INTERFACE FRESNEL ZONE FOR A CURVED REFLECTOR IN ANISOTROPIC MEDIA

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The Fresnel volume concept plays an important role in seismic exploration.


The Interface Fresnel zone (IFZ)

- region of constructive reflection interference surrounding the reflection point of the geometrical ray

- largely contributes to the formation of the reflection and transmission wavefields at an observation point, and more specifically to their amplitude


- determines the lateral resolving power for unmigrated seismic data with which important lithological changes along a seismic profile direction may be observed

(Sheriff 1980 Geophysics)
Analytical and numerical modeling techniques used to determine the IFZ dimensions in various configurations (geometric approaches, ray tracing, isochrons ...)

(Gelchinsky 1985 J. Geophys., Hubral et al. 1993 Geophysics
Lindsey 1989 The Leading Edge
Iversen 2006 Geophysics
Favretto-Cristini et al. 2009 Geophysics
Monk 2010 Geophysics
...

Most studies concerned with zero-offset configurations and plane reflectors

Few works devoted to P-SV reflections and/or anisotropic media (VSP configurations, dipping reflector...)


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Address the issue of deriving simple expressions for the IFZ for multi-offset configurations and a curved interface between general anisotropic media.

Example of anticline-type structure (Ainsa, Spain)

- Transversely Isotropic (TI) media with symmetry axis orthogonal to the curved reflector
- ( Possibly converted) reflected and transmitted waves
- Non-spherically shaped interface between homogeneous anisotropic media

Generalization of the work reported in Favretto-Cristini et al. 2009 Geophysics

Ursin, Favretto-Cristini & Cristini
To appear in Geophysics in 2014

Example of syncline-type structure (Ainsa, Spain)
The FV is defined by

\[ |\delta T(x^S, \delta x) + \delta T(x^U, \delta x)| \leq \frac{1}{2f} (U = R, T) \]

\( f : \) central frequency of the signal

with the traveltime difference \( \delta T \) between the source (respectively, receiver) point and the reflection point

\[ \delta T(x, \delta x) = T(x, \delta x) - T(x, 0) = \frac{1}{V + \delta V} \|x - \delta x\| - \frac{1}{V} \|x\| \]

Neglecting changes in group velocity \( V \) (i.e., weak anisotropy) and considering

\[ X = \frac{d}{V} V : \]

\[ \delta T(V, \delta x) \simeq \frac{1}{2Vd} \left[ \|\delta x\|^2 - 2d \frac{V \cdot \delta x}{V} - \left( \frac{V \cdot \delta x}{V} \right)^2 \right] \]
A curved interface $\Sigma$ may locally be approximated by a 2\textsuperscript{nd}-order expression

$$x_3 = F(x_1, x_2) = \frac{1}{2} (x_1, x_2) F (x_1, x_2)^t = \frac{1}{2} (F_{11} x_1^2 + 2 F_{12} x_1 x_2 + F_{22} x_2^2)$$

$F_{11}, F_{12}$ and $F_{22}$ : interface curvatures

For the FZ at the interface : $$\delta x_3 = F(\delta x_1, \delta x_2)$$

**Approximation for $\partial T$** (keeping terms up to 2\textsuperscript{nd} order)

$$\delta T_\Sigma (V, \delta x_1, \delta x_2) \simeq \frac{1}{2 V d} \left[ \left( 1 - \frac{V_1^2}{V^2} \right) \delta x_1^2 + \left( 1 - \frac{V_2^2}{V^2} \right) \delta x_2^2 - 2 \frac{V_1 V_2}{V^2} \delta x_1 \delta x_2 \right]$$

$$- \frac{V_1}{V^2} \delta x_1 - \frac{V_2}{V^2} \delta x_2 - F(\delta x_1, \delta x_2) \frac{V_3}{V}$$

The IFZ is defined by

$$|\delta T_\Sigma (V^S, \delta x_1, \delta x_2) + \delta T_\Sigma (V^U, \delta x_1, \delta x_2)| \leq \frac{1}{2 f} \quad (U = R, T)$$
Specific case: DTI media

Dip-constrained Transversely Isotropic (DTI) media = local VTI media

- Curved interface between 2 DTI media
  - SH wave and coupled P-S waves
  - all seismic signatures depend only on the angle between the propagation direction and the symmetry axis

- Approximation for $\delta T$

$$
\delta T_\Sigma (\mathbf{V}, \delta x_1, \delta x_2) \simeq \frac{1}{2 V d} \left[ \left( 1 - \frac{V_1^2}{V^2} \right) \delta x_1^2 + \delta x_2^2 \right] - \frac{V_1}{V^2} \delta x_1 - F(\delta x_1, \delta x_2) \frac{V_3}{V^2}
$$

where $F$ defines the interface curvatures, $d = [(x_1)^2 + (x_3)^2]^{\frac{1}{2}}$ and $V = \left( \frac{V_1^2 + V_3^2}{2} \right)^{\frac{1}{2}}$
Explicit expressions for $V_1$ and $V_3$ for P-SV waves in *Tsvankin 2001, Chapman 2004*

... preferable to use them in actual modeling, inversion and processing algorithms.

**Goal:** analytic insight into the effects of anisotropy on the IFZ

Approximate dispersion relation *(Pestana et al. 2012 J. Geophys. Eng.)*

for P-waves:

$$\omega^2 = v_{P0}^2 \left[ (1 + 2\,\epsilon) \, k_1^2 + k_3^2 \right] - 2 \, \frac{v_{P0}^2 \, (\epsilon - \delta)}{k_3^2 + \xi k_1^2} \, k_1^2 \, k_3^2$$

for SV-waves:

$$\omega^2 = v_{S0}^2 \, (k_1^2 + k_3^2) + 2 \, \frac{v_{P0}^2 \, (\epsilon - \delta)}{k_3^2 + \xi k_1^2} \, k_1^2 \, k_3^2$$

with $k_{1,3} = \omega \, p_{1,3}$

where

$$v_{P0} = \left( \frac{c_{33}}{\rho} \right)^{1/2}$$

$$v_{S0} = \left( \frac{c_{44}}{\rho} \right)^{1/2}$$

$$\epsilon = \frac{c_{11} - c_{33}}{2 \, c_{33}}$$

$$\delta = \frac{(c_{13} + c_{44})^2 - (c_{33} - c_{44})^2}{2 \, c_{33} \, (c_{33} - c_{44})}$$

*(Thomsen 1986 Geophysics)*

and

$$\xi = 1 + 2 \, \epsilon \, \frac{v_{P0}^2}{v_{P0}^2 - v_{S0}^2}$$

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From \[ V_i = -\frac{\partial \Omega}{\partial k_i } \frac{\partial k_i}{\partial \omega} \], we get:

for P-waves

\[
\begin{align*}
V_{P1} &= V^2_{P0} p_1 \left[(1 + 2 \epsilon) - 2(\epsilon - \delta) \chi\right] \\
V_{P3} &= V^2_{P0} p_3 \left[1 - 2(\epsilon - \delta) \chi'\right]
\end{align*}
\]

for SV-waves

\[
\begin{align*}
V_{S1} &= V^2_{S0} p_1 \left[1 + 2 \sigma \chi\right] \\
V_{S3} &= V^2_{S0} p_3 \left[1 + 2 \sigma \chi'\right]
\end{align*}
\]

where

\[
\begin{align*}
\chi &= \frac{p^4_3}{(p^2_3 + \xi p^2_1)^2} \\
\chi' &= \frac{\xi p^4_1}{(p^2_3 + \xi p^2_1)^2}
\end{align*}
\]

and \[ \sigma = \frac{V^2_{P0}}{V^2_{S0}} (\epsilon - \delta) \]

The difference \( \epsilon - \delta \)

- governs both P- and SV-wave propagation for non-vertical wave propagation (Pestana et al. 2012 J. Geophys. Eng.)
- controls the shape and the size of the IFZ for both P-P and P-SV reflections

\[
\eta = (\epsilon - \delta) / (1 + 2\delta)
\]

N.B.: The parameter \( \eta = (\epsilon - \delta) / (1 + 2\delta) \), introduced by Alkhalifah & Tsvankin (1995 Geophysics), governs such P-wave signatures (Tsvankin 2001), as dip-dependent NMO velocity, nonhyperbolic reflection moveout, time-migration impulse response, the point-source radiation pattern.
Numerical results

- Focus on the wave reflection at a curved interface
- Shape and size of the IFZ for P-P and P-SV reflections for various anisotropic parameters, incidence angles, and interface curvatures
- Incidence medium:

\[
\begin{align*}
\rho &= 2597 \text{ kg/m}^3 \\
v_{p0} &= 4409 \text{ m/s} \\
v_{s0} &= 2688 \text{ m/s} \\
\varepsilon &= 0.110 \text{ and } \delta = -0.043 \quad (\varepsilon - \delta = 0.153)
\end{align*}
\]

Brine-saturated shales
*(Wang 2002 Geophysics)*

- S and R located at 3000 m from the plane tangent to the interface at the reflection point
- Frequency of the incident P-wave: 25 Hz
- 3 kinds of curved reflectors:
  - Anticline \((R_1 = +5000 \text{ m}, R_2 = +4000 \text{ m})\)
  - Syncline \((R_1 = -5000 \text{ m}, R_2 = -4000 \text{ m})\)
  - Saddle \((R_1 = -5000 \text{ m}, R_2 = +4000 \text{ m})\)

\[
\begin{align*}
F_{11} &= \frac{1}{R_1} \cos^2 \phi + \frac{1}{R_2} \sin^2 \phi \\
F_{12} &= \left(\frac{1}{R_1} - \frac{1}{R_2}\right) \cos \phi \sin \phi \\
F_{22} &= \frac{1}{R_1} \sin^2 \phi + \frac{1}{R_2} \cos^2 \phi
\end{align*}
\]

*(Stavroudis 1972)*
Interface Fresnel zone for PP reflected waves (1/2)

Variation in shape and size of the IFZ in anisotropic (solid line) and isotropic (dashed line) media as a function of incidence angle $\theta$: $\theta = 0$ (dark blue), $\theta = 20^\circ$ (green), $\theta = 35^\circ$ (red), and $\theta = 50^\circ$ (light blue). The size of the IFZ is normalized with respect to the incident P-wavelength for $\theta = 0$. 

$\varepsilon - \delta > 0$  
**Plane reflector**  
$\varepsilon - \delta < 0$  

Anticline-type reflector
Interface Fresnel zone for PP reflected waves (2/2)

\[ \varepsilon - \delta > 0 \quad \text{Syncline-type reflector} \]

\[ \varepsilon - \delta < 0 \]

Rotation of the principal curvature axes of the interface by 20° with respect to the \(x_1\)-axis

Stationary points of hyperbolic type

(Asatryan and Kravtsov 1988)

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Interface Fresnel zone for PS reflected waves (1/2)

Variation in shape and size of the IFZ in anisotropic (solid line) and isotropic (dashed line) media as a function of incidence angle $\theta : \theta = 0$ (dark blue), $\theta = 20^\circ$ (green), and $\theta = 35^\circ$ (red).

The size of the IFZ is normalized with respect to the incident P-wavelength for $\theta = 0$.

Plane reflector

Anticline-type reflector
Interface Fresnel zone for PS reflected waves (2/2)

 synaptic-type reflector

 Saddle-type reflector
Derivation of analytic expressions, based on traveltime approximations, to evaluate the shape and the size of the IFZ for P-P and P-S reflected or transmitted waves by a curved reflector between two homogeneous anisotropic media.

Focus on local VTI media.

The size and the shape of the IFZ for reflected waves:

- predominantly dependent on the curvatures of the isochrons together with the curvatures of the interface.
- exhibit large variation with interface curvature and incidence angle. See the IFZ for the syncline- and the saddle-type reflectors.
- effects much more pronounced for positive values of $\varepsilon - \delta$ \( \Rightarrow \) The difference between the Thomsen parameters $\varepsilon$ and $\delta$ also controls the shape and size of the IFZ for both P-P and P-S reflections.

The spatial resolution of unmigrated seismic data in anisotropic media with curved interface is significantly different from that determined for the same configuration for isotropic models and a planar interface.