emgs

LECTURE 1

Rune Mittet
Chief Scientist, EMGS
Adjunct Professor, NTNU

Spot the difference.

Schedule

Wednesday

08:30 Lecture
10:15 Coffee
10:30 Lecture
12:15 Lunch
13:15 Lecture
15:00 Coffee
15:15 Lecture
16:30 End

Thursday

08:30 Lecture
10:15 Coffee
10:30 Lecture
12:15 Lunch
13:15 Lecture
15:00 Coffee
15:15 Lecture
16:30 End

Skindepth and phase velocity – effects on data acquisition and data processing

Understand amplitude (MVO) and phase (PVO) plots

Basic understanding of propagations paths for EM fields in marine CSEM

Inversion: Data misfit – data uncertainty

Up-down decomposition

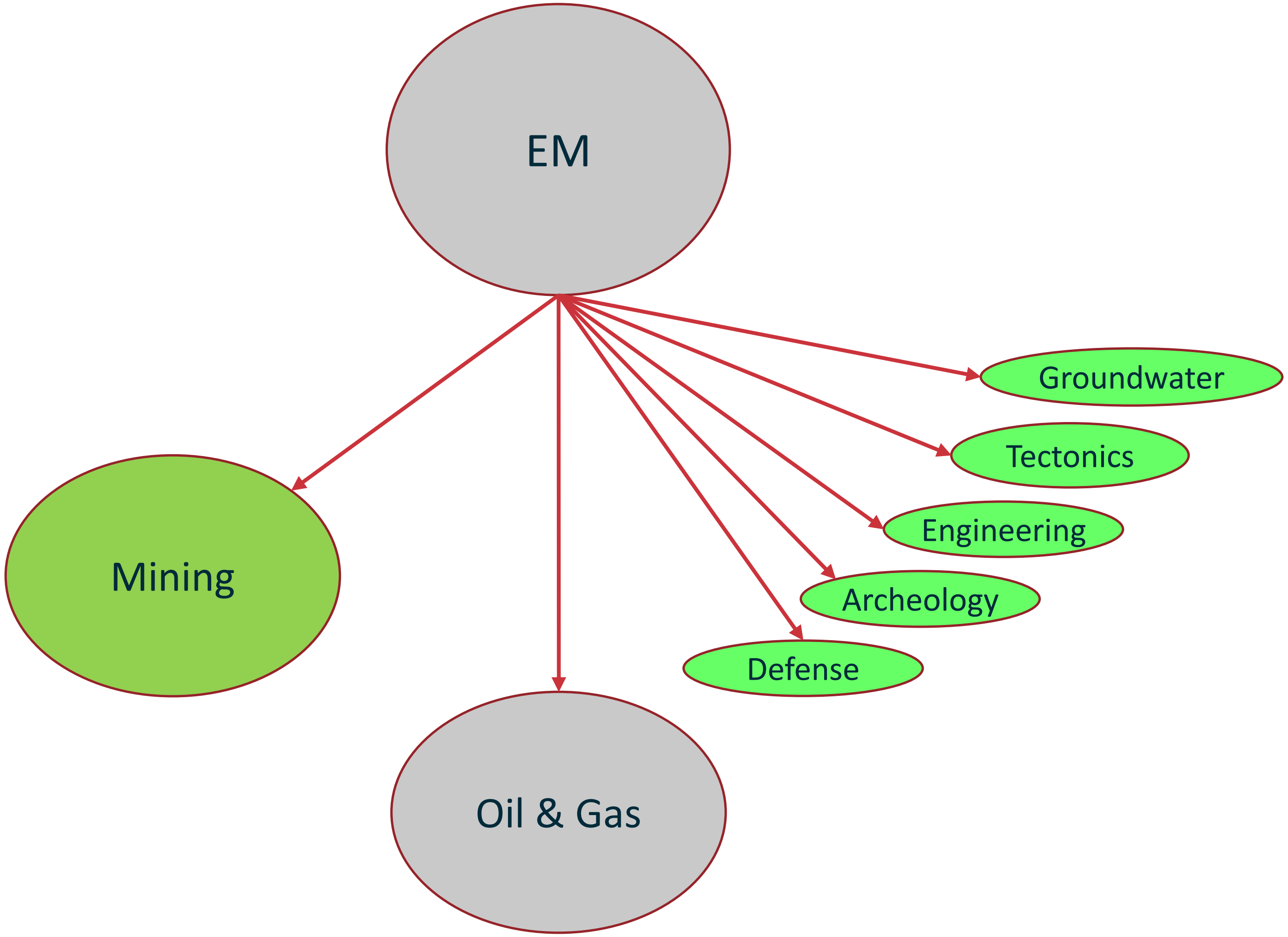
Shallow water

Introduction
Applications
Transmitter
Electric field receiver
Magnetic field receiver

Maxwell equations - Divergence and curl operators
The quasi-static approximation
Maxwell equations in 1D
Skindepth and phase velocity

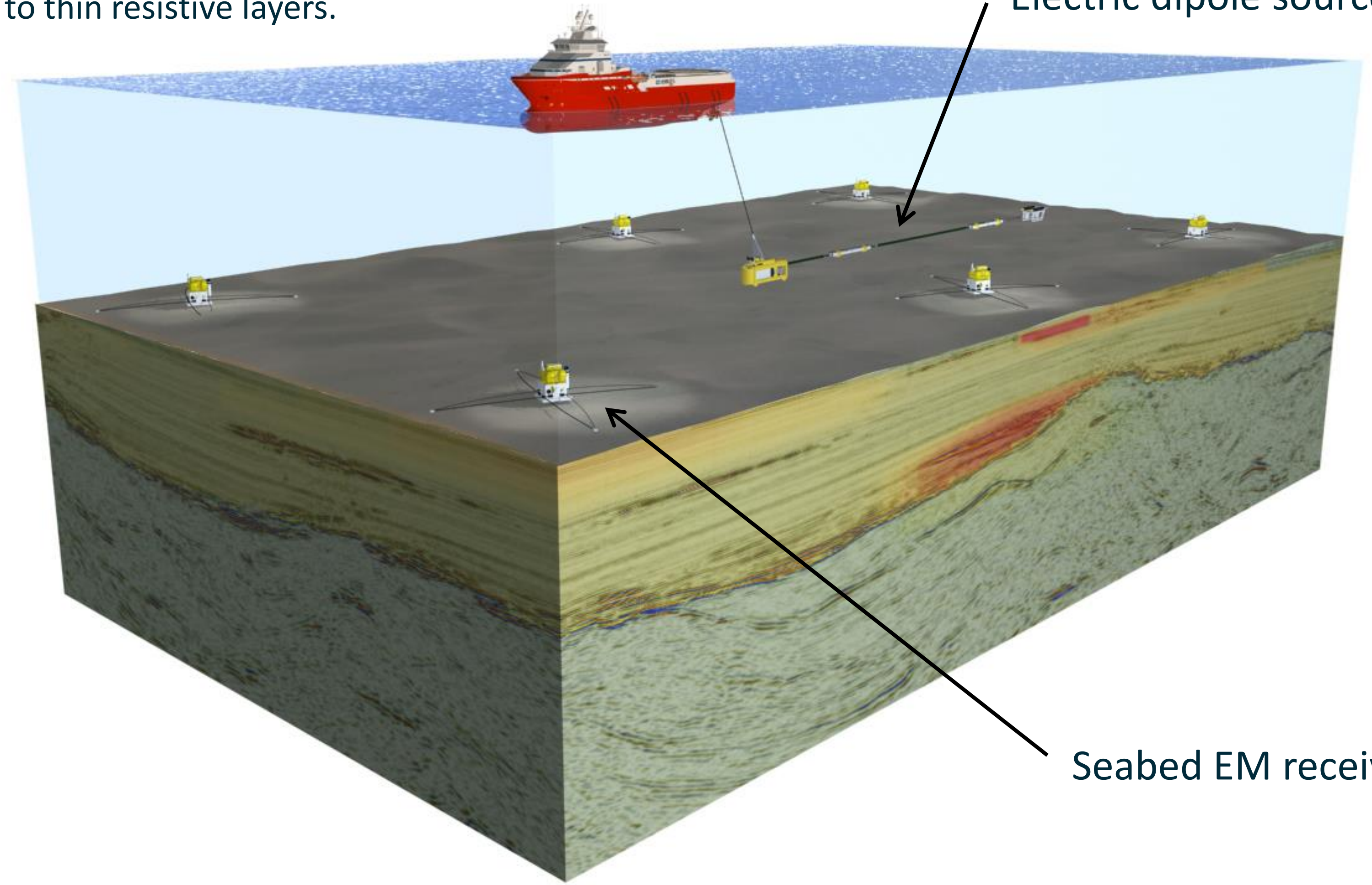
Introduction

Range of applicationS in geophysics



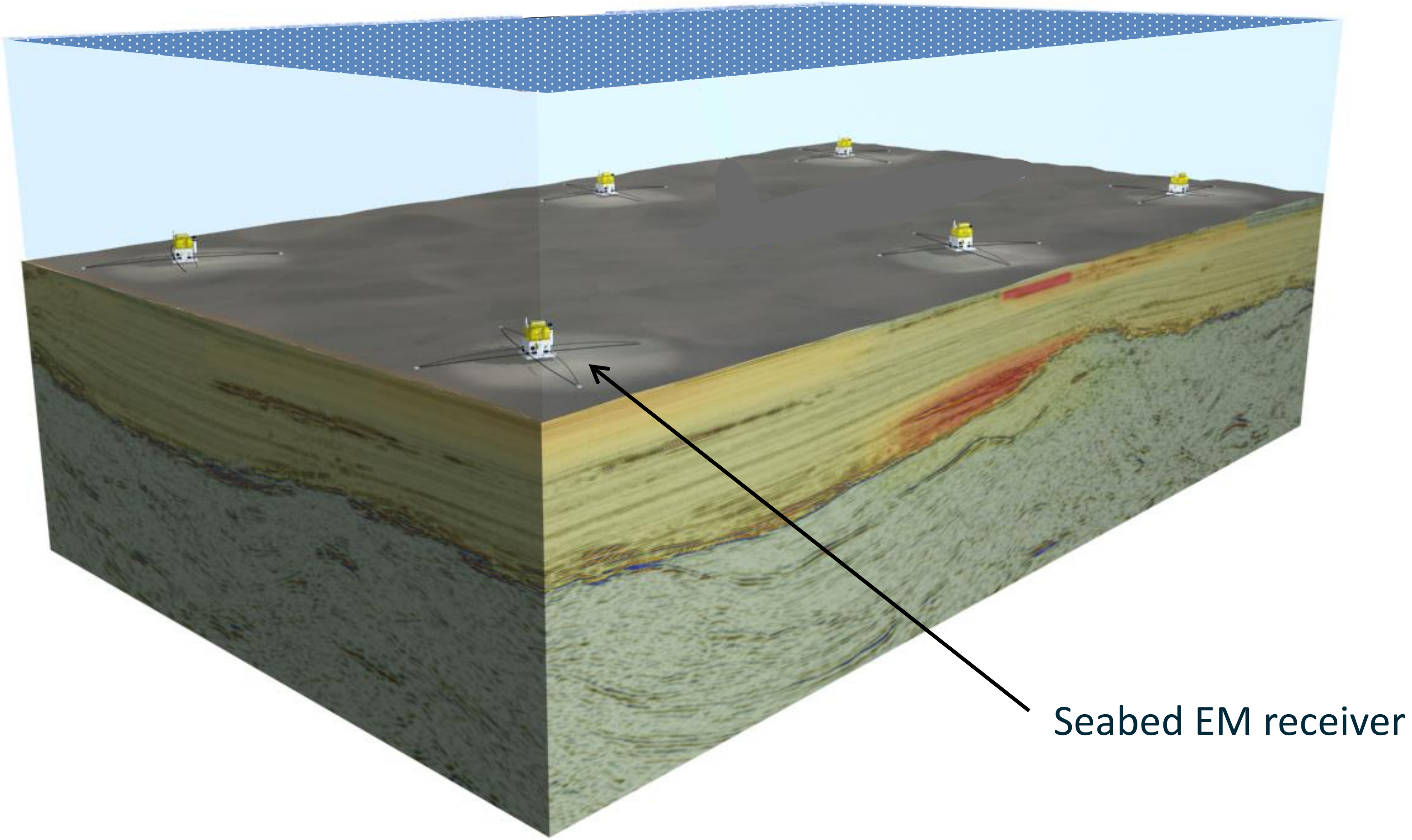
Marine controlled-Source Electromagnetics (CSEM)

Marine CSEM measures formation resistivity remotely from the seabed.
Active source.
Sensitive to thin resistive layers.

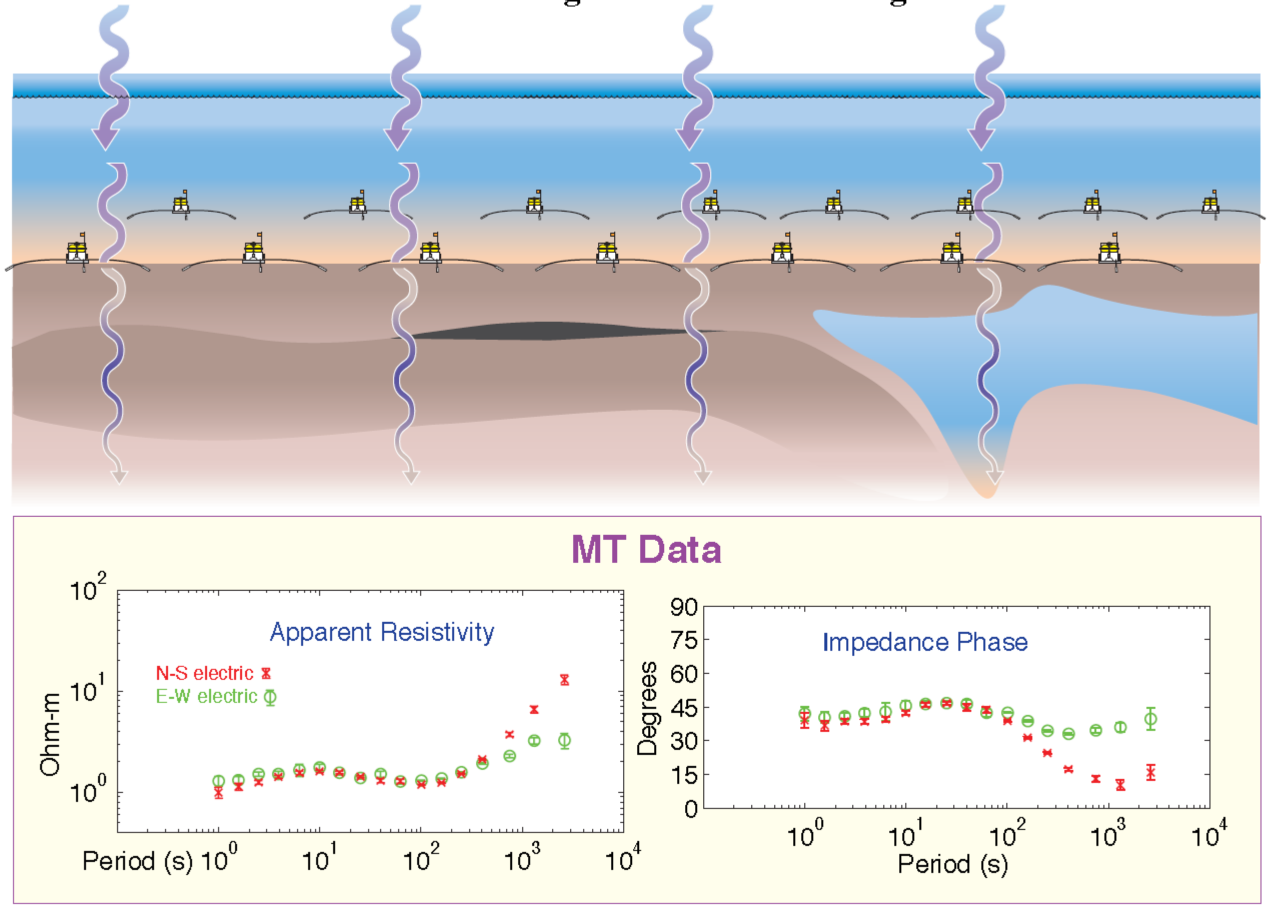


Marine magnetotellurics (MT or MMT)

MT measures formation resistivity remotely from the seabed.
No active source.
Low senstivity to thin resistive layers.

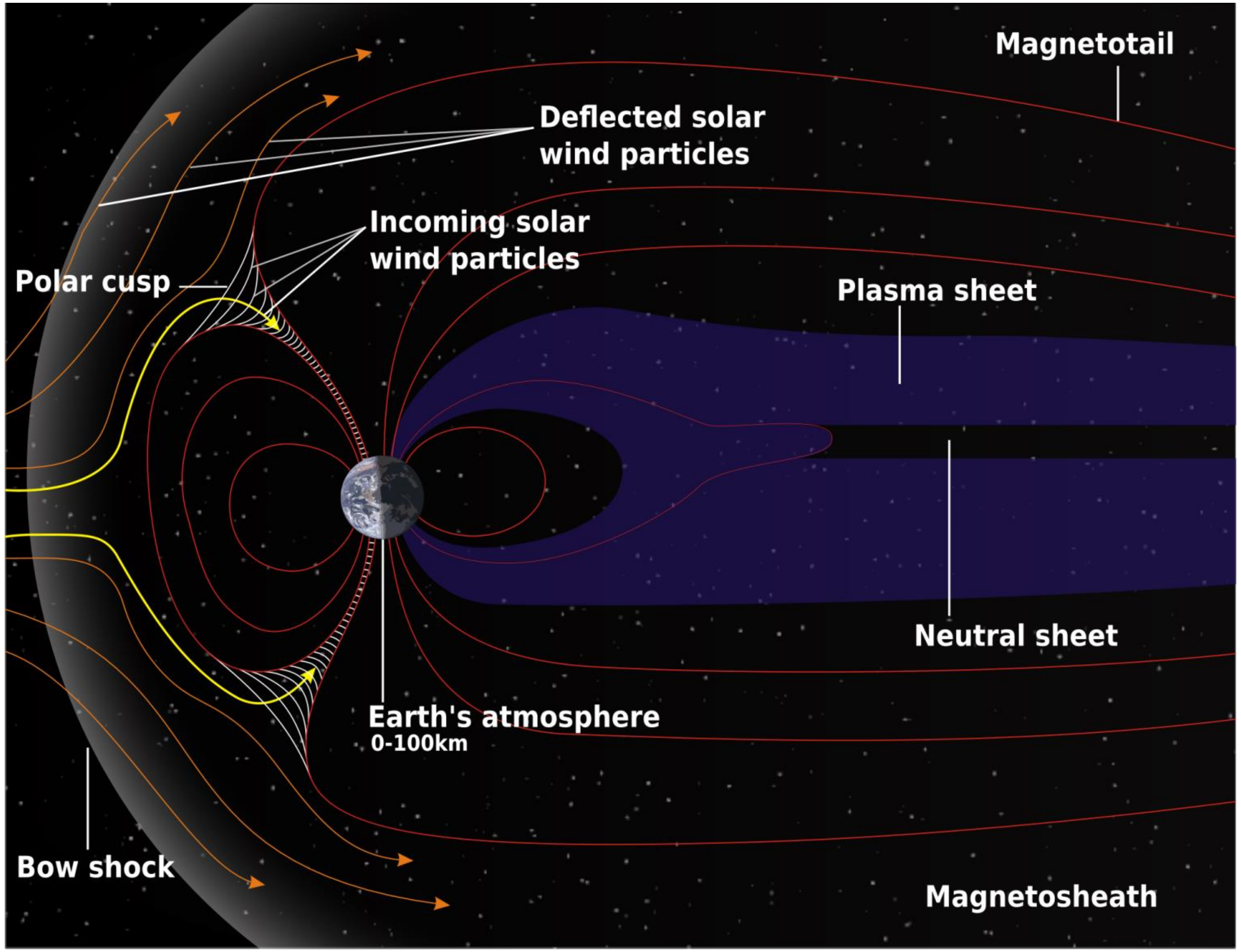


Marine Magnetotelluric Sounding



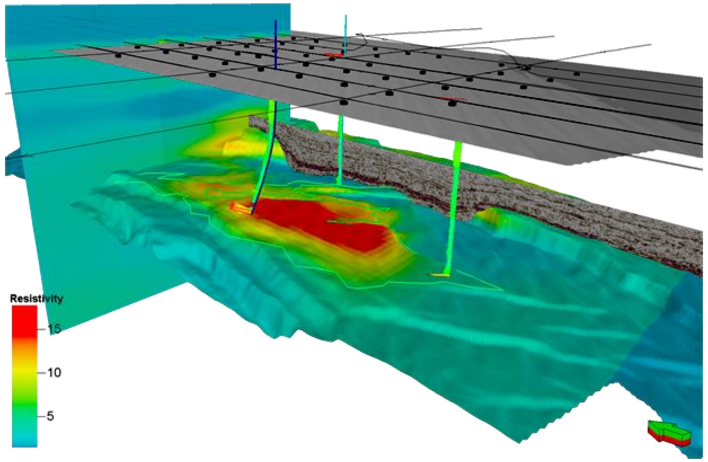
Marine EM for hydrocarbon exploration, Steven Constable and Kerry Key

A-5

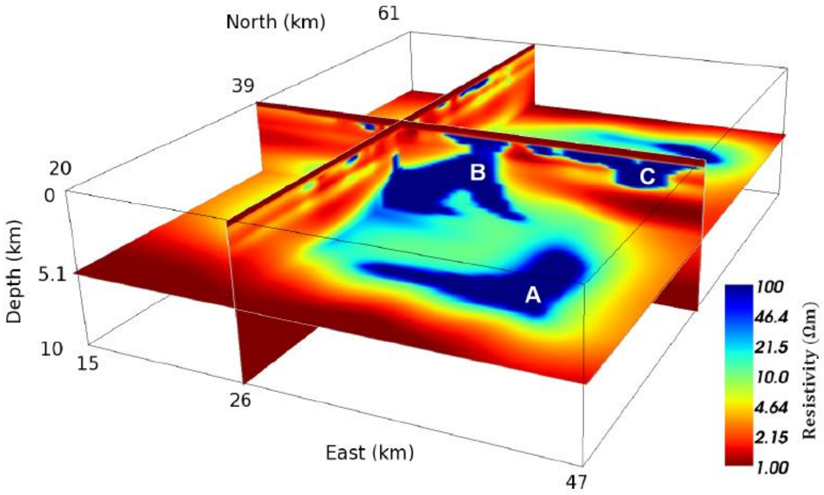

Magnetotail
Deflected solar wind particles
Incoming solar wind particles
Polar cusp
Plasma sheet
Neutral sheet
Earth's atmosphere
0-100km
Bow shock
Magnetosheath

The end product of CSEM and MT processing are resistivity volumes

CSEM

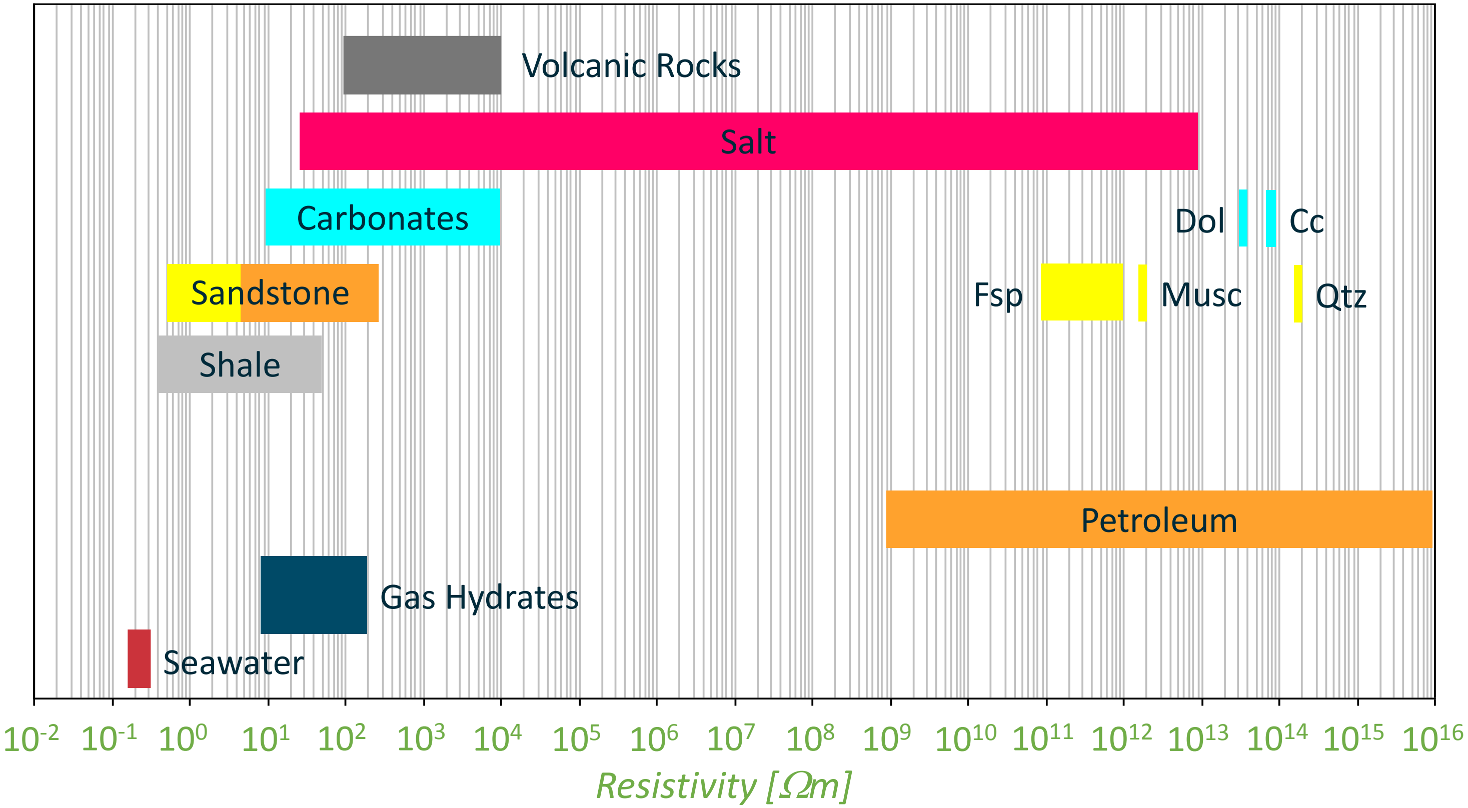


MT



CSEM data sensitive to thin resistive layers.

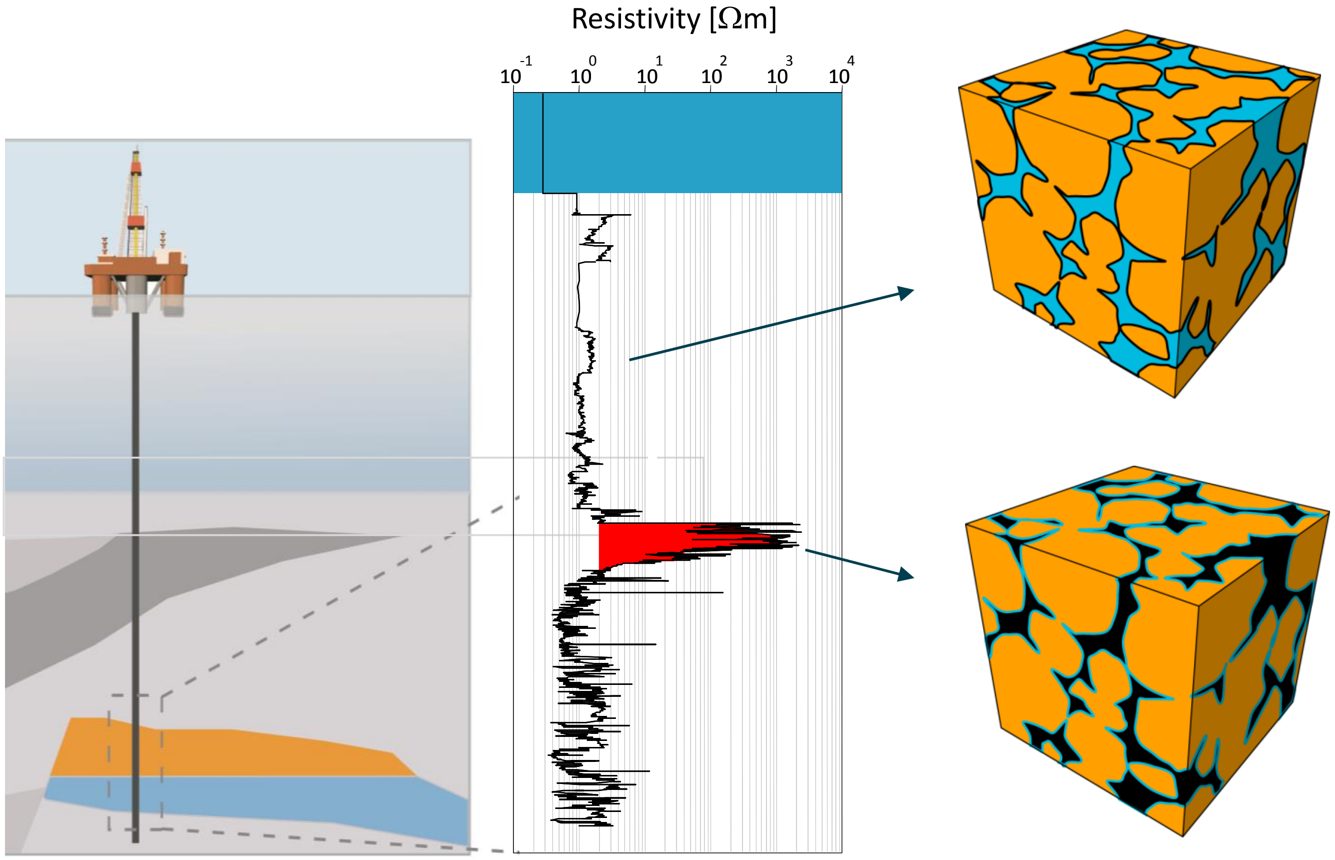
Range of resistivities of Earth materials



Resistivity varies over many orders of magnitude in Earth materials.

The resistivity of a reservoir rock is largely dependent on its porosity and the resistivity of the fluids contained in the pore space.

Resistivity is a hydrocarbon indicator



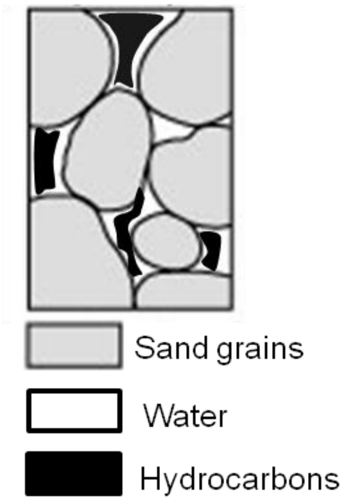
Water-bearing sediments:
1 - 2 Ωm

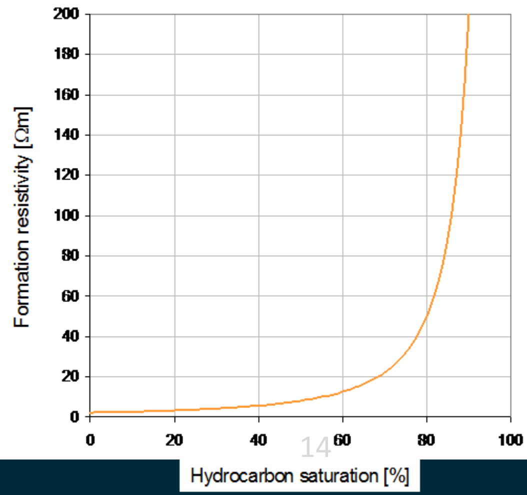
Hydrocarbon reservoirs:
10 - 200 Ωm

Resistivity well logging is a standard measurement performed in all wells drilled into a (potential) hydrocarbon reservoir.

Reservoir resistivity in terms of rock properties

Clean sand with hydrocarbons





Archie's law

Empirical law proposed by Gus Archie of Shell Oil (1942)

True resistivity In terms of the brine saturated formation resistivity and hydrocarbon saturation

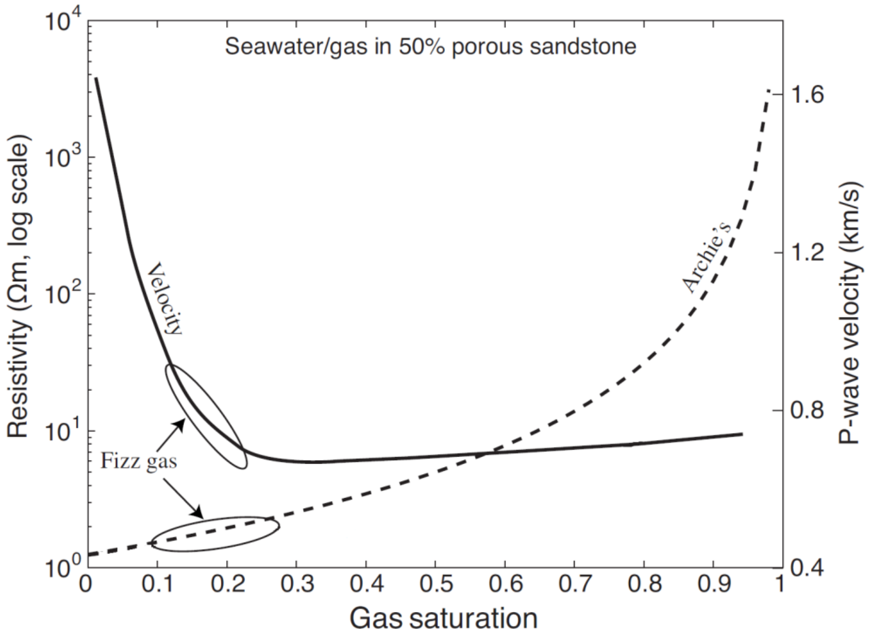
$$\rho_t = \frac{\rho_0}{(1 - S_{HC})^n}$$

ρ_0	Brine saturated formation resistivity

S_{HC}	Hydrocarbon saturation (fraction of pore space filled with hydrocarbons)

Typically $n=2$ is used when no log or core calibration is available.

Distinguishing low from high saturation: The Fizz Gas problem



(Constable, 2010, Geophysics)

P-wave velocity changes drastically when a small amount of gas is introduced into the pore fluid.

→ Risk of drilling dry holes on structures characterized by a seismic amplitude anomaly.

Significant resistivity increase only occurs for high gas saturations.

→ Risk reduction by combining CSEM with seismic.

Applications

Applications

I)	Hydrocarbon indicator	(CSEM)
II)	Prospect ranking	(CSEM)
III)	Structural imaging	(CSEM and MT)
IV)	Appraisal - Volume estimates	(CSEM)
V)	4D - Monitoring	(CSEM)
VI)	Drilling hazards	(CSEM)

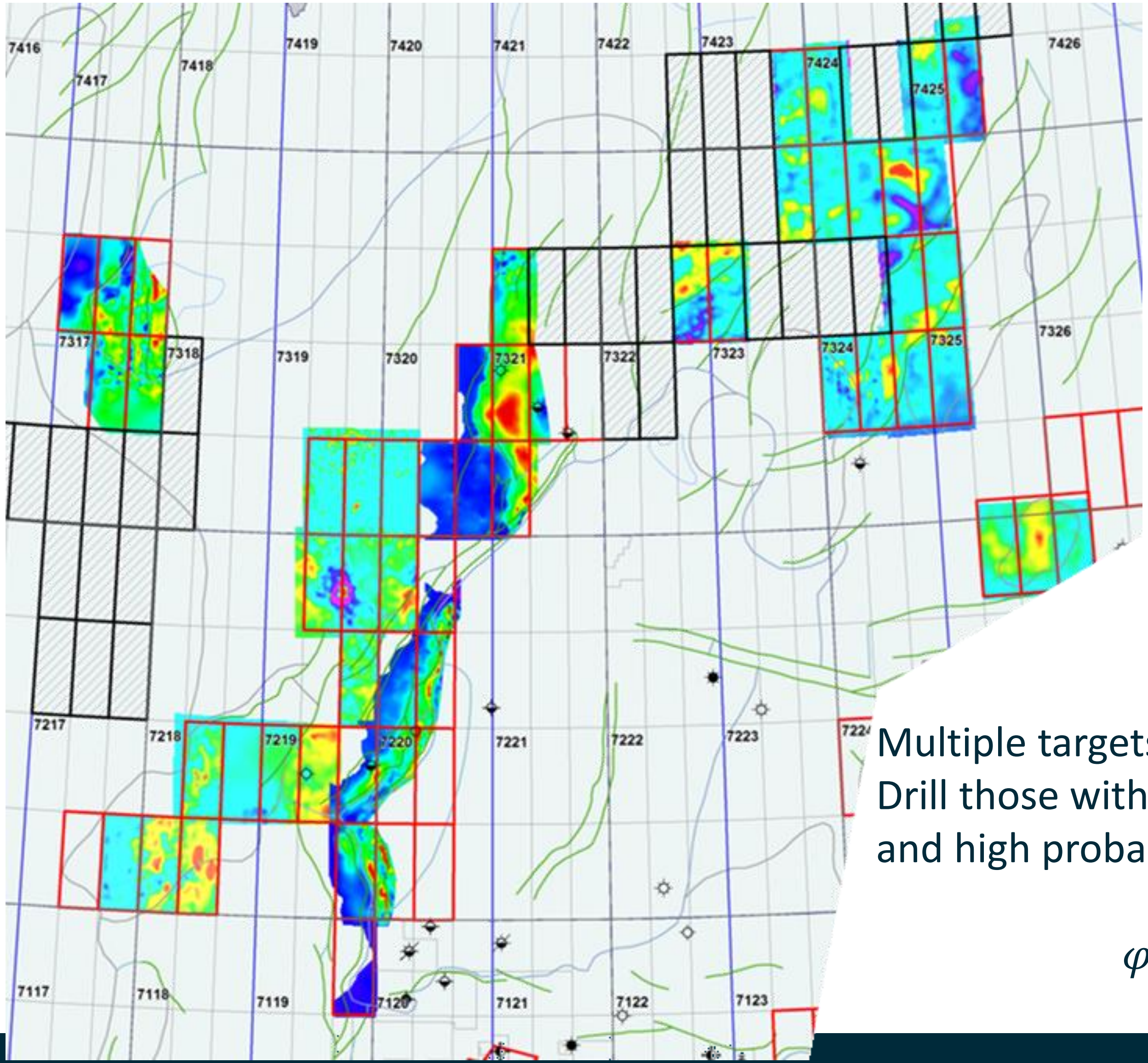
Barents sea - Hydrocarbon indicator

Blue: Low resistivity
Green: Intermediate resistivity
Red: High resistivity

Pre CSEM: Seismic anomaly.

CSEM: High resistivity at depth ~600 m below mudline.
CSEM data used for drill-drop decission.

Prospect ranking in the barents sea

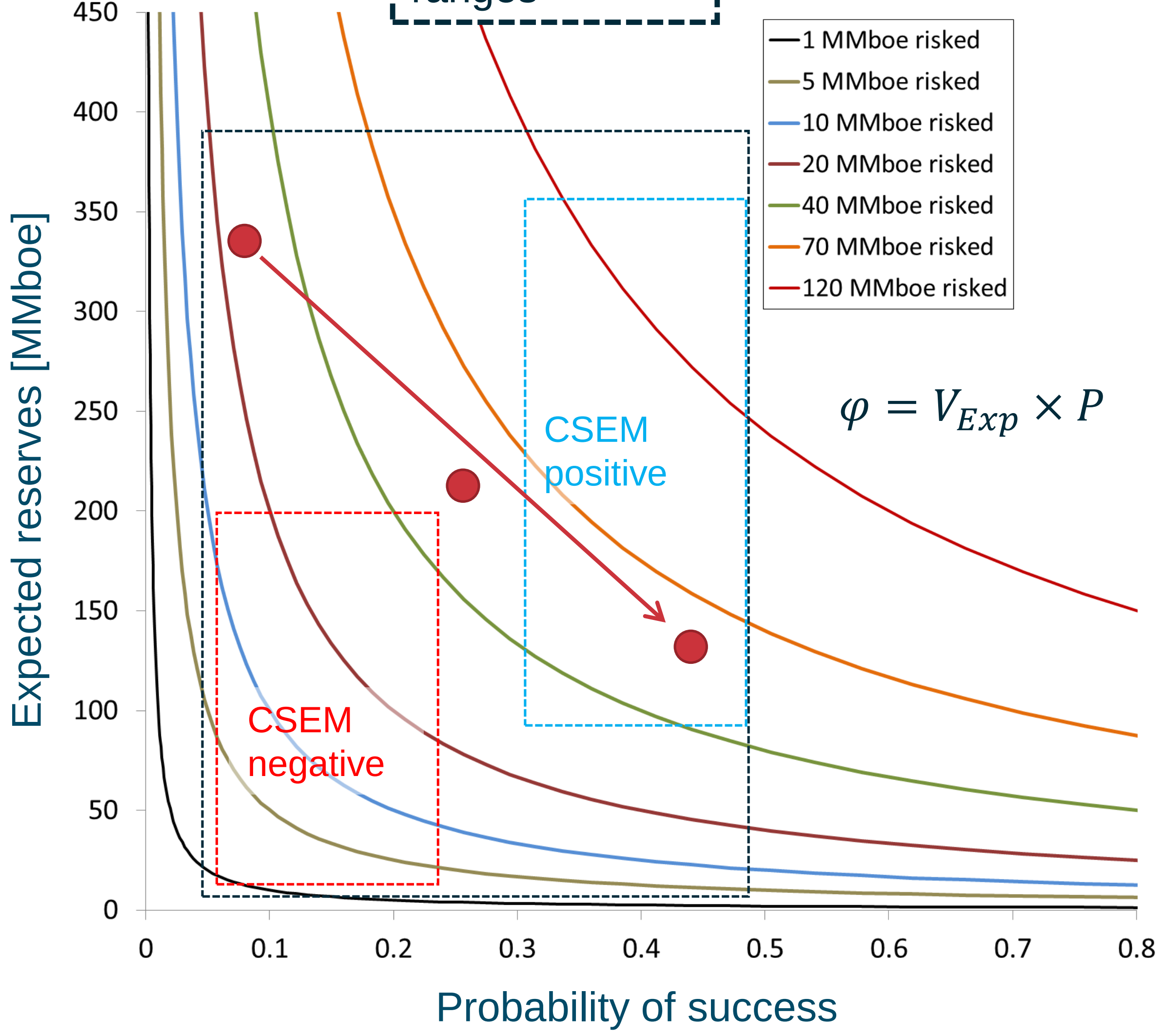


Multiple targets:
Drill those with large volume expectation and high probabillity of success first:

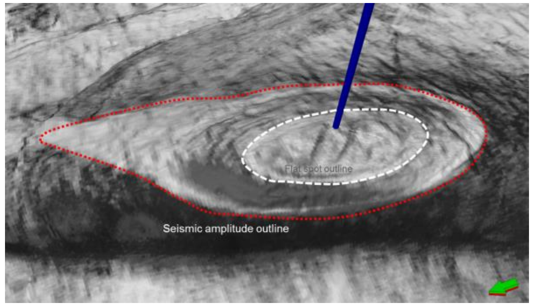
$$\varphi = V_{Exp} \times P$$

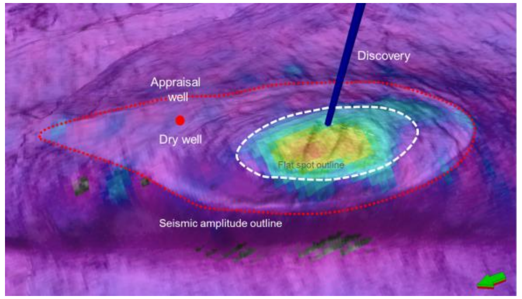
EM in the E&P wo… …ptimization

Uncertainty ranges



$$\varphi = V_{Exp} \times P$$





Seismic imaging problems

Seismic Imaging problems

/ Accurate velocity models at or below top salt/basalt

/ Image base of salt / basalt
/ Identify salt feeders
/ Image sediments below salt
/ Image deeper autochthonous salt

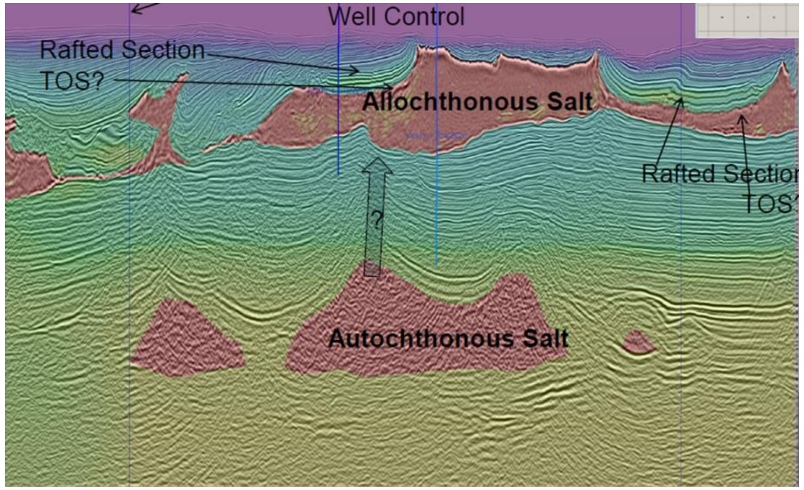
Solutions

/ Wide azimuth,long offset seismic acquisition

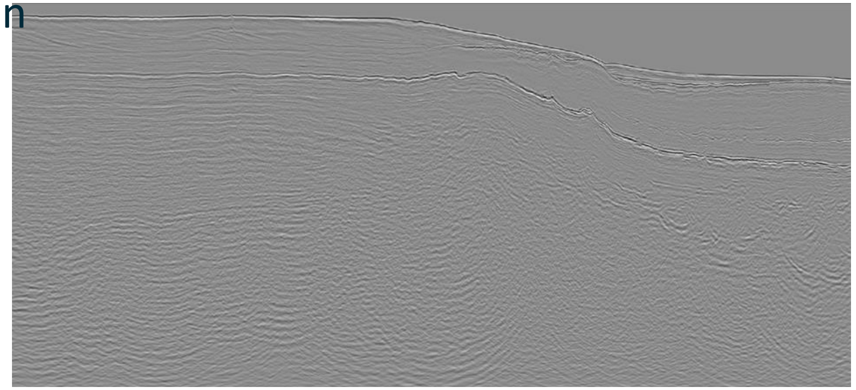
/ Seismic reverse time migration (RTM) and Full waveform inversion (FWI)

/ Acquisition of additional geophysical data (EM, potential fields)
/ Inversion and joint inversion of other geophysical data (EM, potential fields)

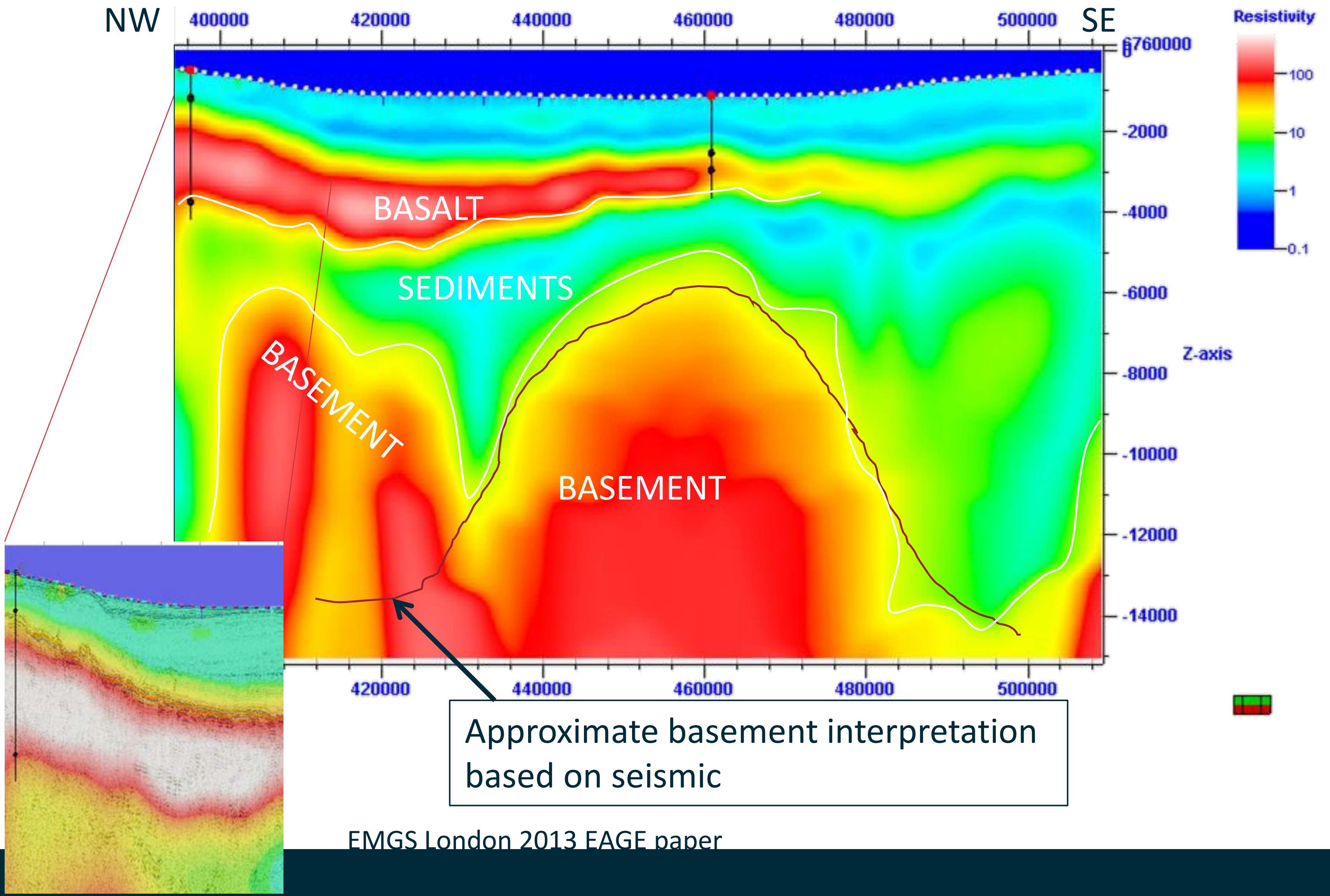
Gulf of Mexico, Keathley Canyon



Basalt – West of Sheteland

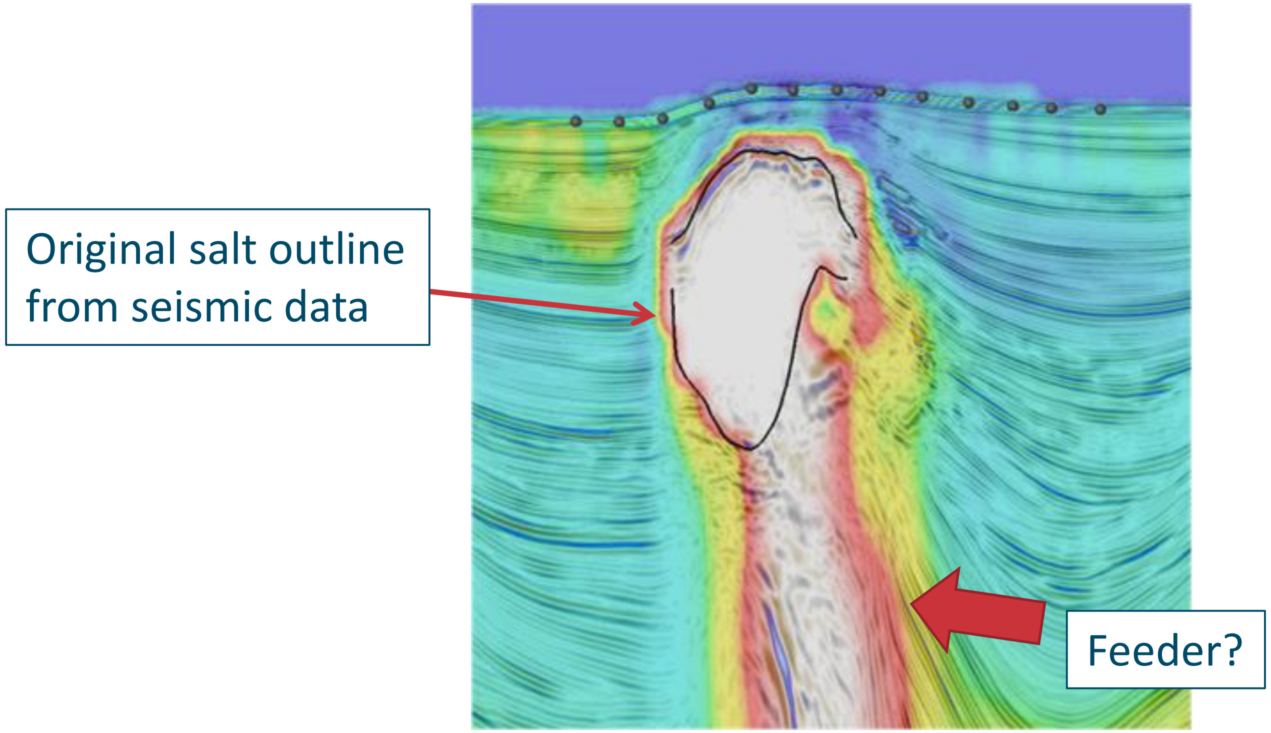


CSEM/MMT joint inversion result



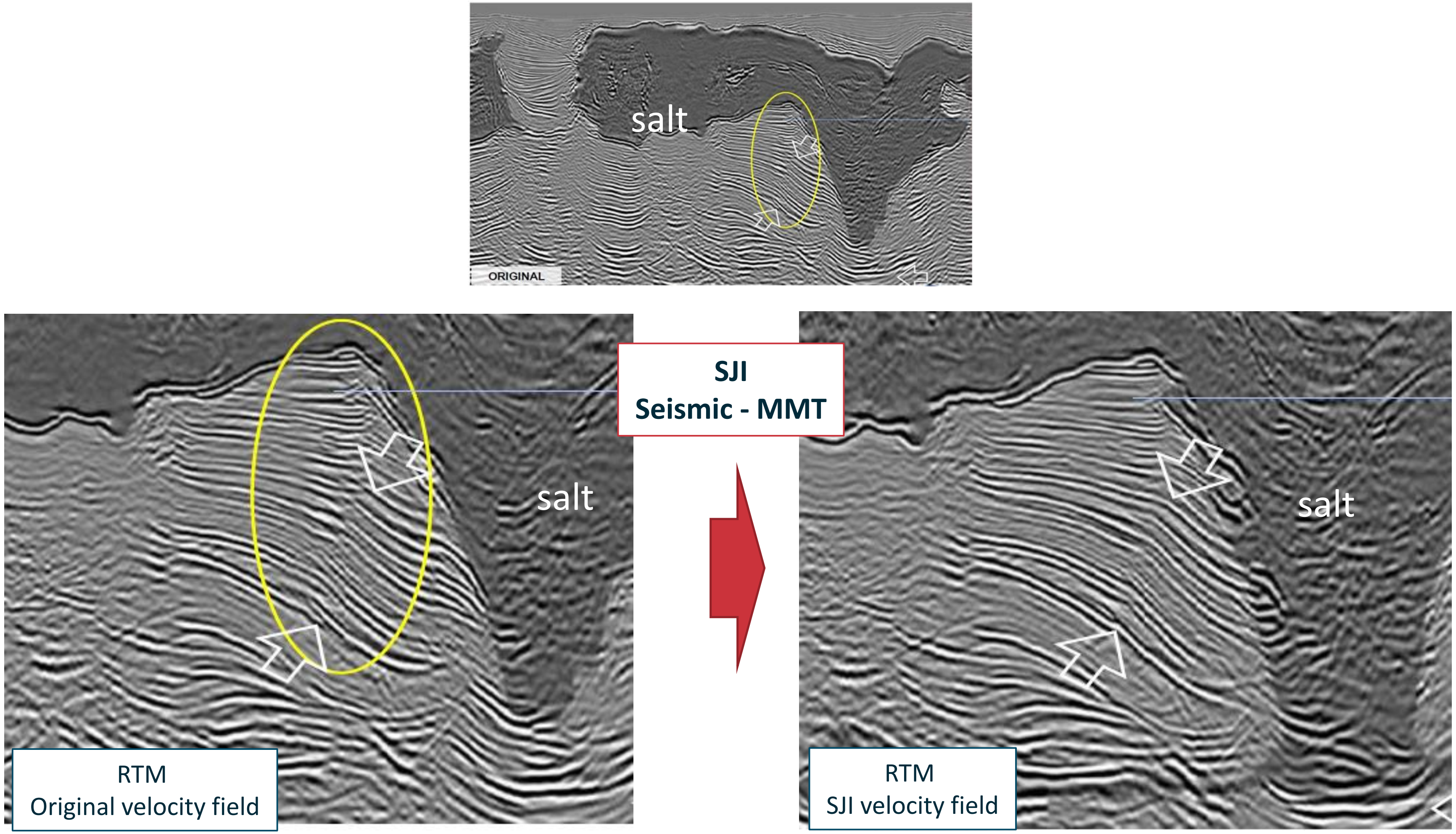
EMGS London 2013 EAGE paper

Gom – salt imaging using 3D CSEM data

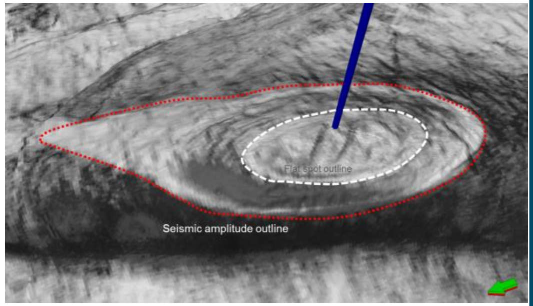


/ Identifying salt feeder
/ Differentiate salt composition with respect to resistivity
/ Identify connections between interpreted individual salt bodies

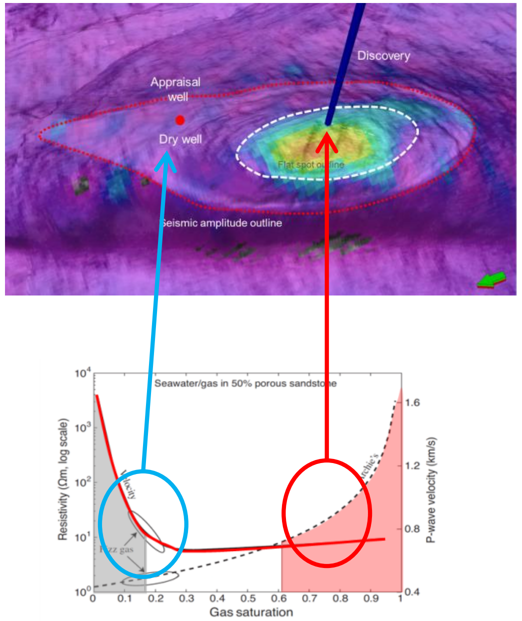
GOM - IMPROVED IMAGING BELOW SALT



CSEM IN APPRAISAL – VOLUME ESTIMATES



Courtesy of Pemex

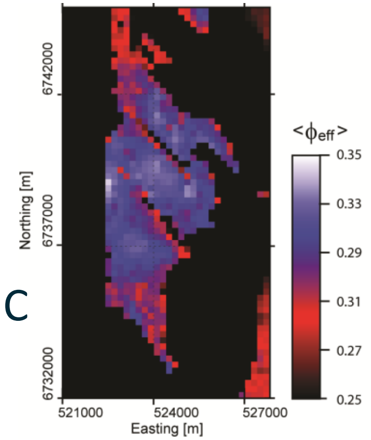


(Constable, 2010, Geophysics)

Reservoir property estimation

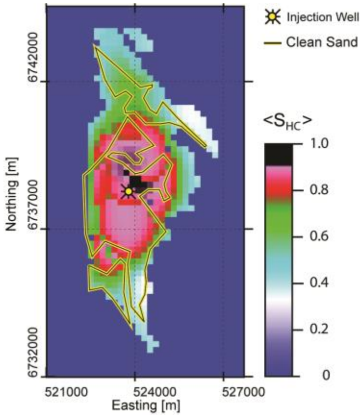
Effective porosity map

FROM SEISMIC



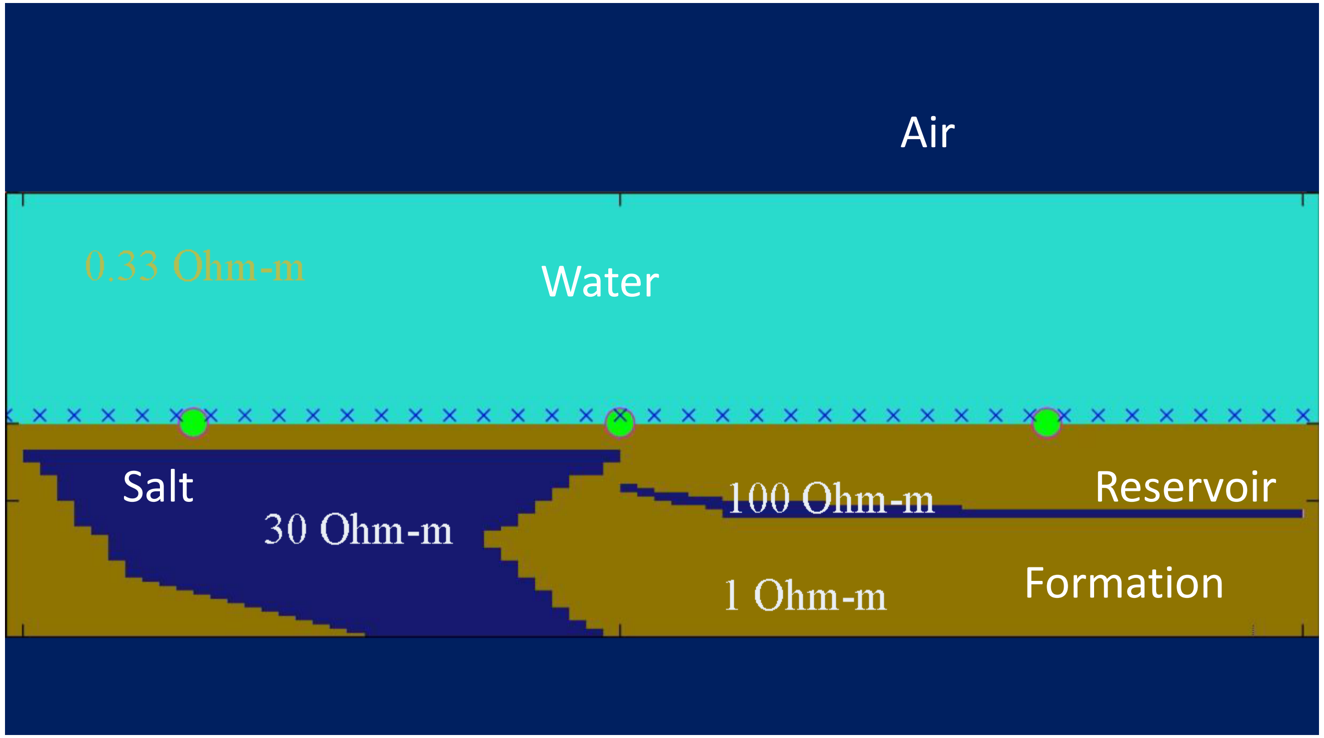
HC saturation map

FROM CSEM

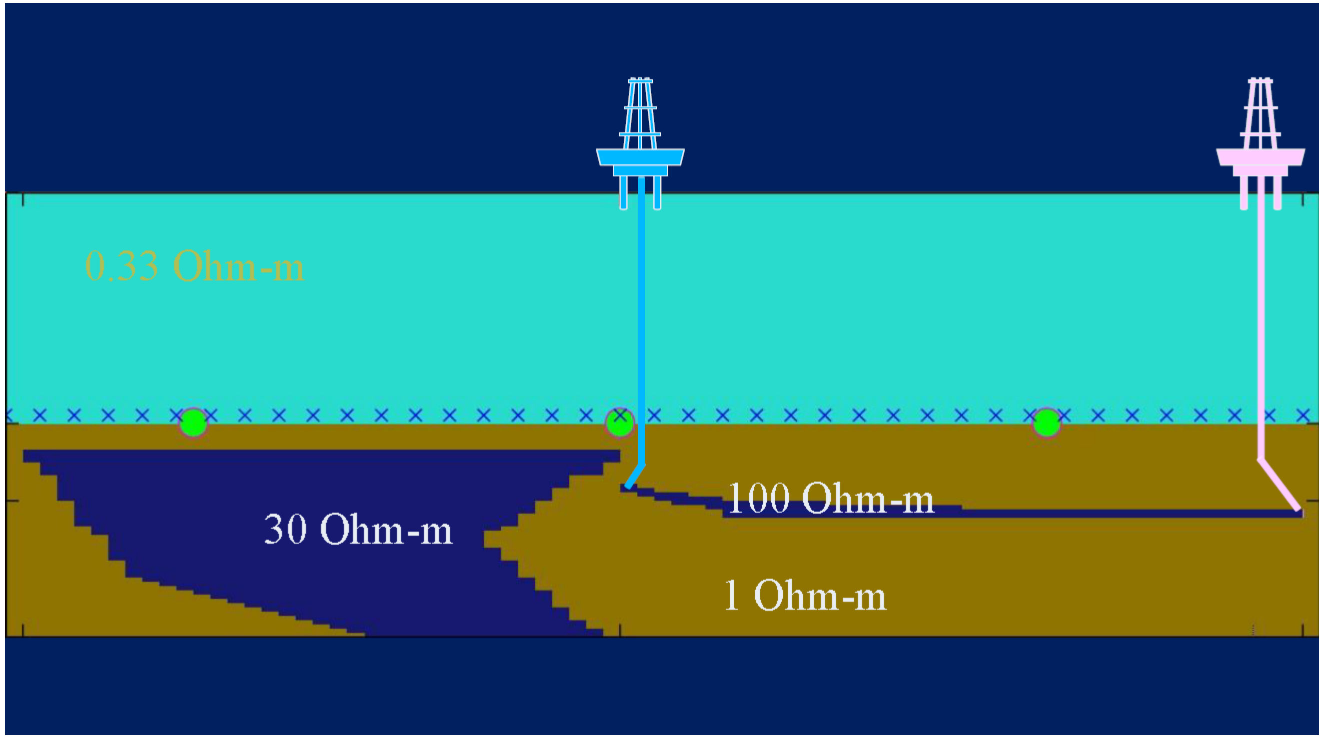


→ **HC volume estimation and saturation**

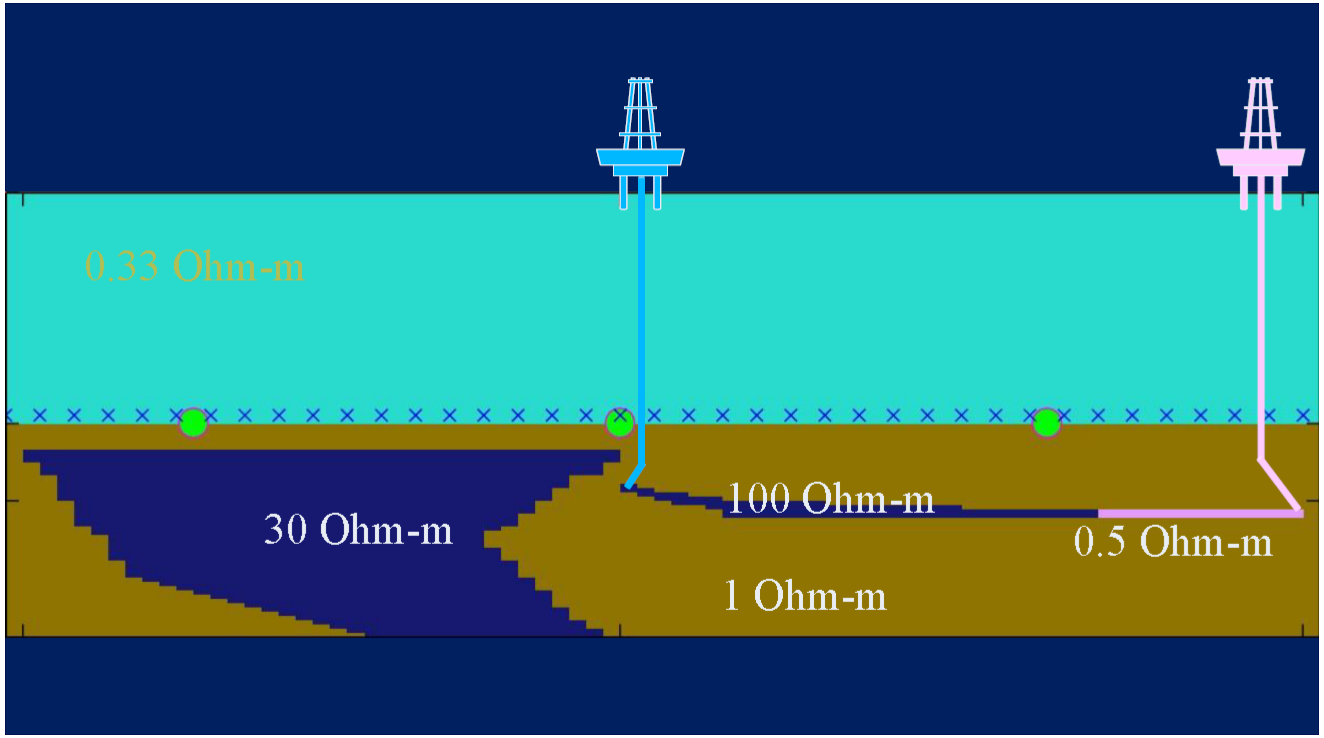
4D - Monitoring



After M. Zhdanov



After M. Zhdanov



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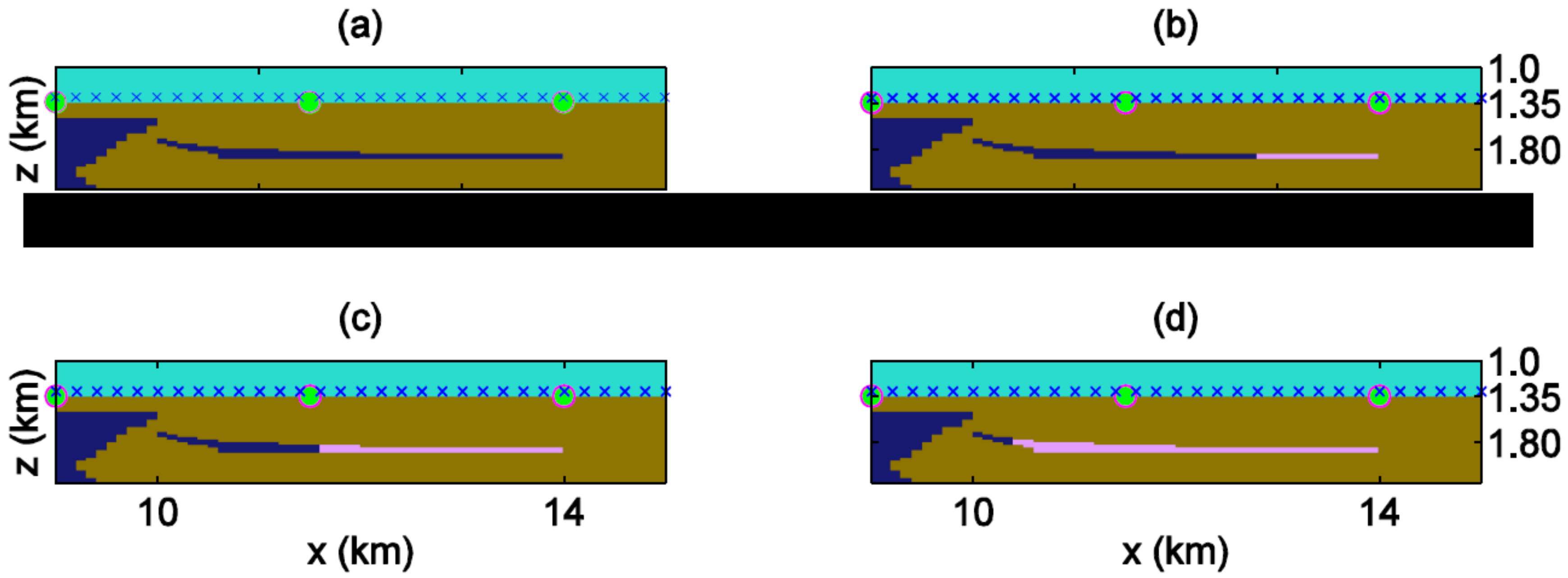
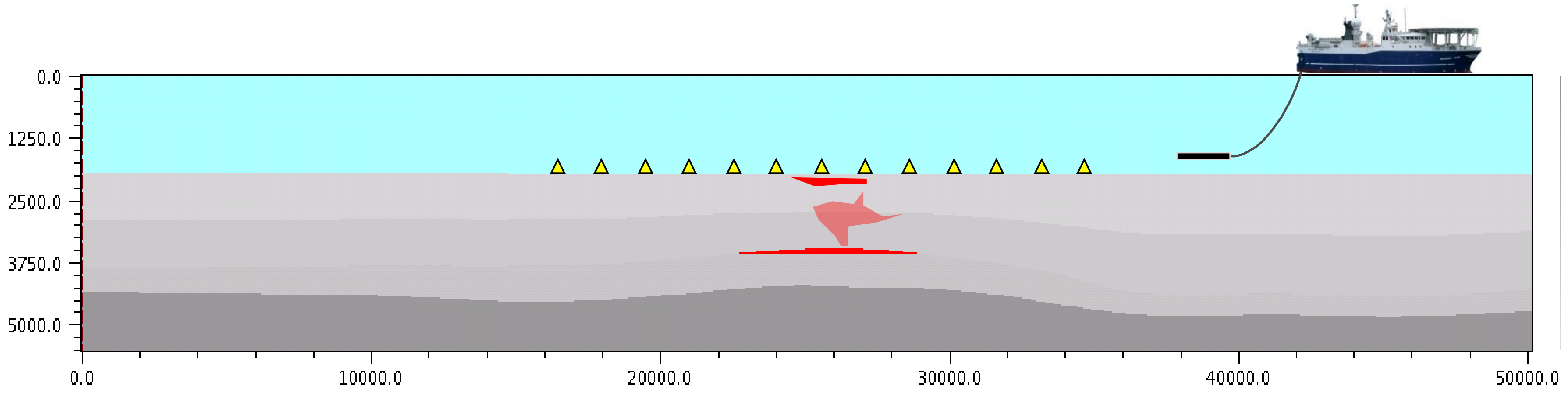


After M. Zhdanov

Drilling Hazards



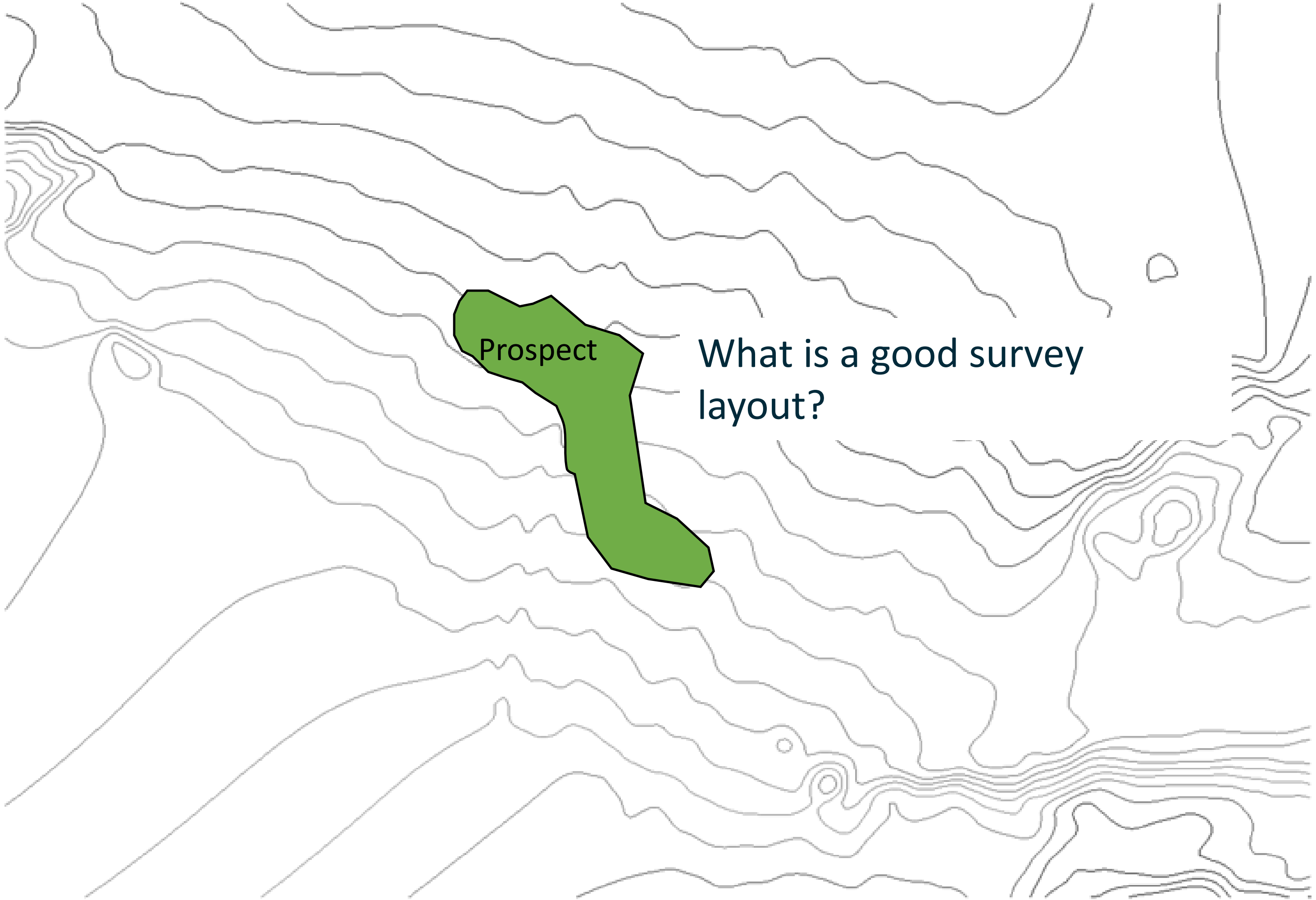
Drilling hazards that can be detected with EM:

Hydrates

Shallow gas

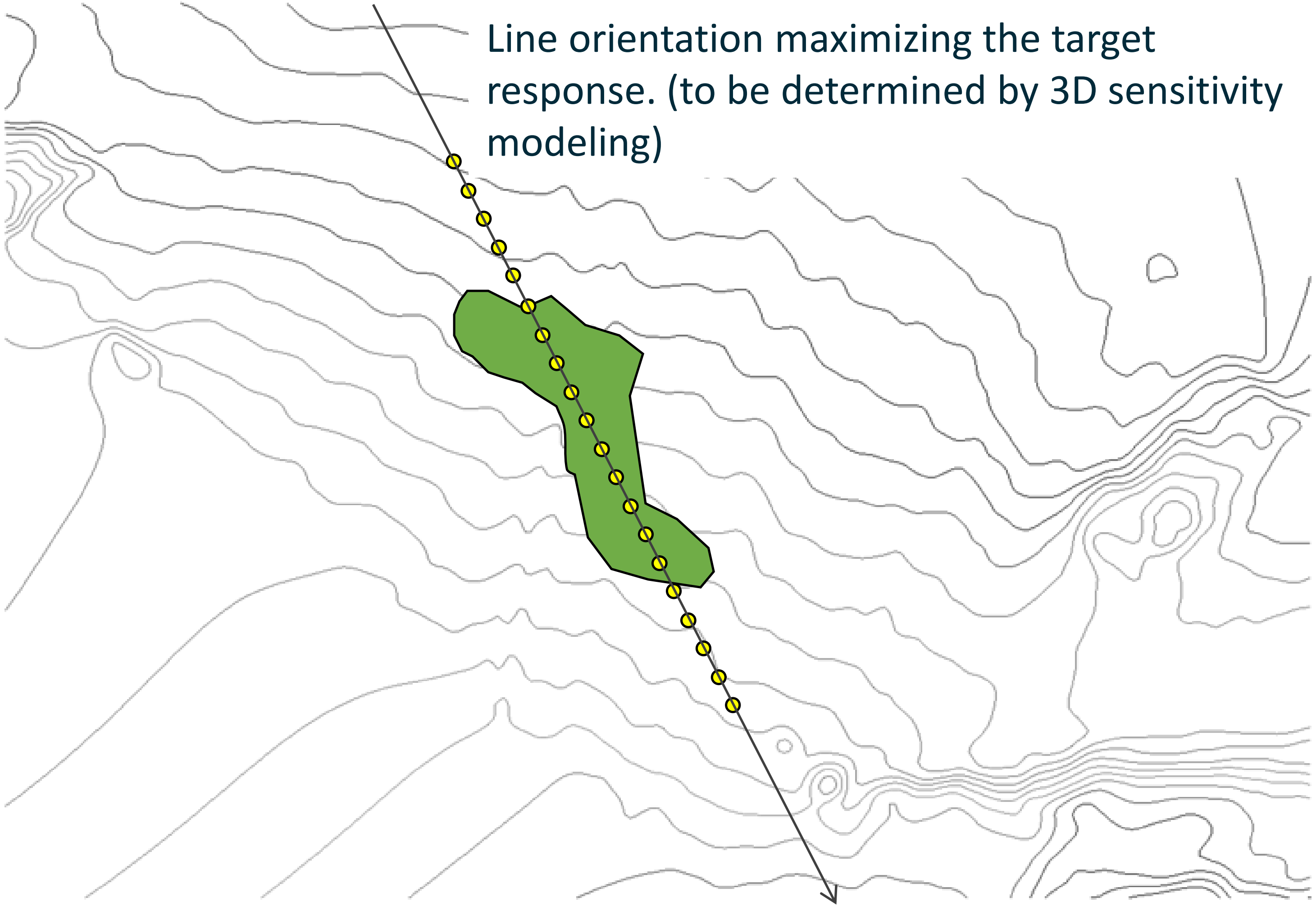
Survey layout

Layout considerations

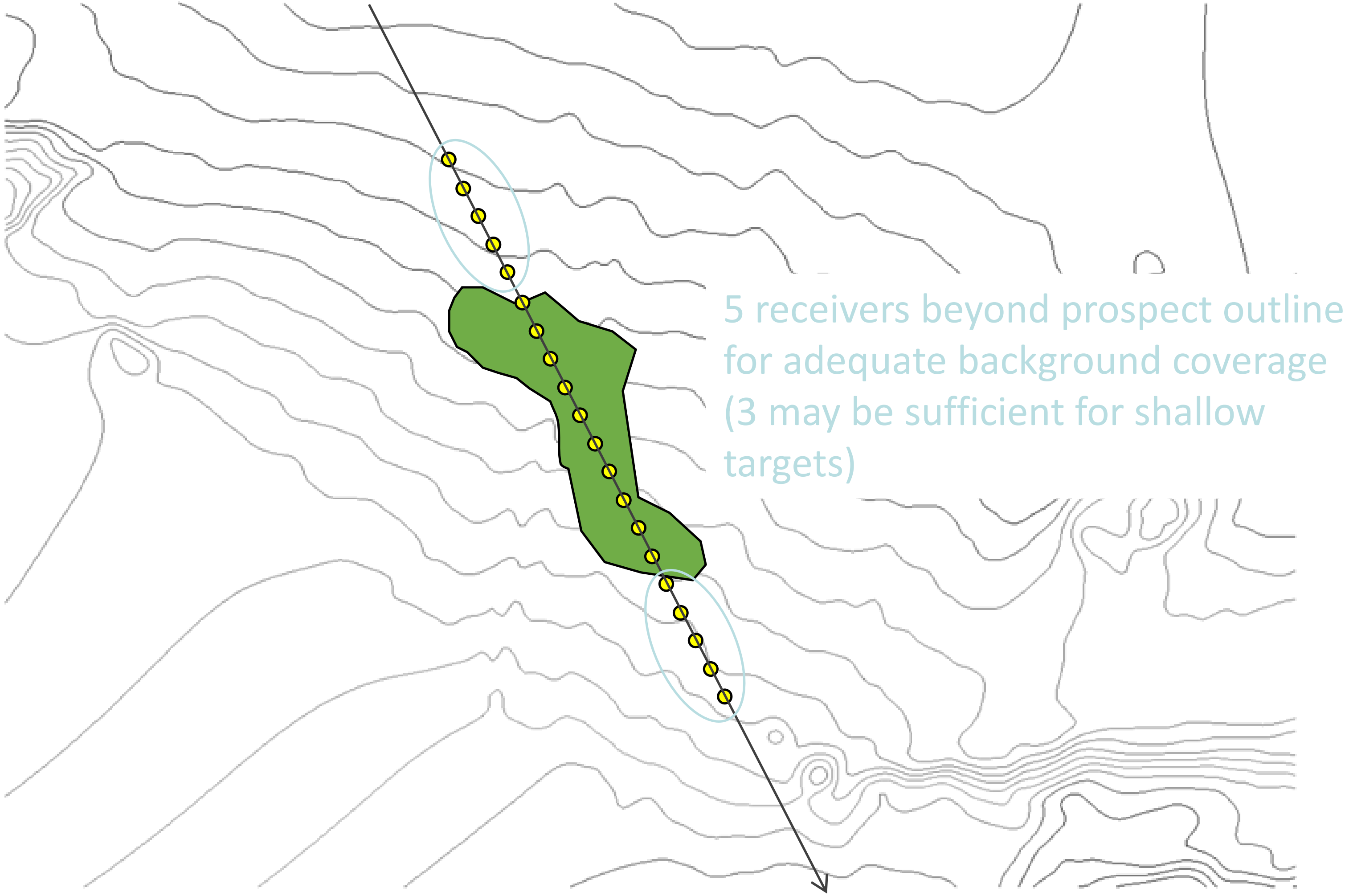


What is a good survey layout?

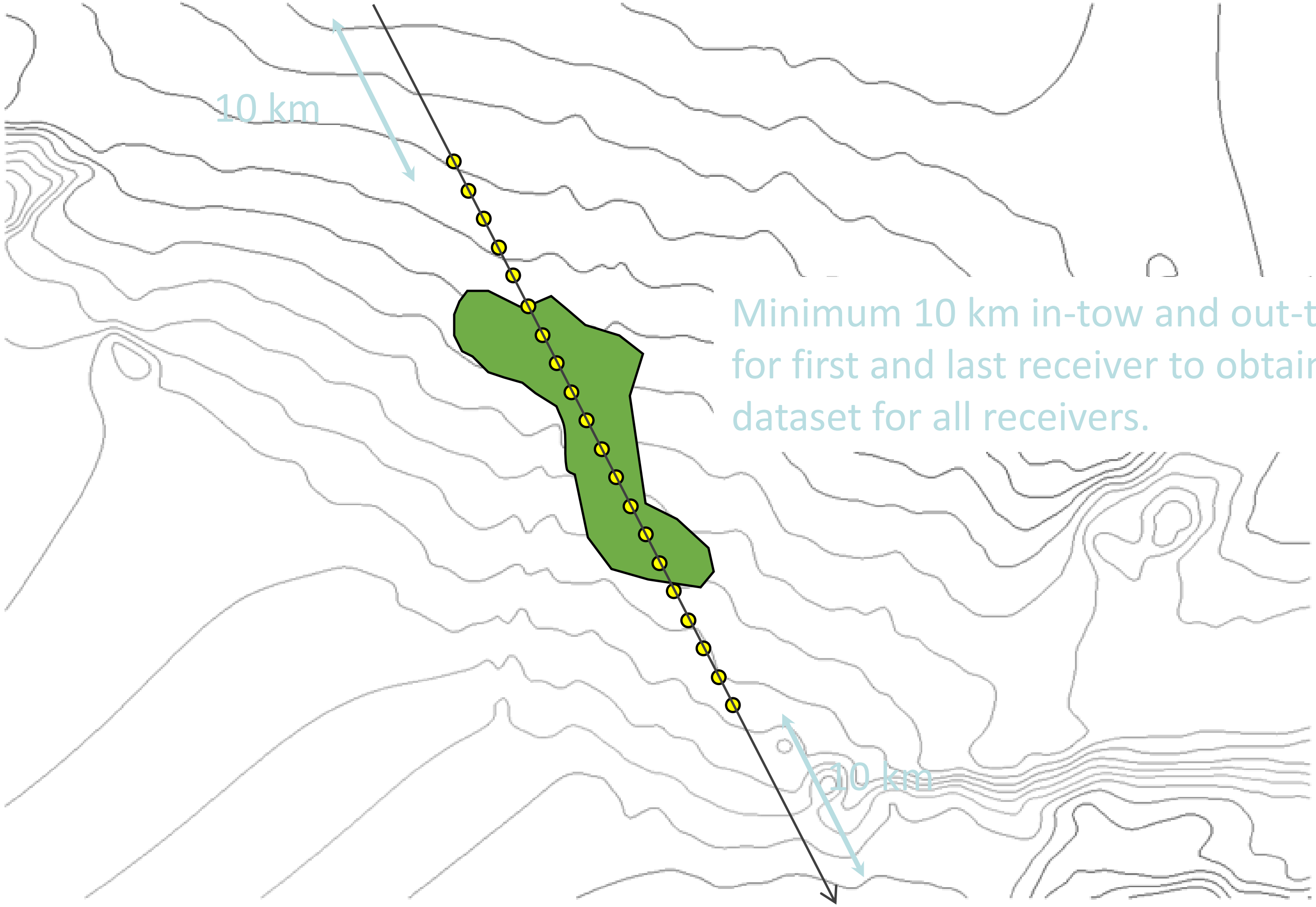
Layout considerations



Layout considerations



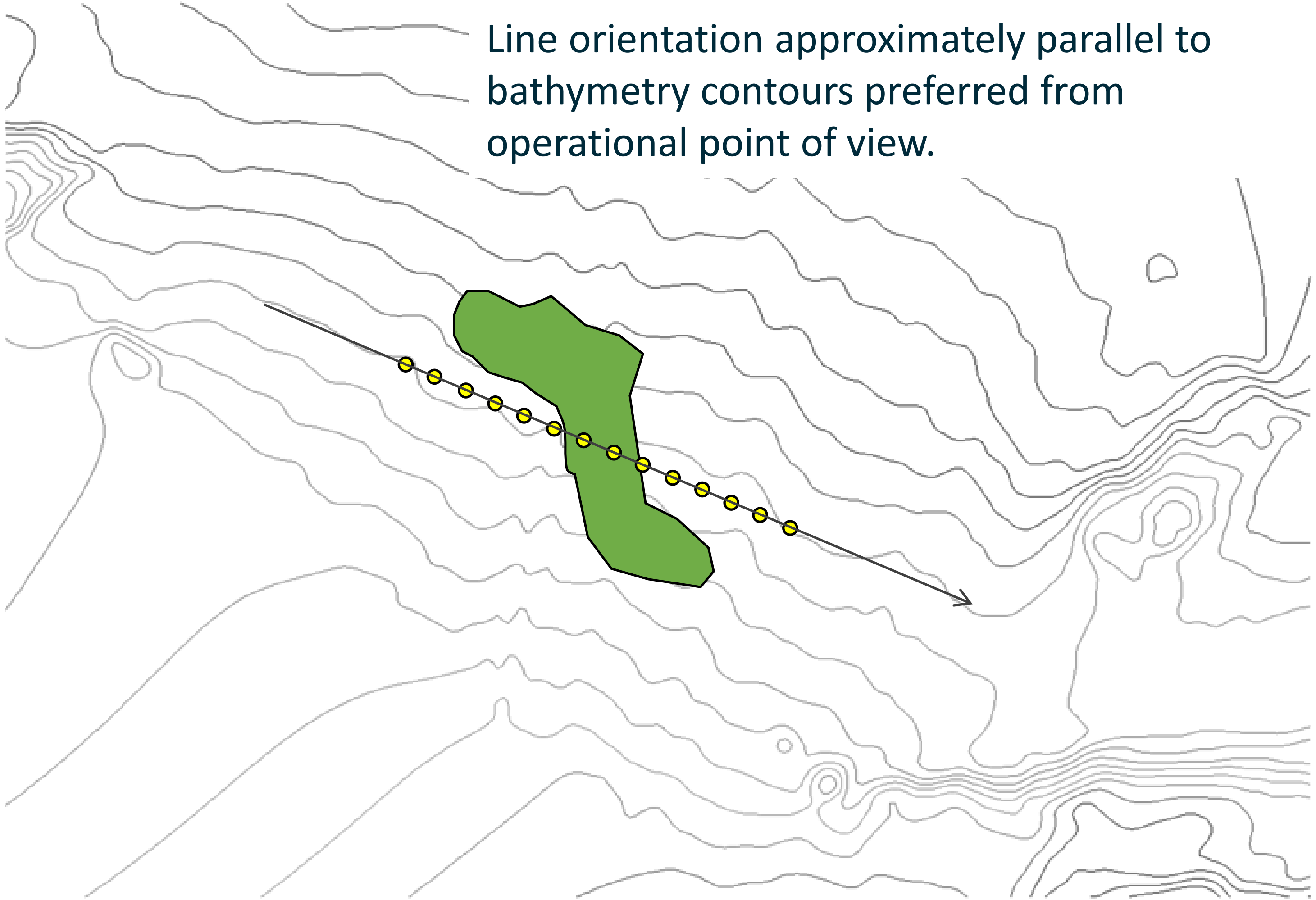
Layout considerations



Minimum 10 km in-tow and out-tow for first and last receiver to obtain full dataset for all receivers.

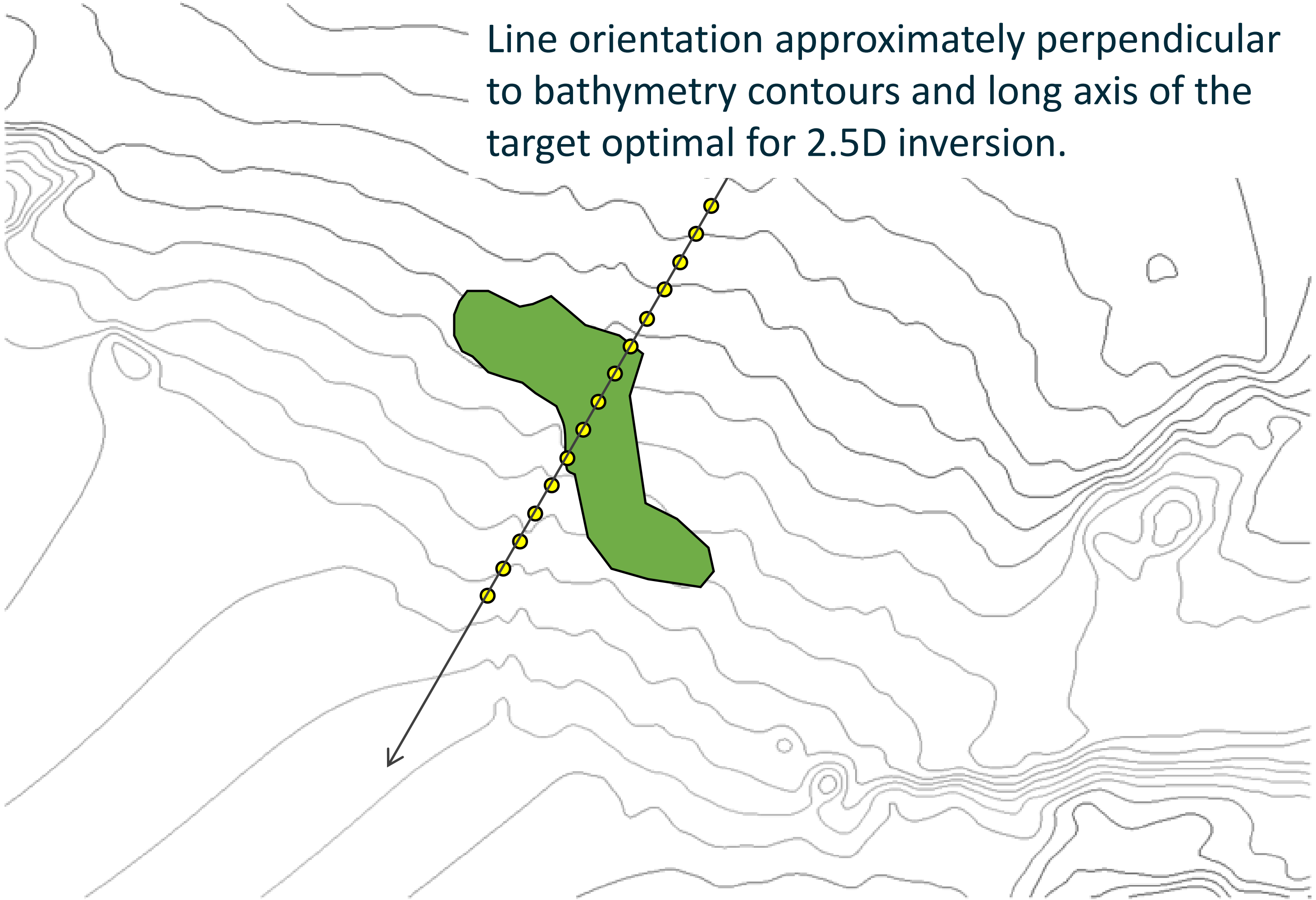
Layout considerations

Line orientation approximately parallel to bathymetry contours preferred from operational point of view.



Layout considerations

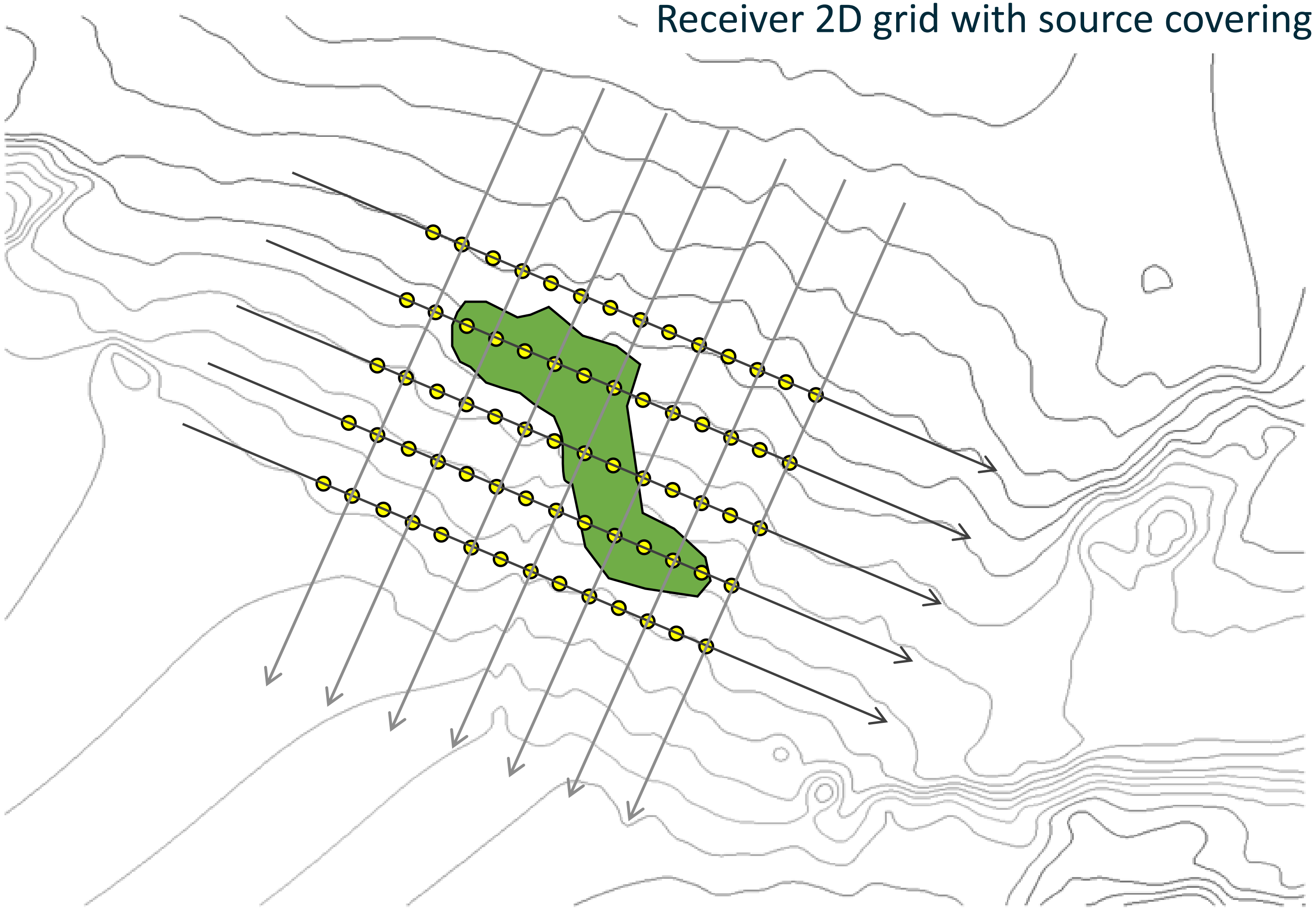
Line orientation approximately perpendicular to bathymetry contours and long axis of the target optimal for 2.5D inversion.



Layout considerations

Proper imaging only from 3D data!

Receiver 2D grid with source covering 2D surface

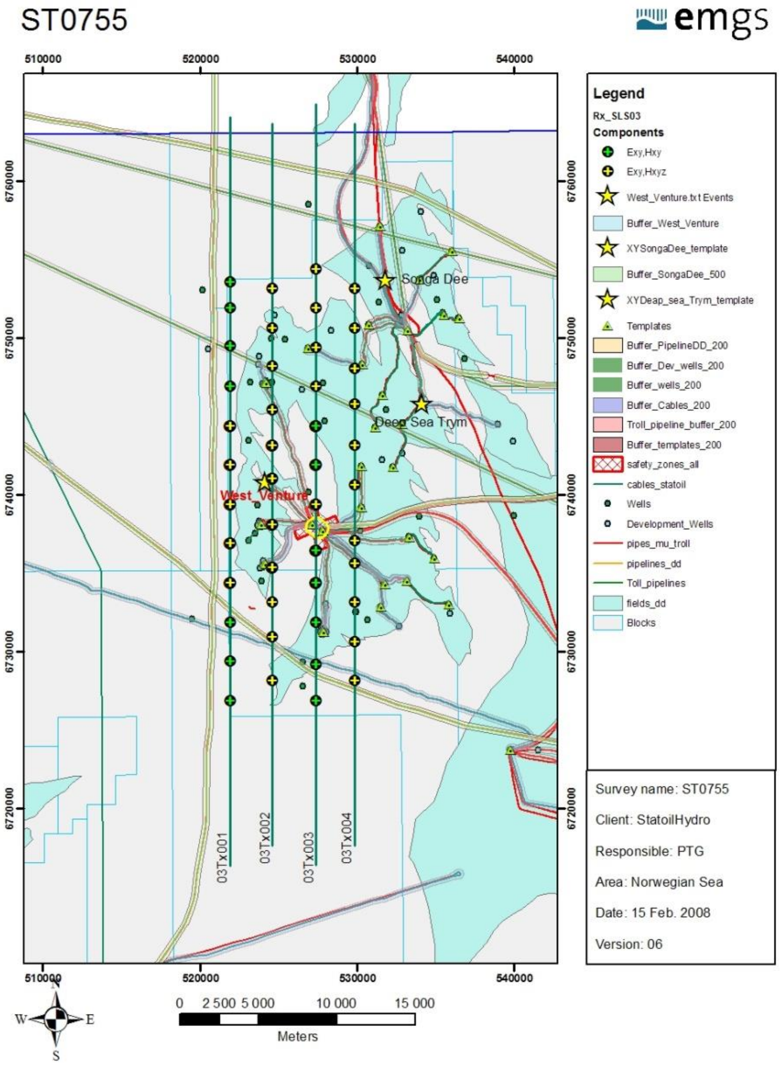


Survey layout sheet (SLS)

Once the survey map and source waveform have been generated, a **Survey Layout Sheet (SLS)** is prepared.

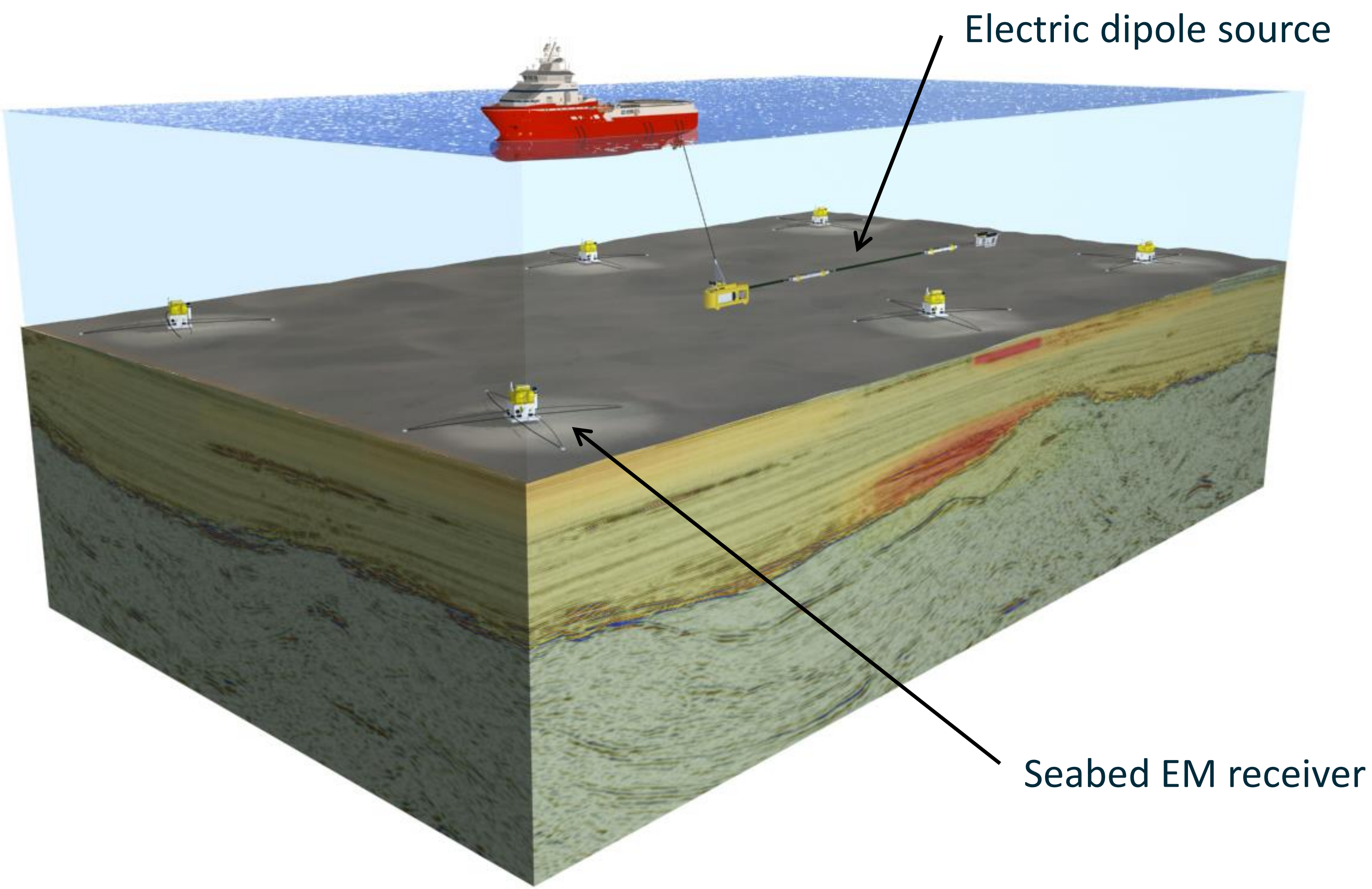
The **SLS** is a formal document sent to the vessel containing all instructions and information required for acquiring the survey:

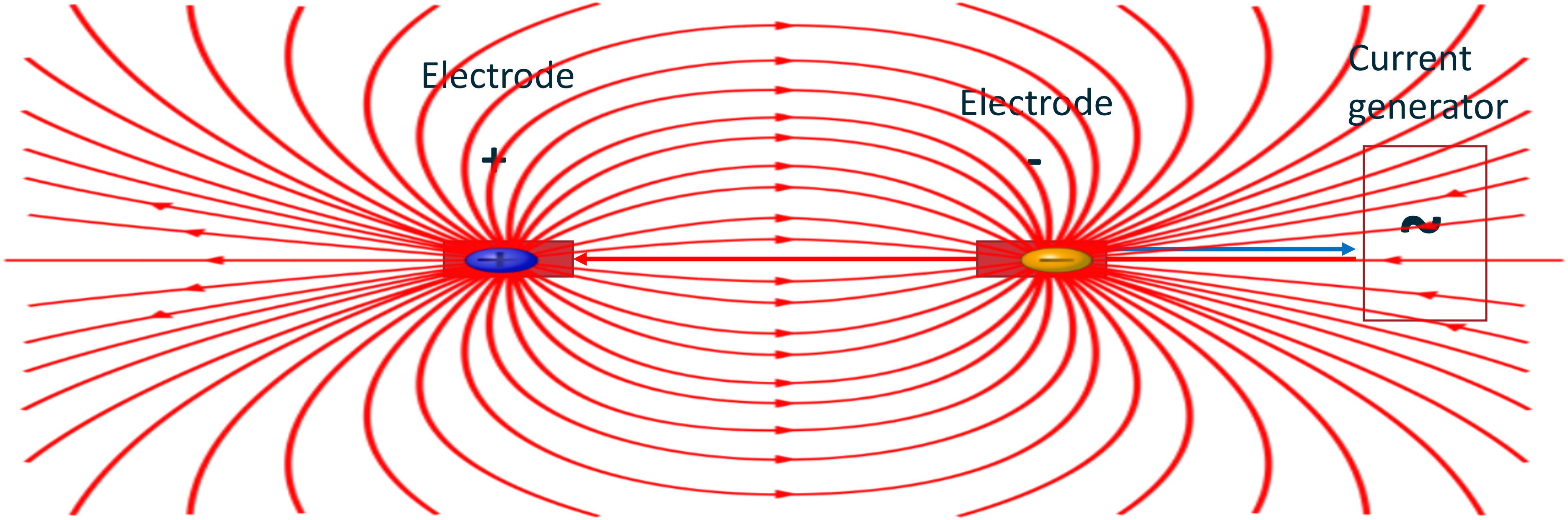
- Survey information
- Rx and Tx positions and specifications
- Source waveform specification
- Obstructions
- Geodetic parameters
- Survey map



Transmitter

Marine controlled-Source Electromagnetics (CSEM or MCSEM)

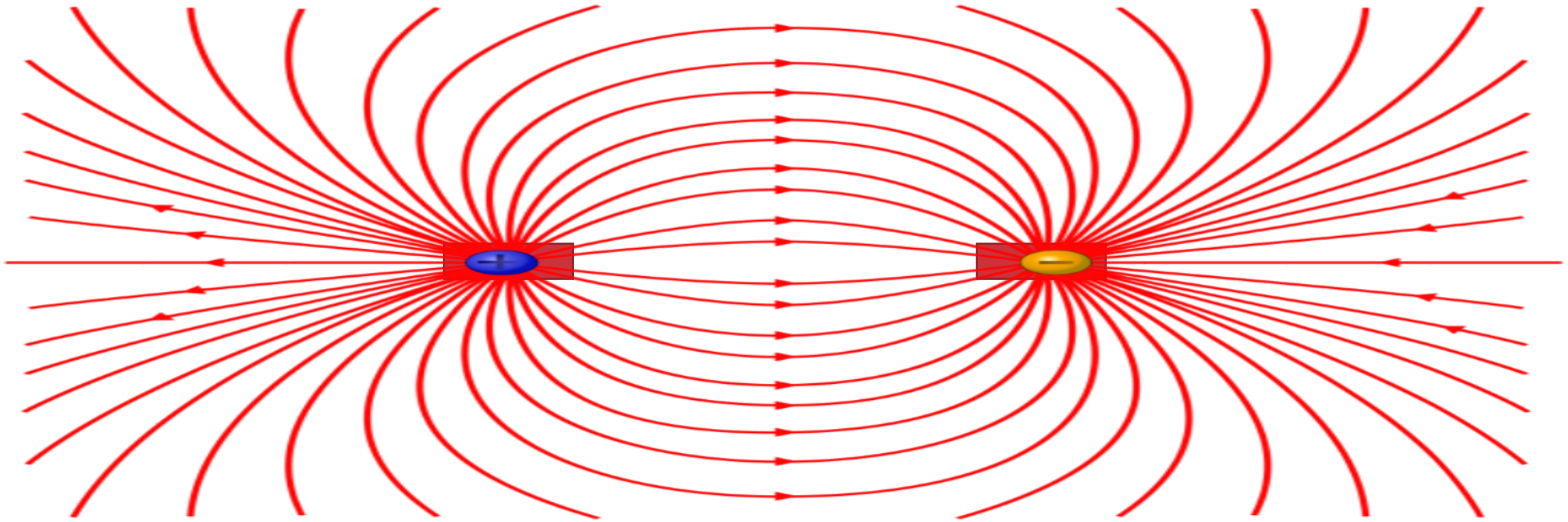



Electrode
+
Electrode
-
Current
generator
~

Conductivity: σ

Resistivity: ρ

$$\sigma = \frac{1}{\rho}$$



Versions of Ohm's law:

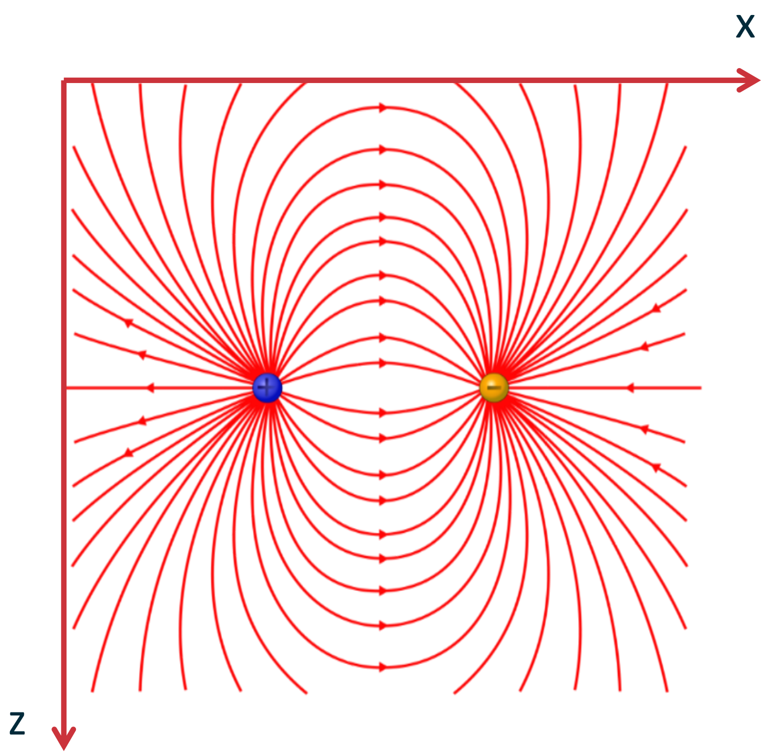
Wire	$U = RI$
Continuum	$E_i = \rho J_i$
Continuum	$J_i = \sigma E_i$

Relation between current density and electric field in continuum a given by Ohm's law at low frequencies

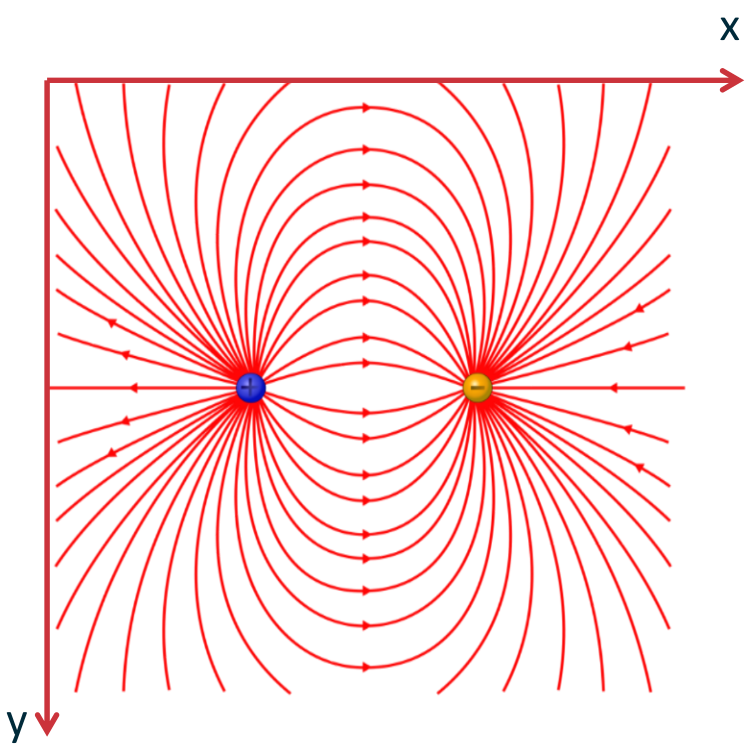
Current and electric field in same direction for isotropic medium

For whole space: Rotational symmetry

Cross section

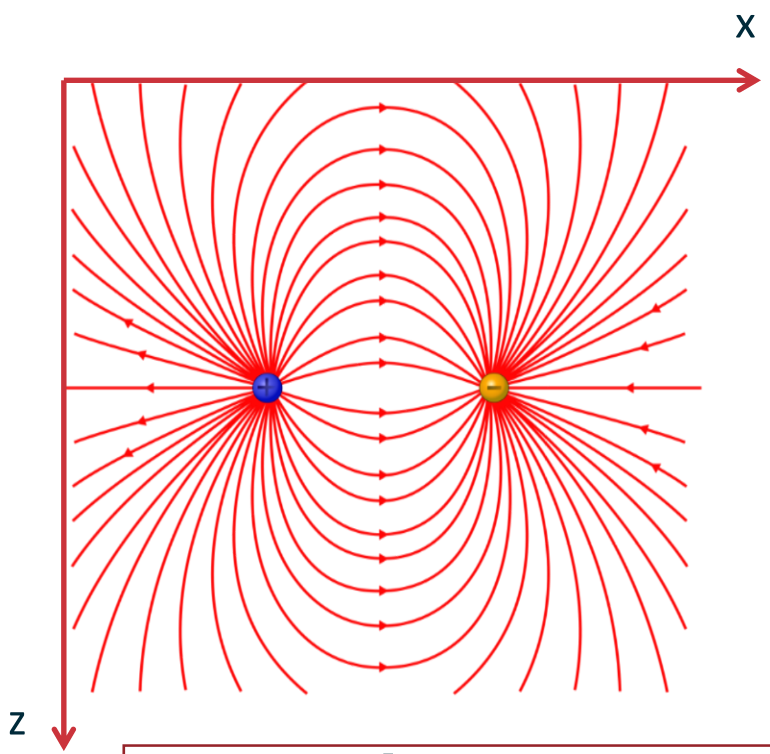


Depth slice

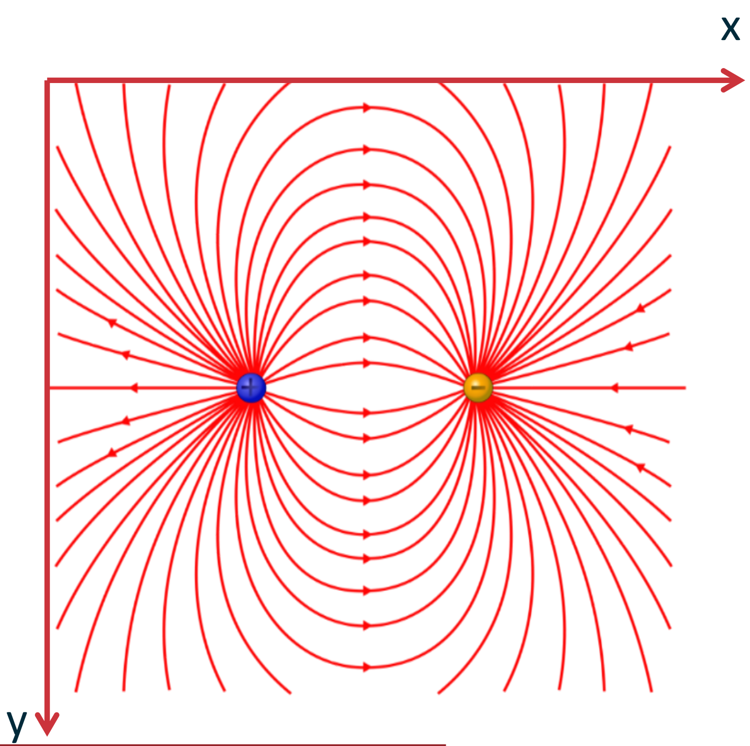


For whole space: Rotational symmetry

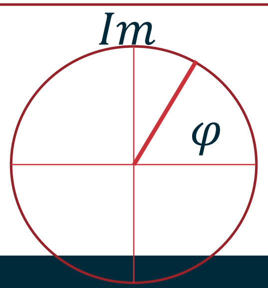
Cross section



Depth slice



$e^{i\varphi}$



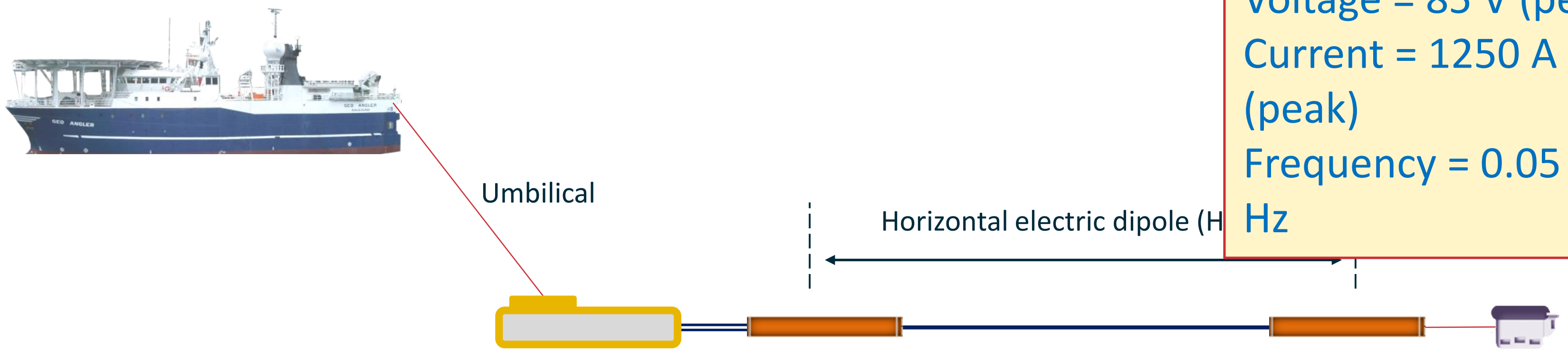
$e^{i0} = 1$ $\quad e^{i\pi} = -1$

$e^{i\frac{\pi}{2}} = i$ $\quad e^{i2\pi} = 1$

A switch of sign is equal to 180 degrees phase shift

the horizontal electric dipole (HED) EM source

Key Specifications
HED Length = 270 m
Power ~ 100 kW
Voltage = 85 V (peak)
Current = 1250 A (peak)
Frequency = 0.05 – 10 Hz

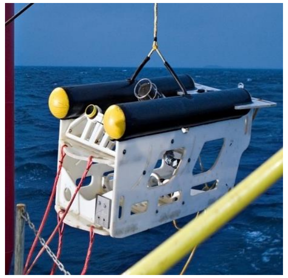


Tow-fish

Lead electrode

Streamer

Tail electrode

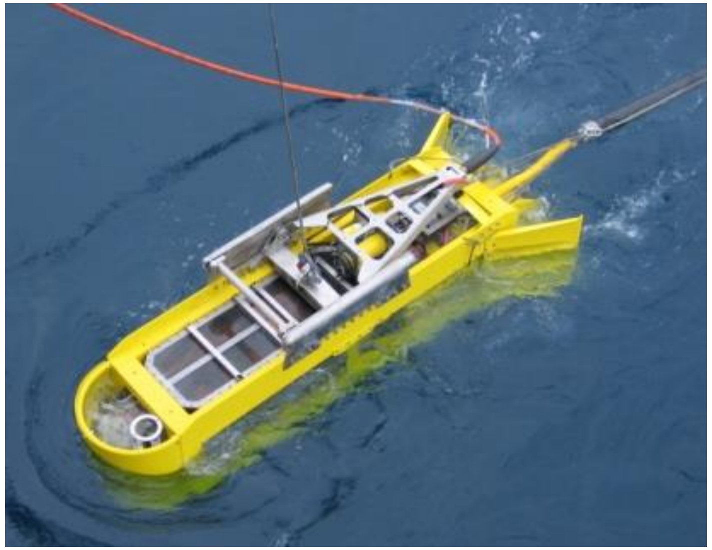


Tail-fish

the tow fish and the tail fish

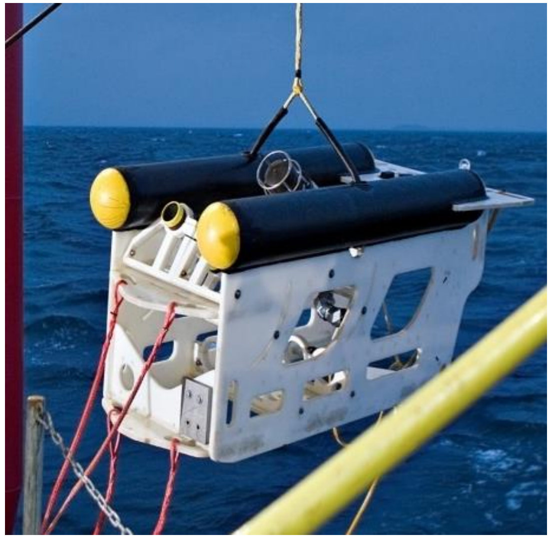
What functions do the tow fish and tail fish perform?

Tow fish



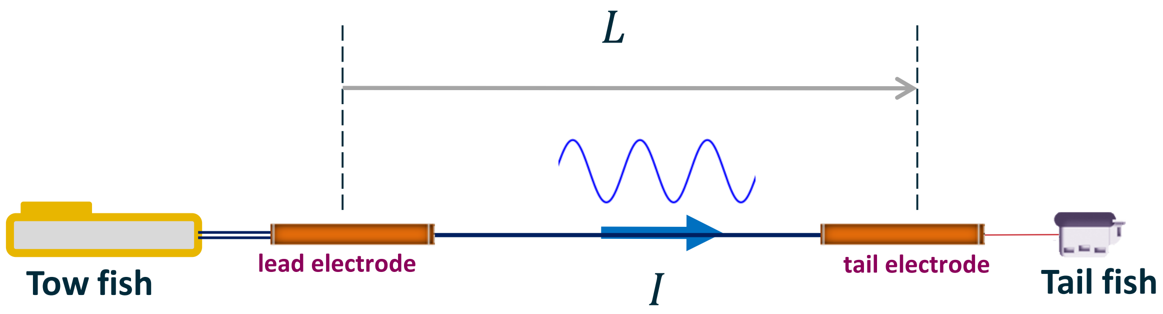
- **Transformer** (from high voltage,
  low current to low voltage, high
  current)
- **Navigation related functions**

Tail fish



- **Brake**
- **Navigation related functions**
- **Receiver for quality control purposes**

THE HED EM source: dipole moment



Dipole moment: $P = IL$

Solution of the Maxwell equations:
The radiated electric and magnetic fields are proportional to the **dipole moment**
i.e. proportional to:

- **Source current**
- **Source length**

EMGS source systems

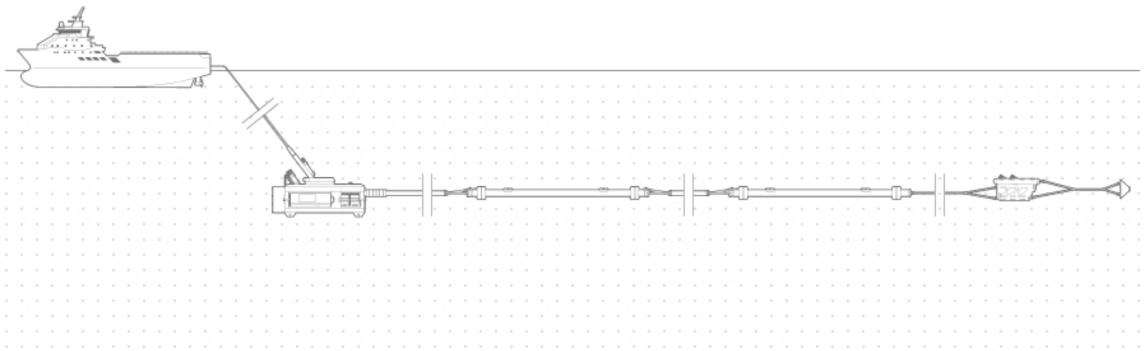
Deep Xpress:

Current: 1500 A

Dipole length: 300 m

Electrode length: 15 m

Towdepth: 30 m – 3500 m



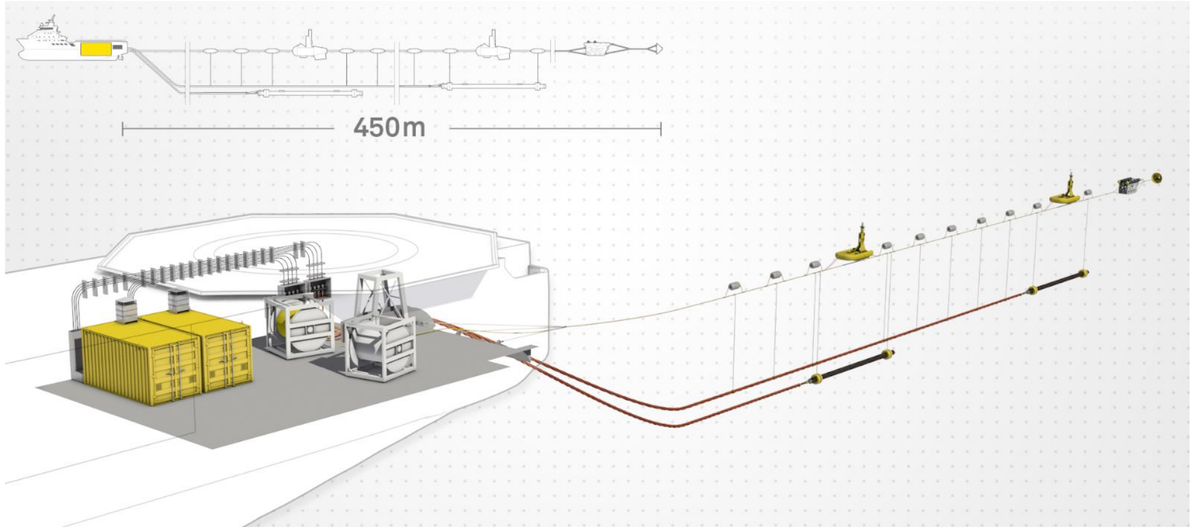
Shelf Xpress:

Current: 7200 A

Dipole length: 280 m

Electrode length: 75 m

Towdepth: 10 m



The difference between marine CSEM and marine MT is due to the difference in sources:

Source geometry

Source frequency content

Dominant modes in the subsurface are different due to the difference in sources

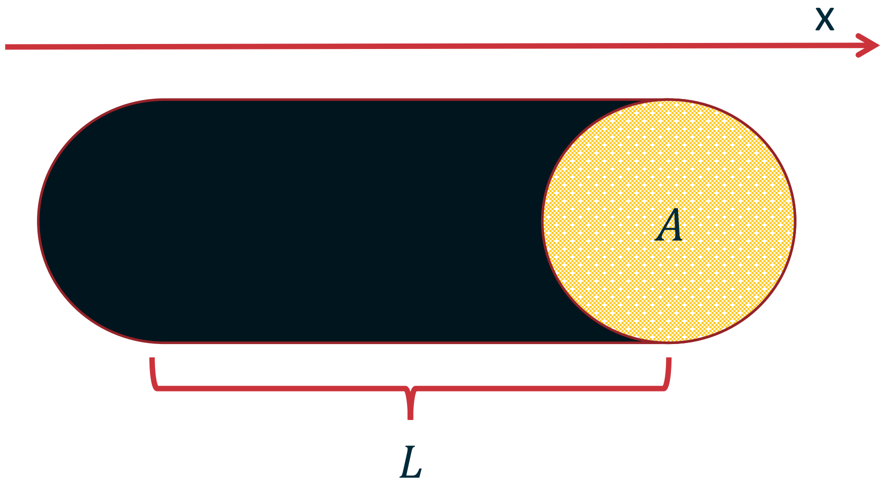
CSEM: Active source – horizontal electric dipole in seawater
Usually in range 0.1 Hz – 3 Hz
Can be in range 0.05 – 10 Hz

MT: Passive source – electric waves/currents in the earths magnetosphere ~ below 1Hz
– electric storms in the earths atmosphere ~ above 1Hz
Usually in range 0.0001 – 1 Hz for marine acquisition
High frequencies problematic in deep water and low lattitudes

Electric field receiver

Ohm's law for a piece of wire: $U = RI$

Ohm's law for a continuum: $E_x = \rho J_x$



$$R = \rho \frac{L}{A}$$

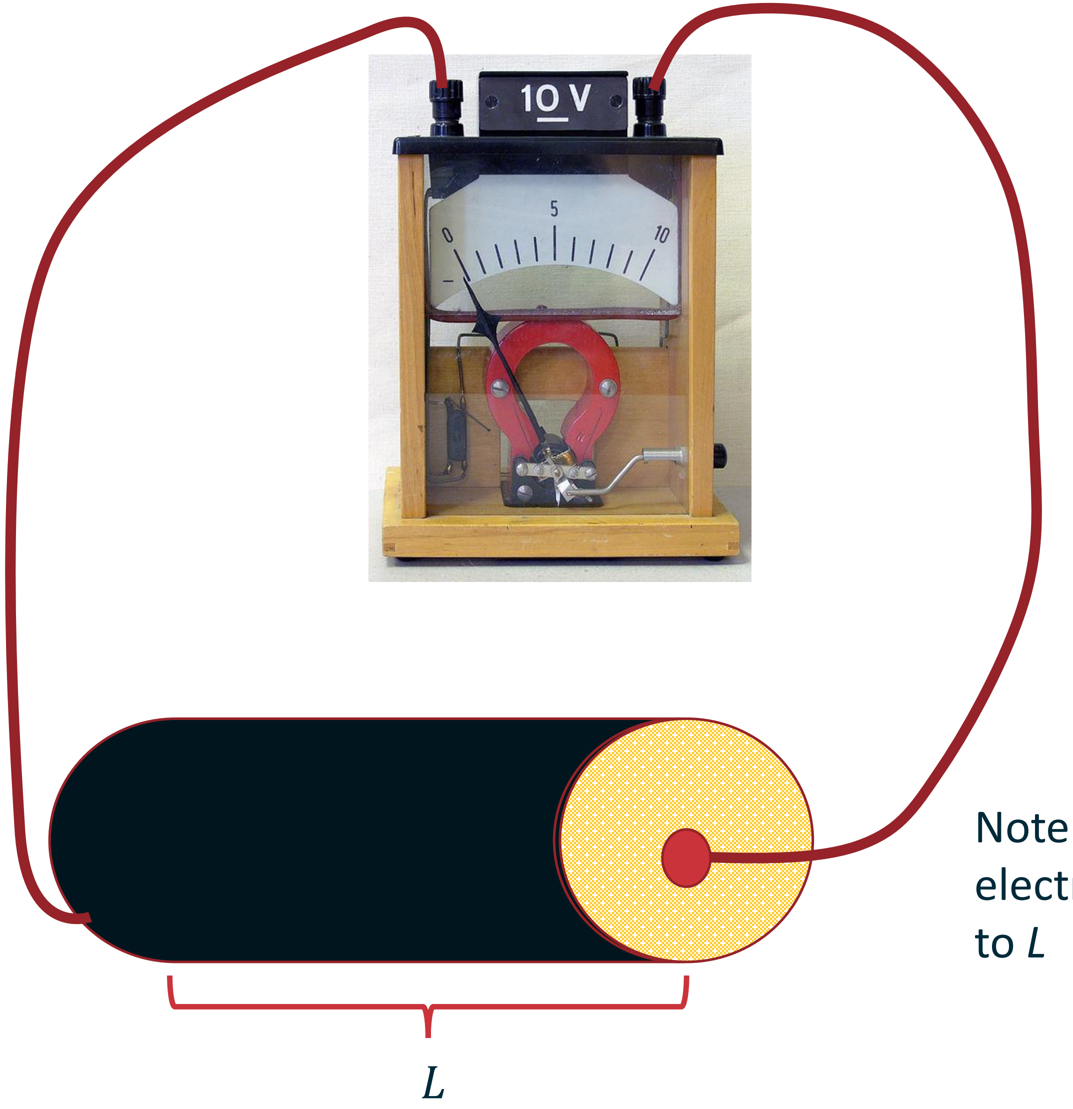
$$I = J_x A$$

$$U = \rho J_x L \Rightarrow \frac{U}{L} = \rho J_x$$

$$E_x = \frac{U}{L}$$

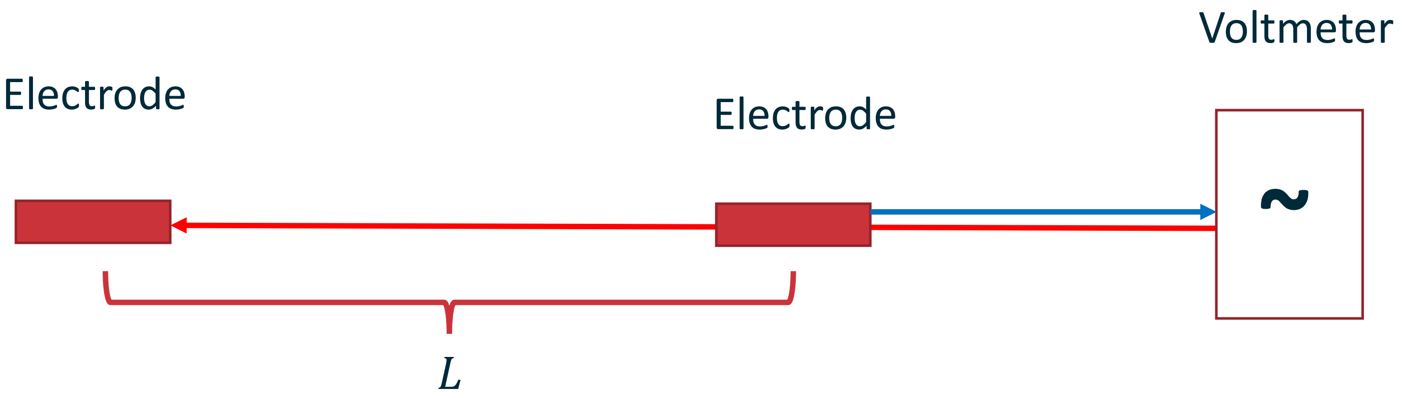
Resistivity is intrinsic property of a material
Resitance depends on resistivity and geometry

Electric field can be measured by a voltmeter



$$E_x = \frac{U}{L}$$

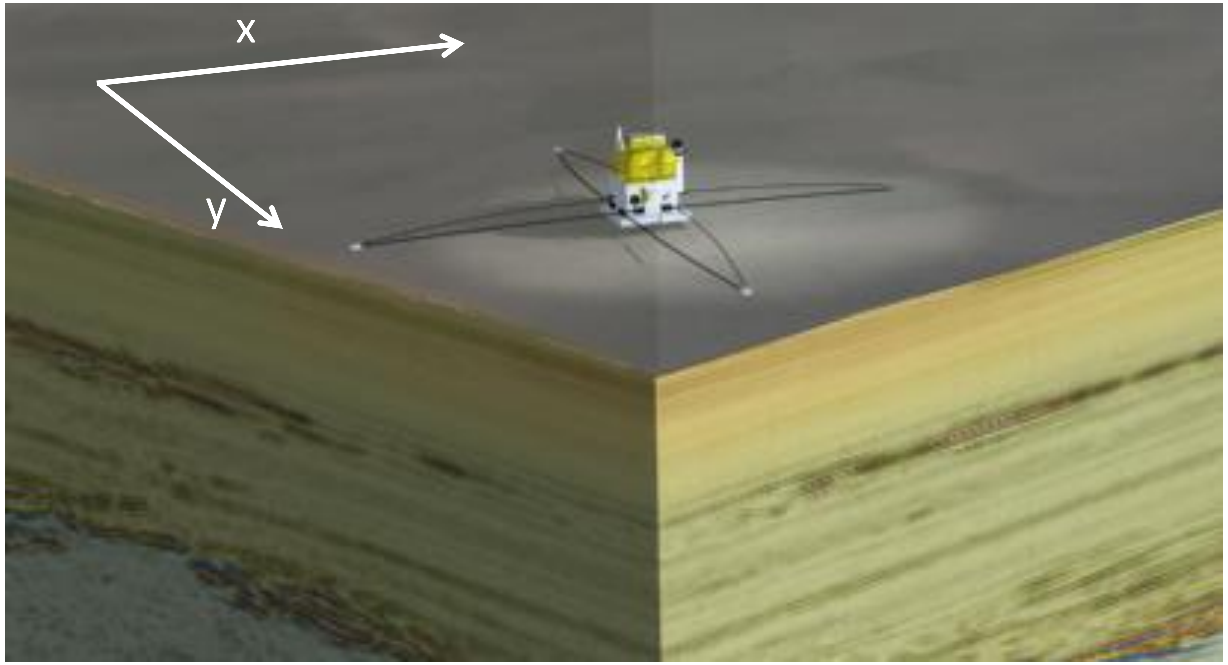
Note: Total voltage over electrode pair proportional to L



Electrodes are placed at seabed with ~ 8 m spacing

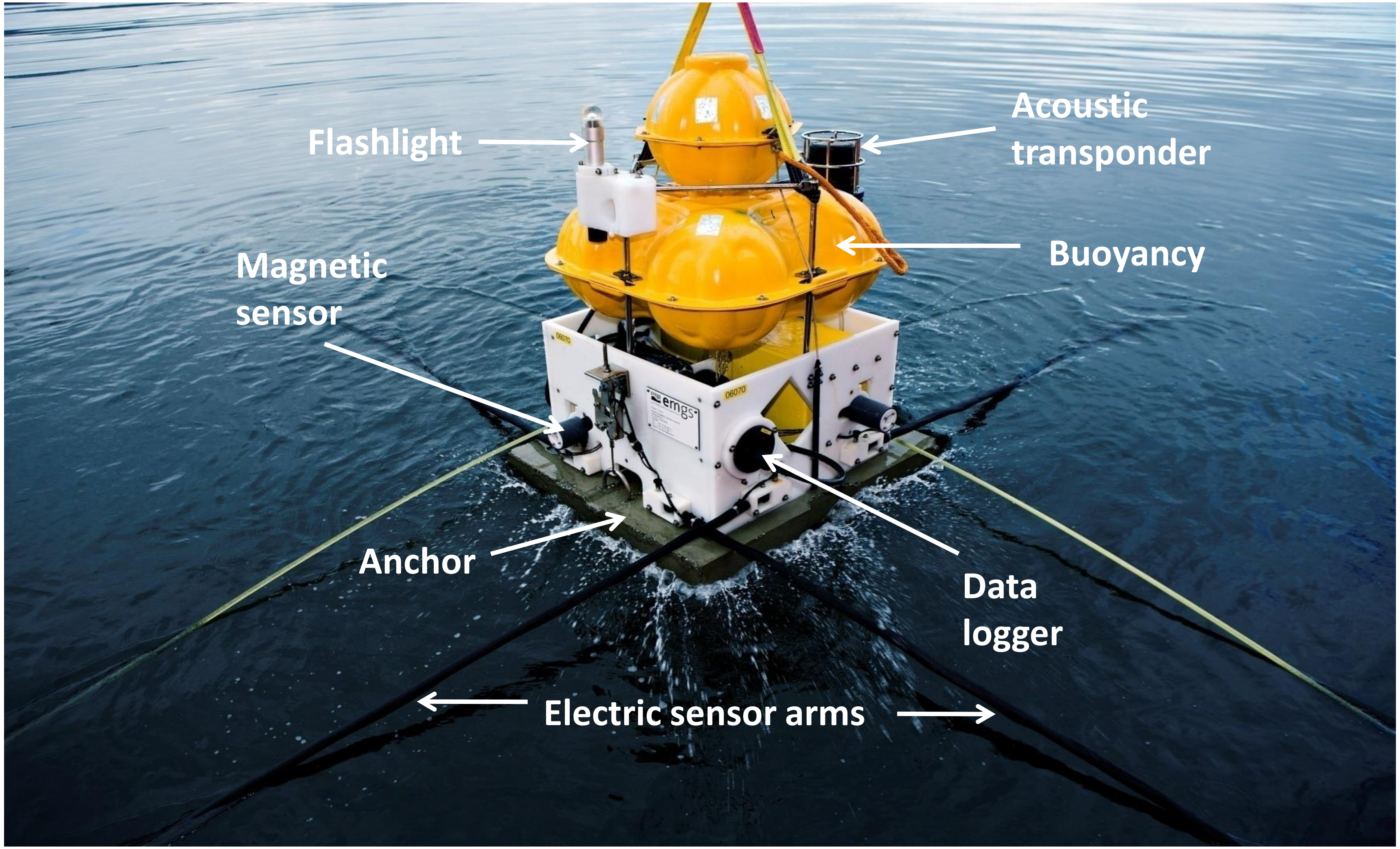
Measure E_x and E_y

Electrodes at the end of each arm
Separation known
Very sensitive voltmeter



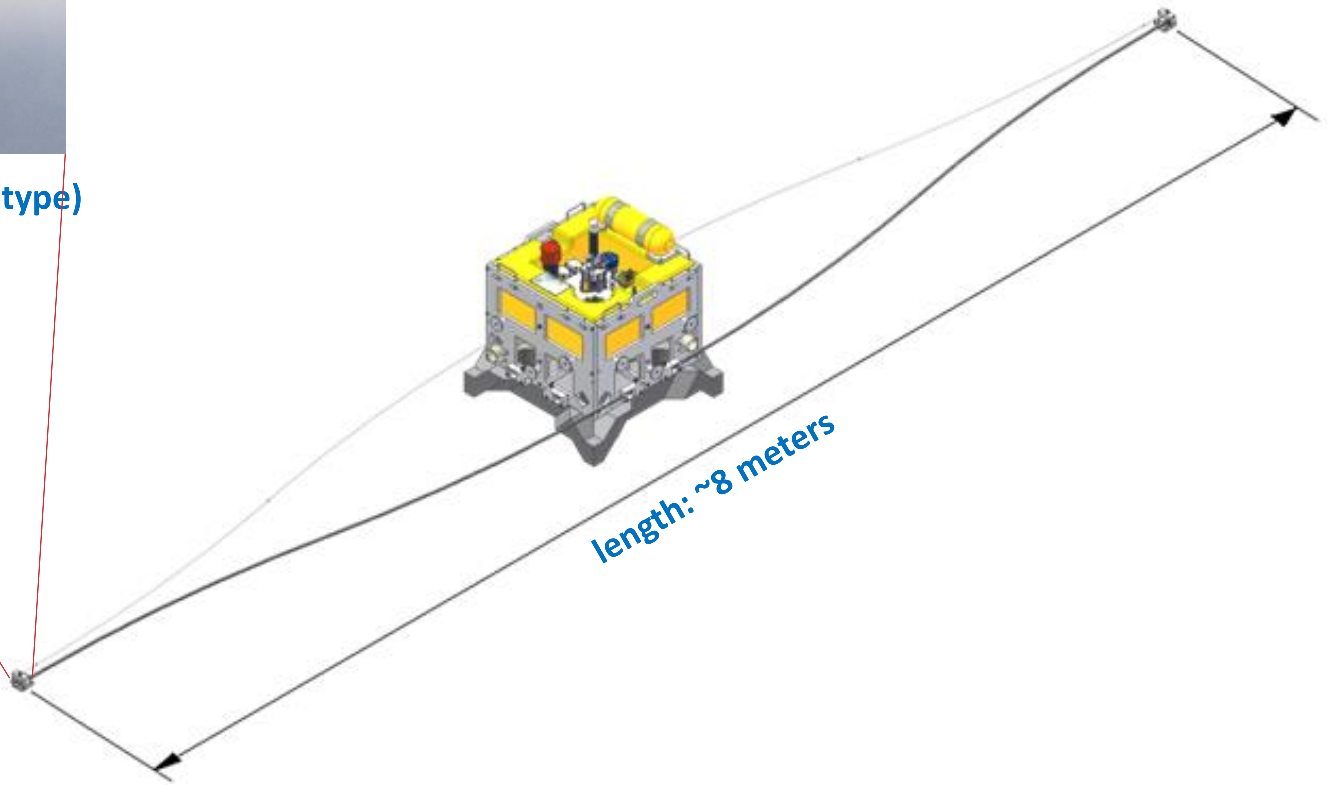

HS.Marine
NOREQ
ATB 80/10 SE1

the csEM receiver

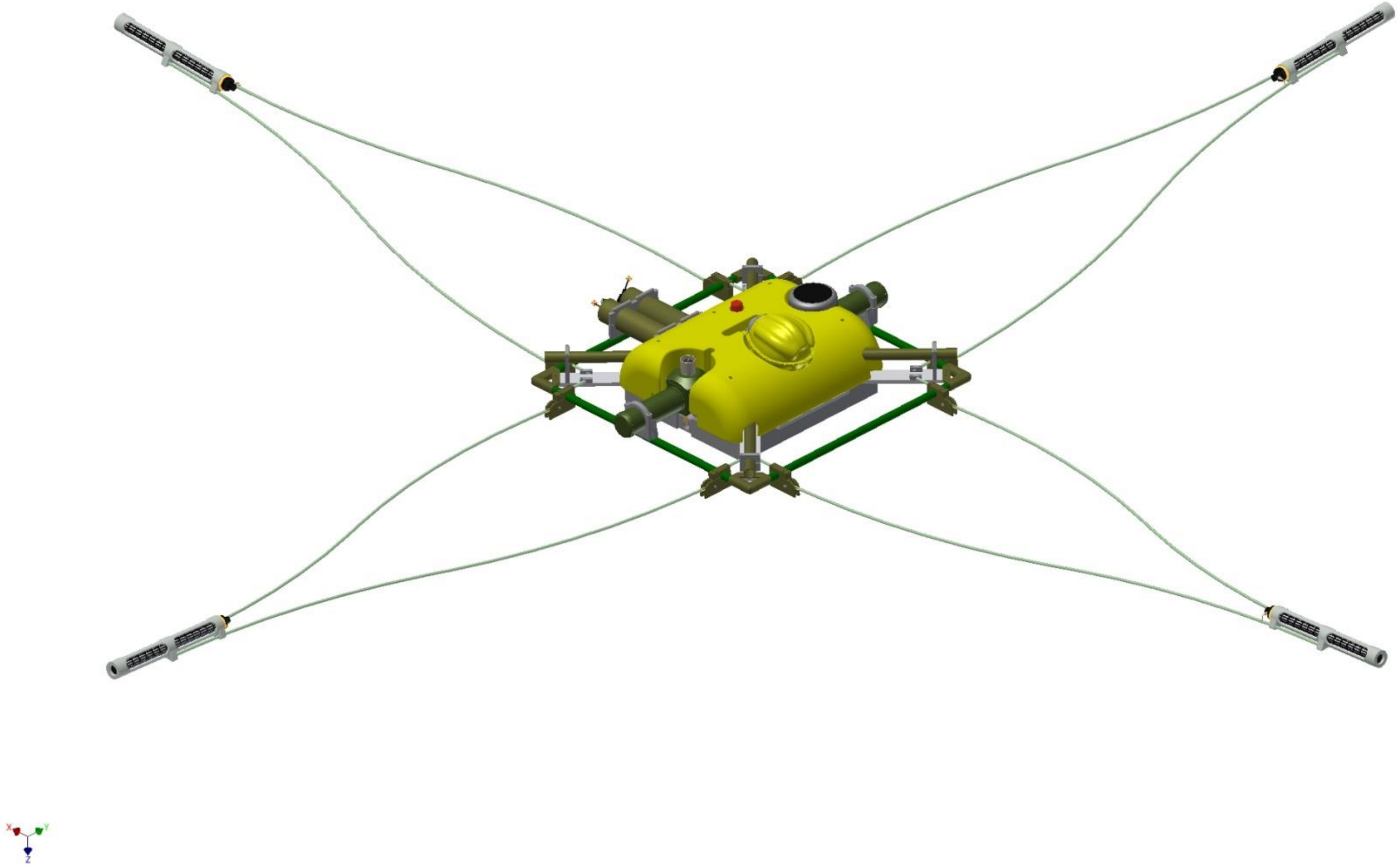


Electric sensors

electrode (old type)

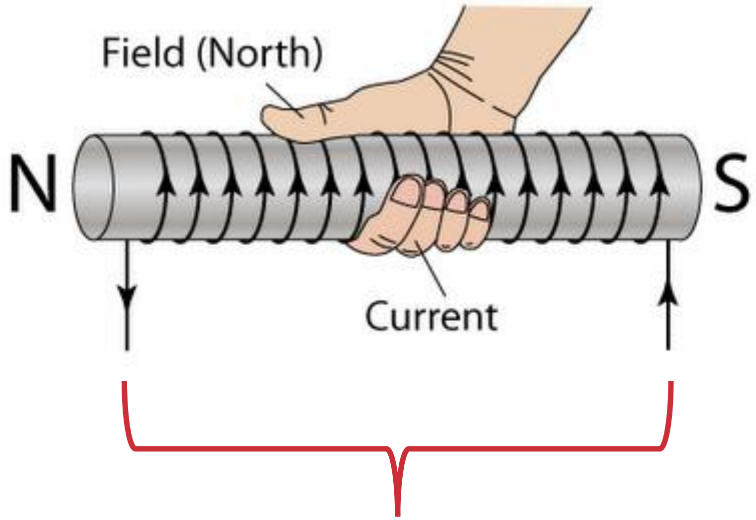


New generation receiver
New electrodes
New coils



Magnetic field receiver

The coil



Surface of coil windings
Is, $\mathrm{A} = |\boldsymbol{A}|$, with direction
defined normal to surface

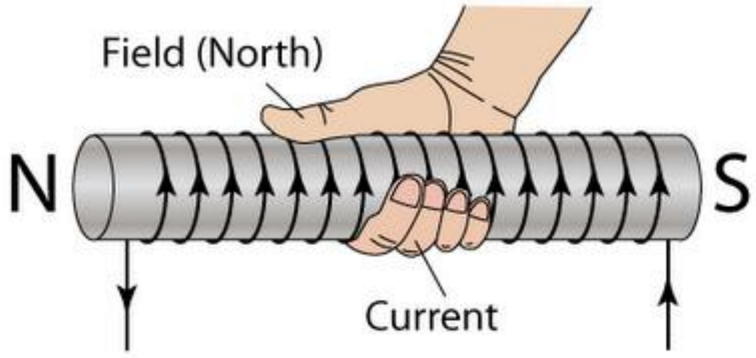
Terminal voltage = $\varepsilon(t)$. Called EMF (electromotive force) measured in units [V]

$$\varepsilon(t) = -\frac{d\Phi_m}{dt} = -\frac{d(\,\boldsymbol{B}(t) \cdot \boldsymbol{A}(t)\,)}{dt}$$

$$\boldsymbol{B}(t) = \mu \boldsymbol{H}(t)$$

Note: A time dependent voltage is measured if
- the magnetic field change
- the coil area absolute value change
- the direction of the coil change

The coil



Surface of coil windings is $|\boldsymbol{A}|$ with direction normal to surface

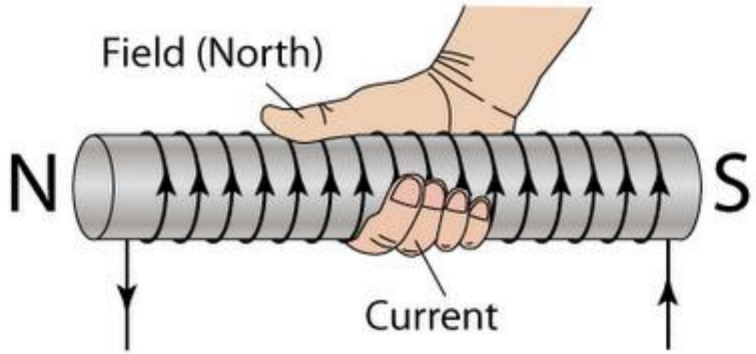
The area absolute value is assumed fixed for a receiver coil, but note that
a change of direction in the static Earth magnetic field will induce a current in the coil

$$\varepsilon(t) = -\frac{d\Phi_m}{dt} = -\mu_0 \mu_c N A \frac{d(\,\boldsymbol{H}(t) \cdot \boldsymbol{n}(t)\,)}{dt} \qquad \boldsymbol{n}(t) = \frac{\boldsymbol{A}(t)}{A}$$

Relative permeabillity of core: μ_c

Number of windings: N

The coil



Surface of coil windings is $|\boldsymbol{A}|$ with direction normal to surface

Assume $\boldsymbol{n}(t) = \boldsymbol{n} = [n_x, n_y, n_z]^T = [1,0,0]^T$ Thus independent of time.

Calibration in frequency domain:

From

$$\varepsilon(t) = -\mu_0 \mu_c N A \frac{d(\boldsymbol{H}(t) \cdot \boldsymbol{n})}{dt}$$

we obtain

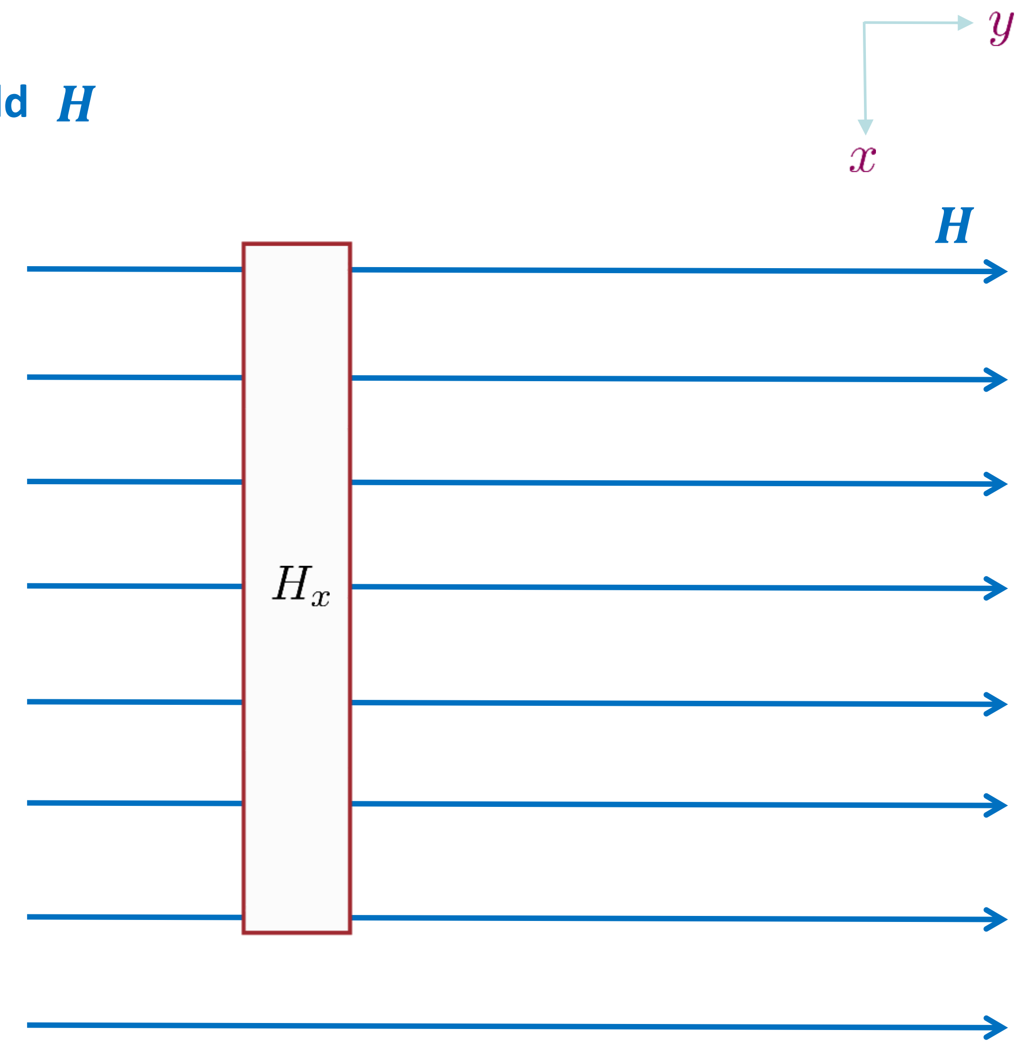
$$H_x(\omega) = -\frac{\varepsilon(\omega)}{i\omega\mu_0\mu_c NA}$$

Coil cross talk

- Sensor H_x **perpendicular** to **external magnetic field** $\boldsymbol{H}$

- Sensor H_x does not sensitive to $\boldsymbol{H}$

$$\varepsilon(t) = -\mu_0 \mu_c N A \frac{d(\boldsymbol{H}(t) \cdot \boldsymbol{n})}{dt}$$



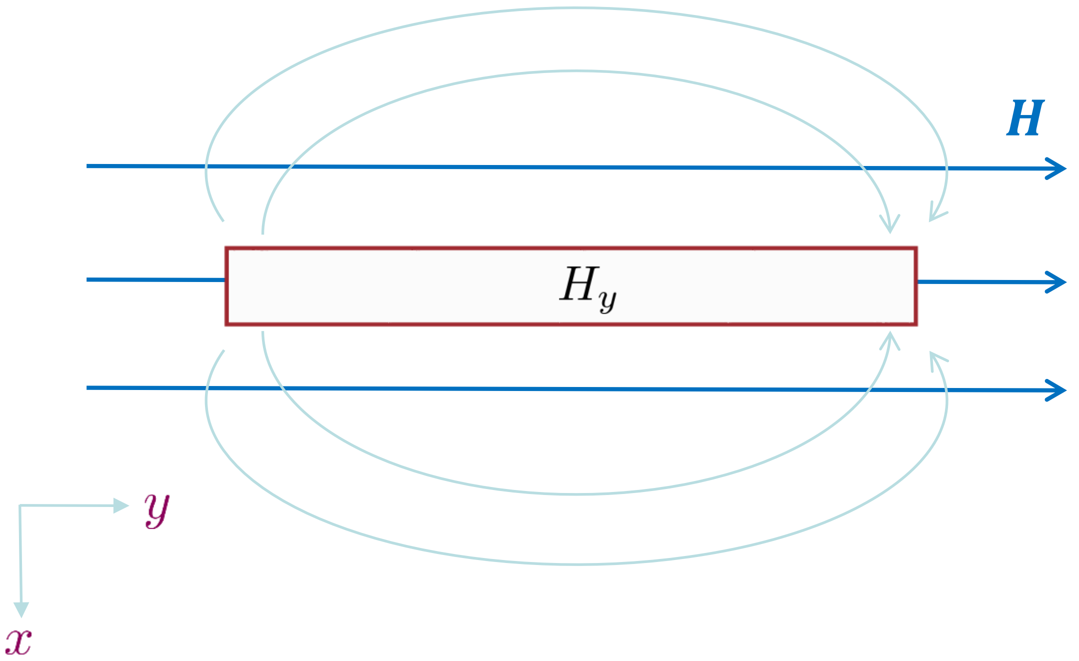
Coil cross talk

- Sensor H_y **parallel** to **external magnetic field** $\boldsymbol{H}$

- The magnetic flux $-d\Phi_H/dt$ through sensor H_y generates an electric current through the coil.

- The electric current induced in sensor H_y generates in turn a **new magnetic field**

$$\varepsilon(t) = -\mu_0\mu_c NA\frac{d(\,\boldsymbol{H}(t)\cdot\boldsymbol{n})}{dt}$$

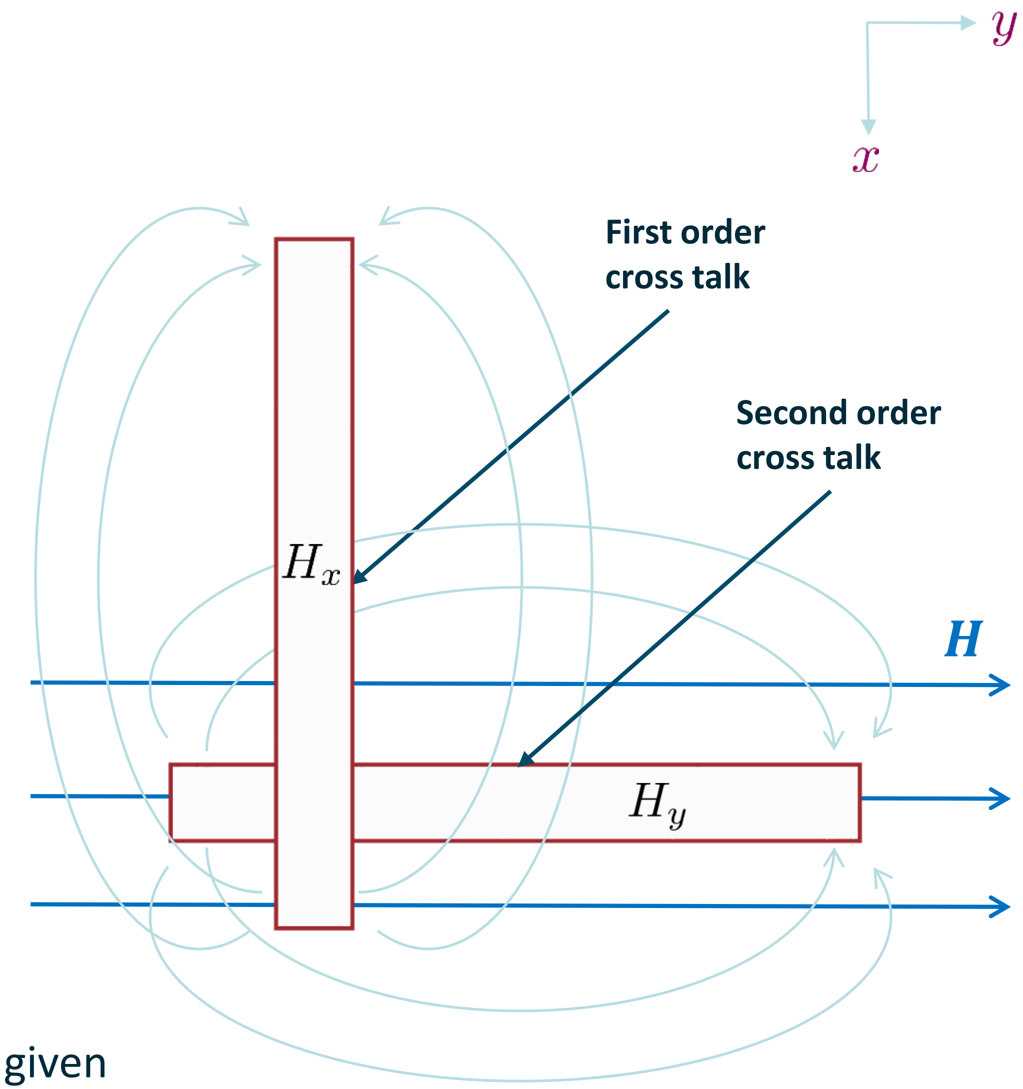


Coil cross talk

- Sensor H_x measures the H_y magnetic field generated by sensor
→ **First order cross talk**

- Sensor H_y measures the magnetic field generated by sensor H_x
→ **Second order cross talk**

- These effects can be **measured in the laboratory**

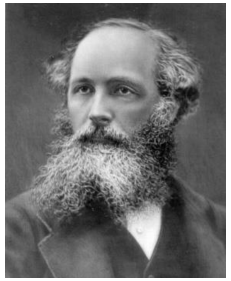


Since they can be quantified for a given setup, they can also be corrected for in receiver calibration procedures.

Maxwell equations
Divergence and curl operators

Maxwell equations

Describe the mutual interaction
between electric and magnetic fields



$$\nabla \times \boldsymbol{H} = \boldsymbol{J}$$

$$\nabla \times \boldsymbol{E} = -\partial_t \boldsymbol{B}$$

$$\nabla \cdot \boldsymbol{D} = q$$

$$\nabla \cdot \boldsymbol{B} = 0$$

Magnetic field: $\boldsymbol{H}$
Magnetic induction: $\boldsymbol{B}$
Electric field: $\boldsymbol{E}$
Electric displacement: $\boldsymbol{D}$
Current density: $\boldsymbol{J}$
Charge density: q

Isotropic: $\boldsymbol{D} = \varepsilon \boldsymbol{E}$ $\boldsymbol{B} = \mu \boldsymbol{H}$ $\boldsymbol{J} = \varepsilon \partial_t \boldsymbol{E} + \sigma \boldsymbol{E} + \boldsymbol{J}^{source}$

Anisotropic: $\boldsymbol{D} = \boldsymbol{\varepsilon} \boldsymbol{E}$ $\boldsymbol{B} = \boldsymbol{\mu} \boldsymbol{H}$ $\boldsymbol{J} = \boldsymbol{\varepsilon} \partial_t \boldsymbol{E} + \boldsymbol{\sigma} \boldsymbol{E} + \boldsymbol{J}^{source}$

Material properties

σ : conductivity [S/m] ⟺ $\rho = \frac{1}{\sigma}$: resistivity [Ωm]

μ : magnetic permeability [H/m]

ε: electric permittivity [F/m]

Notations

$$\boldsymbol{J} = \nabla \times \boldsymbol{H}$$

$$\boldsymbol{J} = \mathrm{curl}\ \boldsymbol{H}$$

$$J_i = \varepsilon_{ijk} \frac{\partial}{\partial_j} H_k = \varepsilon_{ijk} \partial_j H_k$$

The Levi-Cevita tensor: ε_{ijk}

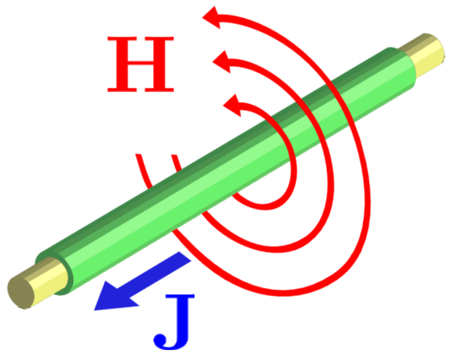
$$\varepsilon_{xyz} = \varepsilon_{zxy} = \varepsilon_{yzx} = 1$$

$$\varepsilon_{xzy} = \varepsilon_{zyx} = \varepsilon_{yxz} = -1$$

Zero for any two indices the same

To calculate:

$$\begin{bmatrix} J_x \\ J_y \\ J_z \end{bmatrix} = \begin{bmatrix} \boldsymbol{e}_x & \boldsymbol{e}_y & \boldsymbol{e}_z \\ \partial_x & \partial_y & \partial_z \\ H_x & H_y & H_z \end{bmatrix}$$

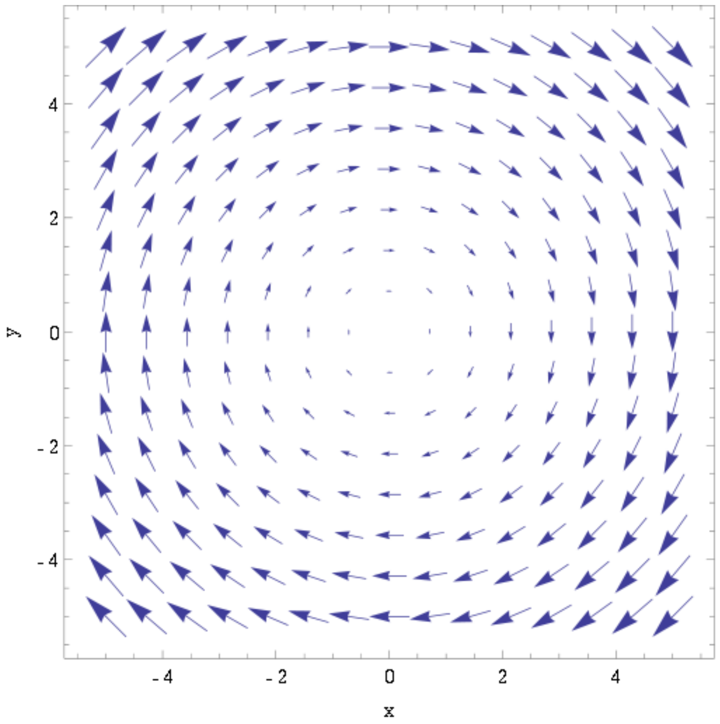


$$\begin{bmatrix} J_x \\ J_y \\ J_z \end{bmatrix} = \begin{bmatrix} \partial_y H_z - \partial_z H_y \\ \partial_z H_x - \partial_x H_z \\ \partial_x H_y - \partial_y H_x \end{bmatrix}$$

Note: J_i independent of H_i

Depends on amplitude and curvature of a vector field

$$\boldsymbol{H} = y\boldsymbol{e}_x - x\boldsymbol{e}_y \qquad |\boldsymbol{H}| = \sqrt{x^2 + y^2} = r$$



Note: Curvature largest for small *r*
Amplitude largest for large *r*

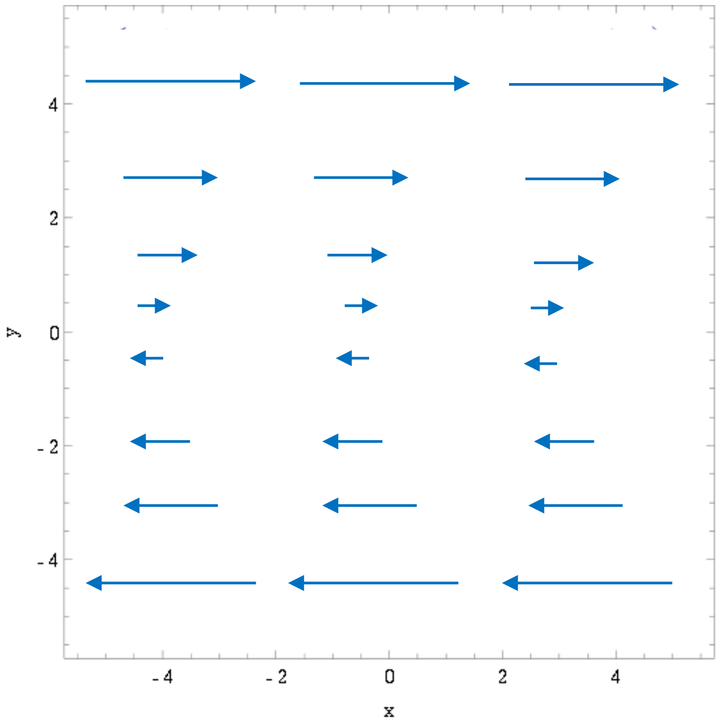
$$\boldsymbol{J} = \nabla \times \boldsymbol{H} \Rightarrow \begin{bmatrix} J_x \\ J_y \\ J_z \end{bmatrix} = \begin{bmatrix} \partial_y H_z - \partial_z H_y \\ \partial_z H_x - \partial_x H_z \\ \partial_x H_y - \partial_y H_x \end{bmatrix}$$

$$\begin{bmatrix} J_x \\ J_y \\ J_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -2 \end{bmatrix}$$

Vector ***J*** normal to horizontal plane and constant

Depends on «shear» of a vector field

$\boldsymbol{H} = y\boldsymbol{e}_x$ $|\boldsymbol{H}| = y$



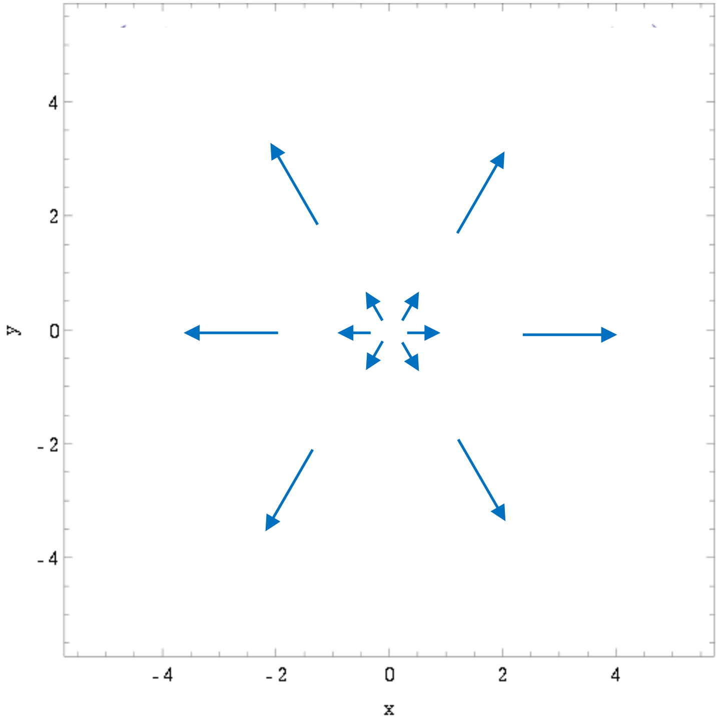
$\boldsymbol{J} = \nabla \times \boldsymbol{H}$ ➡ $\begin{bmatrix} J_x \\ J_y \\ J_z \end{bmatrix} = \begin{bmatrix} \partial_y H_z - \partial_z H_y \\ \partial_z H_x - \partial_x H_z \\ \partial_x H_y - \partial_y H_x \end{bmatrix}$

$$\begin{bmatrix} J_x \\ J_y \\ J_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$$

Vector $\boldsymbol{J}$ normal to horizontal plane and constant

$\boldsymbol{H} = x\boldsymbol{e}_x + y\boldsymbol{e}_y$

$|\boldsymbol{H}| = \sqrt{x^2 + y^2} = r$



$\boldsymbol{J} = \nabla \times \boldsymbol{H}$ ➡

$$\begin{bmatrix} J_x \\ J_y \\ J_z \end{bmatrix} = \begin{bmatrix} \partial_y H_z - \partial_z H_y \\ \partial_z H_x - \partial_x H_z \\ \partial_x H_y - \partial_y H_x \end{bmatrix}$$

$$\begin{bmatrix} J_x \\ J_y \\ J_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Notations

$$q = \nabla \cdot \boldsymbol{D}$$

$$q = \operatorname{div} \boldsymbol{D}$$

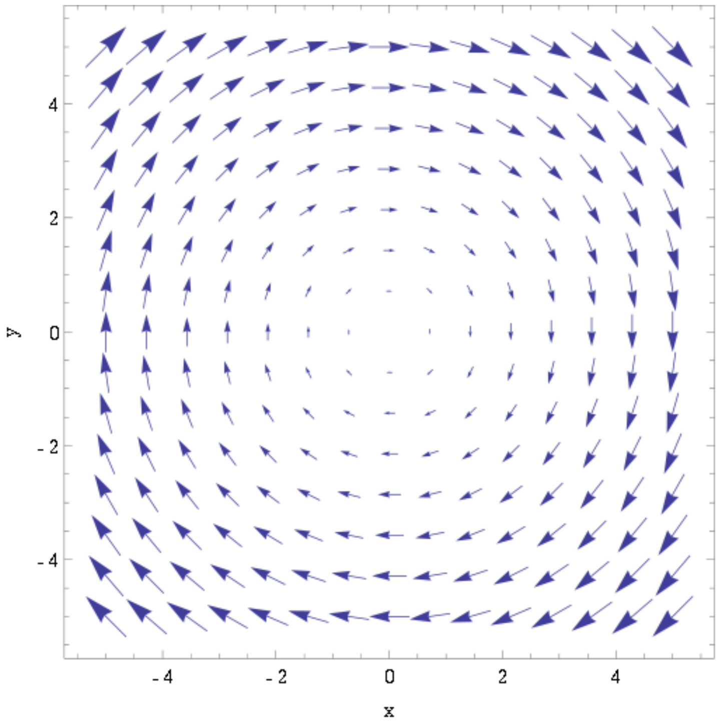
$$q = \partial_i D_i$$

To calculate:

$$q = \partial_x D_x + \partial_y D_y + \partial_z D_z$$

$$\boldsymbol{D} = y\boldsymbol{e}_x - x\boldsymbol{e}_y$$

$$|\boldsymbol{D}| = \sqrt{x^2 + y^2} = r$$

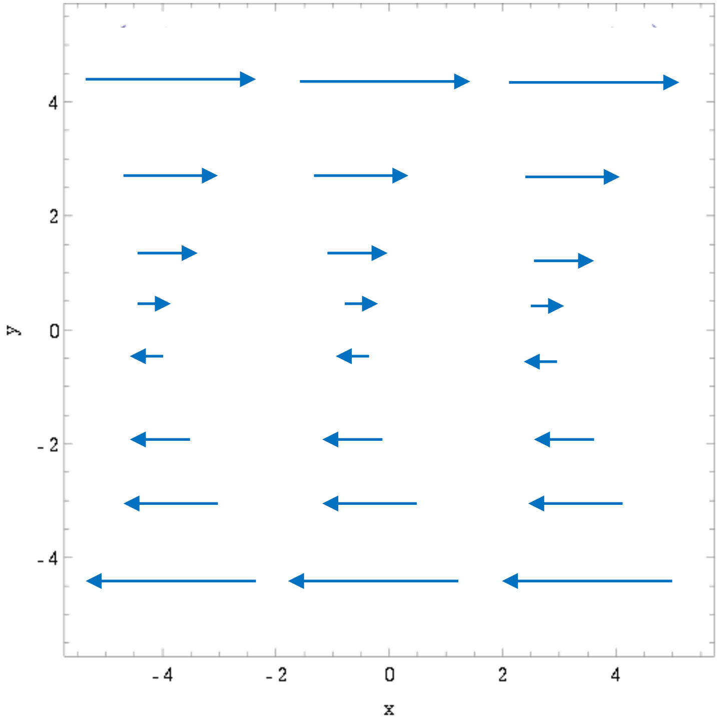


$$q = \partial_x D_x + \partial_y D_y + \partial_z D_z$$

$$q = 0$$

Depends on «shear» of a vector field

$$\boldsymbol{D} = y\boldsymbol{e}_x \qquad |\boldsymbol{D}| = y$$

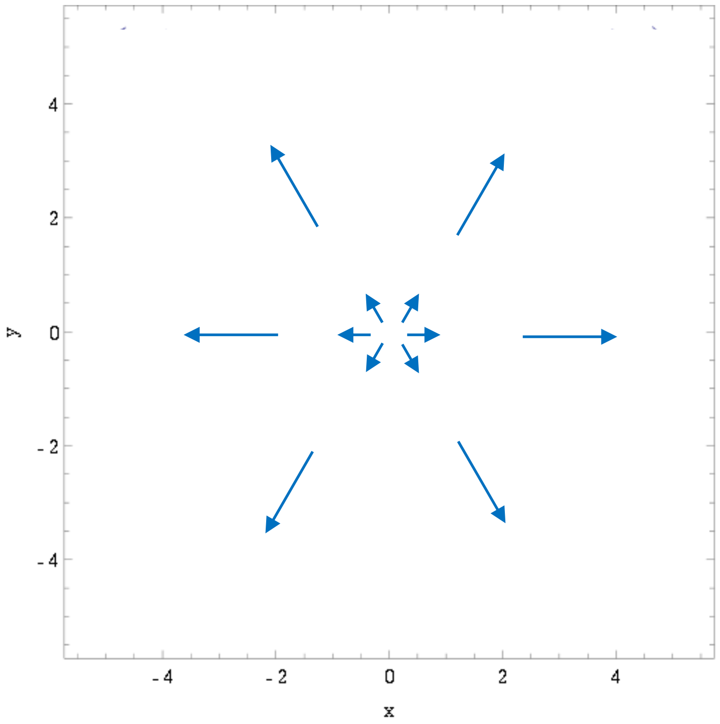


$$q = \partial_x D_x + \partial_y D_y + \partial_z D_z$$

$$q = 0$$

Depends on «shear» of a vector field

$$\boldsymbol{D} = x\boldsymbol{e}_x + y\boldsymbol{e}_y \qquad |\boldsymbol{D}| = \sqrt{x^2 + y^2} = r$$



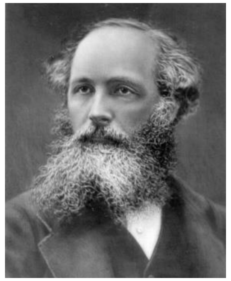
$$q = \partial_x D_x + \partial_y D_y + \partial_z D_z$$

$$q = 2$$

The quasi-static approximation

Maxwell equations

Describe the mutual interaction
between electric and magnetic fields



$$\nabla \times \boldsymbol{H} = \boldsymbol{J}$$

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Magnetic field: $\boldsymbol{H}$
Magnetic induction: $\boldsymbol{B}$
Electric field: $\boldsymbol{E}$
Electric displacement: $\boldsymbol{D}$
Current density: $\boldsymbol{J}$
Charge density: q

Isotropic: $\boldsymbol{D} = \varepsilon \boldsymbol{E}$ $\boldsymbol{B} = \mu \boldsymbol{H}$ $\boldsymbol{J} = \varepsilon \partial_t \boldsymbol{E} + \sigma \boldsymbol{E} + \boldsymbol{J}^{source}$

Anisotropic: $\boldsymbol{D} = \boldsymbol{\varepsilon} \boldsymbol{E}$ $\boldsymbol{B} = \boldsymbol{\mu} \boldsymbol{H}$ $\boldsymbol{J} = \boldsymbol{\varepsilon} \partial_t \boldsymbol{E} + \boldsymbol{\sigma} \boldsymbol{E} + \boldsymbol{J}^{source}$

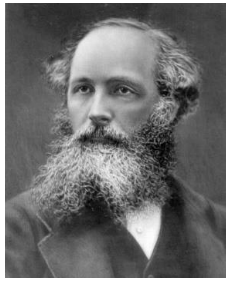
Material properties

σ : conductivity [S/m] ⟺ $\rho = \frac{1}{\sigma}$: resistivity [Ωm]

μ : magnetic permeability [H/m]

ε: electric permittivity [F/m]

Solution to EM problems from 2 of the Maxwell equations



$$\nabla \times \boldsymbol{H} - \varepsilon \partial_t \boldsymbol{E} - \sigma \boldsymbol{E} = \boldsymbol{J}^{source}$$ (Ampere's law)

$$\nabla \times \boldsymbol{E} + \mu_0 \partial_t \boldsymbol{H} = 0$$ (Faraday's law)

$\mu_0 = 4\pi \times 10^{-7}$ H/m

$\varepsilon_0 = 8.85 \times 10^{-12}$ F/m

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 2.998 \times 10^8 \text{ m/s}$$

$$Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 376.7\ \Omega$$

Sedimentary rocks non-magnetic: $\boldsymbol{\mu} \rightarrow \mu_0$

$\varepsilon \partial_t \boldsymbol{E} + \sigma \boldsymbol{E}$ → (Fourier) $i\omega \varepsilon_r \varepsilon_0 \boldsymbol{E} + \sigma \boldsymbol{E}$

Compare $\omega \varepsilon_r \varepsilon_0$ to σ

Note: $\varepsilon_r = 80$ for seawater

Note: $\varepsilon_r < 80$ for sedimentary rocks

Seawater:

f [Hz]	$\omega\varepsilon_r\varepsilon_0$ [S/m]	σ [S/m]	
1	4.4×10^{-9}	3.2	CSEM - Diffusive
10^9	4.4	3.2	GPR - Mixed
5×10^{14}	2.2×10^6	3.2	Visible ligth - Wave

$$\nabla \times \boldsymbol{H} - \varepsilon_r\varepsilon_0\partial_t\boldsymbol{E} - \sigma\boldsymbol{E} = \boldsymbol{J}^{source} \longrightarrow \nabla \times \boldsymbol{H} - \sigma\boldsymbol{E} = \boldsymbol{J}^{source}$$

$$\nabla \times \boldsymbol{E} + \mu_0\partial_t\boldsymbol{H} = 0$$

Safe to neglect displacement currents for CSEM and MT frequency band

Maxwell equations for CSEM and MT

$$\nabla \times \boldsymbol{H} - \sigma \boldsymbol{E} = \boldsymbol{J}^s$$

$$\nabla \times \boldsymbol{E} + \mu_0 \partial_t \boldsymbol{H} = 0$$

Solutions in terms of electric and magnetic fields

Called: The quasi-static approximation
System is diffusive in nature

Typical for diffusive systems:

I) Very strong absorption – loss of amplitude with propagation (here the effect is transformation of electromagentic energy too heat. Resistive heating and induction heating)

II) Dispersion – different frequencies propagate with different velocity

Maxwell equations in 1D

Maxwell equations for CSEM and MT

$$\nabla \times \boldsymbol{H} = \sigma \boldsymbol{E} + \boldsymbol{J}^s$$

$$\nabla \times \boldsymbol{E} = i\omega\mu_0 \boldsymbol{H}$$

Assume earth invariant in x and y direction
Assume source invariant in x and y direction and no vertical current
Electric and magnetic fields invariant in x and y direction as a consequence

$$\begin{bmatrix} J_x^s + \sigma E_x \\ J_y^s + \sigma E_y \\ 0 + \sigma E_z \end{bmatrix} = \begin{bmatrix} \boldsymbol{e}_x & \boldsymbol{e}_y & \boldsymbol{e}_z \\ \partial_x & \partial_y & \partial_z \\ H_x & H_y & H_z \end{bmatrix} = \begin{bmatrix} \partial_y H_z - \partial_z H_y \\ \partial_z H_x - \partial_x H_z \\ \partial_x H_y - \partial_y H_x \end{bmatrix} = \begin{bmatrix} -\partial_z H_y \\ \partial_z H_x \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} i\omega\mu_0 H_x \\ i\omega\mu_0 H_y \\ i\omega\mu_0 H_z \end{bmatrix} = \begin{bmatrix} -\partial_z E_y \\ \partial_z E_x \\ 0 \end{bmatrix}$$

Maxwell equations for 1D MT

$$\begin{bmatrix} J_x^s + \sigma E_x \\ J_y^s + \sigma E_y \\ 0 + \sigma E_z \end{bmatrix} = \begin{bmatrix} -\partial_z H_y \\ \partial_z H_x \\ 0 \end{bmatrix} \qquad \begin{bmatrix} i\omega\mu_0 H_x \\ i\omega\mu_0 H_y \\ i\omega\mu_0 H_z \end{bmatrix} = \begin{bmatrix} -\partial_z E_y \\ \partial_z E_x \\ 0 \end{bmatrix}$$

Obtain two sets of equations that describe two different polarizations:

$\partial_z H_y + \sigma E_x = -J_x^s$

$\partial_z E_x - i\omega\mu_0 H_y = 0$

$\partial_z H_x + \sigma E_y = -J_y^s$

$\partial_z E_y + i\omega\mu_0 H_x = 0$

Equations for both polarizations :

$\partial_z^2 E_x + \mathrm{i}\omega\mu_0\sigma E_x = -\ \mathrm{i}\omega\mu_0\ J_x^s$

$\partial_z^2 E_y + \mathrm{i}\omega\mu_0\sigma E_y = -\ \mathrm{i}\omega\mu_0\ J_y^s$

Sufficient to concentrate on x-polarization to understand the physics.

Type of solution «away» from any current sources

$$\partial_z^2 E_x + \mathrm{i}\omega\mu_0\sigma E_x = 0$$

$$k_\omega^2 = i\omega\mu_0\sigma$$

$$\partial_z^2 E_x + k_\omega^2 E_x = 0$$

Solutions have the general form:

$$E_x = Ae^{ik_\omega z} + Be^{-ik_\omega z}$$

The factors A and B are determined by the source(s) and reflection/transmission properties of the medium

$$k_\omega = \sqrt{i\omega\mu_0\sigma} = (1+i)\sqrt{\frac{\omega\mu_0\sigma}{2}} \qquad \sqrt{i} = \frac{1+i}{\sqrt{2}}$$

Skindepth and phase velocity

$$k_\omega = (1+i)\sqrt{\frac{\omega\mu_0\sigma}{2}}$$

Introduce phase velocity $c(\omega)$ and skin depth $\delta(\omega)$

$$k_\omega = \frac{\omega}{c(\omega)} + \frac{i}{\delta(\omega)}$$

$$c(\omega) = \sqrt{\frac{2\rho\omega}{\mu_0}}$$

$$\delta(\omega) = \sqrt{\frac{2\rho}{\mu_0\omega}}$$

Causal solution:

$$E_x = Ae^{ik_\omega z}$$

$$E_x = Ae^{-\frac{z}{\delta(\omega)}}e^{i\frac{\omega}{c(\omega)}z}$$

The field experience absorption
The absorption is frequency dependent
The phase velocity is frequency dependent – Dispersion

Absorption:

$$E_x = Ae^{-\frac{z}{\delta(\omega)}}e^{i\frac{\omega}{c(\omega)}z}$$

$$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{H/m}$$

$$\omega = 2\pi f$$

$$\delta(\omega) = \sqrt{\frac{2\rho}{\mu_0\omega}} \longrightarrow \delta(f) \approx 500\sqrt{\frac{\rho}{f}}\ [\mathrm{m}]$$

Skin depth vs resistivity

The skin depth δ describes the travel distance after which the magnitude of the EM signal is reduced by a factor of $1/e \cong 0.37$.

$$\delta = \sqrt{\frac{\rho}{\pi \mu f}} \approx 500\,[\mathrm{m}] \sqrt{\frac{\rho\,[\Omega\mathrm{m}]}{f\,[\mathrm{Hz}]}}$$

It was assumed here that the Earth is non-magnetic:

$$\mu \approx \mu_0 = 4\pi \times 10^{-7}\ \mathrm{H/m}$$

Skin depth is larger (attenuation is weaker) for larger resistivity:

e.g. for f = 0.25 Hz

Water (0.3 Ωm): δ = 548 m

Overburden (1.0 Ωm): δ = 1000 m

Overburden (2.0 Ωm): δ = 1414 m

HC-filled reservoir (50 Ωm): δ = 7000 m

Skin depth vs frequency

The skin depth δ describes the travel distance after which the magnitude of the EM signal is reduced by a factor of $1/e \cong 0.37$.

$$\delta = \sqrt{\frac{\rho}{\pi \mu f}} \approx 500\,[\mathrm{m}] \sqrt{\frac{\rho\,[\Omega\mathrm{m}]}{f\,[\mathrm{Hz}]}}$$

$$\mu \approx \mu_0 = 4\pi \times 10^{-7}\ \mathrm{H/m}$$

Skin depth is larger (attenuation is weaker) for smaller frequency:

Overburden (1.0 Ωm)

f = 0.25 Hz: δ = 1000 m

f = 0.75 Hz: δ = 575 m

f = 1.25 Hz: δ = 450 m

Note: Skin depth concept sometimes miss-used as limiting factor.

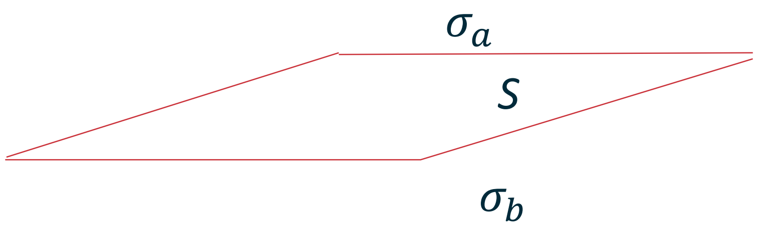
The receiver equipment has sensitivity to measures fields that have propagated several skindepths.

A propagation distance of 4.5 skindepths give an amplitude decay of approximately a factor 100.

Skin depth effects describe amlitude loss as a function of propagation distance for MT to a good approximation.

Note that amplitude loss depends on more than the skindepth
for an electric dipole source. Geometrical spreading effects comes in addition.

Boundary conditions



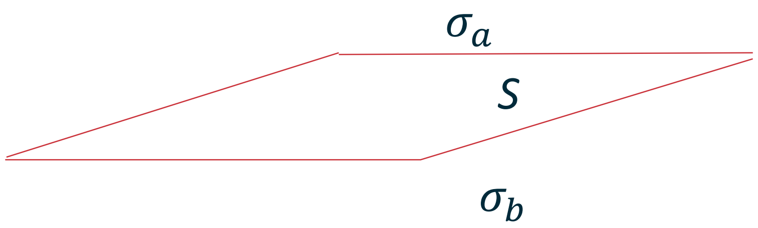
Surface S separates top halfspace with conductivity σ_a from bottom halfspace with conductivity σ_b.
The boundary conditions can be derived from the Maxwell equations

A component of the electric or magnetic field that is parallel to S is contineous over S.

The current normal to S is contineous.

The magnetic field normal to S is contineous if the to halfspaces are non-magnetic

As an example: Assume the surface S is horizontal



$$E_x^a = E_x^b$$
$$E_y^a = E_y^b$$
$$\sigma_a E_z^a = \sigma_b E_z^b$$

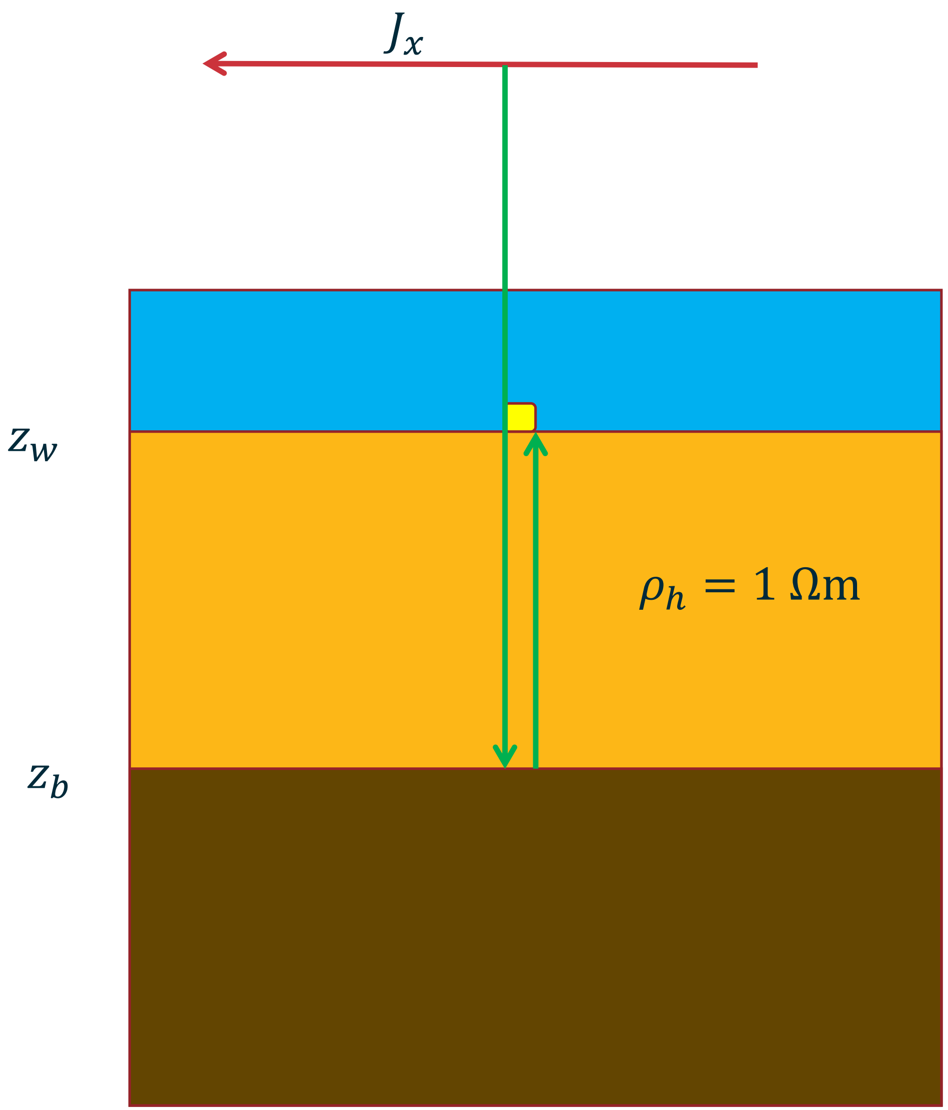
This boundary conditions plays a big role in marine CSEM since the vertical electric field from an electric dipole can be large.

In MT the electric field is dominantly horizontal due to the type of source , however a large vertical conductivity contrast may give introduce large amplitude variations in the field

$$H_x^a = H_x^b$$
$$H_y^a = H_y^b$$
$$H_z^a = H_z^b$$

$$\sigma_a E_z^a = \sigma_b E_z^b$$

Note that of the conductivity goes from a formation value of 1 Ohm-m to a reservoir conductivity of 100 Ohm-m over a short interval, then the vertical electric field must increase with a factor 100 over the same interval



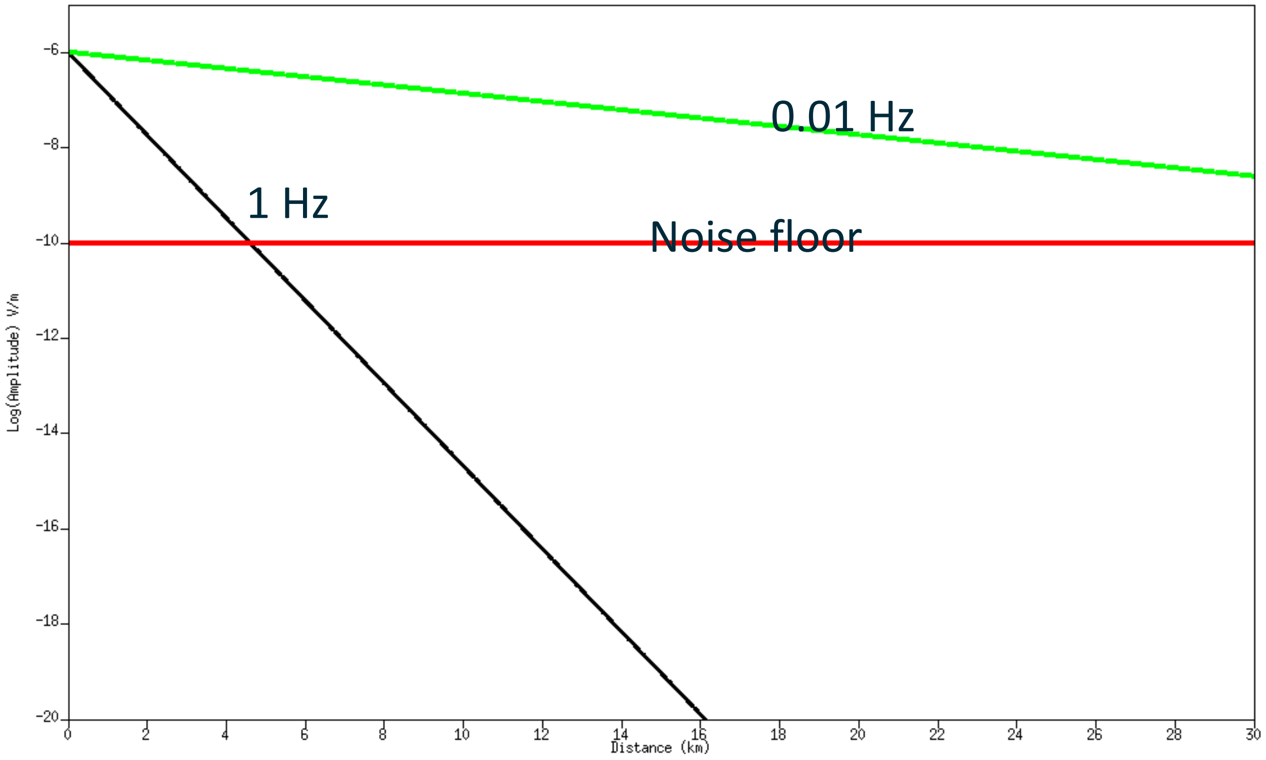
Assume a receiver can measure electric field amplitudes as low as 10^{-10} V/m

The downgoing MT signal has a signal strength of 10^{-6} V/m at receiver level

The reflection strength at z_b is:
$R = 0.1$

How deep can z_b be in order to be observable at 1 Hz?
How deep can z_b be in order to be observable at 0.01 Hz?

The limiting factor for observation is that the reflected field must be above the receiver noise floor of 10^{-10} V/m.



The limiting factor for observation is that the reflected field must be above the receiver noise floor of 10^{-10} V/m.

$$A_0 = 10^{-6} \text{ V/m}$$

$$A_{Noise} = 10^{-4} A_0$$

Critical distance when field amplitude at noise floor value
Must consider propagation down and up plus reflection strength

$$A_0 \, e^{-\frac{(z_b - z_w)}{\delta(f)}} \; R \; e^{-\frac{(z_b - z_w)}{\delta(f)}} = A_{Noise}$$

$$e^{-\frac{2(z_b - z_w)}{\delta(f)}} = \frac{A_{Noise}}{R \, A_0}$$

$$(z_b - z_w) = -\frac{\delta(f)}{2} \ln\left(\frac{A_{Noise}}{R \, A_0}\right) \qquad \delta(f) \approx 500\sqrt{\frac{\rho}{f}} \; [\mathrm{m}]$$

$$(z_b - z_w) = -\frac{\delta(f)}{2} \ln\left(\frac{A_{Noise}}{R\ A_0}\right)$$

$$\delta(f) \approx 500 \sqrt{\frac{\rho_h}{f}}\ [\mathrm{m}]$$

$$\ln\left(\frac{A_{Noise}}{R\ A_0}\right) \approx -6.9$$

$$\rho_h = 1\ \Omega\mathrm{m}$$

$$\delta(1\ Hz) = 500\ \mathrm{m}$$

$$(z_b - z_w) \approx 1700\ \mathrm{m}$$

$$\delta(0.01\ Hz) = 5000\ \mathrm{m}$$

$$(z_b - z_w) \approx 17000\ \mathrm{m}$$

Disperson:

$$E_x = Ae^{-\frac{z}{\delta(\omega)}}e^{i\frac{\omega}{c(\omega)}z}$$

$$c(\omega) = \sqrt{\frac{2\rho\omega}{\mu_0}} \longrightarrow c(f) \approx 3160\sqrt{\rho\, f}\ [\mathrm{m}/s]$$

Phase velocity increase with frequency and resistivity.

Some relations:

Phase velocity $c = \omega\delta$

Wave length $\lambda = 2\pi\delta$

In a 1 Ωm medium:

Frequency	Skindepth	Phase velocity	Wavelength
f [Hz]	δ [m]	c [m/s]	λ [m]
0.01	5000	316	31400
0.25	1000	1580	6280
1.0	500	3160	3140
4.0	250	6320	1570

In a 100 Ωm medium:

f [Hz]	δ [m]	c [m/s]	λ [m]
0.01	50000	3160	314000
0.25	10000	15800	62800
1.0	5000	31600	31400
4.0	2500	63200	15700

The Maxwell equations in a source free region

$$\nabla \times \boldsymbol{H} = \varepsilon_r \varepsilon_0 \partial_t \boldsymbol{E} + \sigma \boldsymbol{E}$$

$$\nabla \times \boldsymbol{E} = -\mu_0 \partial_t \boldsymbol{H}$$

$$\nabla \times \nabla \times \boldsymbol{E} + \mu_0 \varepsilon_r \varepsilon_0 \partial_t^2 \boldsymbol{E} + \mu_0 \sigma \partial_t \boldsymbol{E} = 0$$

Identity:

$$\nabla \times \nabla \times \boldsymbol{E} = \nabla(\nabla \cdot \boldsymbol{E}) - \nabla^2 \boldsymbol{E}$$

Special case: Homogeneous medium and no charges -> $\nabla \cdot \boldsymbol{E} = 0$

$$\nabla^2 \boldsymbol{E} - \mu_0 \varepsilon_r \varepsilon_0 \partial_t^2 \boldsymbol{E} - \mu_0 \sigma \partial_t \boldsymbol{E} = 0$$

- $\nabla^2 \boldsymbol{E} - \mu_0 \varepsilon_r \varepsilon_0 \partial_t^2 \boldsymbol{E} = 0$ Wave equation
- $\nabla^2 \boldsymbol{E} - \mu_0 \sigma \partial_t \boldsymbol{E} = 0$ Diffusion equation

Wave propagation

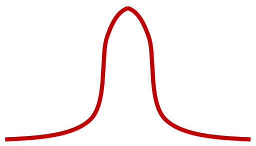
$$2\pi f\varepsilon \gg \sigma$$

$$\nabla^2 \boldsymbol{E} - \mu_0 \varepsilon_r \varepsilon_0 \partial_t^2 \boldsymbol{E} = 0$$

- waveshape doesn't change since

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_r \varepsilon_0}}$$

- real propagation constant (k_ω)
- negligible attenuation
- geometrical spreading



DIFFUSION

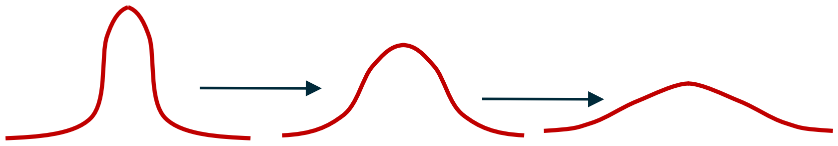
$$2\pi f\varepsilon \ll \sigma$$

$$\nabla^2 \boldsymbol{E} - \mu_0 \sigma \partial_t \boldsymbol{E} = 0$$

- waveshape change since

$$c(\omega) = \sqrt{\frac{2\rho\omega}{\mu_0}}$$

- complex propagation constant (k_ω)
- strong attenuation
- geometrical spreading



emgs

SPOT THE DIFFERENCE.

Thank you