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Magnetotelluric inversion – classic impedance vs. direct field inversion

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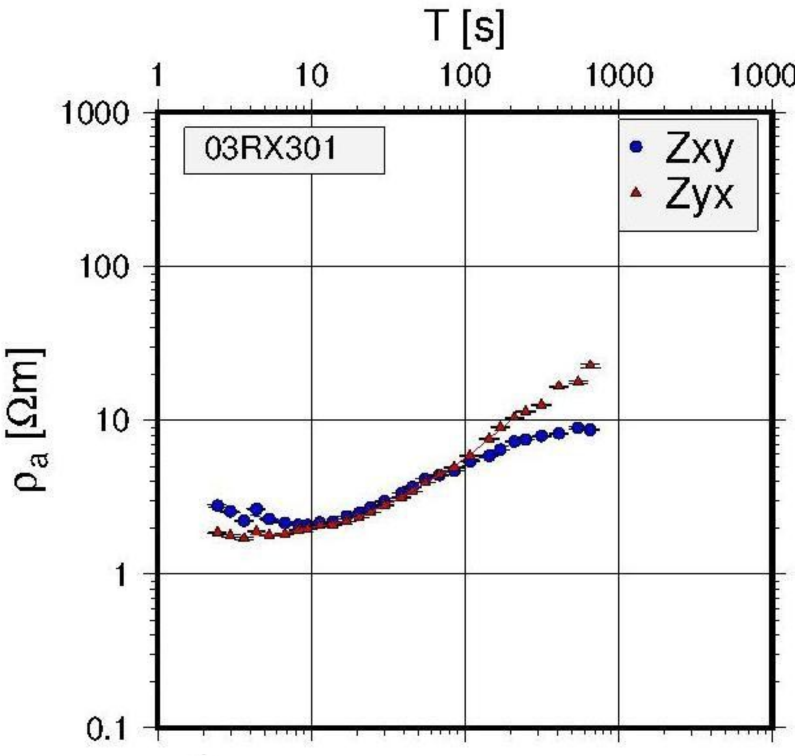
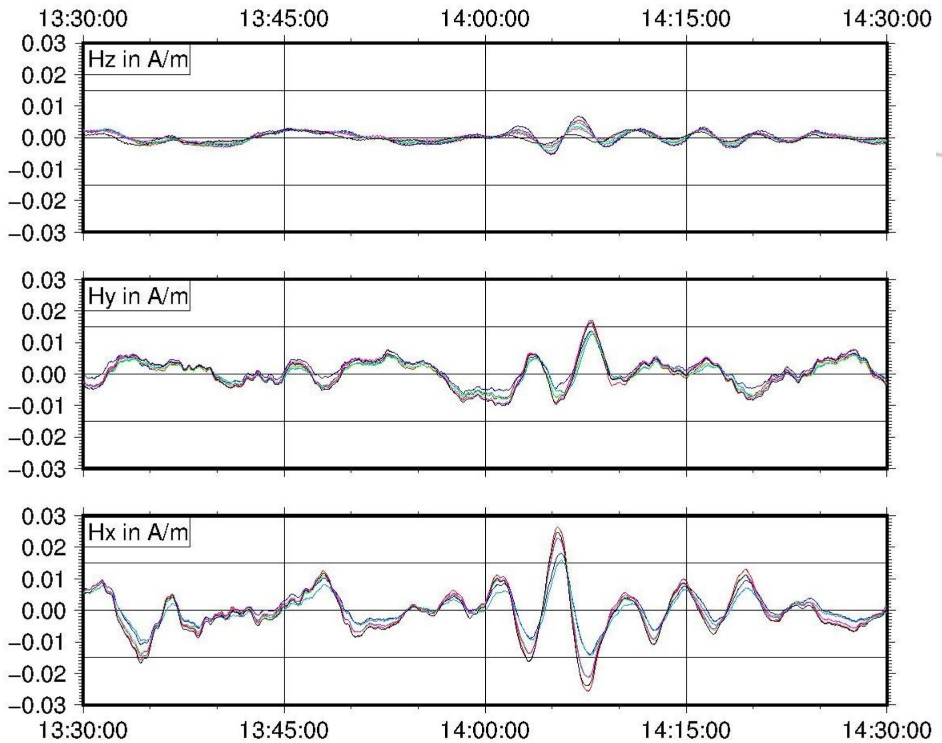
Motivation

- Final goal to develop joint inversion for EM, magnetics, and gravity data for deep basin characterization
- Base is a 3D magnetotelluric inversion
- Compare impedance vs. direct EM field inversion

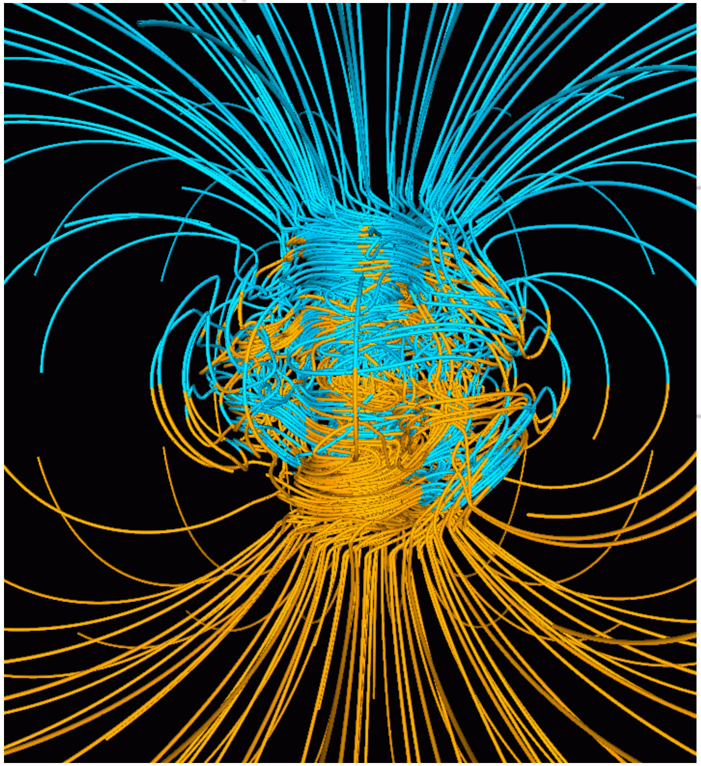


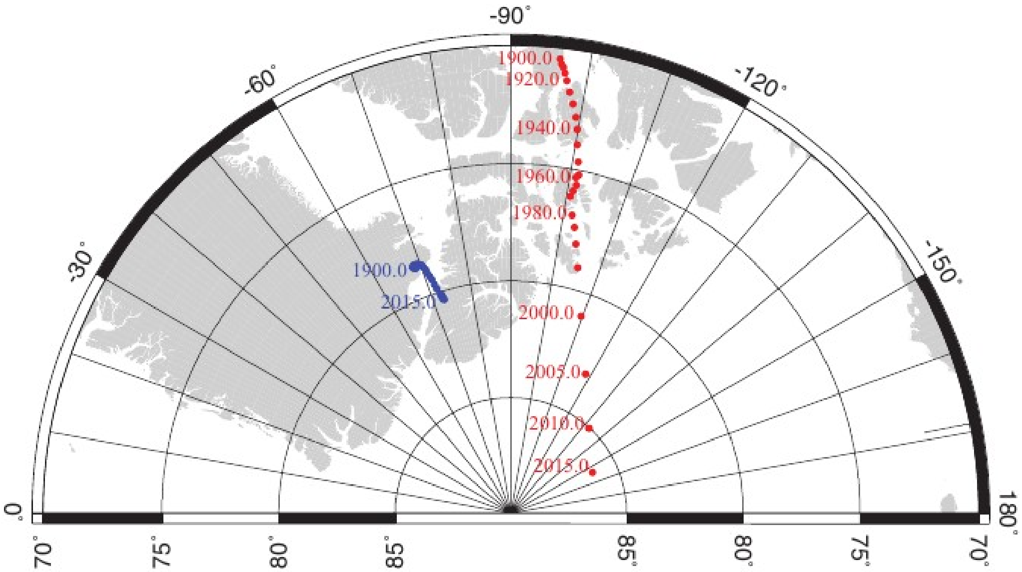


Magnetotelluric field – static part

$$V(r,\theta,\phi,t) = a \sum_{n=1}^{N} \sum_{m=0}^{n} \left(\frac{a}{r}\right)^{n+1} \left[g_n^m(t)\cos m\phi + h_n^m(t)\sin m\phi\right] \times P_n^m(\cos\theta)$$

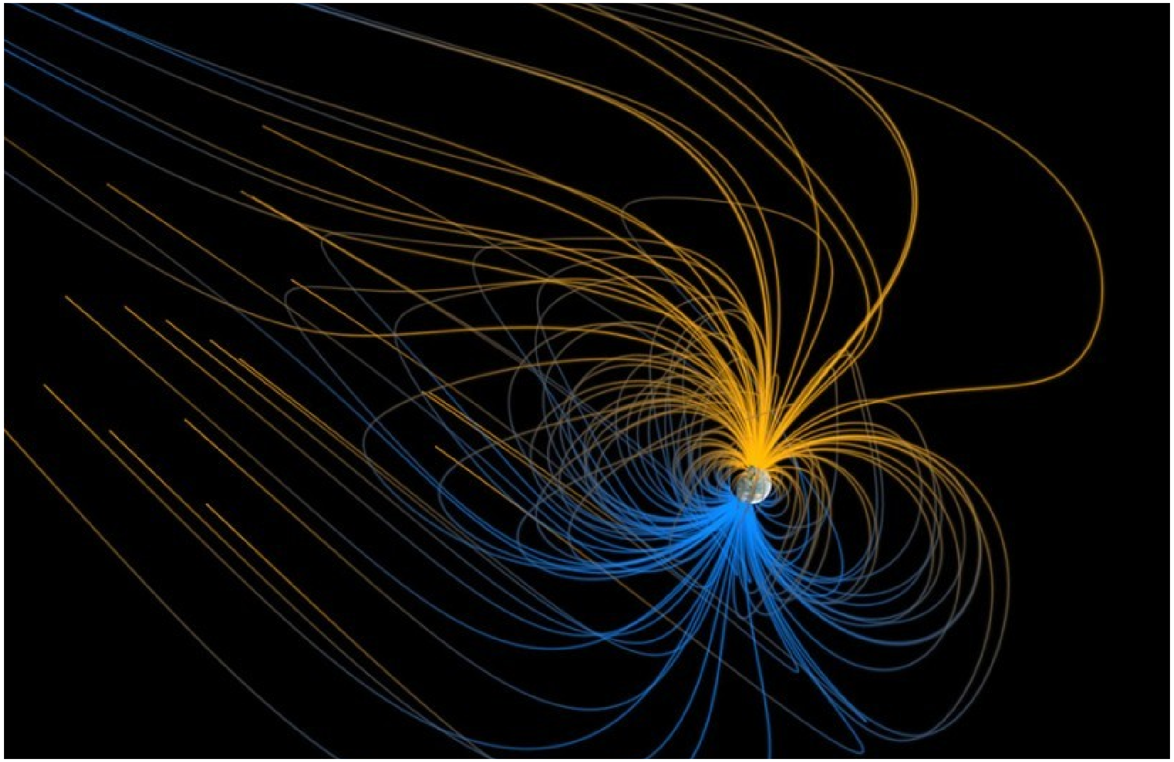
g/h	n	m	1900.0	1920.0	1940.0	1960.0	1980.0	2000.0	2010.0
g	1	0	-31543	-31060	-30654	-30421	-29992	-29619.4	-29496.5
g	1	1	-2298	-2317	-2292	-2169	-1956	-1728.2	-1585.9
h	1	1	5922	5845	5821	5791	5604	5186.1	4945.1
g	2	0	-677	-839	-1106	-1555	-1997	-2267.7	-2396.6
g	2	1	2905	2959	2981	3002	3027	3068.4	3026.0
h	2	1	-1061	-1259	-1614	-1967	-2129	-2481.6	-2707.7
g	2	2	924	1407	1566	1590	1663	1670.9	1668.6
h	2	2	1121	823	528	206	-200	-458.0	-575.4
g	3	0	1022	1111	1240	1302	1281	1339.6	1339.7
g	3	1	-1469	-1600	-1790	-1992	-2180	-2288.0	-2326.3
h	3	1	-330	-445	-499	-414	-336	-227.6	-160.5
g	3	2	1256	1205	1232	1289	1251	1252.1	1231.7
h	3	2	3	103	163	224	271	293.4	251.7
g	3	3	572	839	916	878	833	714.5	634.2
h	3	3	523	293	43	-130	-252	-491.1	-536.8





Total field intensity of 51.781nT for Trondheim

Magnetotelluric field – variational part



sources	Freq.	Period
meteorology	1Hz – 1kHz	0.001s – 1s
Solar wind	1µHz – 1Hz	1s – 10000s
Dead band	0.5Hz – 5Hz	0.2s – 2s

Daily variation ca. 20 – 50nT
(during mag. Storm – 200nT)

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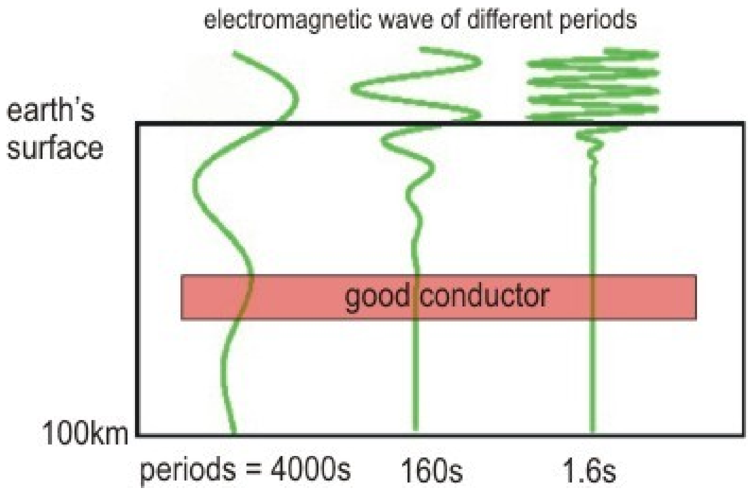
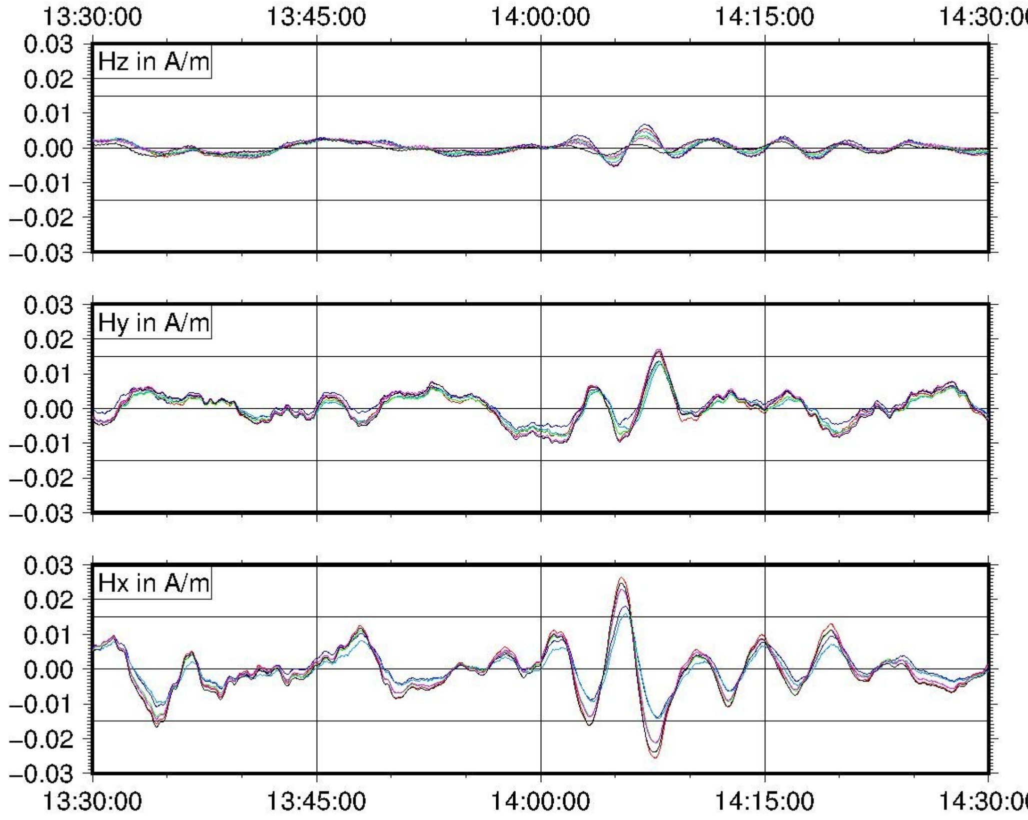
Magnetotelluric field – skin depth

$$\delta = \sqrt{\frac{T\tilde{\rho}}{\Pi\mu_0}} = \sqrt{\frac{1}{\Pi\mu_0\omega\tilde{\sigma}}} \simeq 500\sqrt{T\rho_a}$$

$$\mathbf{E}(z) = \mathbf{E}_0(z=0)e^{-\frac{z}{\delta}}$$

	Period 1s	Period 1h
ρ = 1 Ωm	δ = 500m	δ = 30km
ρ = 100 Ωm	δ = 5000m	δ = 300km





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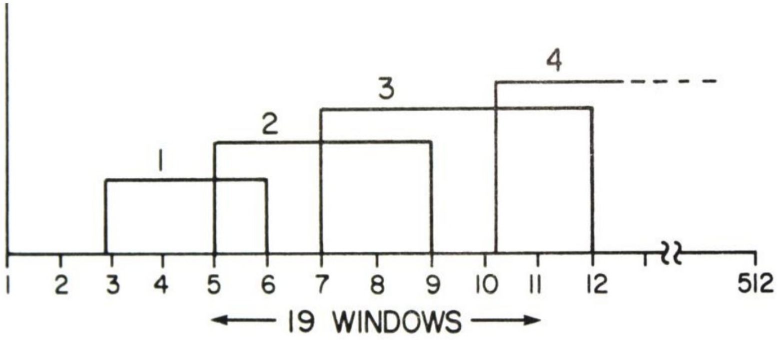
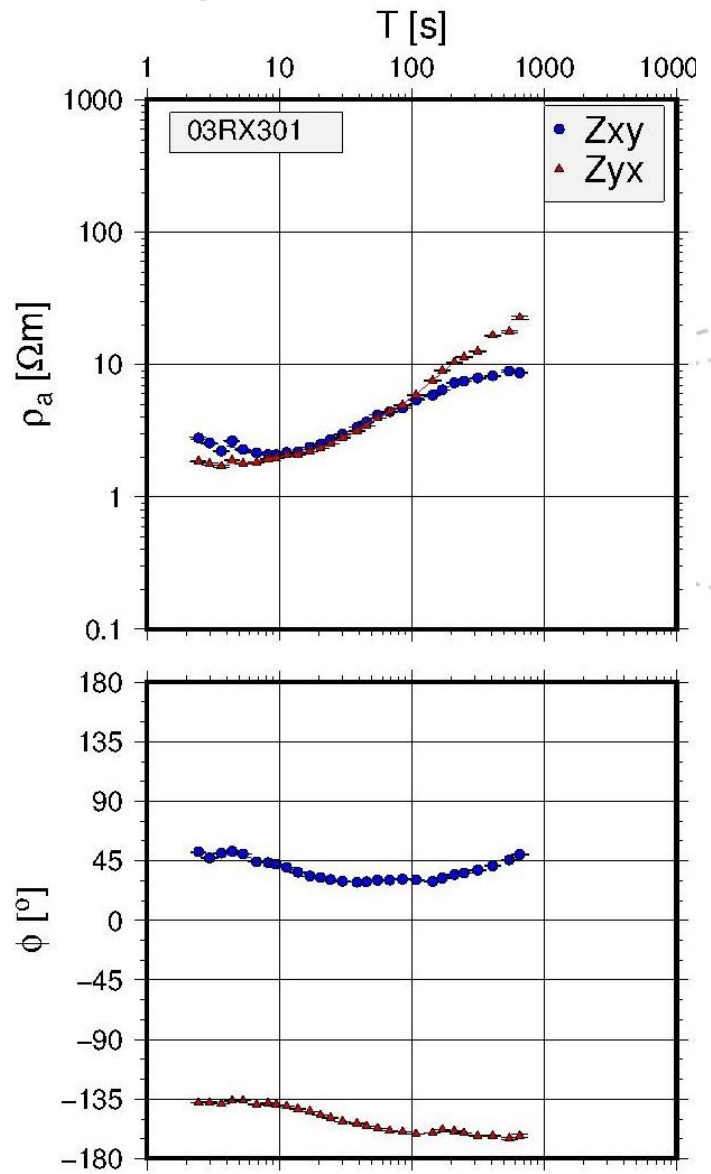
Magnetotelluric field – impedance data

$$\begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{pmatrix} \begin{pmatrix} H_x \\ H_y \end{pmatrix}$$



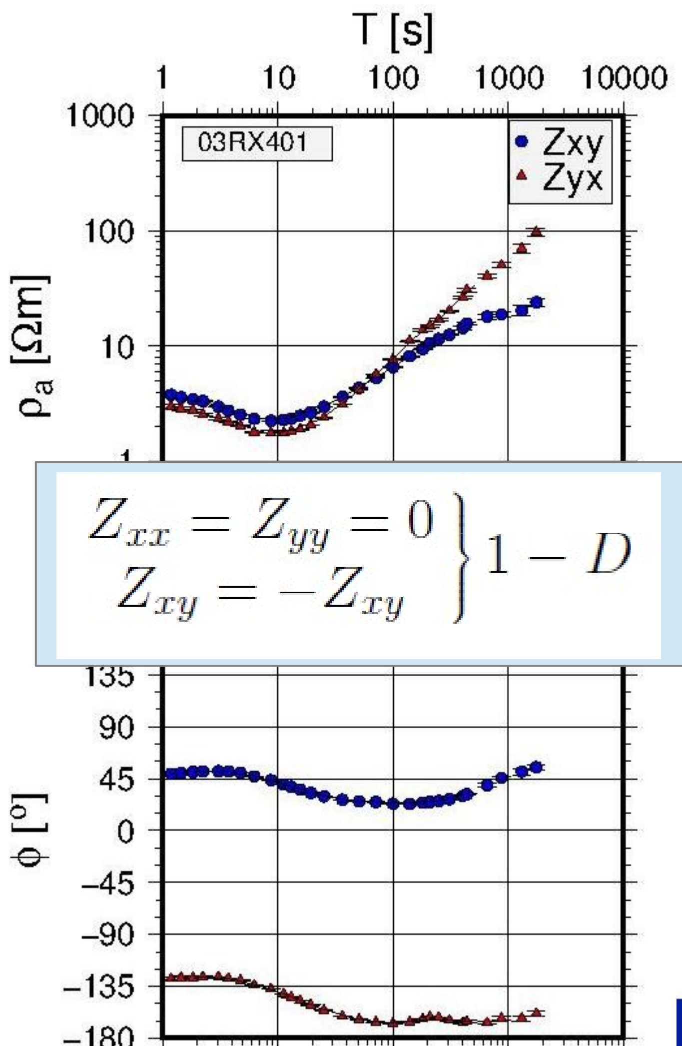
$$Z_{xy} = \frac{\langle E_x H_x^* \rangle \langle H_x H_y^* \rangle - \langle E_x H_y^* \rangle \langle H_x H_x^* \rangle}{\langle H_y H_x^* \rangle \langle H_x H_y^* \rangle - \langle H_y H_y^* \rangle \langle H_y H_x^* \rangle}$$

1)Decimation (down sampling)
2)Receiver orientation
3)Fourier transform
4)Band averaging
5)Transfer function estimation
6)Display



Magnetotelluric field – direct field data

Find unknown source distribution with help of a 1D response receiver



$$\left.\begin{array}{c} Z_{xx} = Z_{yy} = 0 \\ Z_{xy} = -Z_{xy} \end{array}\right\} 1-D$$

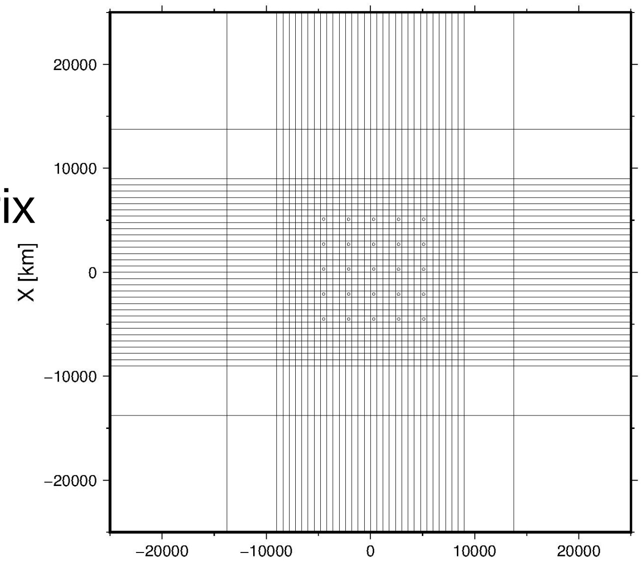
(1)Find 1D model fitting the data
(2)Estimate response inv. model
(3)Estimate source field
(4)Response of 3D start model
(5)Scaling factors

$$E_{\mathrm{receiver}} = E_{\mathrm{incoming}} R_{\mathrm{sb}}^{\mathrm{Total}}$$

$$R_{\mathrm{sb}}^{\mathrm{Total}} = R_{\mathrm{sb}} \left(1 - \frac{r_0 \mathrm{e}^{2\mathrm{i}\kappa_{\mathrm{w}} \Delta z_{\mathrm{w}}}}{1 + r_{\mathrm{sb}} r_0 \mathrm{e}^{2\mathrm{i}\kappa_{\mathrm{w}} \Delta z_{\mathrm{w}}}} \right)$$

Inversion - Forward Modeling

- Finite volume modeling of el. field (Weiss et al. 2006)
- Scattered field solution
- PARDISO sparse direct solver to invert coefficient matrix
- Example 40x40x40 cells -> 201720 x 201720 matrix

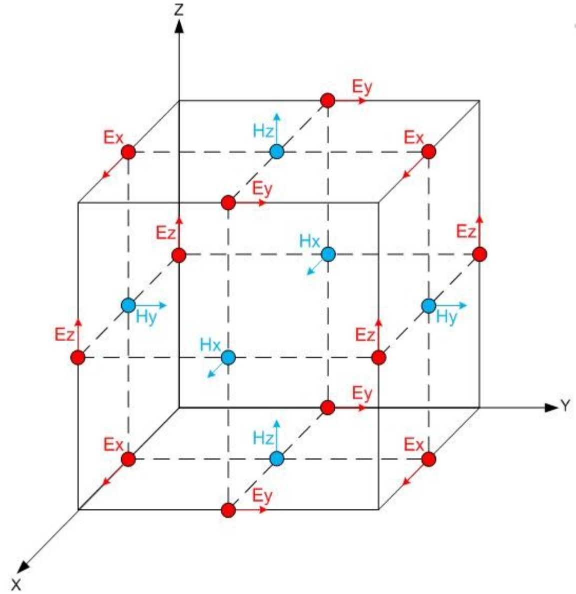


$$\nabla \times \nabla \times \mathbf{E} + i\omega\mu_0\sigma\mathbf{E} = -i\omega\mu_0\sigma\mathbf{J}_s$$

$$\mathbf{E}^{'} = \mathbf{E} - \mathbf{E}^0$$

$$\nabla \times \nabla \times \mathbf{E}^{'} + i\omega\mu_0\sigma\mathbf{E}^{'} = -i\omega\mu_0(\sigma - \sigma_0)\mathbf{E}_0$$

$$\mathbf{A}\mathbf{e} = \mathbf{b}$$



Inversion of MT data

- Gauss – Newton inversion of the scattered field
- Undetermined problem 50000 to 100000 unknowns with ca. 1000 to 3000 data points
- Minimum norm solution

$$\mathbf{m} = [m_1, m_2, m_3, ..., m_M]^T = [\sigma_1, \sigma_2, \sigma_3, ..., \sigma_M]^T$$

$$\mathbf{d} = [d_1, d_2, d_3, ..., d_N]^T$$

$$\mathbf{d} = [E_x|_{per=1}^{sta=1}, E_y|_{per=1}^{sta=1}, H_x|_{per=1}^{sta=1}, ..., H_y|_{per=nper}^{sta=nsta}]^T$$

$$\mathbf{d} = [Z_{xx}|_{per=1}^{sta=1}, Z_{xy}|_{per=1}^{sta=1}, Z_{yx}|_{per=1}^{sta=1}, ..., Z_{yy}|_{per=nper}^{sta=nsta}]^T$$

$$\mathbf{C}_d^{-1} = diag\left[1/err_1^2, 1/err_2^2, \cdots, 1/err_N^2\right]$$

$$\phi = \frac{1}{\lambda}\left[(\mathbf{d} - \mathbf{F}(\mathbf{m}))^T \mathbf{C}_d^{-1} (\mathbf{d} - \mathbf{F}(\mathbf{m}))\right] + (\mathbf{m} - \mathbf{m}_0)^T \mathbf{C}_m^{-1} (\mathbf{m} - \mathbf{m}_0)$$

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Inversion of MT data – Jacobi calculation

- Gauss – Newton inversion

$$\mathbf{m}_{k+1} - \mathbf{m}_0 = \mathbf{C}_m \mathbf{J}_k^T \left(\lambda \mathbf{C}_d + \mathbf{J}_k \mathbf{C}_m \mathbf{J}_k^T\right)^{-1} \left(\mathbf{d} - \mathbf{F}(\mathbf{m}_k) + \mathbf{J}_k(\mathbf{m}_k - \mathbf{m}_0)\right)$$

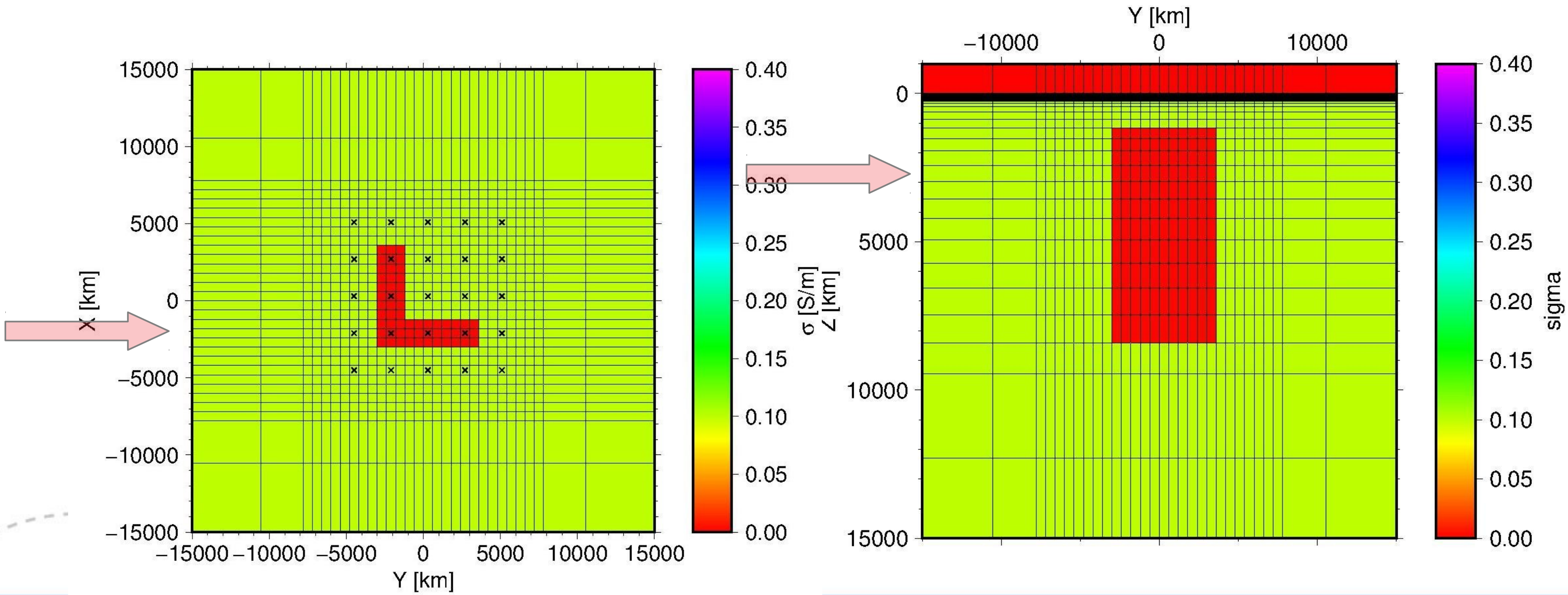
$$\mathbf{J}^T = \left(-\frac{\partial \mathbf{A}}{\partial \mathbf{m}}\mathbf{e} + \frac{\partial \mathbf{b}}{\partial \mathbf{m}}\right)^T \mathbf{A}^{-1} \left(\frac{\partial \psi}{\partial \mathbf{e}}\right)^T$$

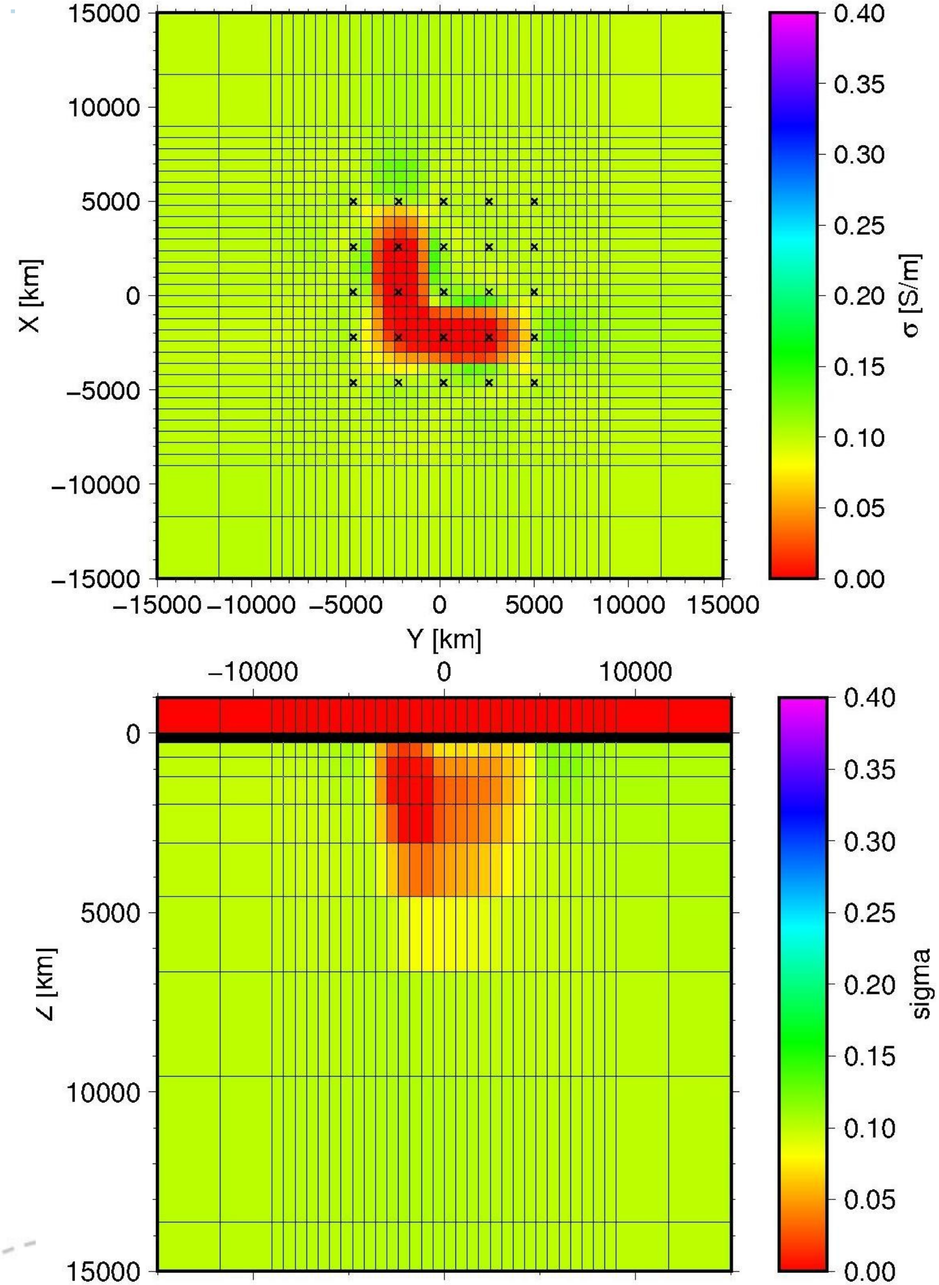
$$Z_{xx} = \frac{E_x^1 H_y^2 - E_x^2 H_y^1}{H_x^1 H_y^2 - H_x^2 H_y^1}$$

$$\frac{\partial Z_{xx}}{\partial e_k} = \left[(H_x^1 H_y^2 - H_x^2 H_y^1)(H_y^2 \frac{\partial E_x^1}{\partial e_k} - E_x^2 \frac{\partial H_y^1}{\partial e_k}) - (E_x^1 H_y^2 - E_x^2 H_y^1)(H_y^2 \frac{\partial H_x^1}{\partial e_k} - H_x^2 \frac{\partial H_y^1}{\partial e_k})\right] / (H_x^1 H_y^2 - H_x^2 H_y^1)^2$$
$$+ \left[(H_x^1 H_y^2 - H_x^2 H_y^1)(E_x^1 \frac{\partial H_y^2}{\partial e_k} - H_y^1 \frac{\partial E_x^2}{\partial e_k}) - (E_x^1 H_y^2 - E_x^2 H_y^1)(H_x^1 \frac{\partial H_y^2}{\partial e_k} - H_y^1 \frac{\partial H_x^2}{\partial e_k})\right] / (H_x^1 H_y^2 - H_x^2 H_y^1)^2$$

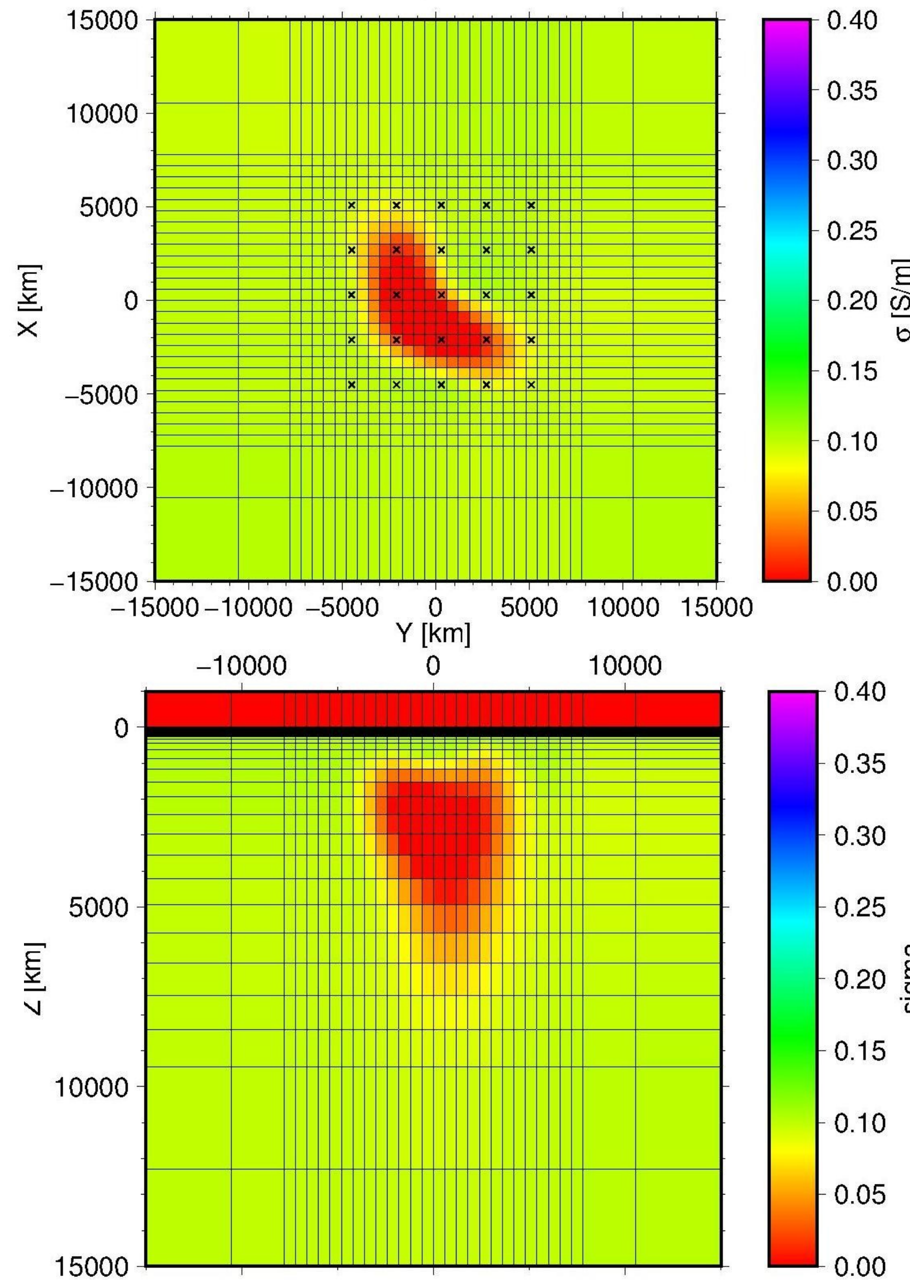
Synthetic example

- L - shaped resistor(0.001 S/m) in a conductive background (0.1 S/m) between 1 – 8km depth
- Model 40x40x40 cells 600m resolution center part
- 25 receiver on the seabed (260m water depth)
- 10 frequencies from 0.5 to 0.002Hz
- data: Zxy and Zyx, or Ex,Ey,Hx,Hy





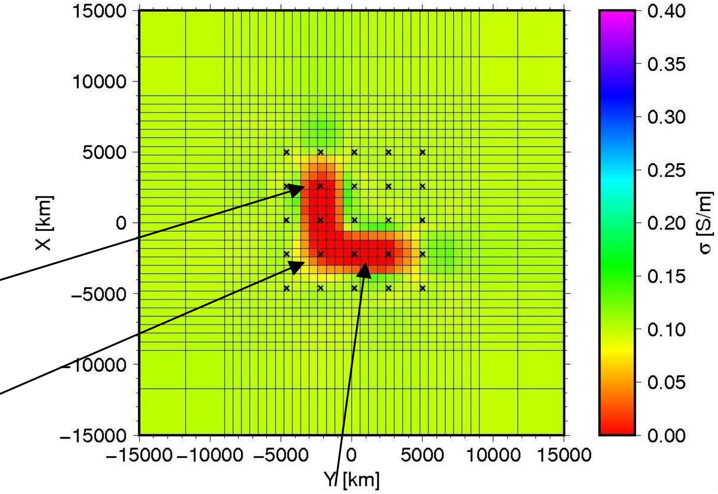
IMP-RMS: 0,102

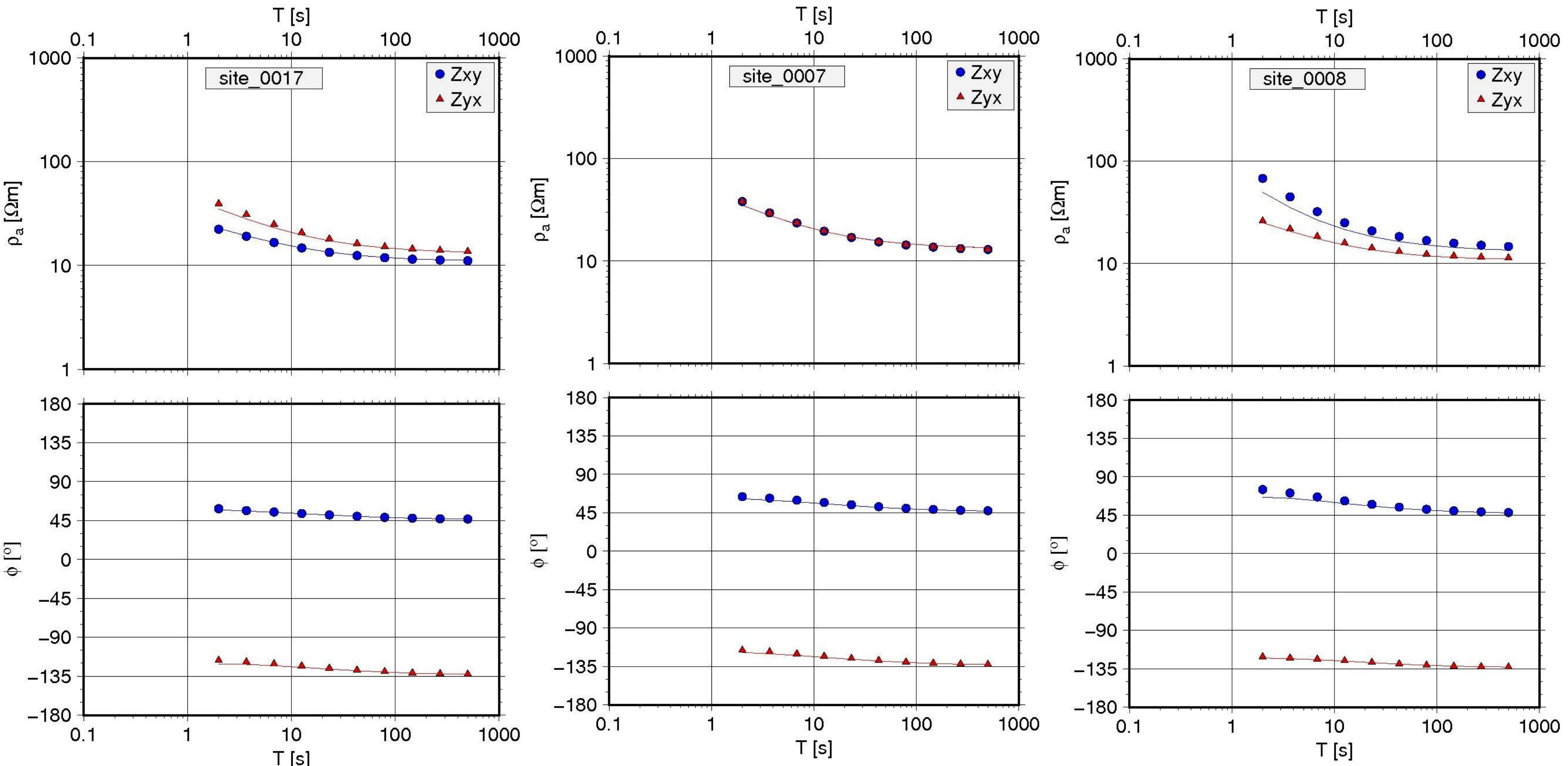


FLD-RMS: 0,056

Synthetic example

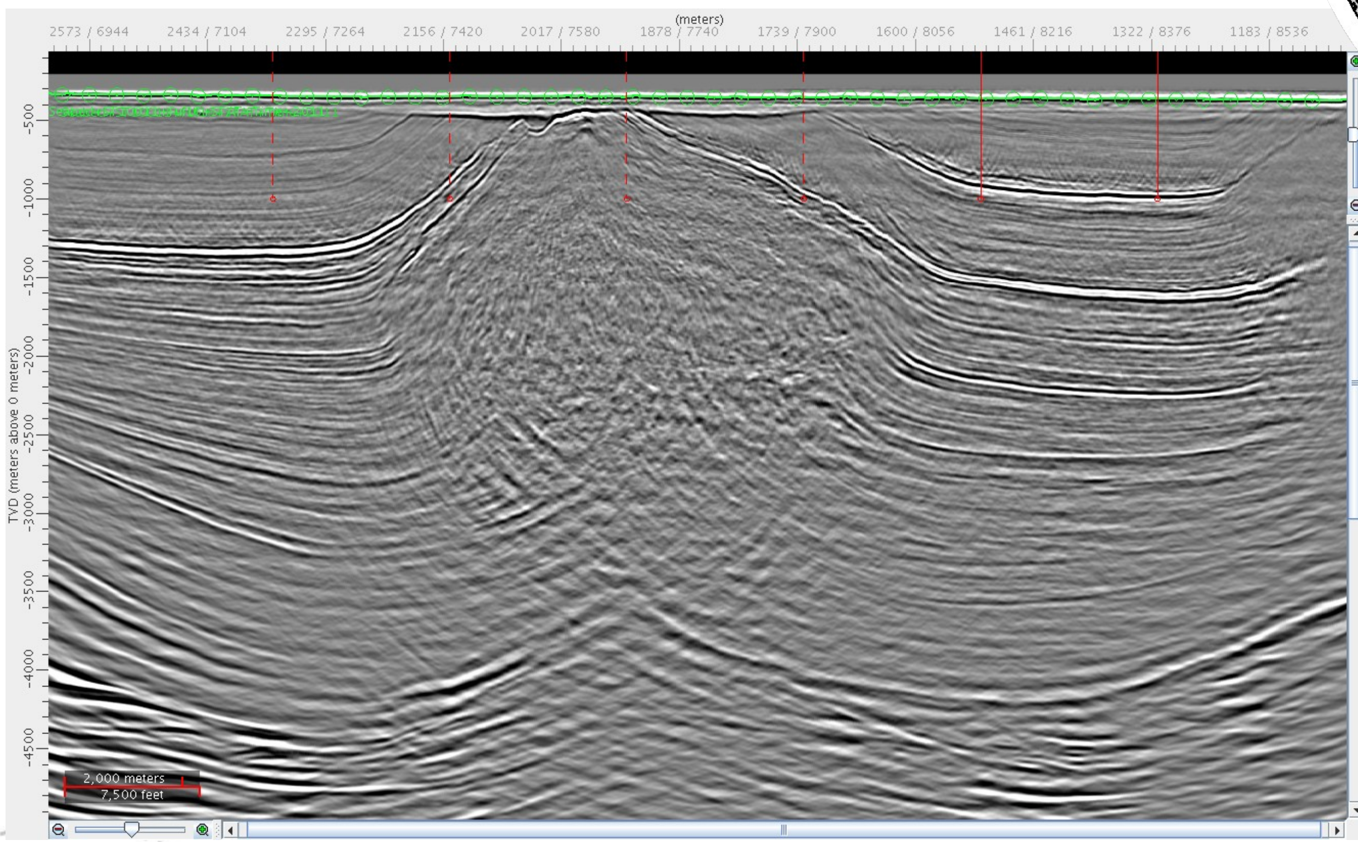
Depth slice at 2.7 – 3.7km

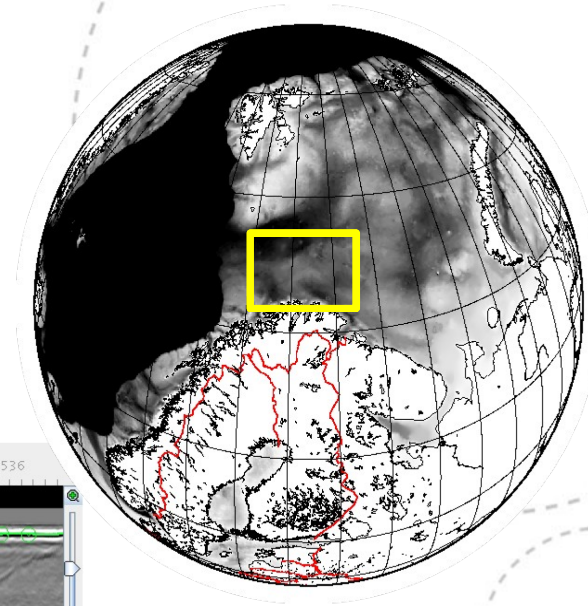




Real data example

Nordkapp basin – Jupiter salt body





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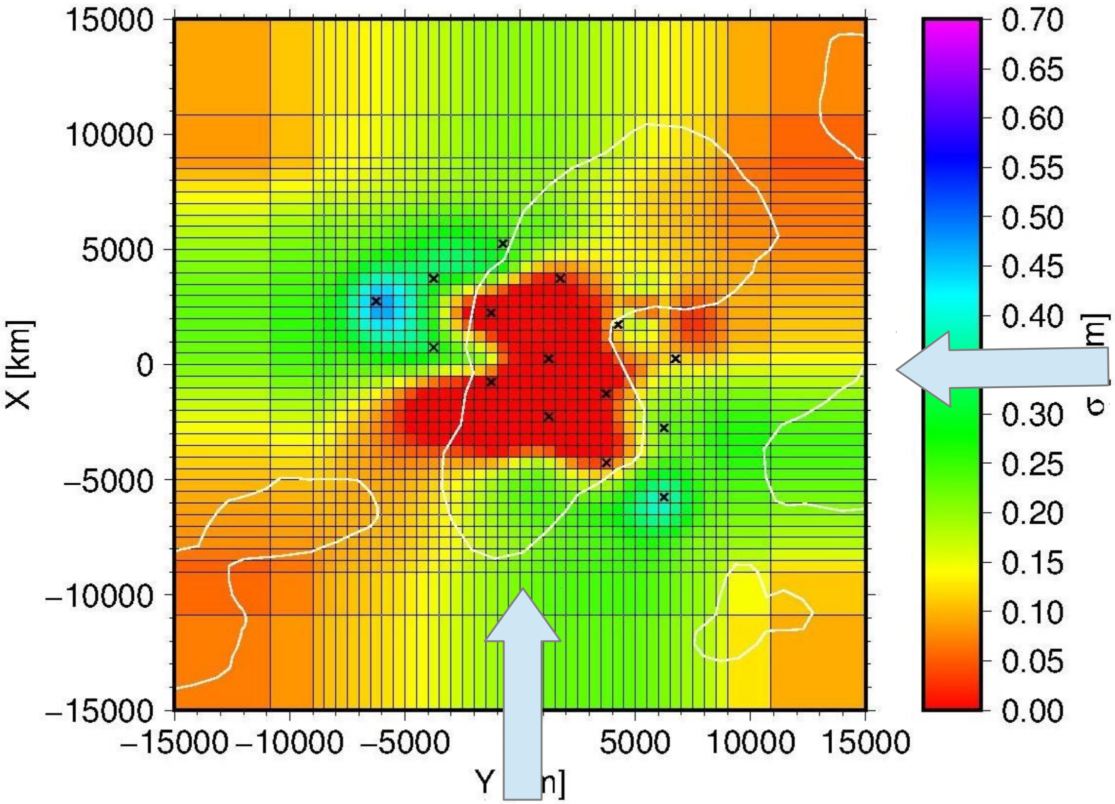
Real data example

- Model 47x47x31 cells
- Homogeneous half-space of 0.1S/m, 260m waterlayer (3.3S/m), airlayer
- 10 frequencies, Zxy, Zyx



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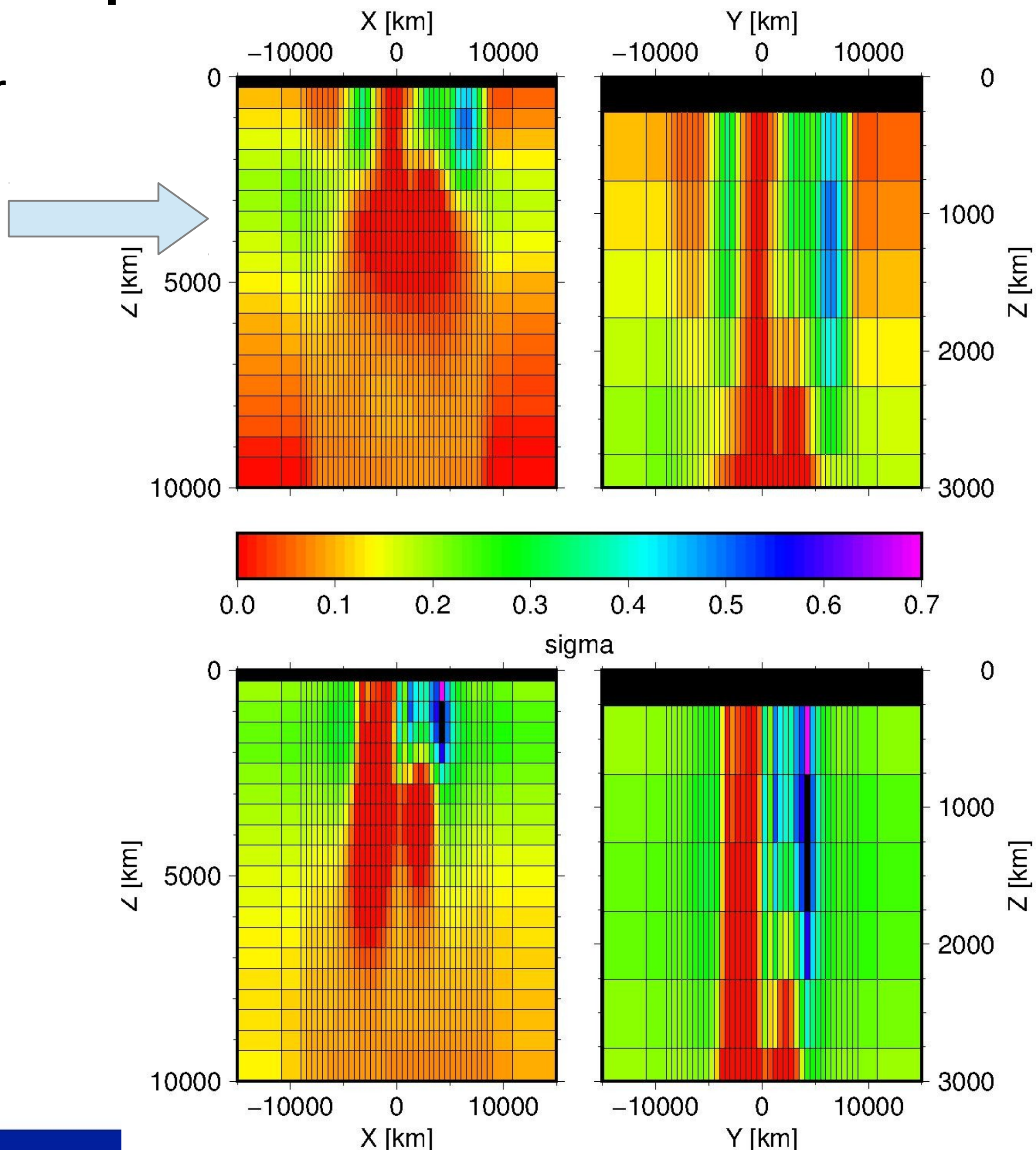
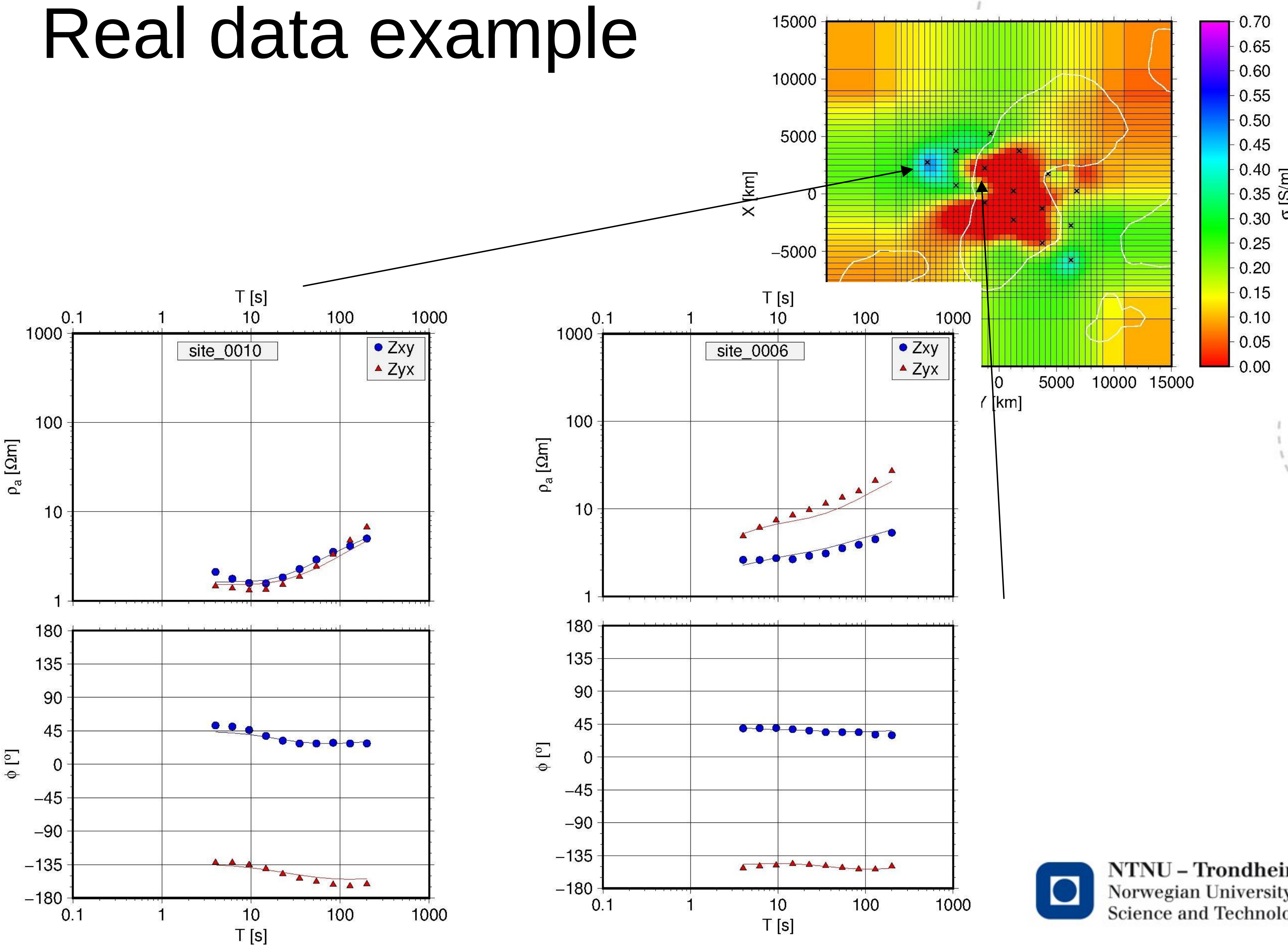
Real data example

- Vertical cut at center



Real data example



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Discussion

pro and contra of direct field inversion

- PRO
- No transfer-function estimation (less processing)
- Simpler (less non-linearity) inversion
- Faster convergence
- Better depth resolution
- CONTRA
- 1D receiver, source estimation

Conclusions

- Alternative imaging methods to help seismic interpretation
- Magnetotellurics offers low resolution but good sensitivity at wider depth range

- Gauss - Newton inversion
- good results for synthetic and real data
- Improve non-linearity and convergence with direct inversion of field components

Acknowledgements

- NFR for financial support to the ROSE project
- Statoil and their partner GDF SUEZ E&P Norge for providing data from the Nordkapp basin survey
- Ketil Hokstad and Bjørn Ursin for their supervision

Literature

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T. Wiik, L. Løseth, B. Ursin and K. Hokstad, 2011, *TIV contrast source inversion of mCSEM data*: Geophysics 76

T. Wiik, K. Hokstad, B. Ursin and Lutz Mütschard, *Joint inversion of mCSEM and MT data*: submitted to Geophysical Prospecting

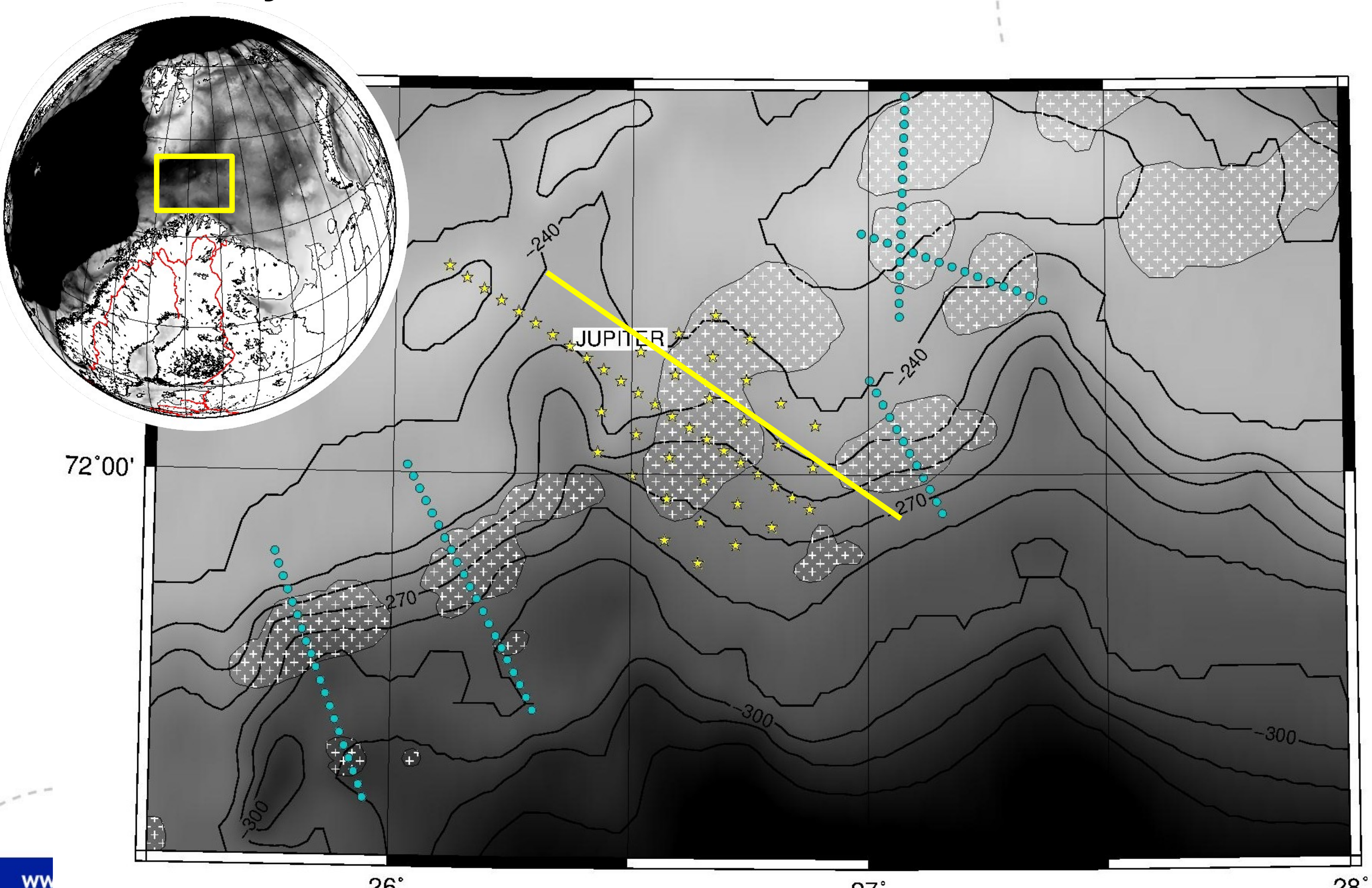
$$\rho_{a,ij}(\omega) = \frac{1}{\mu_0 \omega} \left| Z_{ij}(\omega) \right|^2 \qquad \phi_{ij}(\omega) = tan^{-1} \left(\frac{\Im \left\{ Z_{ij}(\omega) \right\}}{\Re \left\{ Z_{ij}(\omega) \right\}} \right)$$

$$\left.\begin{array}{c} Z_{xx} = Z_{yy} = 0 \\ Z_{xy} = -Z_{xy} \end{array}\right\} 1 - D \qquad \begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{pmatrix} \begin{pmatrix} B_x \\ B_y \end{pmatrix}$$

$$\left.\begin{array}{c} Z_{xx} = -Z_{yy} \\ Z_{xy} \neq Z_{xy} \end{array}\right\} 2 - D$$

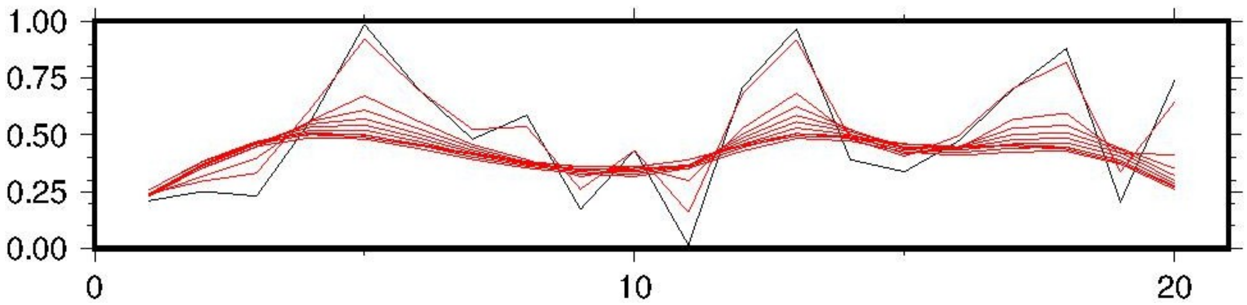
$$\left.\begin{array}{c} Z_{xx} \neq Z_{yy} \\ Z_{xy} \neq Z_{xy} \end{array}\right\} 3 - D$$

Survey Area

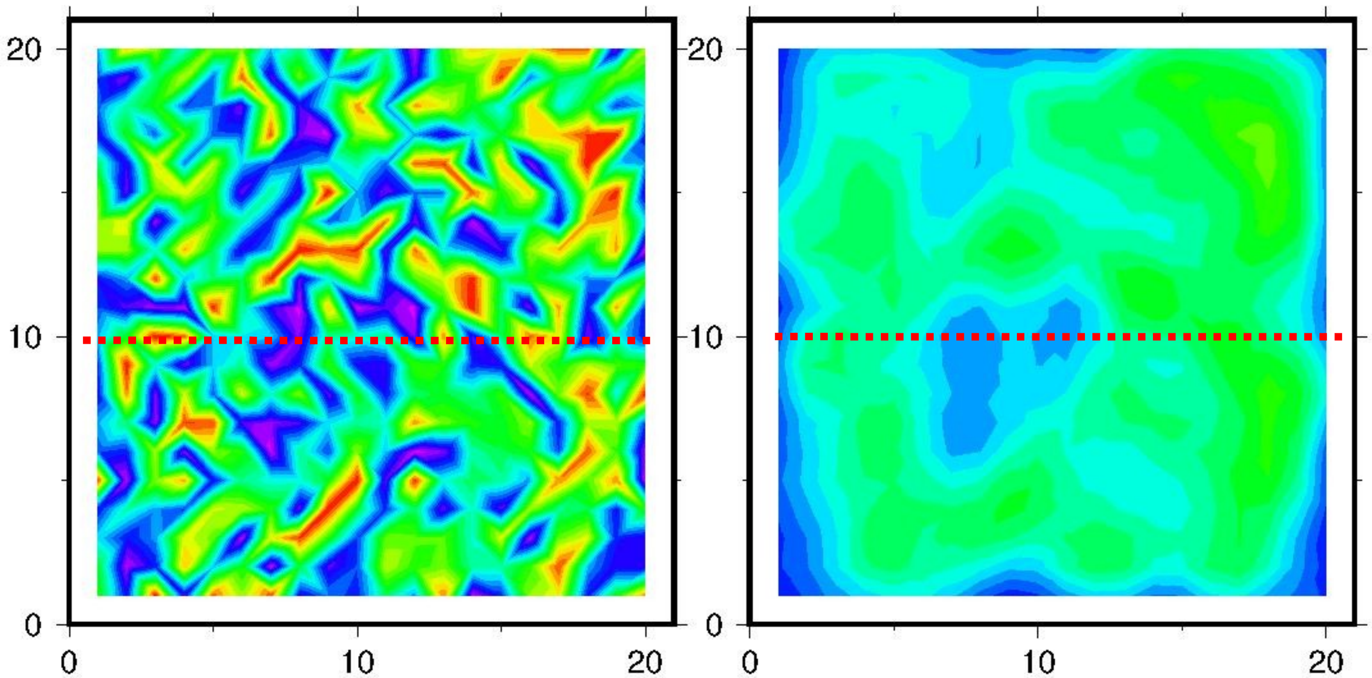


Model covariance C_m

$$u(\vec{x}_i) = \frac{1}{4\Pi\,|\Sigma|^{\frac{1}{2}}} \sum_j^N \mathrm{e}^{-\frac{(x_{x,i}-y_{x,j})^2}{4\eta x}} \cdot \mathrm{e}^{-\frac{(x_{y,i}-y_{y,j})^2}{4\eta y}} \cdot \mathrm{e}^{-\frac{(x_{z,i}-y_{z,j})^2}{4\eta z}} \cdot u(\vec{y}_j)$$



$$|\Sigma| = det \begin{vmatrix} \eta_x & 0 & 0 \\ 0 & \eta_y & 0 \\ 0 & 0 & \eta_z \end{vmatrix}$$



Motivation

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