Transmission-Propagation Operator Theory and Tip Wave Superposition Method for 3D 2-block model with shadow.

Comparison with Finite Difference Method

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Outline

Shadow

Forward modeling methods description

TPOT&TWSM method

- V-model: 3D 2-block with shadow zone
- Analytical solution by Transmission-Propagation Operator Theory (TPOT)
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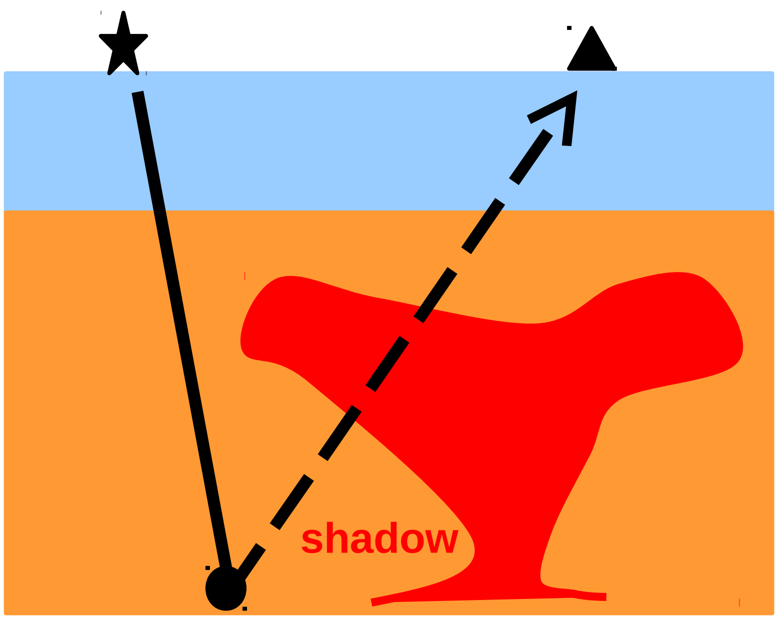
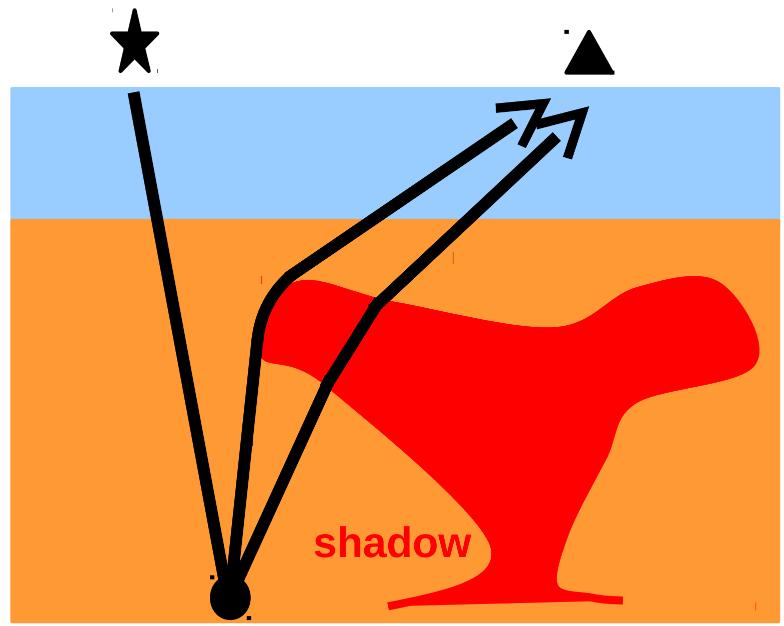
Shadow changes recorded time

Wrong travel time



Correct travel time:
reflection wave will come
at 2 different shifted times



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Forward modeling: TPOT&TWSM vs FD

TPOT&TWSM

Analytical & visualization

Gives separate waves' description

Computation time
proportional number of modelled waves

FD

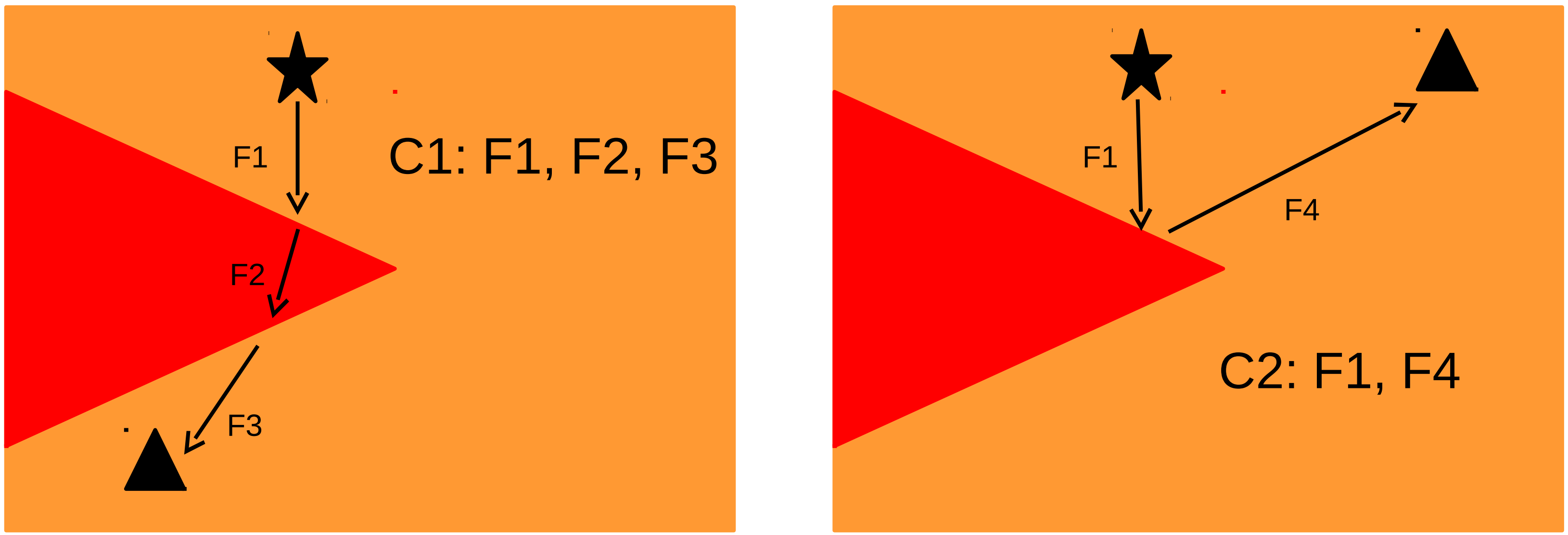
Numerical

Gives total solution

Computation time
depends on equation

TPOT&TWSM gives separate waves' description

Arkady Aizenberg, Milana Ayzenberg and K.D. Klem-Musatov, 2011



Gives flexibility of modeling code (C1, C2 etc.)
and separate description of each wavefield fragment (F1, F2, F3, F4)

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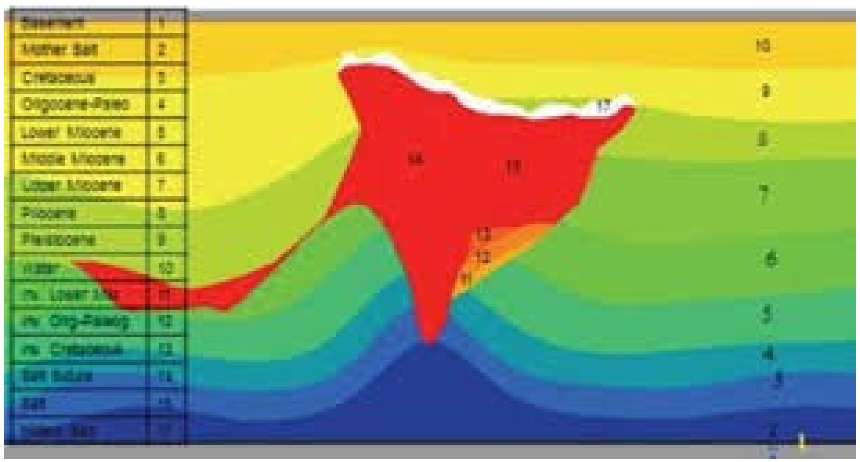
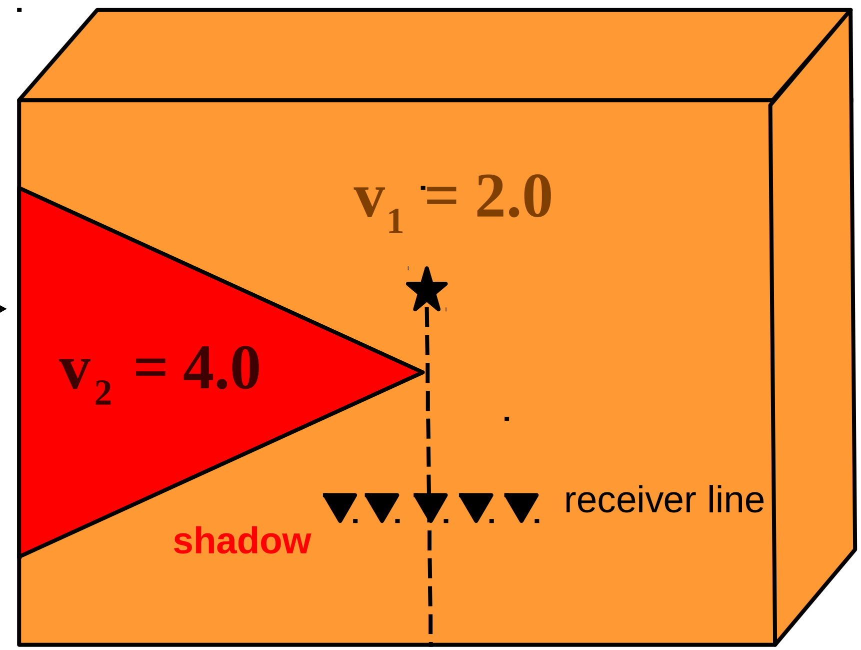
V-model: 3D 2-block with shadow zone

Shadow = velocity contrast

real model



mathematical model



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TPOT statement of problem for particles. V-model

Alena Ayzenberg, N. Zyatkov, W. Weibull, Arkady Aizenberg, A. Stovas, to be submitted

Wave motion equation in media m=1,2

Radiation condition in media m=1,2

Edge condition in media m=1,2

Boundary condition

$$\begin{cases} \mathrm{D_x}\, \mathrm{u}_m(\mathrm{x},\omega) + \left(-i\,k_{P,m}\right) \mathrm{u}_m(\mathrm{x},\omega) = -\,\mathrm{f}_m(\mathrm{x},\omega) \\ \iint\limits_{\mathrm{S}_m^{\infty}} \mathrm{G}_m(\mathrm{x},\mathrm{s},\omega)\, \mathrm{N_s}\, \mathrm{u}_m(\mathrm{s},\omega)\, dS(\mathrm{s}) = 0 \\ \iint\limits_{\{\mathrm{S}_m^{E}\}} \mathrm{G}_m(\mathrm{x},\mathrm{s},\omega)\, \mathrm{N_s}\, \mathrm{u}_m(\mathrm{s},\omega)\, dS(\mathrm{s}) = 0 \\ \begin{cases} \mathrm{C}_1^1\, \mathrm{u}_1^1(\mathrm{s},\omega) = \mathrm{J}\, \mathrm{C}_2^1\, \mathrm{u}_2^1(\mathrm{s},\omega) \\ \mathrm{C}_1^2\, \mathrm{u}_1^2(\mathrm{s},\omega) = \mathrm{J}\, \mathrm{C}_2^2\, \mathrm{u}_2^2(\mathrm{s},\omega) \end{cases} \end{cases}$$

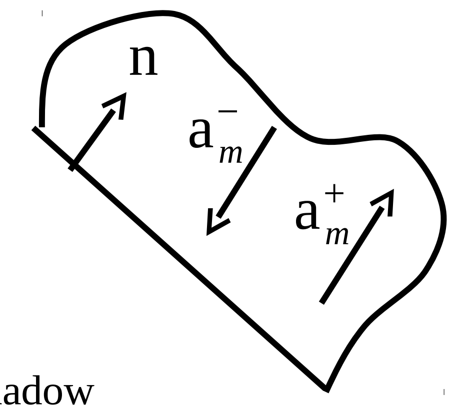
Particle velocity / pressure vector

$$\mathrm{u}_m = \begin{pmatrix} \left(\rho_m v_{P,m}\right) v_{1,m} \\ \left(\rho_m v_{P,m}\right) v_{2,m} \\ \left(\rho_m v_{P,m}\right) v_{3,m} \\ p_m \end{pmatrix}$$

TPOT statement of problem for amplitudes. V-model

Arkady Aizenberg, N. Zyatkov, Alena Ayzenberg, E. Rakshaeva, 2014
Arkady Aizenberg, Milana Ayzenberg and K.D. Klem-Musatov, 2011
Arkady Aizenberg and Alena Ayzenberg, 2015

New equation $$\mathrm{a} = \mathrm{P}\,\mathrm{a} + \mathrm{a}^{(0)}$$



$$\mathrm{a} = \begin{pmatrix} \mathrm{a}_1 \\ \mathrm{a}_2 \end{pmatrix}, \quad \mathrm{a}_m = \begin{pmatrix} \mathrm{a}_m^+ \\ \mathrm{a}_m^- \end{pmatrix} - \text{amplitudes, } m = 1, 2 \text{ (Medium I, II)}$$

$$\mathrm{a}^{(0)} = \sum_{n=0}^{\infty} \left[\mathrm{P}_{\mathrm{G}}\ \mathrm{P}_{h\mathrm{G}} \right]^n \mathrm{a}_{\mathrm{G}}^{(0)} - \text{feasible source wavefield which accounts for shadow}$$

$n = 2$ – approximation of feasible source wavefield

$$\mathrm{P} = \begin{bmatrix} \mathrm{P}_{11} & \mathrm{O} \\ \mathrm{O} & \mathrm{P}_{22} \end{bmatrix}, \quad \mathrm{P}_{mm} = \begin{bmatrix} P_{mm}^{++} & P_{mm}^{+-} \\ P_{mm}^{-+} & P_{mm}^{--} \end{bmatrix}$$

$$P_{mm}^{\pm\pm}\left(\tilde{\sigma}_m, \mathrm{s}'_m, \mathrm{s}_m, \tilde{\sigma}_m\right)\langle \mathrm{K} \rangle = Q^{\pm}\, K_{mm}\, H^{\pm}$$

$$K_{mm}\left(\mathrm{s}'_m, \mathrm{s}_m\right)\langle \mathrm{K} \rangle - \text{feasible Kirchhoff operator which accounts for shadow}$$

TPOT statement of problem for amplitudes. V-model

Arkady Aizenberg, Milana Ayzenberg and K.D. Klem-Musatov, 2011
Milana Ayzenberg, Arkady Aizenberg, H. Helle, K. Klem-Musatov, J. Pajchel, B. Ursin, 2007 Milana Ayzenberg, I. Tsvankin, Arkady Aizenberg, B. Ursin, 2009
E. Rakshaeva, T. Nefedkina, Arkady Aizenberg, R. Vilegzhanin, P. Lykhin, 2015

New boundary condition $\mathrm{a} = \mathrm{T}\,\mathrm{a}$

$$\mathrm{T} = \begin{bmatrix} \mathrm{T}_{11} & \mathrm{T}_{12} \\ \mathrm{T}_{21} & \mathrm{T}_{22} \end{bmatrix}, \quad \mathrm{T}_{mn} = \begin{bmatrix} 0 & T_{mn} \\ T_{mn} & 0 \end{bmatrix}, \quad m,n = 1,2$$

$$T_{jn}(\mathrm{s},\mathrm{s}') = F^{-1}(\mathrm{s},\mathrm{k})\,\hat{T}_{jn}(\mathrm{k})\,F(\mathrm{k},\mathrm{s}')$$

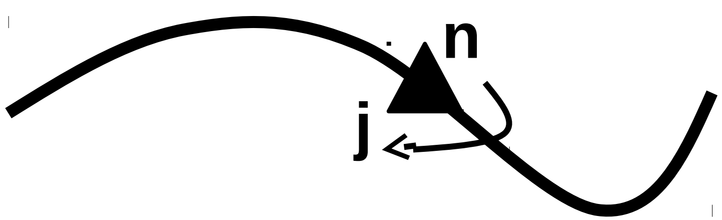
$$\mathrm{k} = (k_1, k_2)$$

$$\mathrm{s} = (s_1, s_2)$$

$$\hat{T}_{jn}(\mathrm{k}) = \frac{2\dfrac{k_{3n}}{\rho_n}}{\dfrac{k_{3n}}{\rho_n} + \dfrac{k_{3j}}{\rho_j}}$$

conventional plane-wave transmission coefficient

3D curved / plane interface



TPOT solution for V-model

Arkady Aizenberg, Milana Ayzenberg and K.D. Klem-Musatov, 2011
Alena Ayzenberg, N. Zyatkov, W. Weibull, Arkady Aizenberg, A. Stovas, to be submitted

New statement of problem

$$\begin{cases} a = P\,a + a^{(0)} \\ a = T\,a \end{cases}$$

$$\downarrow$$

$$a = P\,T\,a + a^{(0)}$$

$$\downarrow \quad \|P\| < 1, \quad \|T\| = 1$$

$$a = [P\,T]^3\,a + a^{(0)} + a^{(1)} + a^{(2)}, \quad a^{(2)} = [P\,T]\,[P\,T]\,a^{(0)}$$

solution:
source wavefield and
double transmitted wavefield

$$a = a^{(0)} + a^{(2)}$$

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Forward modeling by TWSM for V-model

Arkady Aizenberg, 1982, 1993
Arkady Aizenberg, Milana Ayzenberg and K.D. Klem-Musatov, 2011
Alena Ayzenberg, N. Zyatkov, W. Weibull, Arkady Aizenberg, A. Stovas, to be submitted

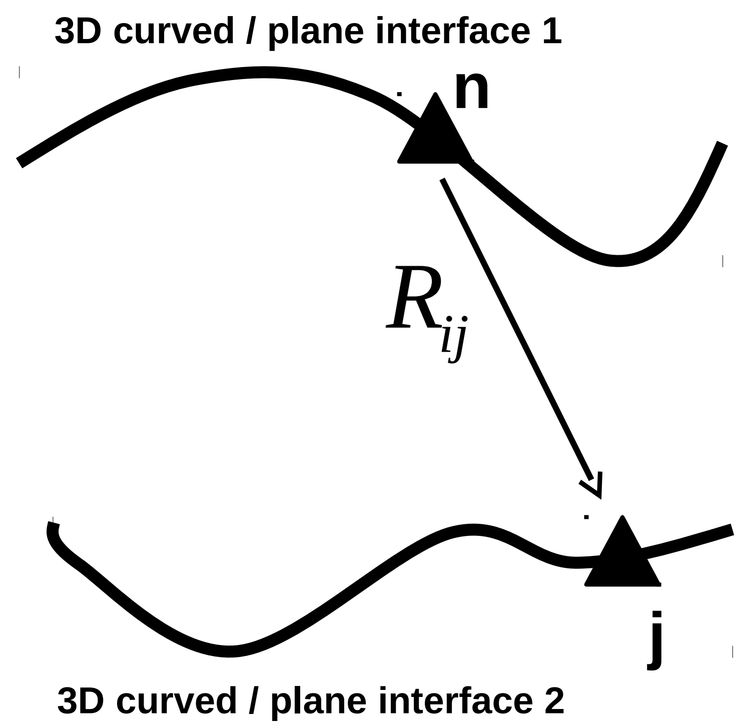
$$\mathrm{a} = \mathrm{a}^{(0)} + \mathrm{a}^{(2)} = \mathrm{a}^{(0)} + [\mathrm{P}\ \mathrm{T}][\mathrm{P}\ \mathrm{T}]\,\mathrm{a}^{(0)}$$

$$\mathrm{a}^{(2)} = \begin{pmatrix} \mathrm{a}_1^{(2)} \\ \mathrm{a}_2^{(2)} \end{pmatrix}, \quad \mathrm{a}^{(0)} = \begin{pmatrix} \mathrm{a}_1^{(0)} \\ 0 \end{pmatrix}$$

$$\begin{cases} \left(\Delta P_{mm}^{\pm\pm}\right)_{Gnj} = TWB_{nj} = -\dfrac{i\,\omega}{2\pi v}\,\dfrac{\cos(\theta_j)\,\Delta S_n}{R_{nj}}\,e^{i\omega \frac{R_{nj}}{v}}\,\sigma_{nj} & \text{aproximation of P} \\ \left(\Delta P_{mm}^{\pm\pm}\right)_{hGnj} = h_{nj} \times TWB_{nj} & \text{approximation of P in shadow} \end{cases}$$

$$T_{mn}\left(\mathrm{s}_m, \tilde{\mathrm{s}}_n\right)\,\mathrm{a}_n^{\pm}\left(\tilde{\mathrm{s}}_n\right) = \chi_{mn}\left(\mathrm{s}_m\right)\,\mathrm{a}_n^{\pm}\left(\mathrm{s}_m\right) \quad \text{approximation of T}$$

$$\chi_{mn}\left(\mathrm{s}_m, \omega\right) = \frac{F^{-1}\left(\mathrm{s}_m, \bar{\mathrm{k}}\right)\,\hat{T}_{mn}\left(\bar{\mathrm{k}}, \omega\right)\,\bar{F}\left(\bar{\mathrm{k}}, \bar{\mathrm{y}}_n\right)\,\bar{\mathrm{a}}_n^{\pm}\left(\bar{\mathrm{y}}_n\right)}{\mathrm{a}_n^{\pm}\left(\mathrm{s}_m\right)}$$

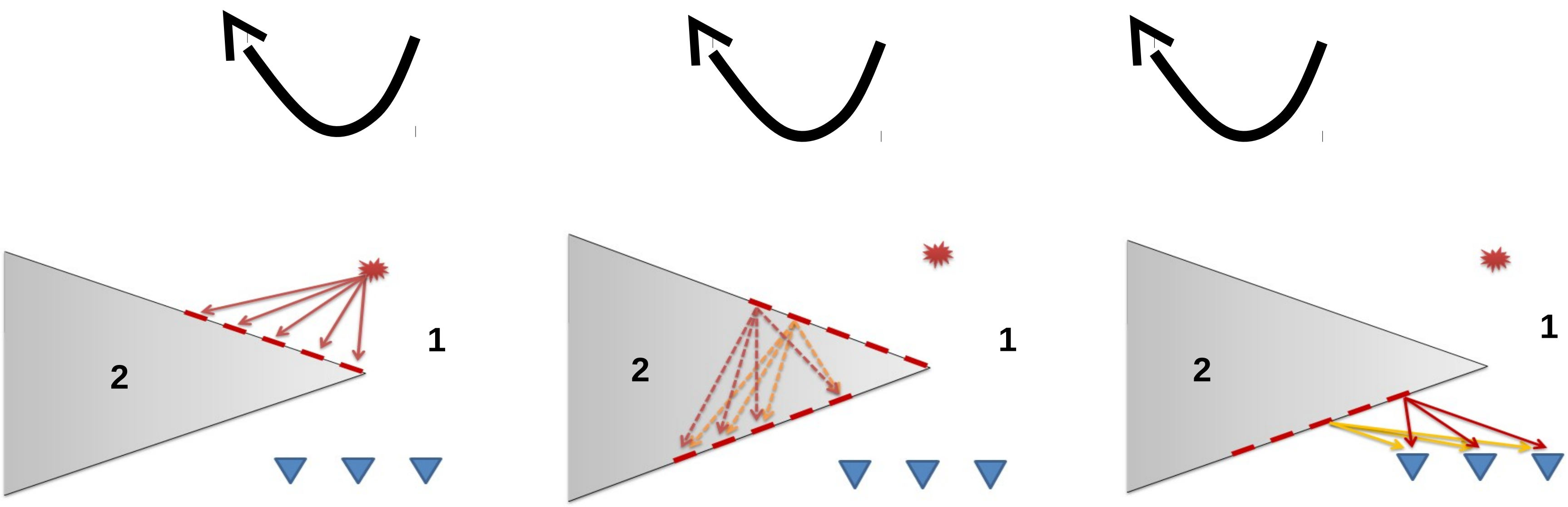


Forward modeling by TWSM for V-model

Alena Ayzenberg, N. Zyatkov, Arkady Aizenberg, A. Stovas, 2014

$$a^{(2)} = (P \times T) \times (P \times T) \times a^{(0)}$$

receiver **propagation transmission** **propagation transmission** **source**

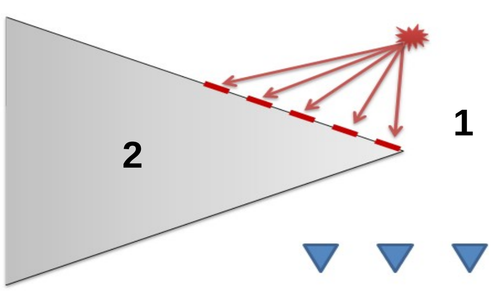


Forward modeling by TWSM for V-model

Alena Ayzenberg, N. Zyatkov, Arkady Aizenberg, A. Stovas, 2014

$$a^{(2)} = \left(P \times T \right) \times \left(P \times T \right) \times a^{(0)}$$

receiver — propagation transmission — propagation transmission — **source**



$$\left(P_{G11} + P_{G11} P_{hG11} P_{G11} \right) T_{12} P_{G22} T_{21} \left(a_G^{(0)} + P_{G11} P_{hG11} a_G^{(0)} \right)$$

2-terms approximation

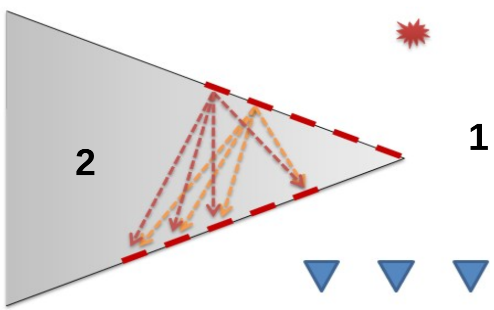
2-terms approximation

Forward modeling by TWSM for V-model

Alena Ayzenberg, N. Zyatkov, Arkady Aizenberg, A. Stovas, 2014

$$a^{(2)} = (P \times T) \times (P \times T) \times a^{(0)}$$

receiver | propagation transmission | propagation transmission | source



$$\left(P_{G11} + P_{G11}\, P_{hG11} P_{G11}\right) T_{12}\; P_{G22}\; T_{21} \left(a_G^{(0)} + P_{G11}\, P_{hG11}\, a_G^{(0)}\right)$$

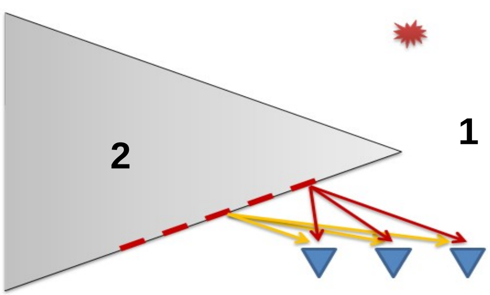
2-terms approximation | 2-terms approximation

Forward modeling by TWSM for V-model

Alena Ayzenberg, N. Zyatkov, Arkady Aizenberg, A. Stovas, 2014

$$a^{(2)} = \left(P \times T\right) \times \left(P \times T\right) \times a^{(0)}$$

receiver **propagation transmission** **propagation transmission** **source**



$$\left(P_{G11} + P_{G11}\,P_{hG11}P_{G11}\right)T_{12}\;P_{G22}\;T_{21}\left(a_G^{(0)} + P_{G11}\;P_{hG11}\;a_G^{(0)}\right)$$

2-terms approximation **2-terms approximation**

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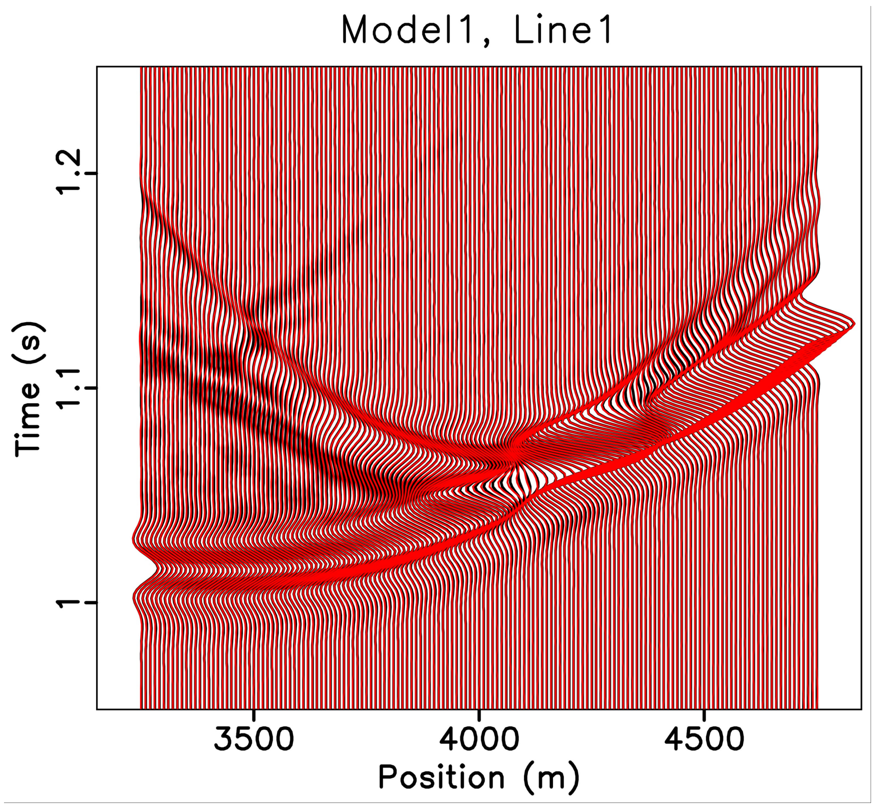
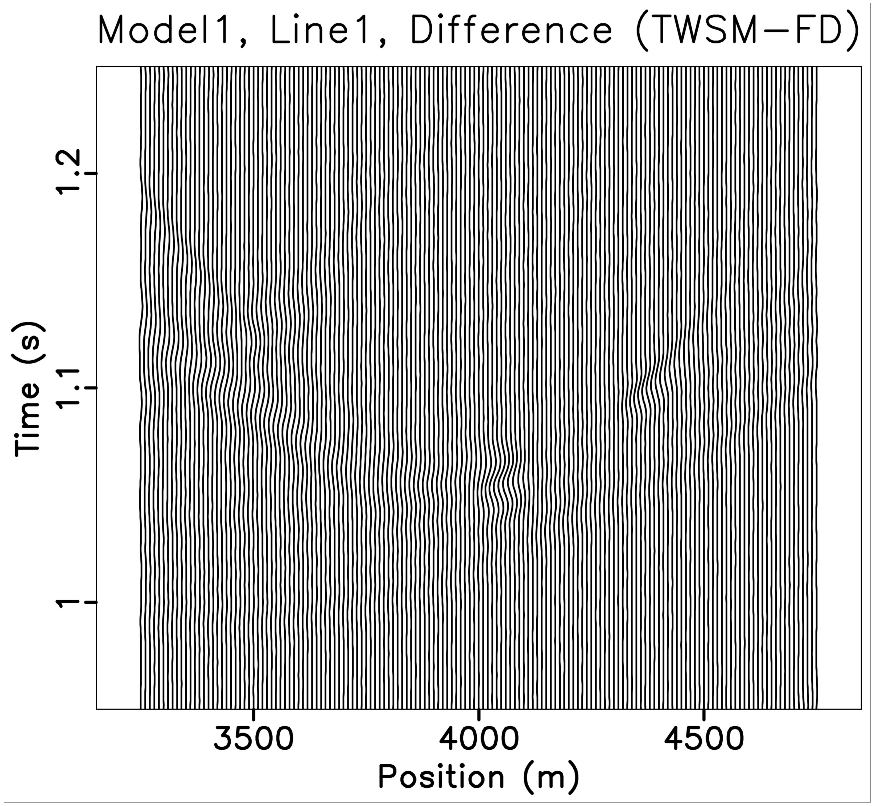
TWSM vs FD. V-model

Alena Ayzenberg, N. Zyatkov, W. Weibull, Arkady Aizenberg, A. Stovas, to be submitted

Red TWSM, Black FD



Difference (TWSM – FD)

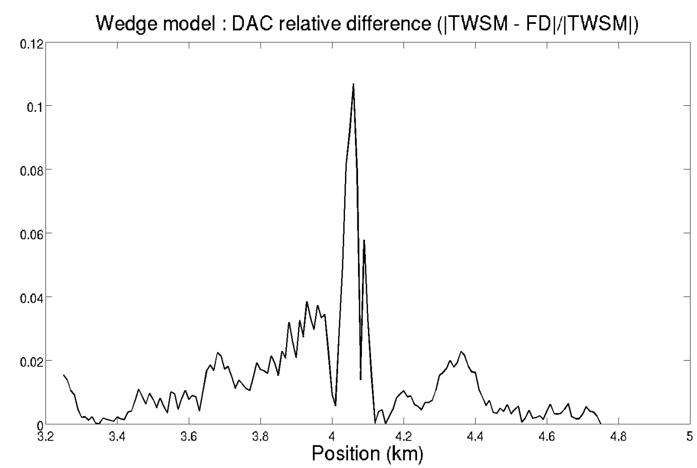


TWSM vs FD. V-model

Alena Ayzenberg, N. Zyatkov, W. Weibull, Arkady Aizenberg, A. Stovas, to be submitted

Red TWSM, Black FD

Relative error



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For V-model:

- TPOT approach is applied
- Statement of problem in terms of particles is rewritten in terms of amplitudes
- Solution for amplitudes is obtained
- TWSM modeling is done to receiver line below V-boundary
- Comparison with FD for gives relative error 3 percent

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