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Similarities and differences between static and dynamic moduli

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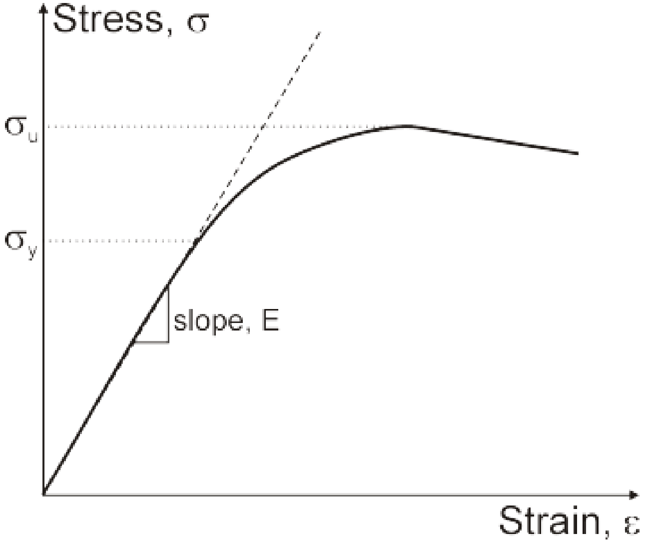
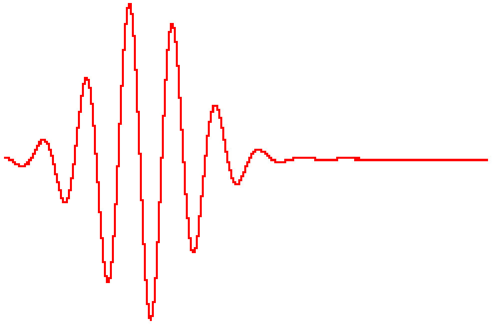
Static & Dynamic Moduli

- (Quasi-) Static modulus given by the slope of a stress-strain curve

- Dynamic modulus = $r\ v^2$ given by the bulk density r and the wave speed v





For uniaxial strain, the static modulus $H=ds_z/de_z$ should be equal to $r\ v_P^2$

Static & Dynamic Moduli: Motivation

Static mechanical behaviour is needed for several engineering applications

- Ø Reservoir compaction & Surface Subsidence (elasticity, plasticity)
- Ø Sand production prediction (strength, plasticity)
- Ø Borehole stability assessment (strength, plasticity)
- Ø Overburden characterization (Cap rock seal; Leakage / fault reactivation / fracturing associated with depletion / injection)
- Ø (Gas) Shale reservoir stimulation: Where to fracture ("Frackability" - Brittleness / Fracture toughness); Where do fractures go?

Can static properties be estimated from seismic / log measurements?

Static & Dynamic Moduli: Equal – and yet Different...

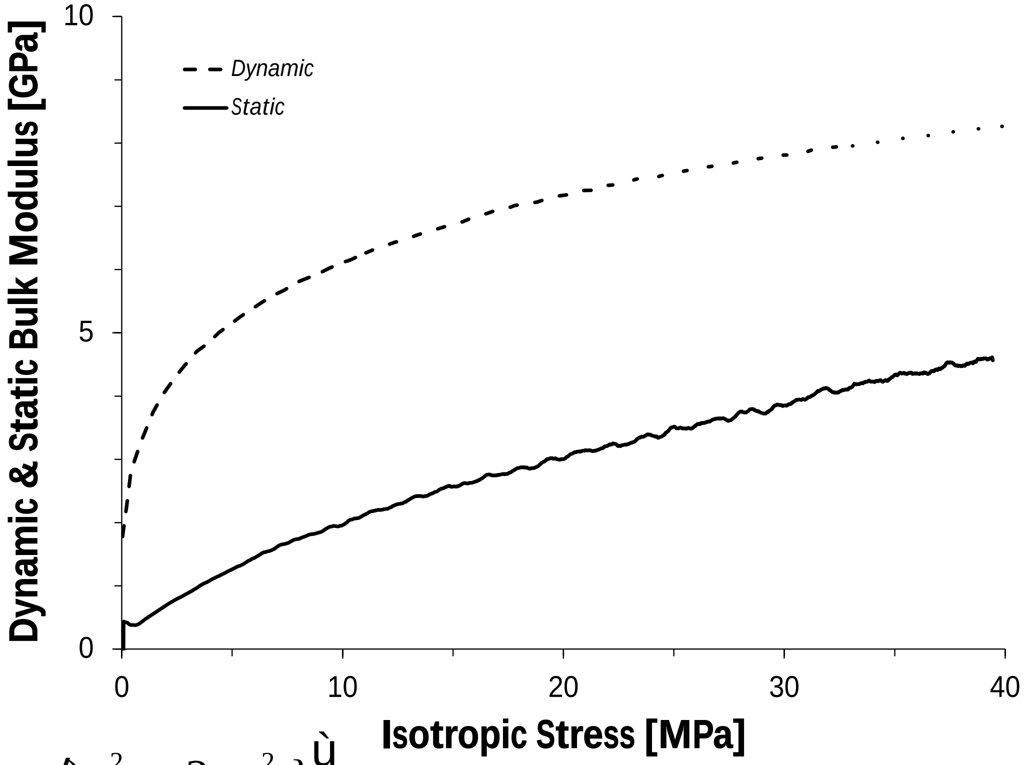
- In solids and fluids static = dynamic moduli (Ledbetter, 1993)
- In rocks they differ, because:
  - Required static moduli are usually drained ("frame") properties, dynamic moduli are undrained
  - Finite strain in static, infinitesimal strain in dynamic measurements
  - Frequency dependence (dispersion)

Typical observation: Castlegate sandstone under hydrostatic loding

Soft, high porosity sandstone:

$K_{dyn} >> K_{stat}$

$$K_{stat} = \frac{\Delta\sigma}{\Delta\varepsilon_{vol}}$$



$$K_{dyn} \approx \rho\left[(v_{Pz}^2 - \frac{4}{3}v_{Sz}^2) + \frac{4}{9}\{(2\varepsilon+\delta)v_{Pz}^2 - 2\gamma v_{Sz}^2\}\right]$$

Castlegate sandstone under uniaxial strain (K_0) loading + un- & re-loading

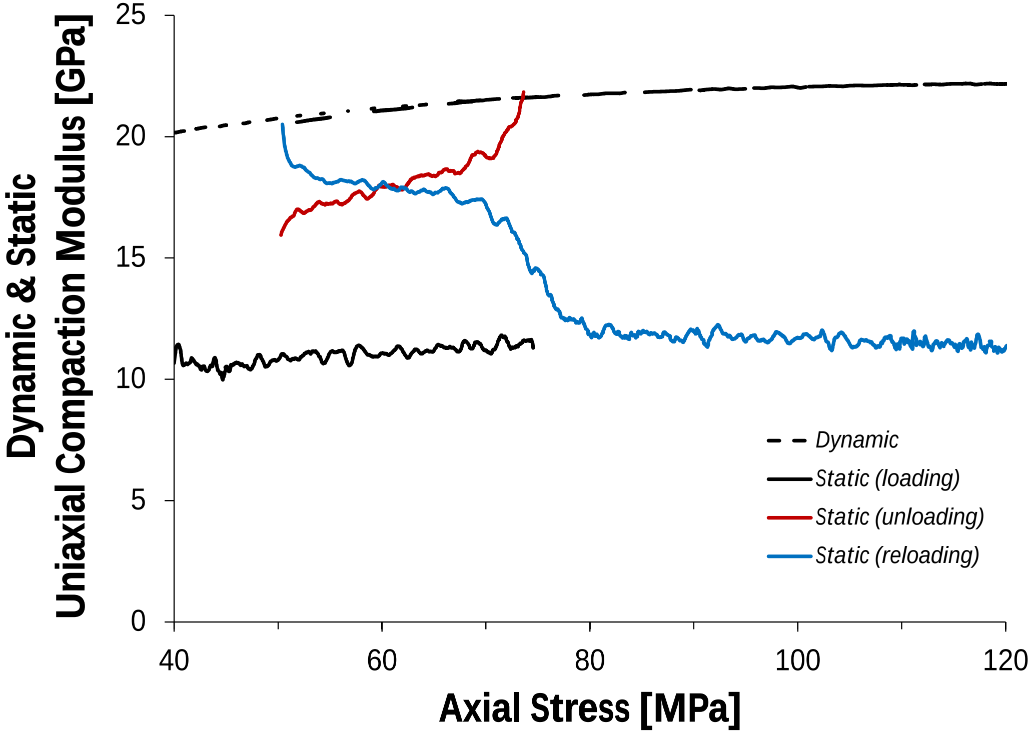
Soft, high porosity sandstone:

$H_{dyn} >> H_{stat}$

Except during initial unloading & initial reloading (turning points of the stress path)

$$H_{stat} = \frac{\Delta\sigma_z}{\Delta\varepsilon_z}$$

$$H_{dyn} = \rho v_{Pz}^2$$



Further discussion in Fjær et al., Rock Mech. & Rock Eng., 2013

Laboratory Simulated Core Behaviour of Stiff Synthetic Sandstone formed under stress

Uniaxial Strain (K_0)

$H_{dyn} >> H_{stat}$

During initial unloading & initial reloading (turning points of the stress path)

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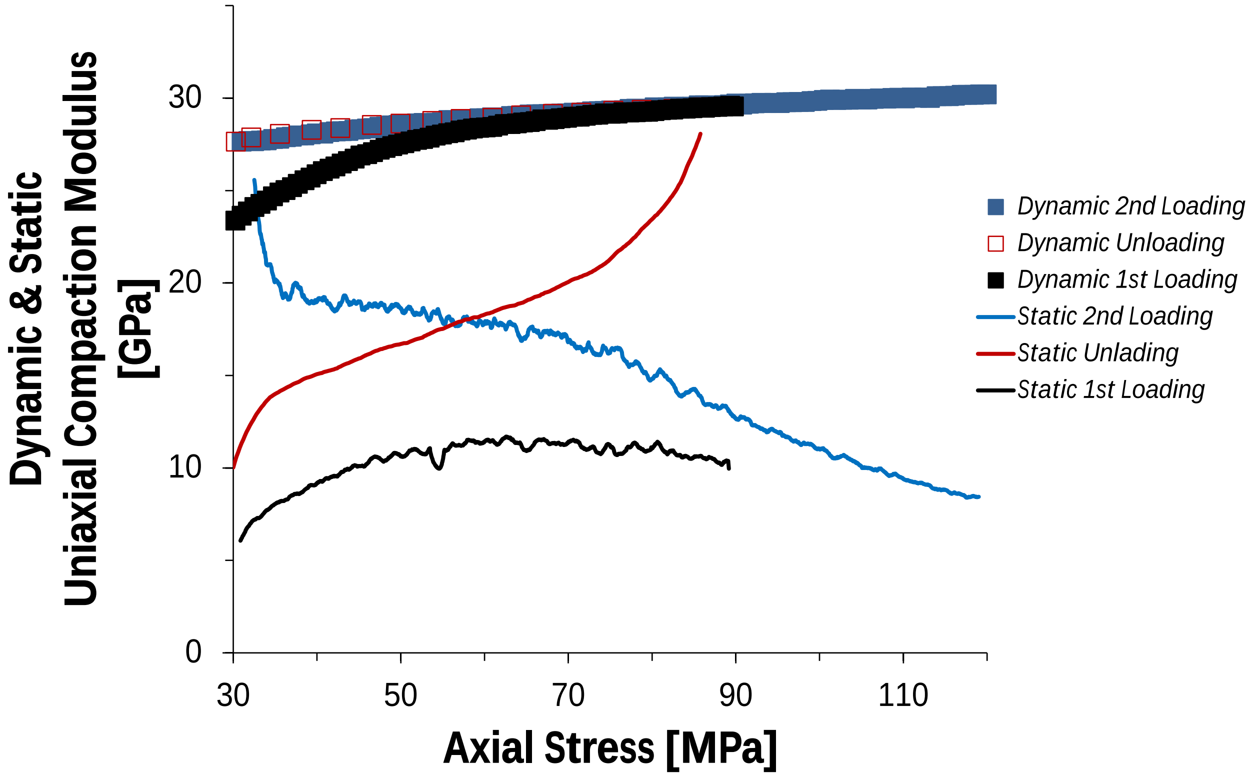
Behaves similar to Castlegate sst

Forming Stress:
30 MPa axial
15 MPa confining



Epoxy-cemented synthetic sandstone

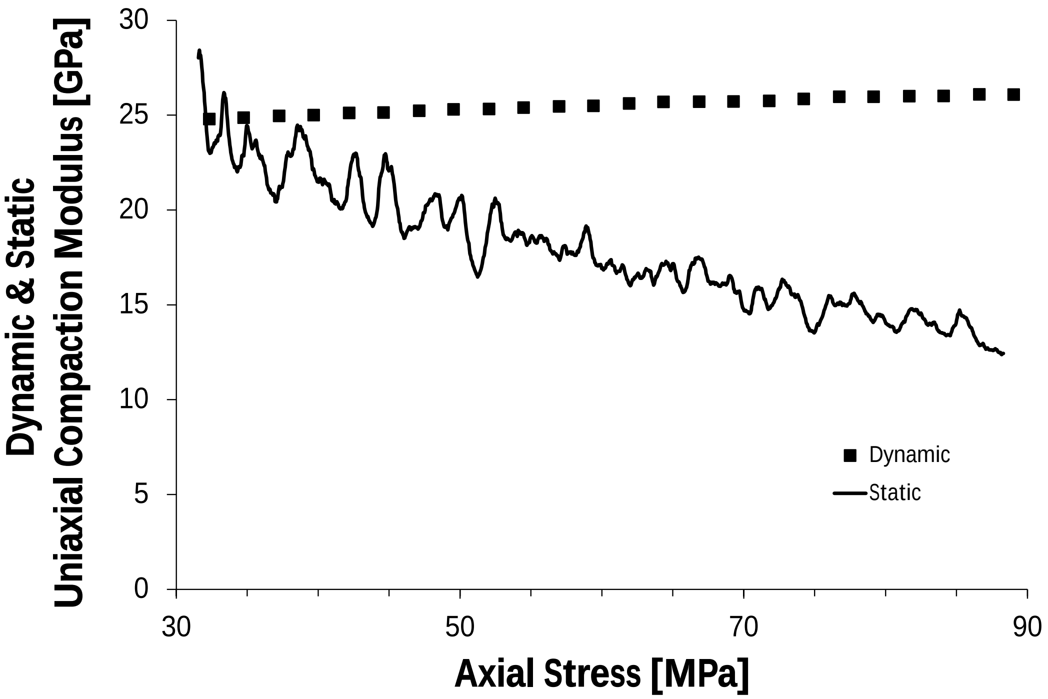
Laboratory Simulated Virgin Behaviour of Stiff Synthetic Sandstone formed under stress

Uniaxial Strain (K_0)

$H_{dyn} >> H_{stat}$

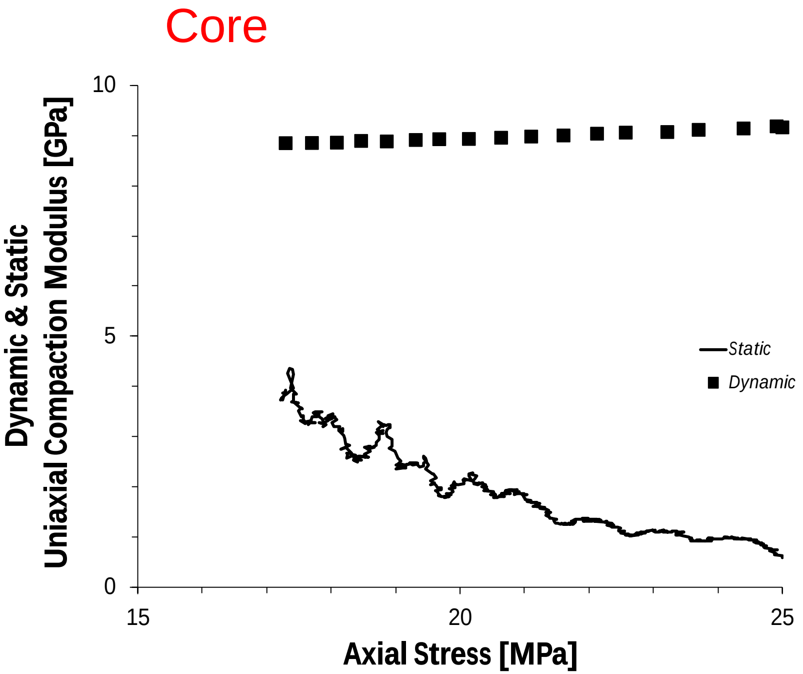
Except during initial loading

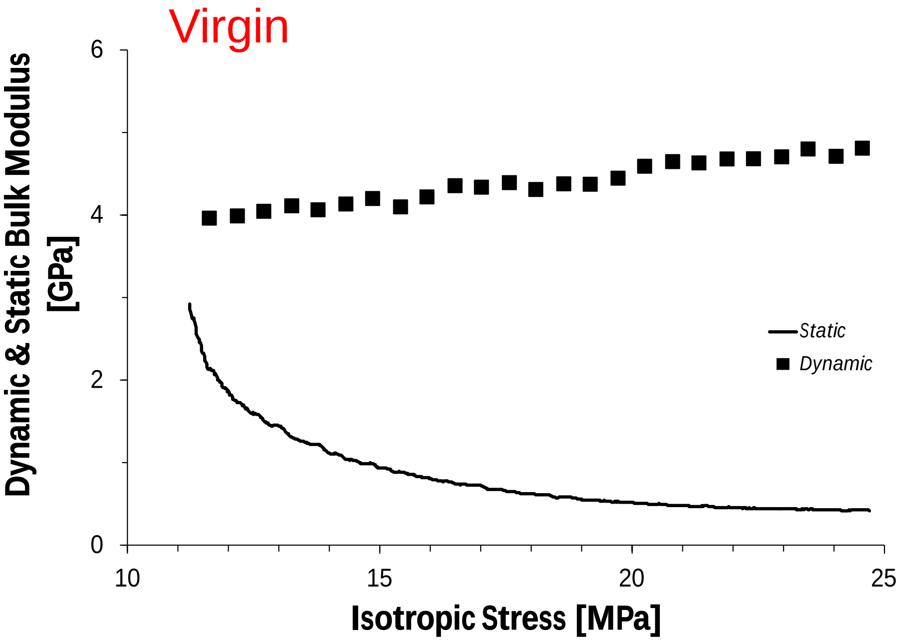
Forming Stress:
30 MPa axial
15 MPa confining



Epoxy-cemented synthetic sandstone

Laboratory simulated Core & Virgin Compaction of Soft Synthetic Sandstone





Forming Stress:
17 MPa axial
8.5 MPa confining

Forming Stress:
11 MPa axial = confining

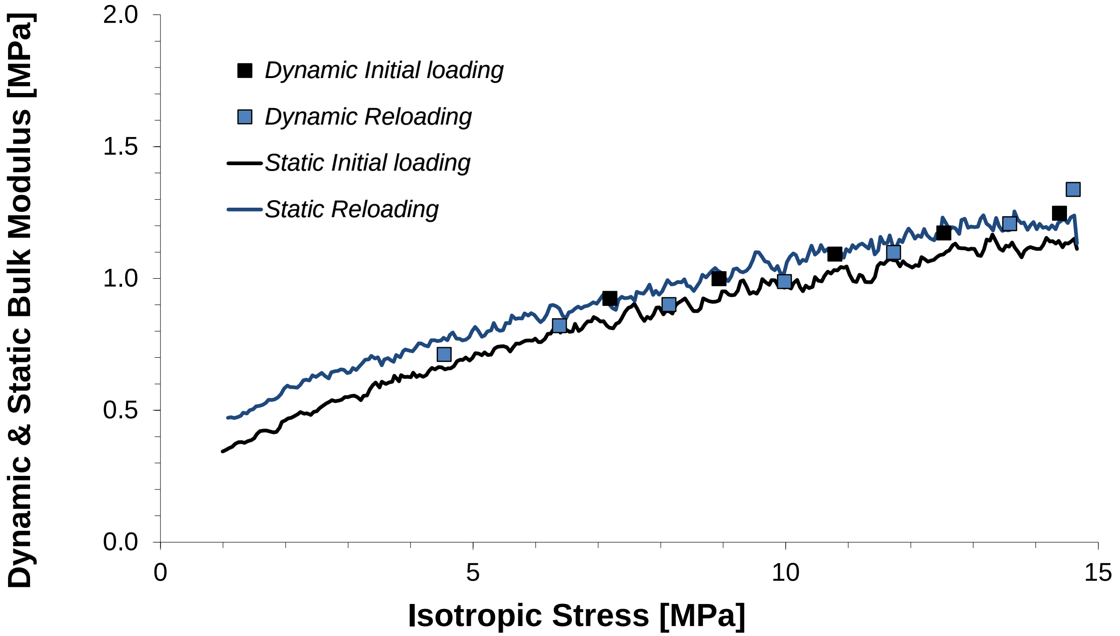
Silicate-cemented synthetic sandstone

… which leads to the hypothesis

Ø **Static = Dynamic moduli for undamaged rock – which behaves purely elastic**

  - Ø Cores are "damaged" because of cement bond breakage during stress release
  - Ø Outcrops are "damaged" because of a.o. weathering
  - Ø "Undamaged" rocks within the Earth, after diagenesis and before any damaging stress change has occurred, will be perfectly elastic
  - Ø Strong & stiff rocks should have static = dynamic moduli, for weak & soft rocks: static < dynamic stiffness… or?

But: Consider uncemented glass beads...

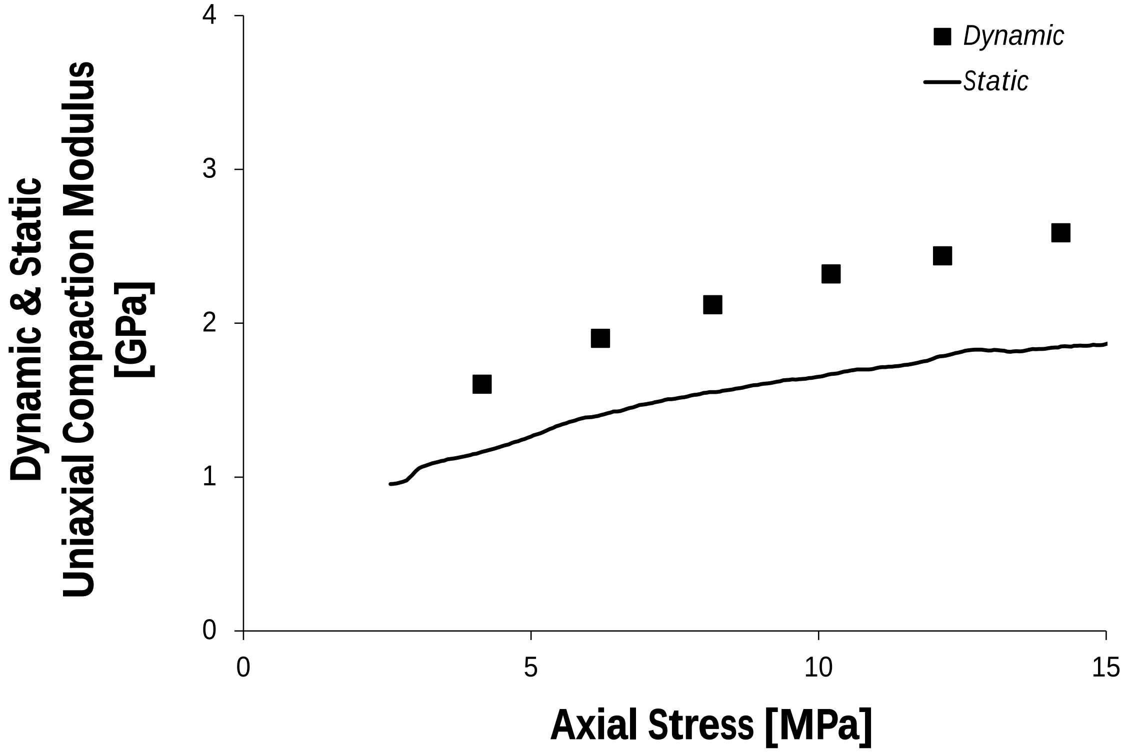


So:

For hydrostatically loaded perfectly spherical particles (with narrow size distribution):
Static = Dynamic Bulk Modulus!

Uncemented glass beads in
Uniaxial compaction

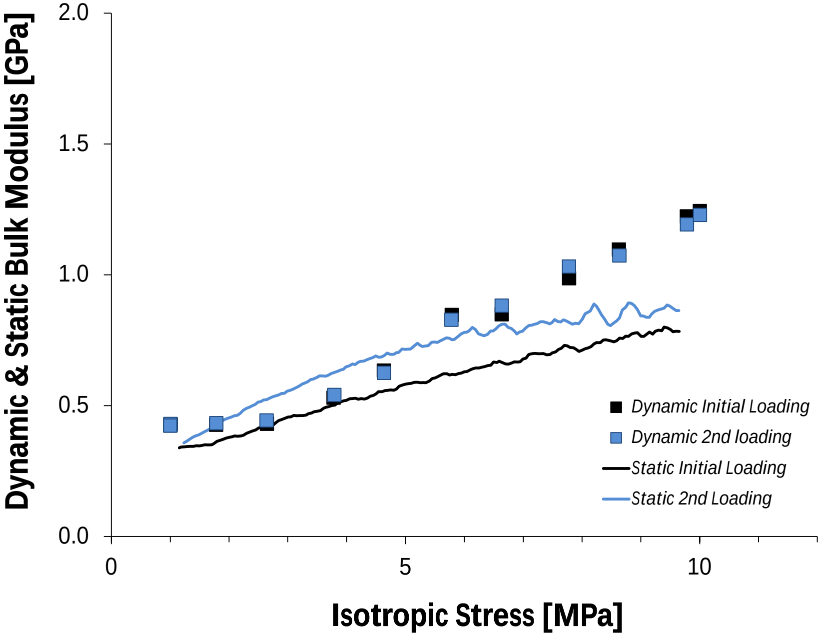
Development of shear stress reduces the static modulus by grain sliding and rearrangement

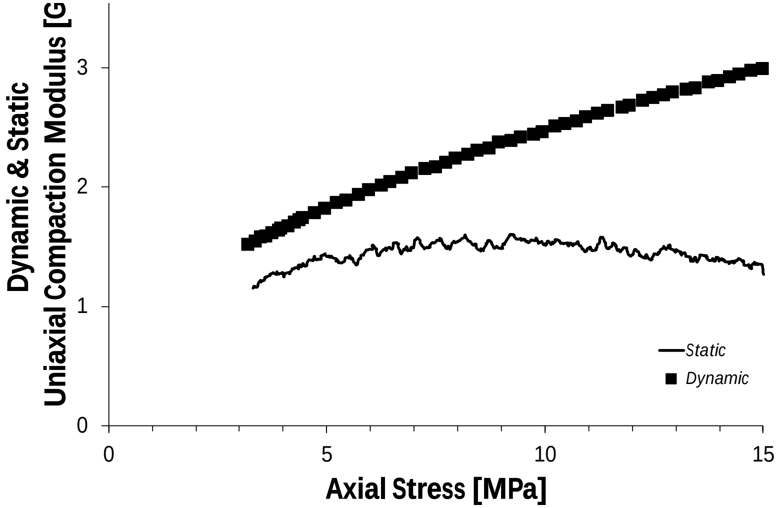


Ottawa Sand

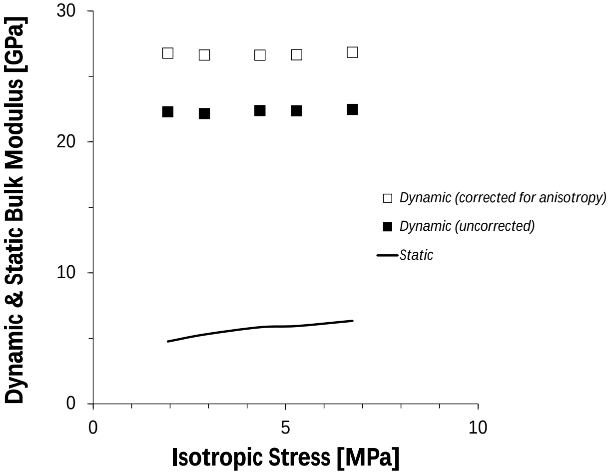
Similar behavior to glass beads, but static & dynamic moduli become more different

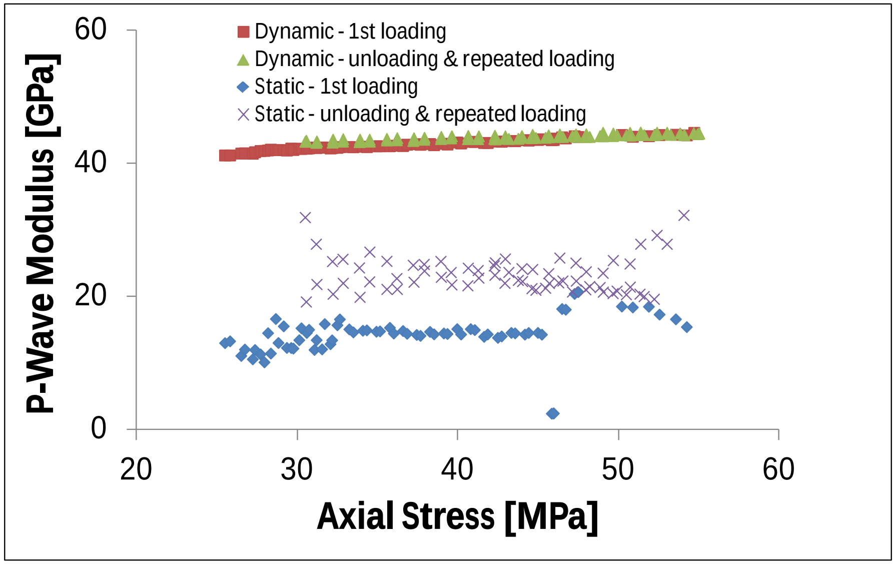
Natural sand is well rounded, but not perfect spheres (also broader grain size distribution)





Mancos Shale





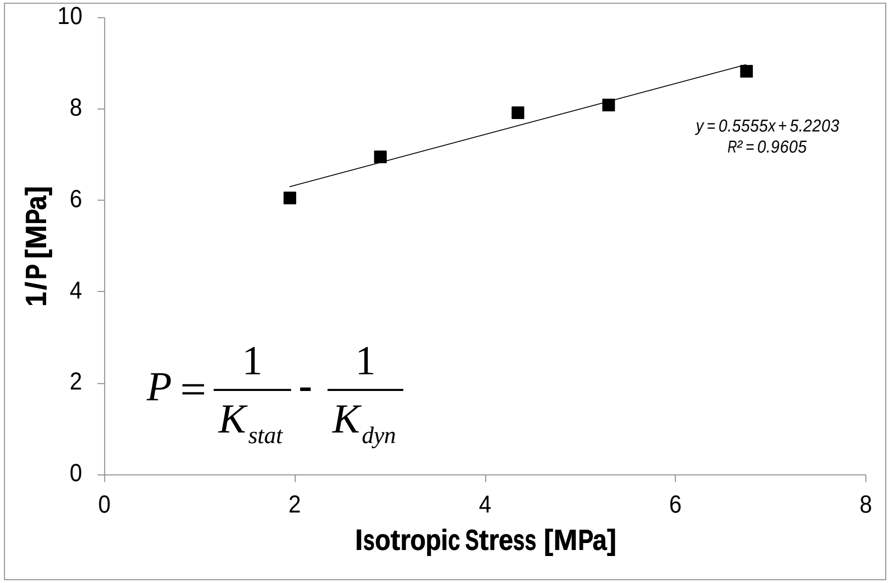
6-8 % porosity, 20-30 % clay, very competent gas shale analogue

Tested as received; i.e. "wet"

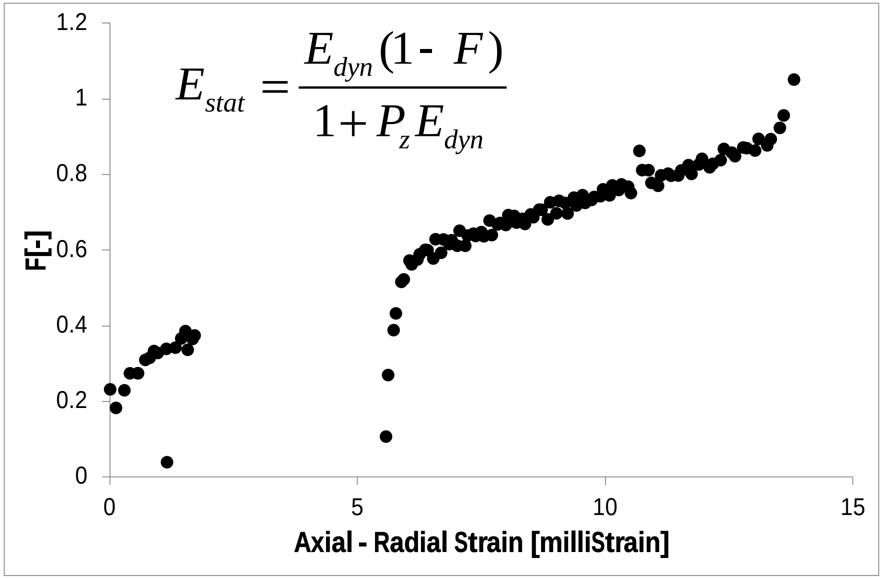
Static drained bulk modulus ~ 5 GPa;
Uniaxial compaction modulus:
15 – 20 GPa (20-25 GPa during unloading)

Ultrasonic bulk modulus: ~ 25 GPa
Ultrasonic P-Wave modulus:
>40 GPa

Strain amplitude effects in shale



$$P = \frac{1}{K_{stat}} - \frac{1}{K_{dyn}}$$



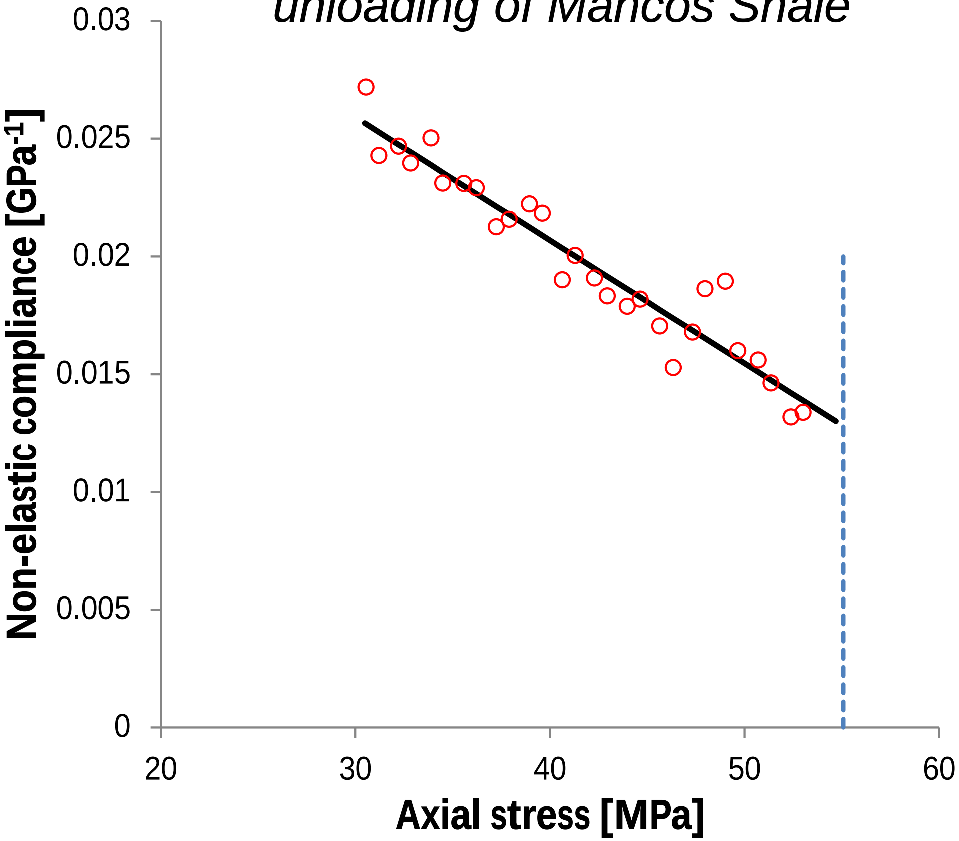
$$E_{stat} = \frac{E_{dyn}(1 - F)}{1 + P_z E_{dyn}}$$

From a triaxial test, the dependence of P and F *("Petroleum Related Rock Mechanics" by Fjær et al., 2008)* on stress and strain as observed in Mancos Shale is resemblant to that seen in soft sandstones =>

Strain amplitude correction for shale may be performed in a similar way

Mancos shale

Data from uniaxial strain (K_0) unloading of Mancos Shale



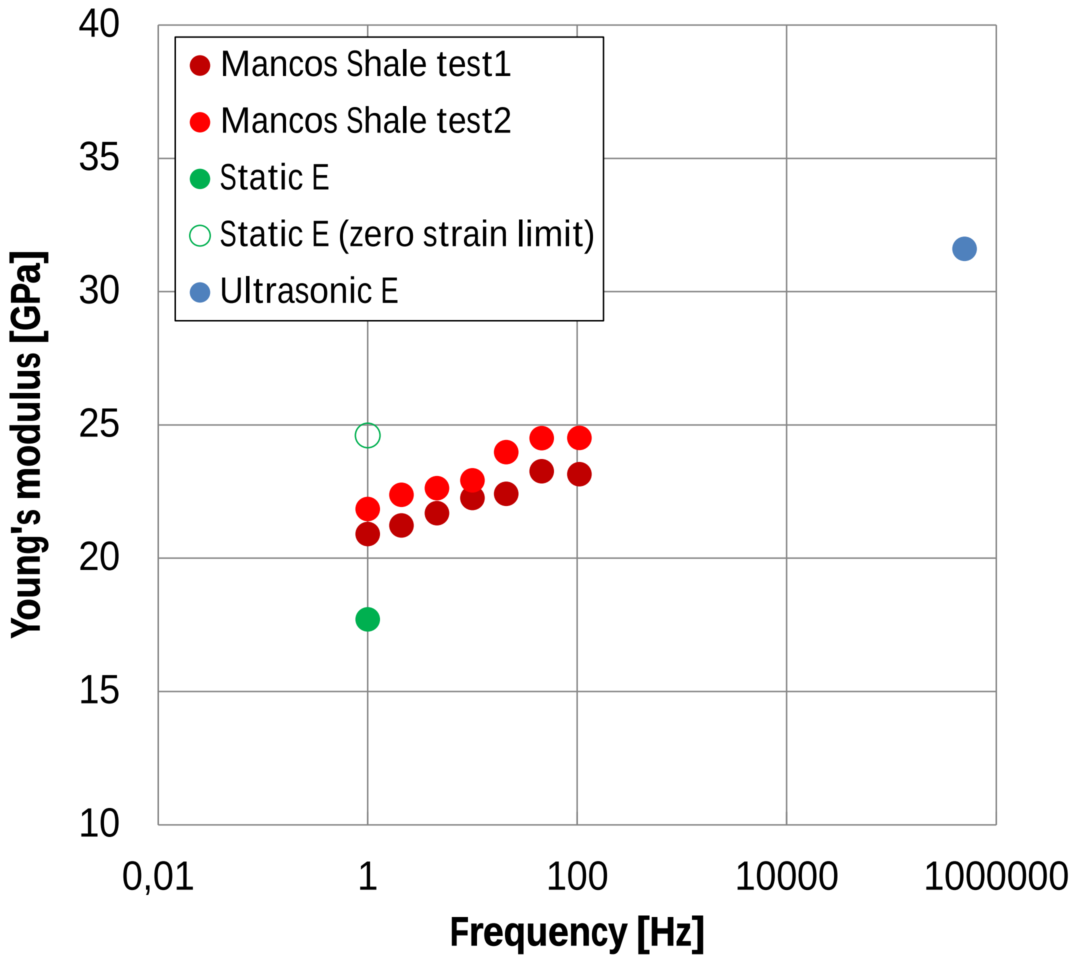
The extrapolated elastic uniaxial compaction modulus estimated at 55 MPa axial (& 18 MPa confining) stress corresponds to a P-wave velocity of ~ 3315 m/s at 1 Hz frequency.

The ultrasonic v_P is ~ 4165 m/s at 500 kHz

=> ~ 25 % velocity dispersion

Based on evidence for purely elastic behavior at the turning point of a stress path (Fjær *et al*., 2013)

Mancos Shale



Measured at ambient conditions in SINTEF's Low Frequency Quasi-Static set-up

Strain amplitude ~ 10^{-6}

Confirms dispersion in Mancos shale

In Mancos Shale, both frequency dependence and strain amplitude effects contribute to the difference between static and dynamic moduli

Concluding remarks

- **Static = Dynamic moduli for undamaged (purely elastic) non-dispersive rock, and for Hertzian granular materials under hydrostatic conditions**
  - "Undamaged" rocks within the Earth, after diagenesis and before any damaging stress change has occurred may have similar static and dynamic moduli
  - Unconsolidated sand: May have quite similar static and dynamic moduli in hydrostatic conditions
  - (Gas) Shale: Both dispersion and non-elastic behavior leads to static-dynamic discrepancy
    - If we can distinguish, this may permit determination of plasticity / brittleness from seismic data

Acknowledgement

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SINTEF NTNU

References

- The main contents of this presentation will be published in:
  - Holt, R.M., Fjær, E., and Bauer, A. (2013) Static and Dynamic Moduli – so equal, and yet so different. ARMA 13-521; 8 pp. To be pres. At 47th US Rock Mechanics / Geomechanics Symposium, San Francisco 23-26 June.
- Other references:
  - Ledbetter H (1993) Dynamic vs. static Young's moduli: a case study. Mater Sci Eng A165:L9–L10
  - Fjær, E., Stroisz, A. M., and Holt, R. M. (2013) Elastic dispersion derived from a combination of static and dynamic measurements. Rock mech Rock Eng; DOI 10.1007/s00603-013-0385-8. 8 pp.
  - Holt, R. M., Nes, O.-M., Stenebråten, J.F., and Fjær, E. (2012) Static vs. Dynamic behavior of shale. ARMA 12-542 7 pp.