INTERFACE FRESNEL ZONE FOR A CURVED REFLECTOR IN ANISOTROPIC MEDIA

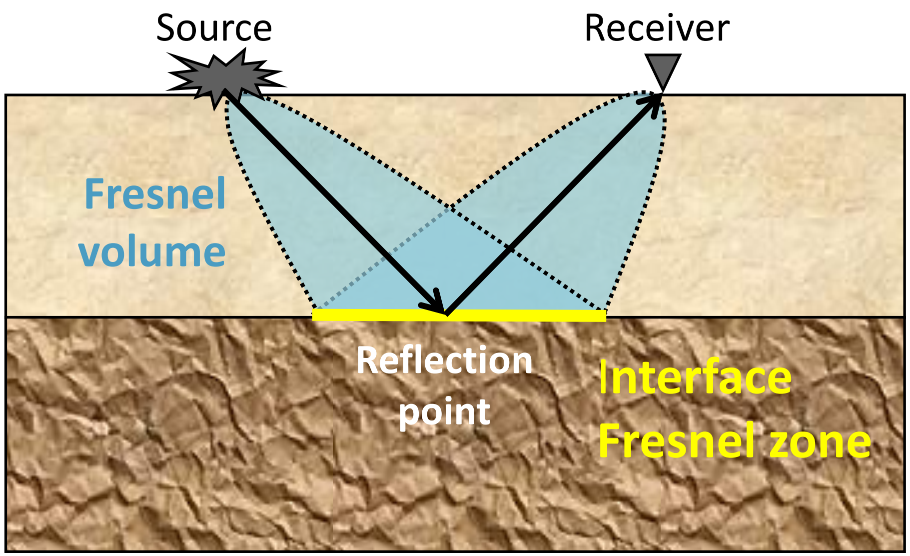
Nathalie FAVRETTO-CRISTINI, Bjørn URSIN & Paul CRISTINI

Rose Meeting – Trondheim (Norway) – May 5, 2014

The Interface Fresnel zone concept

The **Fresnel volume** concept plays an important role in seismic exploration.

(Hagedoorn 1954 Geophys. Prospect.
Kravtsov & Orlov 1990
Červený 2001)



The **Interface Fresnel zone** (IFZ)

- region of constructive reflection interference surrounding the reflection point of the geometrical ray
- largely contributes to the **formation of the reflection and transmission wavefields** at an observation point, and more specifically to their amplitude

  (Spetzler & Snieder 2004 Geophys. J. Int., Favretto-Cristini et al. 2007 Geophys. J. Int.)
- determines the **lateral resolving power for unmigrated seismic data** with which important lithological changes along a seismic profile direction may be observed

  (Sheriff 1980 Geophysics)

Context & Aim (1/2)

- Analytical and numerical modeling techniques used to determine the IFZ dimensions in various configurations *(geometric approaches, ray tracing, isochrons …)*

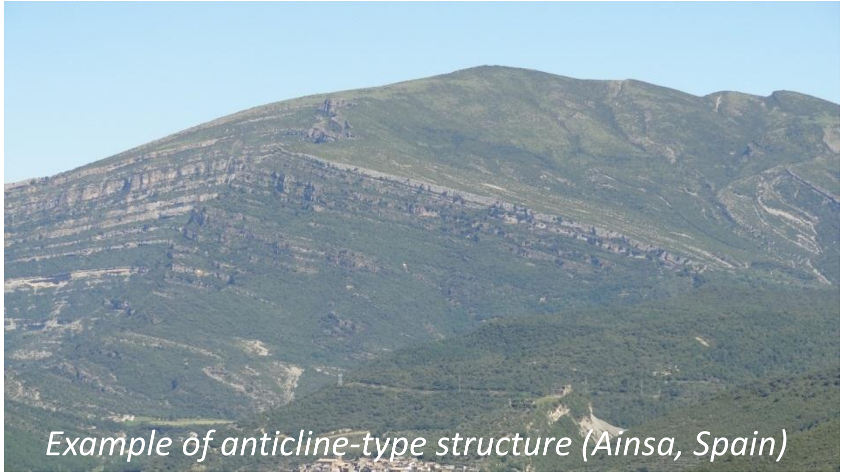
(Gelchinsky 1985 J. Geophys., Hubral et al. 1993 Geophysics
Lindsey 1989 The Leading Edge
Kvasnička & Červený 1996 Stud. Geophys. Geod.
Pulliam & Snieder 1998 Geophys. J. Int.
Červený 2001, Moser & Červený 2007 Geophys. Prospect.
Iversen 2006 Geophysics
Favretto-Cristini et al. 2009 Geophysics
Monk 2010 Geophysics
…)

- Most studies concerned with zero-offset configurations and plane reflectors

- Few works devoted to P-SV reflections and/or anisotropic media *(VSP configurations, dipping reflector…)*

(Eaton et al. 1991 Geophysics, Okoye & Uren 2000 Geophysics …)

Context & Aim (2/2)



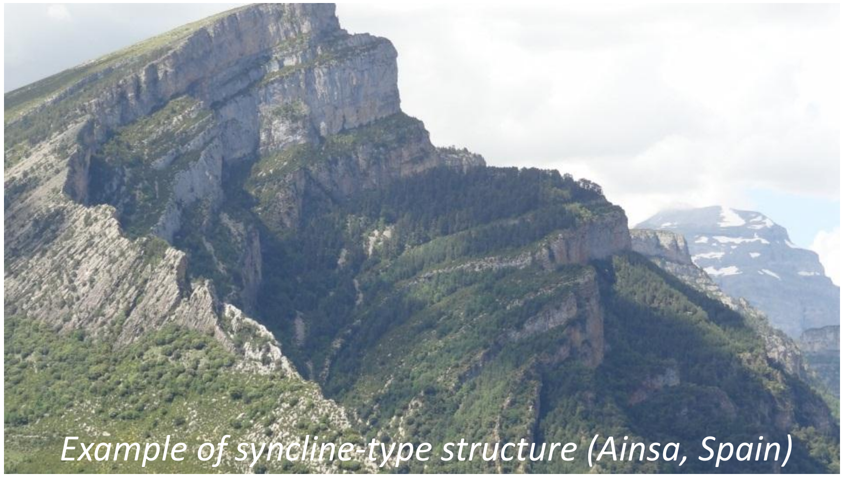
Example of anticline-type structure (Ainsa, Spain)

Address the issue of deriving **simple expressions for the IFZ for multi-offset configurations and a curved interface between general anisotropic media**

Generalization of the work reported in *Favretto-Cristini et al. 2009 Geophysics*

- Transversely Isotropic (TI) media with symmetry axis orthogonal to the curved reflector
- (Possibly converted) reflected and transmitted waves
- Non-spherically shaped interface between homogeneous anisotropic media

Ursin, Favretto-Cristini & Cristini
To appear in Geophysics in 2014



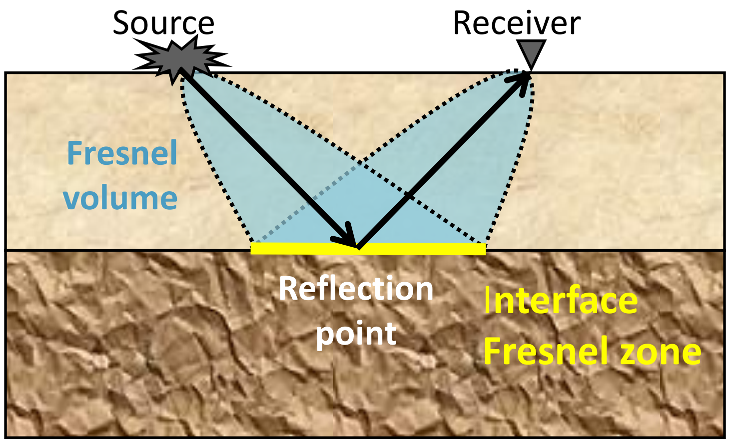
Example of syncline-type structure (Ainsa, Spain)

Fresnel volume & traveltime approximation

The FV is defined by

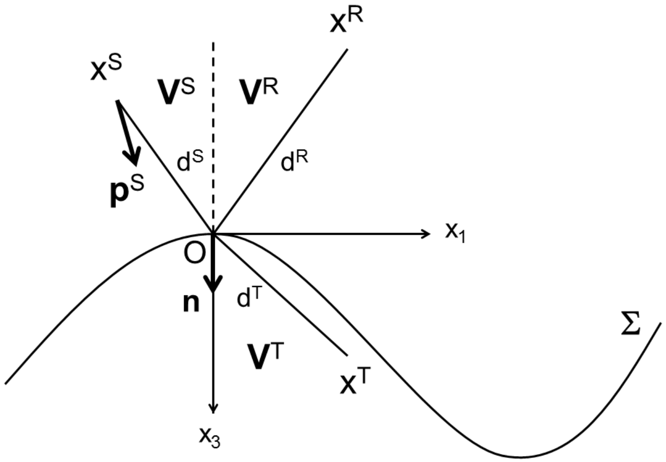
$$\left|\delta T\left(\mathbf{x}^{\mathbf{S}},\,\delta\mathbf{x}\right)+\delta T\left(\mathbf{x}^{\mathbf{U}},\,\delta\mathbf{x}\right)\right|\leq\frac{1}{2\,f}\qquad(U=R,\,T)$$

f : central frequency of the signal



with the traveltime difference δT between the source (respectively, receiver) point and the reflection point

$$\delta T\left(\mathbf{x},\,\delta\mathbf{x}\right)=T\left(\mathbf{x},\,\delta\mathbf{x}\right)-T\left(\mathbf{x},\,0\right)=\frac{1}{V+\delta V}\;\|\mathbf{x}-\delta\mathbf{x}\|-\frac{1}{V}\;\|\mathbf{x}\|$$



Neglecting changes in group velocity V (*i.e.*, **weak anisotropy**) and considering $\mathbf{X}=\dfrac{d}{V}\,\mathbf{V}$:

$$\delta T\left(\mathbf{V},\,\delta\mathbf{x}\right)\simeq\frac{1}{2\,V\,d}\;\left[\|\delta\mathbf{x}\|^2-2\,d\,\frac{\mathbf{V}\cdot\delta\mathbf{x}}{V}-\left(\frac{\mathbf{V}\cdot\delta\mathbf{x}}{V}\right)^2\right]$$

Interface Fresnel zone & traveltime approximation

A curved interface Σ may locally be approximated by a 2nd-order expression

$$x_3 = F\,(x_1\,,\,x_2) = \frac{1}{2}\,(x_1\,,\,x_2)\;\mathbf{F}\;(x_1\,,\,x_2)^t = \frac{1}{2}\,\left(F_{11}\,x_1^2 + 2\,F_{12}\,x_1\,x_2 + F_{22}\,x_2^2\right)$$

F_{11}, F_{12} and F_{22} : interface curvatures

For the FZ at the interface : $\delta x_3 = F\,(\delta x_1\,,\,\delta x_2)$

Approximation for δT (keeping terms up to 2nd order)

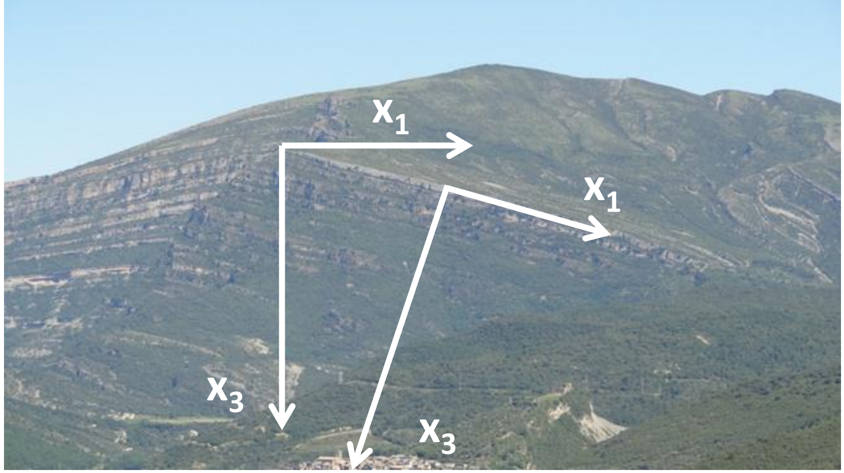
$$\delta T_\Sigma\,(\mathbf{V},\,\delta x_1,\,\delta x_2) \simeq \frac{1}{2\,V\,d}\left[\left(1-\frac{V_1^2}{V^2}\right)\delta x_1^2 + \left(1-\frac{V_2^2}{V^2}\right)\delta x_2^2 - 2\,\frac{V_1\,V_2}{V^2}\,\delta x_1\,\delta x_2\right] - \frac{V_1}{V^2}\,\delta x_1 - \frac{V_2}{V^2}\,\delta x_2 - F\,(\delta x_1\,,\,\delta x_2)\,\frac{V_3}{V^2}$$

The **IFZ** is defined by

$$\left|\delta T_\Sigma\left(\mathbf{V}^{\mathbf{S}},\,\delta x_1,\,\delta x_2\right) + \delta T_\Sigma\left(\mathbf{V}^{\mathbf{U}},\,\delta x_1,\,\delta x_2\right)\right| \le \frac{1}{2\,f} \qquad (U = R,\,T)$$

Specific case : DTI media

Dip-constrained Transversely Isotropic (DTI) media = local VTI media



- Curved interface between 2 DTI media
  - SH wave and coupled P-S waves
  - **all seismic signatures depend only on the angle between the propagation direction and the symmetry axis**

→ $V_2 = 0$, $p_2 = 0$, $x_2 = 0$

- Approximation for δT

$$\delta T_{\Sigma}\left(\mathbf{V},\,\delta x_1,\,\delta x_2\right) \simeq \frac{1}{2\,V\,d}\left[\left(1-\frac{V_1^2}{V^2}\right)\,\delta x_1^2+\delta x_2^2\right]-\frac{V_1}{V^2}\,\delta x_1-F\left(\delta x_1\,,\,\delta x_2\right)\,\frac{V_3}{V^2}$$

where F defines the interface curvatures, $d=\left[(x_1)^2+(x_3)^2\right]^{\frac{1}{2}}$ and $V=\left(V_1^2+V_3^2\right)^{\frac{1}{2}}$

Group velocity components in DTI media (1/2)

- Explicit expressions for V_1 and V_3 for P-SV waves in *Tsvankin 2001*, *Chapman 2004*

  … preferable to use them in actual modeling, inversion and processing algorithms

- Goal : **analytic insight into the effects of anisotropy on the IFZ**

Approximate dispersion relation *(Pestana et al. 2012 J. Geophys. Eng.)*

for P-waves :
$$\omega^2 = v_{P0}^2 \left[(1+2\epsilon)\ k_1^2 + k_3^2\right] - 2\,\frac{v_{P0}^2\ (\epsilon-\delta)\ k_1^2\, k_3^2}{k_3^2 + \xi\, k_1^2}$$

for SV-waves :
$$\omega^2 = v_{S0}^2\ \left(k_1^2 + k_3^2\right) + 2\,\frac{v_{P0}^2\ (\epsilon-\delta)\ k_1^2\, k_3^2}{k_3^2 + \xi\, k_1^2}$$

with $k_{1,3} = \omega\, p_{1,3}$

where
$$\begin{cases} v_{P0} = \left(\frac{c_{33}}{\rho}\right)^{1/2} \\ v_{S0} = \left(\frac{c_{44}}{\rho}\right)^{1/2} \\ \epsilon = \frac{c_{11}-c_{33}}{2\,c_{33}} \\ \delta = \frac{(c_{13}+c_{44})^2-(c_{33}-c_{44})^2}{2\,c_{33}\,(c_{33}-c_{44})} \end{cases}$$
(Thomsen 1986 Geophysics)

and
$$\xi = 1 + 2\,\epsilon\,\frac{v_{P0}^2}{v_{P0}^2 - v_{S0}^2}$$

Group velocity components in DTI media (2/2)

From $V_i = -\frac{\partial\Omega/\partial k_i}{\partial\Omega/\partial\omega}$, we get :

for P-waves

$$\begin{cases} V_{P1} = v_{P0}^2\, p_1\ [(1+2\,\epsilon) - 2\,(\epsilon-\delta)\,\chi] \\ V_{P3} = v_{P0}^2\, p_3\ [1 - 2\,(\epsilon-\delta)\,\chi'] \end{cases}$$

for SV-waves

$$\begin{cases} V_{S1} = v_{S0}^2\, p_1\ [1 + 2\,\sigma\,\chi] \\ V_{S3} = v_{S0}^2\, p_3\ [1 + 2\,\sigma\,\chi'] \end{cases}$$

where $\begin{cases} \chi = \frac{p_3^4}{\left(p_3^2+\xi\, p_1^2\right)^2} \\ \chi' = \frac{\xi\, p_1^4}{\left(p_3^2+\xi\, p_1^2\right)^2} \end{cases}$ and $\sigma = \frac{v_{P0}^2}{v_{S0}^2}(\epsilon-\delta)$

The difference $\varepsilon - \delta$

- governs both P- and SV-wave propagation for non-vertical wave propagation *(Pestana et al. 2012 J. Geophys. Eng.)*
- controls the shape and the size of the IFZ for both P-P and P-SV reflections

N.B. : The parameter $\eta = (\varepsilon - \delta) / (1 + 2\delta)$, introduced by Alkhalifah & Tsvankin (1995 Geophysics), governs such P-wave signatures (Tsvankin 2001), as dip-dependent NMO velocity, nonhyperbolic reflection moveout, time-migration impulse response, the point-source radiation pattern.

Numerical results

- Focus on the wave reflection at a curved interface
- Shape and size of the IFZ for P-P and P-SV reflections for various anisotropic parameters, incidence angles, and interface curvatures
- Incidence medium :

Brine-saturated shales *(Wang 2002 Geophysics)*

$$\begin{cases} \rho = 2\,597 \text{ kg/m}^3 \\ \upsilon_{P0} = 4\,409 \text{ m/s} \\ \upsilon_{S0} = 2\,688 \text{ m/s} \\ \varepsilon = 0.110 \text{ and } \delta = -0.043 \qquad (\varepsilon - \delta = 0.153) \end{cases}$$

- S and R located at 3000 m from the plane tangent to the interface at the reflection point
- Frequency of the incident P-wave : 25 Hz
- 3 kinds of curved reflectors :
  - Anticline (R_1 = + 5000 m, R_2 = + 4000 m)
  - Syncline (R_1 = - 5000 m, R_2 = - 4000 m)
  - Saddle (R_1 = - 5000 m, R_2 = + 4000 m)

$$\begin{cases} F_{11} = \frac{1}{R_1} cos^2\phi + \frac{1}{R_2} sin^2\phi \\ F_{12} = \left(\frac{1}{R_1} - \frac{1}{R_2}\right) cos\phi\, sin\phi \\ F_{22} = \frac{1}{R_1} sin^2\phi + \frac{1}{R_2} cos^2\phi \end{cases}$$

(Stavroudis 1972)

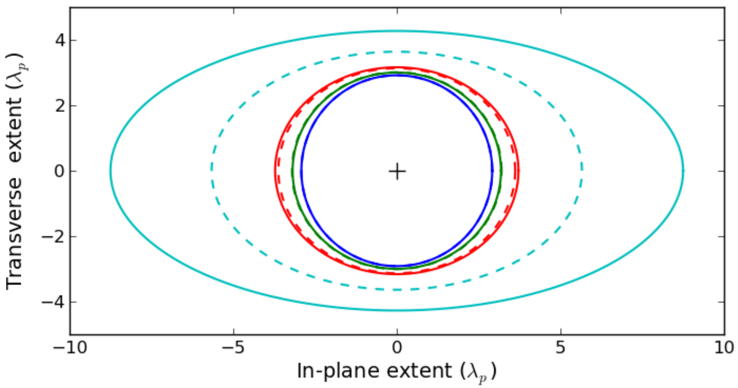
Interface Fresnel zone for PP reflected waves (1/2)

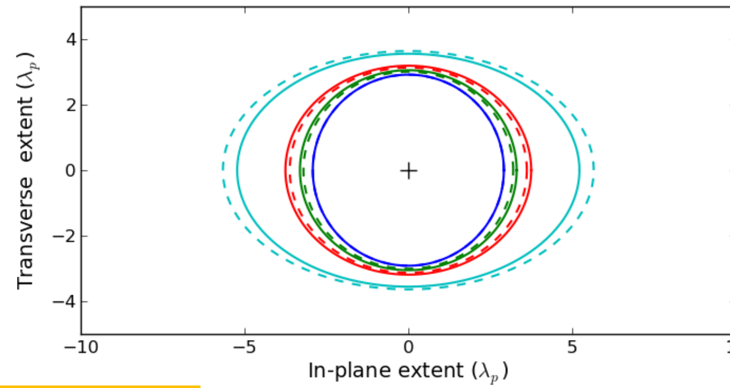
Variation in shape and size of the IFZ in anisotropic (solid line) and isotropic (dashed line) media as a function of incidence angle θ : $\theta = 0$ (dark blue), $\theta = 20°$ (green), $\theta = 35°$ (red), and $\theta = 50°$ (light blue).
The size of the IFZ is normalized with respect to the incident P-wavelength for $\theta = 0$.

$\varepsilon - \delta > 0$

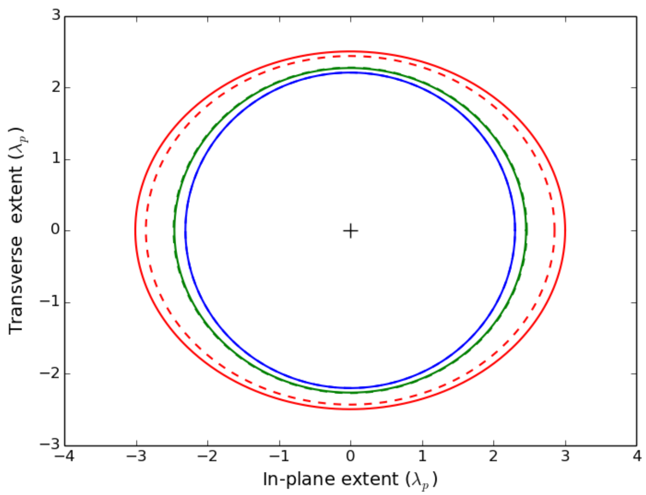
Plane reflector

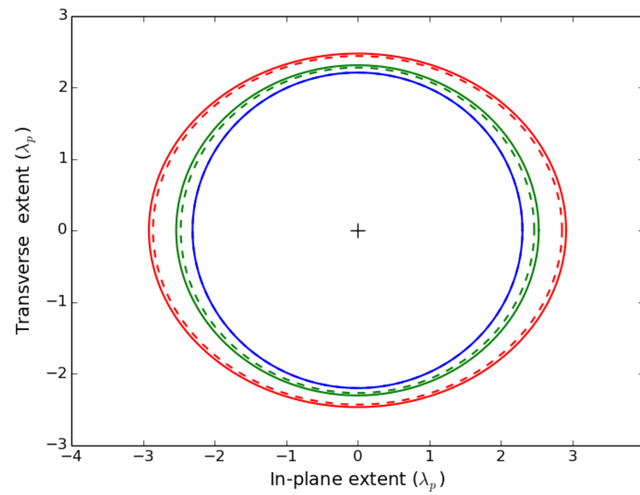
$\varepsilon - \delta < 0$





Anticline-type reflector

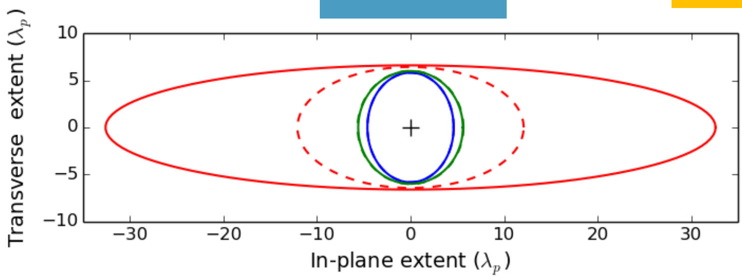




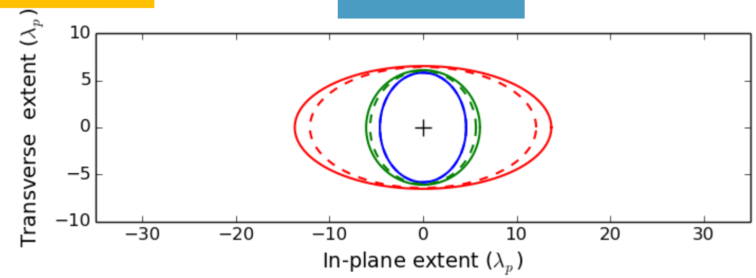
Interface Fresnel zone for PP reflected waves (2/2)

Syncline-type reflector

$\varepsilon - \delta > 0$



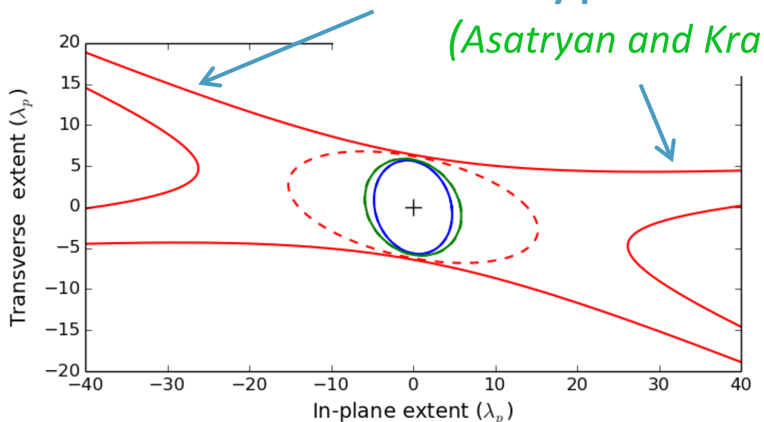
$\varepsilon - \delta < 0$

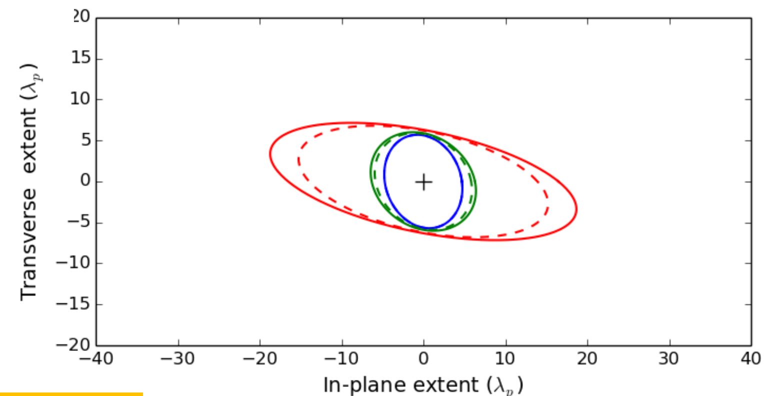


Rotation of the principal curvature axes of the interface by 20° with respect to the x_1-axis

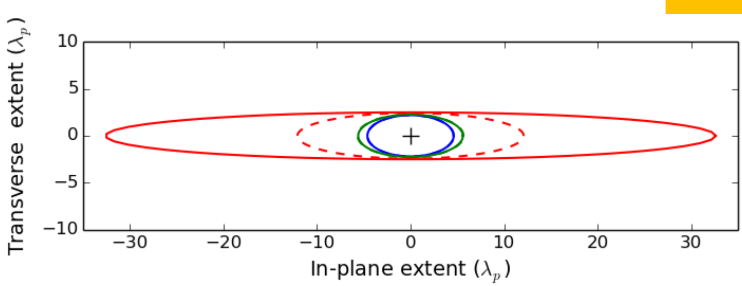
Stationary points of hyperbolic type

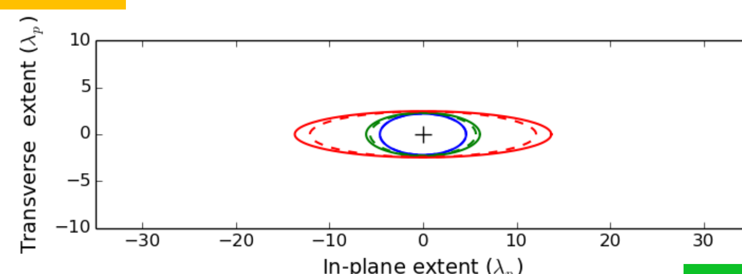
(Asatryan and Kravtsov 1988)





Saddle-type reflector





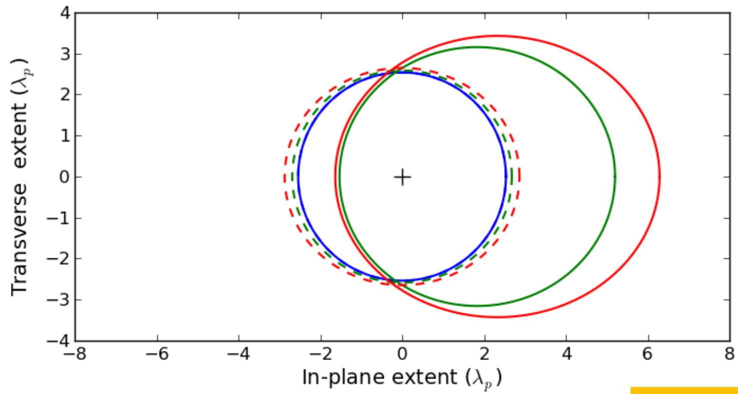
Interface Fresnel zone for PS reflected waves (1/2)

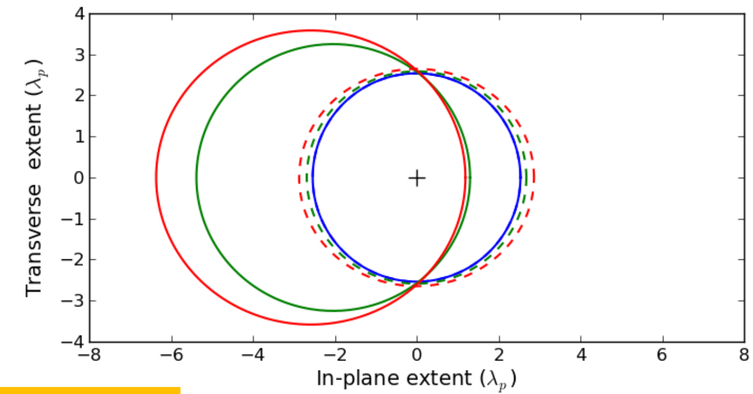
Variation in shape and size of the IFZ in anisotropic (solid line) and isotropic (dashed line) media as a function of incidence angle θ : $\theta = 0$ (dark blue), $\theta = 20°$ (green), and $\theta = 35°$ (red).
The size of the IFZ is normalized with respect to the incident P-wavelength for $\theta = 0$.

$\varepsilon - \delta > 0$

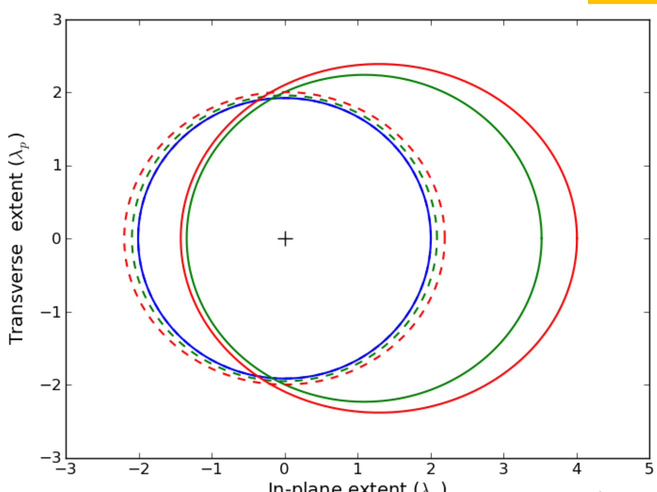
Plane reflector

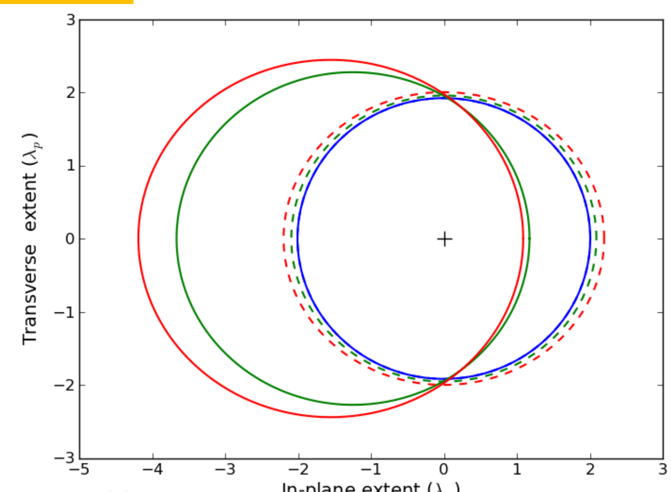
$\varepsilon - \delta < 0$





Anticline-type reflector



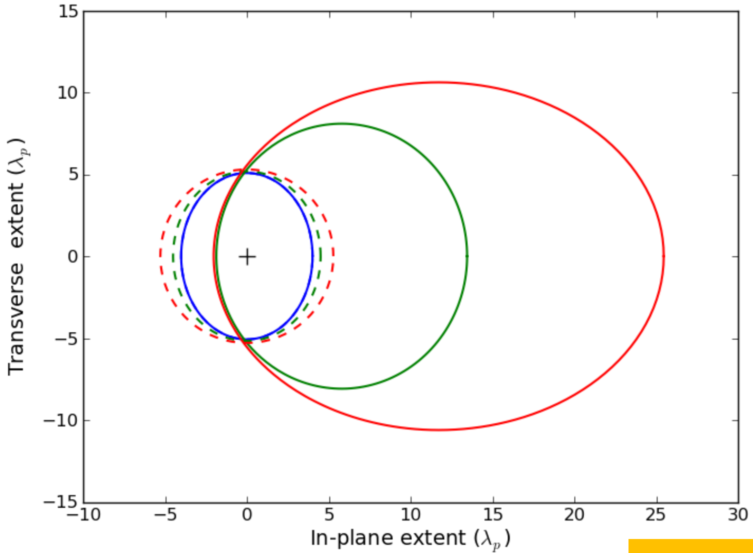


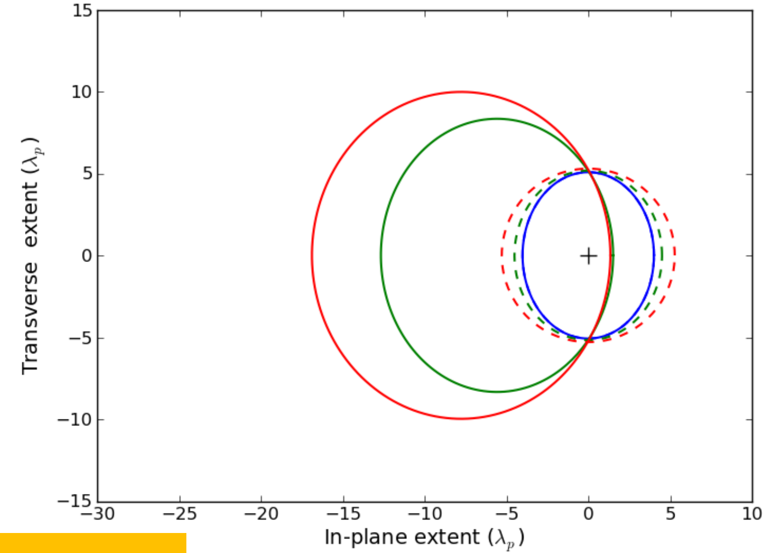
Interface Fresnel zone for PS reflected waves (2/2)

ε – δ > 0

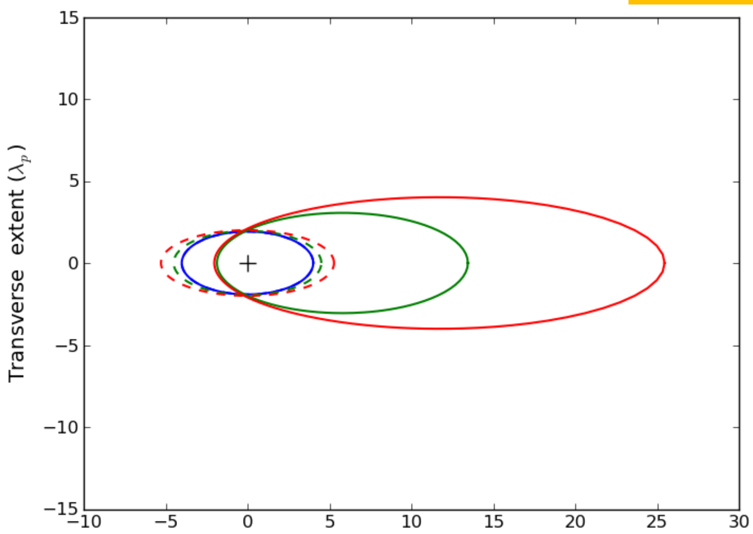
Syncline-type reflector

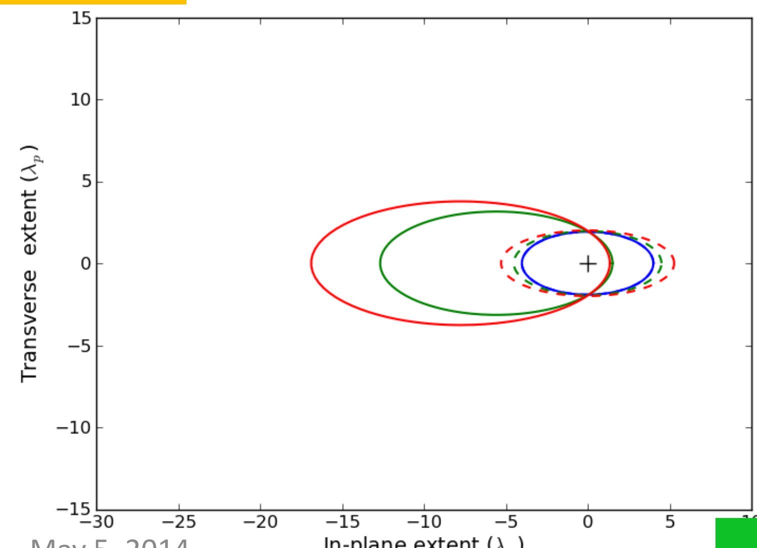
ε – δ < 0





Saddle-type reflector





- Derivation of analytic expressions, based on traveltime approximations, to evaluate the shape and the size of the IFZ for P-P and P-S reflected or transmitted waves by a curved reflector between two homogeneous anisotropic media

- Focus on local VTI media

- The size and the shape of the IFZ for reflected waves

  - predominantly dependent on the curvatures of the isochrons together with the curvatures of the interface.

  - exhibit large variation with interface curvature and incidence angle. See the IFZ for the syncline- and the saddle-type reflectors.

  - effects much **more pronounced for positive values** of $\varepsilon - \delta$ ===> **The difference between the Thomsen parameters ε and δ also controls the shape and size of the IFZ for both P-P and P-S reflections.**

- The spatial resolution of unmigrated seismic data in anisotropic media with curved interface is significantly different from that determined for the same configuration for isotropic models and a planar interface.

Acknowledgements

cnrs
dépasser les frontières
INSIS

INSTITUT CARNOT
STAR

Rock
SEismic
Project

Statoil

The Research Council
of Norway