Diving waves for velocity model estimation

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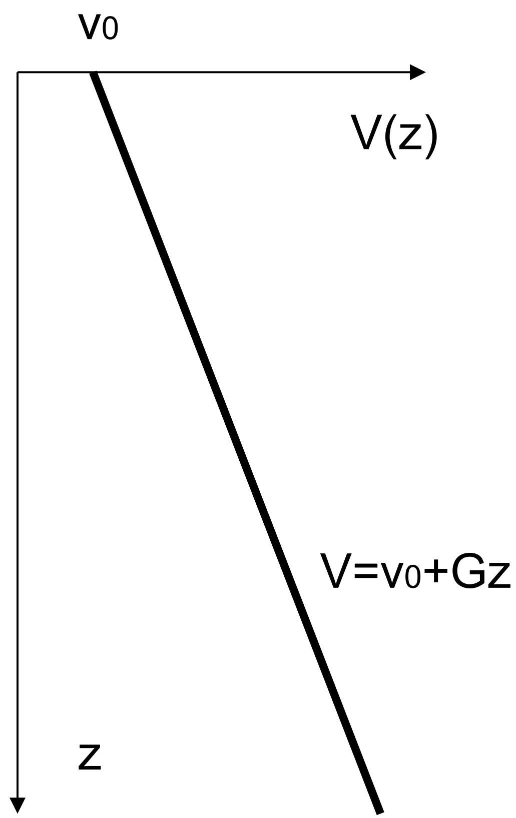
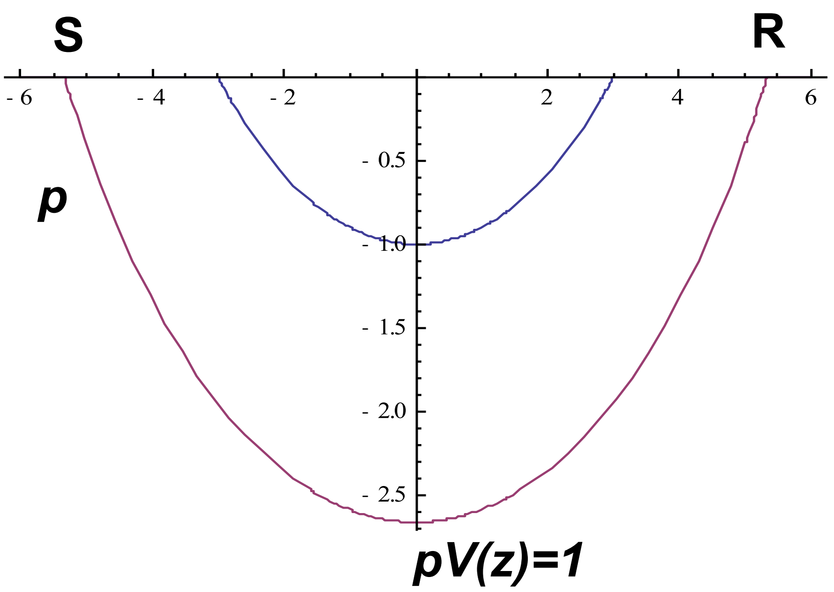
Objectives

- Diving waves are widely used in FWI
- All velocity update methods are purely kinematic

Diving wave





$$t(x) = \frac{2}{G}\log\left[\frac{Gx}{2v_0} + \sqrt{1 + \frac{G^2x^2}{4v_0^2}}\right]$$

$$= \frac{x}{v_0} - \frac{G^2x^3}{24v_0^3} + \frac{3G^4x^5}{640v_0^5} + \dots$$

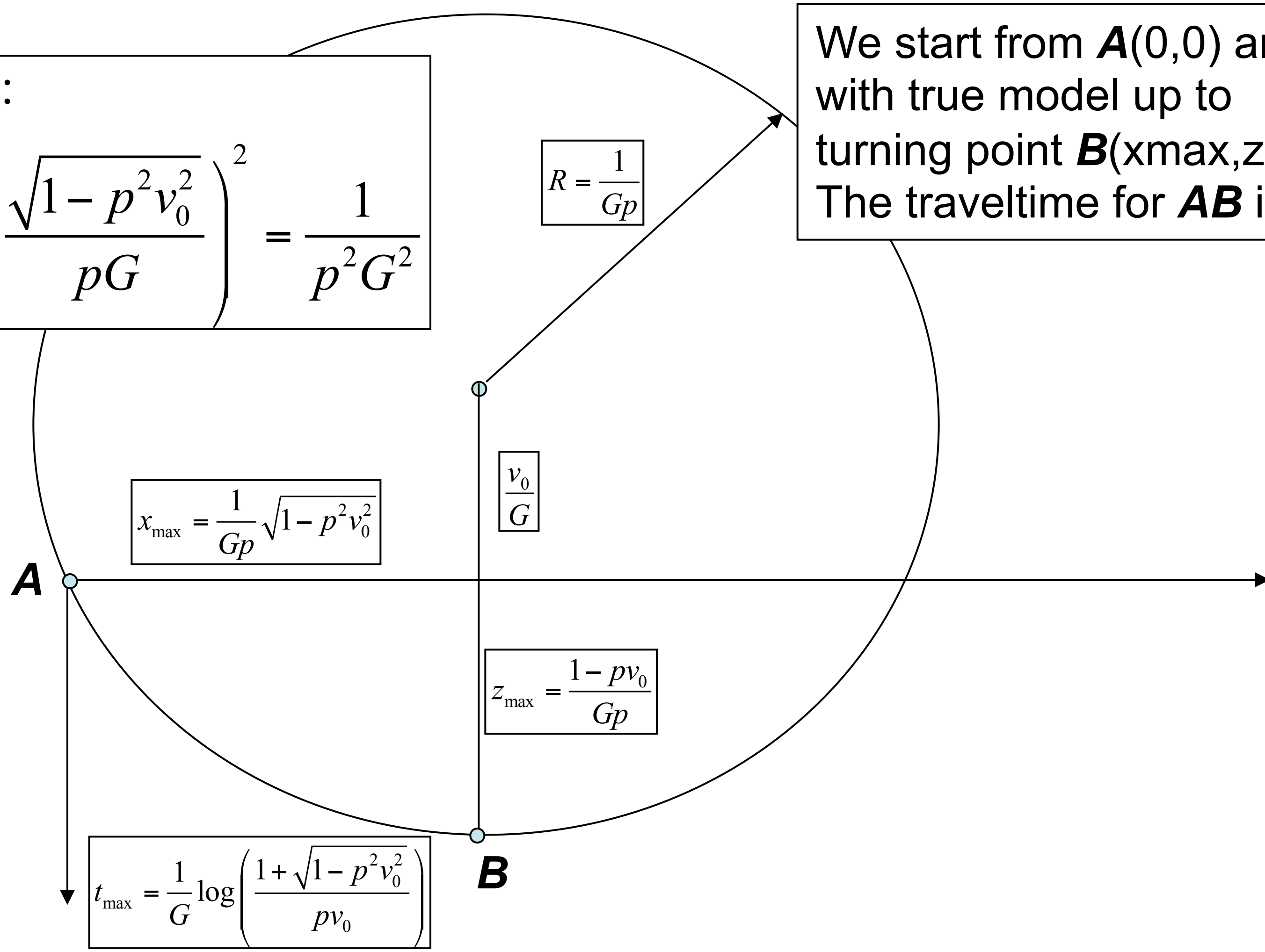
Direct wave

Geometry of of the ray for diving wave in isotropic medium

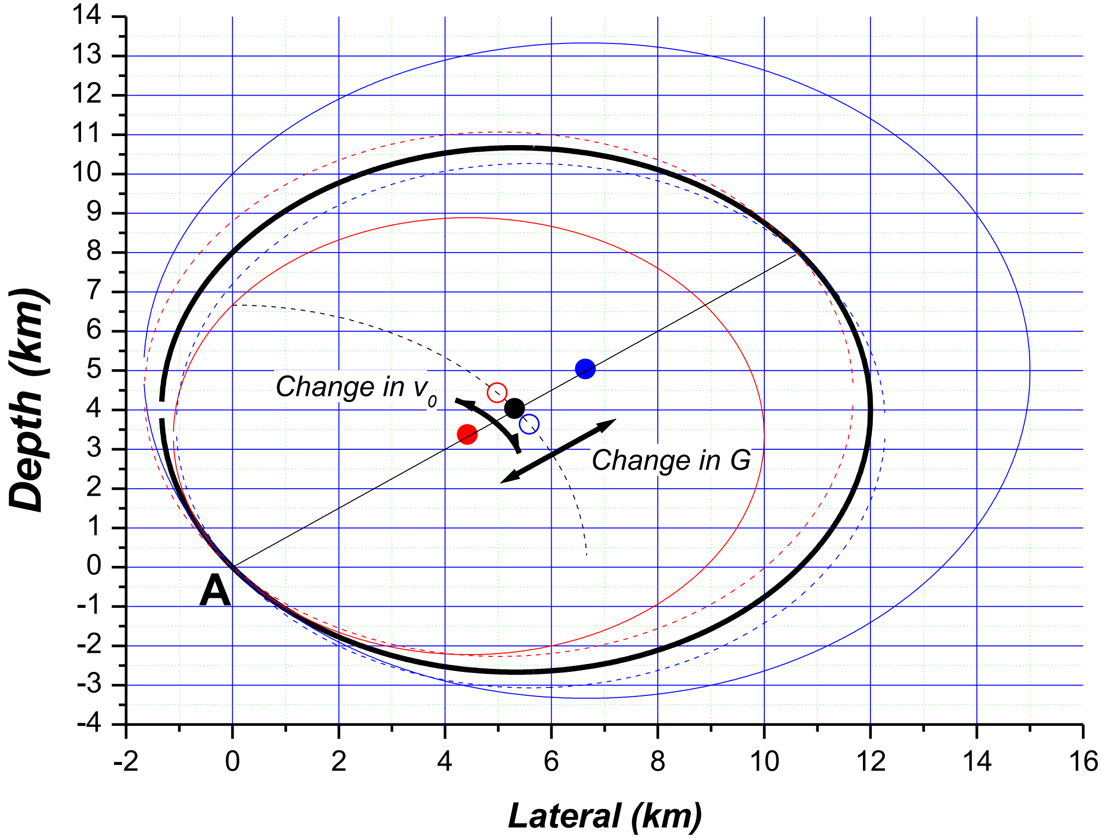
Circle equation:

$$\left(z+\frac{v_0}{G}\right)^2+\left(x-\frac{\sqrt{1-p^2v_0^2}}{pG}\right)^2=\frac{1}{p^2G^2}$$

We start from ***A***(0,0) and go with true model up to turning point ***B***(xmax,zmax). The traveltime for ***AB*** is tmax.



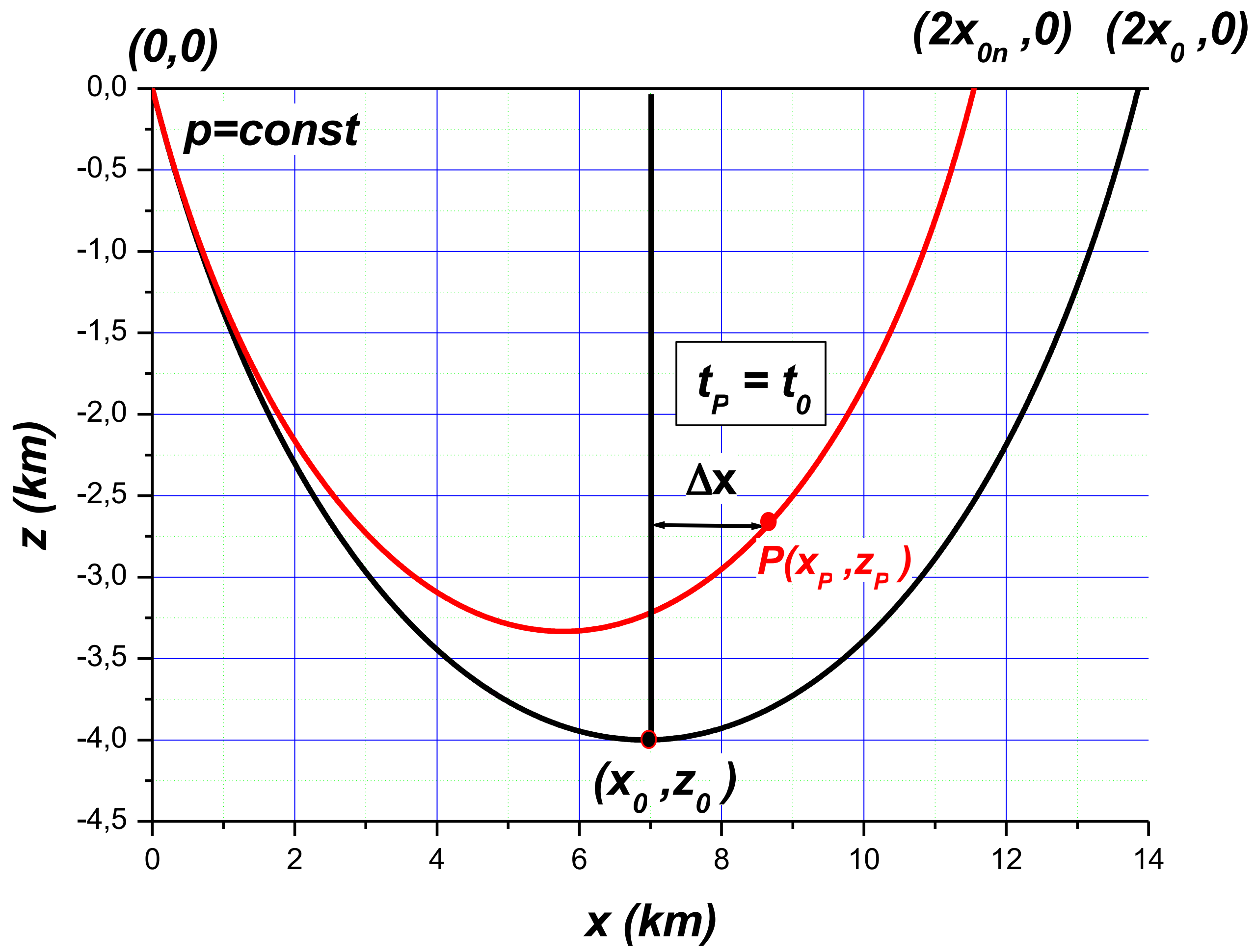
Rays vs change in velocity model



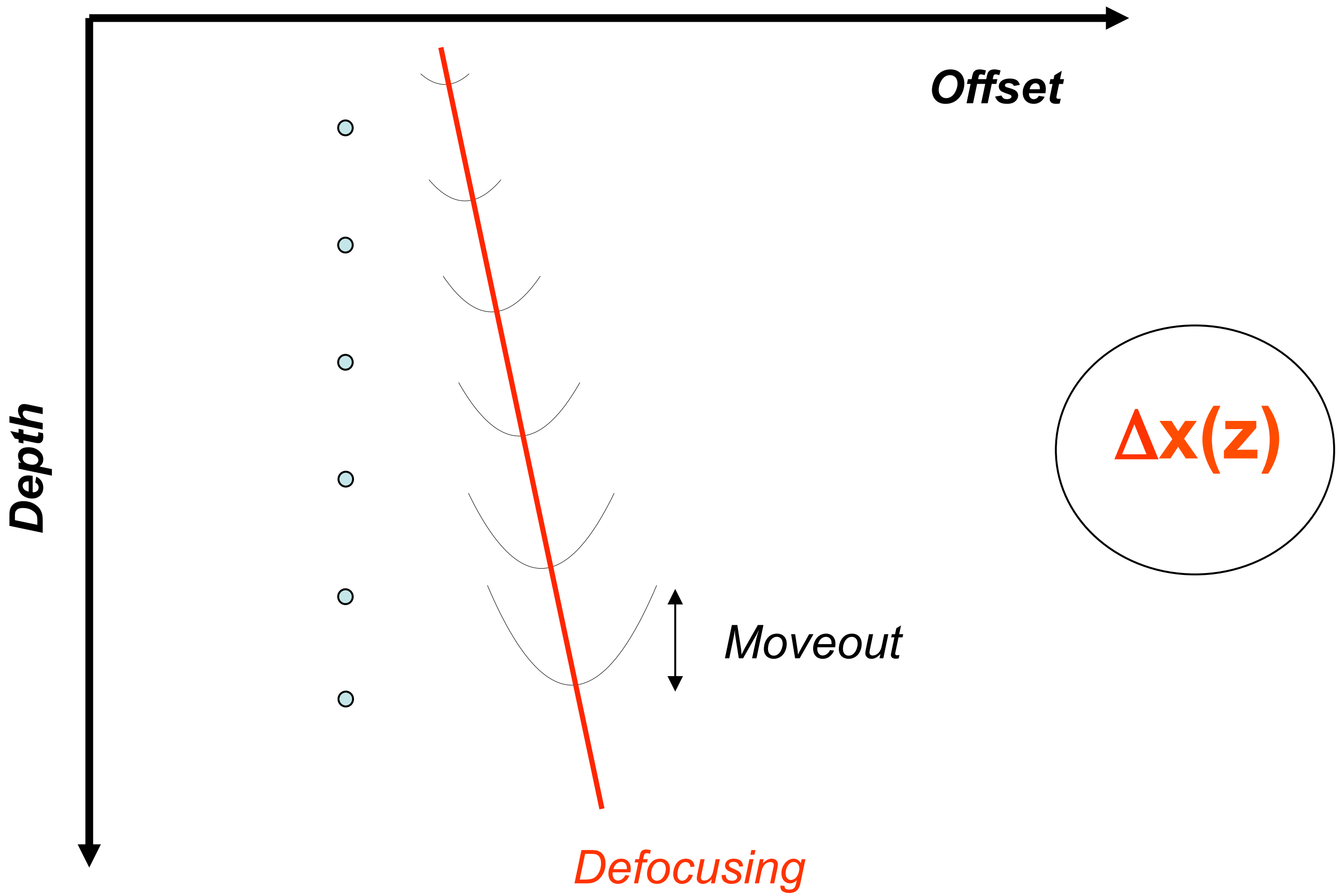
$$Z = X \frac{p v_0}{\sqrt{1 - p^2 v_0^2}}$$

$$X^2 + Z^2 = \frac{1}{p^2 G^2}$$

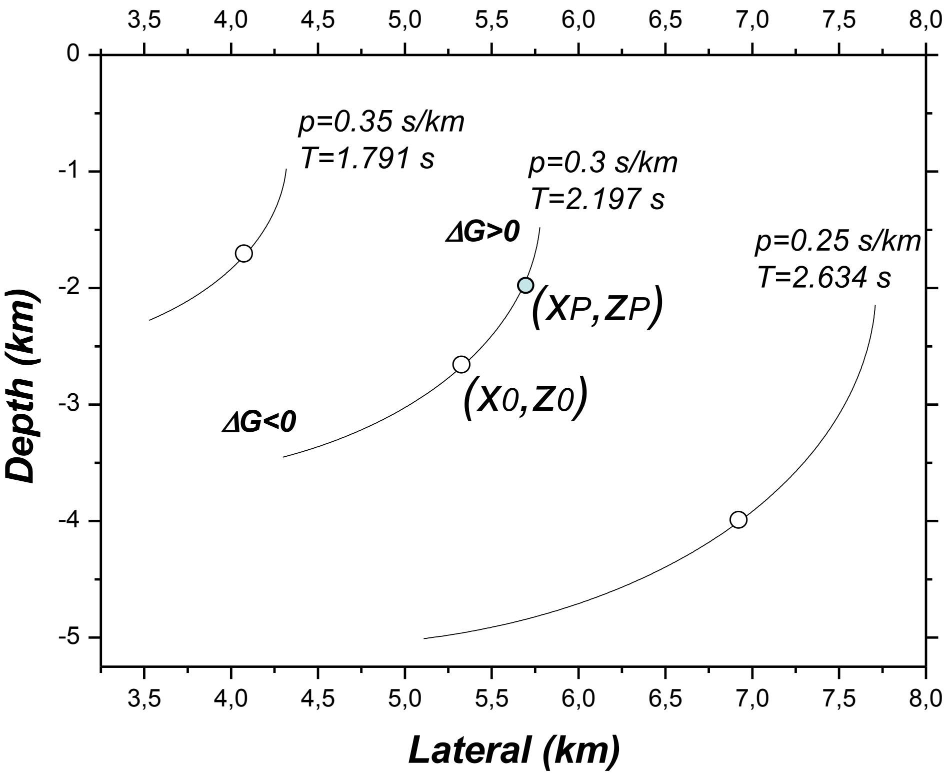
Focusing principle



Defocusing after RTM



”G-wave”



$$z_P = \frac{v_0}{(G+\Delta G)} \frac{\left(e^{GT}e^{\Delta GT}-1\right)\left(e^{GT}-e^{\Delta GT}\right)}{e^{GT}\left(e^{2\Delta GT}+1\right)}$$

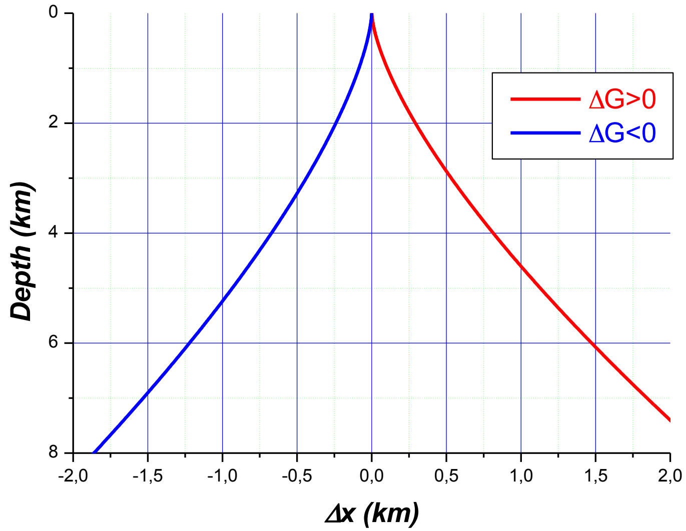
$$x_P = \frac{v_0}{(G+\Delta G)} \frac{\left(e^{2GT}e^{2\Delta GT}-1\right)}{e^{GT}\left(e^{2\Delta GT}+1\right)}$$

$$z_P\left(\Delta G=0\right) = \frac{v_0}{G}\frac{\left(e^{GT}-1\right)^2}{2e^{GT}} = z_0$$

$$x_P\left(\Delta G=0\right) = \frac{v_0}{G}\frac{\left(e^{2GT}-1\right)}{2e^{GT}} = x_0$$

The deeper penetration the larger defocusing.
Effect is larger if ΔG<0.

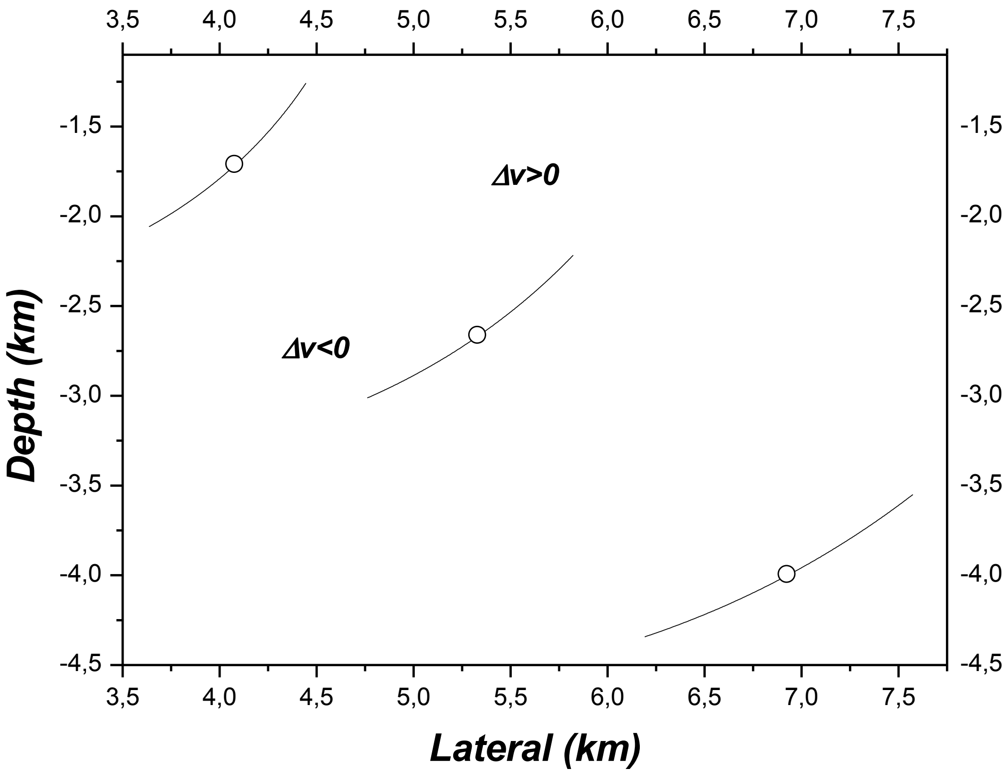
Defocusing in depth



Curves are non-symmetric

$$\Delta x^2 = \frac{8}{9}\frac{\Delta G^2}{v_0\left(G-\Delta G\right)} z_P^3 - \frac{4}{45}\frac{\Delta G^2\left(G^2-9\Delta G^2\right)}{v_0^2\left(G-\Delta G\right)^2} z_P^4 + \ldots$$

"v_0-wave"



$$z_P = \frac{v_0}{G}\frac{\sqrt{1-r_2^2}\left(\sqrt{1-r_1^2}-1-r_1r_2\right)}{\sqrt{1-r_1^2}\left(1-r_1r_2\right)}$$

$$\Delta x = \frac{v_0}{G}\frac{r_1r_2\left(r_1-r_2\right)}{\sqrt{1-r_1^2}\left(1-r_1r_2\right)}$$

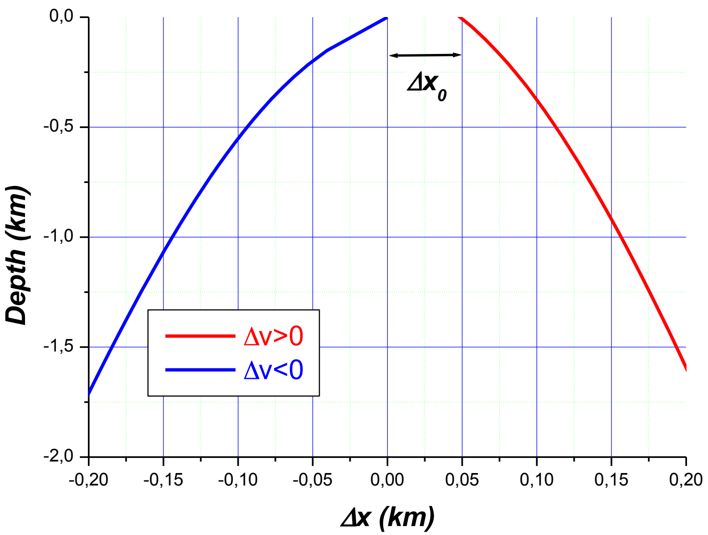
$$r_1 = \sqrt{1-p^2v_0^2} \qquad r_2 = \sqrt{1-p^2\left(v_0+\Delta v\right)^2}$$

$$z_0 = \frac{v_0}{G}\frac{\left(1-\sqrt{1-r_1^2}\right)}{\sqrt{1-r_1^2}}$$

$$x_0 = \frac{v_0}{G}\frac{r_1}{\sqrt{1-r_1^2}}$$

The deeper penetration, a bit larger defocusing.
Effect is similar for both $\Delta v_0<0$ and $\Delta v_0>0$.

Defocusing in depth



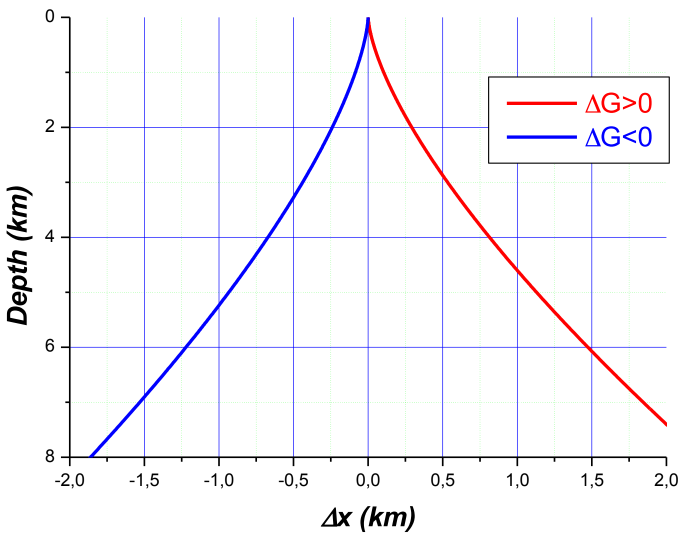
$$\Delta x_0 \approx \frac{4\sqrt{2}v_0}{3\sqrt{3}G}\left(\frac{\Delta v}{v_0}\right)^{3/2}$$

$$\Delta x\left(z_P\right) = \frac{v_0}{G}\frac{\Delta v}{v_0}\sqrt{\left(2+\frac{\Delta v}{v_0}+\frac{Gz}{v_0}\right)\left(\frac{\Delta v}{v_0}+\frac{Gz}{v_0}\right)},\ \Delta v > 0$$

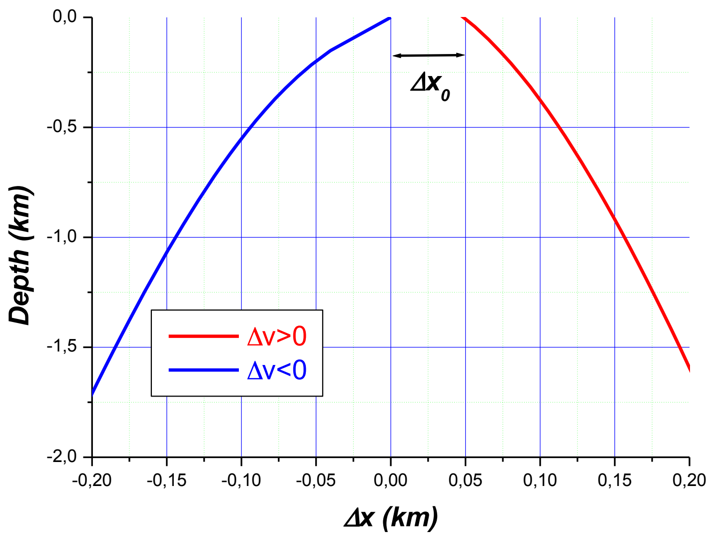
Equations are different depending on the sign of Δv_0

Using series is not good…

Imaging moveout curvature



Error in gradient

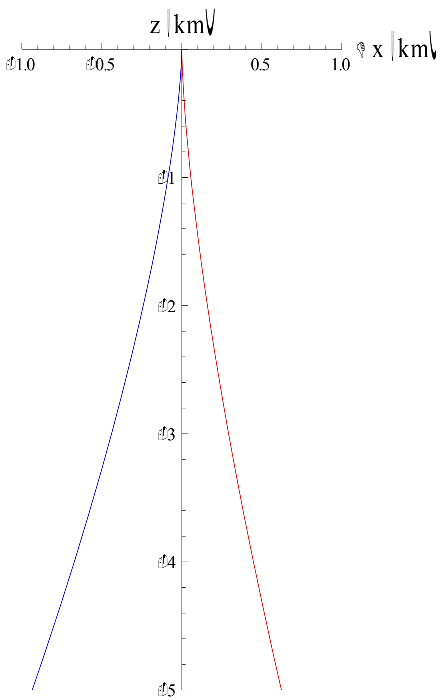


Error in velocity

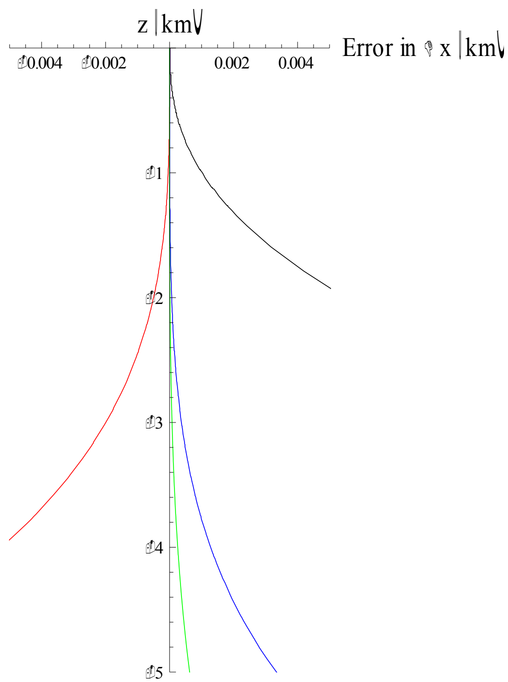
Moveout approximations-1

$$\Delta x^2 = a_3 z_P^3 + a_4 z_P^4 + a_5 z_P^5 + \ldots$$

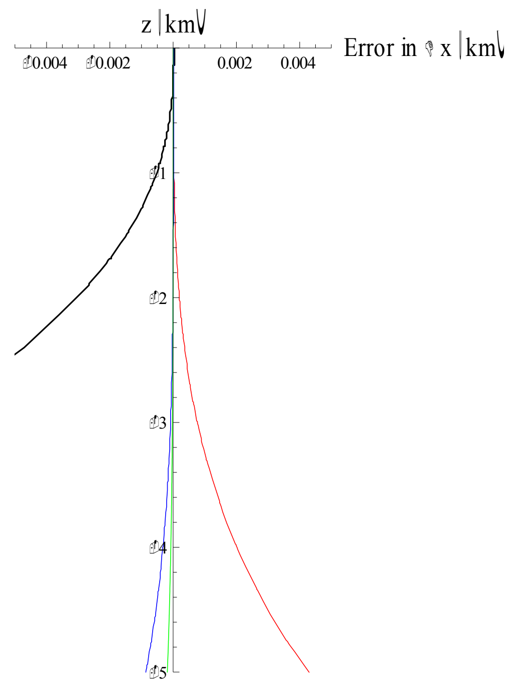
$$\Delta x = \frac{2}{3}\Delta G z_P \sqrt{\frac{2 z_P}{(G-\Delta G)v_0}\,\frac{1+\dfrac{(9G^2-\Delta G^2)z_P}{105(G-\Delta G)v_0}}{1+\dfrac{(39G^2-191\Delta G^2)z_P}{210(G-\Delta G)v_0}}}$$



$\Delta x(z_P)$



$\Delta G > 0$



$\Delta G < 0$

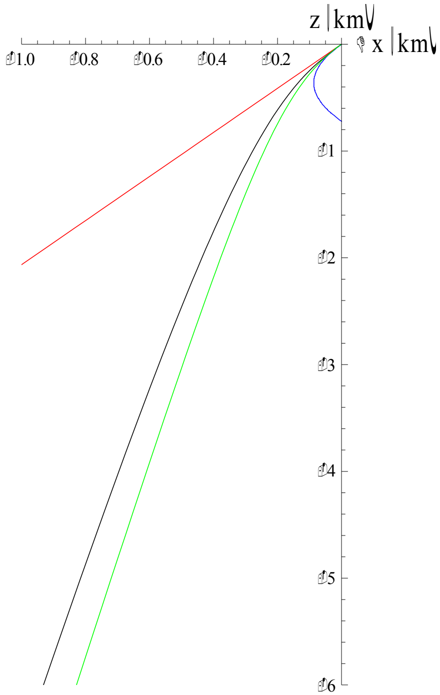
Moveout approximations-2

$$\Delta x(z) = \Delta x(0) + a_1 z_P + a_2 z_P^2$$

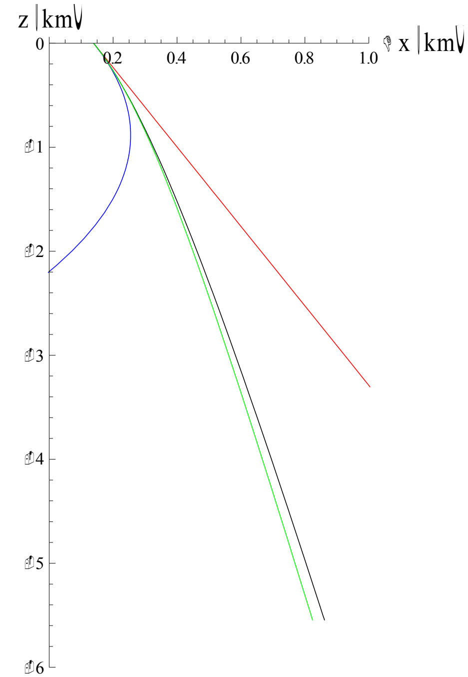
$$\boxed{\Delta x(z) = \Delta x(0) + a_1 z_P \frac{1 + c z_P}{1 + d z_P}}$$

$$c = \frac{a_\infty a_2}{a_1 (a_\infty - a_1)}$$

$$d = \frac{a_2}{(a_\infty - a_1)}$$

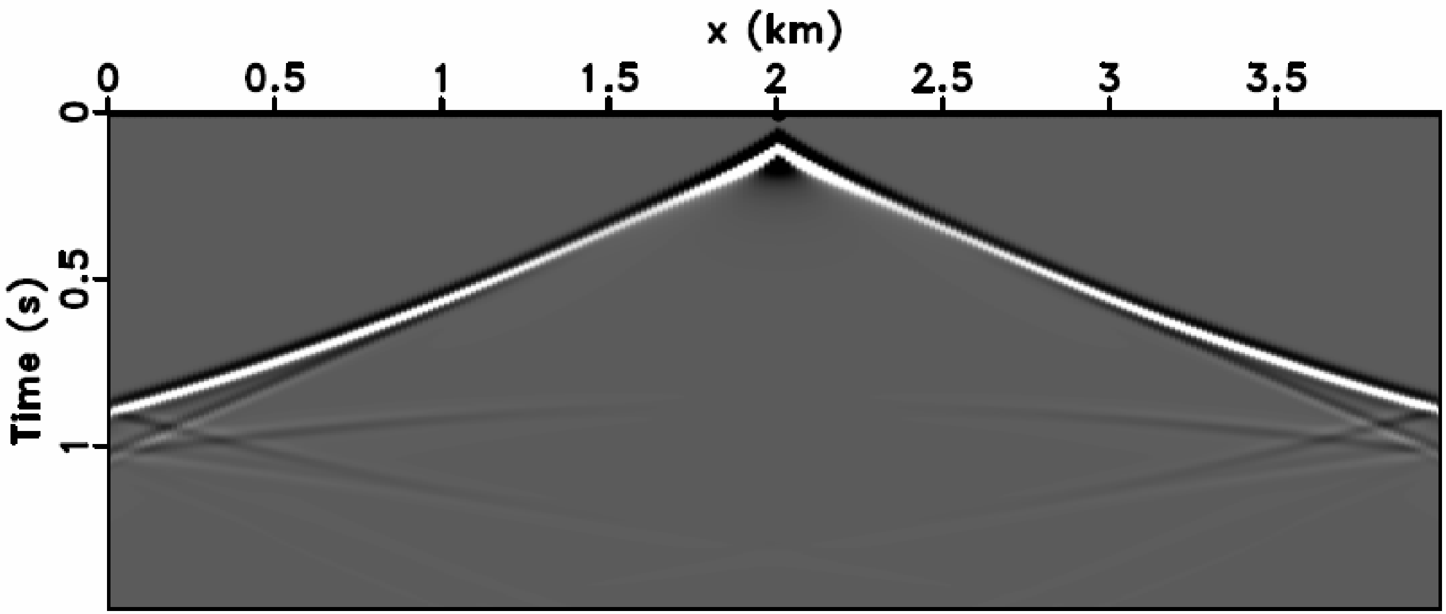


$\Delta v < 0$



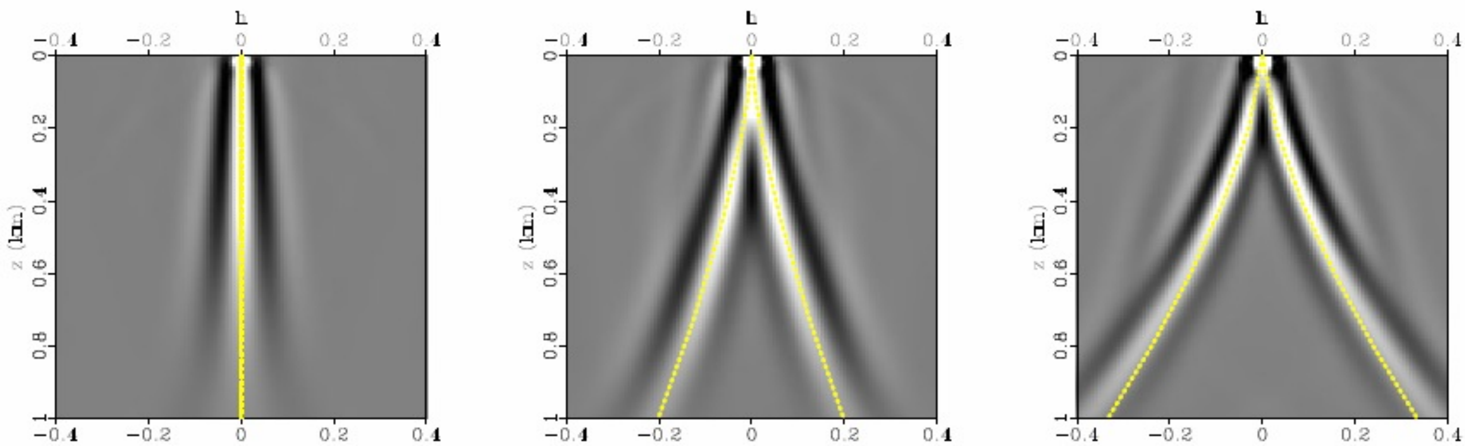
$\Delta v > 0$

Synthetics



V_0=2 km/s G=3 1/s

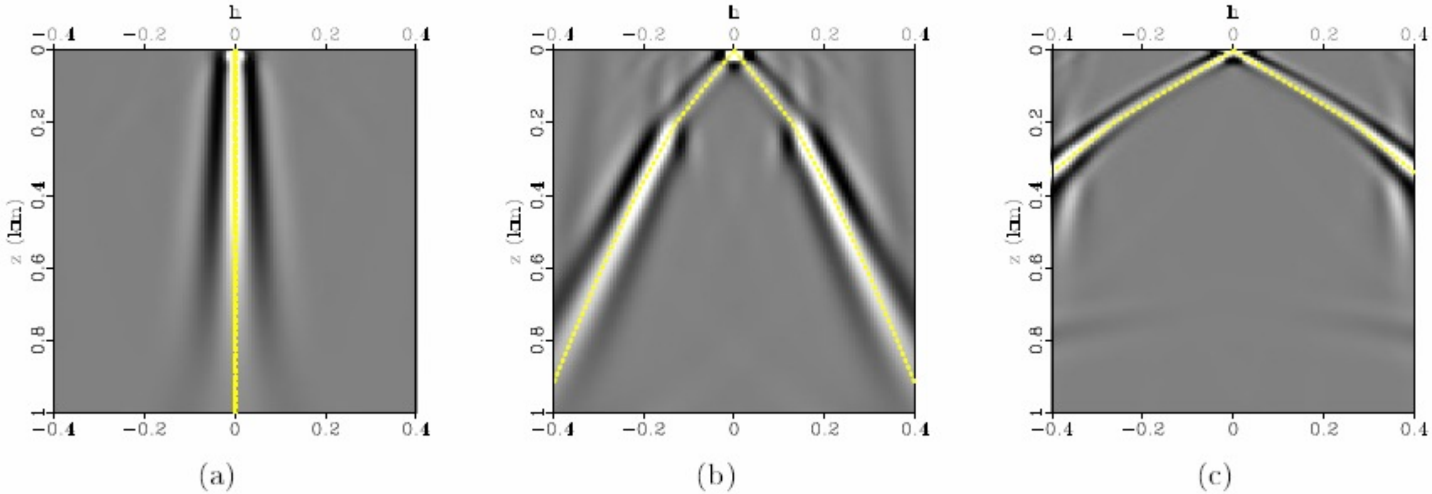
Error in gradient



(a) (b) (c)

G=3 1/s G=2.5 1/s G=2 1/s

Error in v_0



V_0=2 km/s V_0=1.5 km/s V_0=1 km/s

Conclusions

- Diving waves can be used to evaluate the gradient velocity model using ”defocusing” principle
- In case of errors both in velocity and gradient, the procedure can be applied in steps.
- Can be extended for anisotropy…

- The work is still ongoing…

Acknoledgements

- We would like to acknowledge ROSE