NTNU – Trondheim
Norwegian University of
Science and Technology

3D inversion of magnetotelluric data

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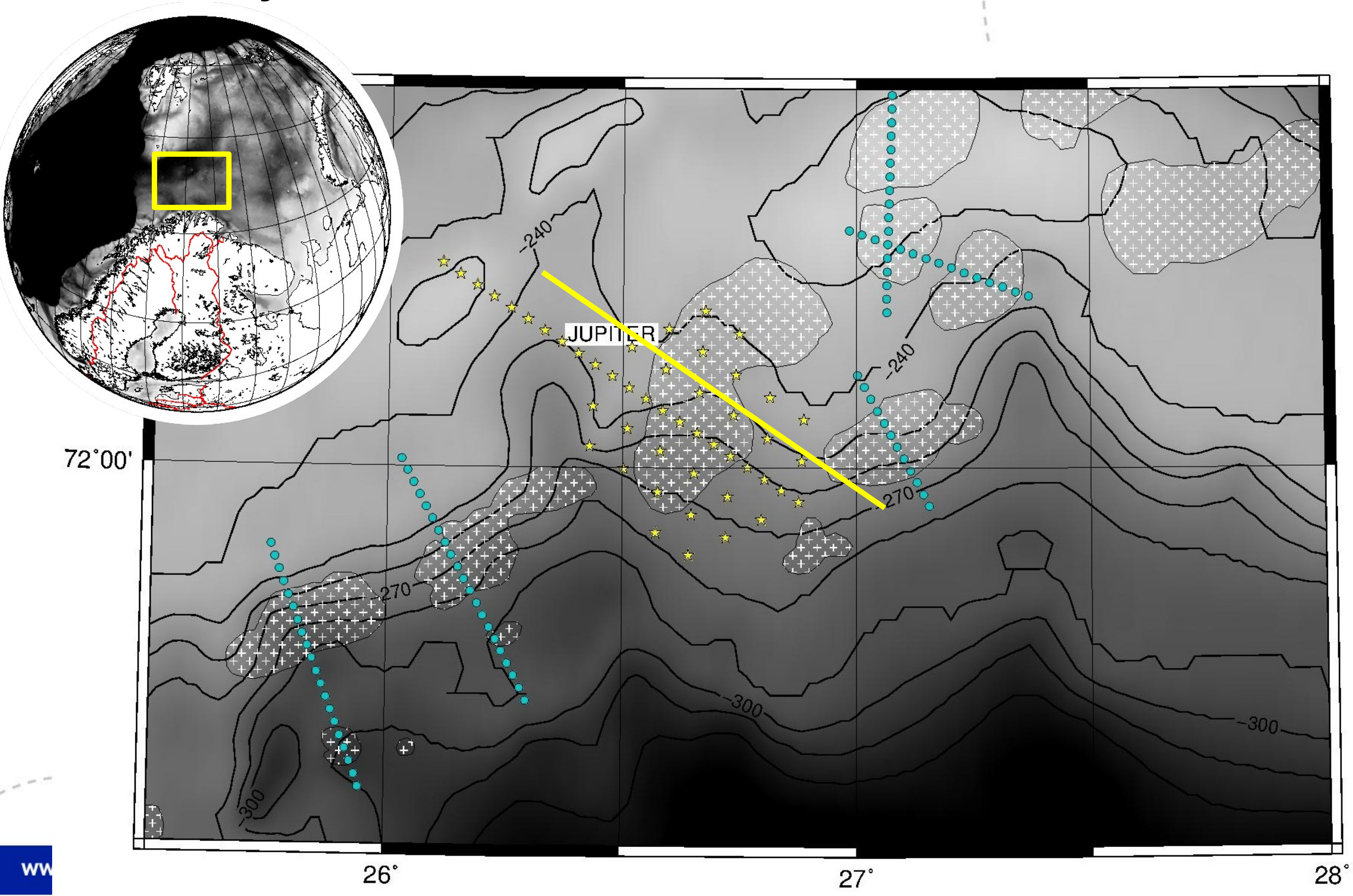
Survey Area



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Motivation

SE

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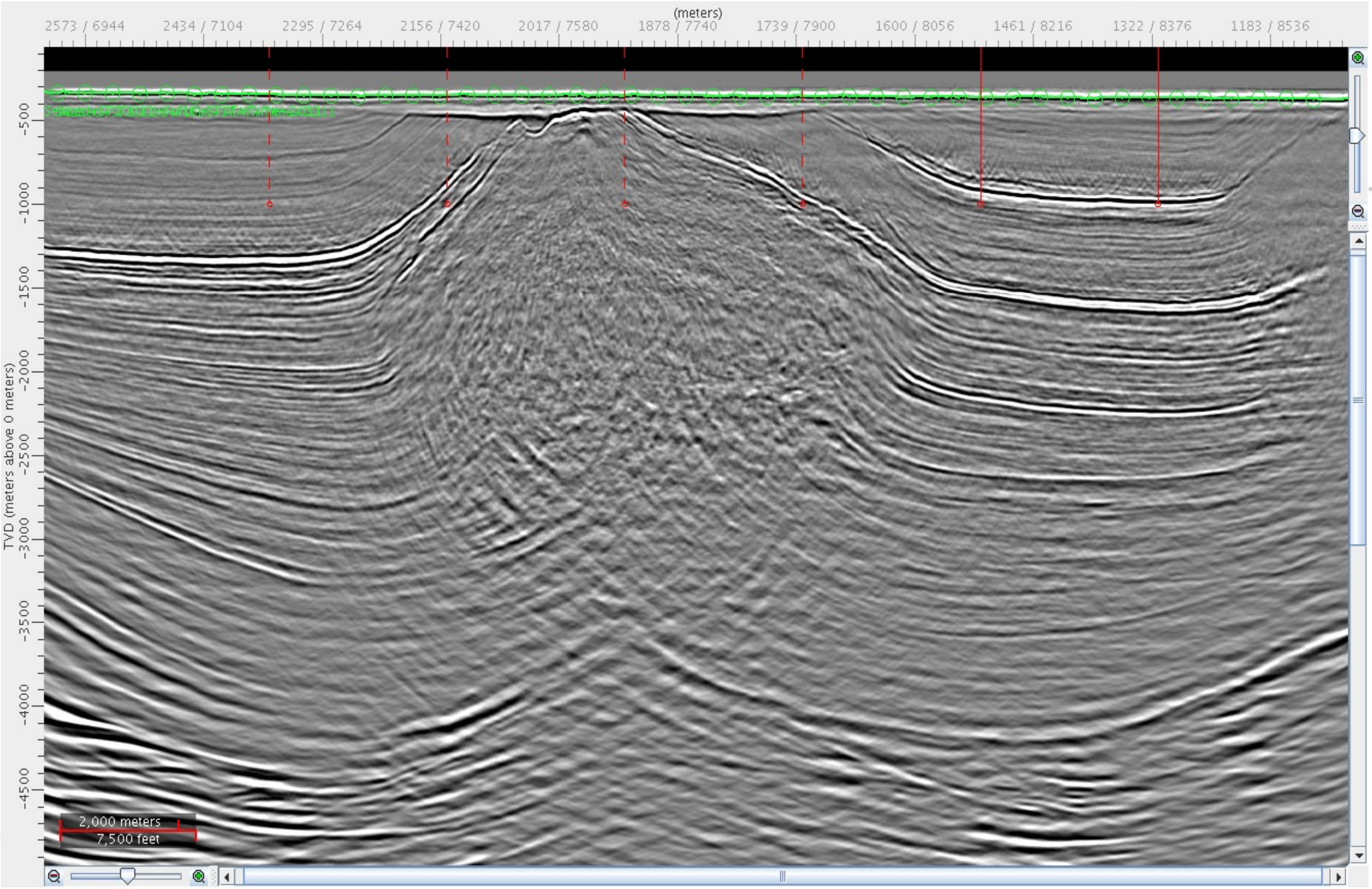


Motivation

Problems:

- Weak primaries, strong multiples,density contrasts, diffraction
- Limited imaging quality close to the salt

→ Alternative methods like CSEM, MT, gravity

Goal:

- Develop joint inversion of EM and gravity data
- Base is a 3D magnetotelluric inversion

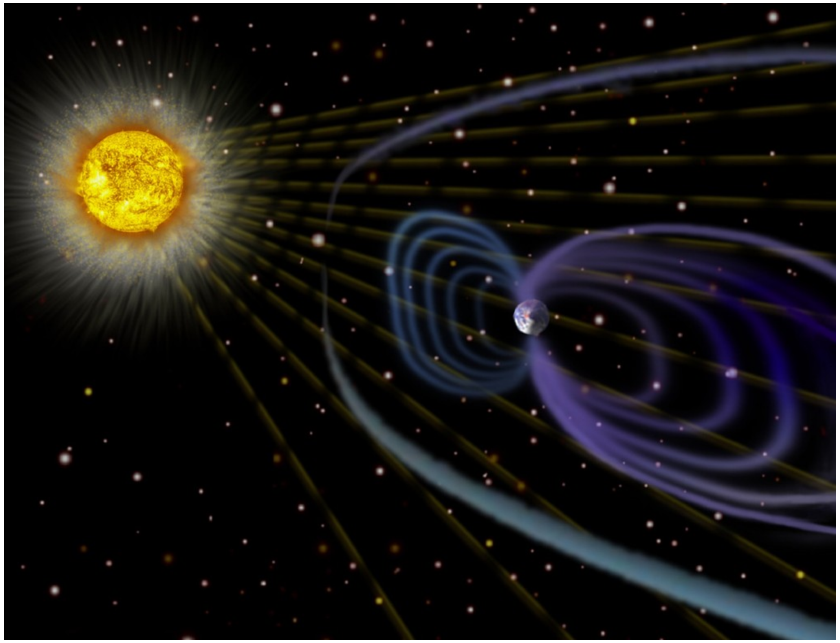
Short introduction to Magnetotellurics

- Passive method
- Source are variations of the natural electromagnetic field
- Plane wave with vertical incedence
- Frequency range: 10 – 0.001Hz
- Low resolution, receiver spacing
- Penetration depth - down to 50km
- Horizontal resistivity
- Impedance tensor

$$\begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{pmatrix} \begin{pmatrix} B_x \\ B_y \end{pmatrix}$$

$$\rho_{a,ij}(\omega) = \frac{1}{\mu_0 \omega} |Z_{ij}(\omega)|^2 \qquad \phi_{ij}(\omega) = tan^{-1}\left(\frac{\Im\{Z_{ij}(\omega)\}}{\Re\{Z_{ij}(\omega)\}}\right)$$

Short introduction to Magnetotellurics



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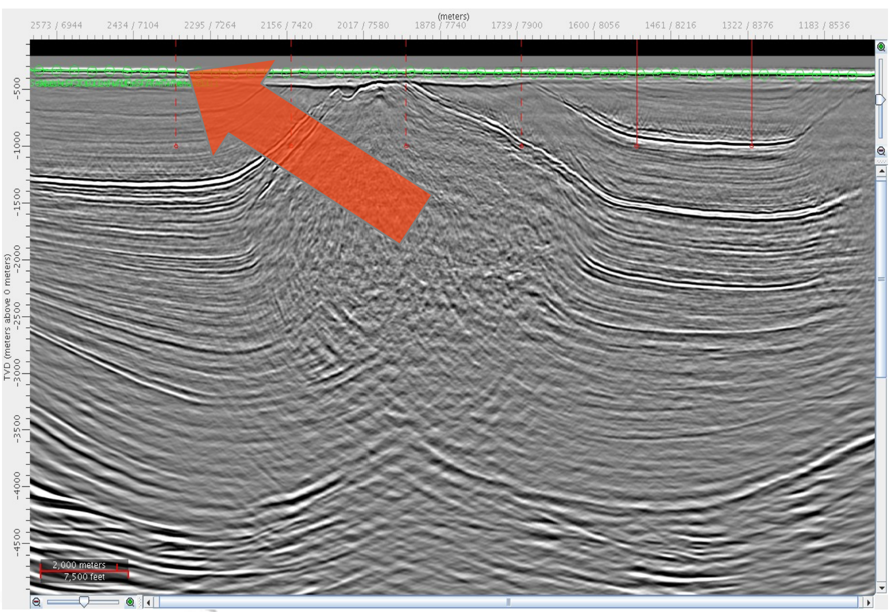
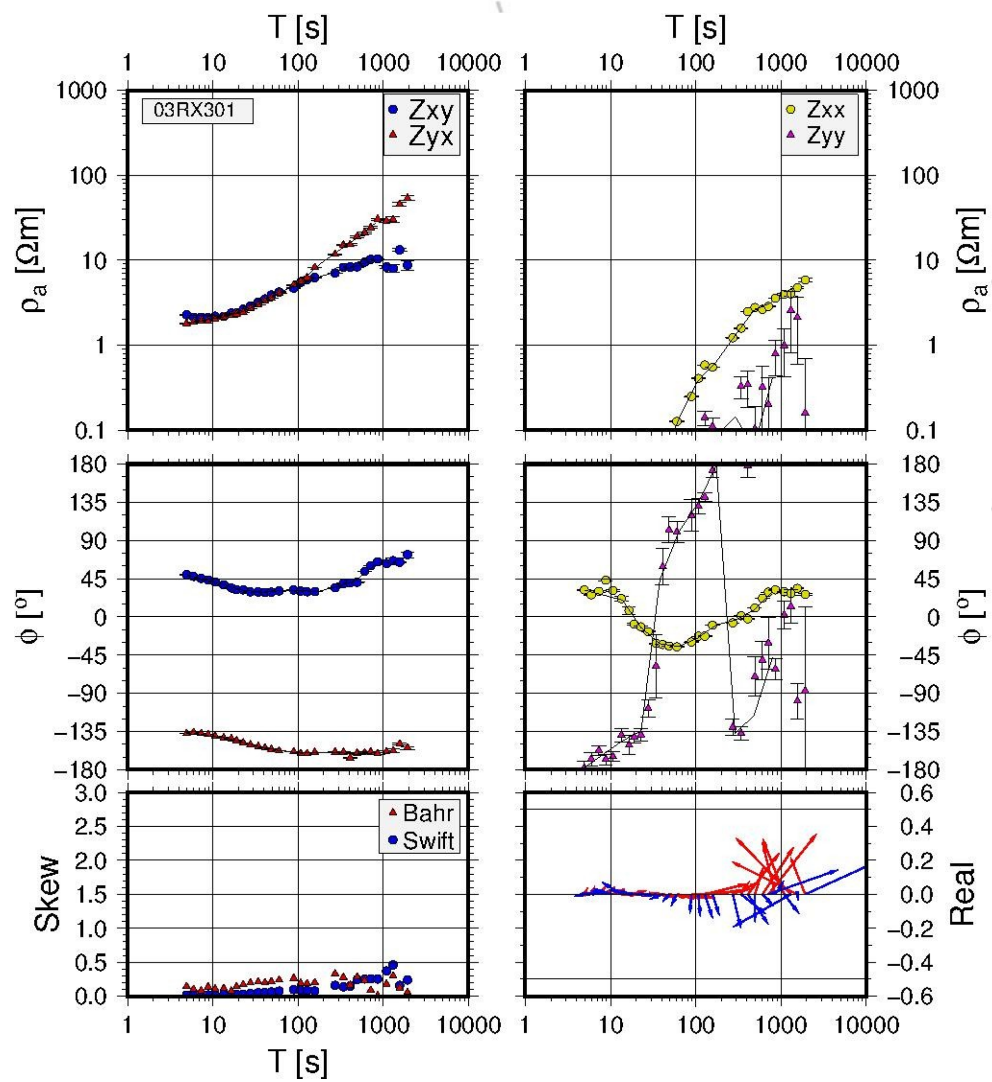
$$\left. \begin{matrix} Z_{xx} = Z_{yy} = 0 \\ Z_{xy} = -Z_{xy} \end{matrix} \right\} 1 - D$$





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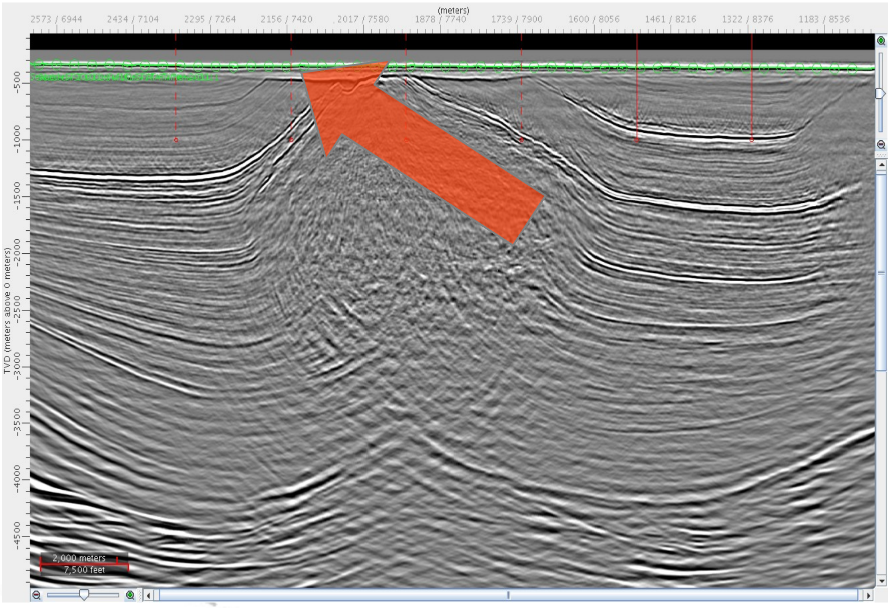

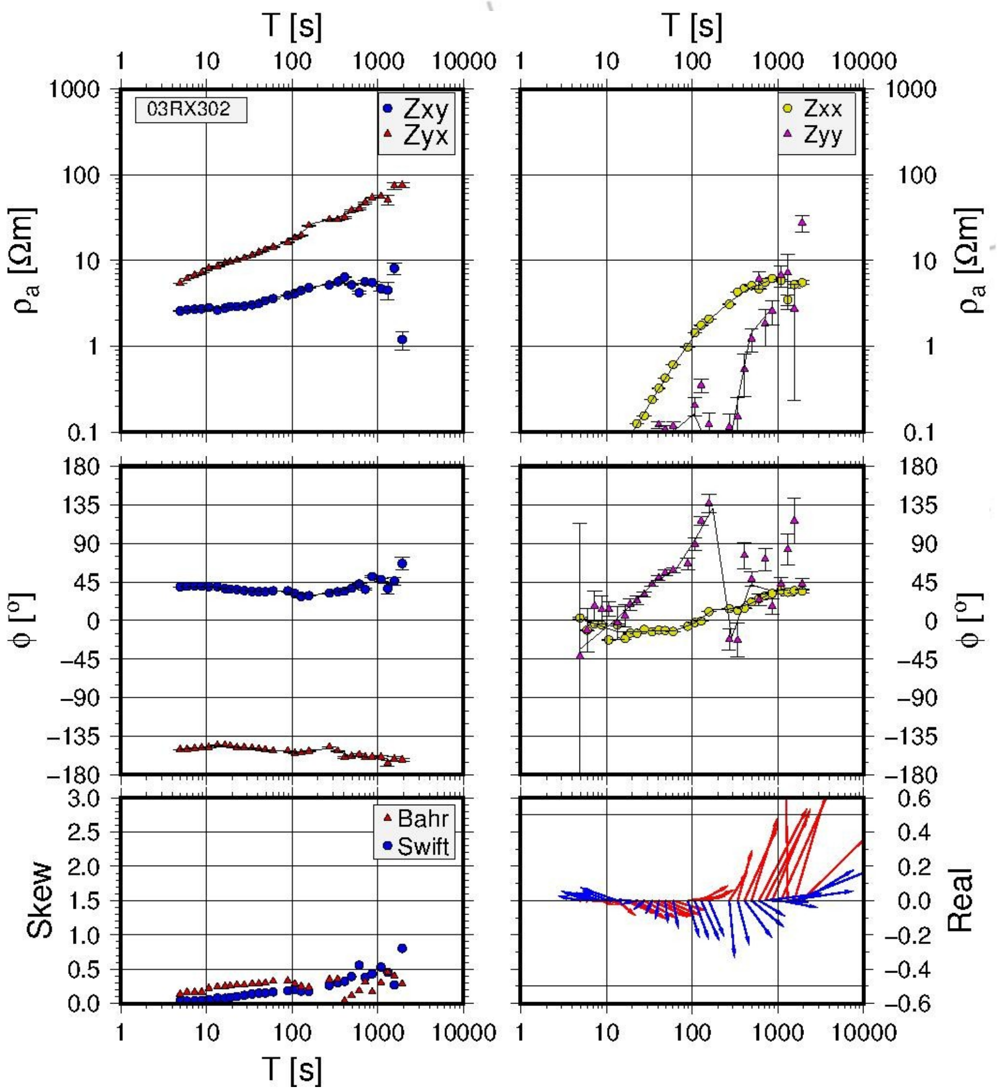

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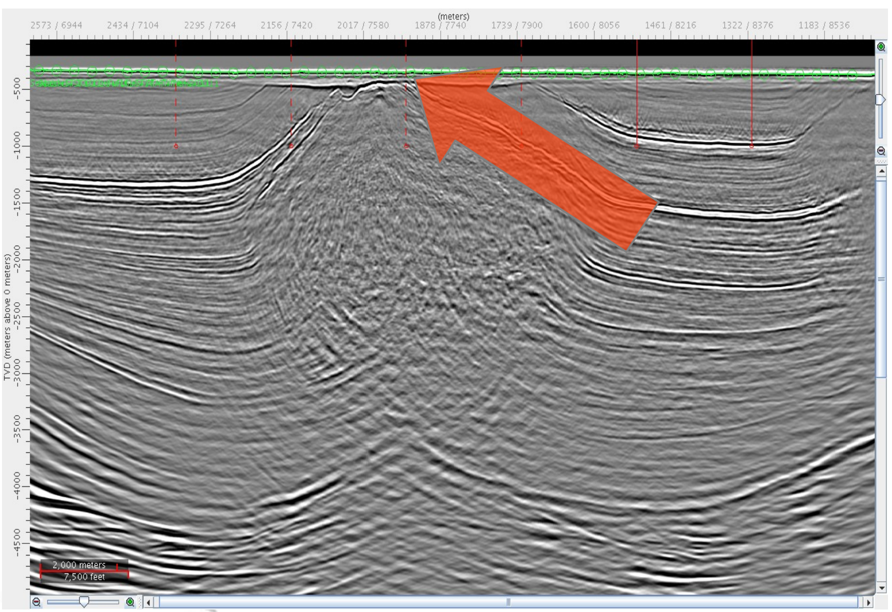
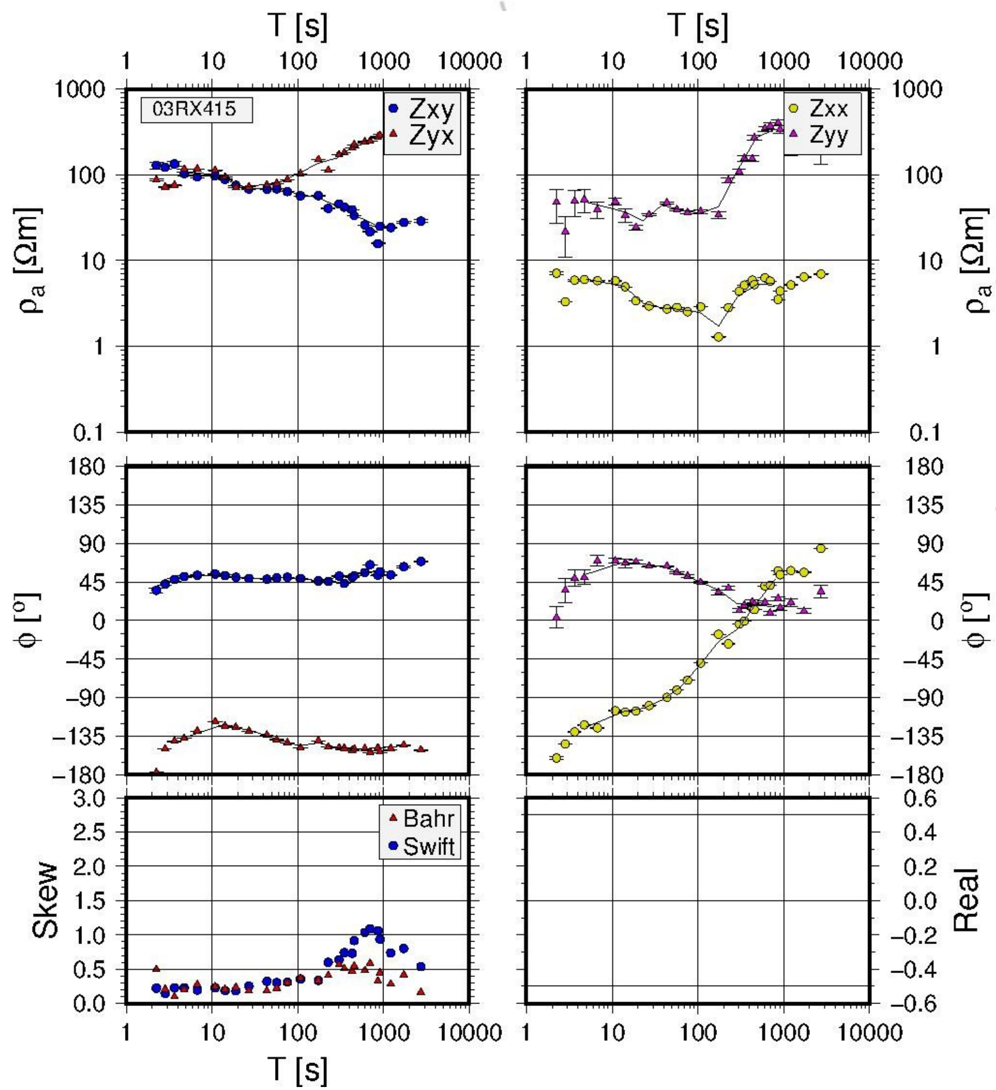
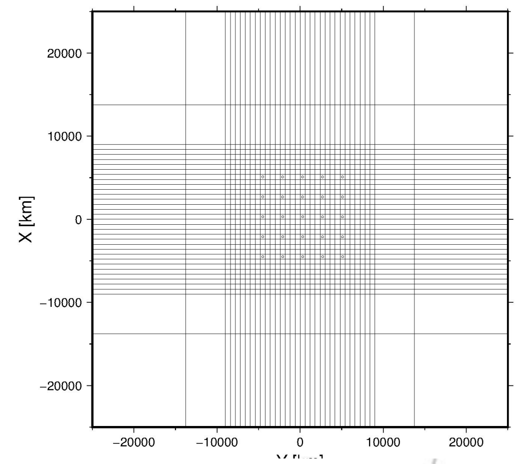




Forward Modeling

- Finite volume modeling of the electromagnetic field (Weiss et al. 2006)
- Scattered field solution
- Dirichlet boundary conditions
- graded staggered grid (Yee et al., 1966)
- Electric field the center of the edges of the model cubes
- Magnetic field calculated from Faraday's (induction) law only at nodes surrounding the receivers $\nabla \times \mathbf{E} = -i\omega \mathbf{B}$



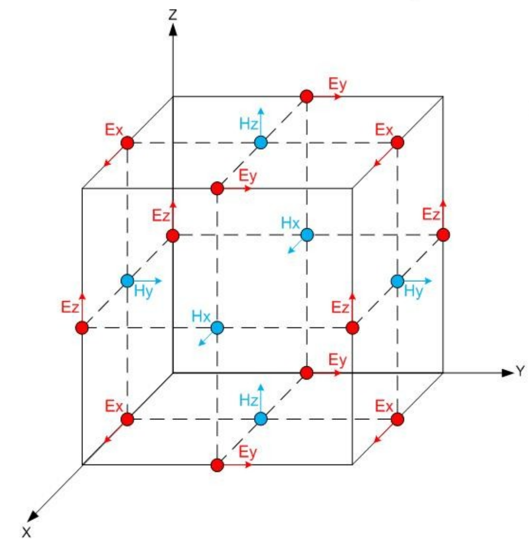
$$\nabla \times \nabla \times \mathbf{E} + i\omega\mu_0\sigma\mathbf{E} = -i\omega\mu_0\sigma\mathbf{J}_s$$

$$\mathbf{E}' = \mathbf{E} - \mathbf{E}^0$$

$$\nabla \times \nabla \times \mathbf{E}' + i\omega\mu_0\sigma\mathbf{E}' = -i\omega\mu_0(\sigma - \sigma_0)\mathbf{E}_0$$

written out into a linear system

$$\mathbf{Ae} = \mathbf{b}$$



A - coefficient matrix

e - electric field solution

number of elements $((nx-1)nynz + nx(ny-1)nz + nxny(nz-1)) \times 3$

b - boundary conditions

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Inversion of MT data

- Gauss – Newton inversion of the scattered field
- Undetermined problem 50000 to 100000 unknowns with ca. 1000 to 3000 data points
- Minimum norm solution

$$\phi = \frac{1}{\lambda}\left[(\mathbf{d}-\mathbf{F}(\mathbf{m}))^T\,\mathbf{C}_d^{-1}\,(\mathbf{d}-\mathbf{F}(\mathbf{m}))\right] + (\mathbf{m}-\mathbf{m}_0)^T\,\mathbf{C}_m^{-1}\,(\mathbf{m}-\mathbf{m}_0)$$

$$\mathbf{m}_{k+1}-\mathbf{m}_0 = \mathbf{C}_m\mathbf{J}_k^T\left(\lambda\mathbf{C}_d+\mathbf{J}_k\mathbf{C}_m\mathbf{J}_k^T\right)^{-1}(\mathbf{d}-\mathbf{F}(\mathbf{m}_k)+\mathbf{J}_k(\mathbf{m}_k-\mathbf{m}_0))$$

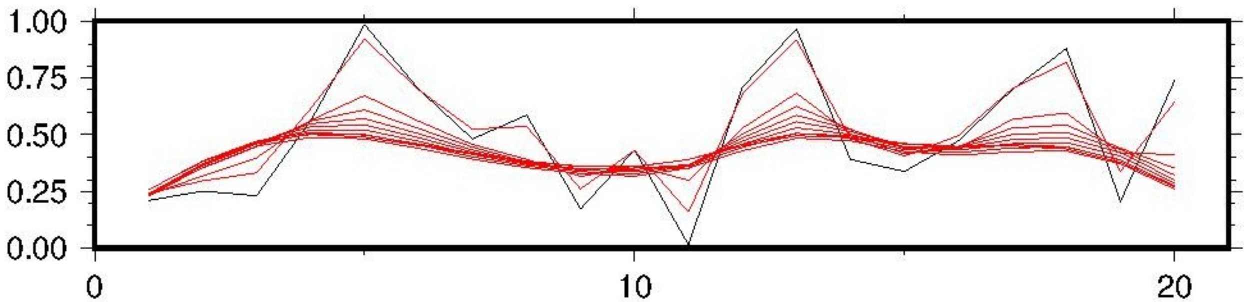
$$\mathbf{m} = [m_1, m_2, m_3, ..., m_M]^T = [\sigma_1, \sigma_2, \sigma_3, ..., \sigma_M]^T$$

$$\mathbf{d} = [d_1, d_2, d_3, ..., d_N]^T = [Z_{xx}|_{per=1}^{sta=1}, Z_{xy}|_{per=1}^{sta=1}, Z_{yx}|_{per=1}^{sta=1}, ..., Z_{yy}|_{per=nper}^{sta=nsta}]^T$$

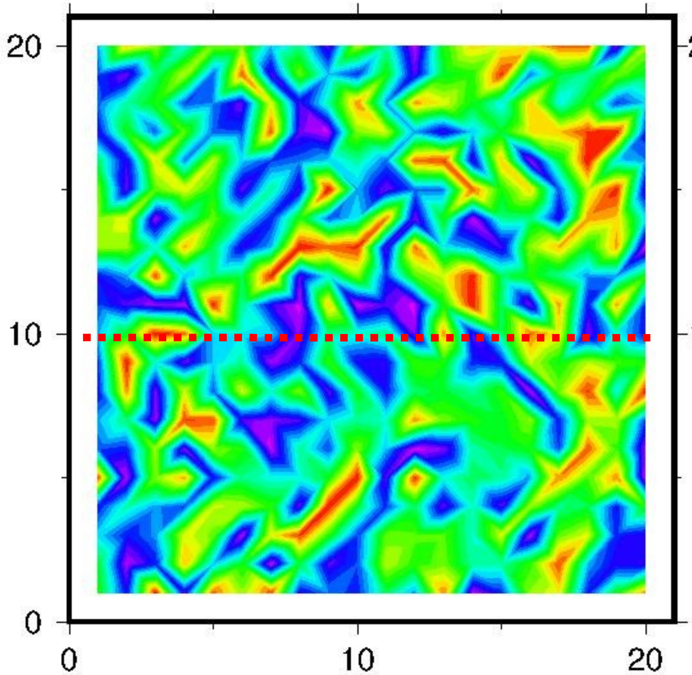
$$\mathbf{C}_d^{-1} = diag\left[1/err_1^2, 1/err_2^2, \cdots, 1/err_N^2\right]$$

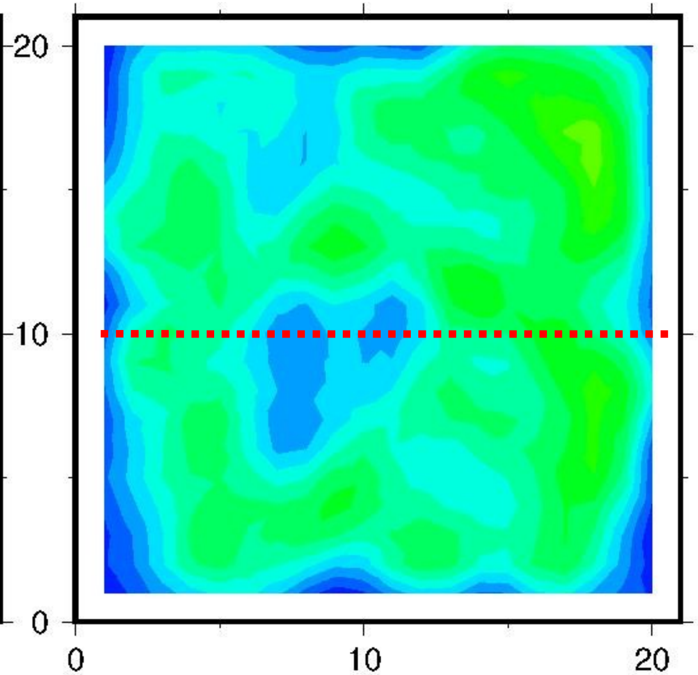
Model covariance C_m

$$u(\vec{x}_i) = \frac{1}{4\Pi\,|\Sigma|^{\frac{1}{2}}} \sum_{j}^{N} \mathrm{e}^{-\frac{(x_{x,i}-y_{x,j})^2}{4\eta x}} \cdot \mathrm{e}^{-\frac{(x_{y,i}-y_{y,j})^2}{4\eta y}} \cdot \mathrm{e}^{-\frac{(x_{z,i}-y_{z,j})^2}{4\eta z}} \cdot u(\vec{y}_j)$$



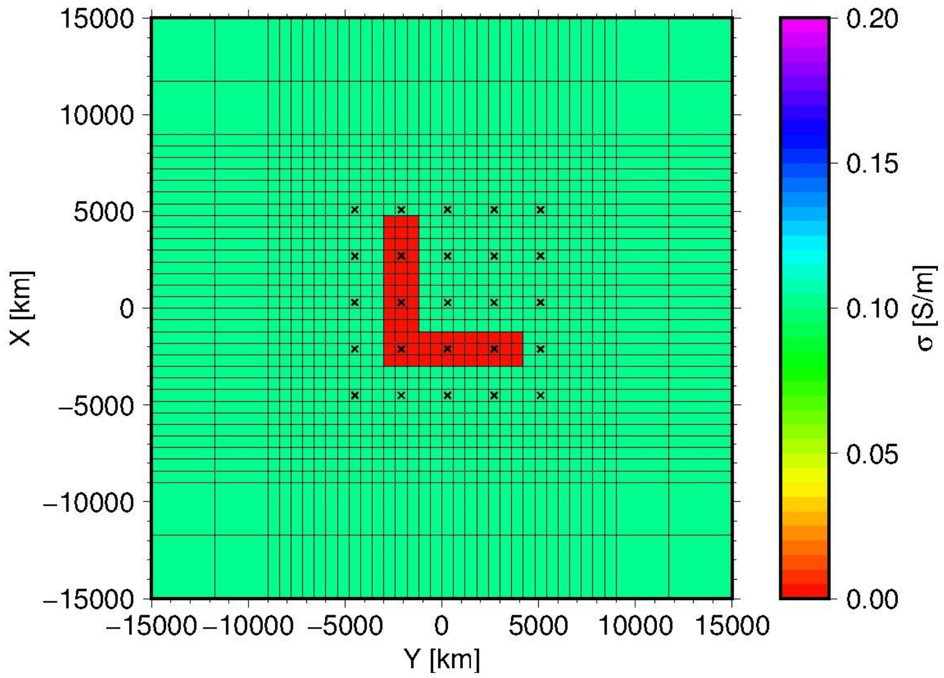
$$|\Sigma| = det \begin{vmatrix} \eta_x & 0 & 0 \\ 0 & \eta_y & 0 \\ 0 & 0 & \eta_z \end{vmatrix}$$





Synthetic example

- L - shaped resistor(0.0002 S/m) in a conductive background (0.1 S/m) between 1 – 8km depth
- Model 39x39x31 cells 600m resolution center part
- 25 receiver on the seabed (260m water depth)
- 10 frequencies from 0.5 to 0.002Hz
- Zxy and Zyx



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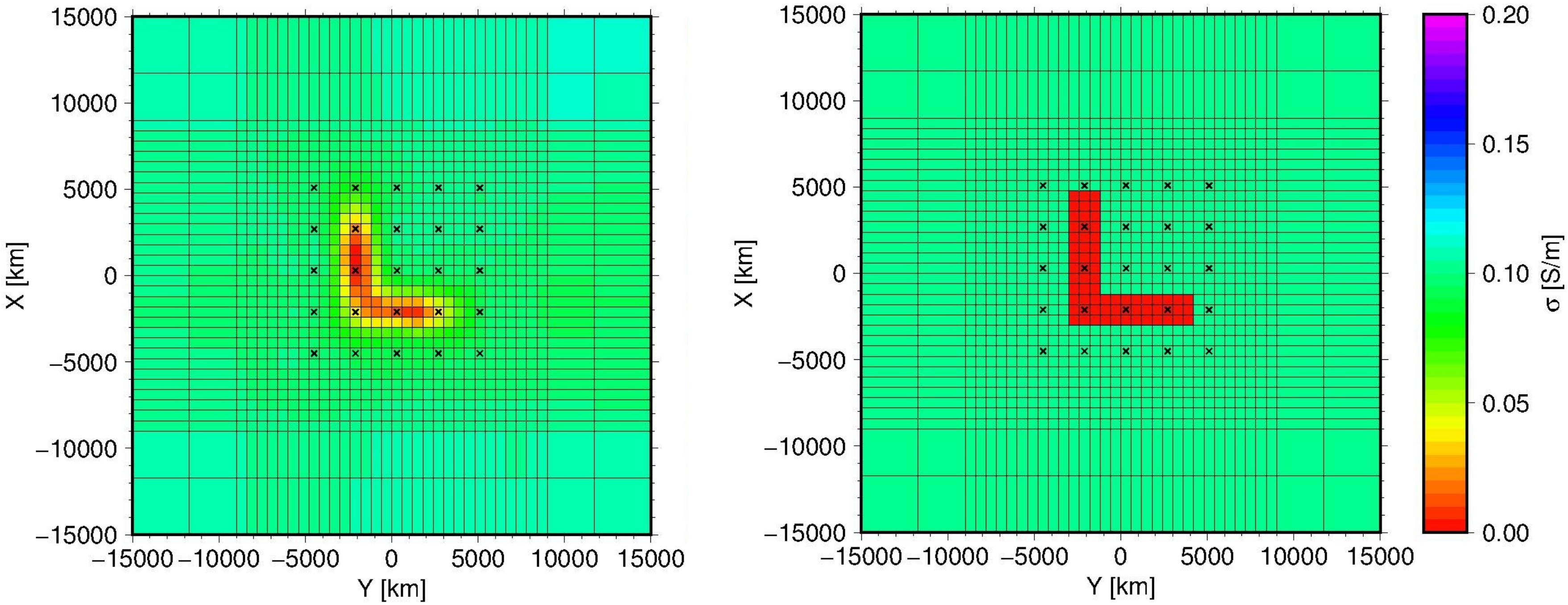
Synthetic example

Depth slice at 2.7 – 3.7km
3 iterations



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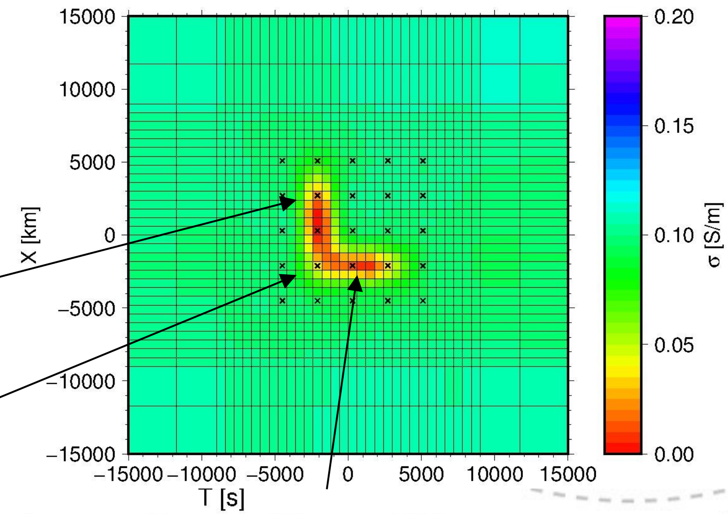
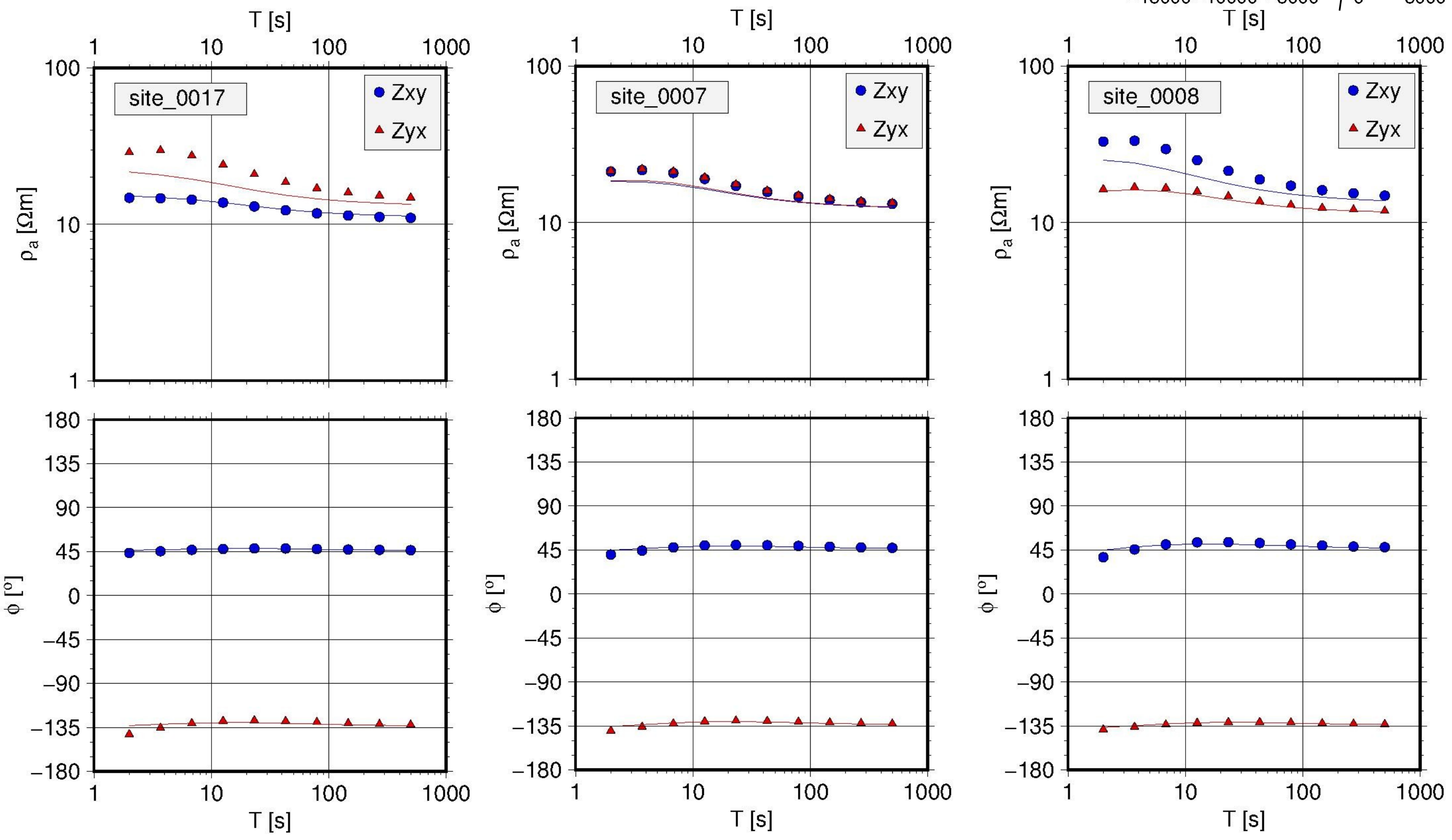
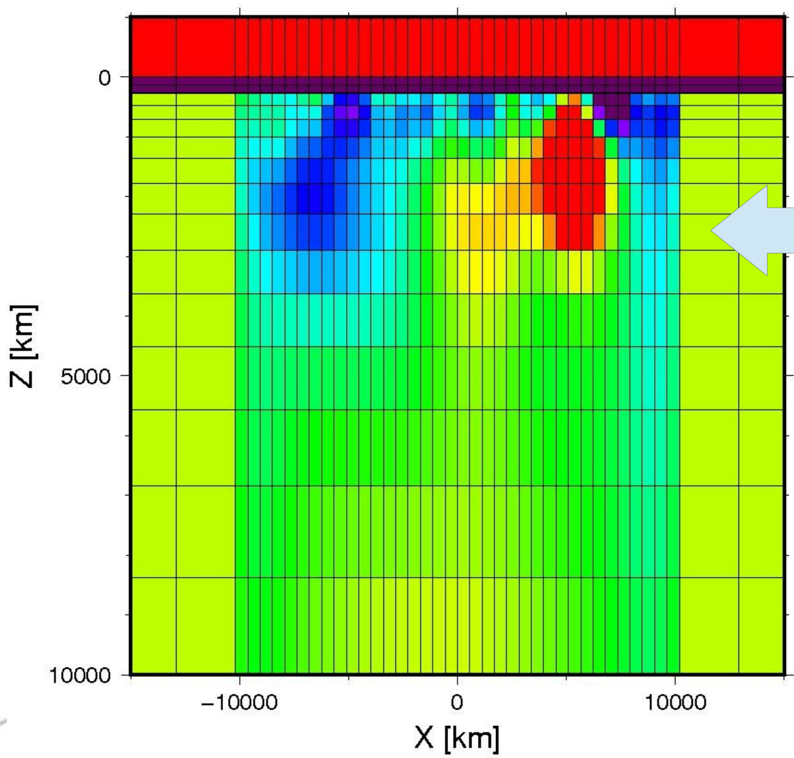
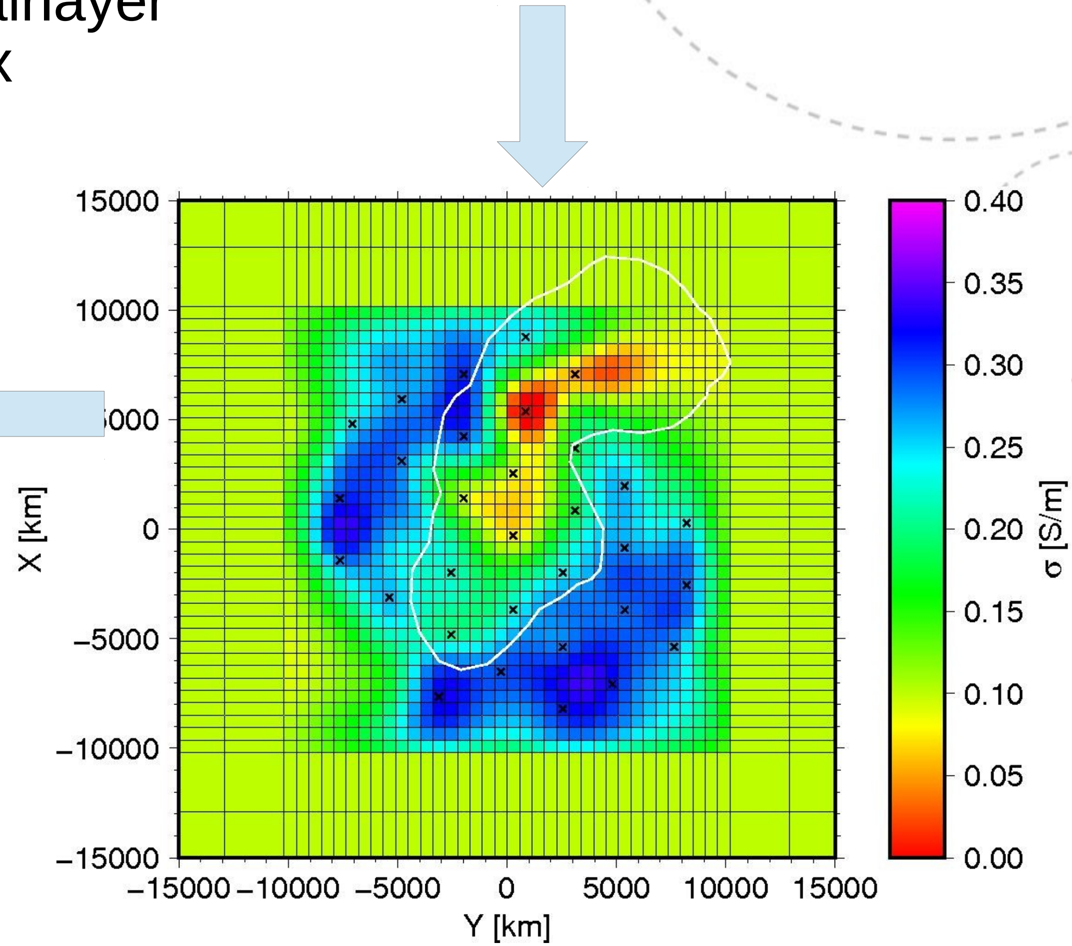




Real data example

- Model 45x45x39 cells
- Homogeneous half-space of 0.1S/m, 260m waterlayer (3.3S/m), airlayer
- 12 frequencies, Zxy, Zyx





Conclusions

- Alternative imaging methods to help seismic interpretation
- Magnetotellurics offers low resolution but good sensitivity at wider depth range

- Gauss - Newton inversion
- Good results for synthetic data

- Improve results for real data
- Incorporate gravity data

Acknowledgements

- NFR for financial support to the ROSE project
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- Ketil Hokstad and Bjørn Ursin for their supervision

Forskningsrådet

Statoil

GDF SUEZ

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Literature

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T. Wiik, L. Løseth, B. Ursin and K. Hokstad, 2011, *TIV contrast source inversion of mCSEM data*: Geophysics 76

T. Wiik, K. Hokstad, B. Ursin and Lutz Mütschard, *Joint inversion of mCSEM and MT data*: submitted to Geophysical Prospecting

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