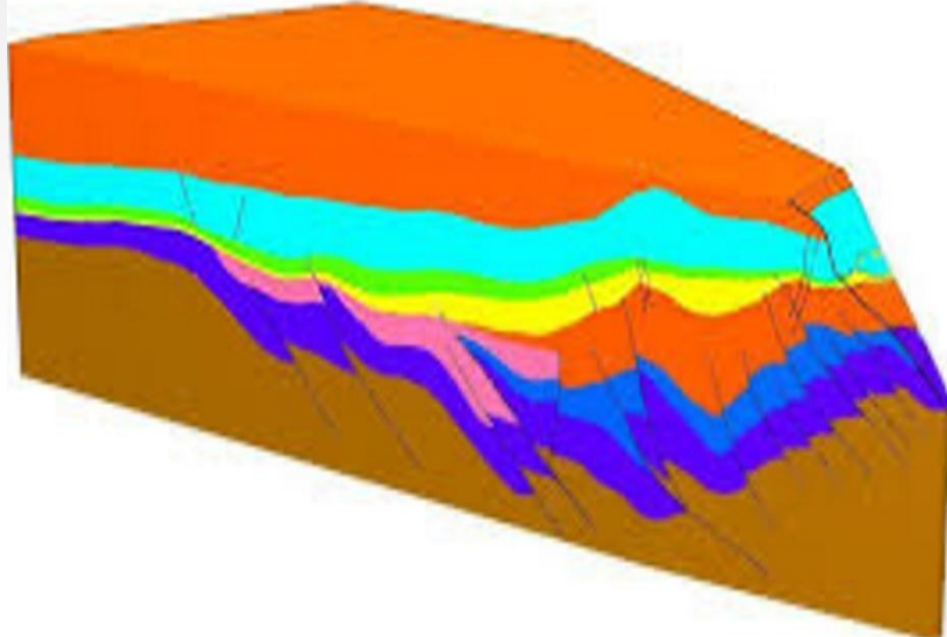
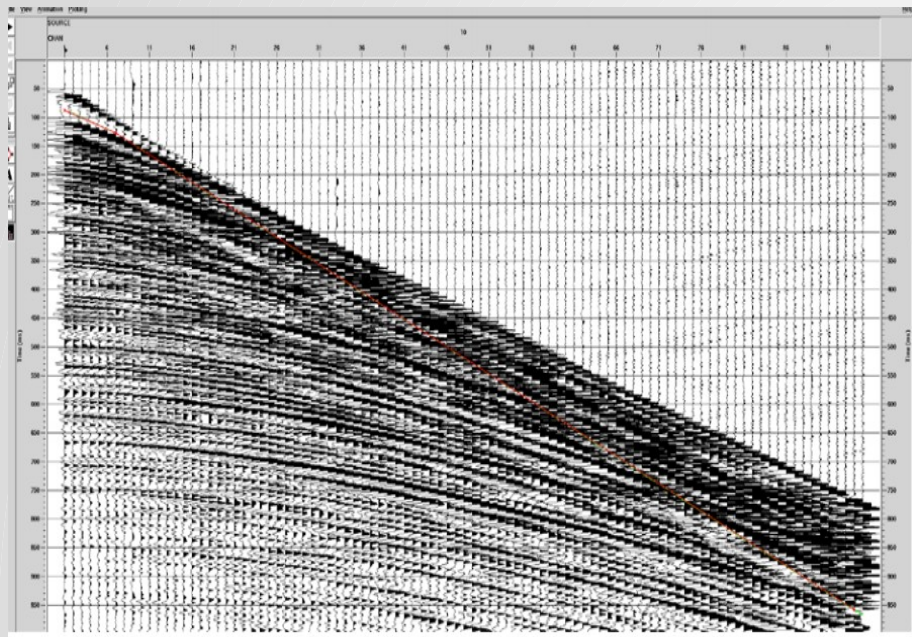


ROSE meeting



Shibo Xu & Alexey Stovas, NTNU
27-30th.04.2015, Trondheim



Seismic data

$$T(m, h)$$

Curvature, Anisotropy & Heterogeneity



Reflector & model

$$R, z_0$$

$$\delta, \eta, V$$

Estimation of anisotropy parameters from CRS approximation

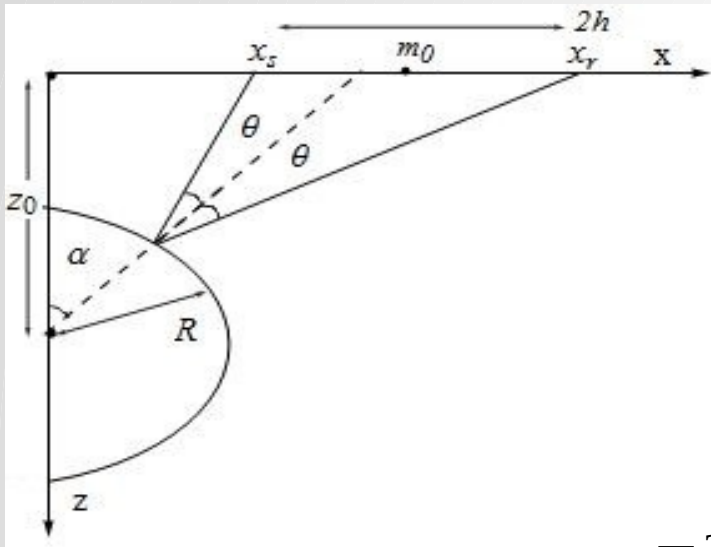
Objectives:

1. Distinguish curvature and anisotropy from CRS approximation
2. Investigate the effect from anisotropy and inhomogeneity to CRS attributes and model parameters
3. Estimation of anisotropy and model parameters

Outline

- ✦ 1 The CRS operator for a circular reflector in isotropic medium
- 2 Extend for more complicated media
- 3 Numerical examples & sensitivity analysis
- 4 Curvature and anisotropy estimation
- 5 Conclusions

CRS approximation

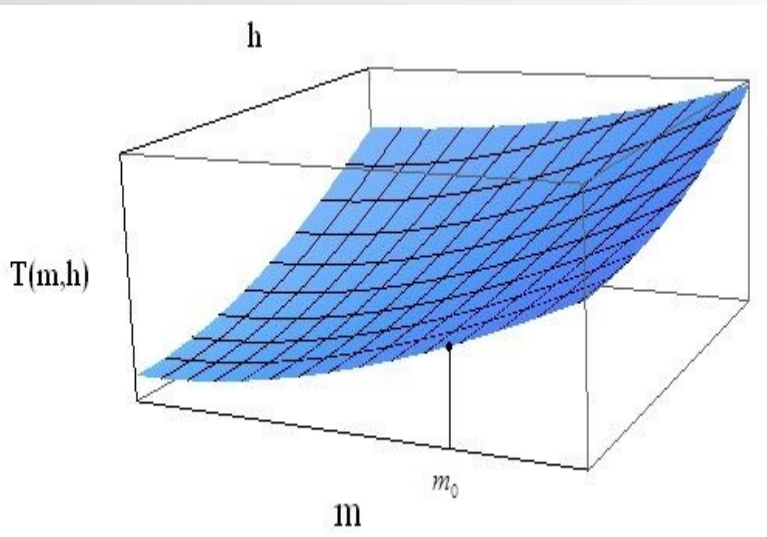


$$m = \frac{x_r + x_s}{2},$$

$$h = \frac{x_r - x_s}{2},$$

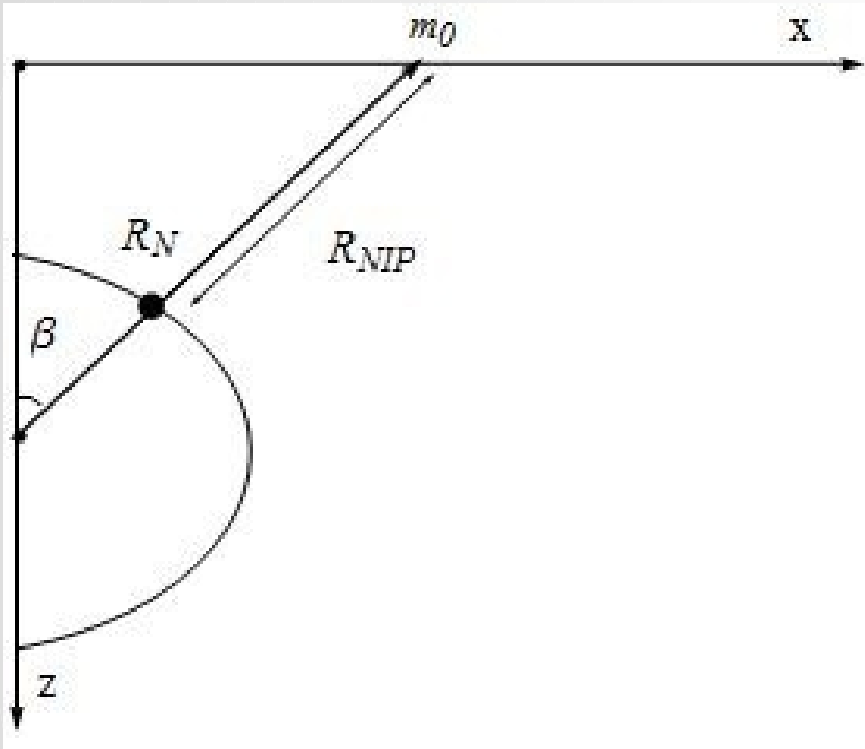
$$T = T_1 + T_2 = \frac{(z_0 - R \cos \alpha)}{V \cos(\alpha - \theta)} + \frac{(z_0 - R \cos \alpha)}{V \cos(\alpha + \theta)}$$

$$T_{CRS}^2(\Delta m, h) = A_0 + A_1 \Delta m + A_2 \Delta m^2 + B_2 h^2.$$



$$A_0, A_1, A_2, B_2 \text{ --- } (R, z_0, V, m_0)$$

Inversion of CRS attributes



$$A_0 = \frac{4R_{NIP}^2}{V^2}, \quad A_1 = \frac{8R_{NIP} \sin \beta}{V^2},$$

$$A_2 = \frac{4(R_{NIP} \cos^2 \beta + R_N \sin^2 \beta)}{V^2 R_N}, \quad B_2 = \frac{4 \cos^2 \beta}{V^2}.$$



$$\hat{R}_{NIP}, \hat{R}_N, \sin \hat{\beta},$$



$$\hat{R}, \hat{V}, \hat{z}_0.$$

Outline

1 The CRS operator for a circular reflector in isotropic medium

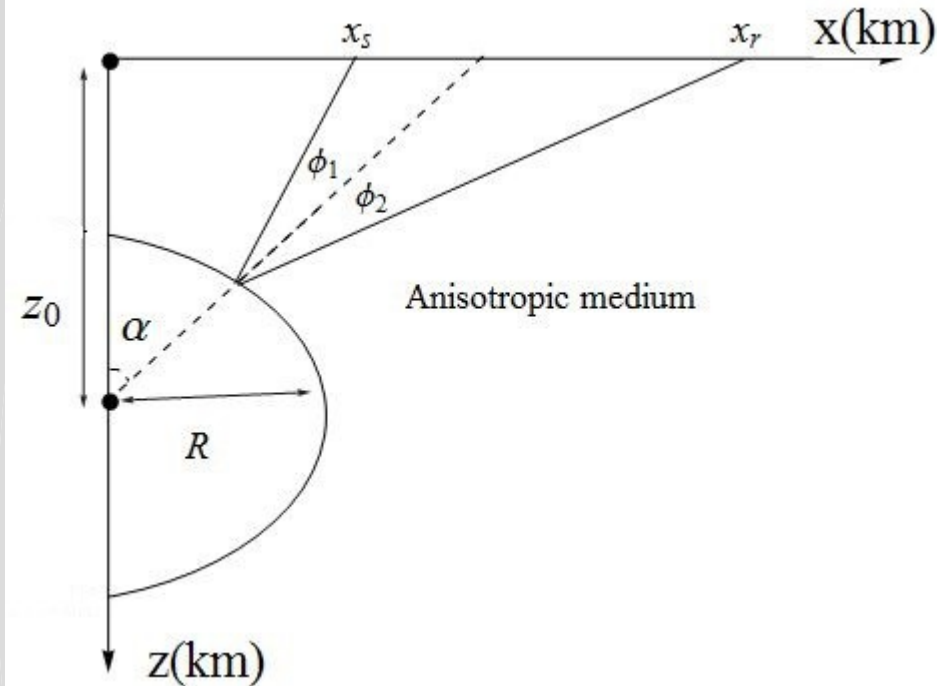
✦ 2 Extend for more complicated media

3 Numerical examples & sensitivity analysis

4 Curvature and anisotropy estimation

5 Conclusions

Extension for anisotropic media



For geometrical relation

$$m = \frac{x_r + x_s}{2}, h = \frac{x_r - x_s}{2}.$$

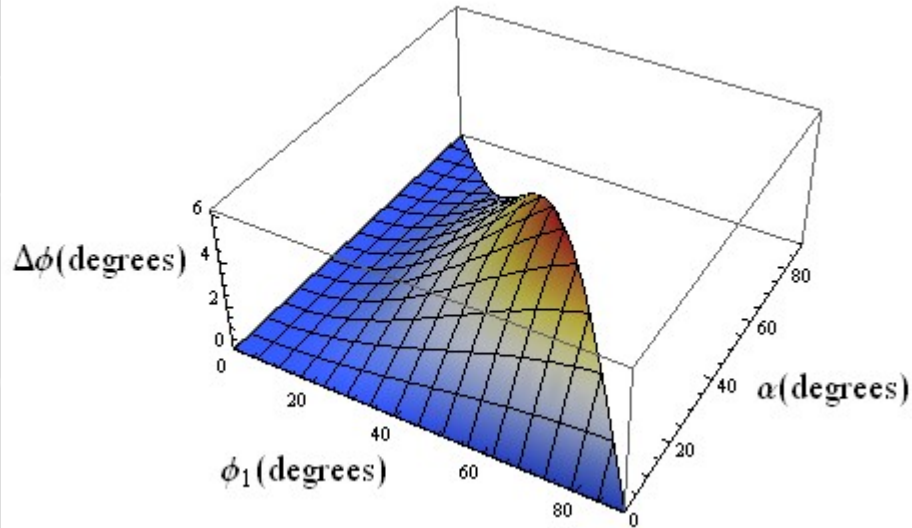
For group velocity

$$V_1 = V(\alpha - \phi_1), \quad V_2 = V(\alpha + \phi_2)$$

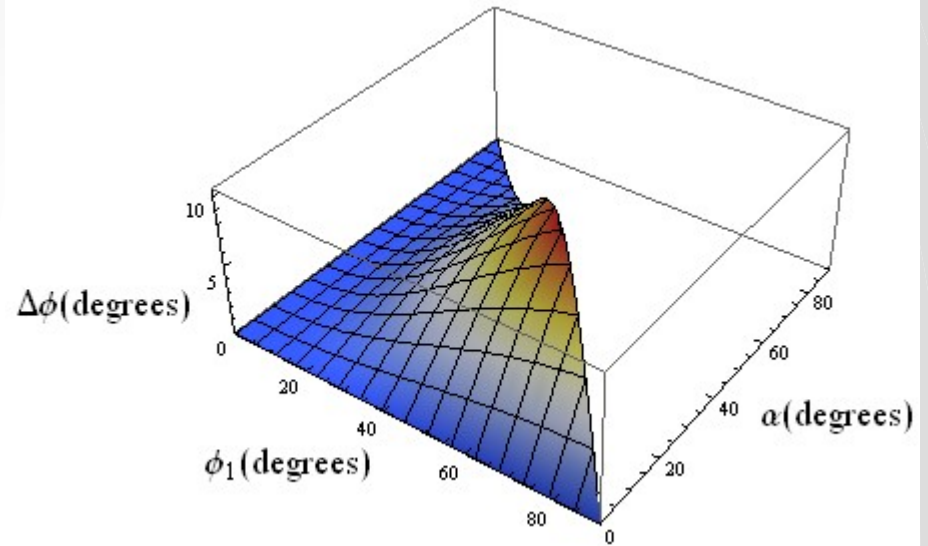
For travelttime

$$T = \frac{(z_0 - R \cos \alpha)}{\cos(\alpha - \phi_1)} \frac{1}{V(\alpha - \phi_1)} + \frac{(z_0 - R \cos \alpha)}{\cos(\alpha + \phi_2)} \frac{1}{V(\alpha + \phi_2)}.$$

EI medium



$\delta = 0.1$



$\delta = 0.2$

$$T = \frac{(z_0 - R \cos \alpha)}{\cos(\alpha - \theta)} \frac{1}{V(\alpha - \theta)} + \frac{(z_0 - R \cos \alpha)}{\cos(\alpha + \theta)} \frac{1}{V(\alpha + \theta)}.$$

←

GMA group velocity

$$\frac{1}{V(\phi)^2} = \frac{\cos^2 \phi}{V_0^2} + \frac{\sin^2 \phi}{V_N^2} + \frac{A \sin^2 \phi \tan^2 \phi}{V_N^4 \left(\frac{1}{V_0^2} + \frac{B \tan^2 \phi}{V_N^2} + \sqrt{\frac{1}{V_0^4} + \frac{2B \tan^2 \phi}{V_0^2 V_N^2} + \frac{C \tan^4 \phi}{V_N^4}} \right)},$$

(Fomel and Stovas, 2010)

Effective anisotropy

1 Homogeneous anisotropic media

- Elliptical isotropic model (EI)
- Transversely isotropic model with vertical symmetry axis (VTI)

2 Heterogeneous isotropic media

- Two layered isotropic model (2LI)

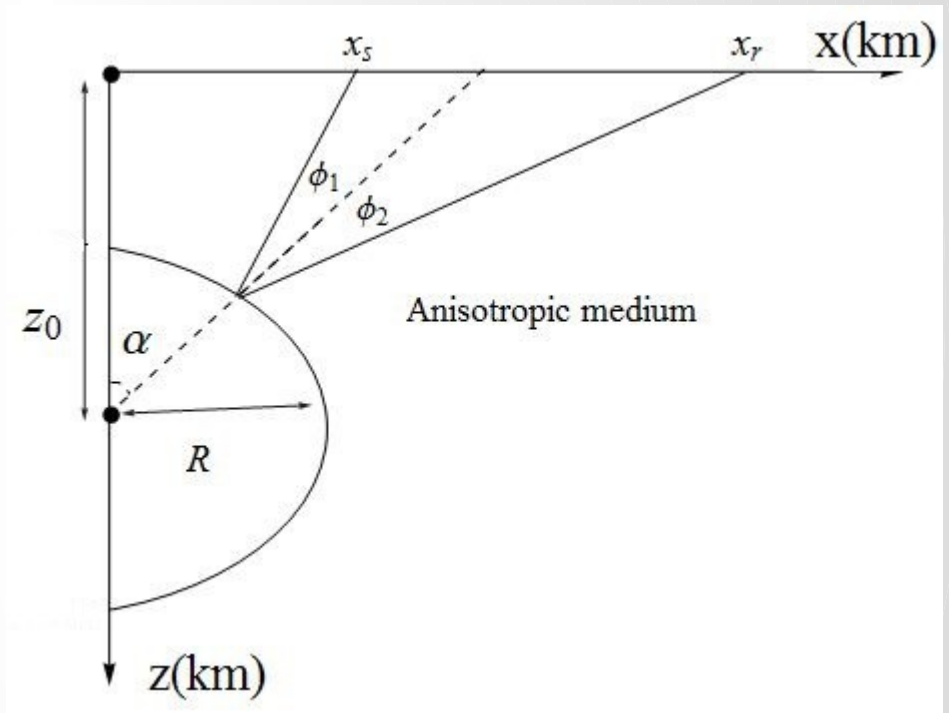
$$\frac{1}{V(\phi)^2} = \frac{\cos^2 \phi}{V_0^2} + \frac{\sin^2 \phi}{V_N^2} + \frac{A \sin^2 \phi \tan^2 \phi}{V_N^4 \left(\frac{1}{V_0^2} + \frac{B \tan^2 \phi}{V_N^2} + \sqrt{\frac{1}{V_0^4} + \frac{2B \tan^2 \phi}{V_0^2 V_N^2} + \frac{C \tan^4 \phi}{V_N^4}} \right)},$$

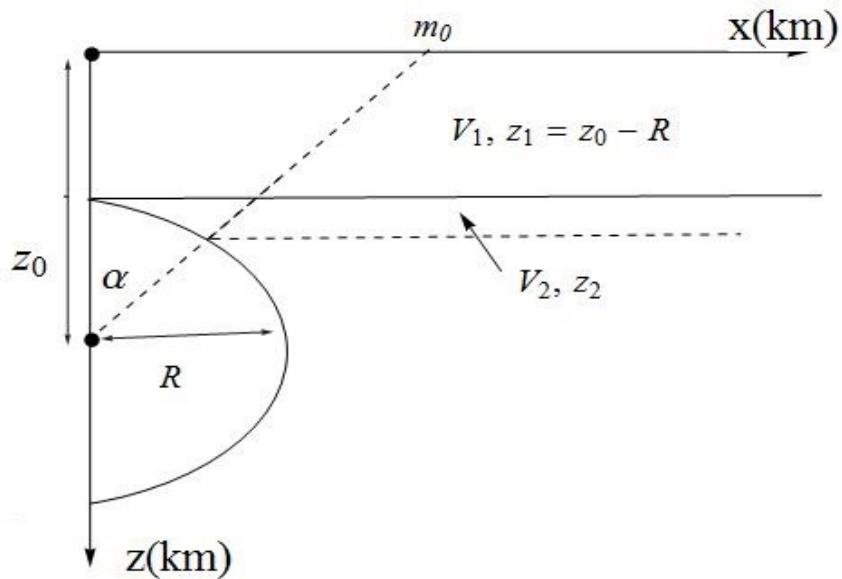
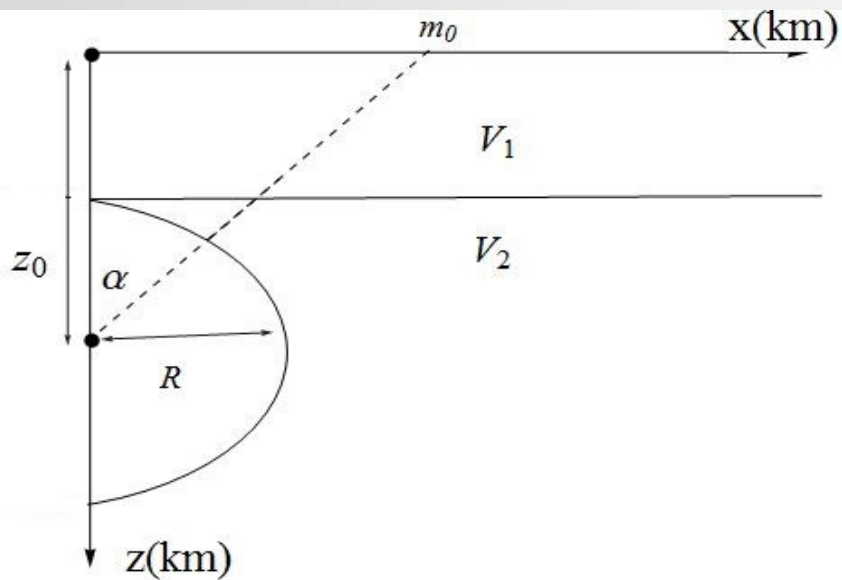
EI model:

$$A = 0.$$

VTI model:

$$A = -4\eta, \quad B = \frac{1 + 8\eta + 8\eta^2}{1 + 2\eta}, \quad C = \frac{1}{(1 + 2\eta)^2}.$$





2LI model:

$$\gamma = \frac{V_2}{V_1} \quad \varepsilon = \frac{z_2}{z_1} = \frac{R - R \cos \alpha}{z_0 - R}$$

$$V_0 = V_1 \frac{1 + \varepsilon}{1 + \frac{\varepsilon}{\gamma}}$$

$$V_N = V_1 \sqrt{\frac{1 + \varepsilon \gamma}{1 + \frac{\varepsilon}{\gamma}}}$$

$$A = -\frac{\varepsilon(\gamma^2 - 1)^2}{2\gamma(1 + \varepsilon\gamma)^2}$$

$$B = \frac{(\gamma^2 - 1)(1 + \frac{\varepsilon}{\gamma})}{2(1 + \varepsilon\gamma)^2}$$

$$C = 0.$$

Group velocity comparison

Reflector $R = 1$ km
 : $z_0 = 2$ km
 ISO : $V_0 = 2$ km/s
 EI : $V_N = V_0 \sqrt{1 + 2\delta}$
 $\delta = 0.1$

VTI $\eta = 0.2$

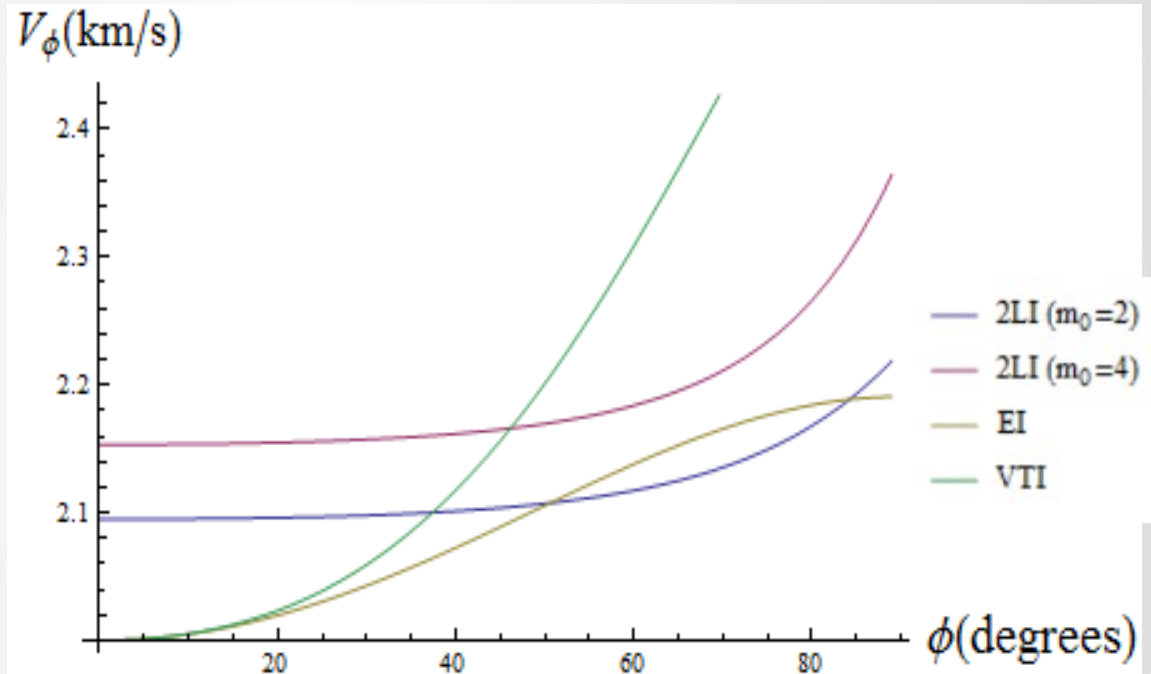
2LI : $V_1 = 2$ km/s

$V_2 = 2.5$ km/s

$m_0 = 2$ km

$m_0 = 4$ km

$\gamma = 1.25$ $\varepsilon_1 \approx 0.29$ $\varepsilon_2 \approx 0.55$



Outline

1 The CRS operator for a circular reflector in isotropic medium

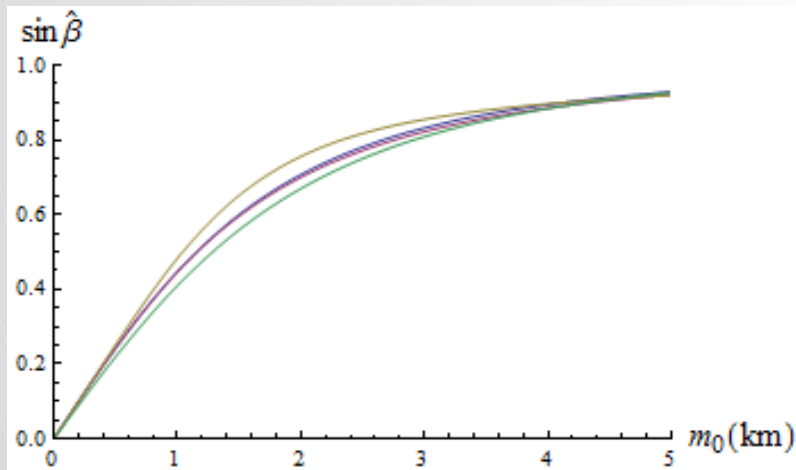
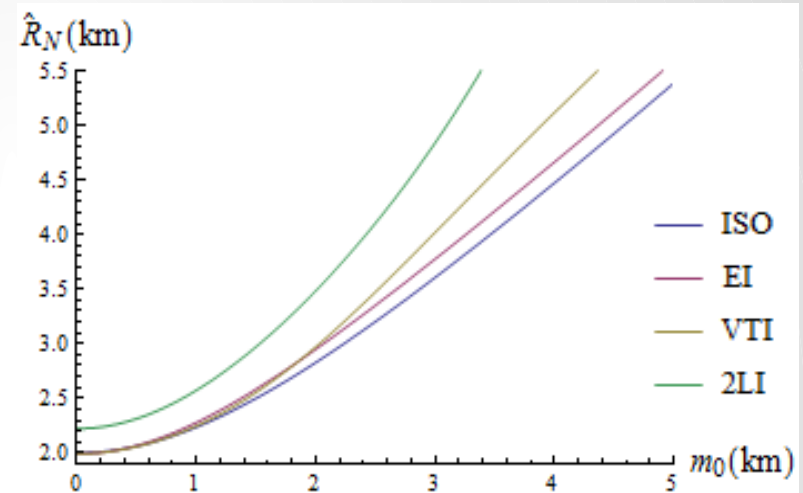
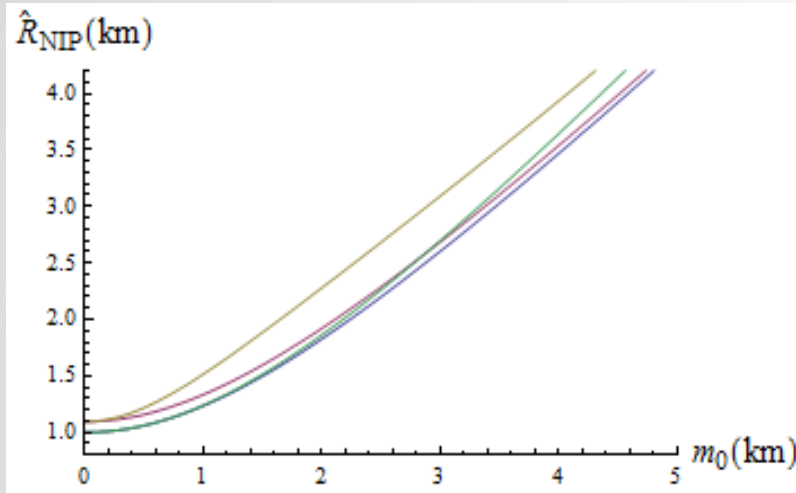
2 Extend for more complicated media

✦ 3 Numerical examples & sensitivity analysis

4 Curvature and anisotropy estimation

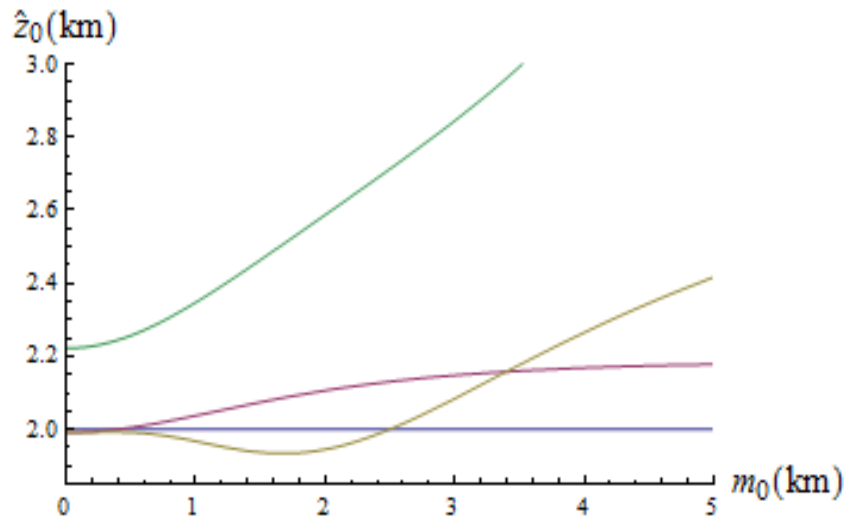
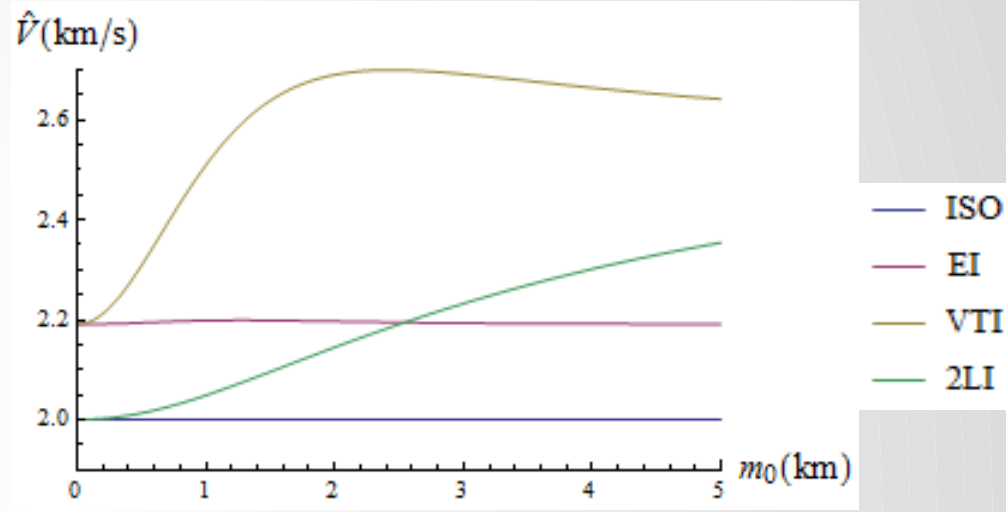
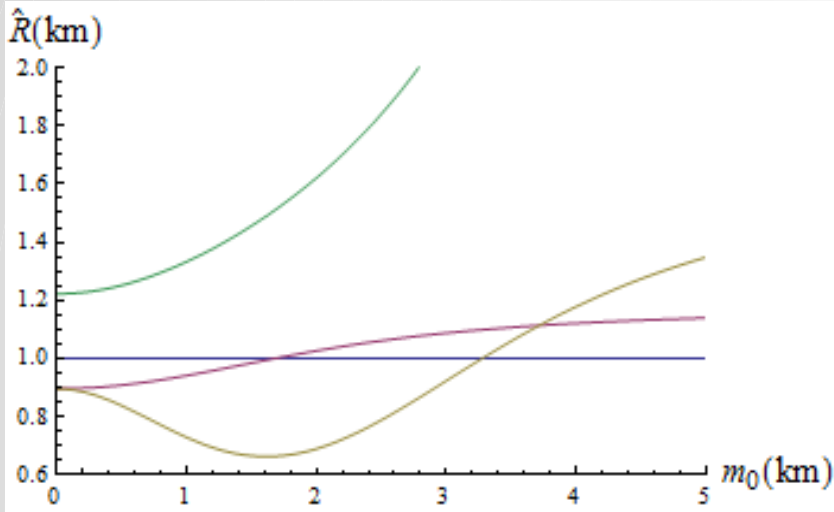
5 Conclusions

Numerical examples –CRS attributes



The plots of CRS attributes versus m_0 . The ISO, EI, VTI and 2LI cases are shown by blue, red, yellow and green colours, respectively.

Numerical examples – model parameters

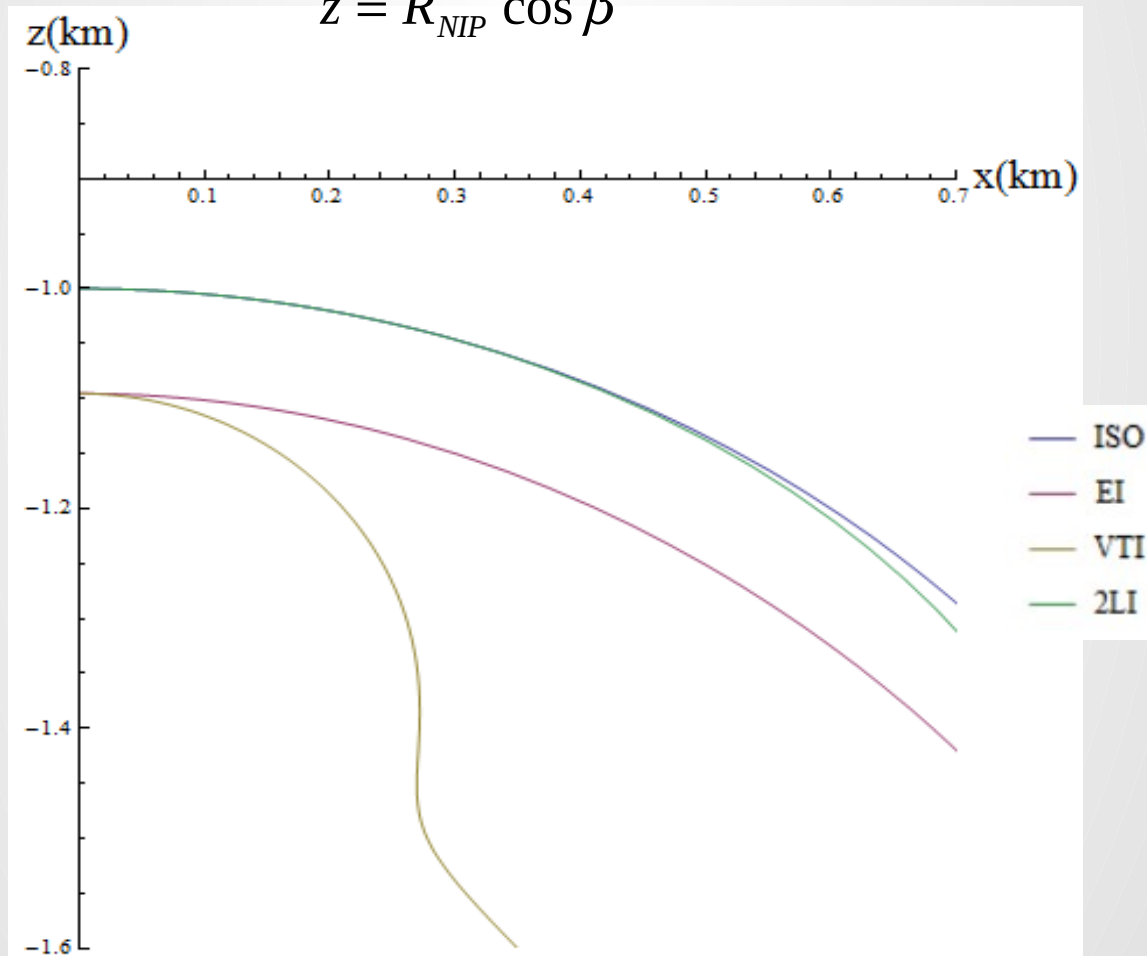


The plots of model parameters versus m_0 . The ISO, EI, VTI and 2LI cases are shown by blue, red, yellow and green colours, respectively.

Reconstructed reflector

$$\hat{x} = m_0 - \hat{R}_{NIP} \sin \hat{\beta}$$

$$\hat{z} = \hat{R}_{NIP} \cos \hat{\beta}$$



Linearization

CRS attributes

$$\hat{R}_{NIP} = R_{NIP} (1 + a_1 \delta + a_2 \eta),$$

$$\hat{R}_N = R_N (1 + b_1 \delta + b_2 \eta),$$

$$\sin \hat{\beta} = \sin \beta (1 + c_1 \delta + c_2 \eta).$$

Model parameters

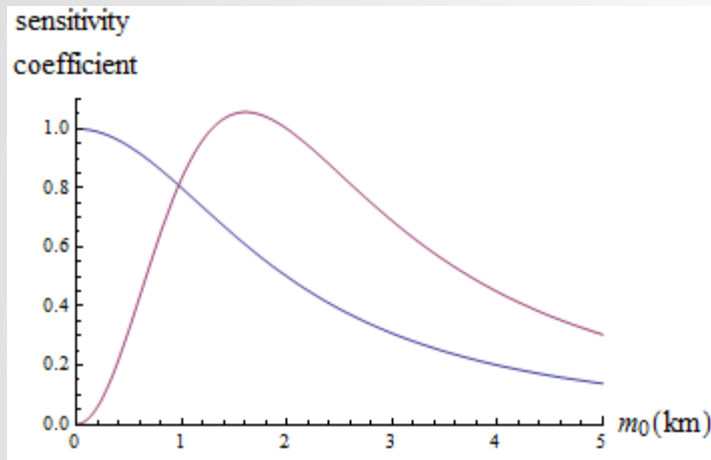
$$\hat{R} = R (1 + d_1 \delta + d_2 \eta),$$

$$\hat{V} = V_0 (1 + e_1 \delta + e_2 \eta),$$

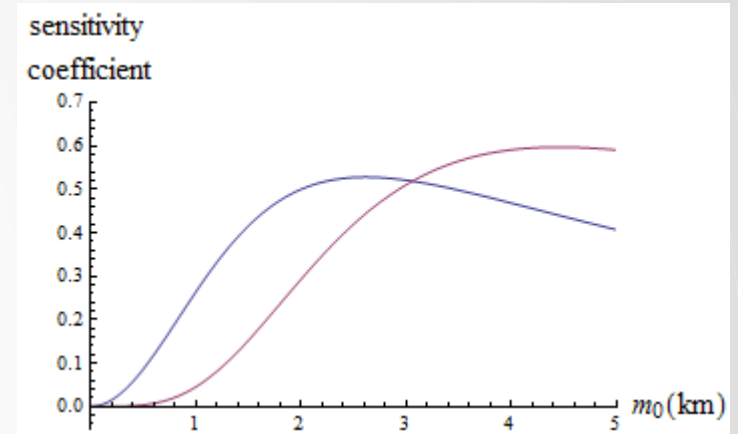
$$\hat{z}_0 = z_0 (1 + f_1 \delta + f_2 \eta).$$

Sensitivity analysis

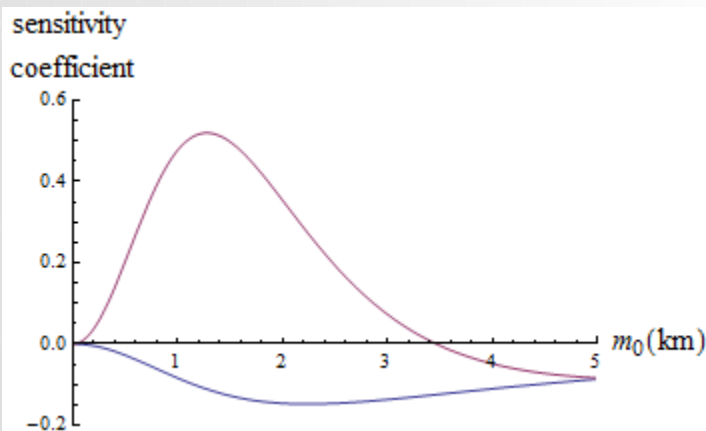
R_{NIP}



R_N



$\sin \beta$

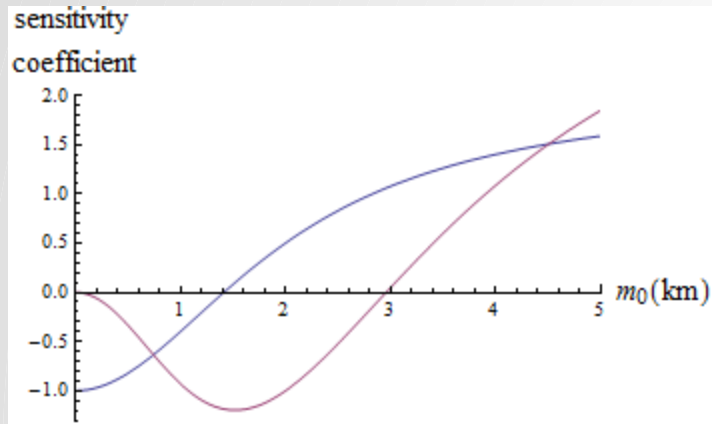


δ – blue

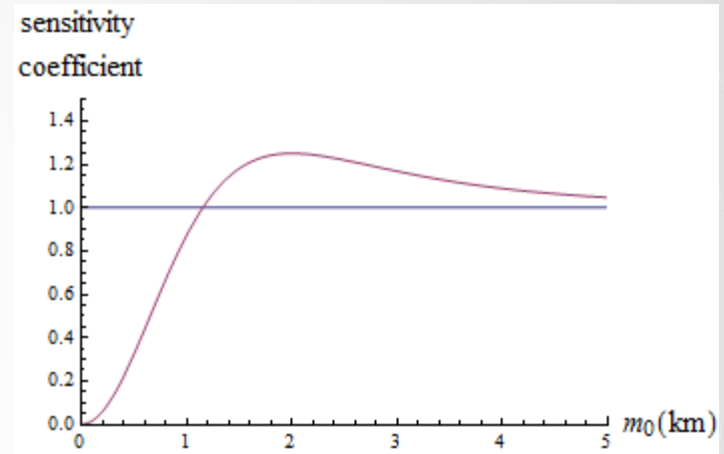
η – red

Sensitivity analysis

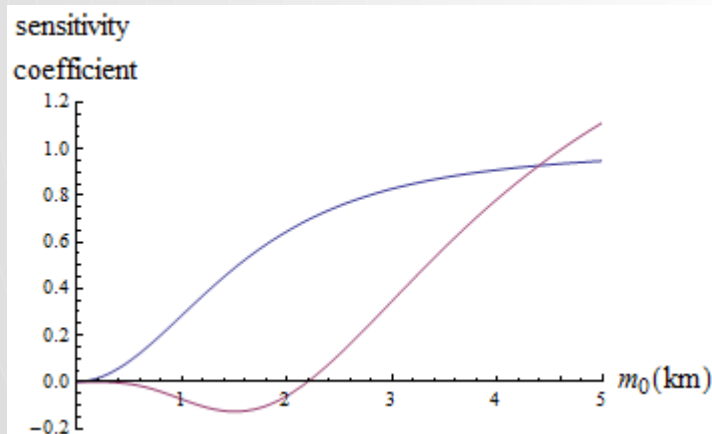
R



V



Z_0



δ – blue

η – red

Outline

1 The CRS operator for a circular reflector in isotropic medium

2 Extend for more complicated media

3 Numerical examples & sensitivity analysis

✦ 4 Curvature and anisotropy estimation

5 Conclusions

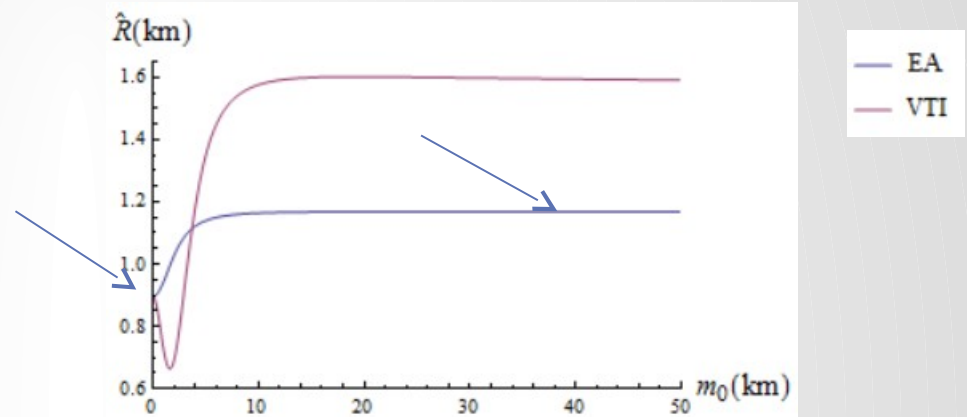
Estimation

$$\hat{R}^{(0)} = \hat{R}(m_0 = 0)$$

$$\hat{R}^{(\infty)} = \hat{R}(m_0 \rightarrow \infty)$$

$$\hat{z}_0^{(0)} = \hat{z}_0(m_0 = 0)$$

$$\hat{z}_0^{(\infty)} = \hat{z}_0(m_0 \rightarrow \infty)$$



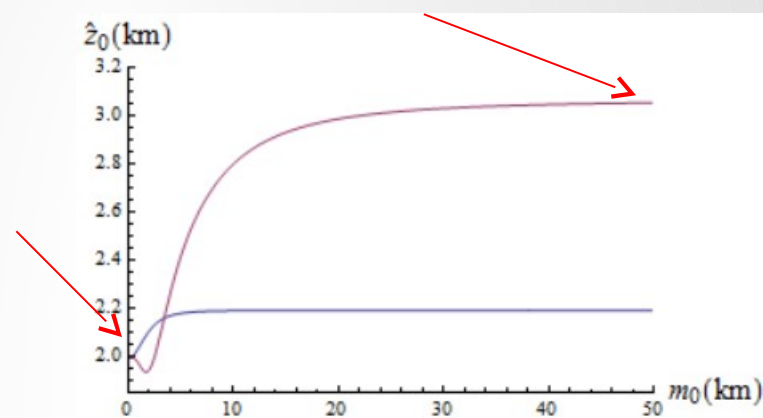
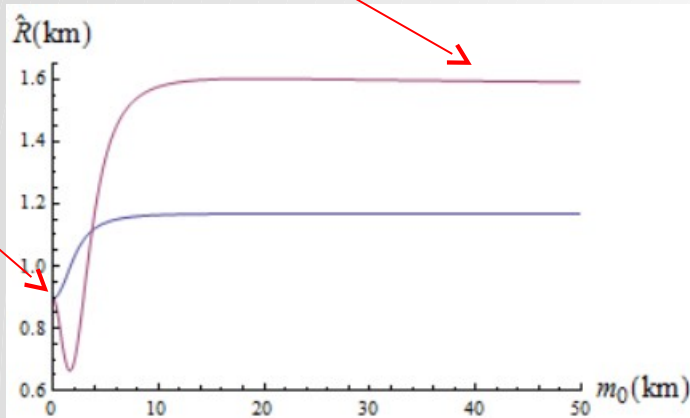
For EI model

$$\hat{R}^{(0)} = \lim_{m_0 \rightarrow 0} \hat{R} = R \left(\frac{z_0 + 2R\delta - 2z_0\delta}{z_0 + 2R\delta} \right) \sqrt{1 + 2\delta} \approx R(1 - \delta),$$

$$\hat{R}^{(\infty)} = \lim_{m_0 \rightarrow \infty} \hat{R} = R \frac{1 + 4\delta}{1 + 2\delta} \approx R(1 + 2\delta).$$

$$\longrightarrow \tilde{R} = \frac{2\hat{R}^{(0)} + \hat{R}^{(\infty)}}{3}, \quad \tilde{\delta} = \frac{\hat{R}^{(\infty)} - \hat{R}^{(0)}}{2\hat{R}^{(0)} + \hat{R}^{(\infty)}}.$$

Estimation



$$\hat{R}^{(0)} = \lim_{m_0 \rightarrow 0} \hat{R} = R \left(\frac{z_0 + 2R\delta - 2z_0\delta}{z_0 + 2R\delta} \right) \sqrt{1 + 2\delta} \approx R(1 - \delta),$$

For VTI model

$$\hat{R}^{(\infty)} = \lim_{m_0 \rightarrow \infty} \hat{R} = R \left(2 - \frac{1}{(1 + 2\delta)(1 + 2\eta)^2} \right) \approx R(1 + 2\delta + 4\eta),$$

$$\hat{z}_0^{(0)} = \lim_{m_0 \rightarrow 0} \hat{z}_0 = \frac{z_0^2 \sqrt{1 + 2\delta}}{z_0 + 2R\delta} \approx z_0(1 + \delta) - 2R\delta,$$

$$\hat{z}_0^{(\infty)} = \lim_{m_0 \rightarrow \infty} \hat{z}_0 = z_0 \sqrt{1 + 2\delta}(1 + 2\eta) \approx z_0(1 + \delta + 2\eta).$$

➡ $\tilde{R}, \tilde{z}_0, \tilde{\delta}, \tilde{\eta}$

Estimation test

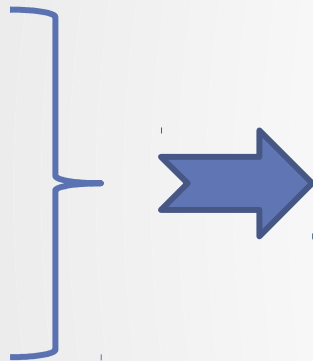
$$m_0 = 0 \text{ km} \quad m_0 = 5 \text{ km}$$

$$\hat{R}^{(0)} = 0.896 \text{ km}$$

$$\hat{R}^{(\infty)} = 1.348 \text{ km}$$

$$\hat{z}_0^{(0)} = 1.992 \text{ km}$$

$$\hat{z}_0^{(\infty)} = 2.417 \text{ km}$$

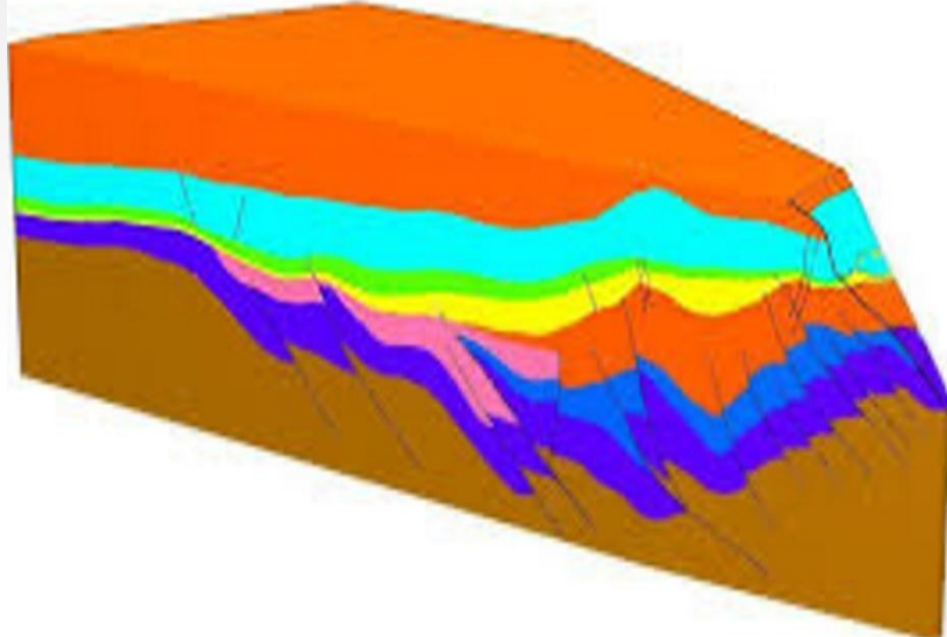
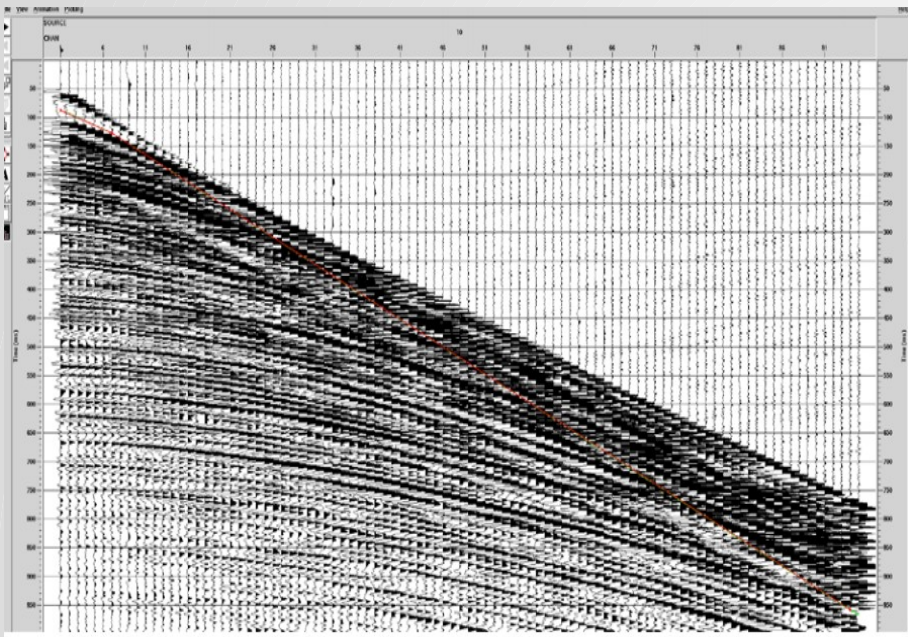


$$R = 0.941 \text{ km}$$

$$z_0 = 1.987 \text{ km}$$

$$\delta = 0.048$$

$$\eta = 0.084$$



Seismic data



Reflector & model

$$T(m, h)$$

$$R, z_0, V$$

$$\delta, \eta$$

Step 1. $T(\Delta m, h)$ ---data



Step 2. A_0, A_1, A_2, B_2 --- $\left(\frac{\partial T^2}{\partial \Delta m}\right), \left(\frac{\partial^2 T^2}{\partial \Delta^2 m}\right), \left(\frac{\partial^2 T^2}{\partial h^2}\right)$, CRS approximation



Step 3. $\hat{R}^{(0)} = \hat{R}(m_0 = 0)$ ---isotropic inversion from CRS approximation

$$\hat{R}^{(\infty)} = \hat{R}(m_0 \rightarrow \infty)$$

$$\hat{z}_0^{(0)} = \hat{z}_0(m_0 = 0)$$

$$\hat{z}_0^{(\infty)} = \hat{z}_0(m_0 \rightarrow \infty)$$



Step 4. $\tilde{R}, \tilde{z}_0, \tilde{\delta}, \tilde{\eta}$ ---analysis sensitivity

Outline

1 The CRS operator for a circular reflector in isotropic medium

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4 Curvature and anisotropy estimation

✦ 5 Conclusions

Conclusions

- 1 Investigate the effect of effective anisotropy (anisotropy & heterogeneity) on CRS attributes and the estimated model parameters.
- 2 Propose a method to estimate the structural and anisotropy parameters by using CRS approximation from a circular reflector.

References

- Alkhalifah T 1998 Acoustic approximations for processing in transversely isotropic media *Geophysics* **63** 623-631
- Fomel S and Kazinnik R 2013 Non-hyperbolic common reflection surface *Geophysical Prospecting* **61** 21-27
- Fomel S and Stovas A 2010 Generalized nonhyperbolic moveout approximation *Geophysics* **75** U9-U18
- Höcht G, E de Bazelaire, P Majer, and P Hubral, 1999 Seismics and optics: Hyperbolae and curvatures *Journal of Applied Geophysics* **42** 261-281
- Jäger R, Mann J, Höcht G and Hubral P 2001 Common-reflection-surfacestack: Image and attributes *Geophysics* **66** 97-109
- Landa E, Keydar S and Moser T J 2010 Multifocusing revisited: Inhomogeneous media and curved interfaces *Geophysical Prospecting* **58** 925-938
- Vanelle C, Bobsin M, Schemmert P, Kashtan B and Gajewski D 2012 i-CRS: A new multiparameter stacking operator for an/isotropic media 82nd Annual International Meeting *SEG Expanded Abstracts*

End

Thanks for attention!