Effect of anelastic sands layers on R/T responses of a periodically layered medium

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Idea

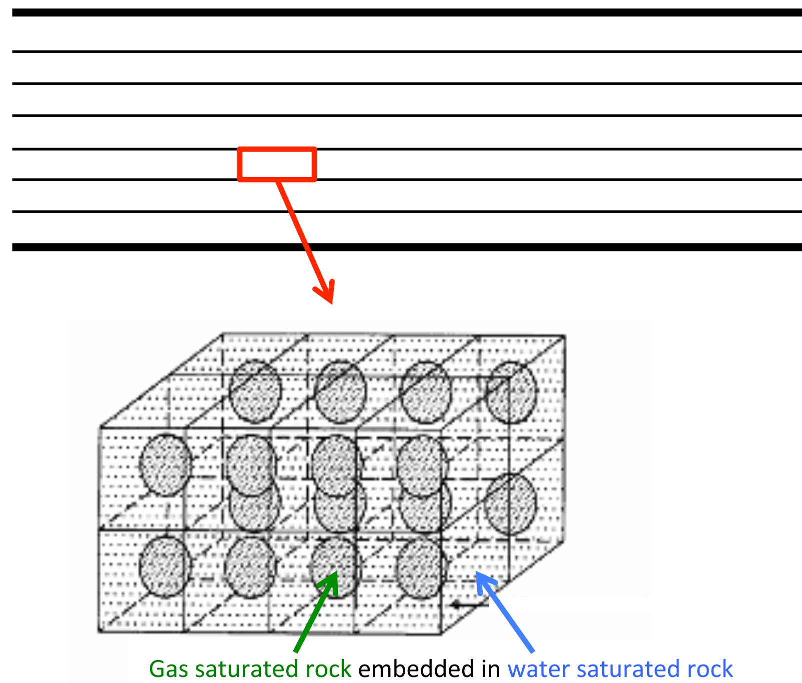
- Paper of Alexey Stovas and Bjørn Ursin on: *Equivalent time-average and effective medium for periodic layers*, **Geophysical Prospecting**, 2007

➔ The reflection and transmission (R/T) responses from a layered periodic medium are frequency-dependent. This dispersion and attenuation of the effective medium is controlled by the layering and the medium parameters.

- What could happen if we mix the frequency dependence due to the layering with the one coming from patchy saturated porous media ?

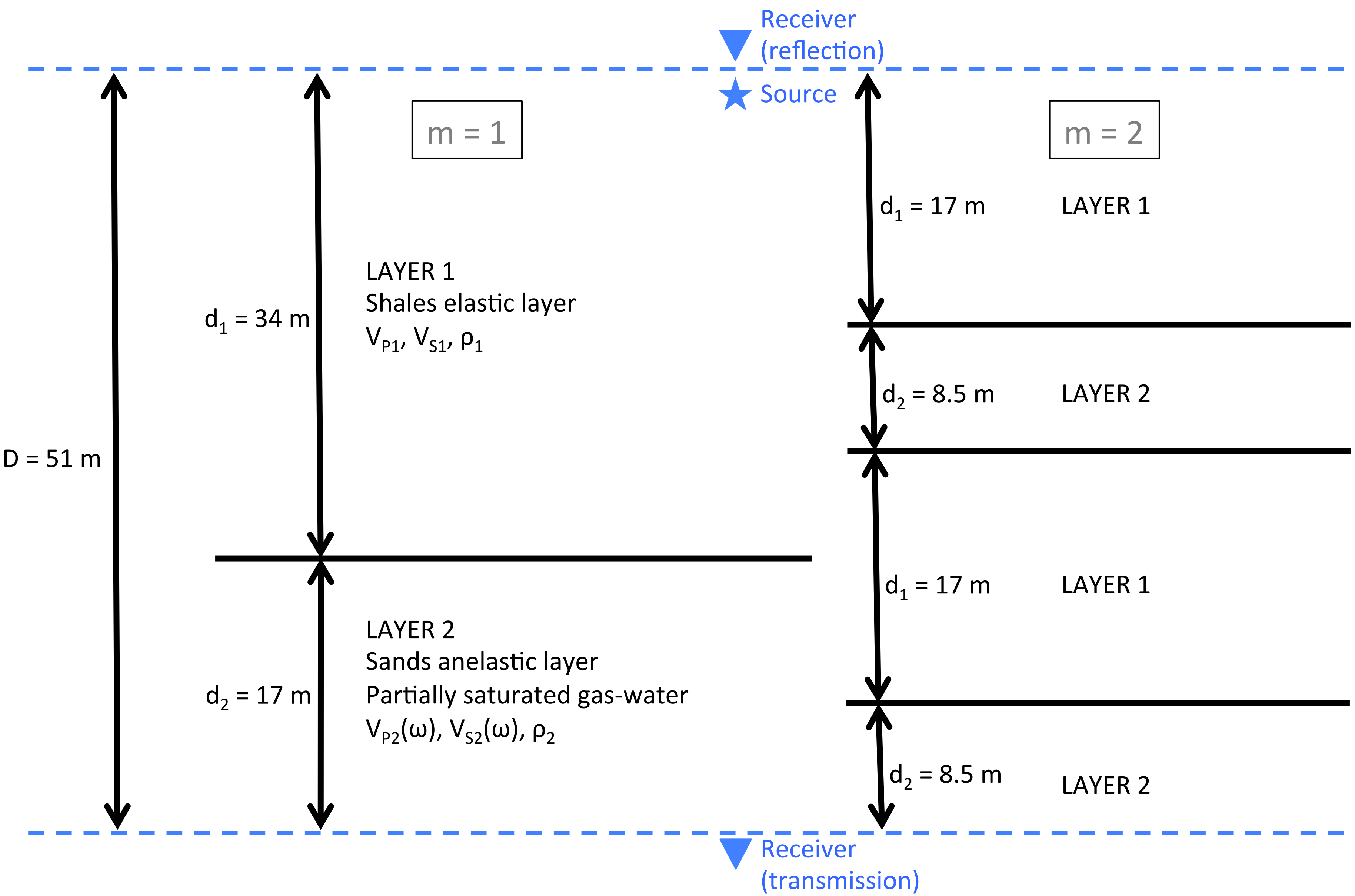
Two scales of periodicity

- Layering ➔ macroscale (wavelength)
- Patchy saturation ➔ mesoscale (gas inclusions)



➔ Both can affect wave propagation (dispersion, attenuation…) at seismic frequencies (1 to 1000 Hz)

Layout



Receiver
(reflection)
Source
m = 1
m = 2
D = 51 m
d1 = 34 m
LAYER 1
Shales elastic layer
VP1, VS1, ρ1
d2 = 17 m
LAYER 2
Sands anelastic layer
Partially saturated gas-water
VP2(ω), VS2(ω), ρ2
d1 = 17 m
LAYER 1
d2 = 8.5 m
LAYER 2
d1 = 17 m
LAYER 1
d2 = 8.5 m
LAYER 2
Receiver
(transmission)

Model

Layers 1 = shales
V_P = 2550 m/s
V_S = 1450 m/s
ρ = 2300 kg/m^3

Elastic

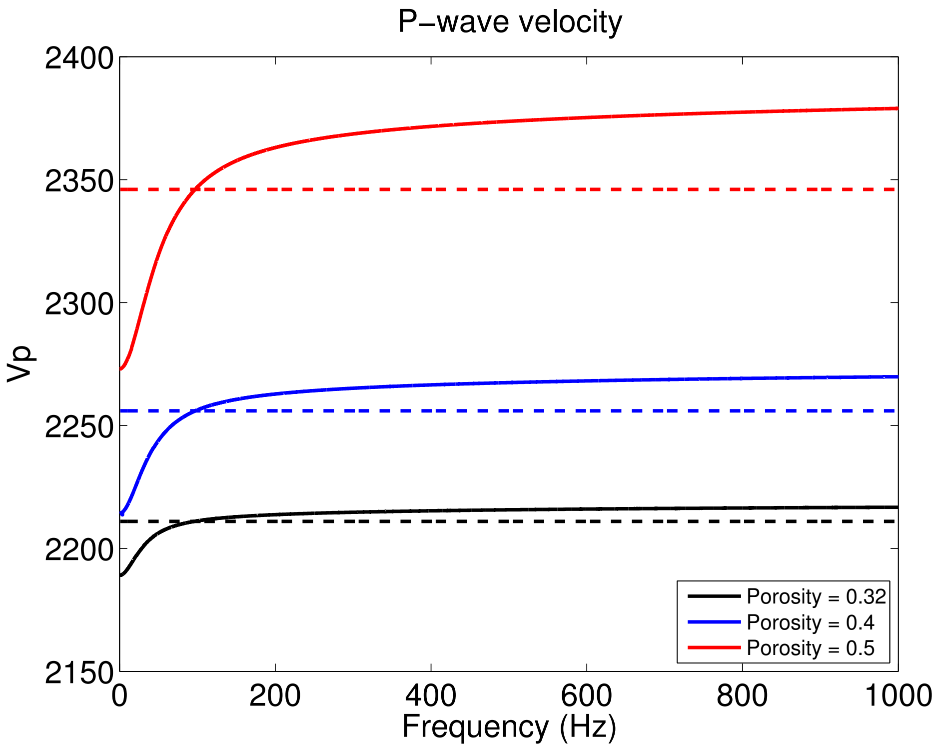
Layers 2 = partially saturated sandstones
Gas saturation = 10 %

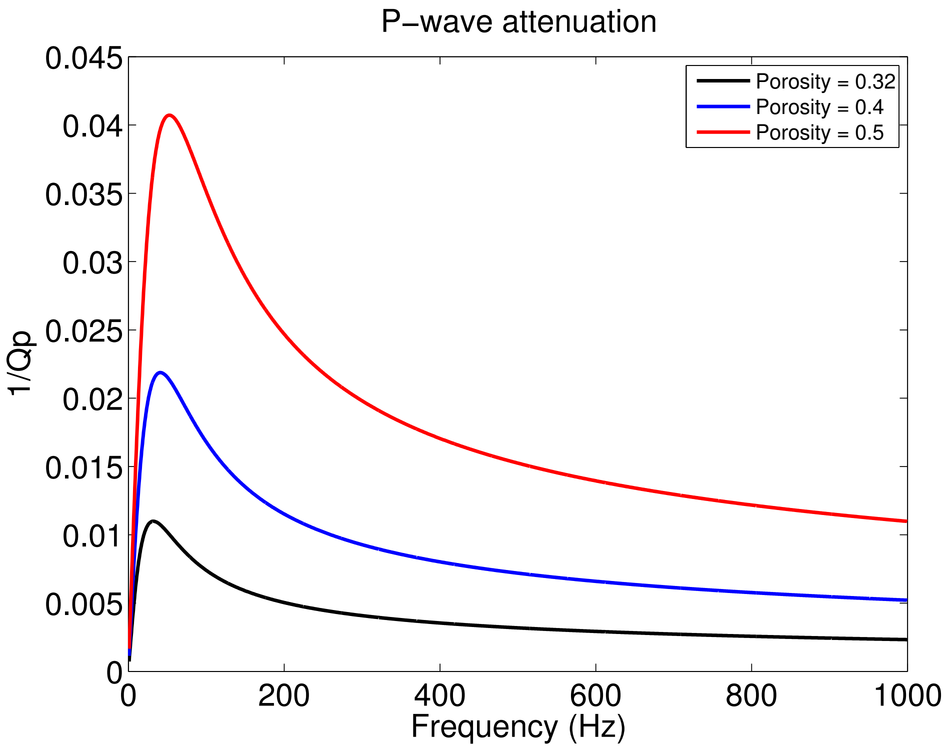
Patchy saturation: ANELASTIC
OR
Equivalent medium: ELASTIC

3 porosity models (= 3 levels of dispersion)

ϕ = 32 %	ϕ = 40 %	ϕ = 50 %
V_P = 2211 m/s	V_P = 2256 m/s	V_P = 2346 m/s
V_S = 1227 m/s	V_S = 1282 m/s	V_S = 1358 m/s
ρ = 2144 kg/m^3	ρ = 2007 kg/m^3	ρ = 1837 kg/m^3

P-wave dispersion and attenuation





$V_P(\omega)$ $1 / Q_P(\omega)$

V_P dispersion between 1.5 % and 5 %

Q_P max between 25 and 100

Computation of R/T responses

Propagator matrix for 1 cycle (2 layers):

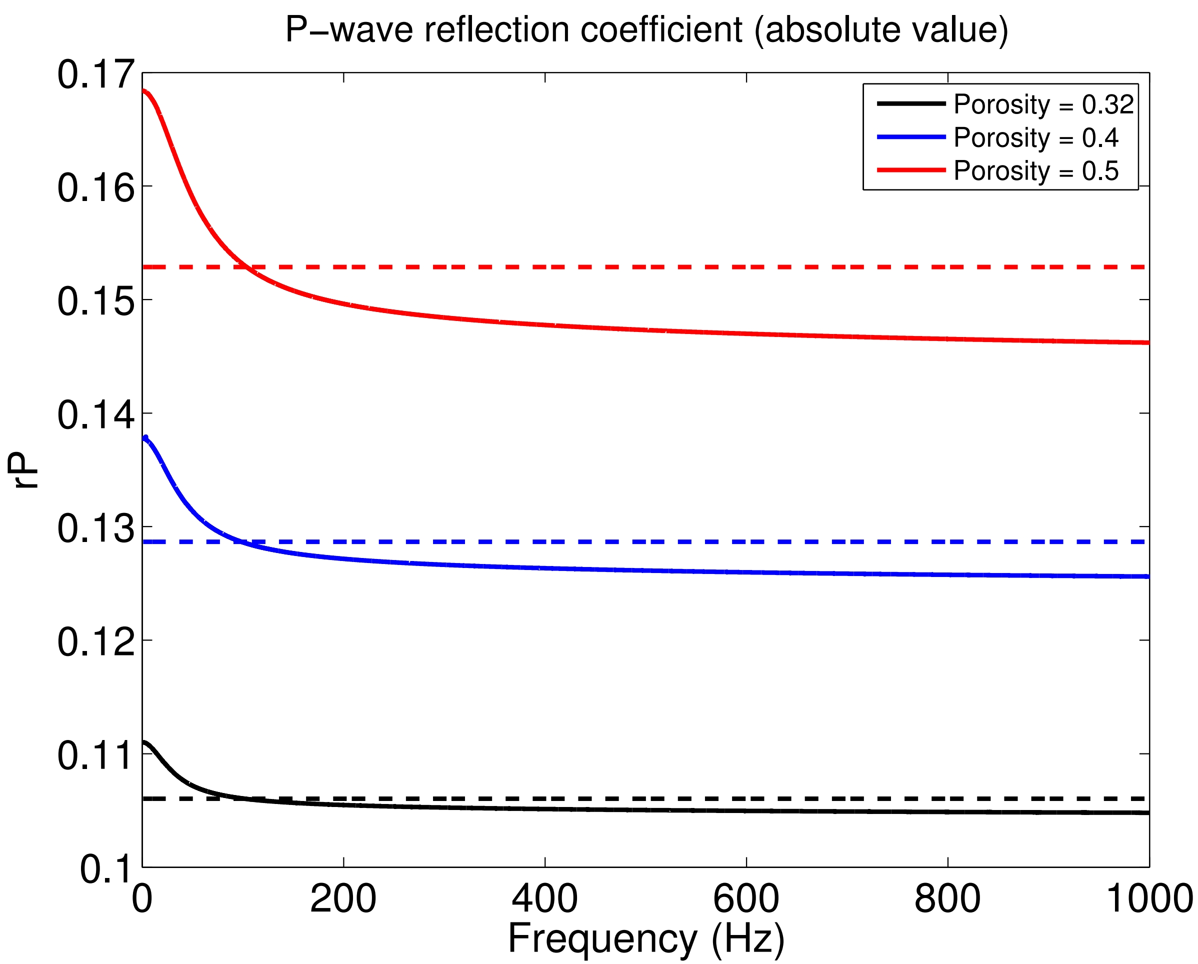
$$\mathbf{S}_k(\omega) = \frac{1}{1-r^2(\omega)} \begin{pmatrix} e^{i\theta_1} & 0 \\ 0 & e^{-i\theta_1} \end{pmatrix} \begin{pmatrix} 1 & r(\omega) \\ r(\omega) & 1 \end{pmatrix} \begin{pmatrix} e^{i\theta_2(\omega)} & 0 \\ 0 & e^{-i\theta_2(\omega)} \end{pmatrix} \begin{pmatrix} 1 & -r(\omega) \\ -r(\omega) & 1 \end{pmatrix}$$

$r(\omega)$ = reflection coefficient
t_1 and $t_2(\omega)$ = traveltimes
θ_1 and $\theta_2(\omega)$ = phase factors

$$\begin{aligned} r(\omega) &= \frac{\rho_2 V_2(\omega) - \rho_1 V_1}{\rho_2 V_2(\omega) + \rho_1 V_1} \ , \\ \theta_1 &= \frac{2\pi f d_1}{V_1} = 2\pi f t_1 \ , \\ \theta_2(\omega) &= \frac{2\pi f d_2}{V_2(\omega)} = 2\pi f t_2(\omega) \ , \end{aligned}$$

Equations valid for P- or S-waves independently

Reflection coefficients



$r_P(\omega)$

Computation of R/T responses

Propagator matrix for 1 cycle (2 layers):

$$\mathbf{S}_k(\omega) = \frac{1}{1-r^2(\omega)} \begin{pmatrix} e^{i\theta_1} & 0 \\ 0 & e^{-i\theta_1} \end{pmatrix} \begin{pmatrix} 1 & r(\omega) \\ r(\omega) & 1 \end{pmatrix} \begin{pmatrix} e^{i\theta_2(\omega)} & 0 \\ 0 & e^{-i\theta_2(\omega)} \end{pmatrix} \begin{pmatrix} 1 & -r(\omega) \\ -r(\omega) & 1 \end{pmatrix}$$

Total response for m cycles:

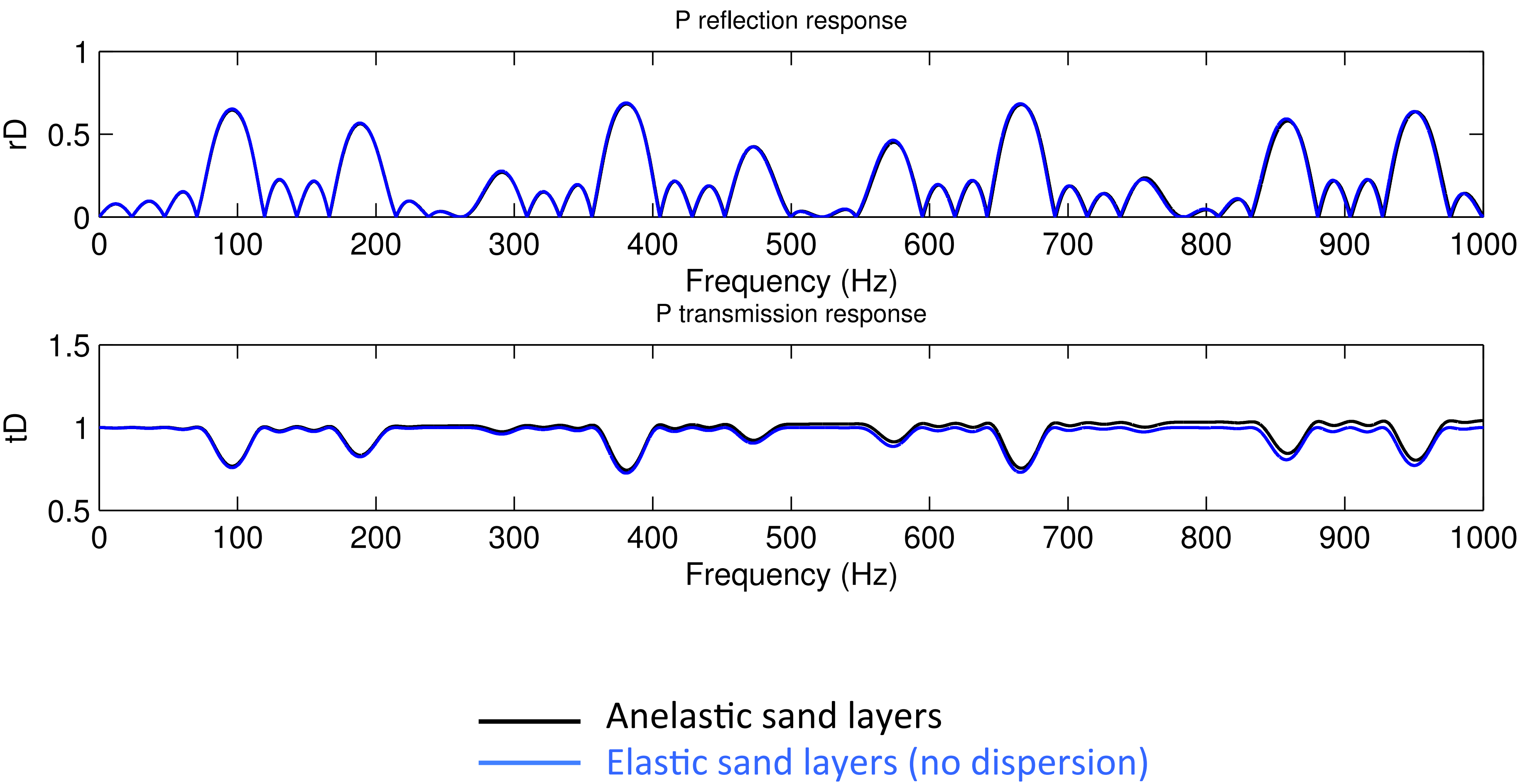
$$\mathbf{S}_m(\omega) = \prod_{k=1}^{m} \mathbf{S}_k = \begin{pmatrix} a_m(\omega) & b_m(\omega) \\ c_m(\omega) & d_m(\omega) \end{pmatrix}$$

Transmission $t_D(\omega)$ and reflection $r_D(\omega)$ responses for a downgoing wave:

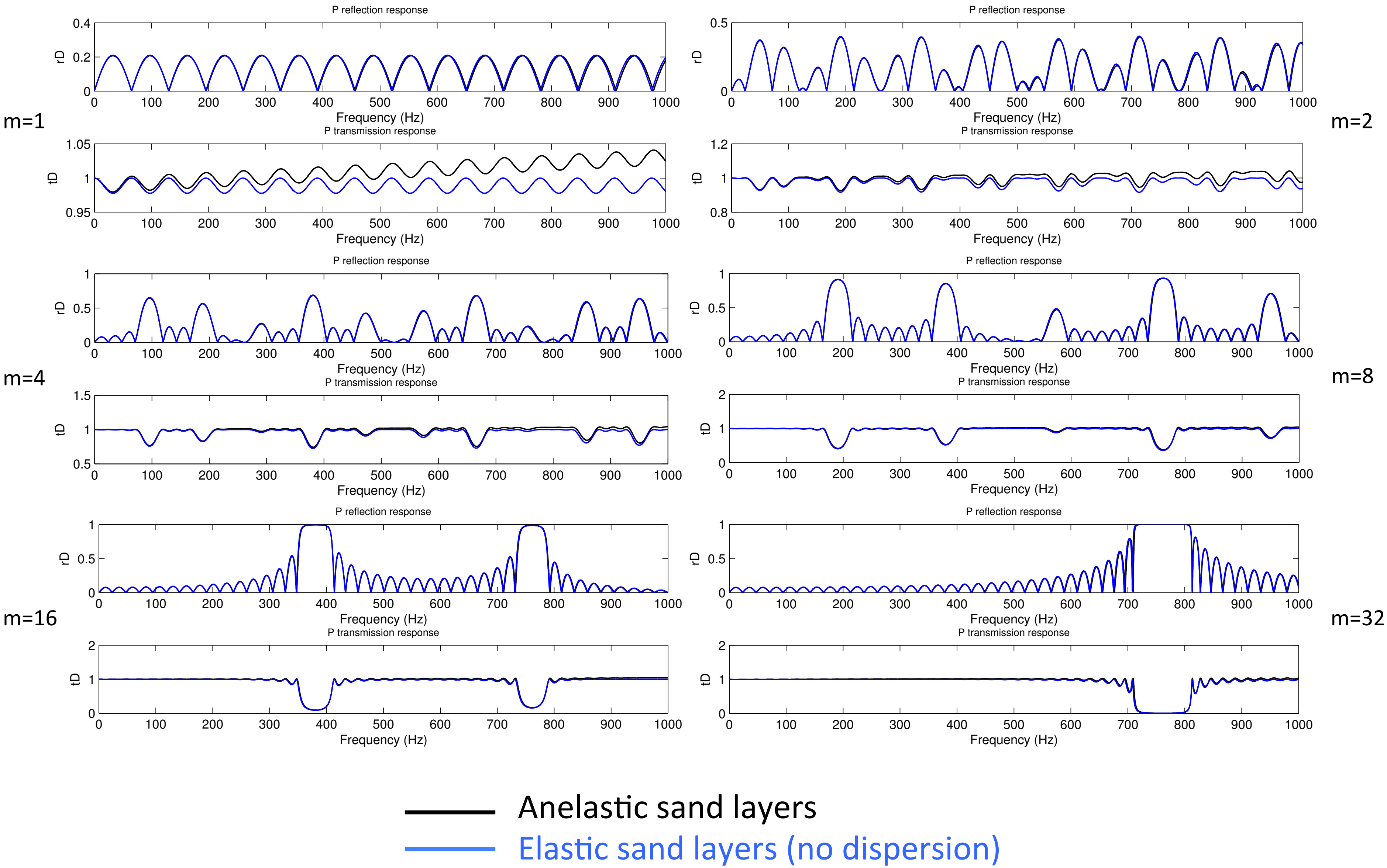
$$t_D(\omega) = \frac{1}{d_m(\omega)},$$

$$r_D(\omega) = \frac{b_m(\omega)}{d_m(\omega)}.$$

R/T responses for m=4 and ϕ=32%



R/T responses (ϕ=32%)

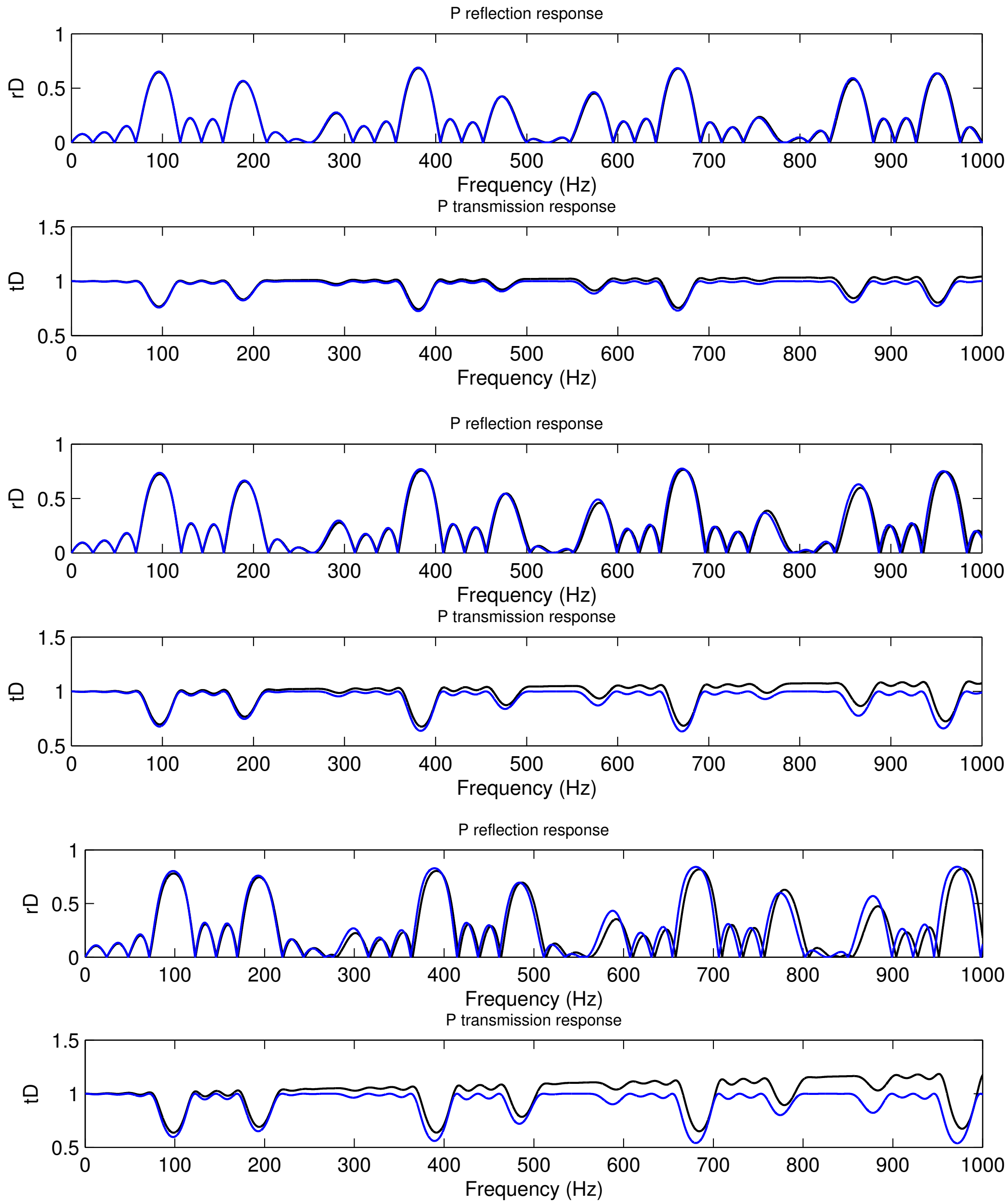


R/T responses (m=4)

ϕ=32%

ϕ=40%

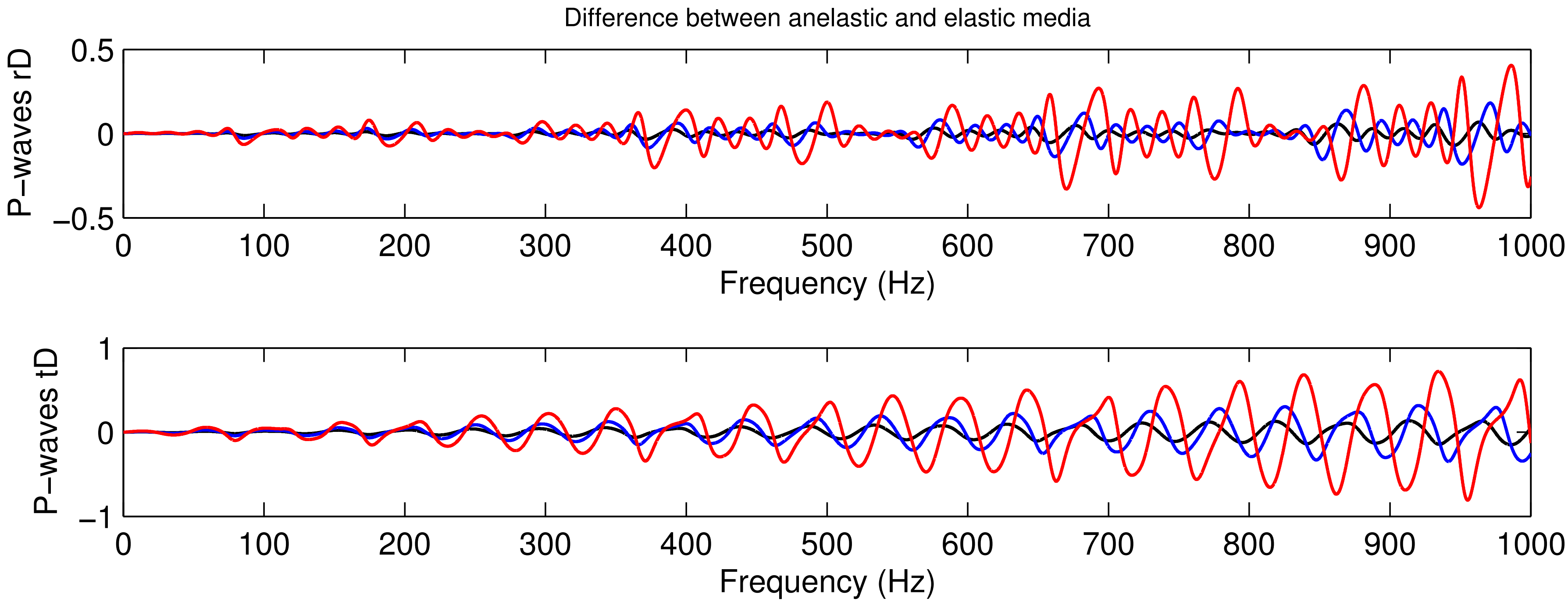
ϕ=50%



Anelastic sand layers

Elastic sand layers (no dispersion)

R/T responses differences (m=4)



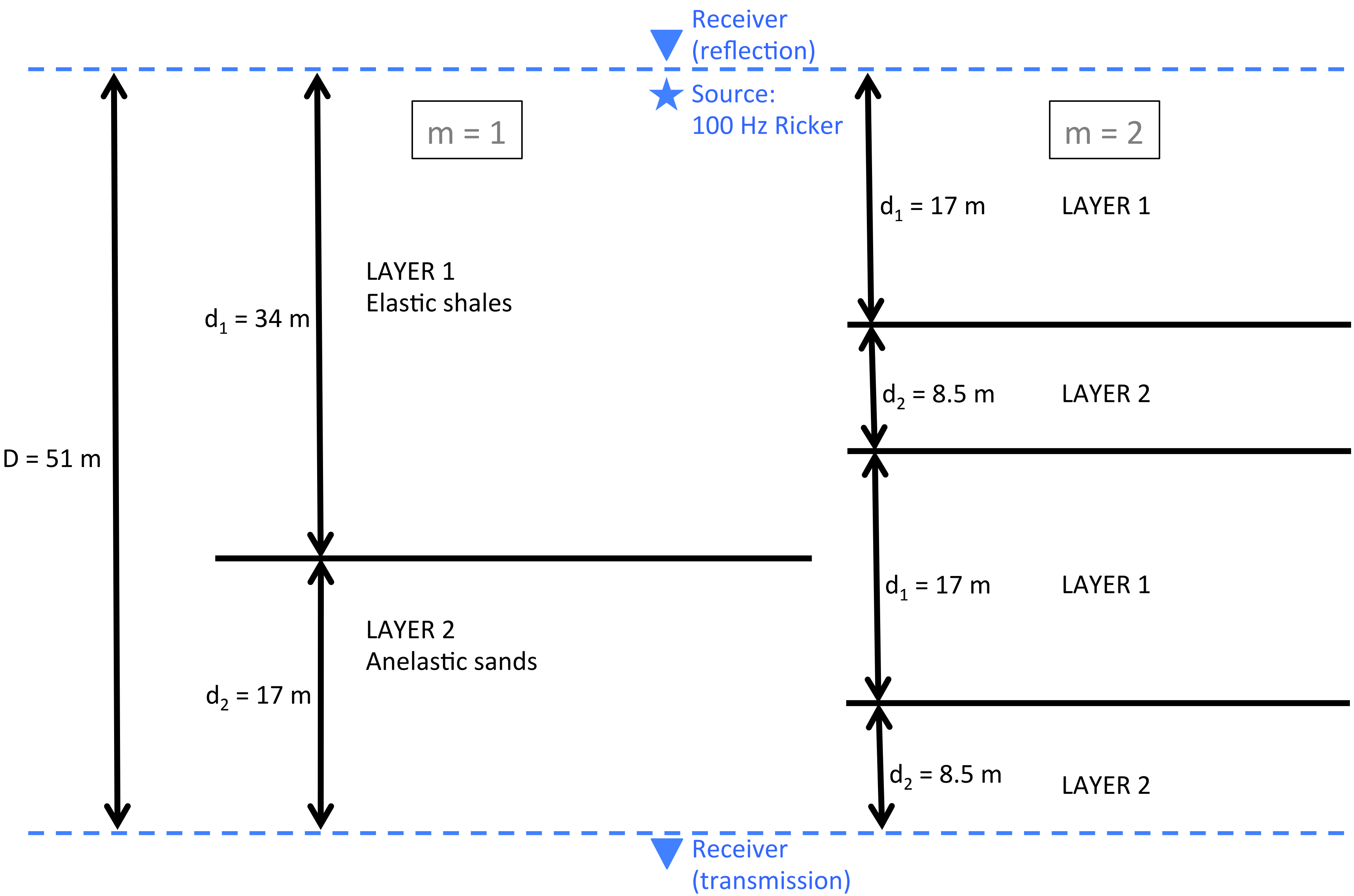
Difference between anelastic and equivalent elastic layers for:

ϕ = 32 %

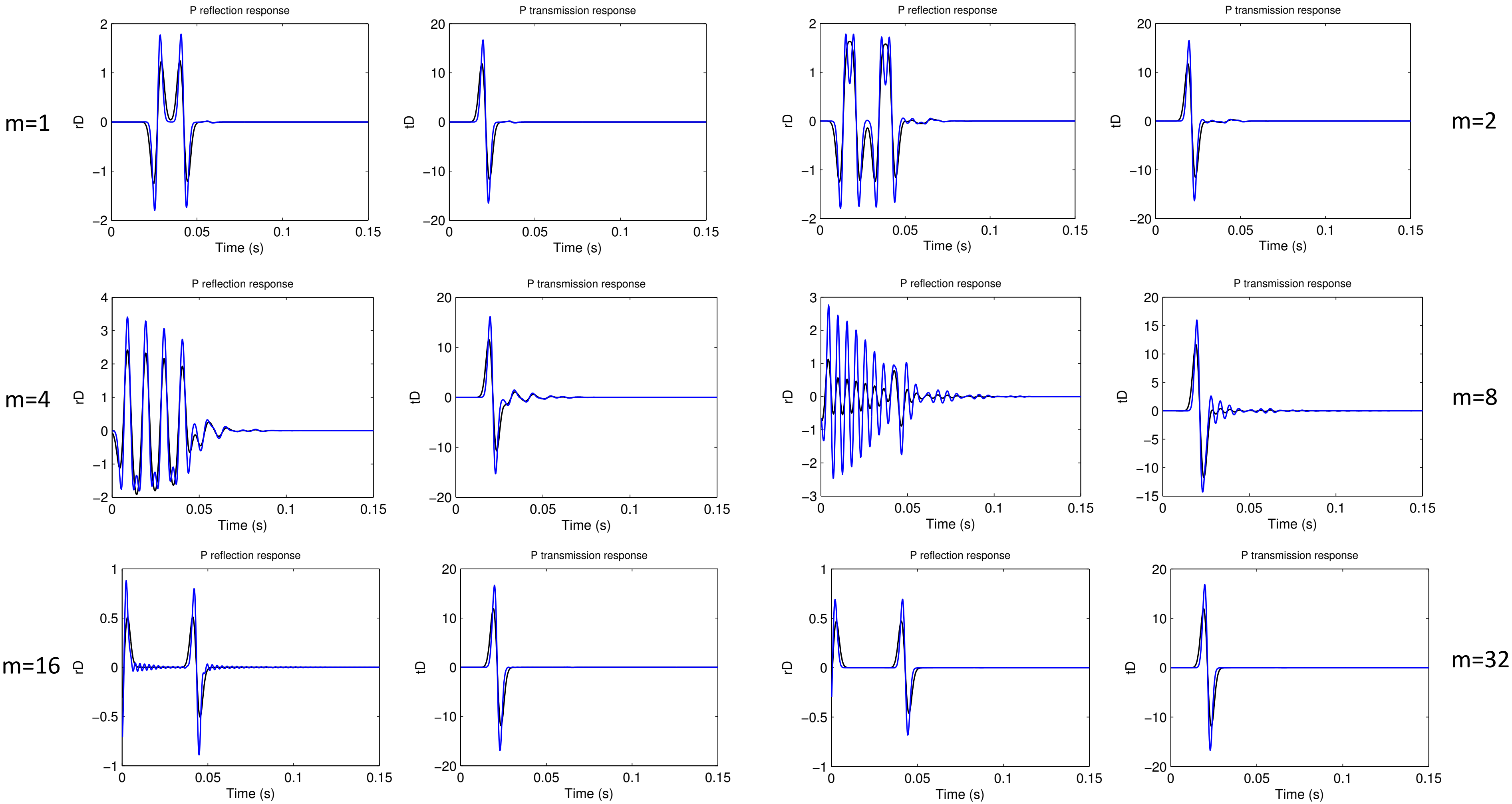
ϕ = 40 %

ϕ = 50 %

Time domain responses



R/T time responses (ϕ=32%)

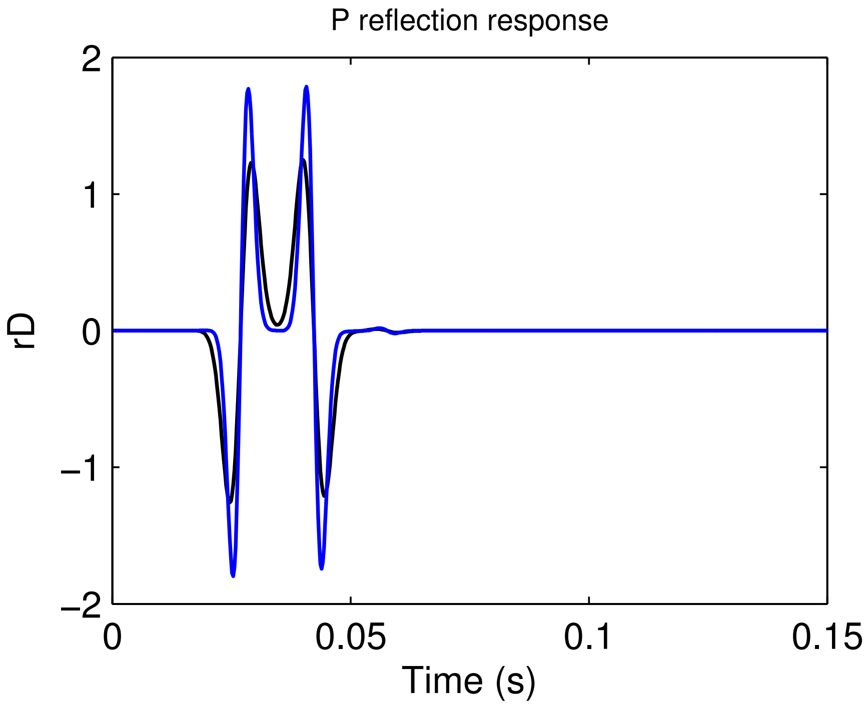


Anelastic sand layers

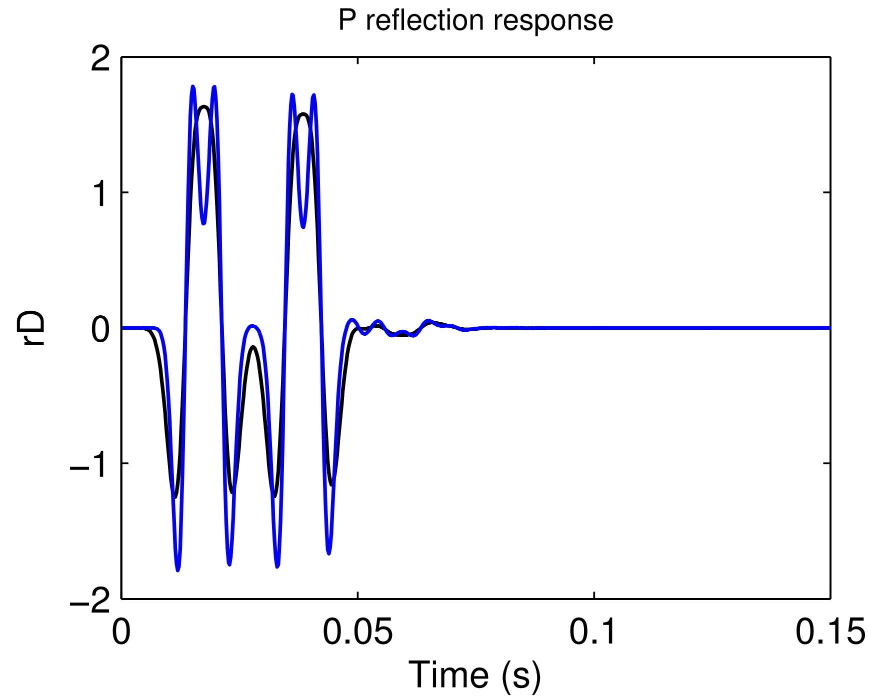
Elastic sand layers (no dispersion)

R/T time responses (ϕ=32%):

reflected events for m=1 and m=2



m=1

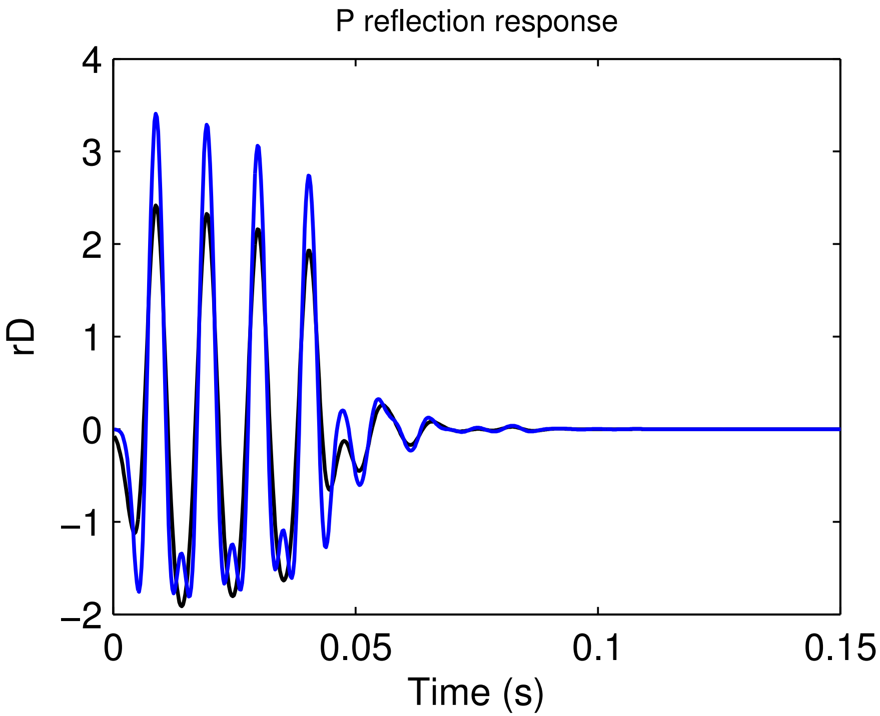


m=2

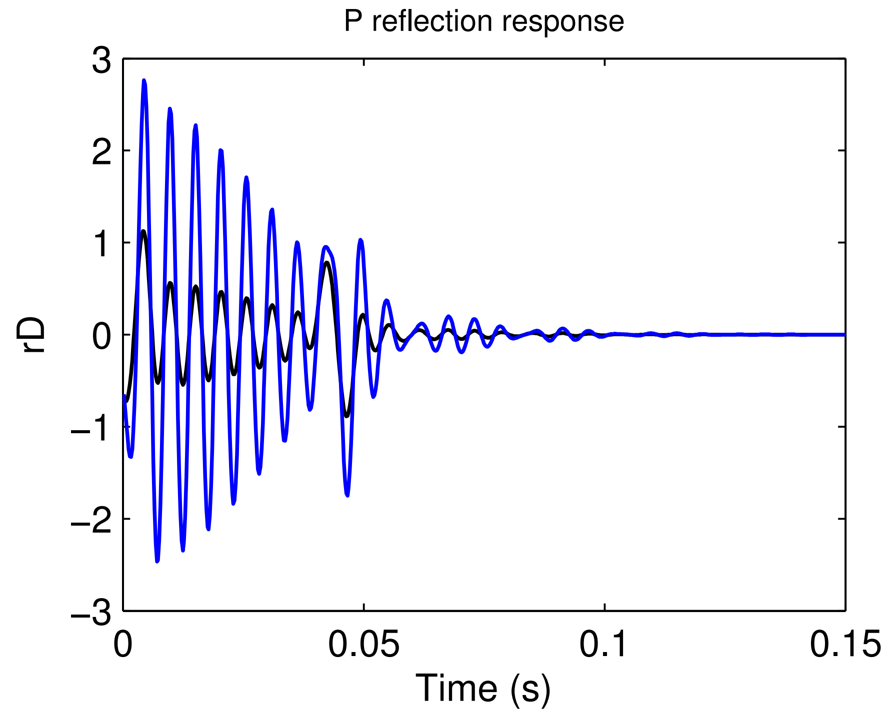
Anelastic sand layers

Elastic sand layers (no dispersion)

R/T time responses (ϕ=32%):
resonance for m=4 and m=8



m=4

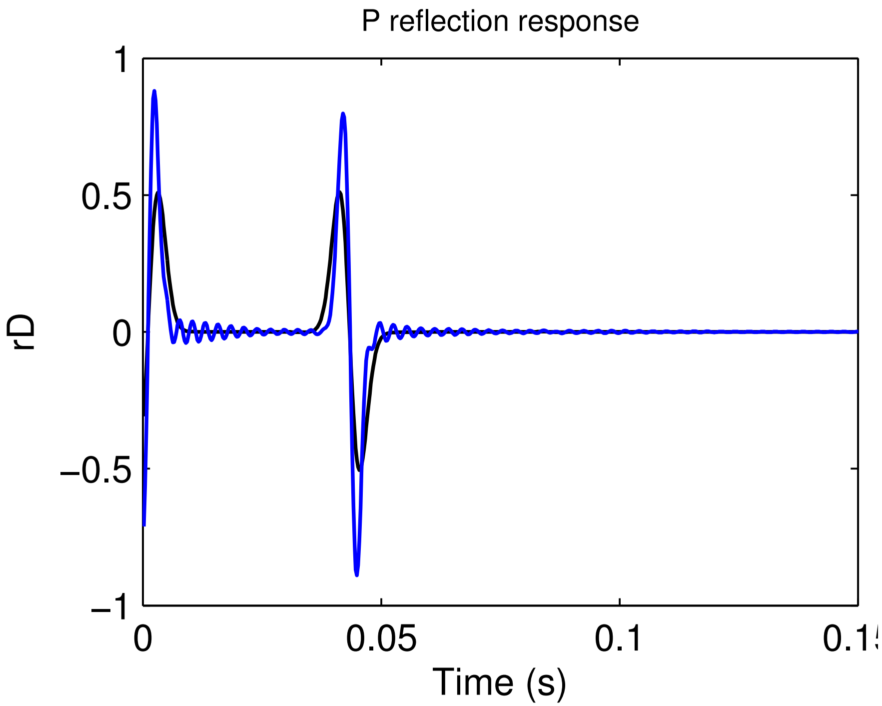


m=8

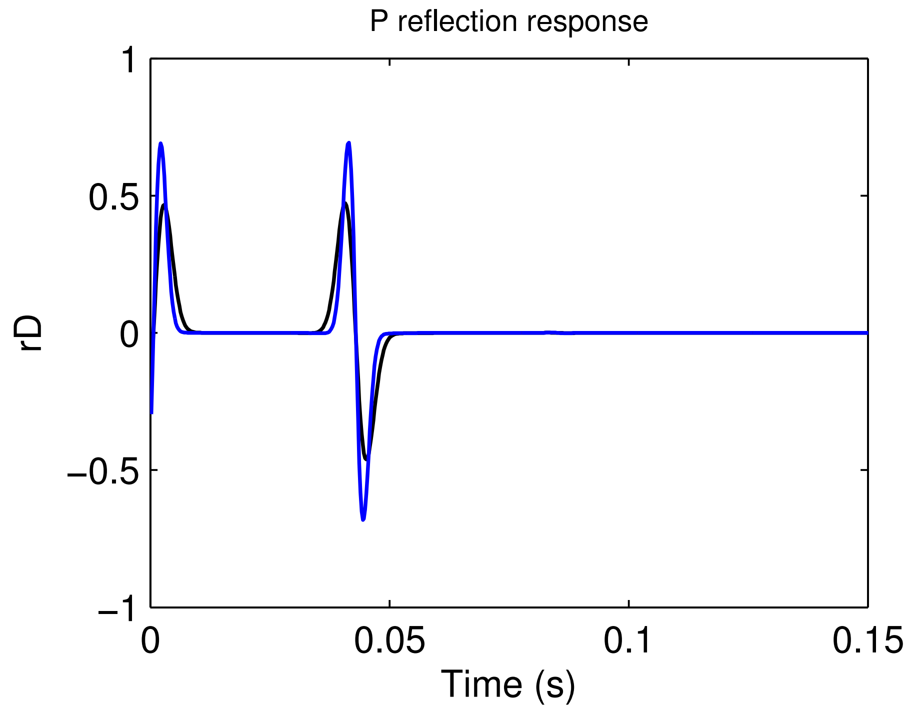
Anelastic sand layers

Elastic sand layers (no dispersion)

R/T time responses (ϕ=32%):
effective medium for m=16 and m=32



m=16

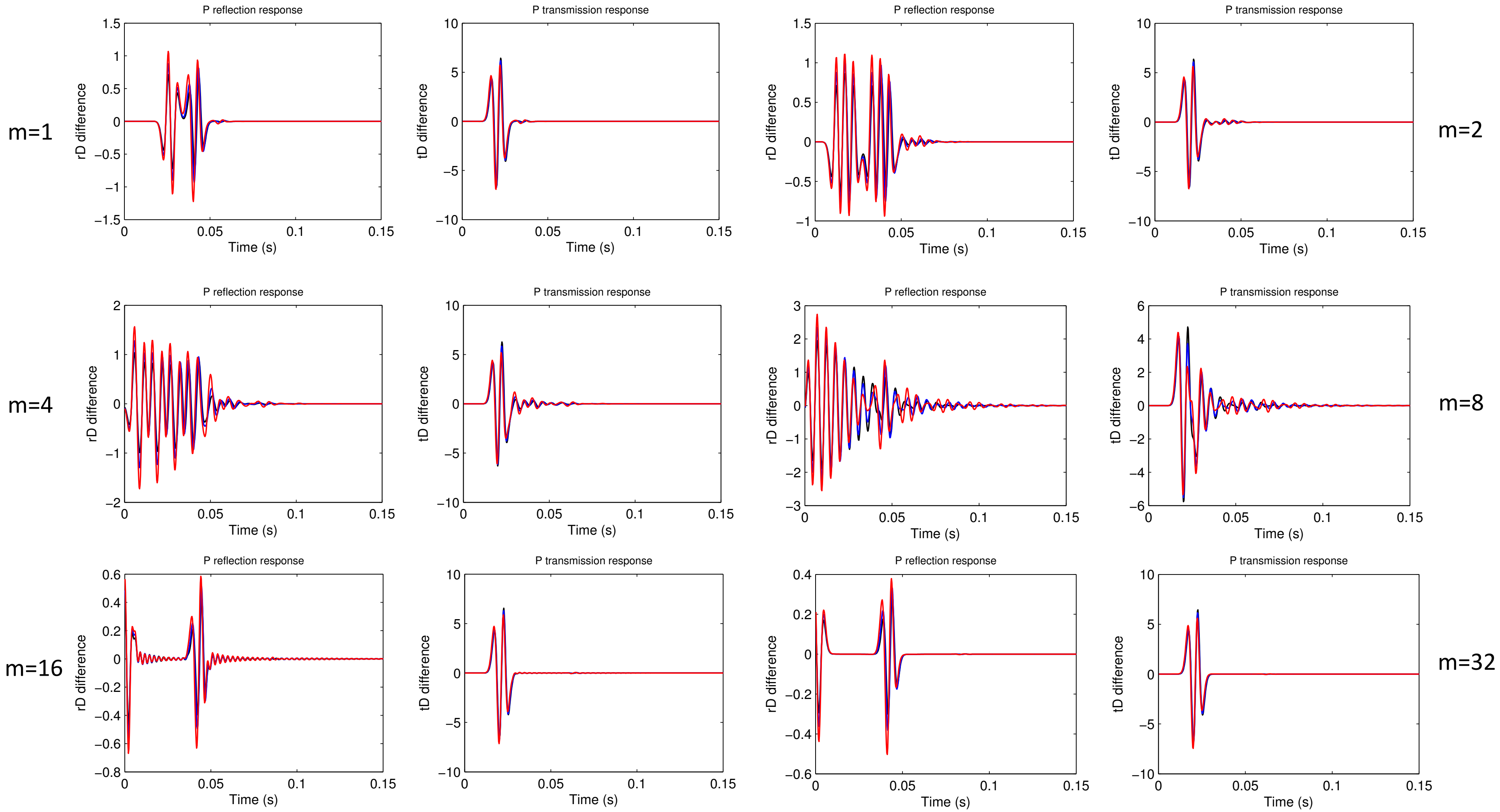


m=32

Anelastic sand layers

Elastic sand layers (no dispersion)

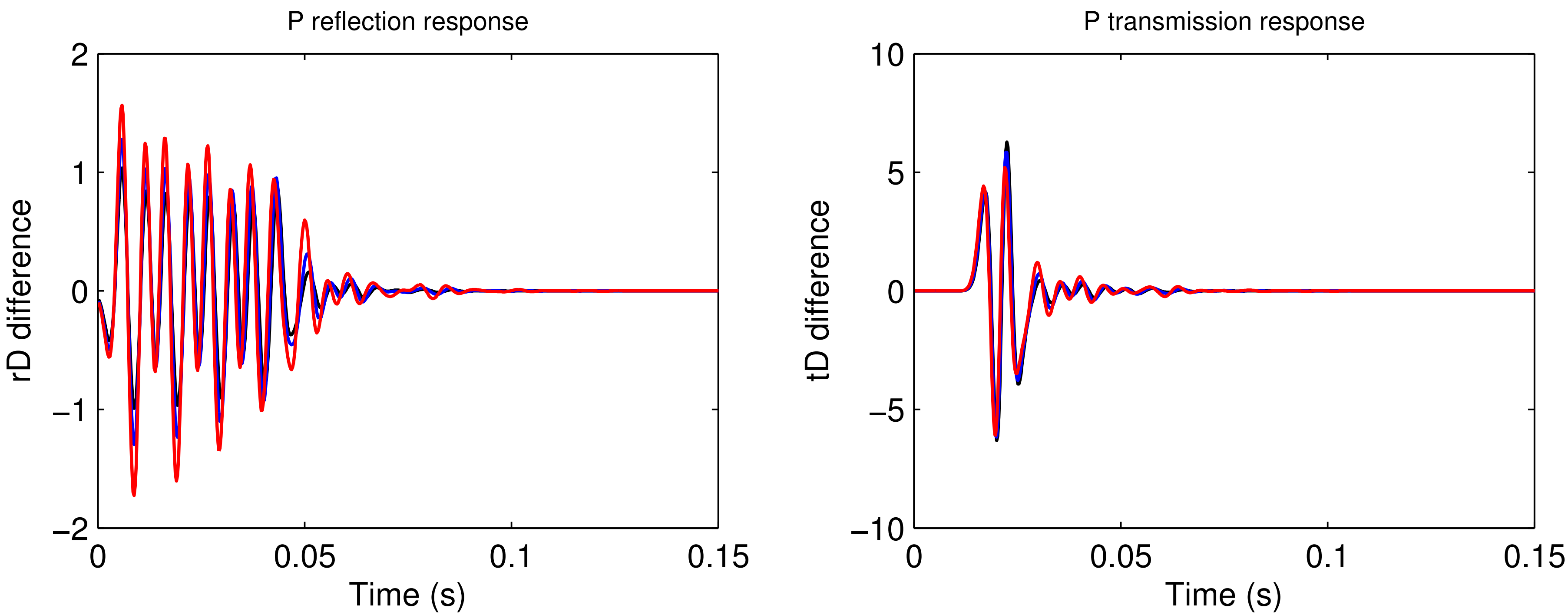
R/T time responses differences



Difference between anelastic and equivalent elastic layers for:

- ϕ = 32 %
- ϕ = 40 %
- ϕ = 50 %

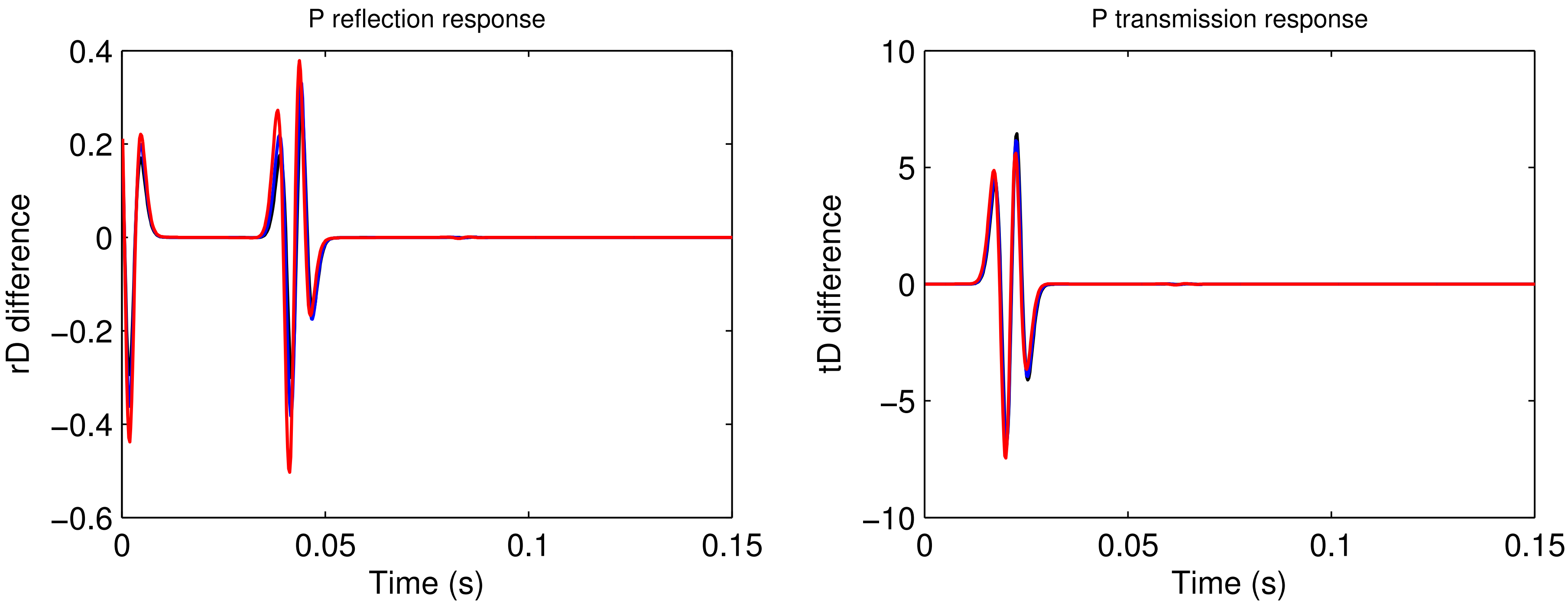
R/T time responses differences (m=4)



Difference between anelastic and equivalent elastic layers for:

ϕ = 32 %
ϕ = 40 %
ϕ = 50 %

R/T time responses differences (m=32)



Difference between anelastic and equivalent elastic layers for:

- ϕ = 32 %
- ϕ = 40 %
- ϕ = 50 %

Conclusions

- 3 porosity models = 3 levels of dispersion ➔ dispersion and attenuation peak frequency around 50 Hz

- Frequency domain:
  - The differences between anelastic and elastic R/T responses increase when porosity increases
  - The layering generally dominates the responses (for realistic porosities)
  - The number of resonance peaks is higher when the number of cycles decreases. These peaks are more pronounced when m increases.
  - If we consider weaker contrast between the 2 layers (here, rP is about 0.11 to 0.17), the effect of anelasticity could be stronger.

Conclusions

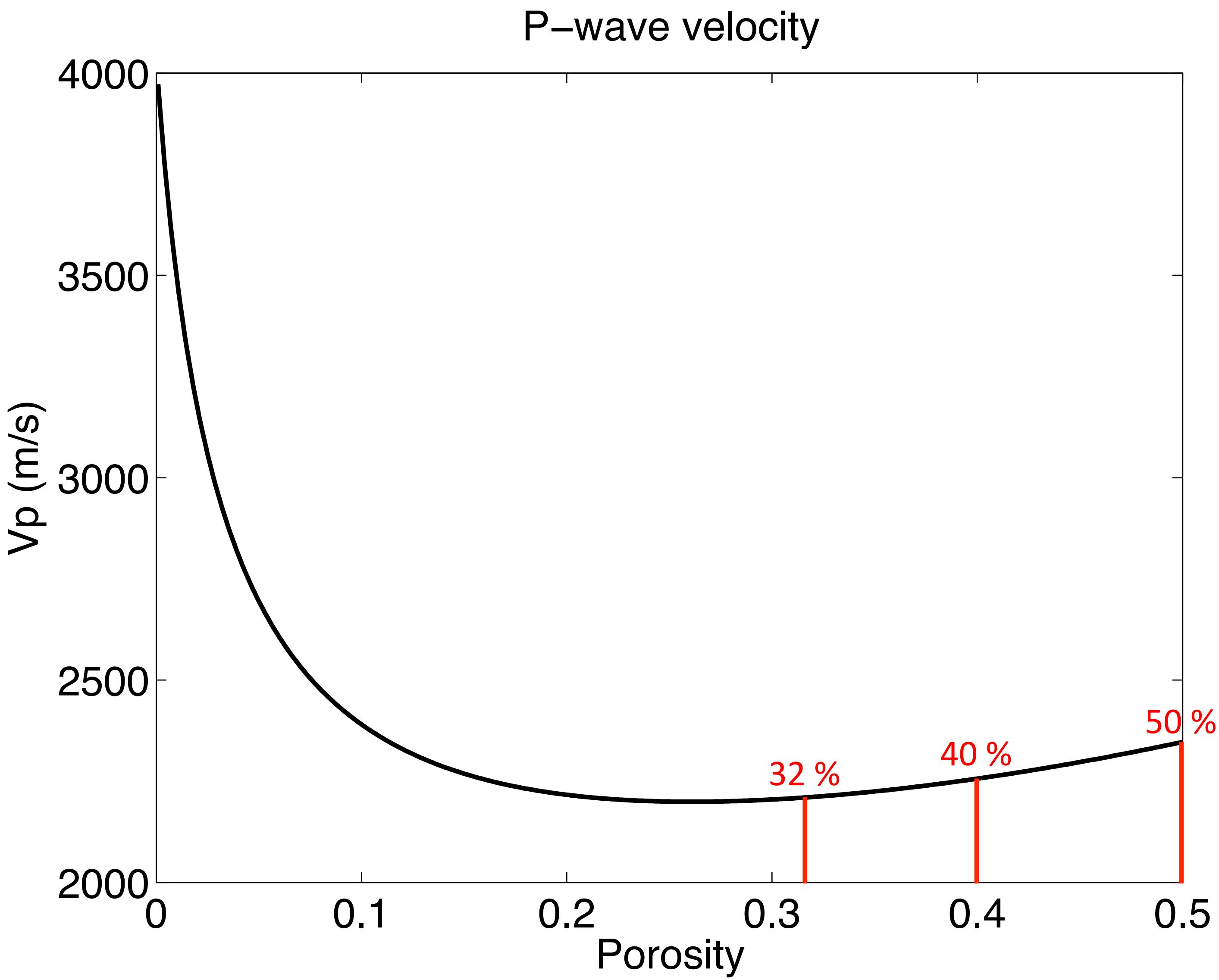
- Time domain:
  - Typical elastic behavior: reflected events, resonance system and then, effective medium are observed when m increases.
  - On the contrary to frequency domain results, the anelasticity has a strong influence on the amplitude of responses

➔ Important conclusion when we deal with seismic interpretation of a layered system: the dispersion and attenuation of the effective R/T responses **are not only due to the layering and can be controlled by intrinsic anelasticity of layers.**

- <u>Acknowledgements:</u>
  – The ROSE project for financial support

- <u>Bibliography:</u>
  – Stovas and Ursin (2003). *Reflection and transmission responses of a layered transversely isotropic visco-elastic media*, GP
  – Pride, Berryman and Harris (2004). *Seismic attenuation due to wave-induced flow*, JGR
  – White (1975), *Computed seismic speeds and attenuation in rocks with partial gas saturation*, Geophysics
  – Dupuy and Stovas (2014). *Reflection and Transmission Responses of a Periodic Layered Medium Constituted by Shales and patchy Saturated Sands*, EAGE expanded Abstract 2014

V_P with respect to the porosity



Model

Layers 1 = shales
V_P = 5500 m/s
V_S = 3000 m/s
ρ = 2700 kg/m^3

Elastic, frequency-independent

Layers 2 = partially saturated sandstones
Gas saturation = 10 %

Patchy saturation: frequency-dependent
OR
Equivalent elastic layer: frequency-independent

→

K_s	(GPa)	40			
ρ_s	(kg/m^3)	2690			
m		1			
ϕ		0.32			
k_0	(m^2)	$9\ 10^{-10}$			
K_D	(GPa)	4			
G_D	(GPa)	3			
K_{f_1} (water)	(GPa)	3.01			
K_{f_2} (gas)	(GPa)	0.13			
ρ_{f_1} (water)	(kg/m^3)	1055			
ρ_{f_2} (gas)	(kg/m^3)	336			
η_1 (water)	$(Pa.s)$	0.001			
η_2 (gas)	$(Pa.s)$	0.00004			
a	(cm)	1			
V_1		0.1	0.2	0.8	0.9
V_P	(m/s)	2085	2077	2107	2192
V_S	(m/s)	1259	1253	1197	1189
V_{Biot}	(m/s)	580	640	311	414
Q_P		74	70	94	153
Q_S		44	39	44	39
Q_{Biot}		1.70	1.57	0.83	0.72
ρ	(kg/m^3)	1960	1983	2121	2144
f_c	(Hz)	7.3	8.1	22	32

Computation of R/T responses

Propagator matrix for 1 cycle (2 layers):

$$\mathbf{S}_k(\omega) = \frac{1}{1-r^2(\omega)} \begin{pmatrix} e^{i\theta_1} & 0 \\ 0 & e^{-i\theta_1} \end{pmatrix} \begin{pmatrix} 1 & r(\omega) \\ r(\omega) & 1 \end{pmatrix} \begin{pmatrix} e^{i\theta_2(\omega)} & 0 \\ 0 & e^{-i\theta_2(\omega)} \end{pmatrix} \begin{pmatrix} 1 & -r(\omega) \\ -r(\omega) & 1 \end{pmatrix}$$

$$\begin{aligned}
\mathbf{S}_k(\omega) &= \begin{pmatrix} a_k(\omega) & b_k(\omega) \\ c_k(\omega) & d_k(\omega) \end{pmatrix} , \\
a_k(\omega) &= \frac{1}{t_1 t_2(\omega)} \left(e^{i(\theta_1+\theta_2(\omega))} \left(1 - r^2(\omega) e^{-2i\theta_2(\omega)} \right) \right) , \\
b_k(\omega) &= \frac{1}{t_1 t_2(\omega)} \left(-r e^{i(\theta_1+\theta_2(\omega))} + r(\omega) e^{i(\theta_1-\theta_2(\omega))} \right) , \\
c_k(\omega) &= \frac{1}{t_1 t_2(\omega)} \left(r e^{i(\theta_1-\theta_2(\omega))} - r(\omega) e^{-i(\theta_1+\theta_2(\omega))} \right) , \\
d_k(\omega) &= \frac{1}{t_1 t_2(\omega)} \left(e^{-i(\theta_1+\theta_2(\omega))} \left(1 - r^2(\omega) e^{2i\theta_2(\omega)} \right) \right) .
\end{aligned}$$

$r(\omega)$ = reflection coefficient
t_1 and $t_2(\omega)$ = traveltimes
θ_1 and $\theta_2(\omega)$ = phase factors

$$\begin{aligned}
r(\omega) &= \frac{\rho_2 V_2(\omega) - \rho_1 V_1}{\rho_2 V_2(\omega) + \rho_1 V_1} , \\
\theta_1 &= \frac{2\pi f d_1}{V_1} = 2\pi f t_1 , \\
\theta_2(\omega) &= \frac{2\pi f d_2}{V_2(\omega)} = 2\pi f t_2(\omega) ,
\end{aligned}$$

Equations valid for P- or S-waves independently