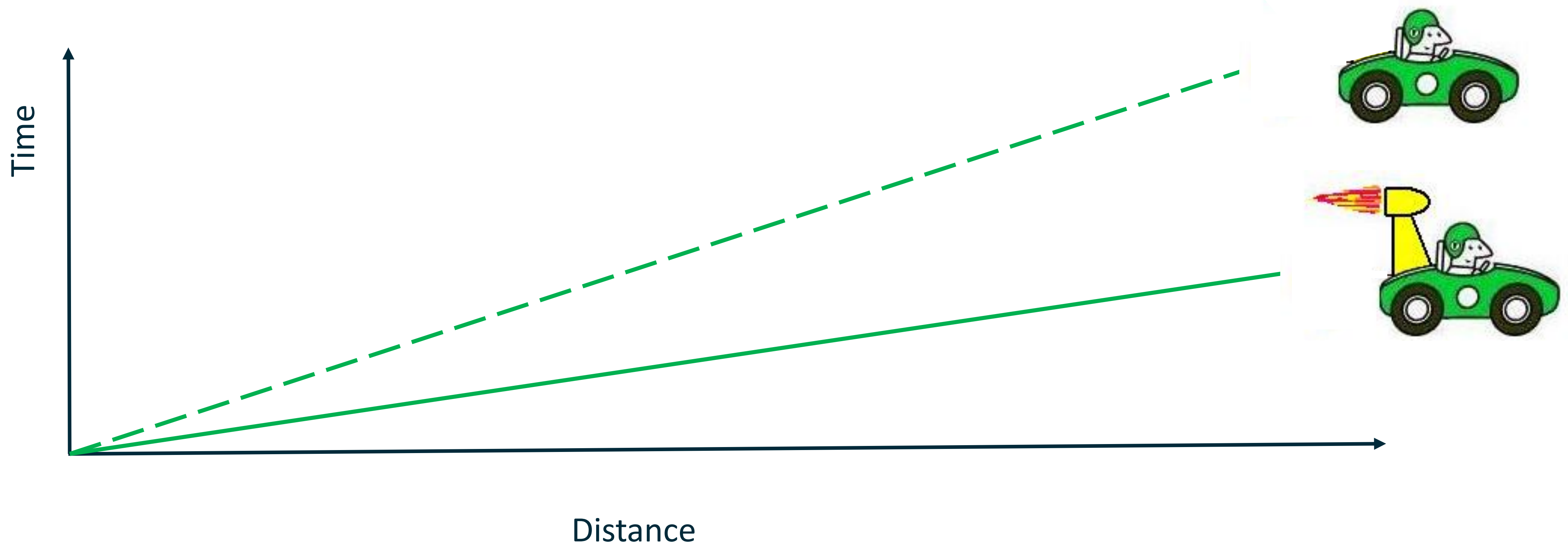
$$R = ct$$

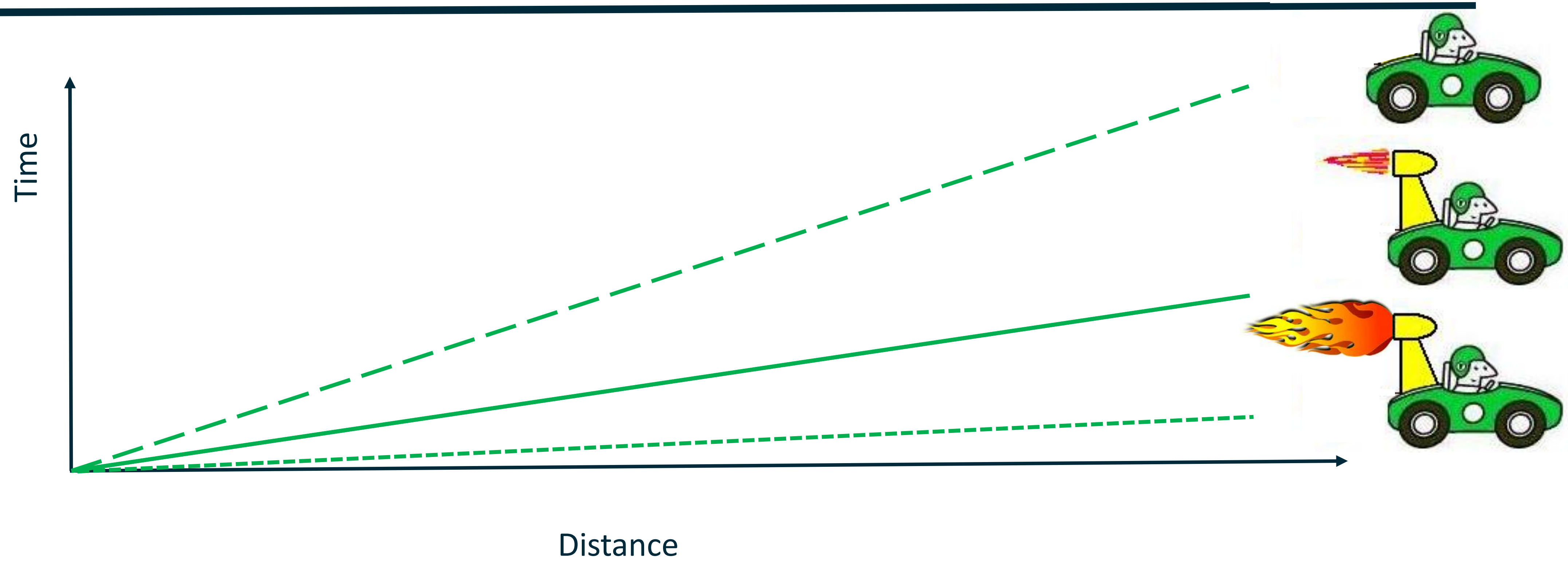
$$t = \frac{R}{c}$$



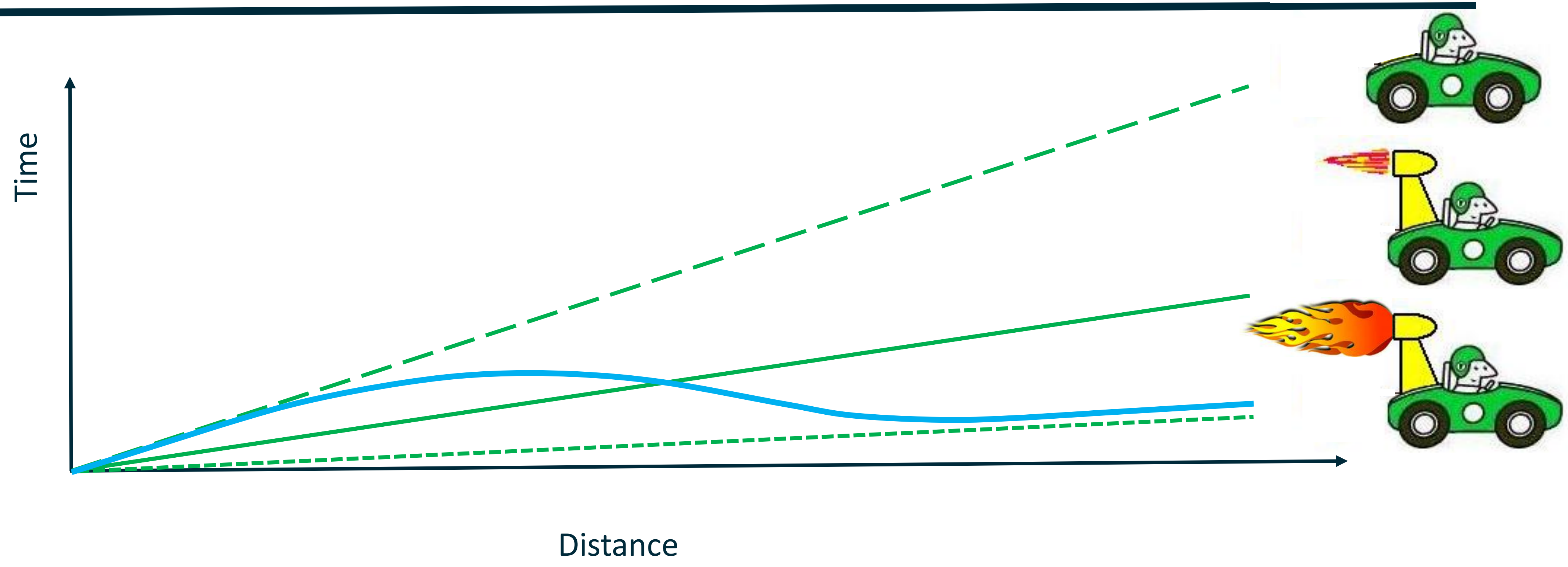


Reduced velocity → Steeper gradient
Time t
Distance R


Time
Distance



Time
Distance


Time
Distance

Fourier transforms

Contineous:

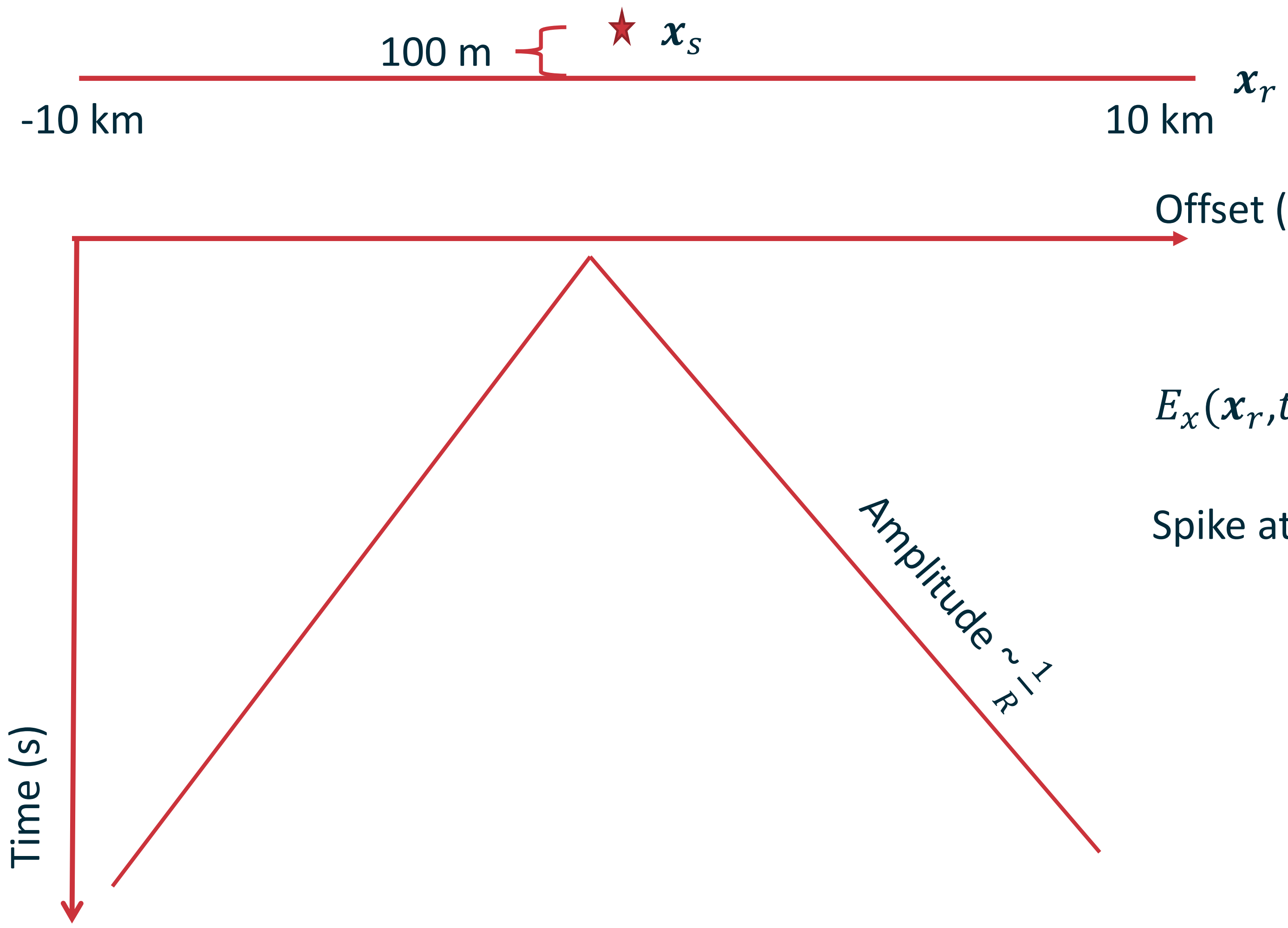
$$F(\omega) = \int_{-\infty}^{\infty} dt\, F(t) e^{i\omega t} \qquad F(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega\, F(\omega) e^{-i\omega t}$$

Suppose electric field behaves as broadband wave:

$$E_x(\boldsymbol{x}_r, t \,|\, \boldsymbol{x}_s) = \frac{\delta\left(t - \frac{|x_r - x_s|}{c}\right)}{4\,\pi\,|\boldsymbol{x}_r - \boldsymbol{x}_s|} = \frac{\delta(t - \frac{R}{c})}{4\,\pi\,R} \qquad R = |\boldsymbol{x}_r - \boldsymbol{x}_s|$$

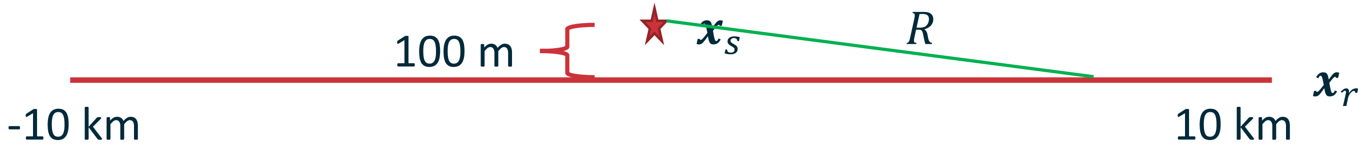
One of many representations of the Dirac delta distribution:

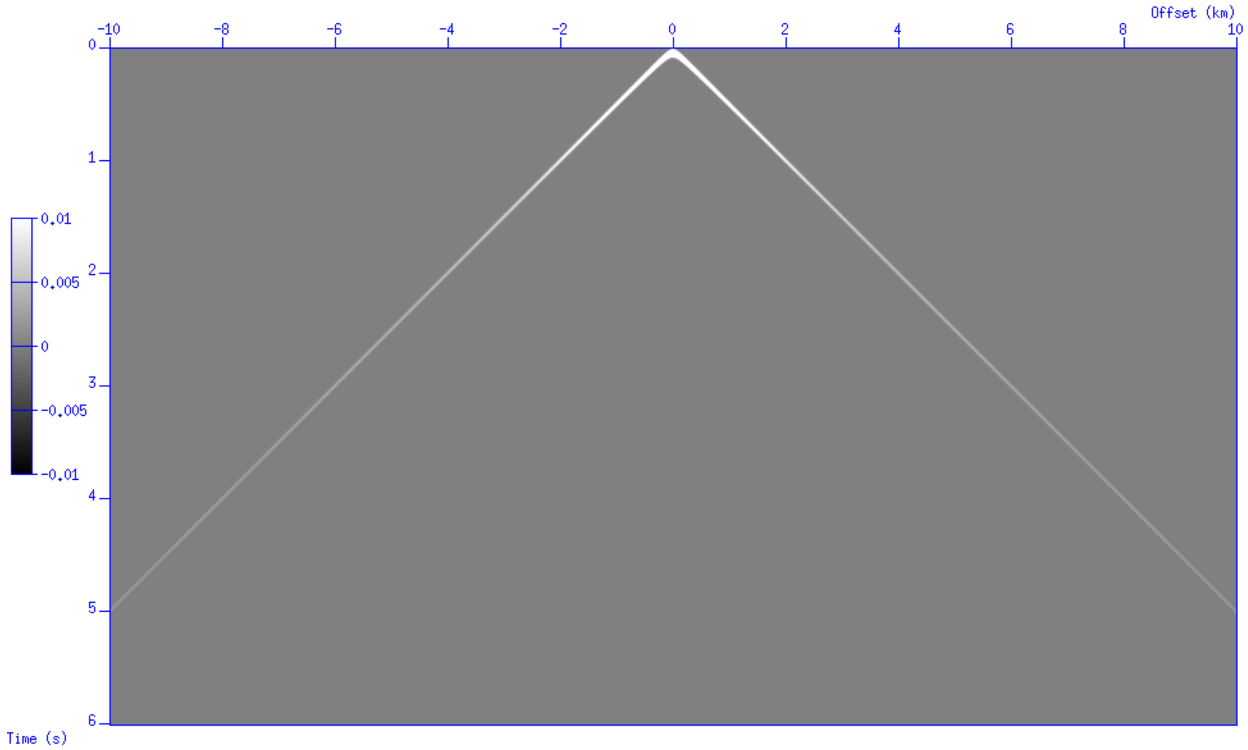
$$\delta(t - \tau) = \frac{1}{\sqrt{\pi}} \, lim_{\varepsilon \to 0} \, \frac{1}{\sqrt{\varepsilon}} e^{-\frac{(t-\tau)^2}{\varepsilon}}$$



$$E_x(\boldsymbol{x}_r, t \,|\, \boldsymbol{x}_s) = \frac{\delta(t - \frac{R}{c})}{4\,\pi\,R}$$

Spike at: $R = c\,t$





$$E_x(\boldsymbol{x}_r, t \,|\, \boldsymbol{x}_s) = \frac{\delta(t - \frac{R}{c})}{4\,\pi\,R}$$

Fourier transforms

Contineous:

$$F(\omega) = \int_{-\infty}^{\infty} dt\, f(t) e^{i\omega t} \qquad f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega\, F(\omega) e^{-i\omega t}$$

$$E_x(R,t) = \frac{\delta(t - \frac{R}{c})}{4\,\pi\,R} \qquad R = |\boldsymbol{x}_r - \boldsymbol{x}_s|$$

Fourier transformed from time to frequency: $E_x(R,\omega) = \frac{e^{i\frac{\omega}{c}R}}{4\,\pi\,R}$

$$E_x(R,\omega) = A(R) e^{i\varphi(R)}$$

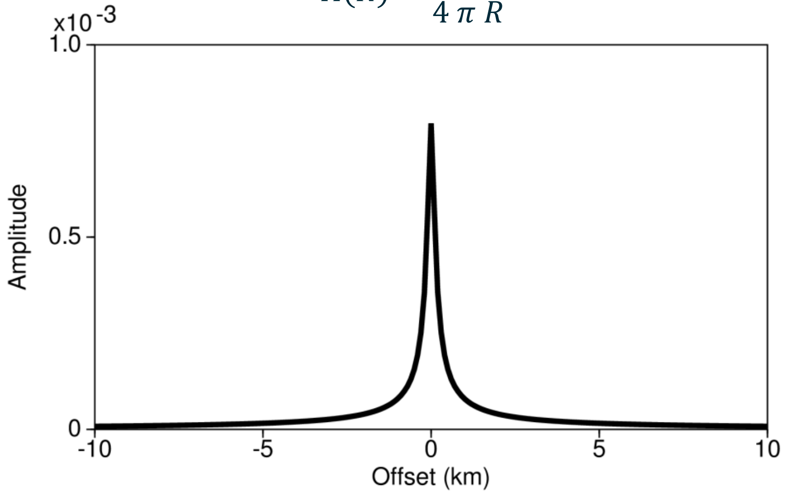
$$A(R) = \frac{1}{4\,\pi\,R} \qquad \varphi(R) = \frac{\omega}{c} R$$

Gradient of phase curve reduced with increased velocity for fixed frequency

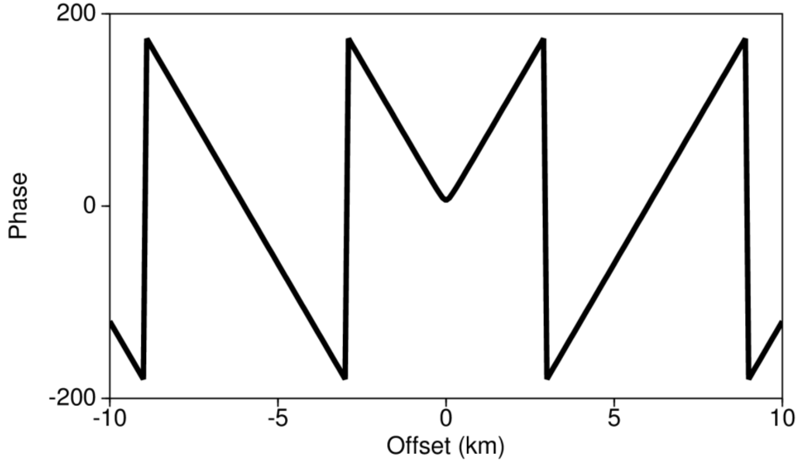
$$E_x(R,\omega) = \frac{e^{i\frac{\omega}{c}R}}{4\,\pi\,R}$$

$$E_x(R,\omega) = A(R)e^{i\varphi(R)}$$

$$A(R) = \frac{1}{4\,\pi\,R}$$



$$\varphi(R) = \frac{\omega}{c}R$$



Tougth experiment: What if strong absorption present?

$$E_x(R,\omega) = \frac{e^{i\frac{\omega}{c}R}}{4\,\pi\,R} \quad \Rightarrow \quad E_x(R,\omega) = \frac{e^{-\frac{\omega}{c}R}\; e^{i\frac{\omega}{c}R}}{4\,\pi\,R}$$

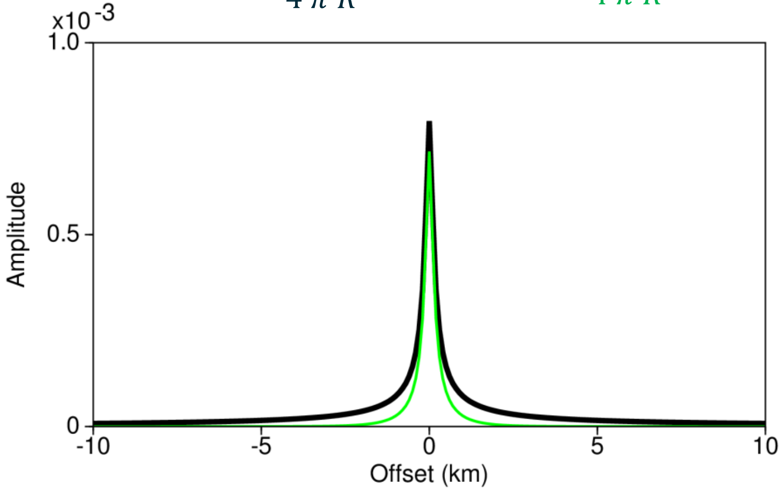
$$E_x(R,\omega) = A(R)e^{i\varphi(R)} \qquad A(R) = \frac{e^{-\frac{\omega}{c}R}}{4\,\pi\,R} \qquad \varphi(R) = \frac{\omega}{c}R$$

Skin depth (amplitude drop by factor $1/e$): $\delta = \frac{c}{\omega}$

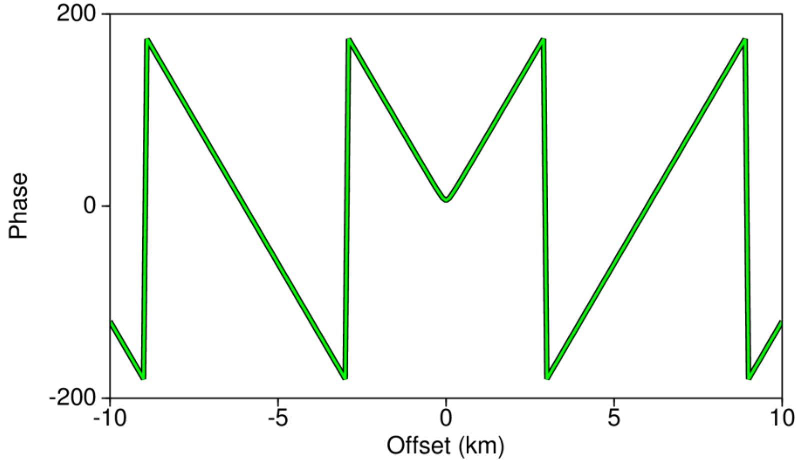
(Neglected effect of geometrical spreading for simplicity)

$$A(R) = \frac{e^{-\frac{R}{\delta}}}{4\,\pi\,R}$$

$A(R) = \frac{1}{4\,\pi\,R}$ $\qquad$ $A(R) = \frac{e^{-\frac{R}{\delta}}}{4\,\pi\,R}$



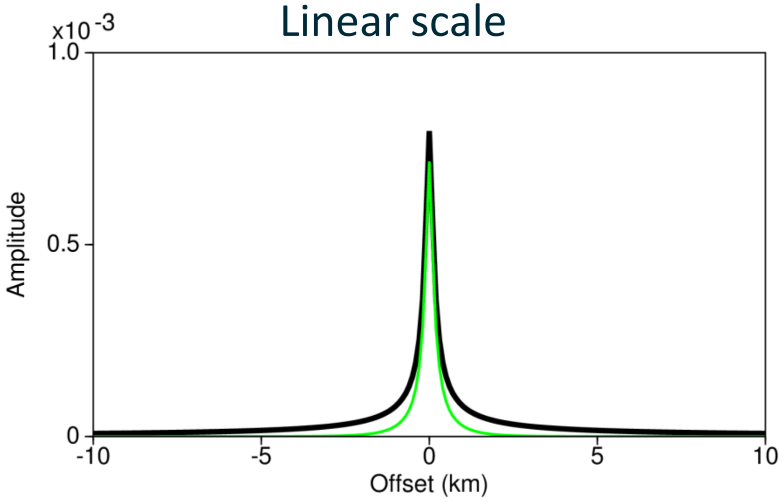
$\varphi(R) = \frac{\omega}{c}R$



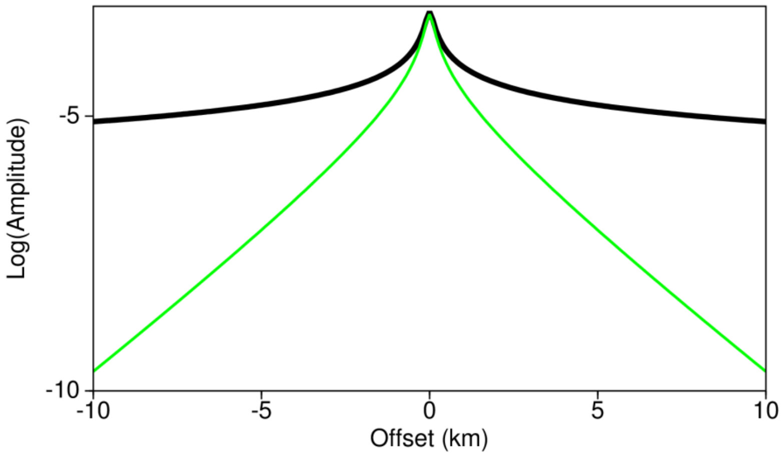
Marine CSEM: Log scale is used for data plots

Strong absorption have the effect that amplitudes drop by several orders of magnitude over a 10 km offset range

Linear scale



Log scale

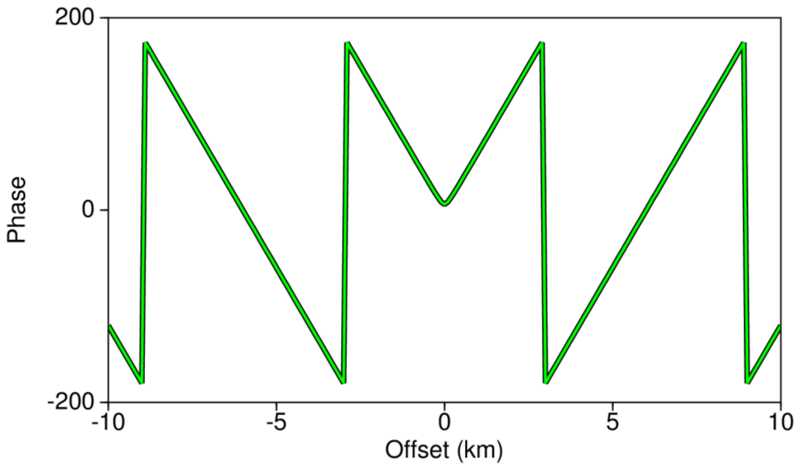


Phase is normally **not** unwrapped when plotting

Phase is normally extracted with the «atan2» function

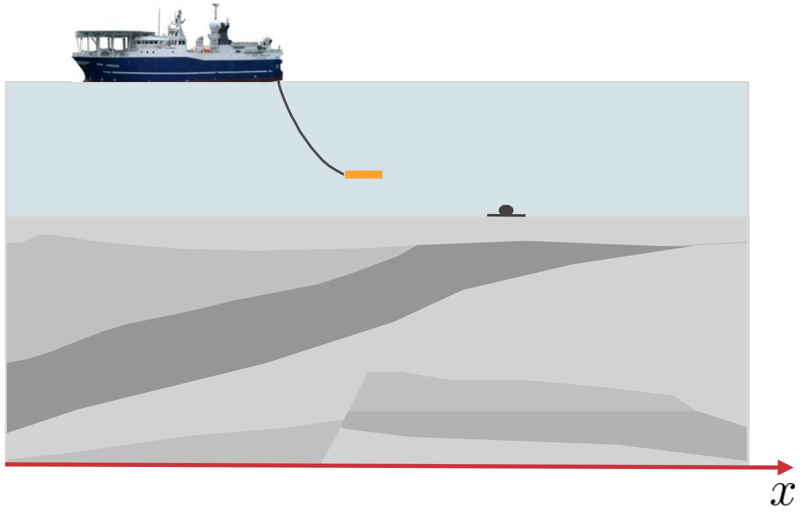
Phase is on the interval $[-\pi,\pi]$ in radians
or on the interval [-180,180] in degrees

A phase function with increasing offset x will grow from initial value to 180 degrees, drop to -180 degrees before reaching 180 degrees again



Note: Phase behavior versus offset is more complicated for CSEM data.

What does typical CSEM data look like?



CSEM data is acquired in the time-domain and transformed into the frequency-domain

In the frequency domain, each data point is a complex number consisting of a magnitude and a phase.

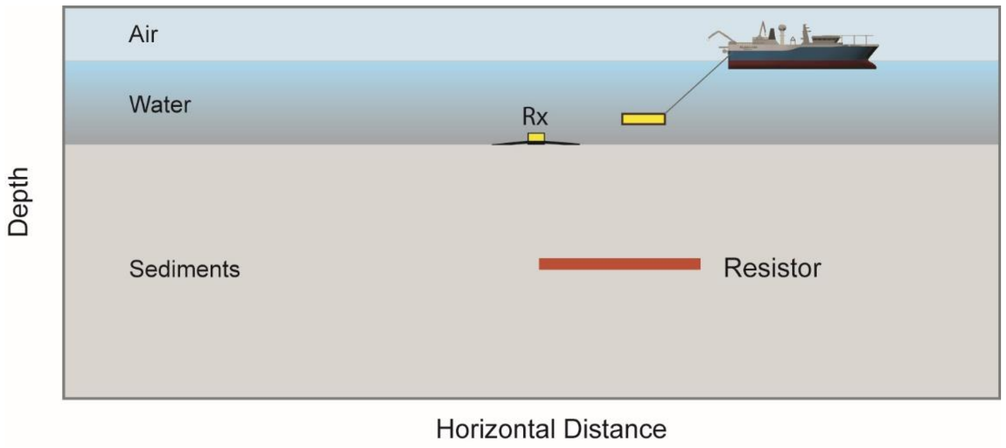
Data from a given receiver is presented as MvO and PvO curves displaying $|E_x|(x), \quad \varphi(x)$

MvO and PvO curves are obtained for each frequency and each electric and magnetic field component.

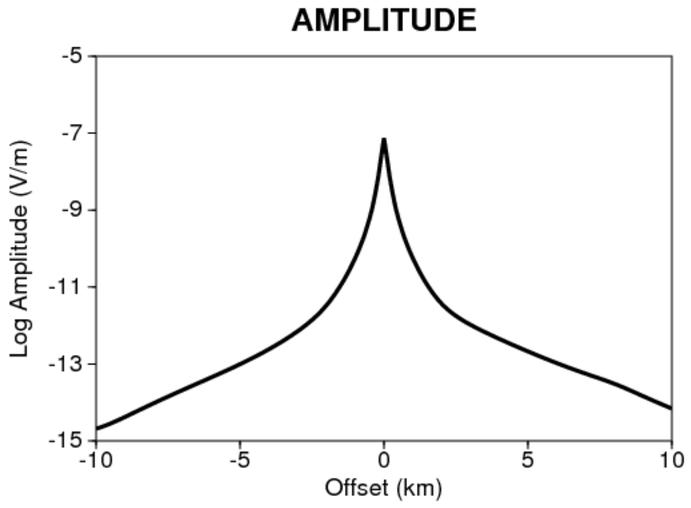
$$\underbrace{E_x(x, t)}_{\text{time-domain}} \rightarrow \underbrace{E_x(x, f)}_{\text{frequency-domain}} = |E_x| \, e^{i\varphi}$$

Magnitude (or amplitude): $|E_x|$

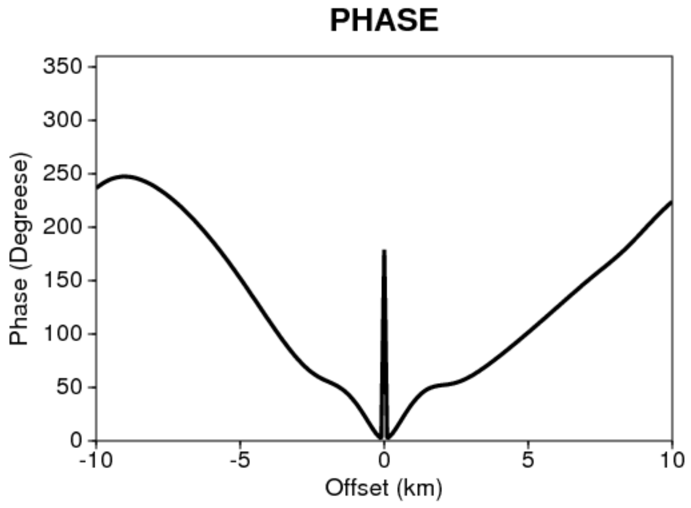
Phase: φ



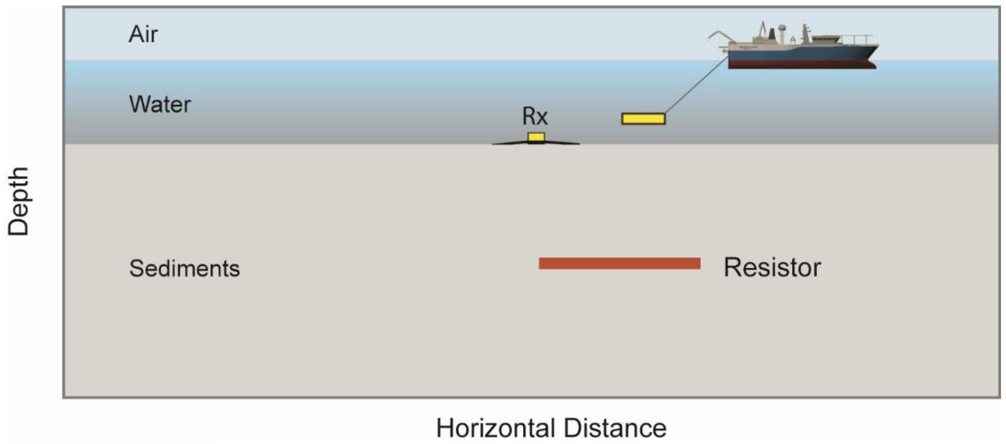
Air
Water
Rx
Depth
Sediments
Resistor
Horizontal Distance

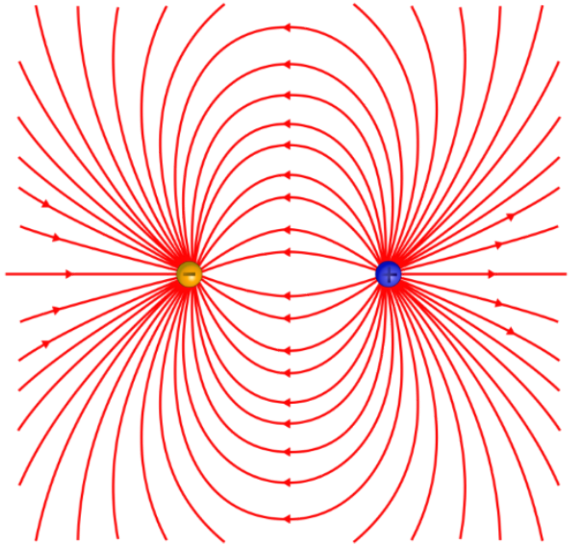


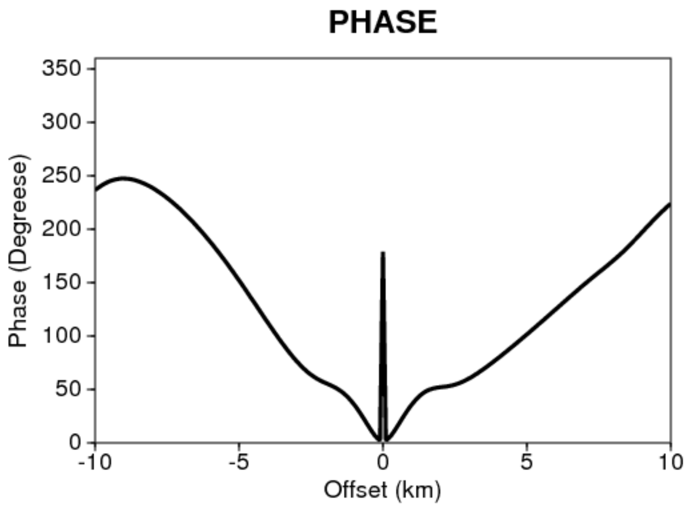
AMPLITUDE
Log Amplitude (V/m)
-5
-7
-9
-11
-13
-15
-10
-5
0
5
10
Offset (km)



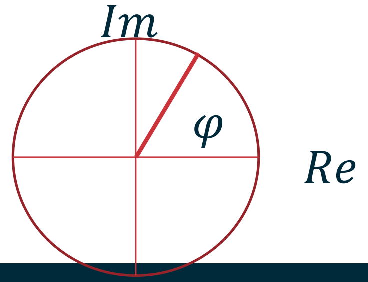
PHASE
Phase (Degreese)
350
300
250
200
150
100
50
0
-10
-5
0
5
10
Offset (km)







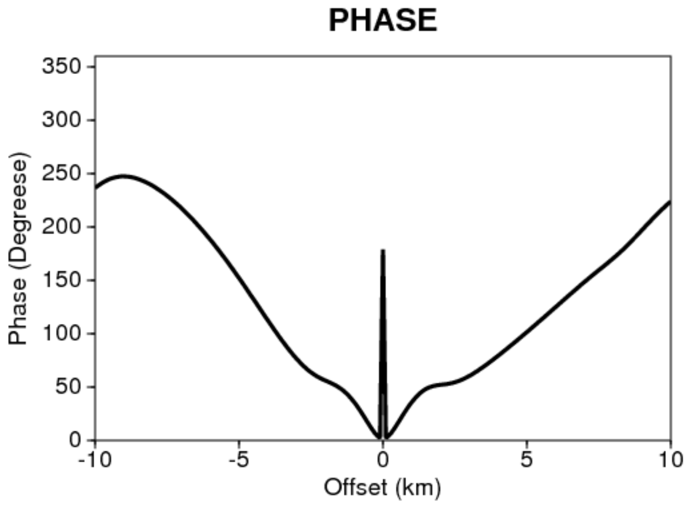
$e^{i\varphi}$



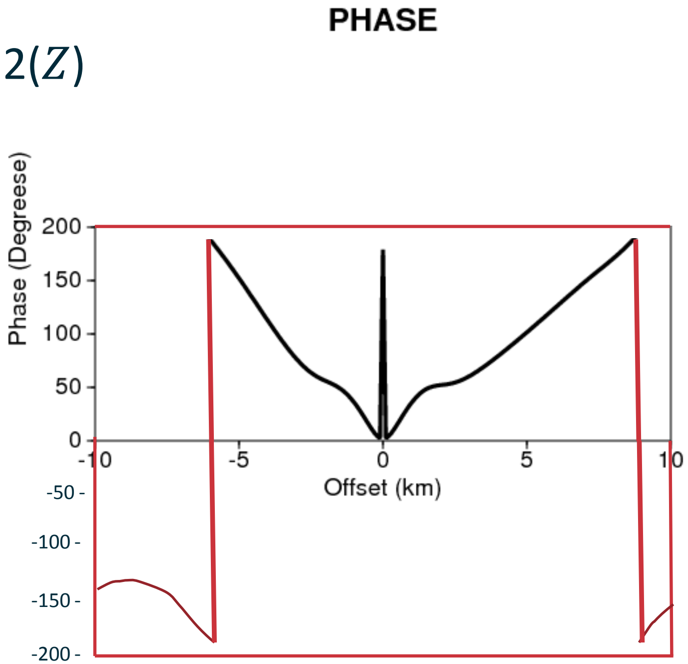
$e^{i0} = 1$ $\quad e^{i\pi} = -1$

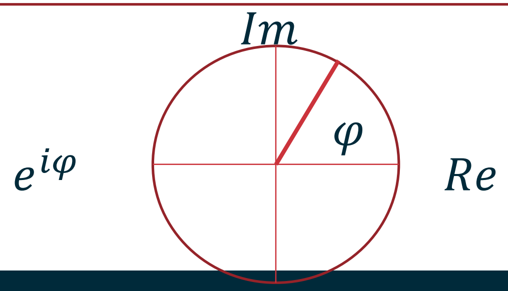
$e^{i\frac{\pi}{2}} = i$ $\quad e^{i2\pi} = 1$

Unwrapped phase



Phase with atan2(Z)





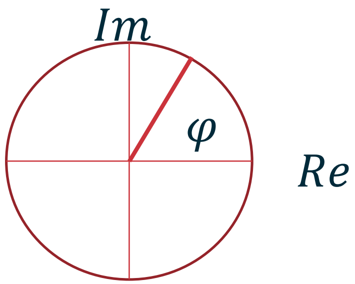
$e^{i0} = 1$ $\quad$ $e^{i\pi} = -1$

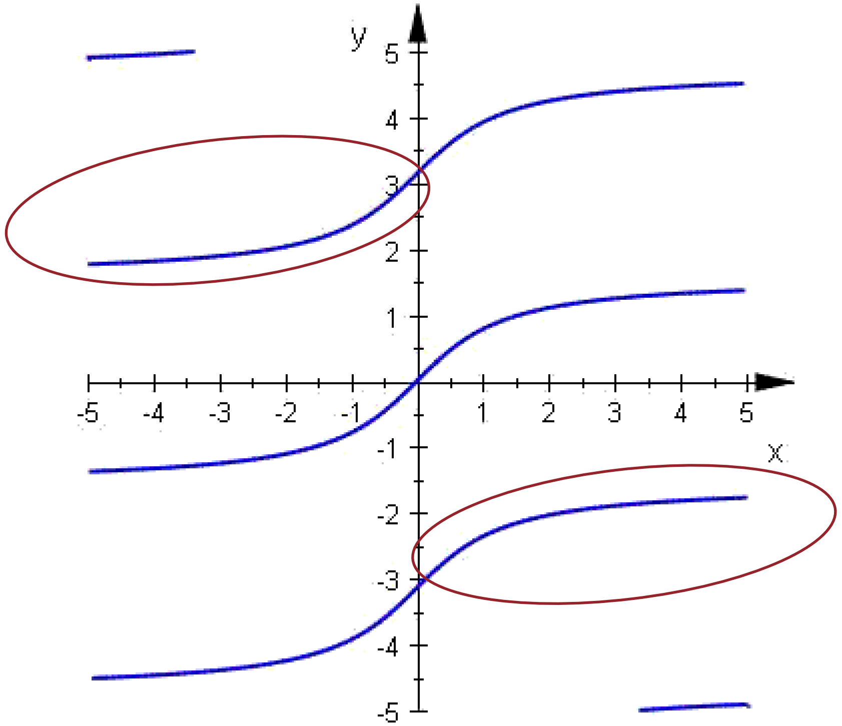
$e^{i\frac{\pi}{2}} = i$ $\quad$ $e^{i2\pi} = 1$

Phase with atan2(Z)

Phase of a complex number Z:

$$tg(\varphi) = \frac{Im(z)}{Re(z)}$$



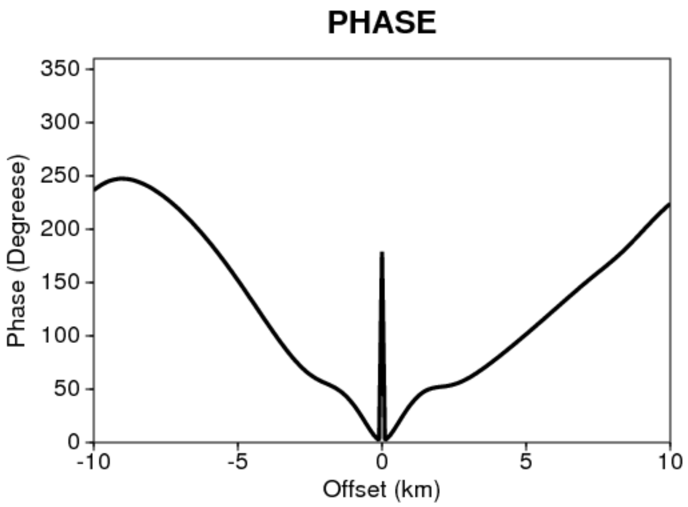


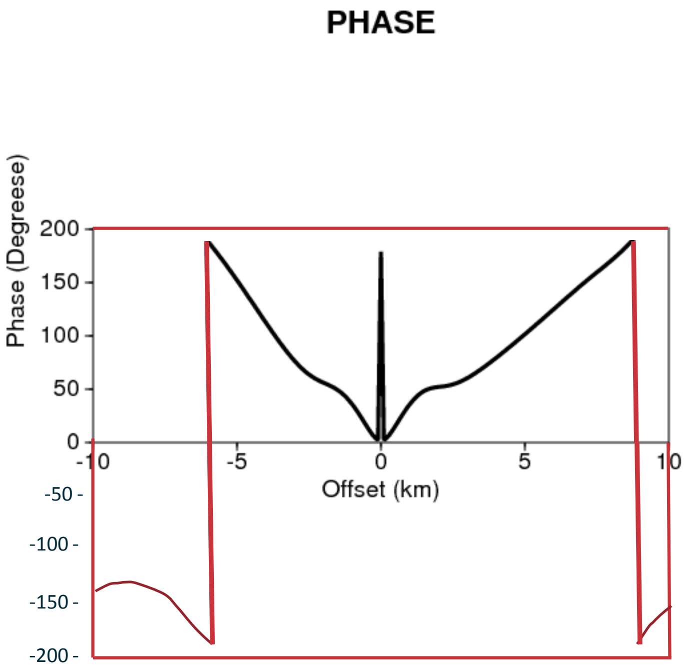
$$\varphi = \arctan(\frac{Im(z)}{Re(z)})$$

$$x = tg(\varphi)$$

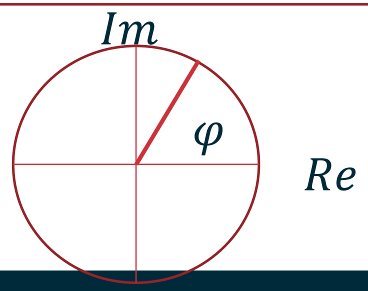
$$\varphi = \arctan(x)$$

$$Z = Ae^{i\varphi} = Ae^{i\varphi}e^{\pm in2\pi} = Ae^{i(\varphi\pm 2\pi n)}$$





$e^{i\varphi}$

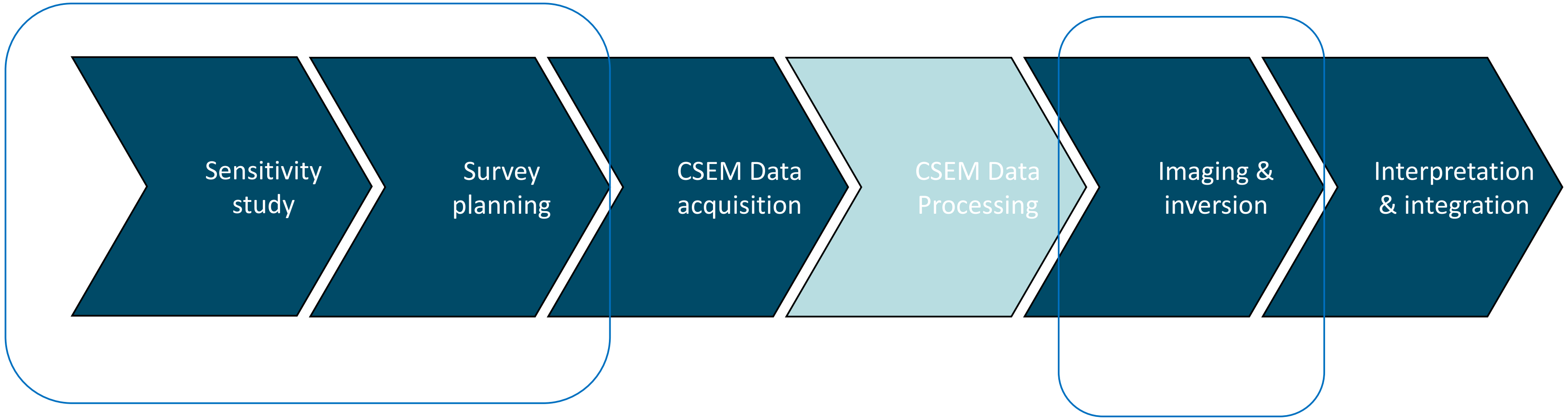


$e^{i0} = 1$ $e^{i\pi} = -1$

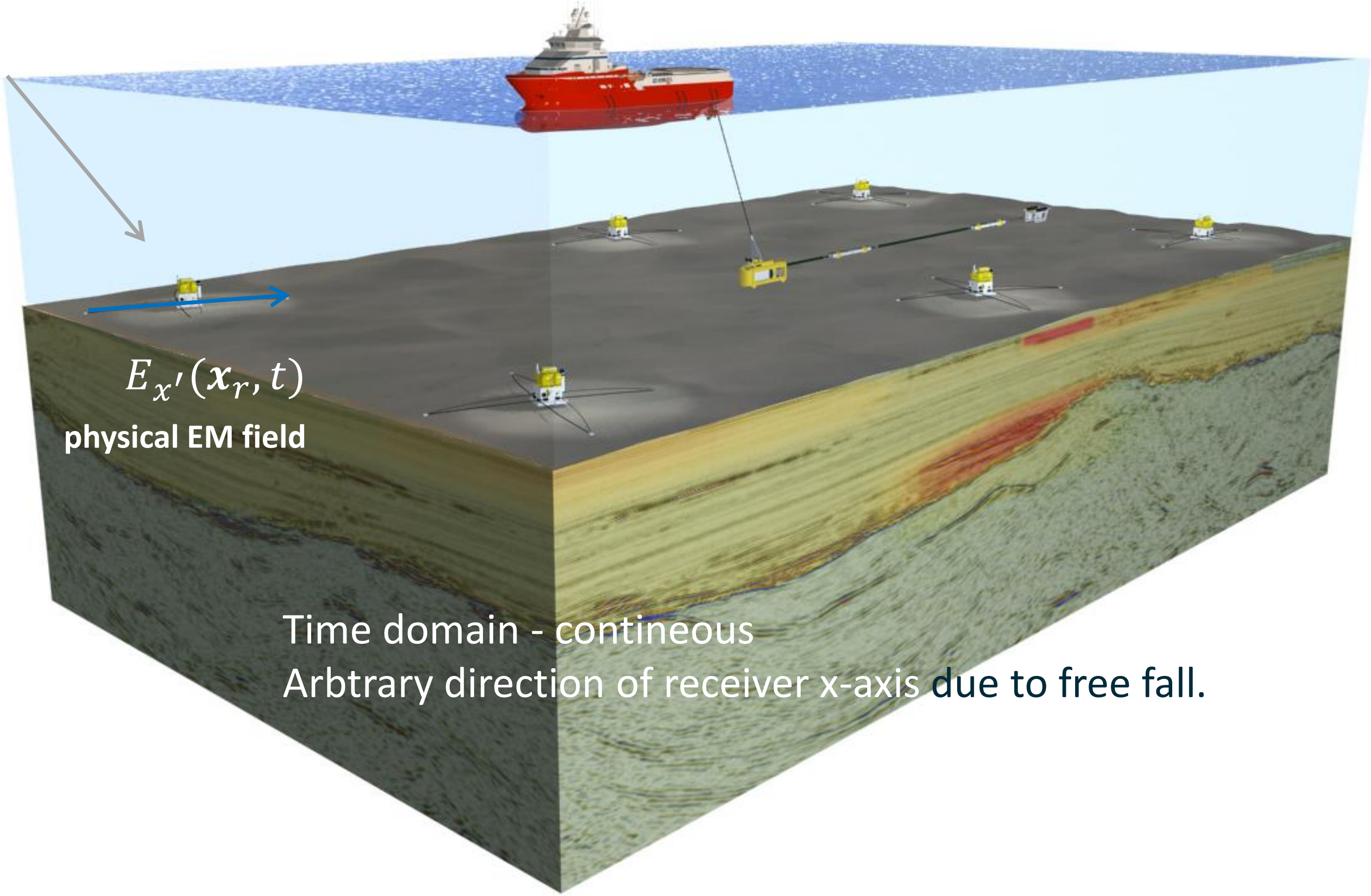
$e^{i\frac{\pi}{2}} = i$ $e^{i2\pi} = 1$

Preprocessing

Typical CSEM Workflow



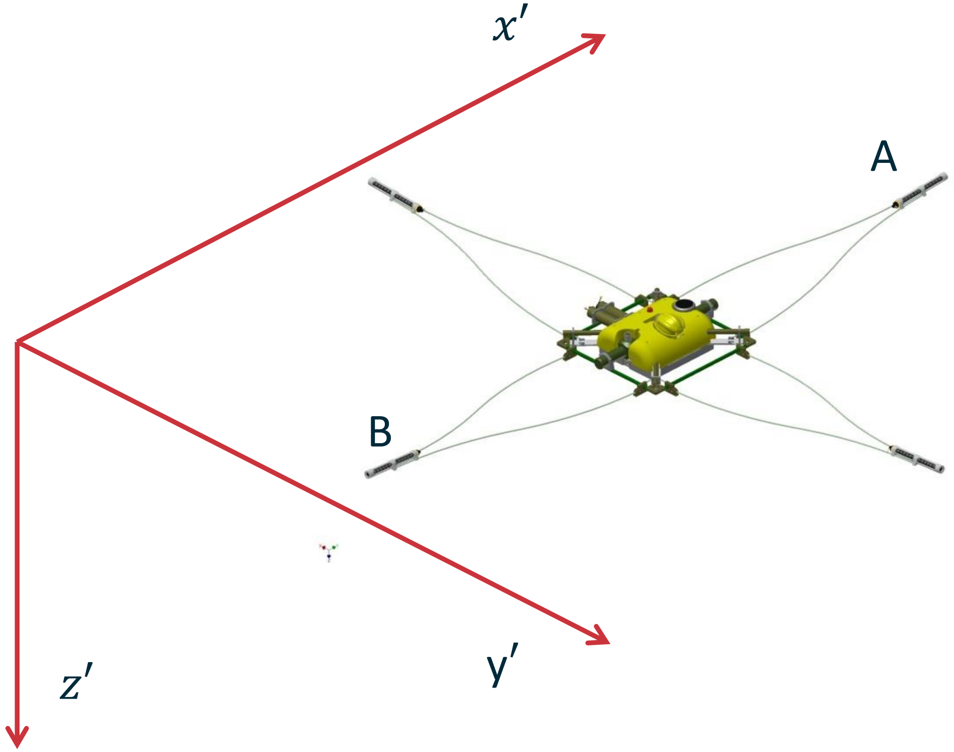
Physical electromagnetic fields



Time domain - contineous
Arbtrary direction of receiver x-axis due to free fall.

Receiver configured in right-handed coordinate system.

Arbitrary x' direction on seabed due to free fall.



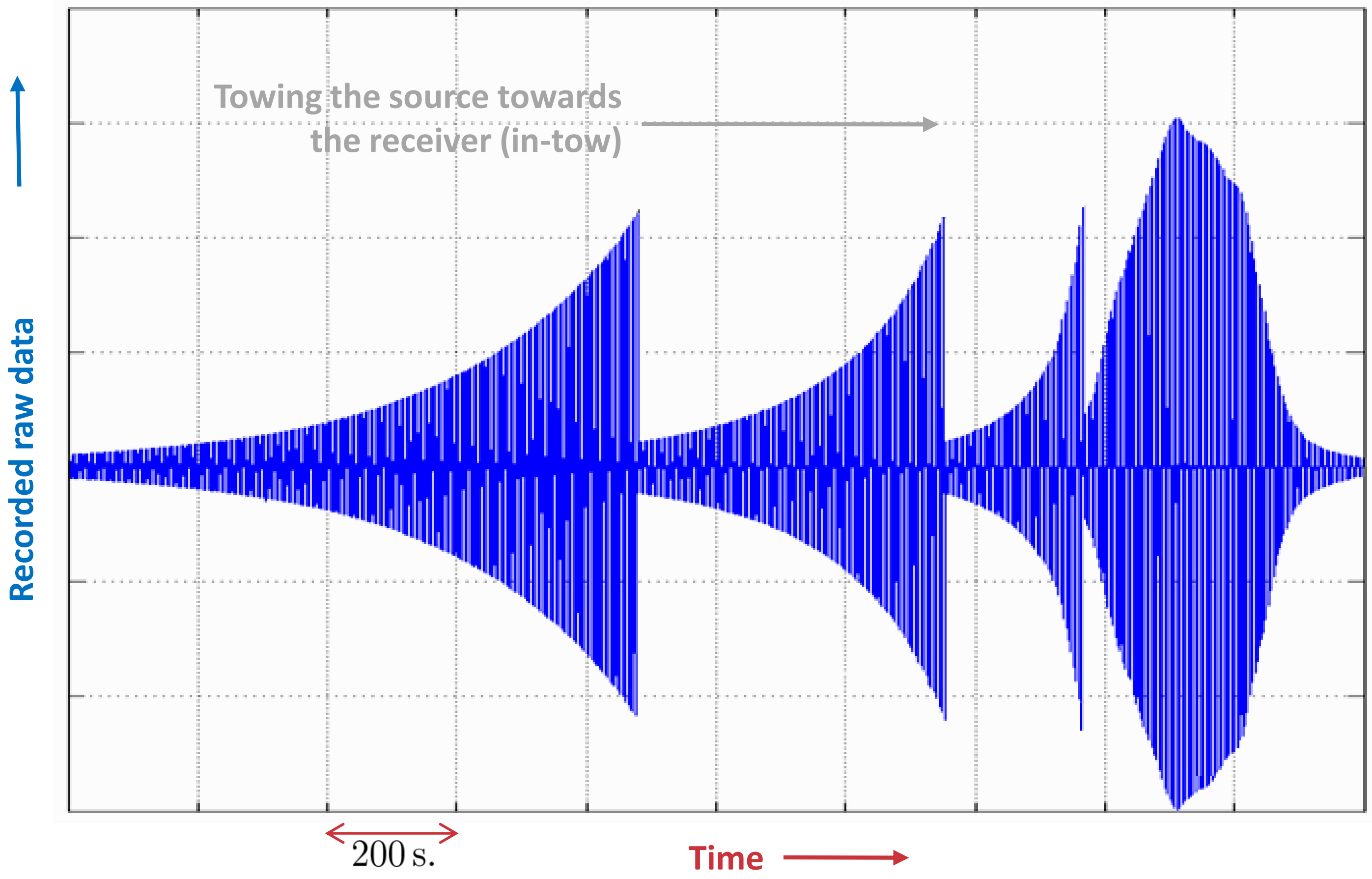
Physical field $E_{x'}(\boldsymbol{x}_r, t)$ measured as voltage over A – B electrode pair

On seabed:

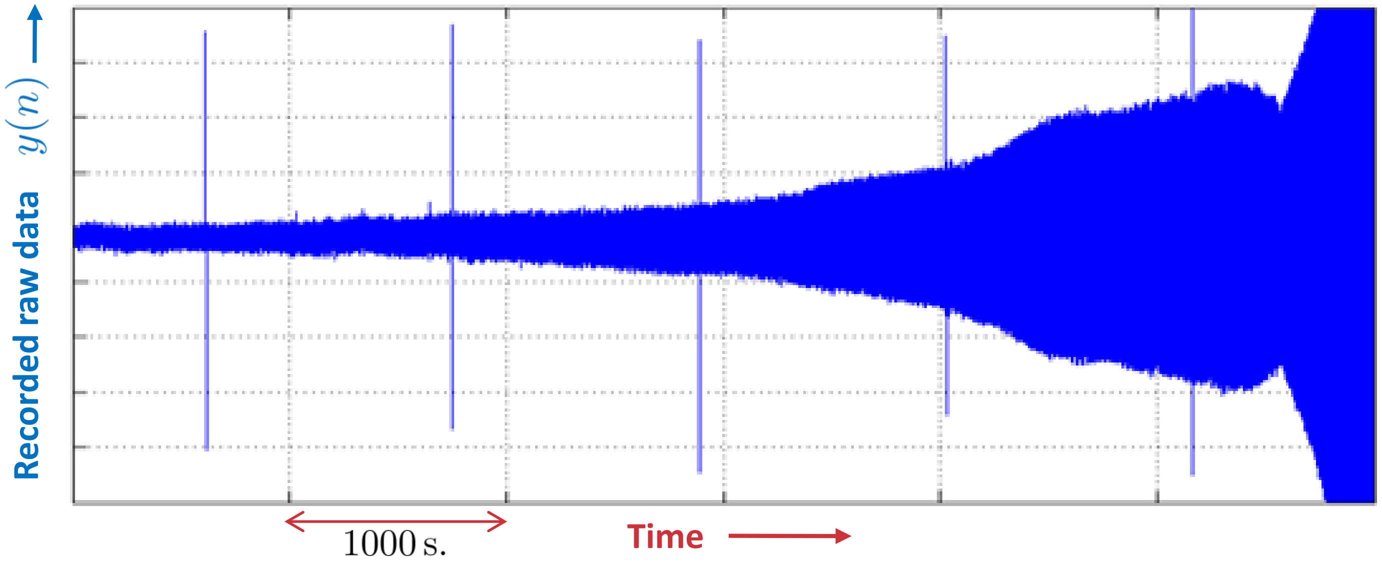
$E_{x'}(\boldsymbol{x}_r, t)$ → Amplifier → ADC → RAM → Flash: $\hat{E}_{x'}(\boldsymbol{x}_r, n\Delta t)$

Automatic gain control

Large dynamic range of EM data but 24-Bit ADC -> AGC.
Avoid saturation at short offset.



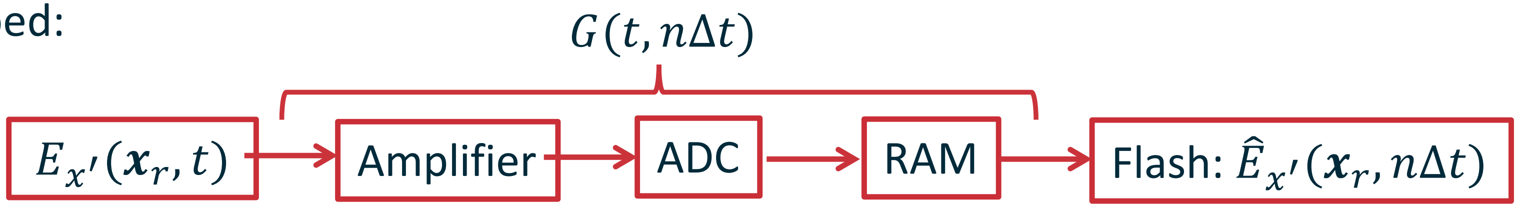
Spikes



Spikes can for example be observed when writing data to flash

Data is stored in RAM and written to Flash every 20 min.

On seabed:



The recorded field on flash is uncalibrated

$$\hat{E}_{x'}(\boldsymbol{x}_r, n\Delta t) = E_{x'}(\boldsymbol{x}_r, t) * G(t, \Delta t)$$

The transfer function $G(t, \Delta t)$ is known. The influence can be described as a convolution. In general:

$$E(t) * G(t) = \int d\tau E(t - \tau) G(\tau)$$

By definition of Fourier transform:

$$\int dt\, E(t) * G(t)\; e^{i\omega t} = E(\omega) G(\omega)$$

Assume $G(t)$ known, then $G(\omega)$ known:

Onboard download and Fourier transform:

Flash: $\hat{E}_{x'}(\boldsymbol{x}_r, n\Delta t)$ → Computer: $\hat{E}_{x'}(\boldsymbol{x}_r, n\Delta t)$ → $\hat{E}_{x'}(\boldsymbol{x}_r, \omega)$

Onboard calibration:

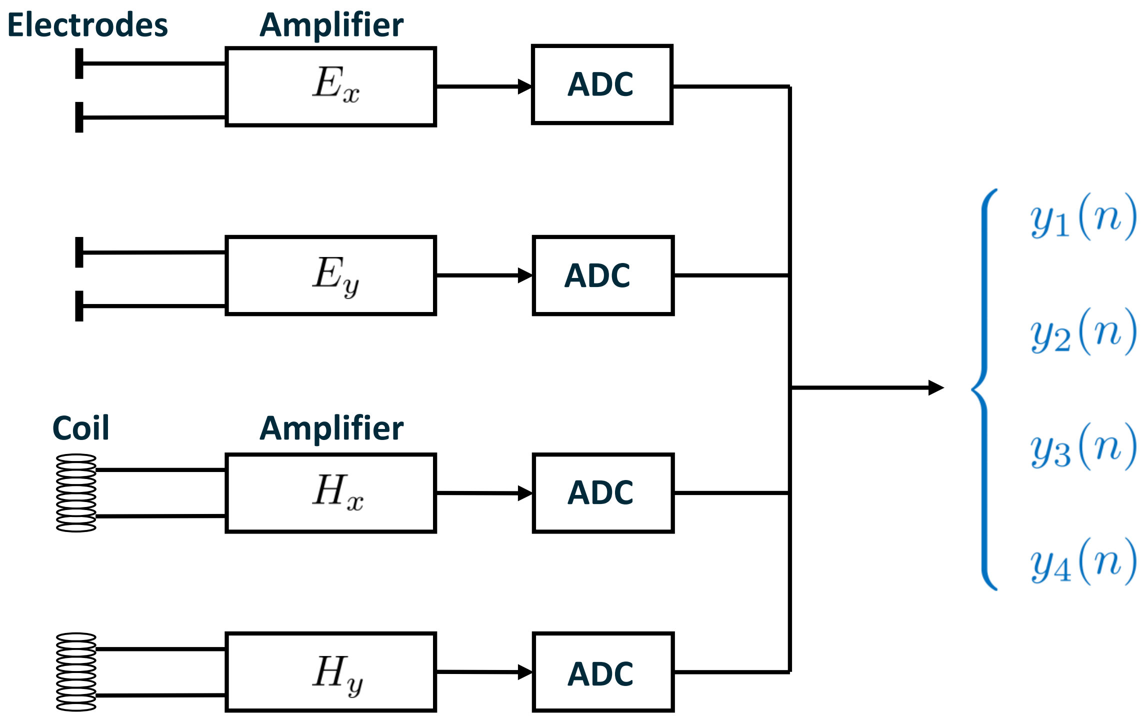
$\hat{E}_{x'}(\boldsymbol{x}_r, \omega)$ → $E_{x'}(\boldsymbol{x}_r, \omega) = \hat{G}^{-1}(\omega, \Delta t)\hat{E}_{x'}(\boldsymbol{x}_r, \omega)$

The data is now calibarated, electric fields are in units [V/m] and magnetic fields are in [A/m]

The direction of the receiver x-axis is unknown at this stage.

Calibration

- Compensation for the sensor frequency responses
- Coil cross-coupling correction
- Compensation for the frequency response of each sensor channel (amplifier & ADC)
- Time drift correction
- Despiking



Calibration

- Compensation for the sensor frequency responses
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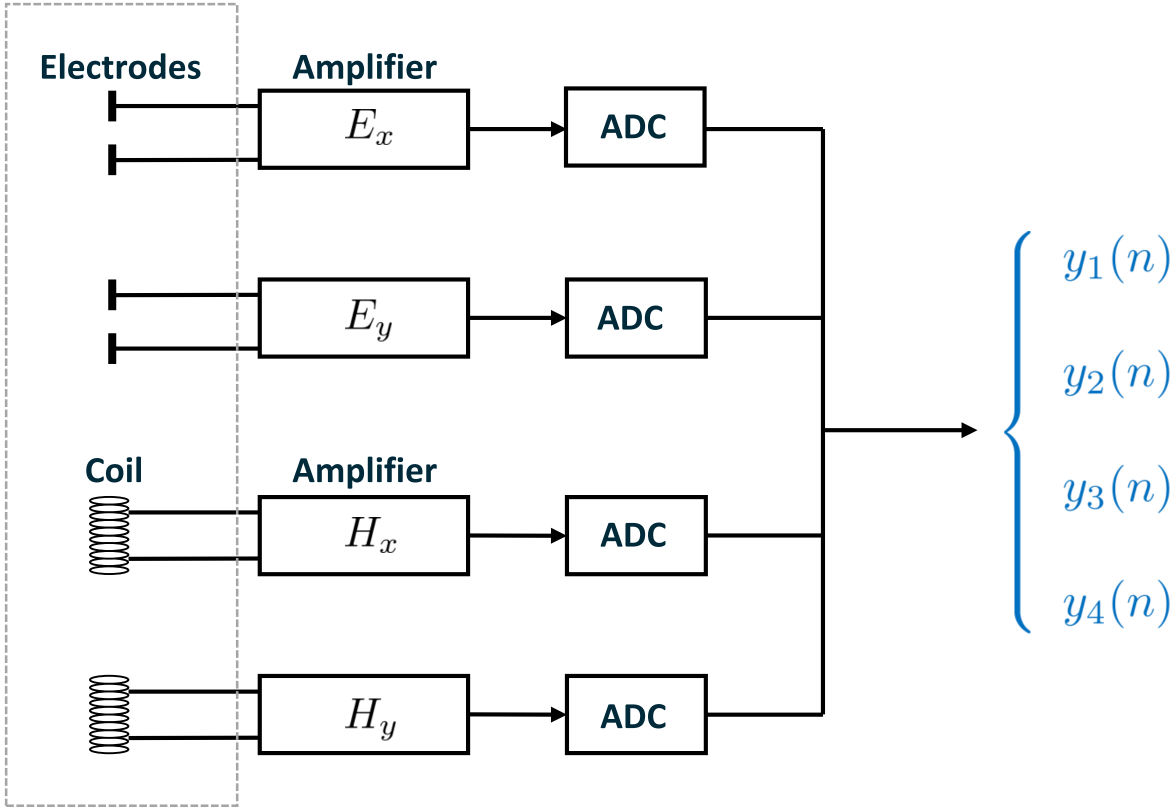

Calibration

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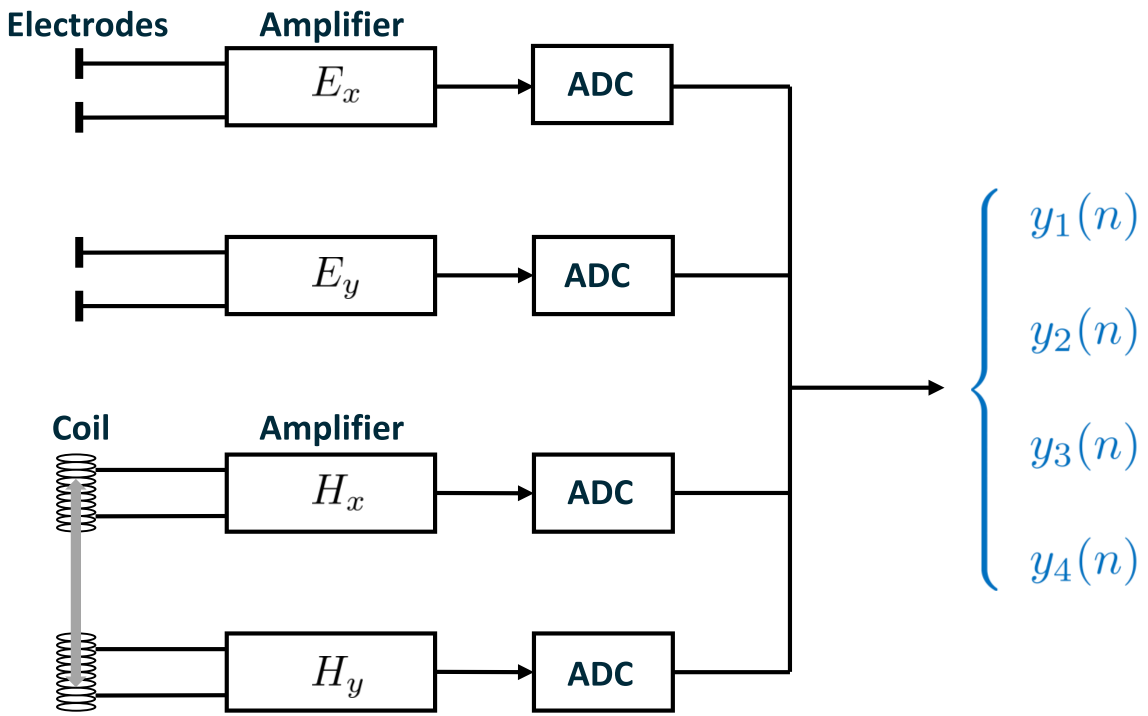
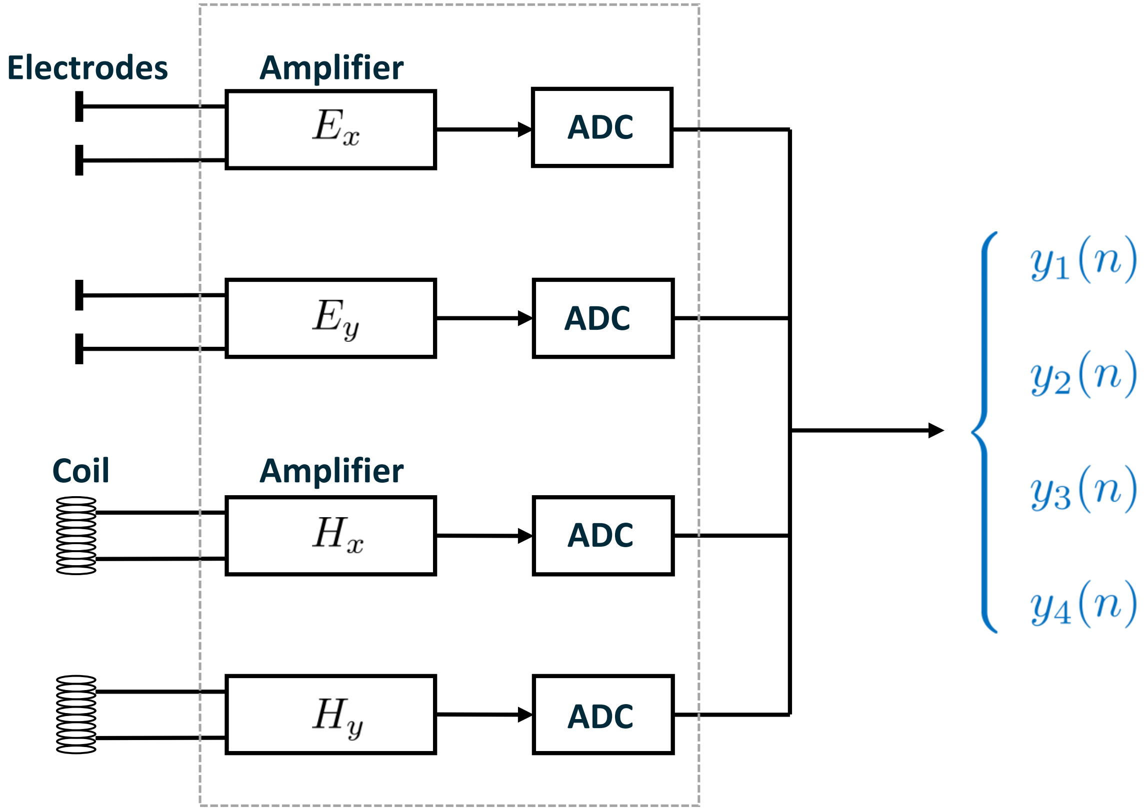


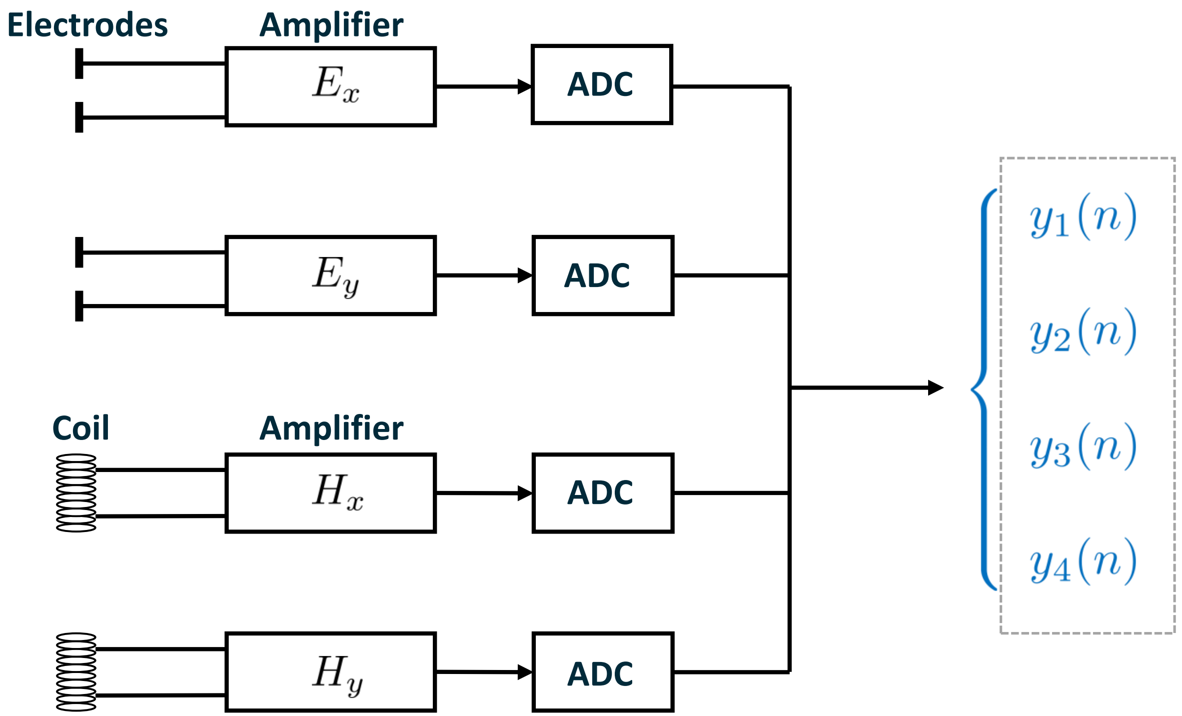
Calibration

- Compensation for the sensor frequency responses
- Coil cross-coupling correction
- Compensation for the frequency response of each sensor channel (amplifier & ADC)
- Time drift correction
- Despiking



Calibration

- Compensation for the sensor frequency responses
- Coil cross-coupling correction
- Compensation for the frequency response of each sensor channel (amplifier & ADC)
- Time drift correction (difference between receiver clock and actual time)
- Despiking



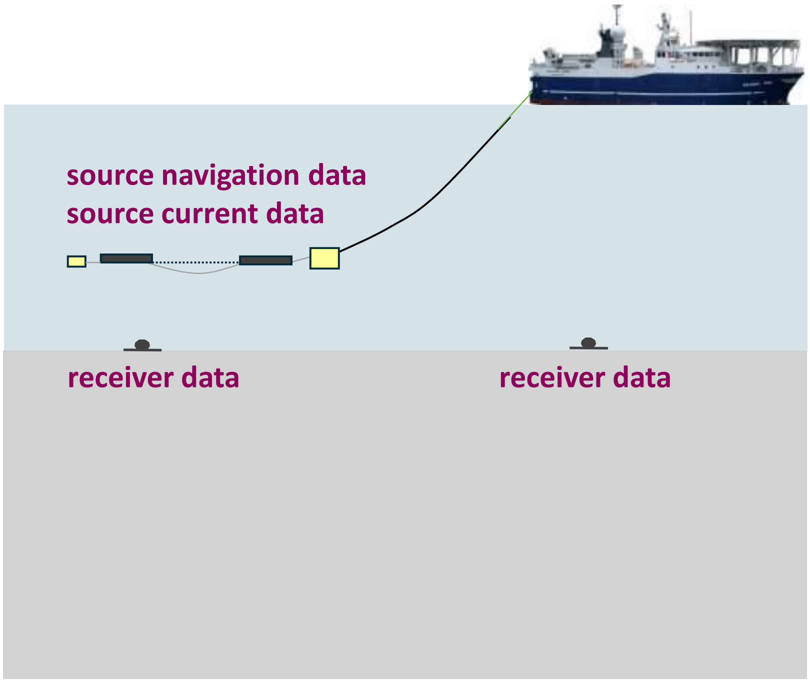
time drift correction

Since **time** is the variable that is used to link the data from the different acquisition units, viz.

- source navigation data
- source current data
- receiver data

it is important for time drift correction to be as accurate as possible

Time stamps are in Unix time.

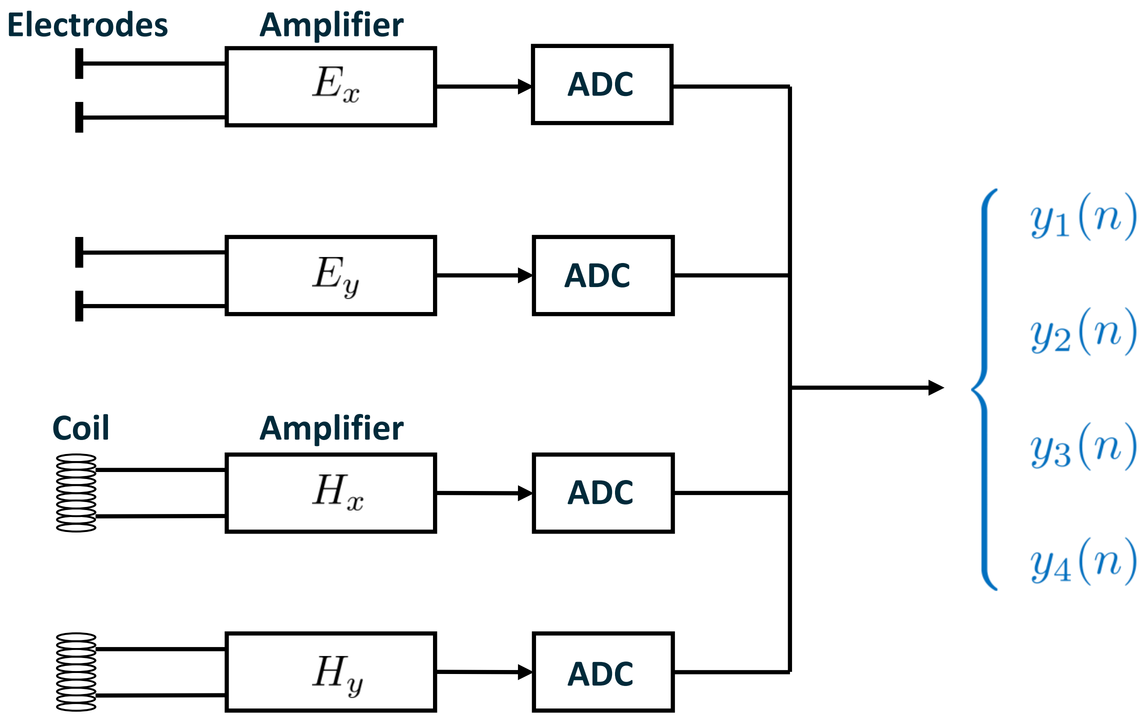


Unix (POSIX) time: Elapsed time in seconds since Unix epoch.
Unix epoch: 00:00:00 UTC, January 1, 1970

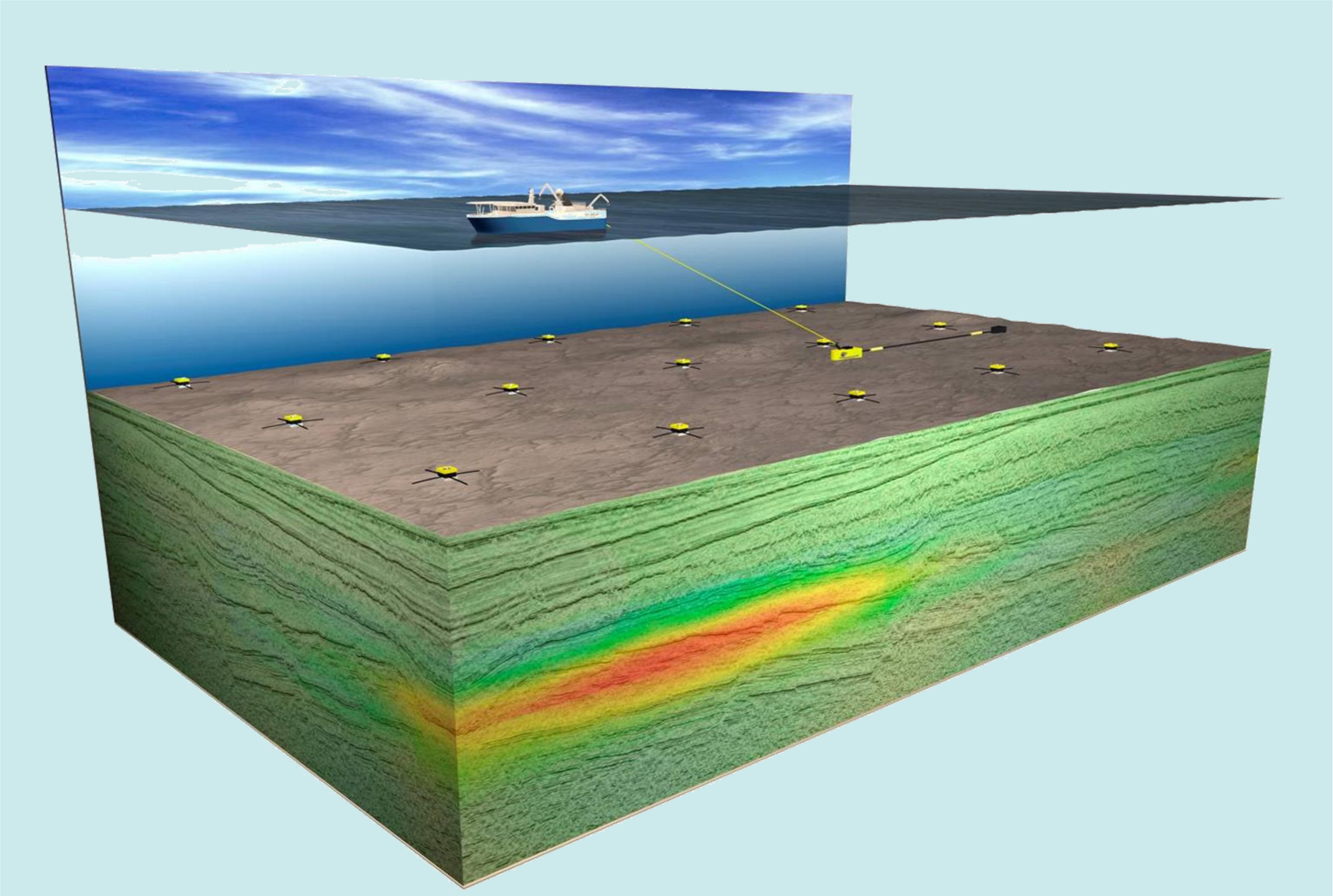
Calibration

- Compensation for the sensor frequency responses
- Coil cross-coupling correction
- Compensation for the frequency response of each sensor channel (amplifier & ADC)
- Time drift correction
- Despiking

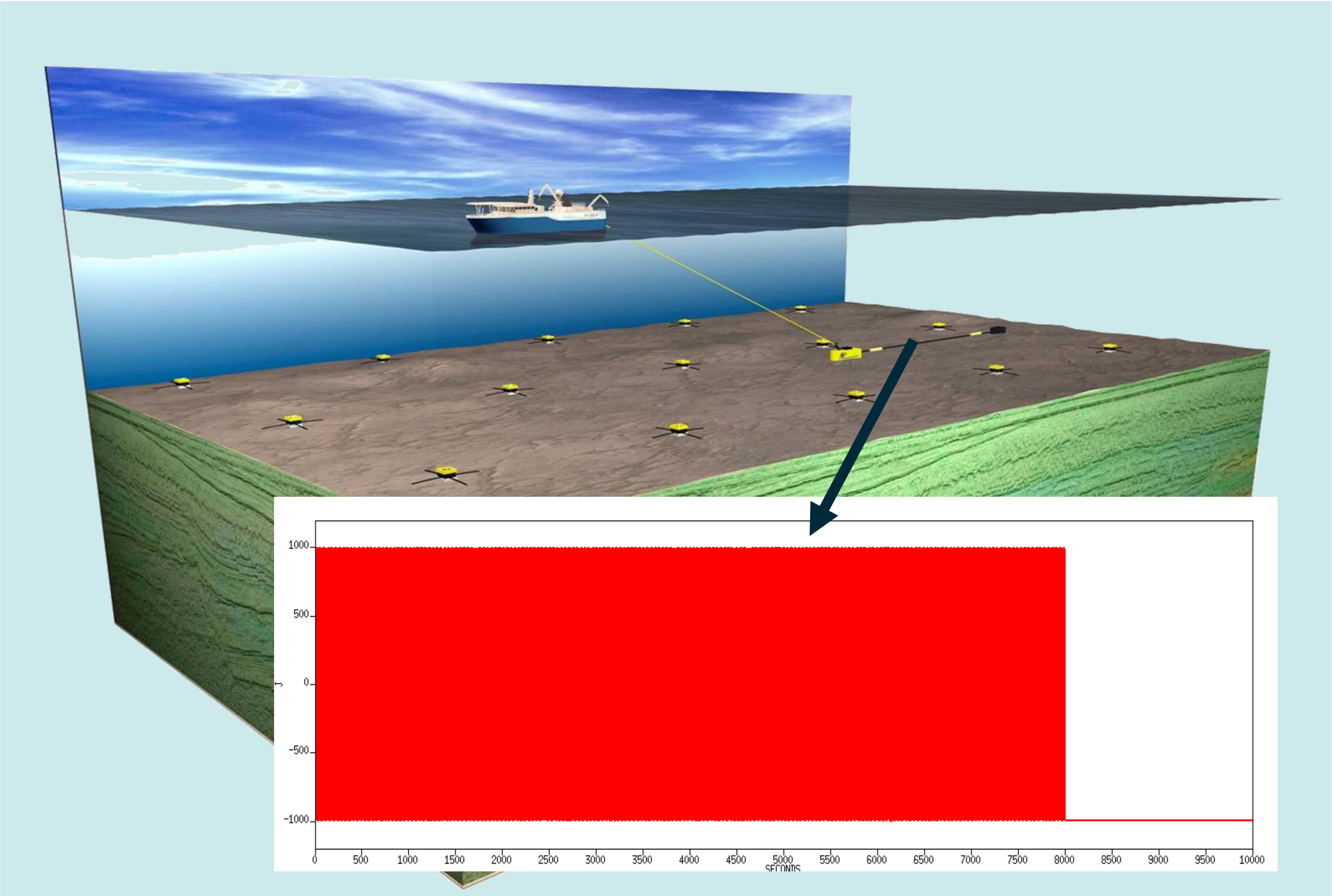
• magnetotelluric bursts
• writing data to Flash



Data



Data



1000
500
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-500
-1000
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1500
2000
2500
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3500
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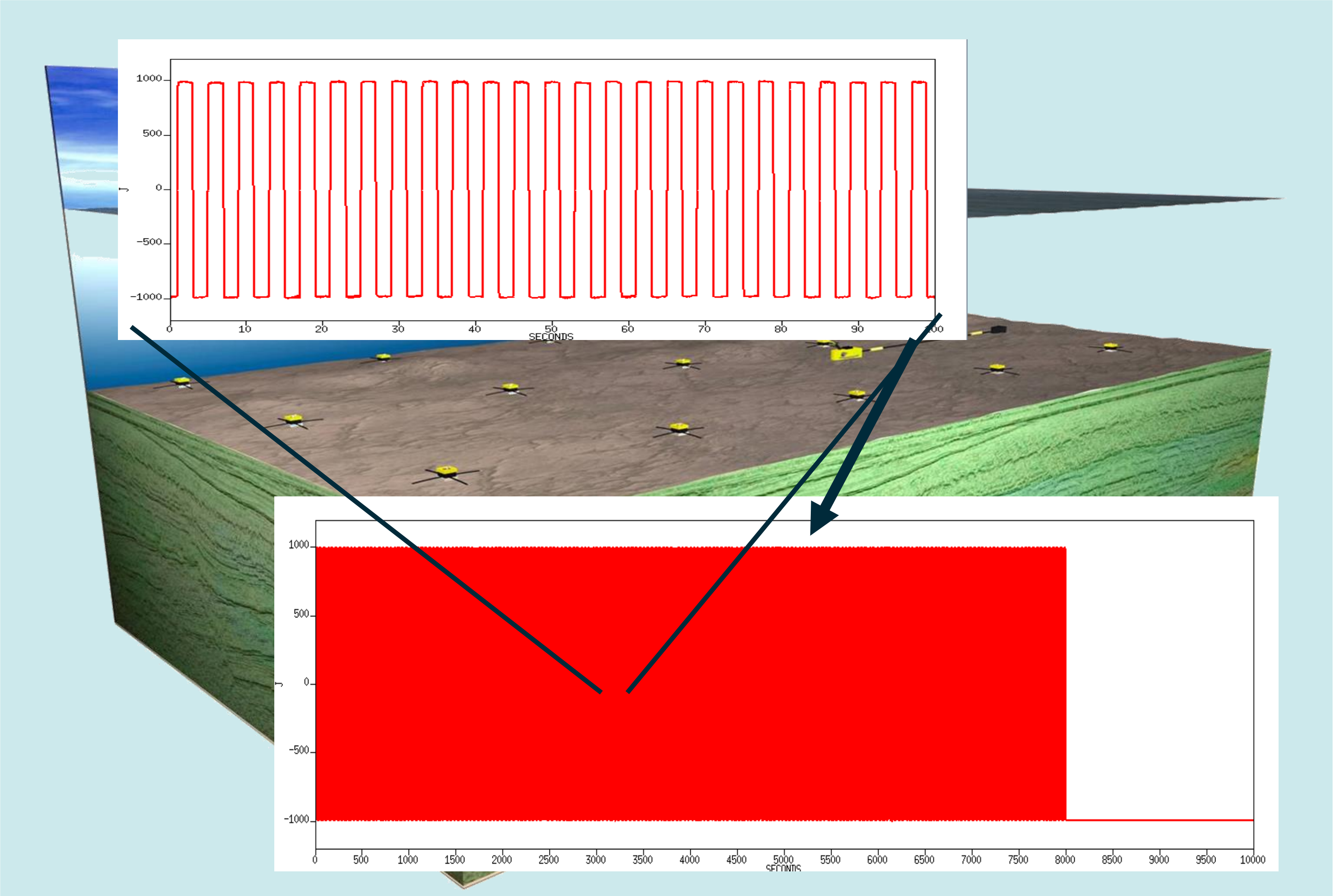
6000
6500
7000
7500
8000
8500
9000
9500
10000
SECONDS

Data


1000
500
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3000
3500
4000
4500
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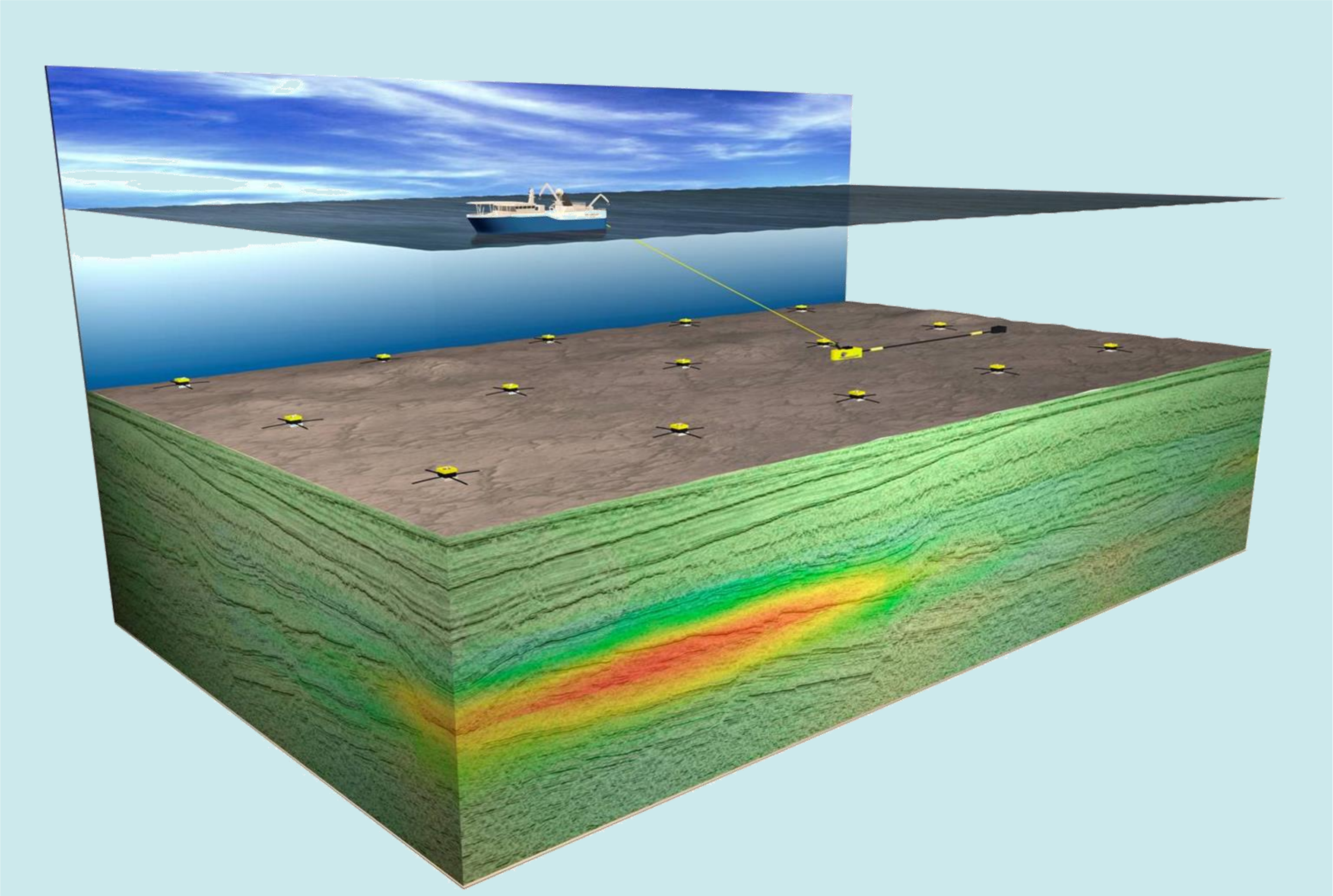
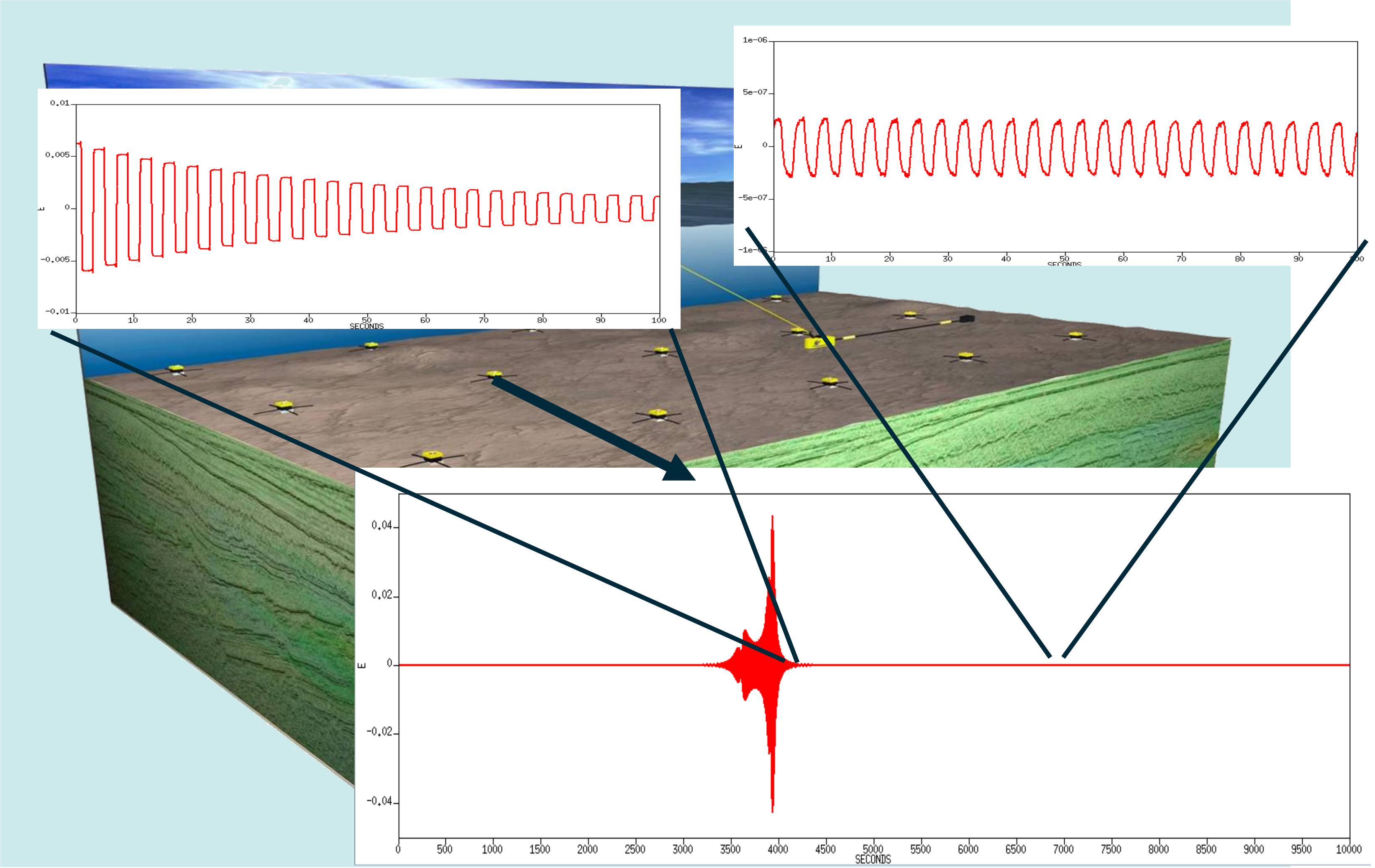
5500
6000
6500
7000
7500
8000
8500
9000
9500
10000
SECONDS

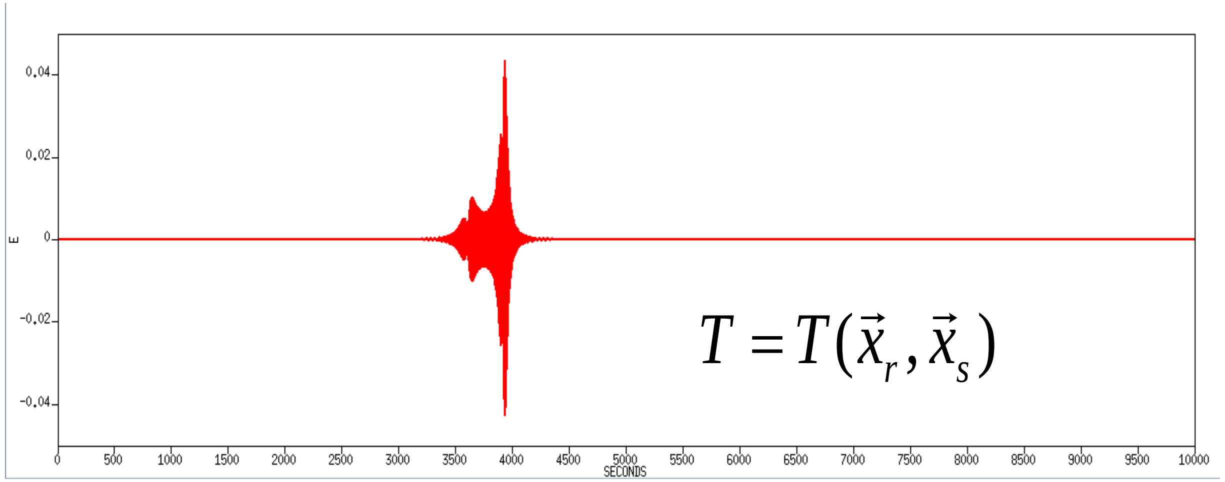
Data



Data



Data

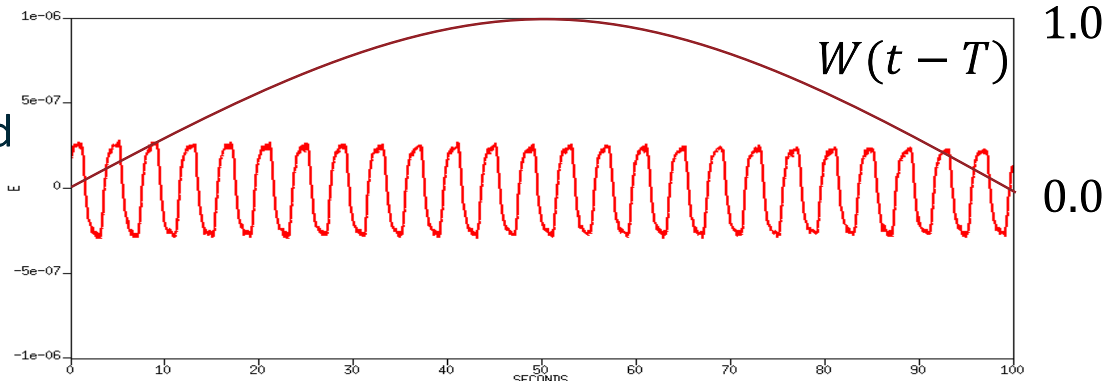


$$T = T(\vec{x}_r, \vec{x}_s)$$

Source position corresponds to time T.

Data

Weight function reduce effect of MT background field.



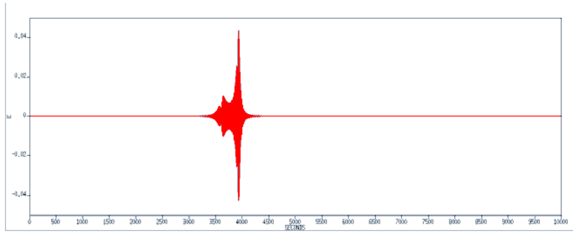
Source position corresponds to time T.

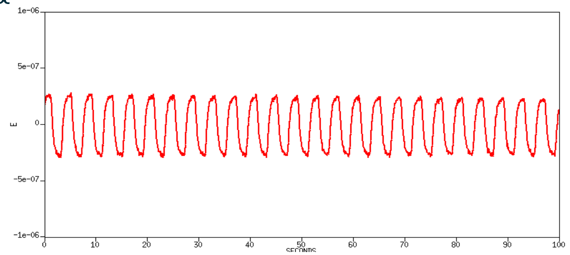
Perform following transform for desired set of source receiver coordinates:

$$\tilde{E}_x(\boldsymbol{x}_r,\omega|\boldsymbol{x}_s) = \frac{1}{\tau}\int_{T-\frac{\tau}{2}}^{T+\frac{\tau}{2}} dt W(t-T) E_x(\boldsymbol{x}_r, t|\boldsymbol{x}_s) e^{i\omega t}$$

Have

$$\tilde{E}_x(\boldsymbol{x}_r,\omega|\boldsymbol{x}_s) = \frac{1}{\tau}\int_{T-\frac{\tau}{2}}^{T+\frac{\tau}{2}} dt W(t-T) E_x(\boldsymbol{x}_r,t|\boldsymbol{x}_s)e^{i\omega t}$$





Phase of $\widetilde{E}_x$ dependes on time T (Unix time)

.

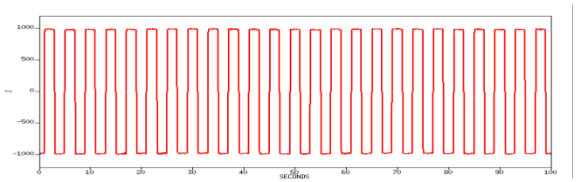
Thus arbtrary!

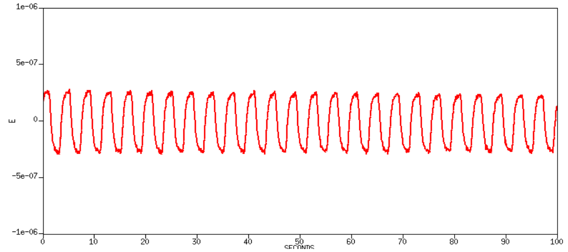
$$\tilde{E}_x(\boldsymbol{x}_r,\omega|\boldsymbol{x}_s) = \frac{1}{\tau}\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} dt W(t') E_x(\boldsymbol{x}_r,t'+T|\boldsymbol{x}_s)e^{i\omega(t'+T)}$$

$$\tilde{E}_x(\boldsymbol{x}_r,\omega|\boldsymbol{x}_s) = \frac{e^{i\omega T}}{\tau}\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} dt W(t') E_x(\boldsymbol{x}_r,t'+T|\boldsymbol{x}_s)e^{i\omega t\prime}$$

Have

$$\tilde{E}_x(\boldsymbol{x}_r, \omega|\boldsymbol{x}_s) = \frac{e^{i\omega T}}{\tau}\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} dt W(t') E_x(\boldsymbol{x}_r, t' + T|\boldsymbol{x}_s) e^{i\omega t\prime}$$





Correct for effect of W

$$E'_x(\boldsymbol{x}_r, \omega|\boldsymbol{x}_s) = e^{i\omega T} A(\boldsymbol{x}_r, \omega|\boldsymbol{x}_s) e^{i\varphi_R(\boldsymbol{x}_r, \omega|\boldsymbol{x}_s)}$$

Source is transformed over same time interval

$$J_x(\boldsymbol{x}_s, \omega) = \frac{1}{\tau}\int_{T-\frac{\tau}{2}}^{T+\frac{\tau}{2}} dt\, J_x(\boldsymbol{x}_s, t) e^{i\omega t}$$

$$J_x(\boldsymbol{x}_s, \omega) = \frac{e^{i\omega T}}{\tau}\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} dt\, J_x(\boldsymbol{x}_s, t' + T) e^{i\omega t\prime}$$

$$J_x(\boldsymbol{x}_s, \omega) = e^{i\omega T} B(\boldsymbol{x}_s, \omega) e^{i\varphi_S(\boldsymbol{x}_s, \omega)}$$

Have

$$E'_x(\boldsymbol{x}_r, \omega|\boldsymbol{x}_s) = e^{i\omega T} A(\boldsymbol{x}_r, \omega|\boldsymbol{x}_s) e^{i\varphi_R(\boldsymbol{x}_r, \omega|\boldsymbol{x}_s)}$$

$$J_x(\boldsymbol{x}_s, \omega) = e^{i\omega T} B(\boldsymbol{x}_s, \omega) e^{i\varphi_S(\boldsymbol{x}_s,\omega)}$$

Give

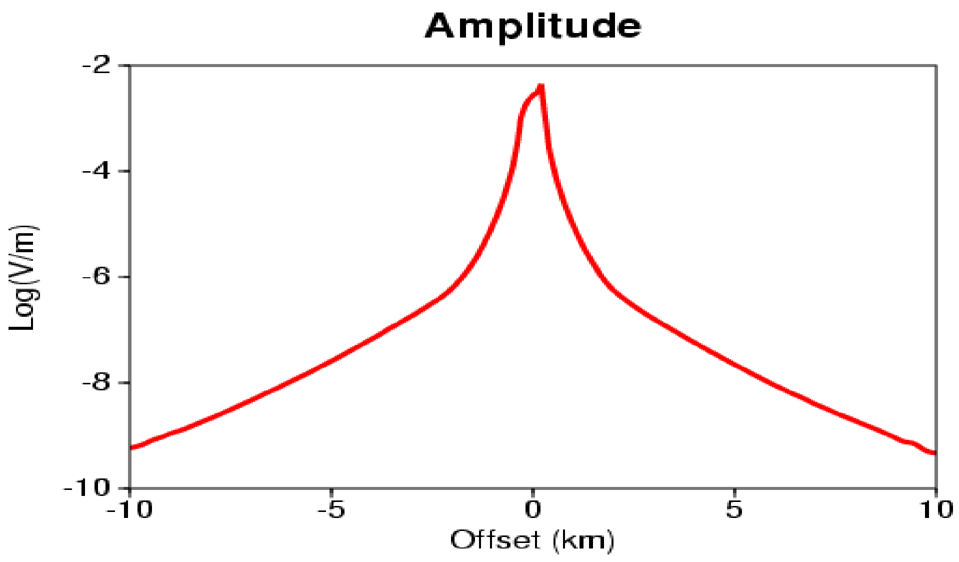
$$E_x(\boldsymbol{x}_r, \omega|\boldsymbol{x}_s) = \frac{e^{i\omega T} A(\boldsymbol{x}_r, \omega|\boldsymbol{x}_s) e^{i\varphi_R(\boldsymbol{x}_r, \omega|\boldsymbol{x}_s)}}{e^{i\omega T} e^{i\varphi_S(\boldsymbol{x}_s,\omega)}}$$

or

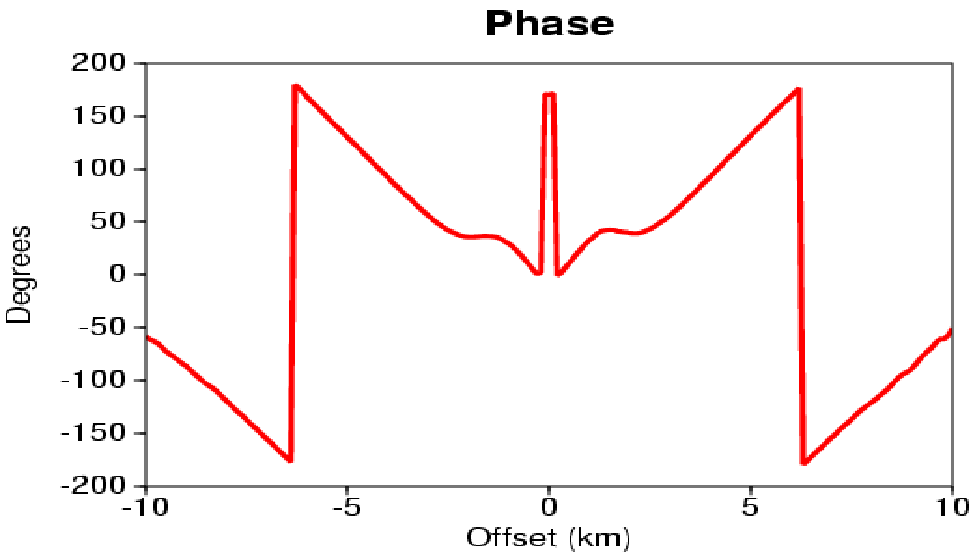
$$E_x(\boldsymbol{x}_r, \omega|\boldsymbol{x}_s) = A(\boldsymbol{x}_r, \omega|\boldsymbol{x}_s) e^{i(\varphi_R(\boldsymbol{x}_r, \omega|\boldsymbol{x}_s) - \varphi_S(\boldsymbol{x}_s,\omega))} = A(\boldsymbol{x}_r, \omega|\boldsymbol{x}_s) e^{i\varphi(\boldsymbol{x}_r, \omega|\boldsymbol{x}_s)}$$

If phase approximately equals angular frequency times traveltime ($\varphi = \omega\tau$) then phase difference is a measure of propagation time from source to receiver.

Data

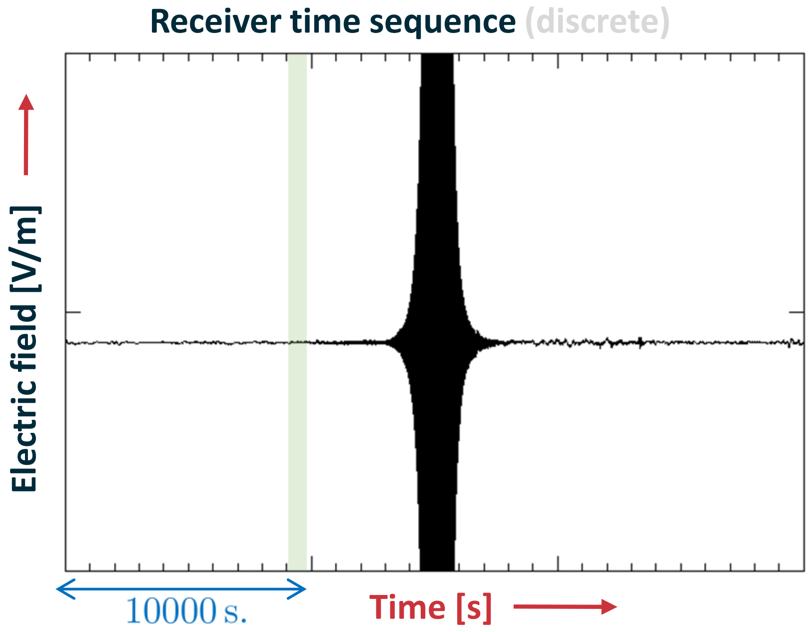


Final results are corrected for influence of weight function



Demodulation

- The receiver time series can be transformed from the time-domain into the frequency-domain via a **discrete-time** short time Fourier transform (STFT)

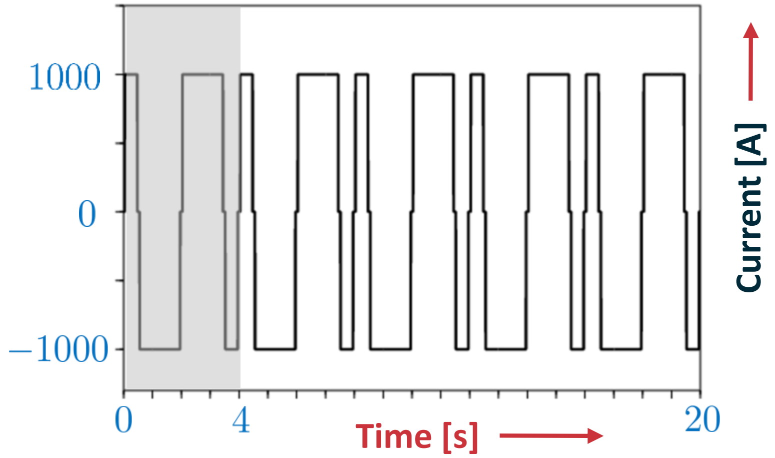


Demodulation

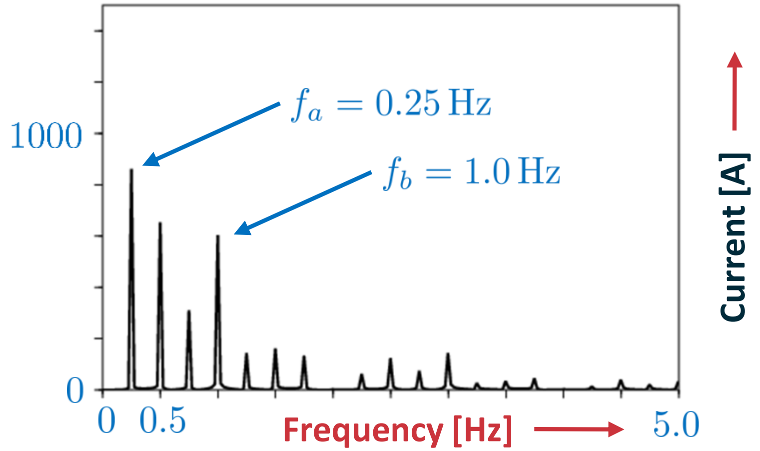
- The receiver time series can be transformed from the time-domain into the frequency-domain via a **discrete-time** **short time Fourier transform (STFT)**

- We are interested in the **frequencies from the source spectrum**

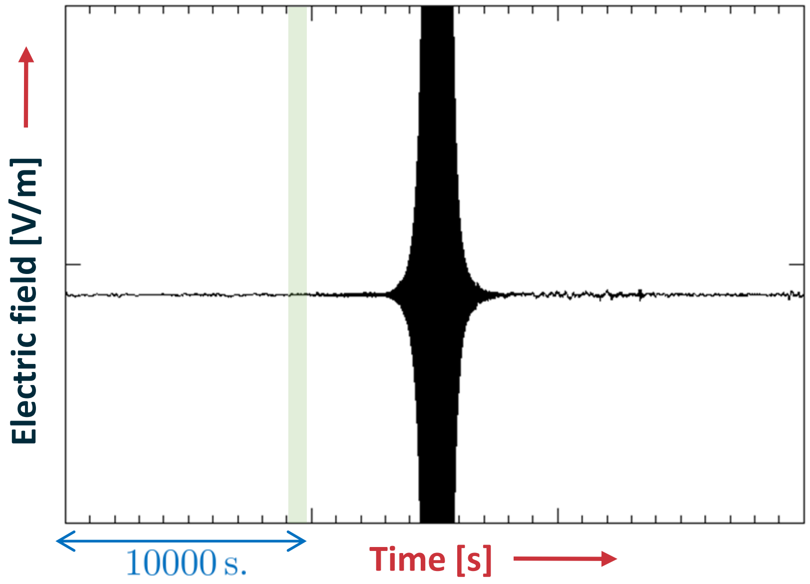
Source signal



Source spectrum

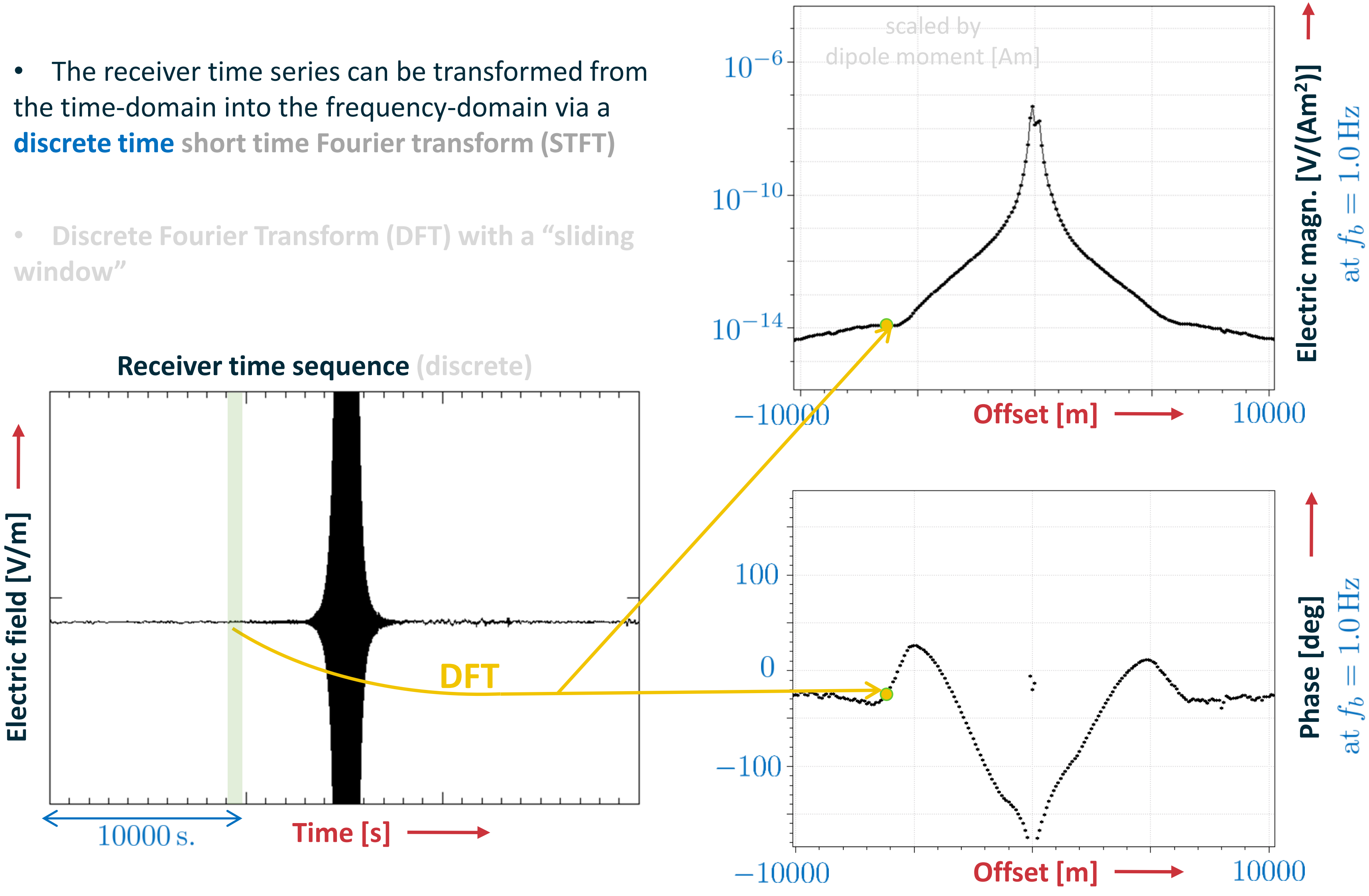


Receiver time sequence (discrete)



Demodulation

- The receiver time series can be transformed from the time-domain into the frequency-domain via a **discrete time** **short time Fourier transform (STFT)**
- Discrete Fourier Transform (DFT) with a "sliding window"

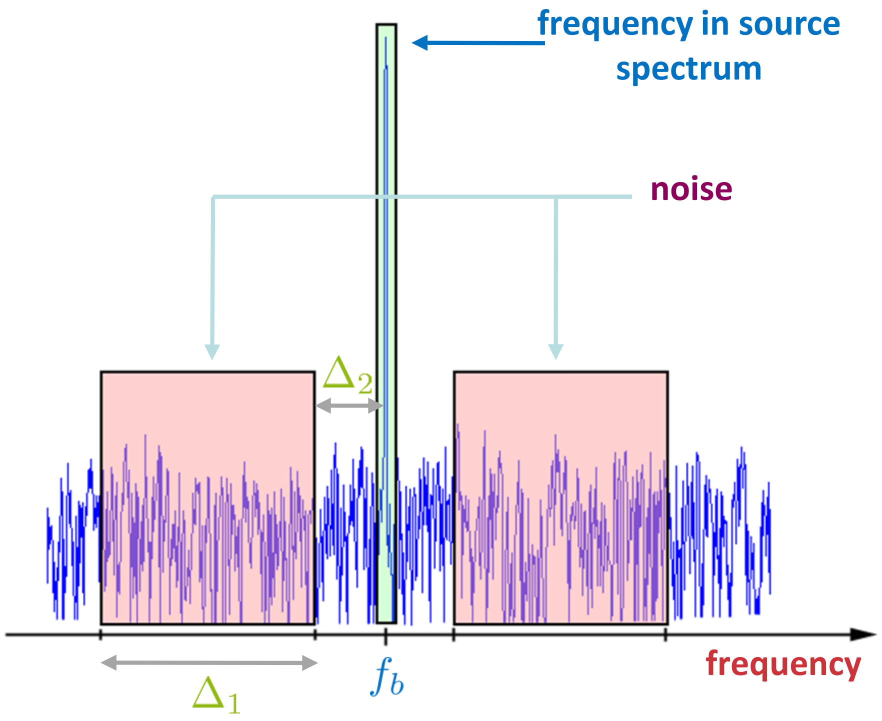


Noise estimation

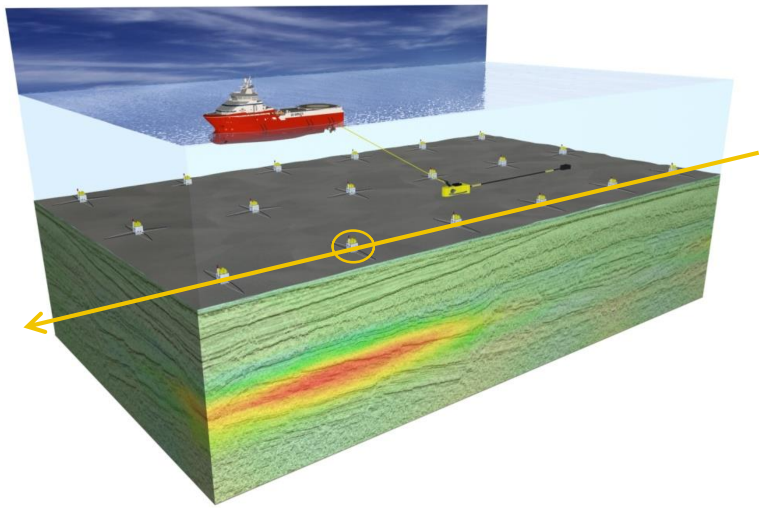
- Noise estimation can be performed using frequencies that are close to the frequency of interest

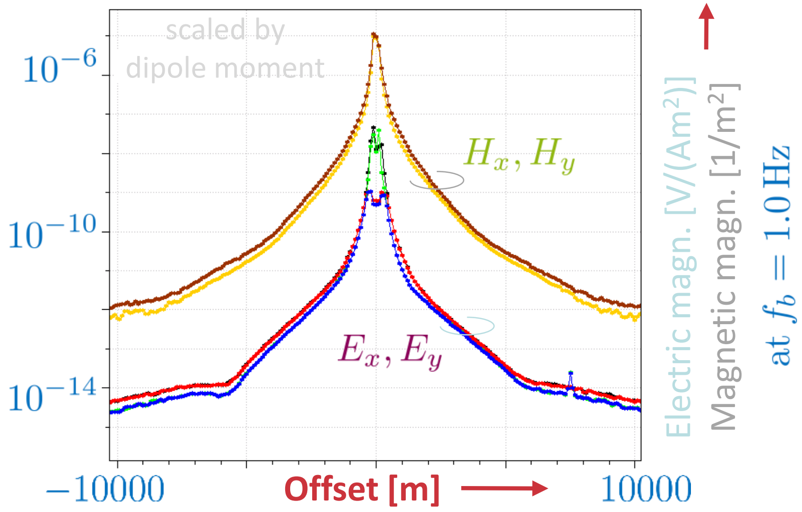
- The noise estimate can be used to compute **inversion weights**

- The noise estimate can be used for **spike detection**

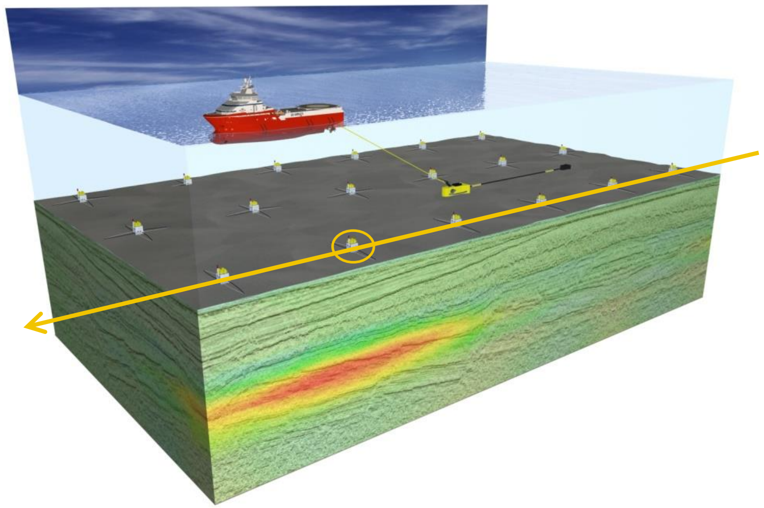


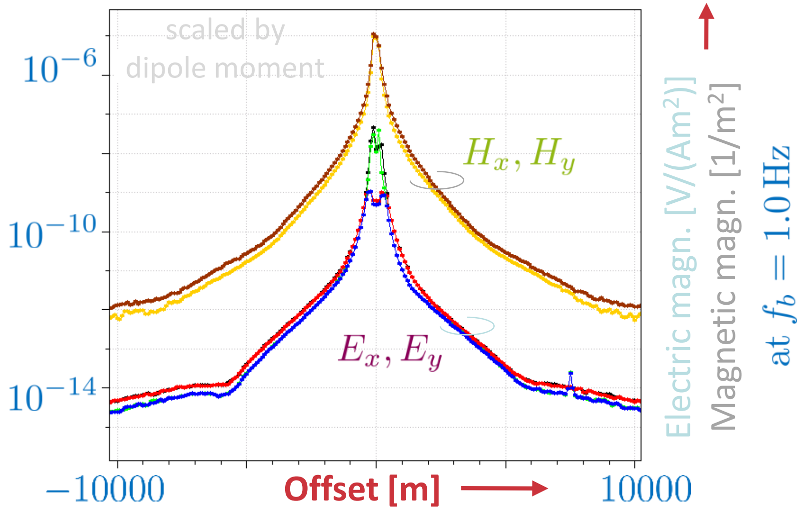
The procedure is **repeated** at **each frequency of interest** and **each offset**

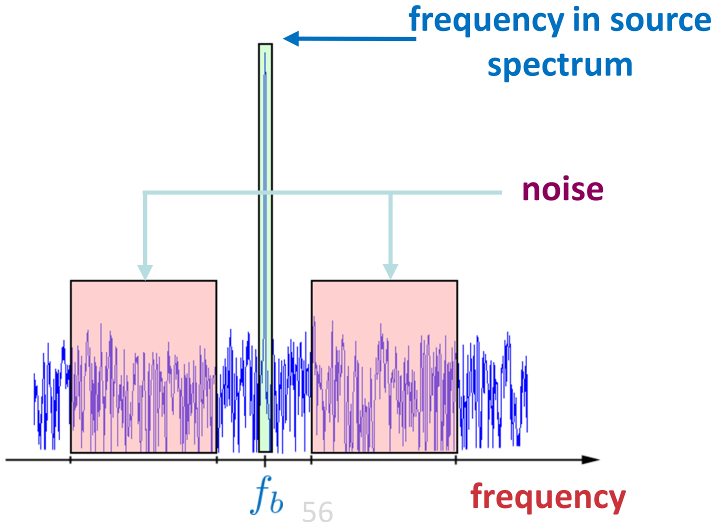


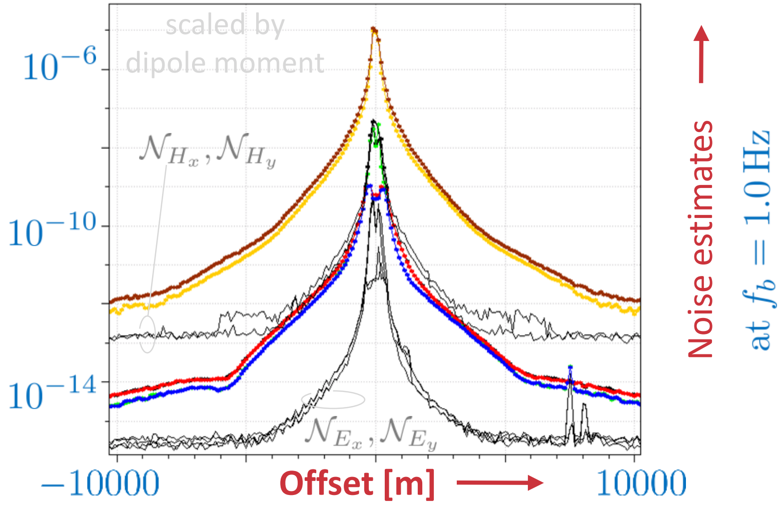


scaled by
dipole moment
10^{-6}
10^{-10}
10^{-14}
H_x, H_y
E_x, E_y
-10000
Offset [m]
10000
Electric magn. [V/(Am^2)]
Magnetic magn. [1/m^2]
at $f_b = 1.0\,\mathrm{Hz}$




scaled by
dipole moment
10^{-6}
10^{-10}
10^{-14}
H_x, H_y
E_x, E_y
-10000
Offset [m]
10000
Electric magn. [V/(Am^2)]
Magnetic magn. [1/m^2]
at $f_b = 1.0\,\mathrm{Hz}$


frequency in source
spectrum
noise
f_b
frequency

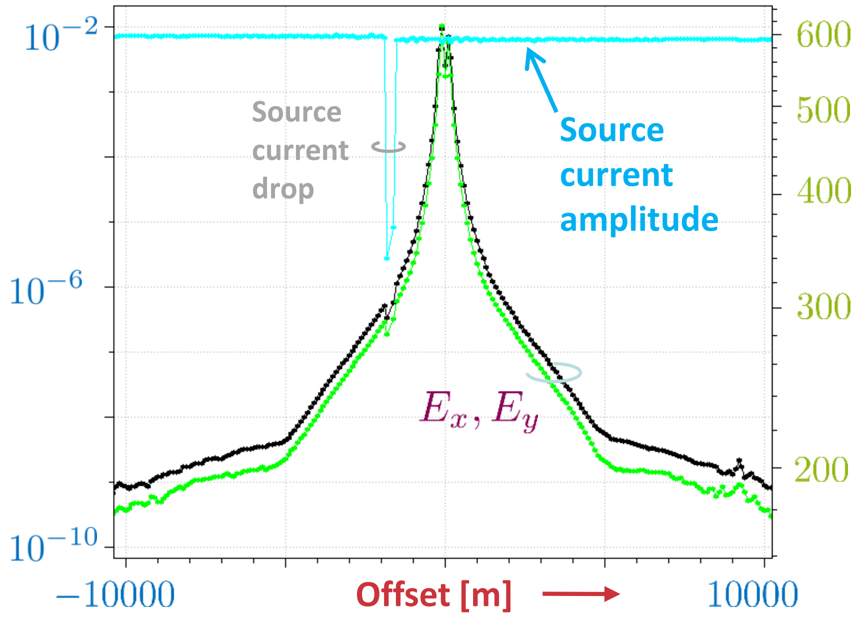

scaled by
dipole moment
10^{-6}
10^{-10}
10^{-14}
$\mathcal{N}_{H_x}, \mathcal{N}_{H_y}$
$\mathcal{N}_{E_x}, \mathcal{N}_{E_y}$
-10000
Offset [m]
10000
Noise estimates
at $f_b = 1.0\,\mathrm{Hz}$

Source dipole moment scaling

$$E_x(\boldsymbol{x}_r, \omega|\boldsymbol{x}_s) = \mathrm{A}(\boldsymbol{x}_r, \omega|\boldsymbol{x}_s)e^{i\varphi(\boldsymbol{x}_r, \omega|\boldsymbol{x}_s)}$$

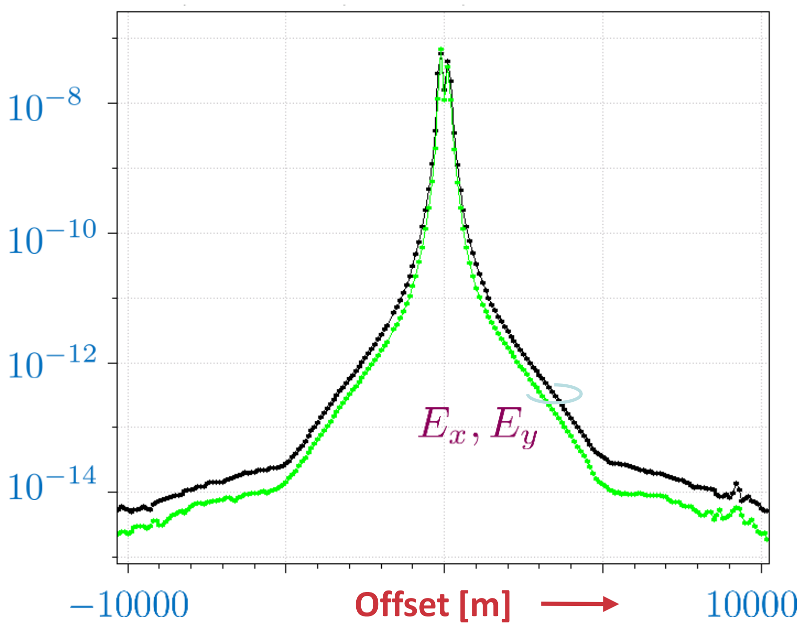
$$E_x(\boldsymbol{x}_r, \omega|\boldsymbol{x}_s) = \frac{\mathrm{A}(\boldsymbol{x}_r, \omega|\boldsymbol{x}_s)e^{i\varphi(\boldsymbol{x}_r, \omega|\boldsymbol{x}_s)}}{LJ(\omega)}$$

Before scaling



Electric magnitude [V/m]
at $f_b = 1.0\,\mathrm{Hz}$

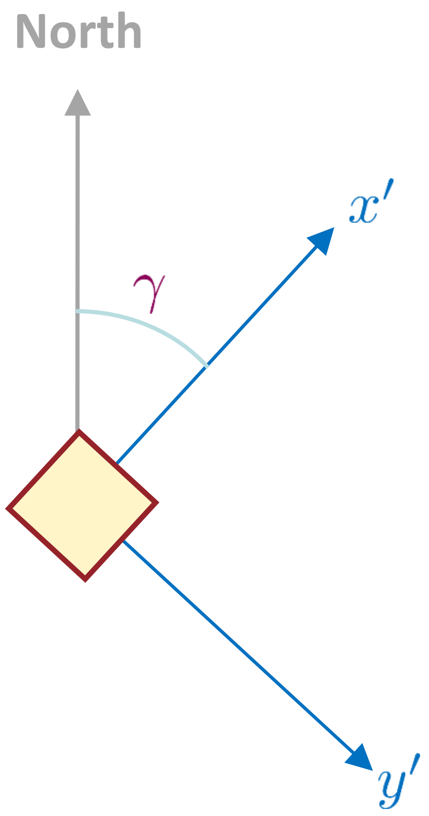
After scaling



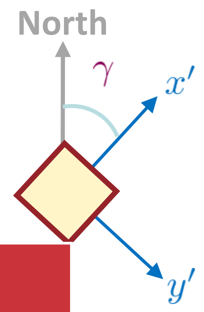
Electric magnitude [V/(Am²)]
at $f_b = 1.0\,\mathrm{Hz}$

CSEM receiver orientation

- How do we determine the CSEM receiver orientation relative to north or relative to towline?

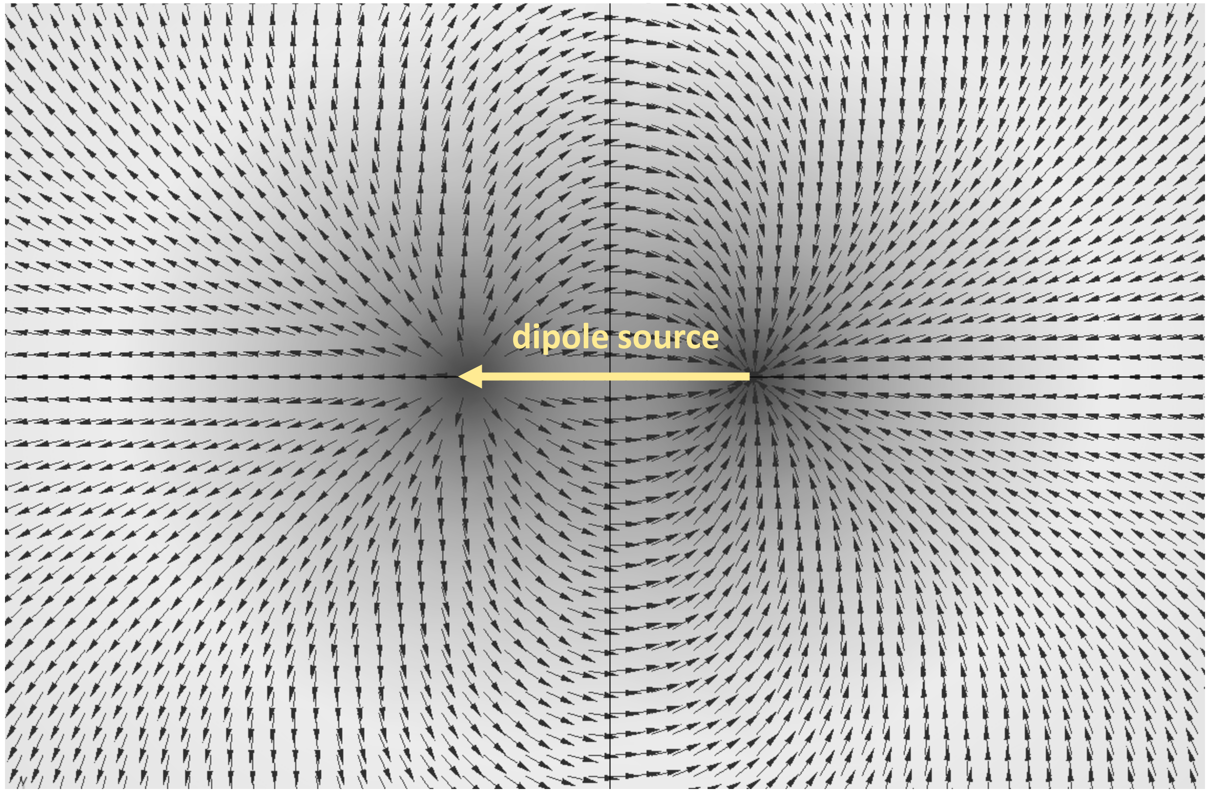


CSEM receiver orientation

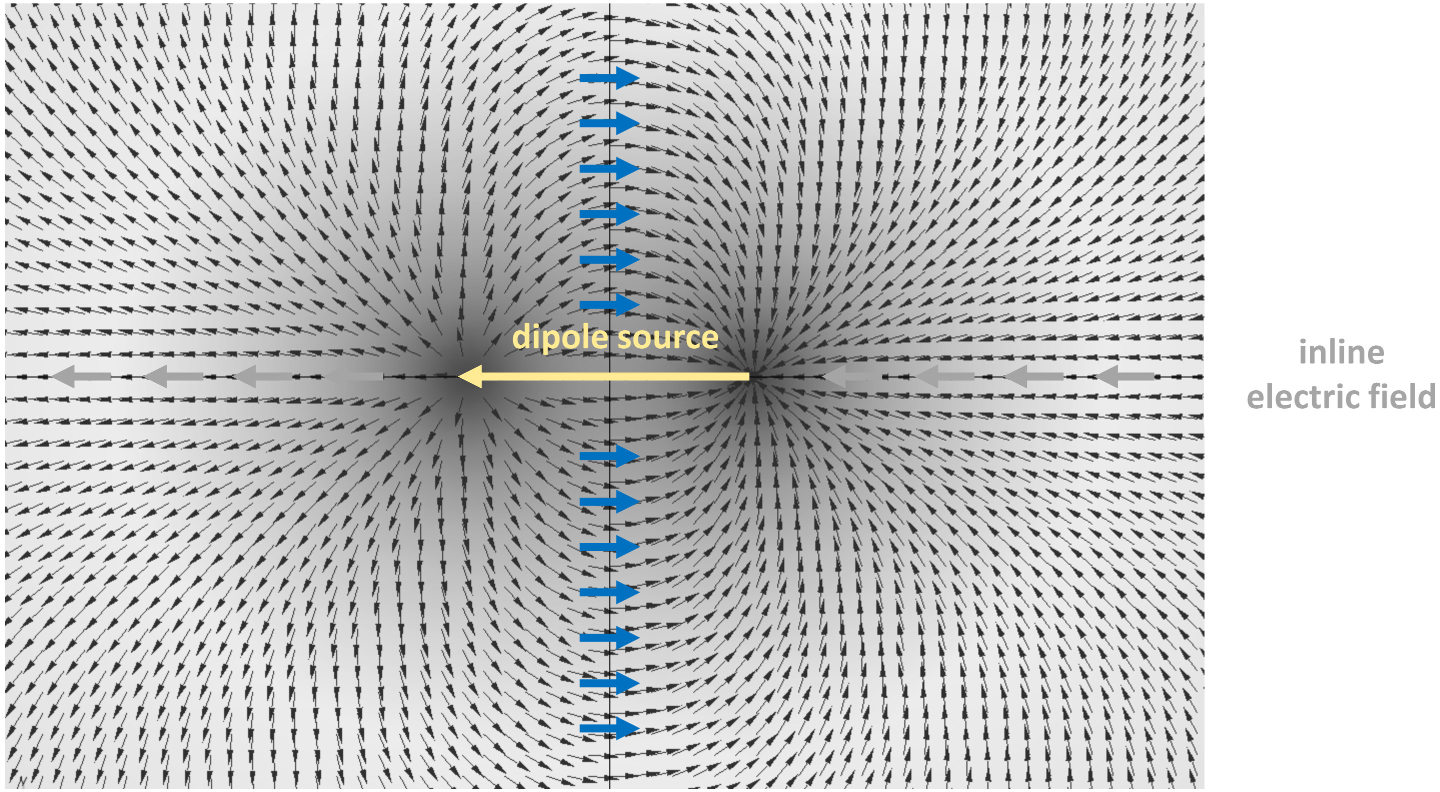


Method	Advantages	Disadvantages
compass	standard	• accuracy, calibration
gyroscope or acoustic	accuracy	• cost • battery requirements
inline rotation		• sensitive to local geology • source needs to be towed over every receiver
broadside rotation	sensitive to local geology	
1D inversion	takes geology into account	local geology may not be 1D
3D inversion	takes source orientation and local geology into account	one needs to run a 3D inversion

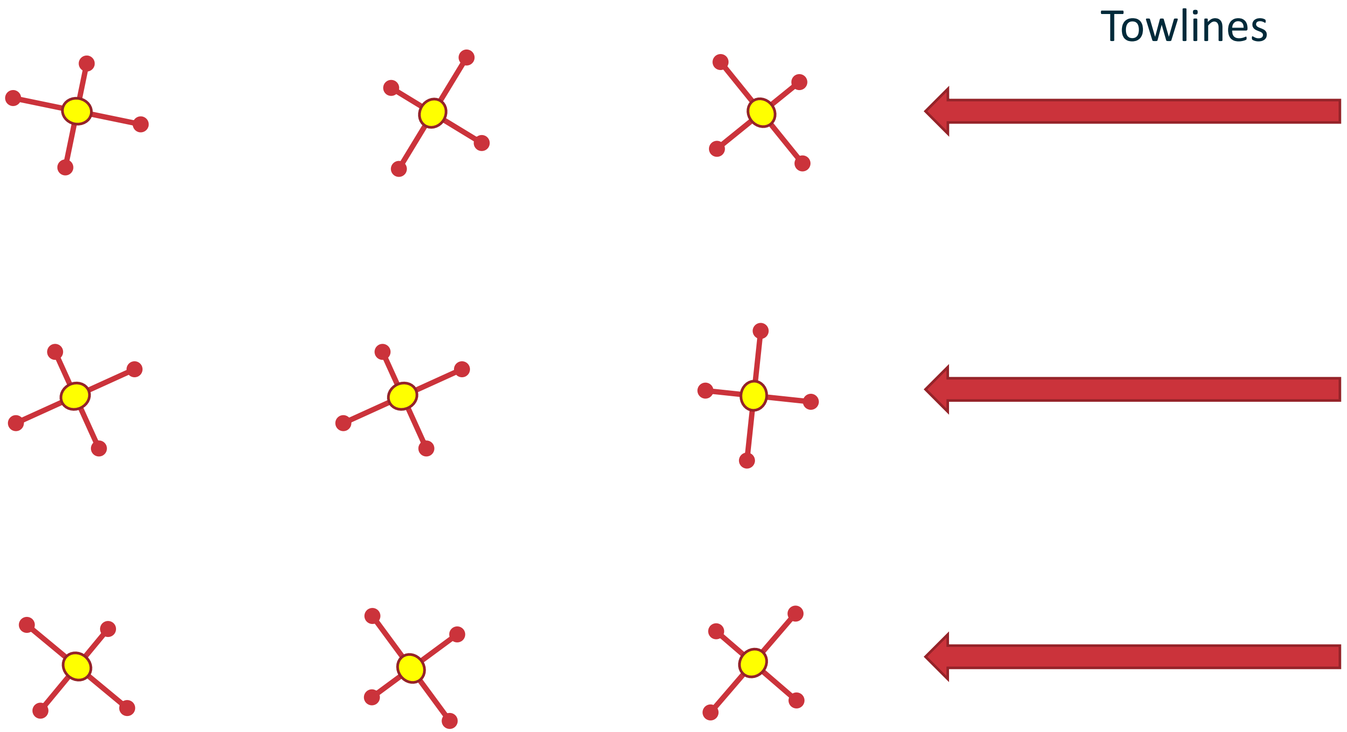
EM dipole source radiation: electric field



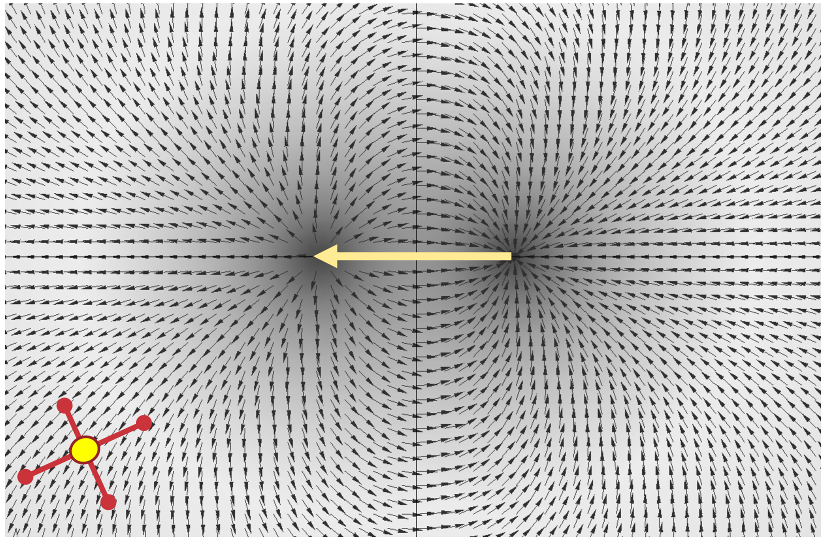
EM dipole source radiation: electric field



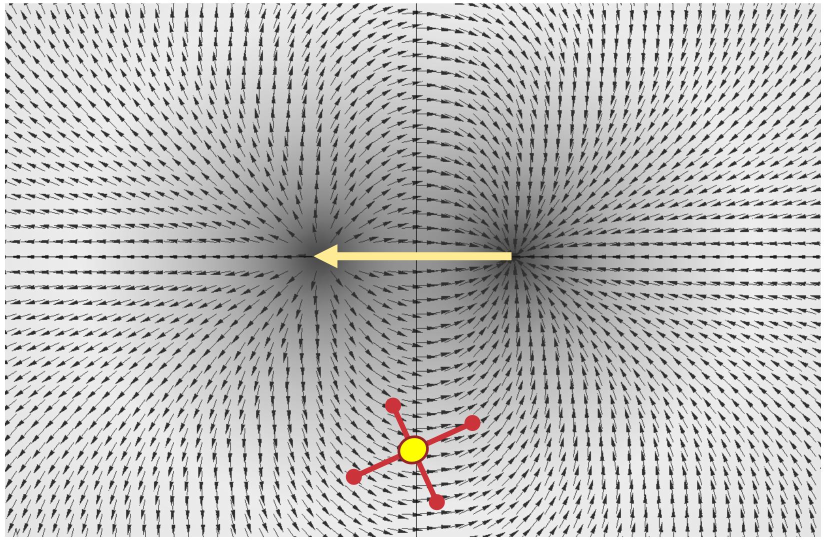
1D inversion (standalone)
3D inversion (integrated)



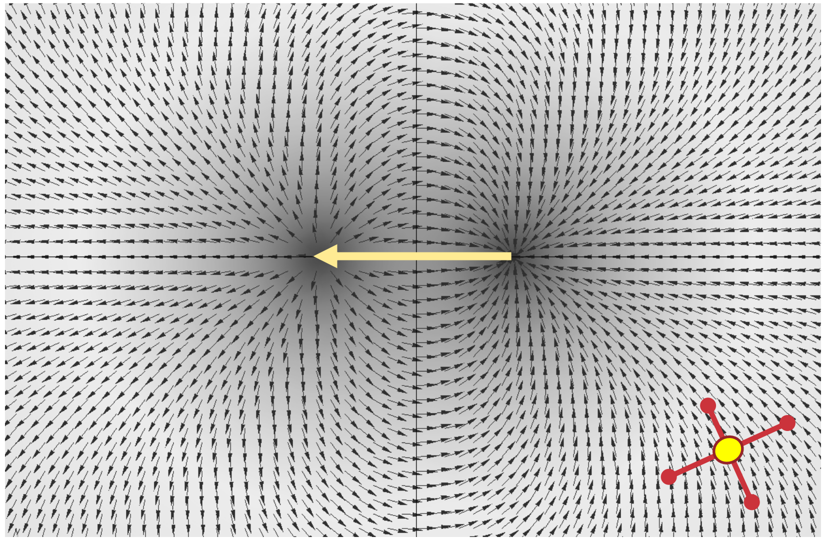
EM dipole source radiation: electric field



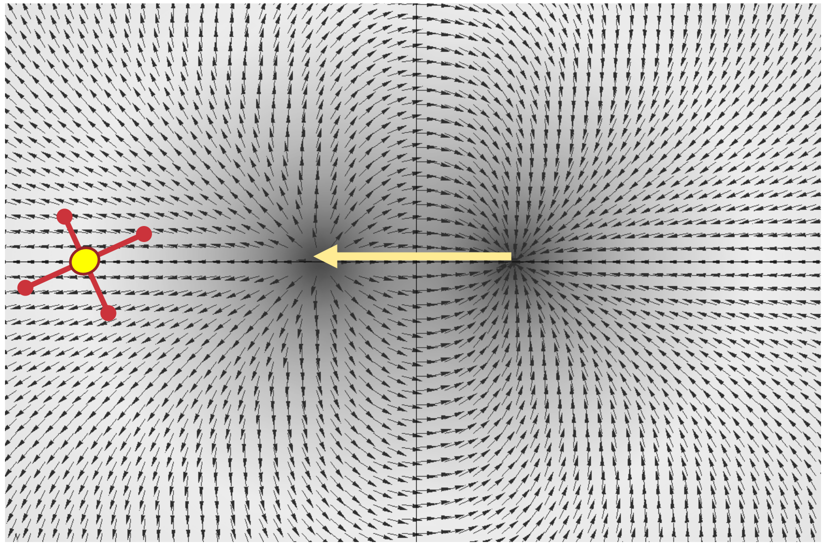
EM dipole source radiation: electric field



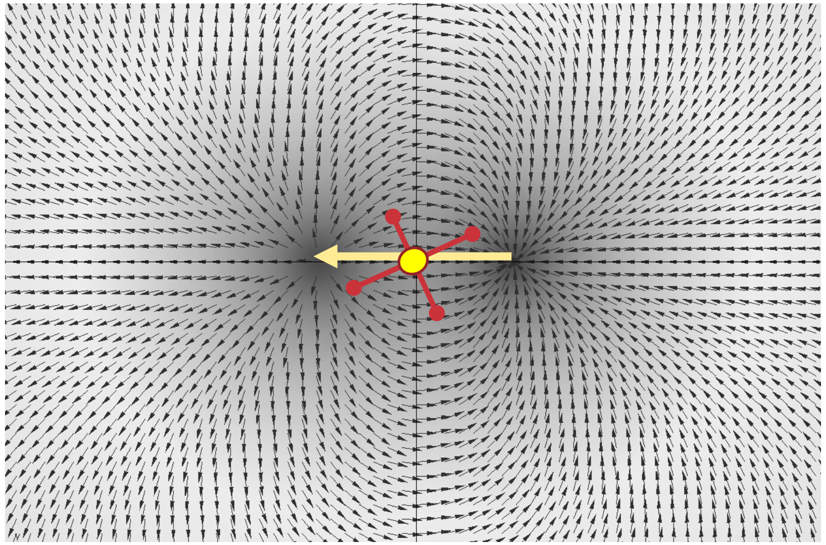
EM dipole source radiation: electric field



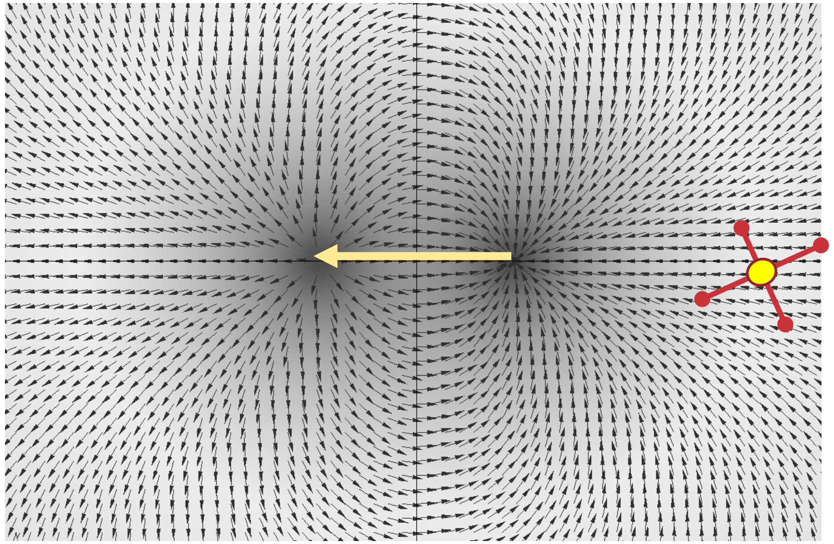
EM dipole source radiation: electric field



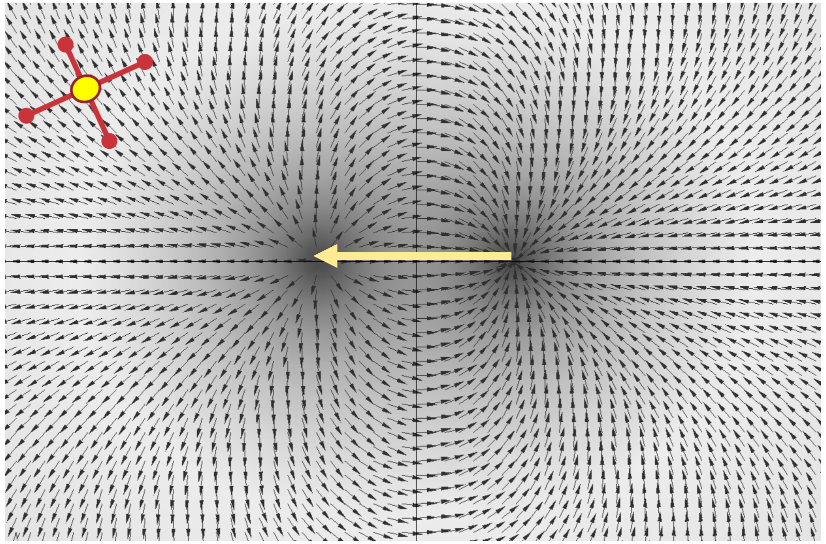
EM dipole source radiation: electric field



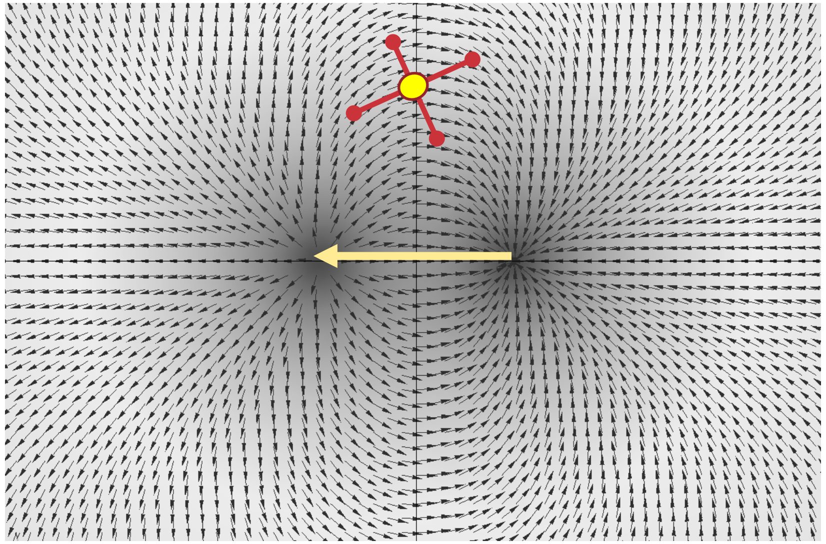
EM dipole source radiation: electric field



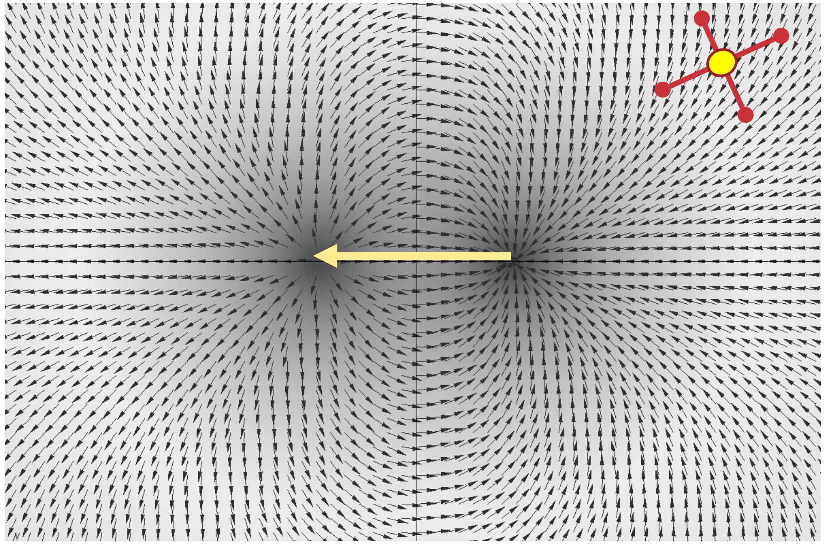
EM dipole source radiation: electric field



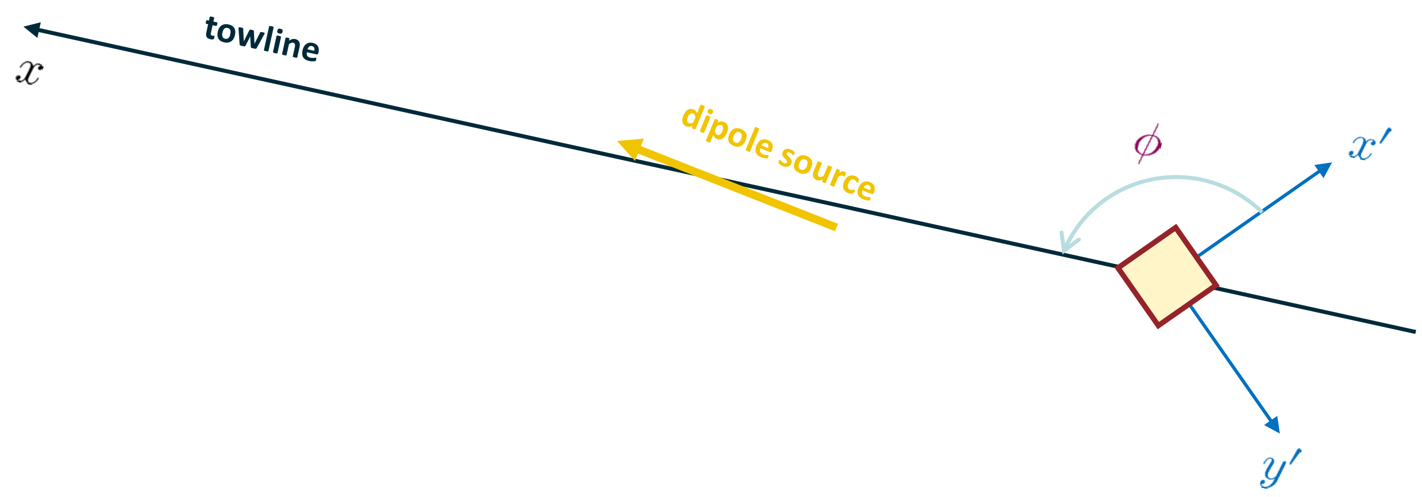
EM dipole source radiation: electric field



EM dipole source radiation: electric field



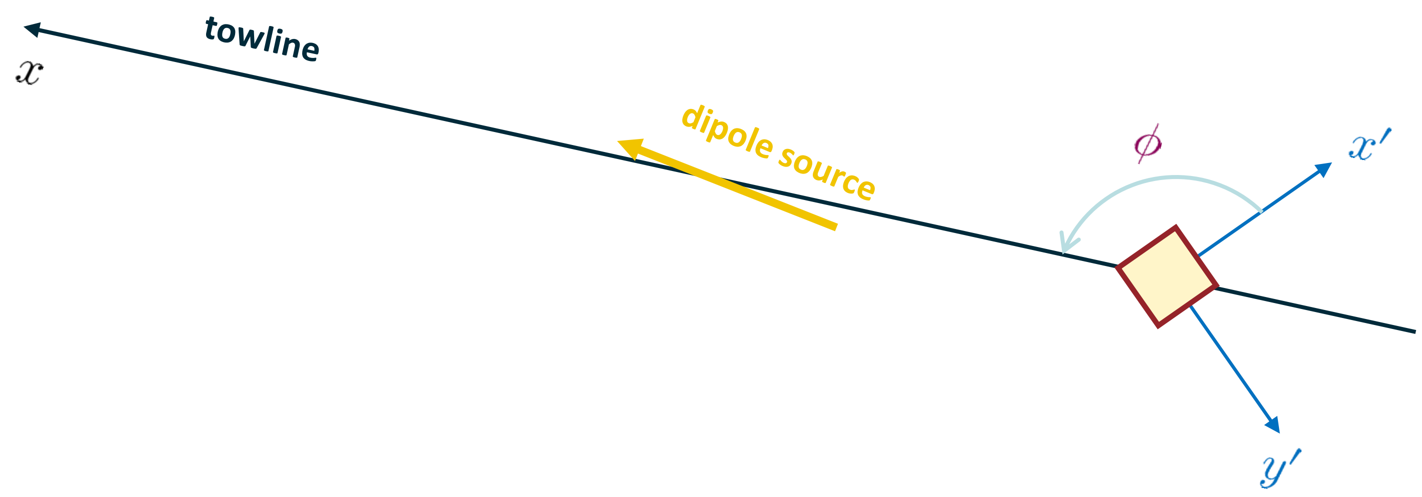
CSEM receiver orientation w.r.t. towline



The source heading relative to north is known from navigation.

The task therefore reduces to estimating the unknown receiver orientation relative to the known source dipole axis.

CSEM receiver orientation w.r.t. Dipole



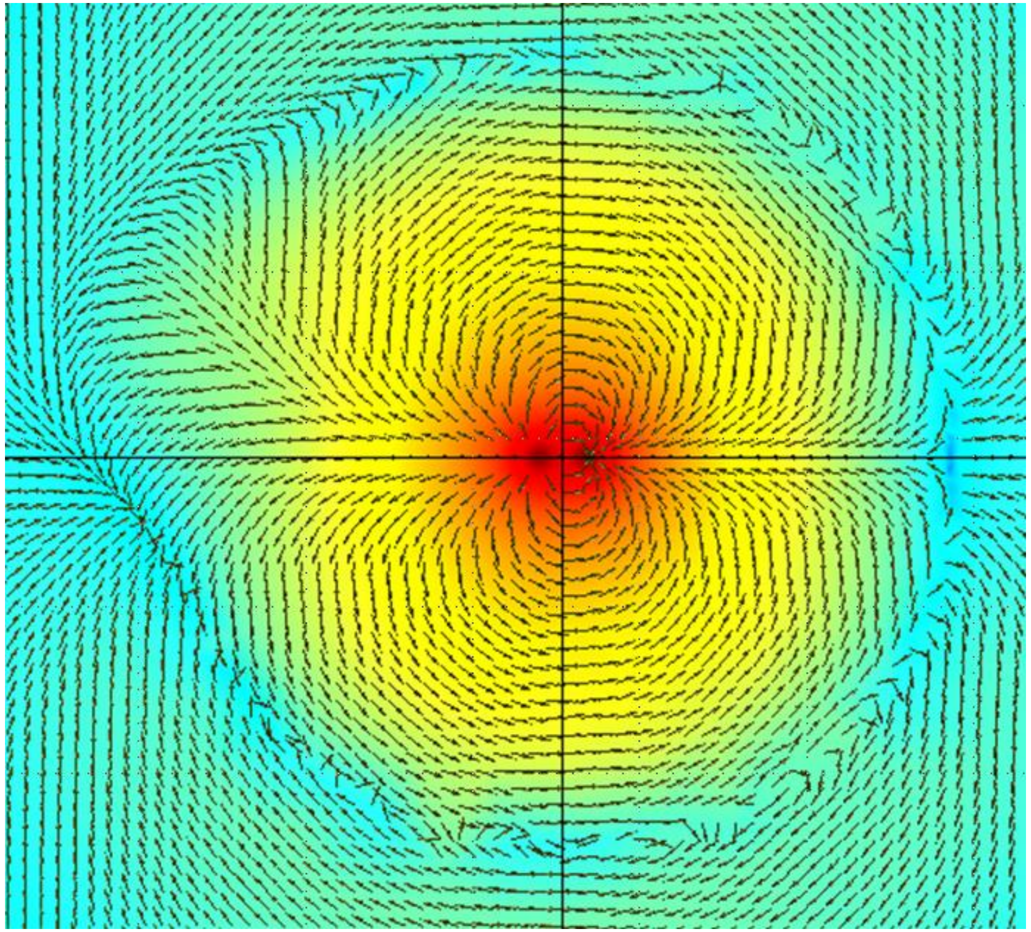
1D inversion: Predict data for 3 layer model.

Dipole direction and source and receiver positions are known.

Minimize difference between observed and predicted electric and magnetic data

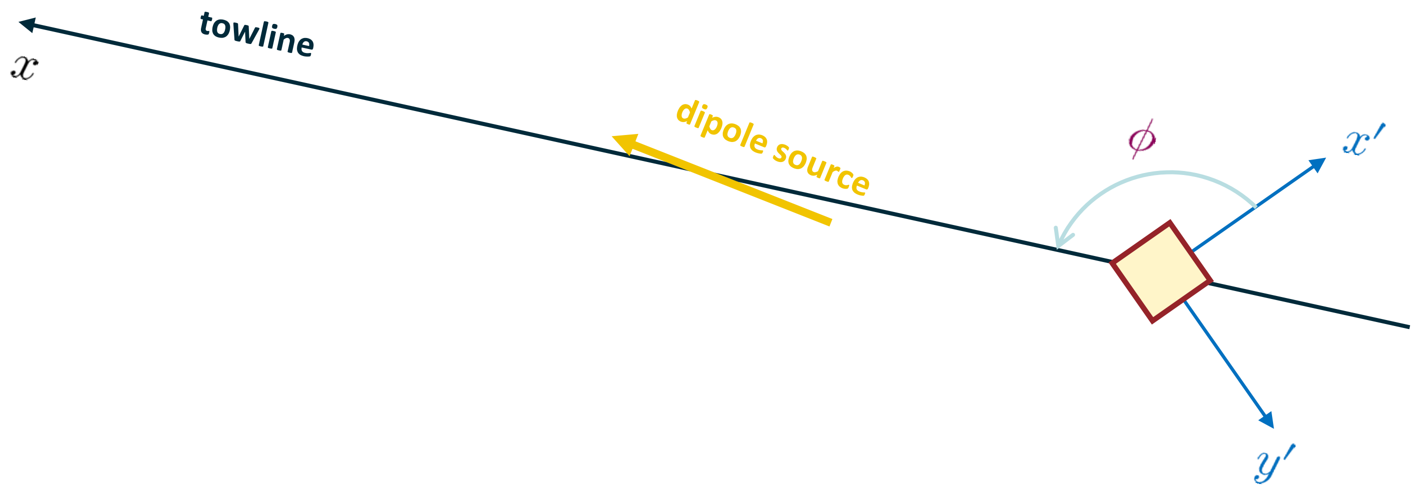
Only unknown is φ.

Local geology (e.g. shallow resistors, bathymetry) perturbs the
polarization of electric and magnetic fields



New generation of CSEM receivers will measure orientation by Independent method

rotation of field components



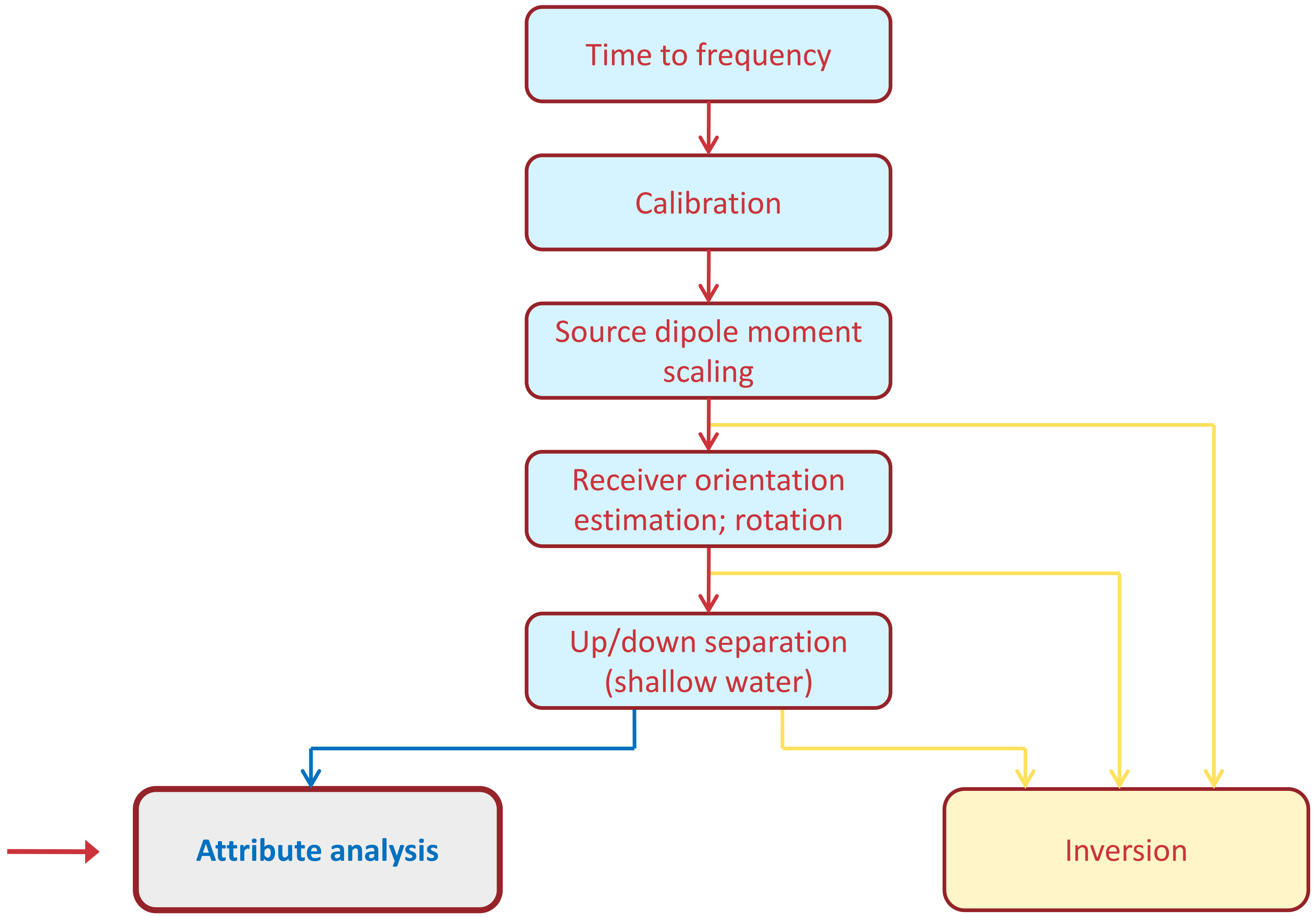
- Once the estimate for the CSEM receiver orientation w.r.t. the dipole has been obtained, we rotate to x-axis pointing in towline direction

$$\begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} E_{x'} \\ E_{y'} \\ E_{z'} \end{pmatrix} \qquad \begin{pmatrix} H_x \\ H_y \\ H_z \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} H_{x'} \\ H_{y'} \\ H_{z'} \end{pmatrix}$$

- Alternative, rotate to x-axis pointing north

- The **rotated fields** $E_x, E_y, E_z, H_z, H_y, H_z$ are used further in the workflow

CSEM data processing: overview



SPOT THE DIFFERENCE.

Thank you