Wavefield modeling in shadow zones

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Outline

1. Shadow

2. Conventional and feasible solution
    - V-shaped model
    - slit model
    - U-shaped model

3. Conventional and feasible Kirchhoff integral

4. Tests
    - V-shaped model
    - U-shaped model

Motivation

Testing the solution which describes separate waves

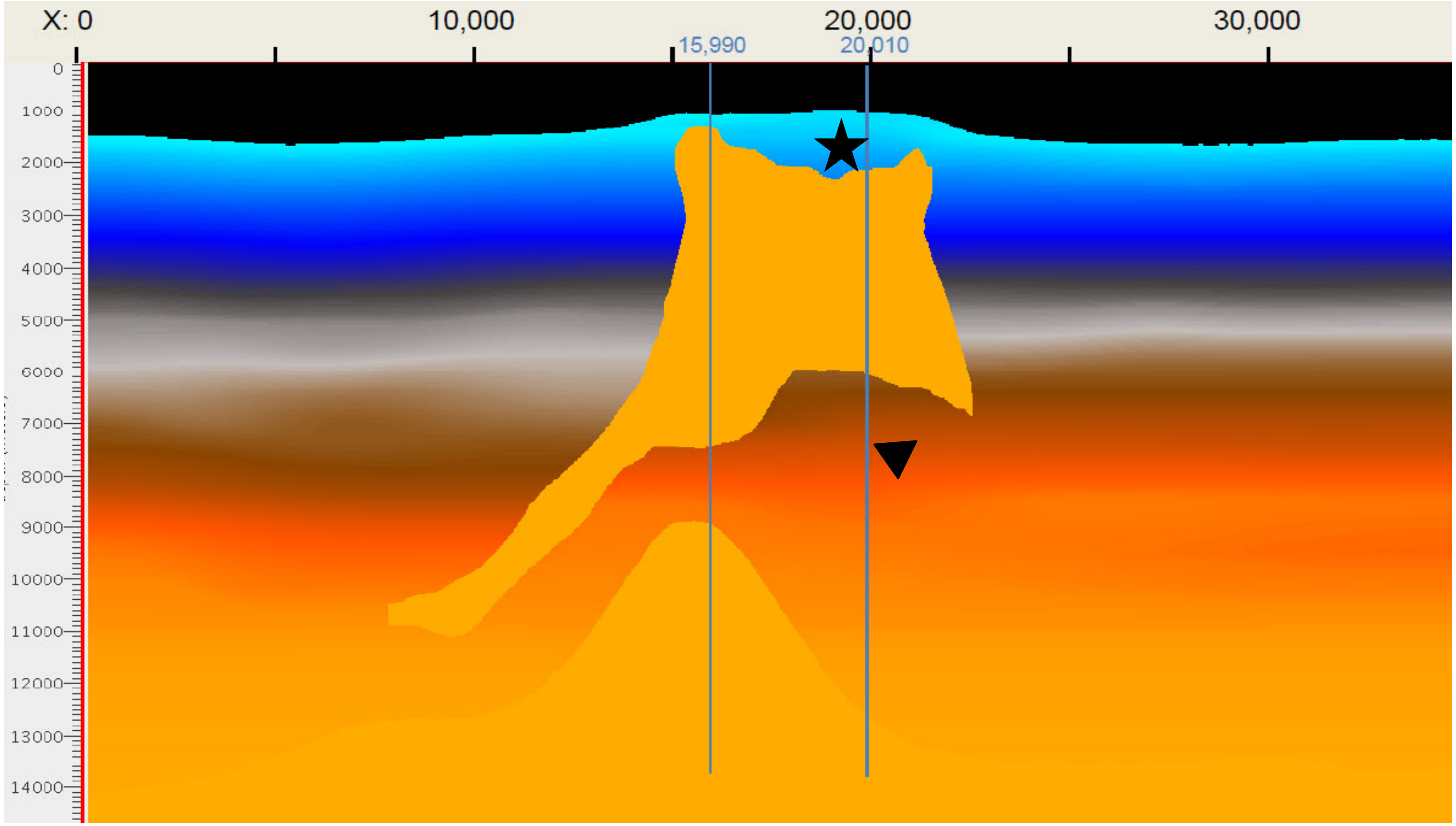
(for example direct, reflected/transmitted, diffracted, creeping etc.)

Improvement of Kirchhoff-type modeling in shadow zones

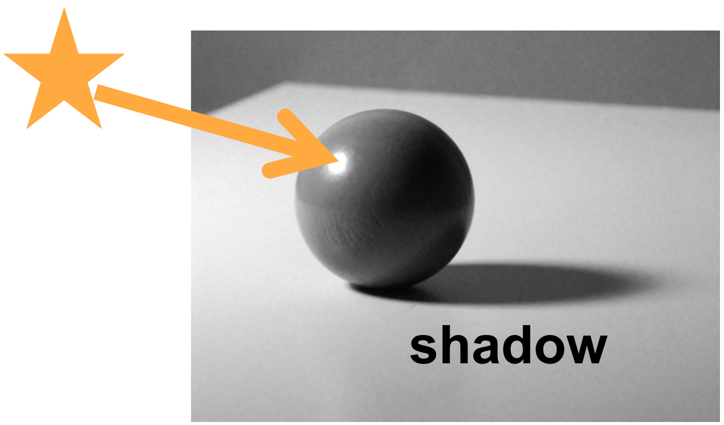
1. Shadow. Statement of the problem

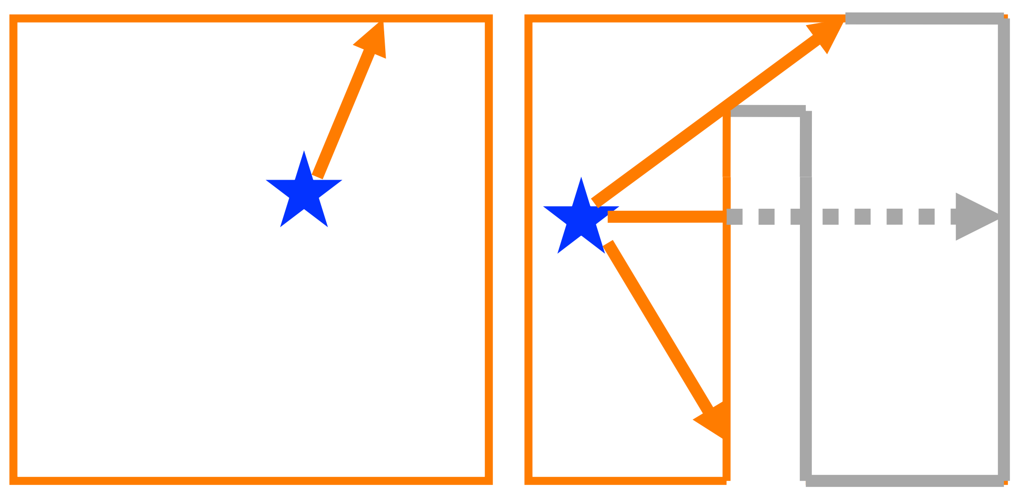
Problem: to find pressure at a point located in shadow

Method: modeling by feasible Kirchhoff integrals

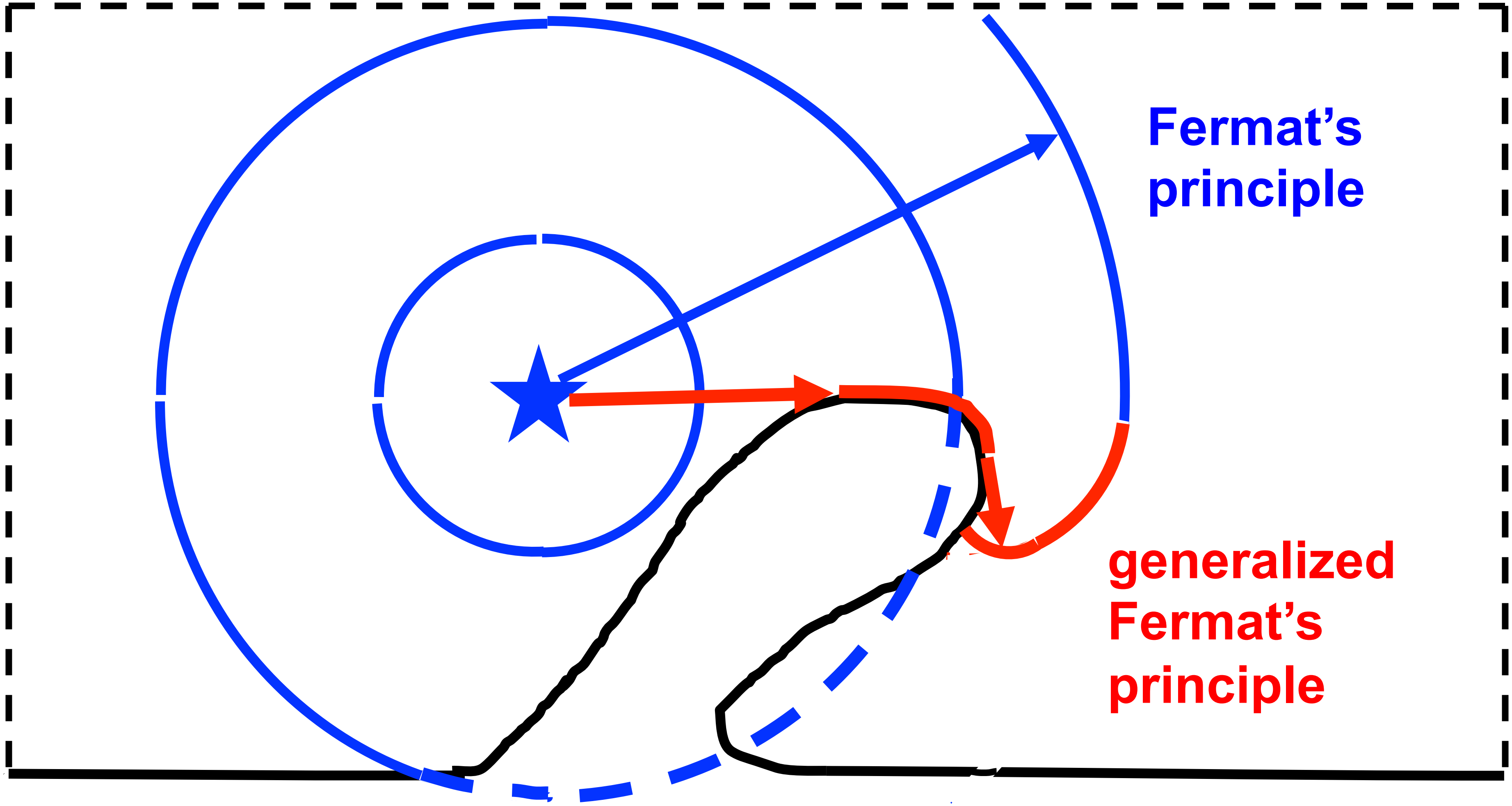


1. Shadow
(Kirchhoff 1891, Kottler 1923, 1965)





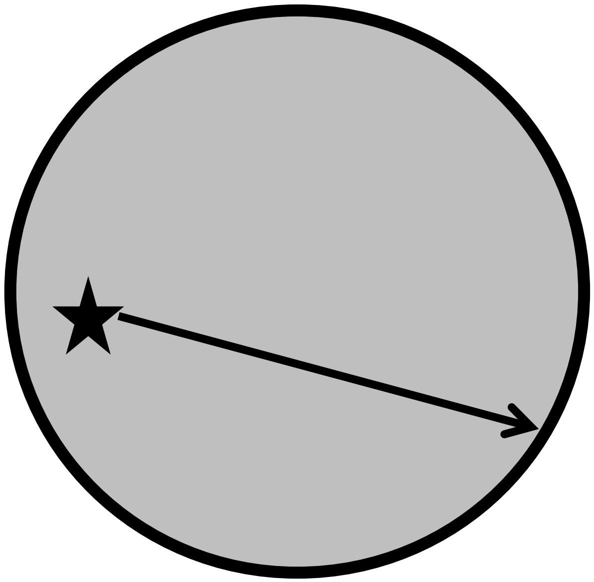
1. Shadow. Generalized Fermat's principle (Hadamard 1910; Friedlander 1958)



1. Shadow. Shadow operator
(Aizenberg and Ayzenberg, 2008)

$$\mathrm{A}\langle\ \rangle = \iint_S h\, \mathrm{G}\, \mathrm{N}\langle\ \rangle \mathrm{d}S$$

convex domain

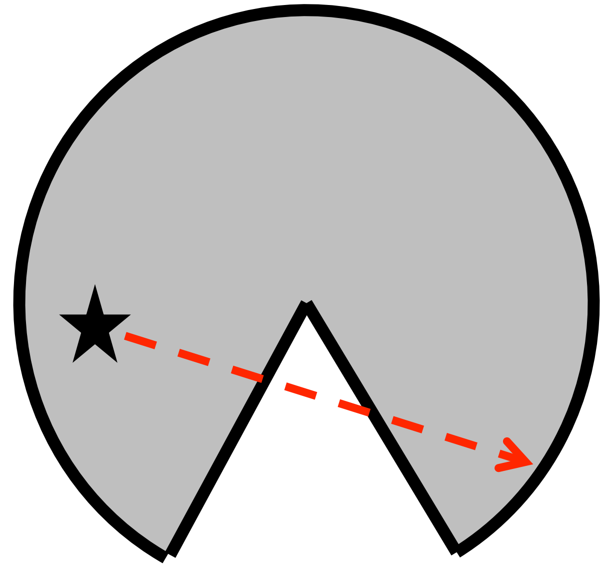


$h = 0 \Rightarrow \mathrm{A} = 0$

$\mathrm{F} = \mathrm{G}$

$\mathrm{K}^{\mathrm{F}} = \mathrm{K}^{\mathrm{G}}$

concave domain

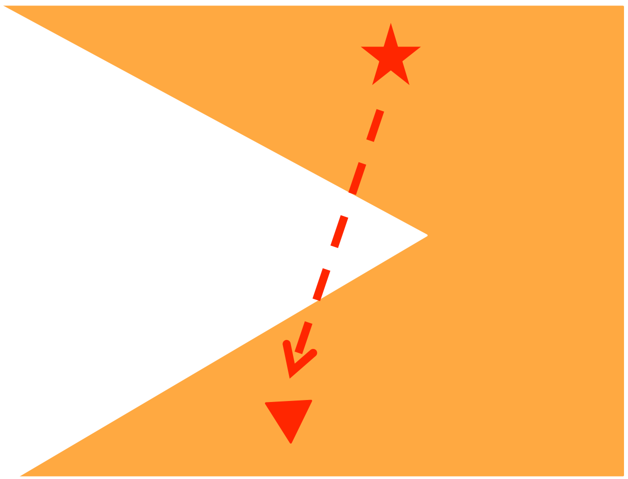


$h \neq 0 \Rightarrow \mathrm{A} \neq 0$

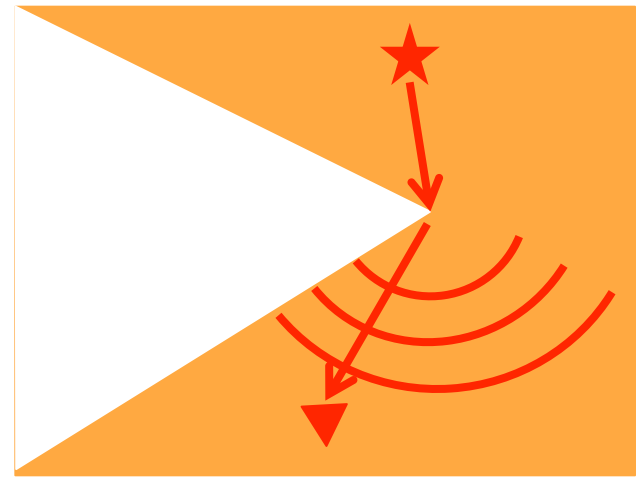
$\mathrm{F} \neq \mathrm{G}$

$\mathrm{K}^{\mathrm{F}} \neq \mathrm{K}^{\mathrm{G}}$

2. Conventional and feasible solution



G - conventional Green's function



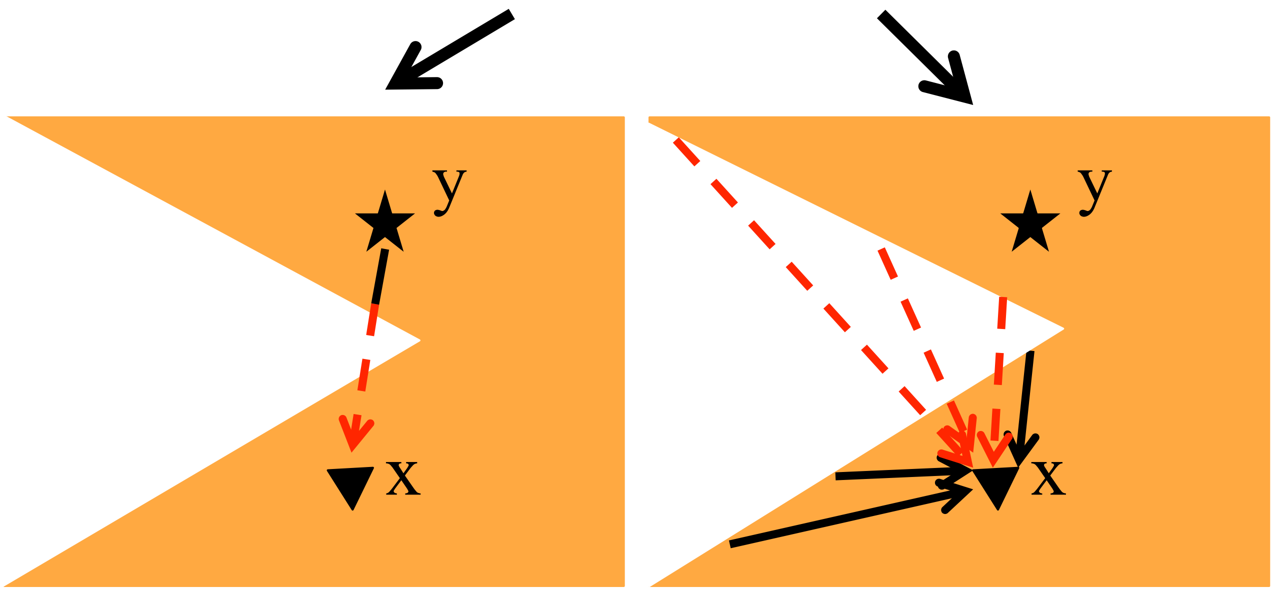
F - feasible Green's function

$$G \neq F$$

2. Conventional and feasible solution

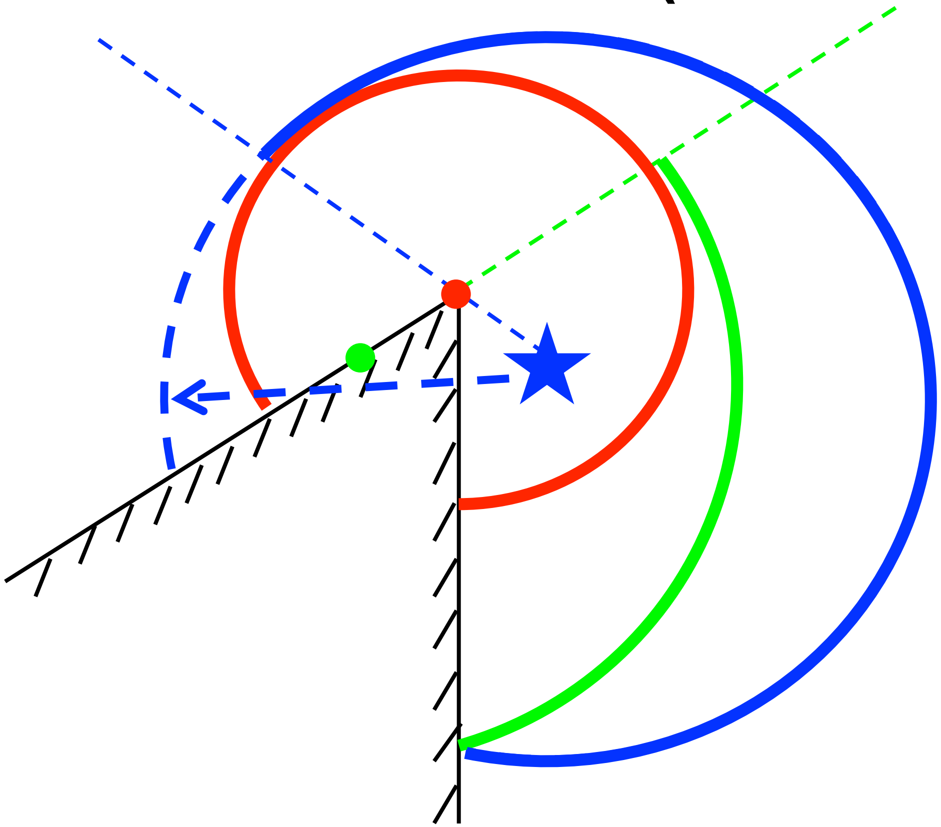
$$\mathrm{p}(\mathrm{x},\mathrm{y}) = \mathrm{p}_0(\mathrm{x},\mathrm{y}) + \iint_{\mathrm{S}} \left[\frac{1}{\rho(\mathrm{s})} \frac{\partial \mathrm{g}(\mathrm{x},\mathrm{s},\omega)}{\partial n(\mathrm{s})} \mathrm{p}(\mathrm{s},\mathrm{y},\omega) - \mathrm{g}(\mathrm{x},\mathrm{s},\omega) \frac{1}{\rho(\mathrm{s})} \frac{\partial \mathrm{p}(\mathrm{s},\mathrm{y},\omega)}{\partial n(\mathrm{s})} \right] \mathrm{d}S(\mathrm{s})$$

$$\mathrm{p}(\mathrm{x},\mathrm{y}) = \mathrm{p}_0(\mathrm{x},\mathrm{y}) + \mathrm{K}^{\mathrm{G}}(\mathrm{x},\mathrm{s})\,\mathrm{p}(\mathrm{s},\mathrm{y})$$



The conventional solution does not describe separate waves

2. Conventional and feasible solution. V-shaped model (Friedlander 1958)



$$\text{Wavefield} = h \cdot G + D_1 + D_2 + R$$

$$h = \begin{cases} 1, & \text{in illuminated zone} \\ 0, & \text{in shadow zone} \end{cases} \text{– shadow function}$$

G – Green's function

D_1 – diffraction

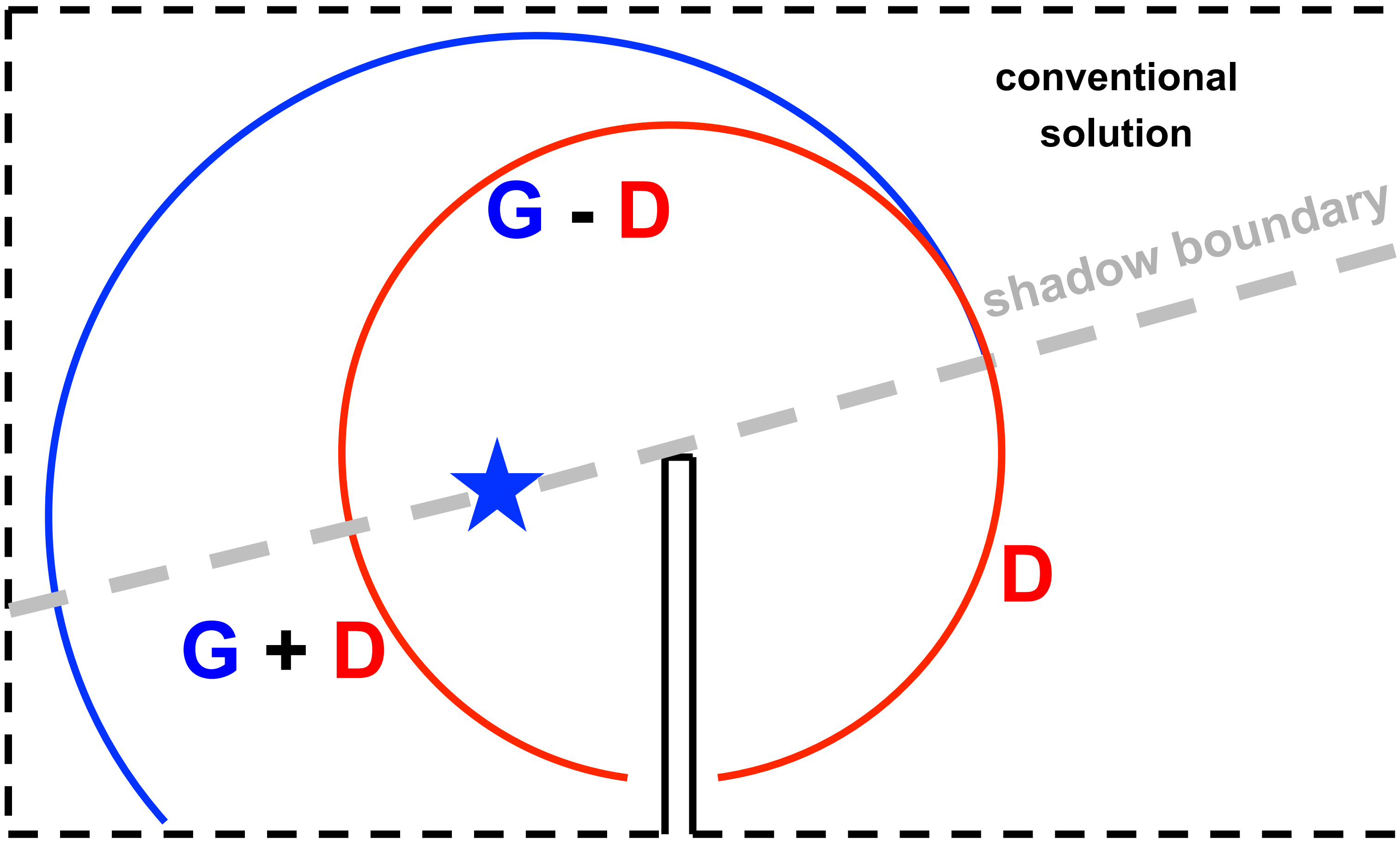
D_2 – diffraction

R – reflection

$h \cdot G \neq G \quad \rightarrow \quad$ Green's function cannot be used in this case $\quad \rightarrow$

(generalized Fermat's principle is applicable instead of Fermat's principle)

2. Conventional and feasible solution. Slit model (Aizenberg and Ayzenberg 2008)

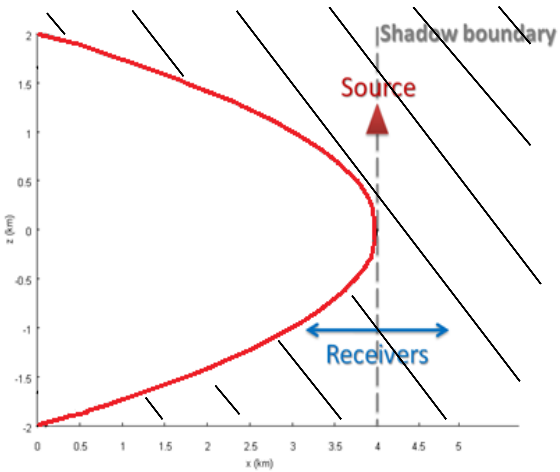


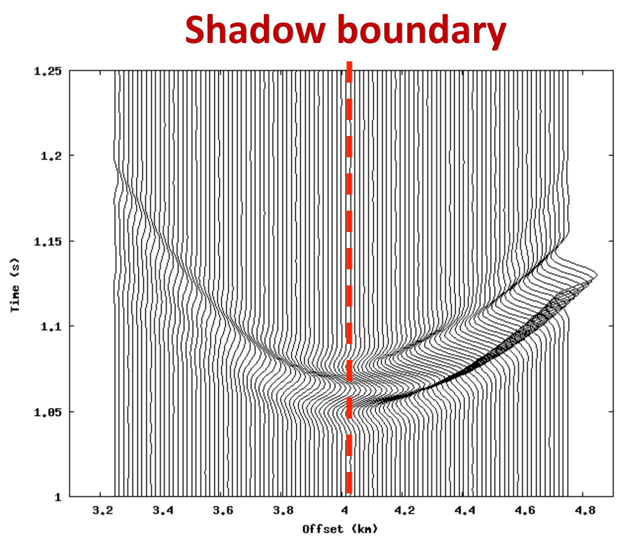
feasible solution

$$F(s, y) \cong G(s, y) + K(s, s'')\, A(s'', s')\, G(s', y) + K$$

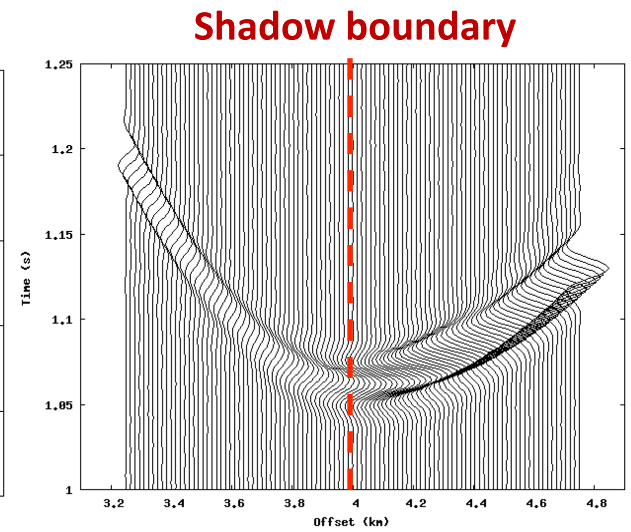
$$F(s, y) \cong [1 - h(s, y)]\, G(s, y) + D(s, y) + K(s, s')\, [2\, h(s', y) - 1]\, D(s', y)$$

2. Conventional and feasible solution. U-shaped model (Ayzenberg A et al. 2012, Zyatkov et al. 2012)





conventional
Kirchhoff and Maggi-Rubinowicz
solution



feasible
solution

3. Conventional and feasible Kirchhoff operator

Conventional Kirchhoff operator

$$K^{G}\langle\ \rangle = \iint_{S} G\, N \langle\ \rangle dS$$

Feasible Kirchhoff operator

$$K^{F}\langle\ \rangle = \iint_{S} F\, N \langle\ \rangle dS$$

$$K^{F} = K^{G} + \left(K^{G} A\right) K^{G} + \left(K^{G} A\right)^{2} K^{G} + K$$

G - Green's function

K^G - Kirchhoff-type operator based on Green's function

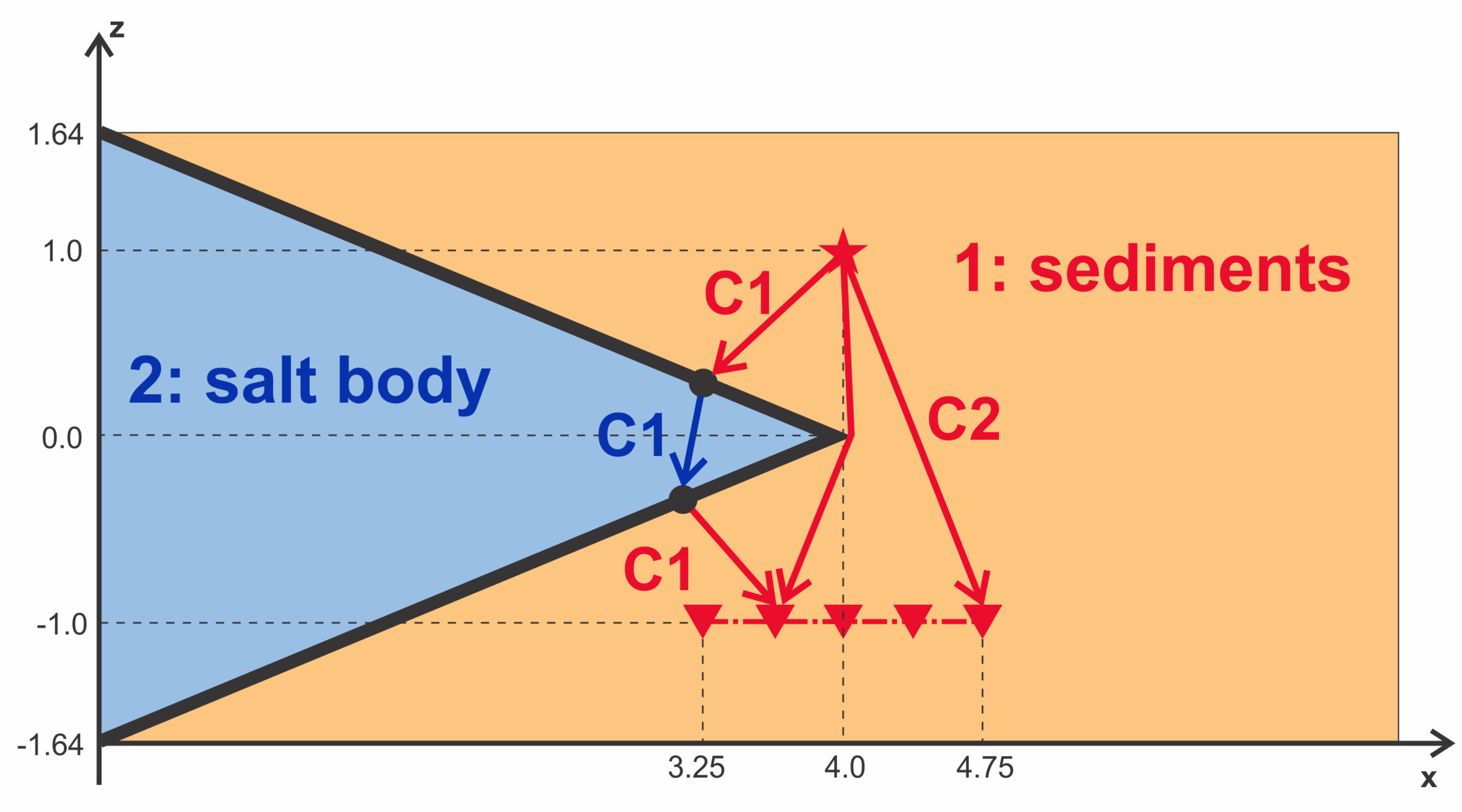
K^F - Kirchhoff-type operator based on feasible Green's function

A - absorption/shadow integral

N - matrix of normals

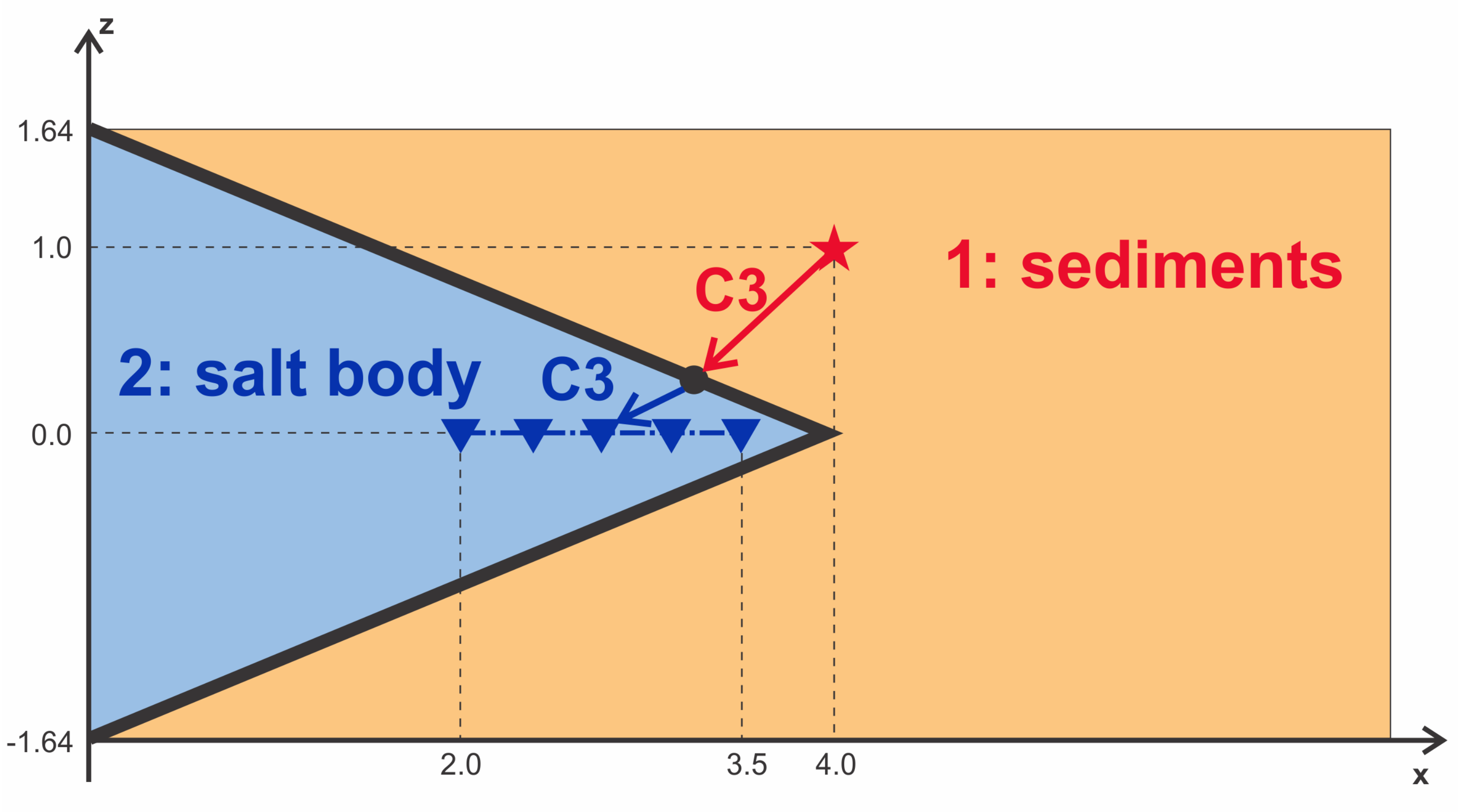
4. Tests. V-shaped model

code C1+C2



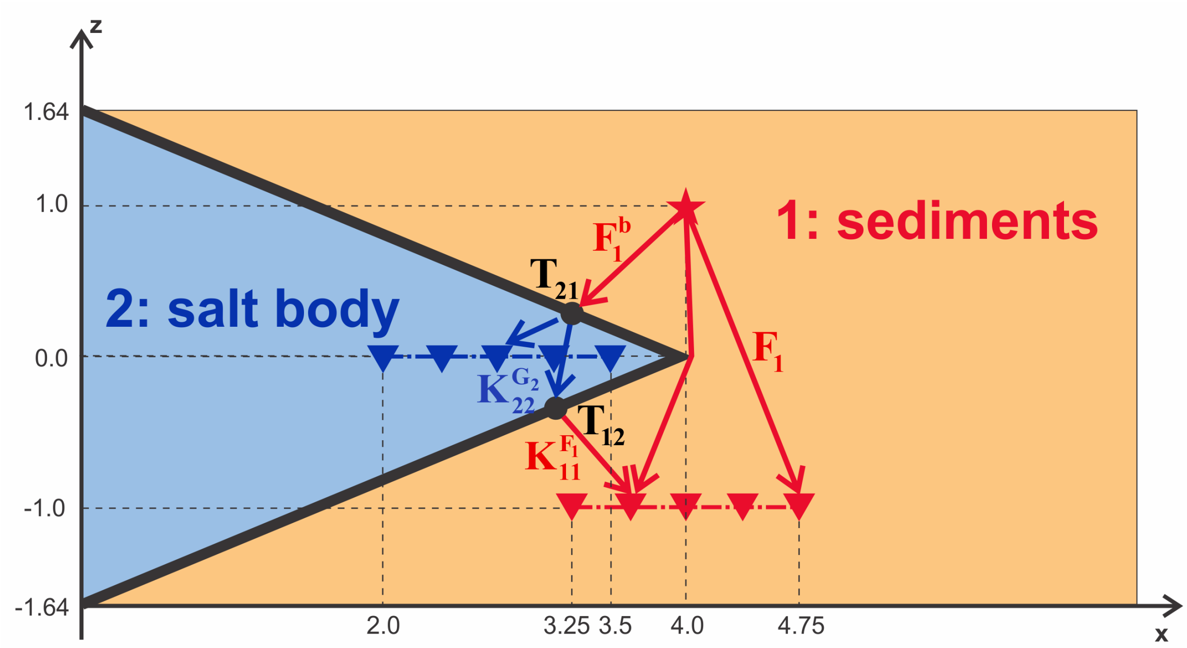
4. Tests. V-shaped model

code C3



4. Tests. V-shaped model

$$W_1^{feas} = F_1 + K_{11}^{F_1} T_{12} K_{22}^{G_2} T_{21} F_1^b$$



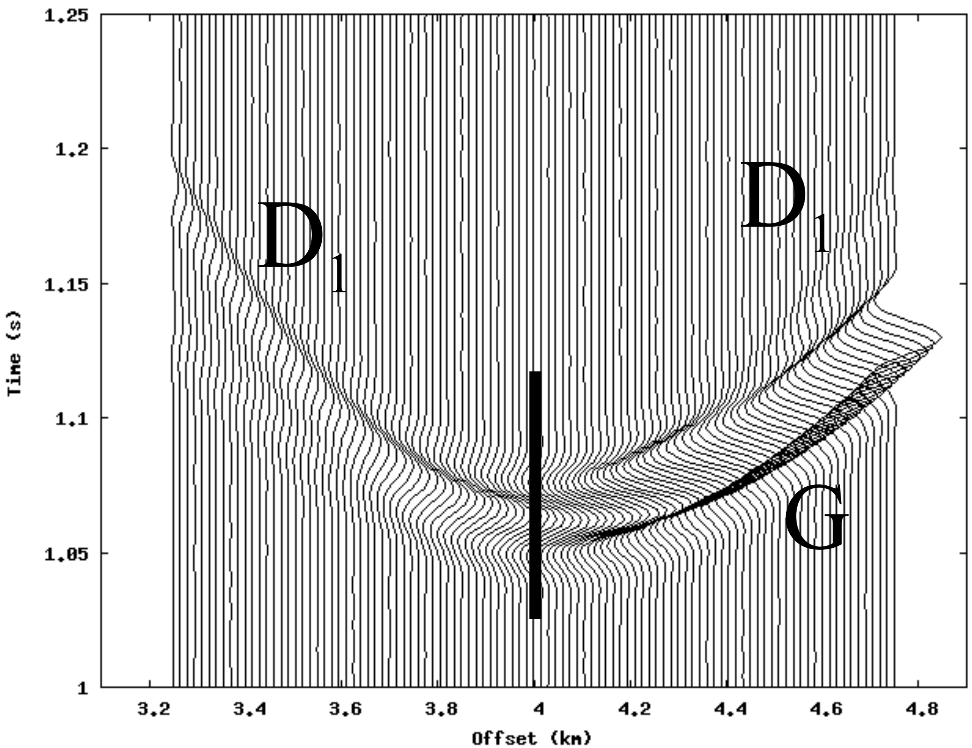
$$F_1 = G_1 + K_{11}^{G_1} A_{11} G_1$$

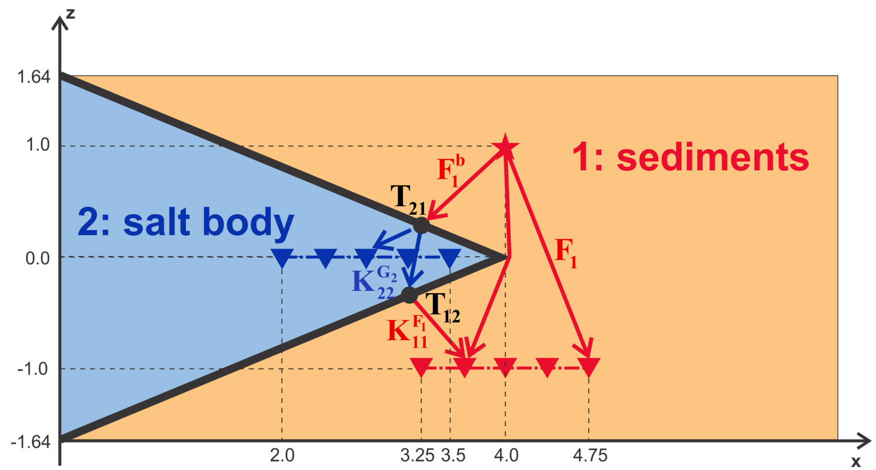
$$F_1^b = G_1^b + K_{11}^{G_1} A_{11} G_1^b$$

$$K_{11}^{F_1} = K_{11}^{G_1} + K_{11}^{G_1} A_{11} K_{11}^{G_1}$$

4. Tests. V-shaped model

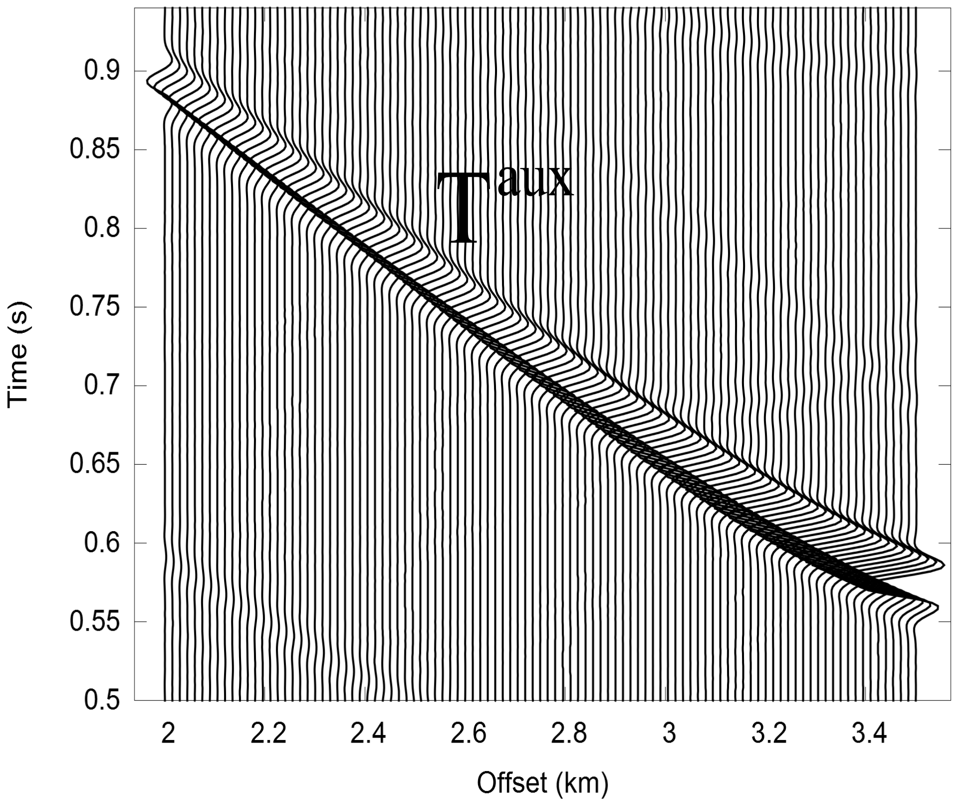
$$W_1^{feas} = F_1 + K_{11}^{F_1} T_{12} K_{22}^{G_2} T_{21} F_1^b$$

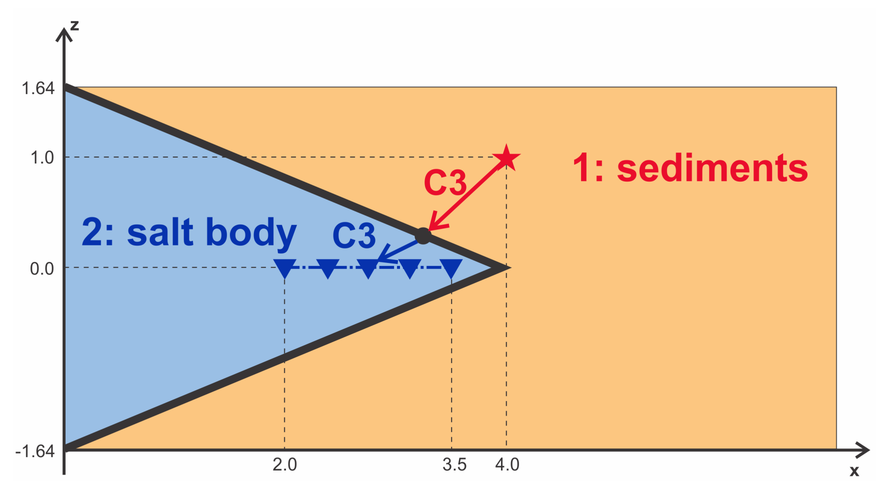




4. Tests. V-shaped model

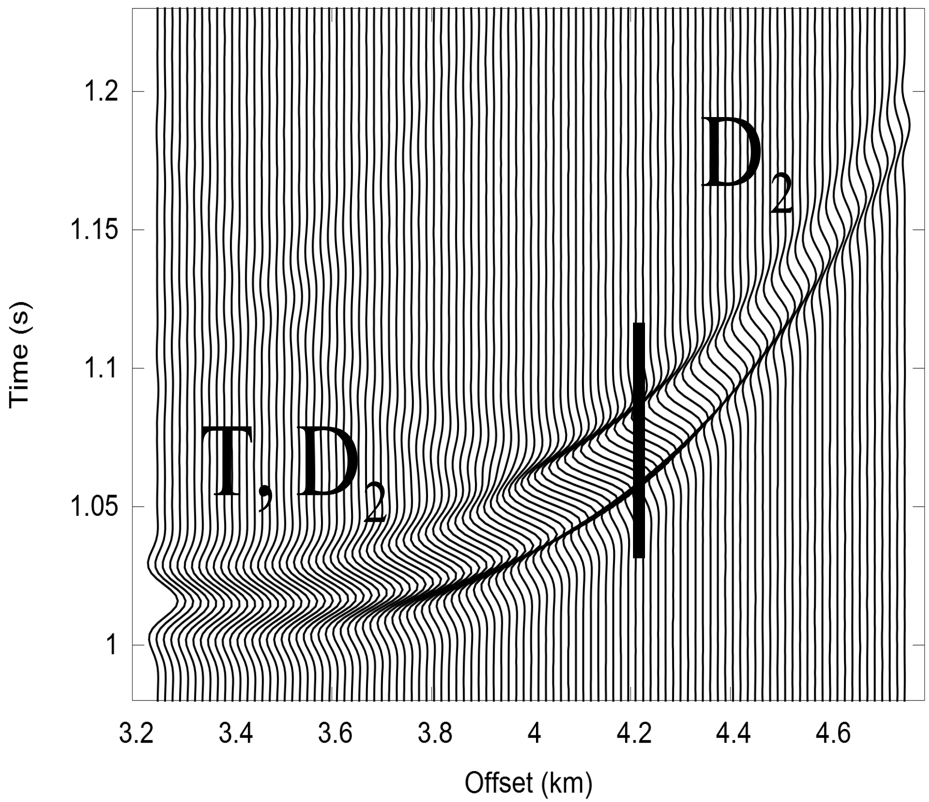
$$W_1^{feas} = F_1 + K_{11}^{F_1} T_{12} K_{22}^{G_2} T_{21} F_1^b$$

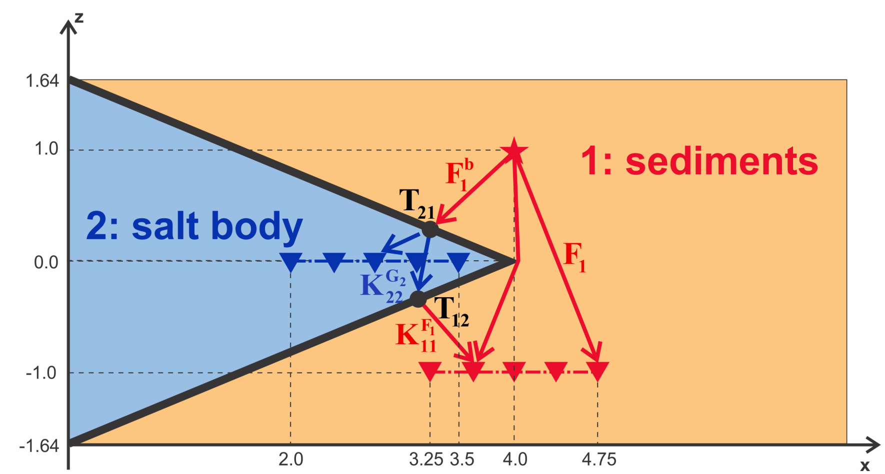




4. Tests. V-shaped model

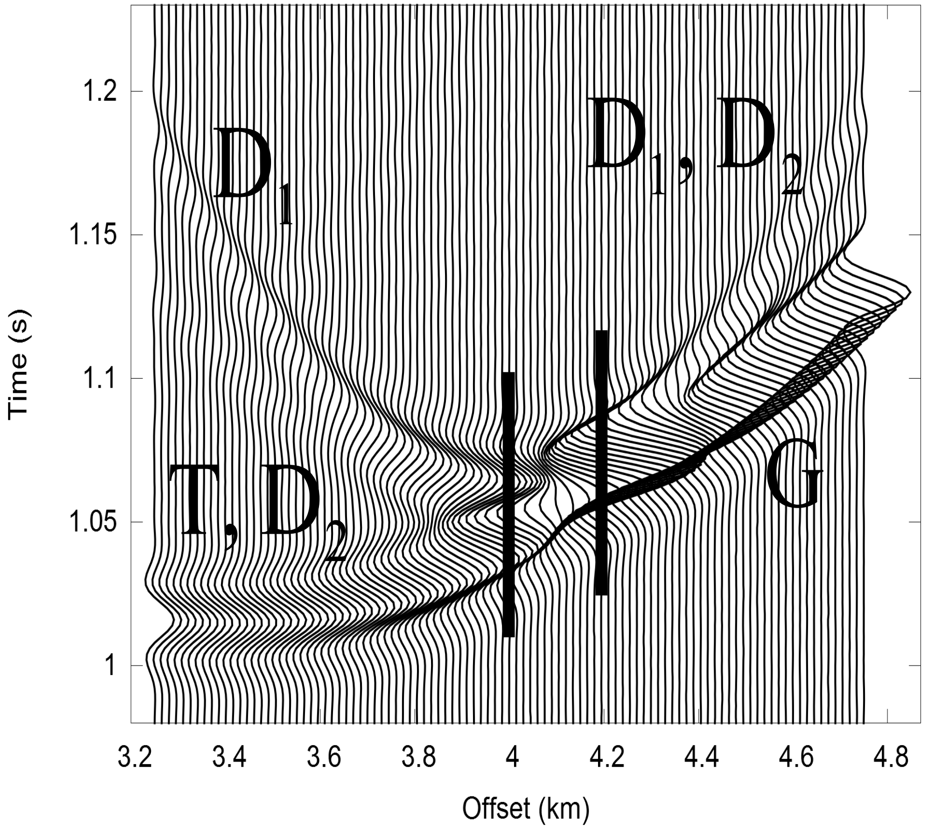
$$W_1^{feas} = F_1 + K_{11}^{F_1} T_{12} K_{22}^{G_2} T_{21} F_1^b$$

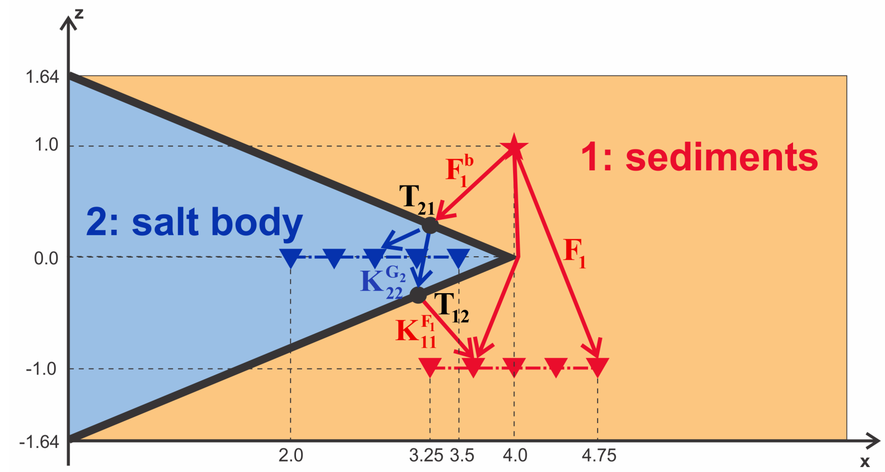




V-shaped model

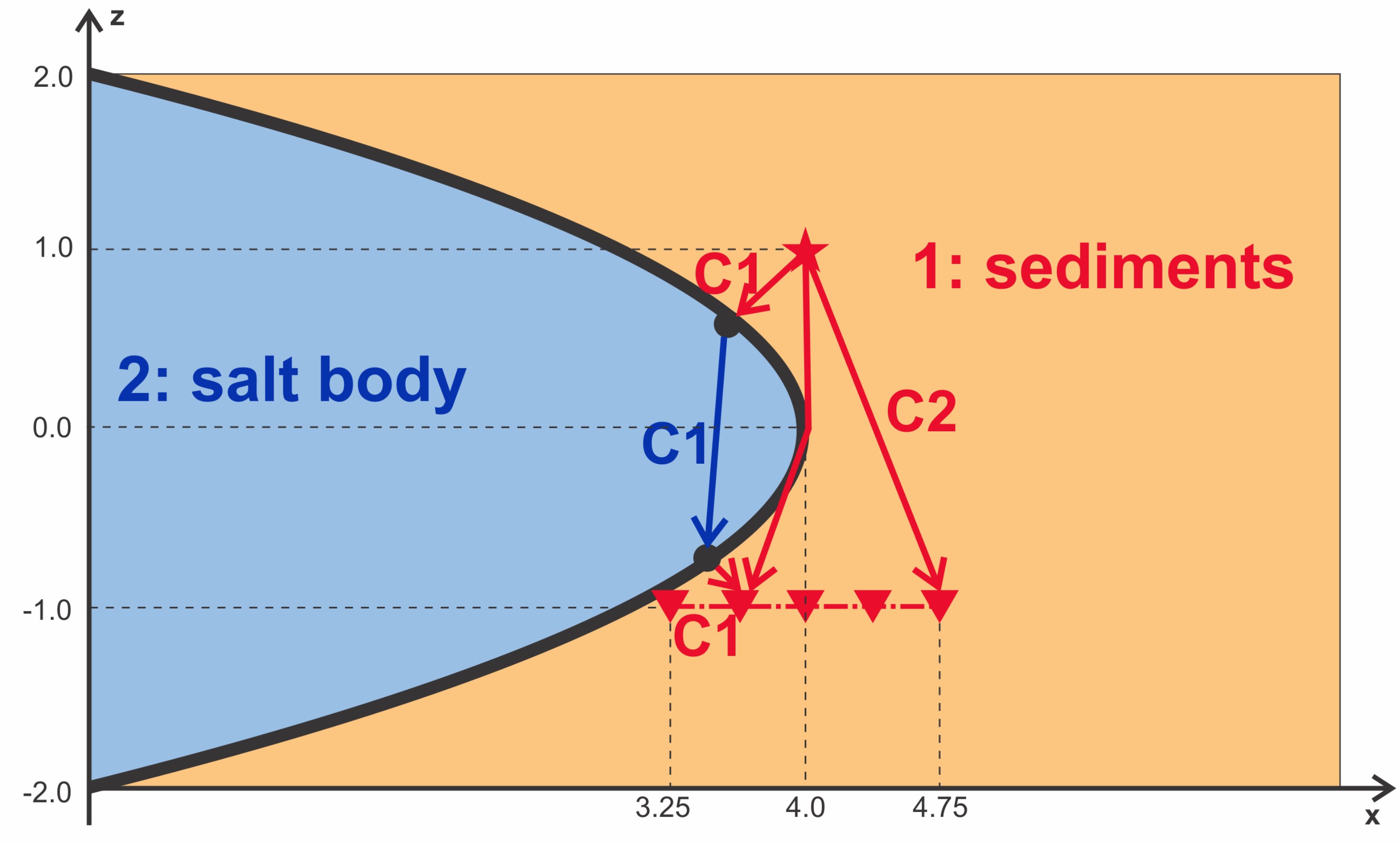
$$W_1^{feas} = F_1 + K_{11}^{F_1} \, T_{12} \, K_{22}^{G_2} \, T_{21} \, F_1^b$$





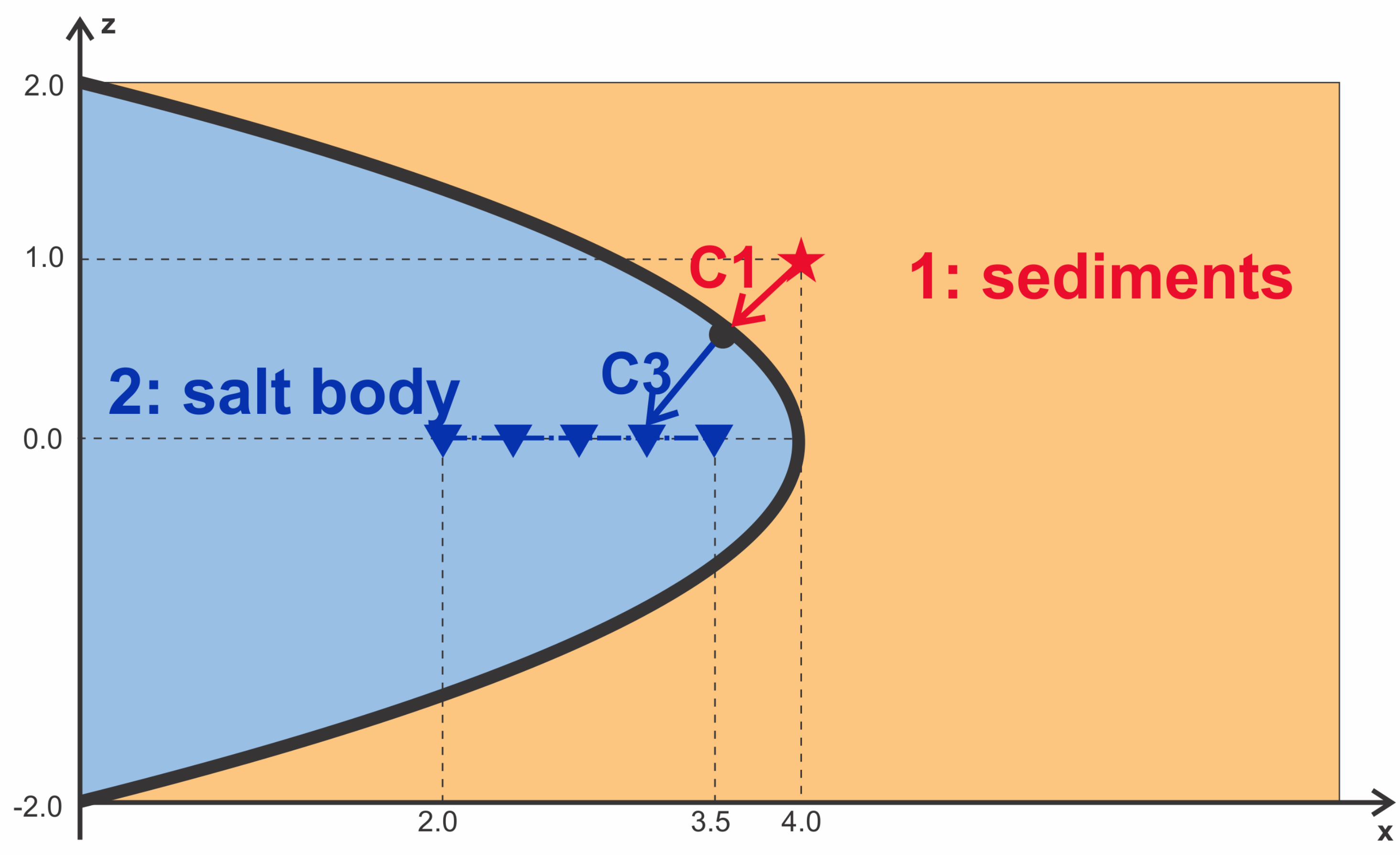
4. Tests. U-shaped model

modeling C1+C2



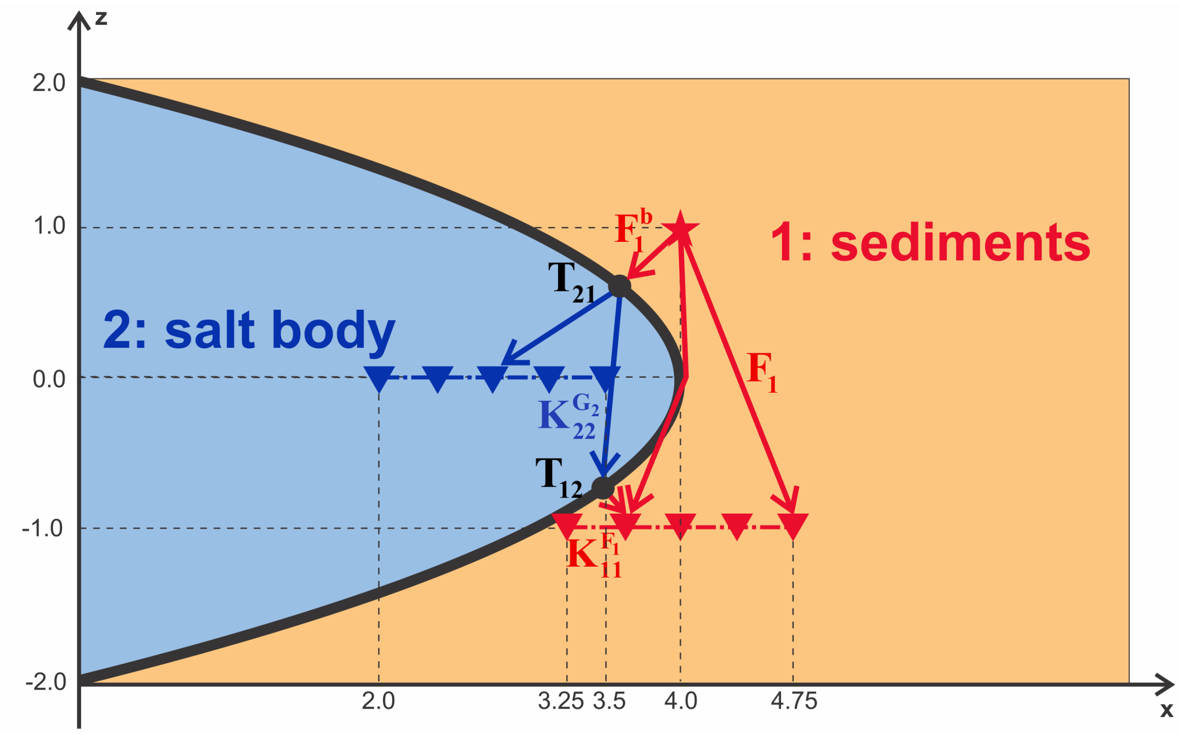
4. Tests. U-shaped model

modeling C3



4. Tests. U-shaped model

$$W_1^{feas} = F_1 + K_{11}^{F_1} T_{12} K_{22}^{G_2} T_{21} F_1^b$$



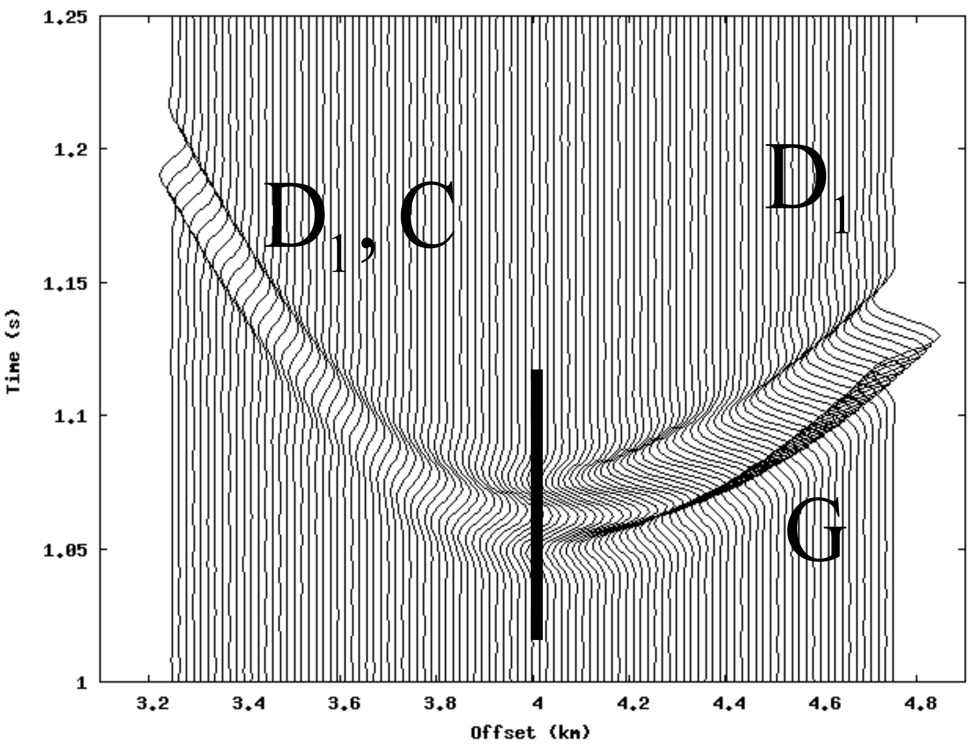
$$F_1 = G_1 + K_{11}^{G_1} A_{11} G_1$$

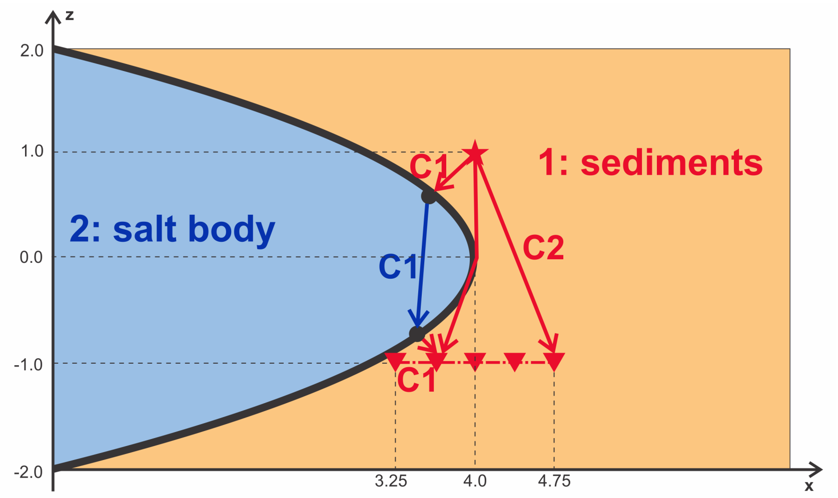
$$F_1^b = G_1^b + K_{11}^{G_1} A_{11} G_1^b$$

$$K_{11}^{F_1} = K_{11}^{G_1} + K_{11}^{G_1} A_{11} K_{11}^{G_1}$$

4. Tests. U-shaped model

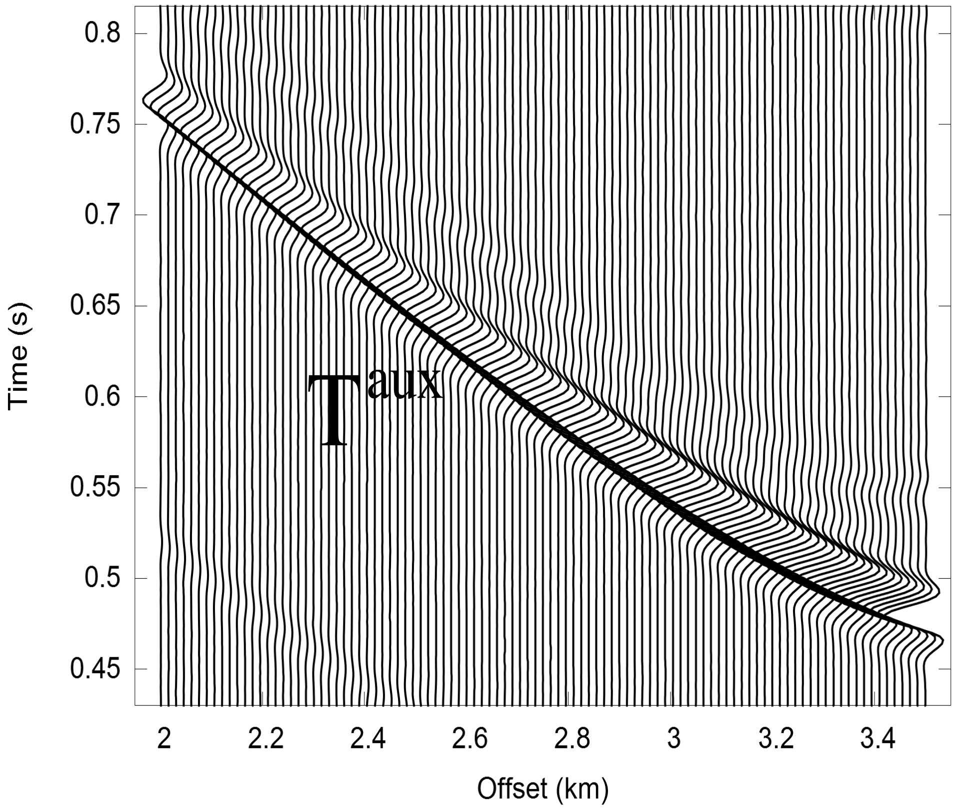
$$W_1^{feas} = F_1 + K_{11}^{F_1} T_{12} K_{22}^{G_2} T_{21} F_1^b$$

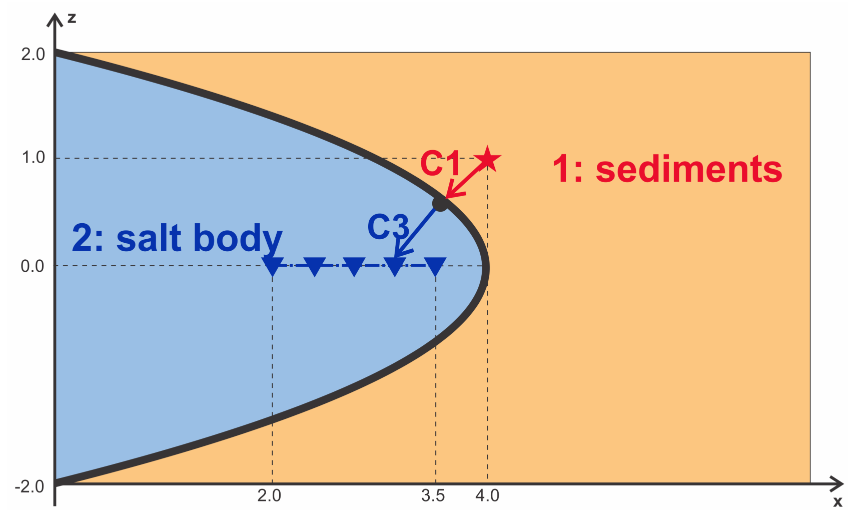




4. Tests. U-shaped model

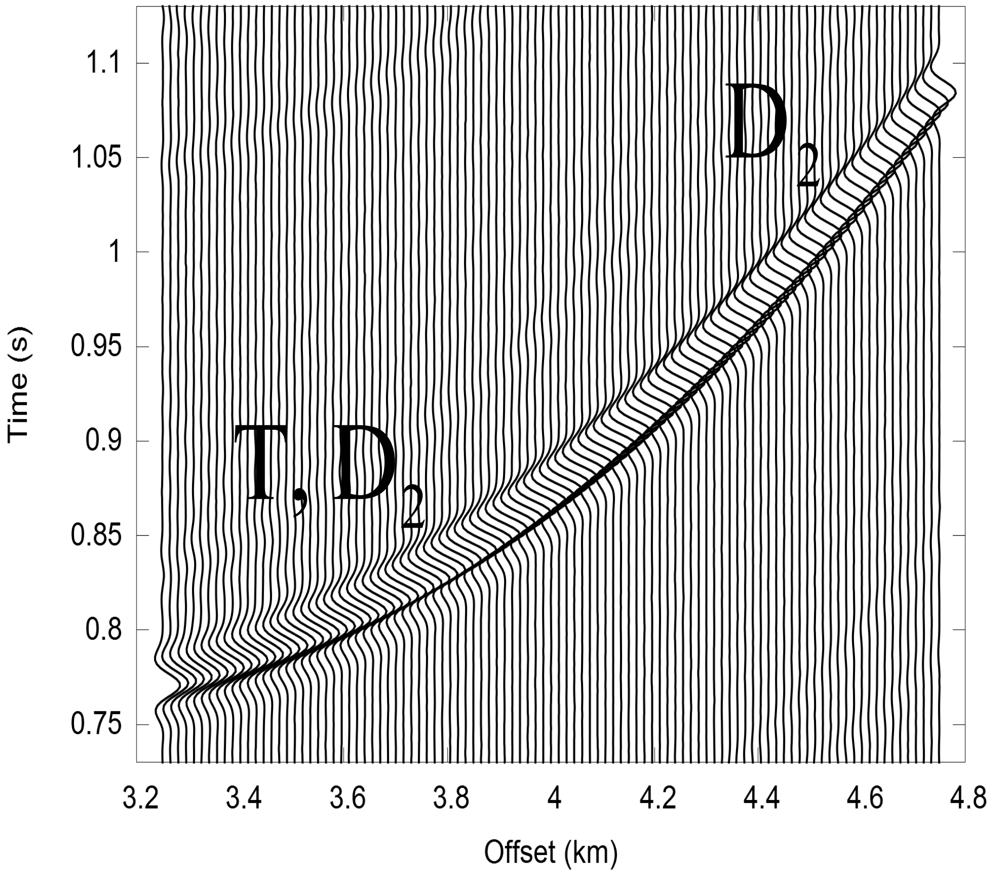
$$W_1^{feas} = F_1 + K_{11}^{F_1} T_{12} K_{22}^{G_2} T_{21} F_1^b$$

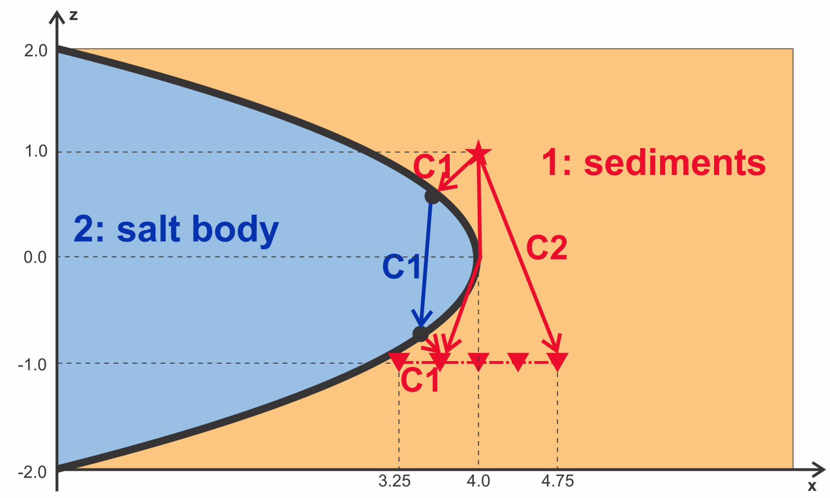




4. Tests. U-shaped model

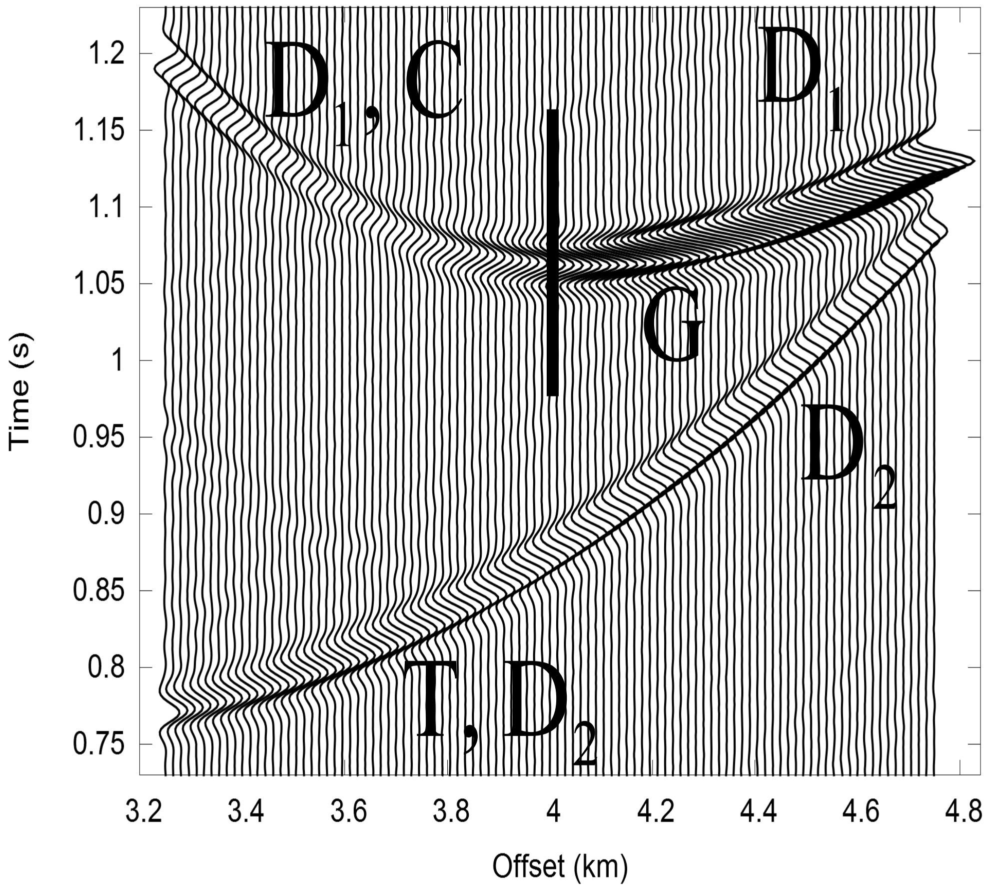
$$W_1^{feas} = F_1 + K_{11}^{F_1} \, T_{12} \, K_{22}^{G_2} \, T_{21} \, F_1^b$$

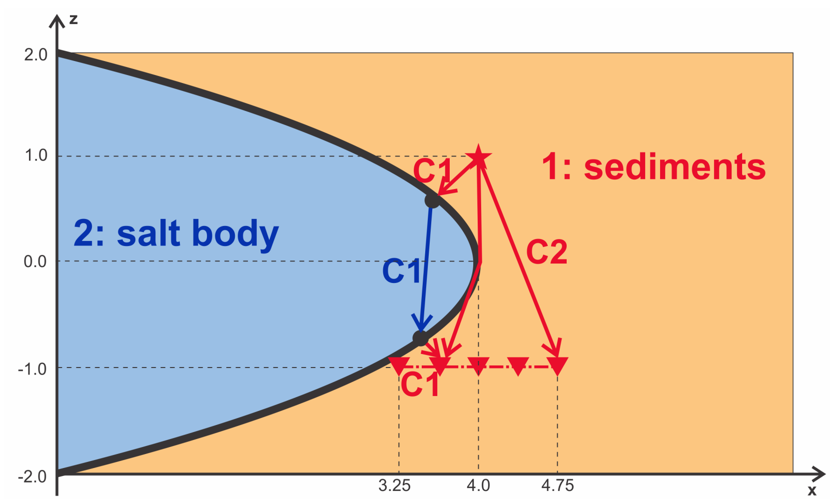




4. Tests. U-shaped model

$$W_1^{feas} = F_1 + K_{11}^{F_1} T_{12} K_{22}^{G_2} T_{21} F_1^b$$





Discussion

Conventional modeling is apllicable for convex domains

Feasible modeling is applicable for concave domains
(real example: salt body and sediments)

Acknowledgements

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