Bayesian inversion of time-lapse seismic data for porosity, pressure and saturation changes

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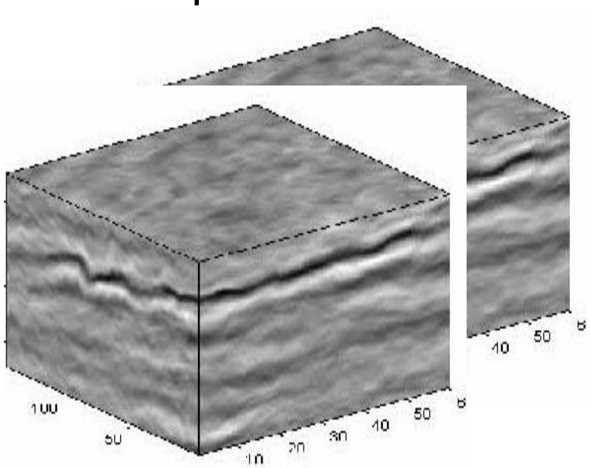
Introduction

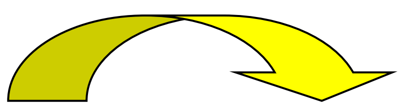
- In time-lapse studies we aim to estimate *pressure and saturation changes*

- Changes in dynamic properties can be measured from well/lab data. The main source of information are time-lapse seismic data.

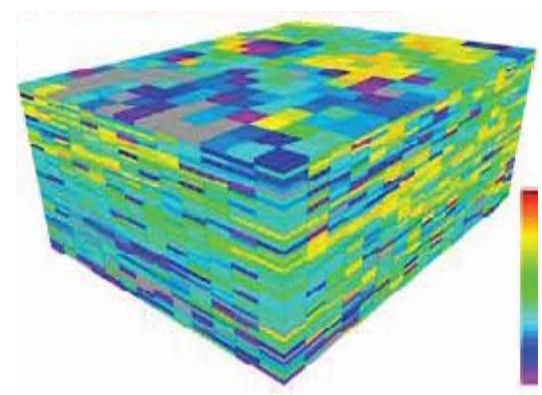
Time-lapse seismic data



Inverse problem



Dynamic property changes



Motivation

- Inverted data can be used in seismic history matching to improve the reservoir description

- The probabilistic approach allows to quantify the uncertainty in predicted data

Introduction

- In pressure-saturation estimation the physical model is not linear and the dynamic property changes are not normally distributed.
- Changes in dynamic properties are not independent of initial rock properties (static reservoir model)

Bayesian inversion

- We propose a hierarchical ***Bayesian approach*** for the simultaneous estimation of porosity jointly with pressure and saturation changes from time-lapse seismic data.

Bayesian inversion

- We propose a hierarchical ***Bayesian approach*** for the simultaneous estimation of porosity jointly with pressure and saturation changes from time-lapse seismic data.

$$\begin{bmatrix} \mathbf{S}^{t_1} \\ \vdots \\ \mathbf{S}^{t_N} \end{bmatrix} = \mathbf{F}\left(\begin{bmatrix} \phi \\ \Delta sw \\ \Delta p \end{bmatrix} \right)$$

Bayesian inversion

- We propose a hierarchical ***Bayesian approach*** for the simultaneous estimation of porosity jointly with pressure and saturation changes from time-lapse seismic data.

$\mathbf{S}, \Delta\mathbf{S}$	Seismic data, Changes in seismic data
$\mathbf{m}, \Delta\mathbf{m}$	Elastic properties, Changes in elastic properties (velocities or impedances)
$\mathbf{R}$	Reservoir properties (porosity)
$\Delta\mathbf{D}$	Changes in dynamic properties (saturation and pressure)

Bayesian inversion

- Seismic data **S** depend on reservoir properties **R** through elastic properties **m**

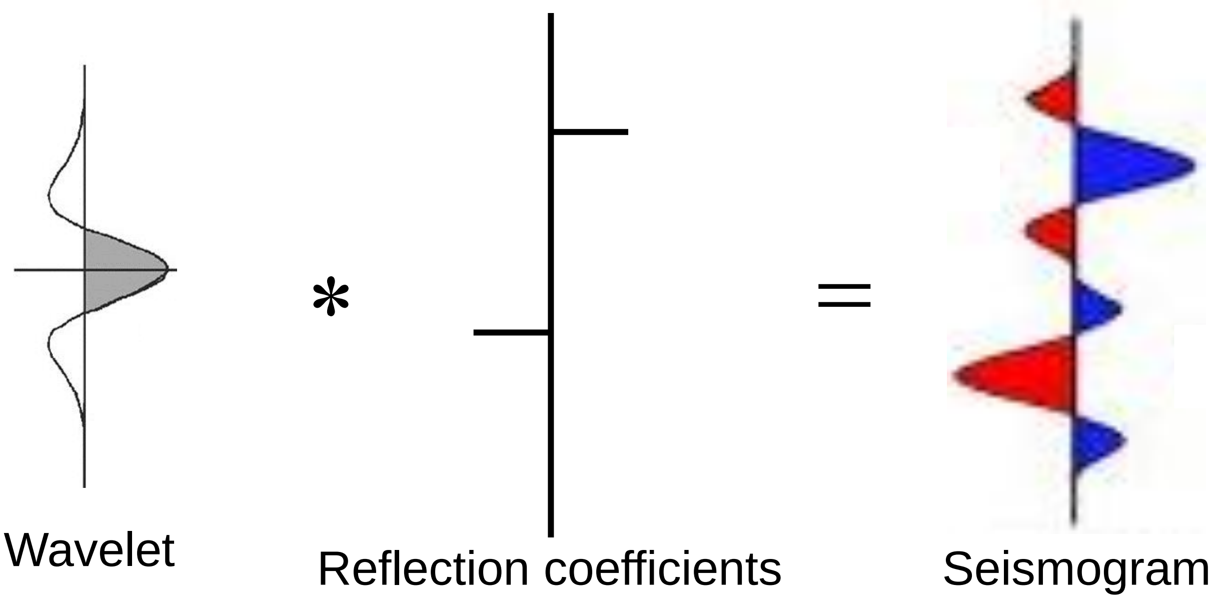
- We can split the inverse problem into two sub-problems:
  - $\mathbf{m} = g(\mathbf{S})$ — g seismic linearized modeling
  - $\mathbf{R} = f(\mathbf{m})$ — f rock physics model

$$\mathbf{R}(x,y,z) = f(g(\mathbf{S}(x,y,z)))$$

Physical model

Seismic forward model:

- Wavelet convolution
- Linearized Aki-Richards approximation of Zoeppritz equations



$$r_{PP}(\theta) = h(\mathbf{m}, \theta)$$

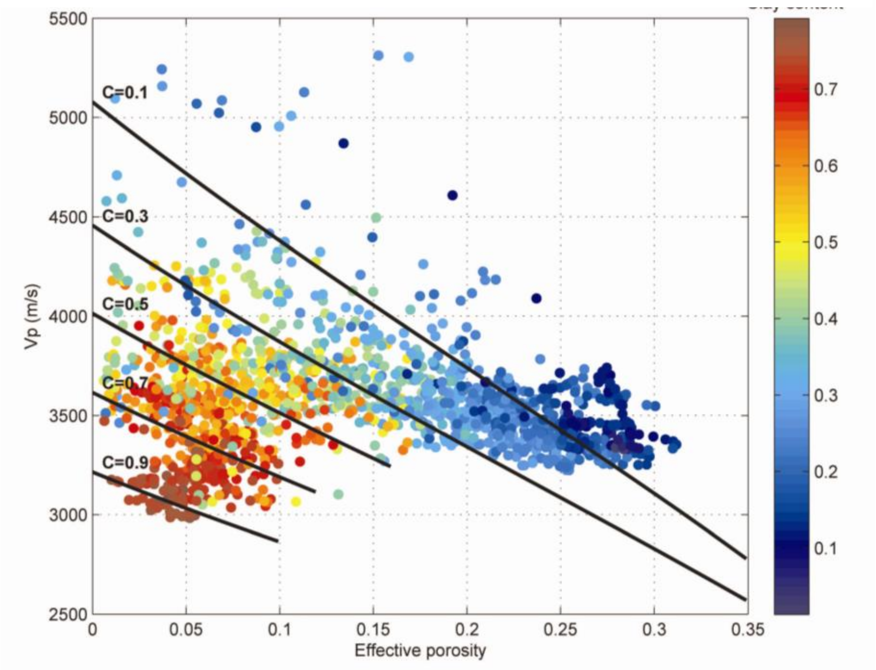
Physical model

Rock physics forward model:

- Granular media models (Hertz-Mindlin contact theory)
- Gassmann's equations
- Velocity-pressure relations (modified MacBeth eq.)

$$\begin{bmatrix} V_P \\ V_S \\ \rho \end{bmatrix} = \mathbf{f}_{RPM}\left(\begin{bmatrix} \phi \\ sw \\ p \end{bmatrix}\right)$$

P-wave velocity versus effective porosity



Outline

- Introduction
- Theory and Inversion workflow
- Application
- Conclusions

Inversion workflow

1. We first estimate

$$P\left(\begin{bmatrix}\mathbf{m}\\ \Delta\mathbf{m}\end{bmatrix} \middle| \begin{bmatrix}\mathbf{S}\\ \Delta\mathbf{S}\end{bmatrix}\right) \propto P\left(\begin{bmatrix}\mathbf{S}\\ \Delta\mathbf{S}\end{bmatrix} \middle| \begin{bmatrix}\mathbf{m}\\ \Delta\mathbf{m}\end{bmatrix}\right) P\left(\begin{bmatrix}\mathbf{m}\\ \Delta\mathbf{m}\end{bmatrix}\right)$$

Buland and Omre, 2003
Buland and El Ouair, 2006

Inversion workflow

1. We first estimate

$$P(\begin{bmatrix} \mathbf{m} \\ \Delta\mathbf{m} \end{bmatrix} | \begin{bmatrix} \mathbf{S} \\ \Delta\mathbf{S} \end{bmatrix}) \propto P(\begin{bmatrix} \mathbf{S} \\ \Delta\mathbf{S} \end{bmatrix} | \begin{bmatrix} \mathbf{m} \\ \Delta\mathbf{m} \end{bmatrix}) P(\begin{bmatrix} \mathbf{m} \\ \Delta\mathbf{m} \end{bmatrix})$$

Buland and Omre, 2003
Buland and El Ouair, 2006

2. We then estimate

$$P(\begin{bmatrix} \mathbf{R} \\ \Delta\mathbf{D} \end{bmatrix} | \begin{bmatrix} \mathbf{m} \\ \Delta\mathbf{m} \end{bmatrix}) \propto P(\begin{bmatrix} \mathbf{m} \\ \Delta\mathbf{m} \end{bmatrix} | \begin{bmatrix} \mathbf{R} \\ \Delta\mathbf{D} \end{bmatrix}) P(\begin{bmatrix} \mathbf{R} \\ \Delta\mathbf{D} \end{bmatrix})$$

Inversion workflow

1. We first estimate

$$P\left(\begin{bmatrix}\mathbf{m}\\ \Delta\mathbf{m}\end{bmatrix} \middle| \begin{bmatrix}\mathbf{S}\\ \Delta\mathbf{S}\end{bmatrix}\right) \propto P\left(\begin{bmatrix}\mathbf{S}\\ \Delta\mathbf{S}\end{bmatrix} \middle| \begin{bmatrix}\mathbf{m}\\ \Delta\mathbf{m}\end{bmatrix}\right) P\left(\begin{bmatrix}\mathbf{m}\\ \Delta\mathbf{m}\end{bmatrix}\right)$$

Buland and Omre, 2003
Buland and El Ouair, 2006

2. We then estimate

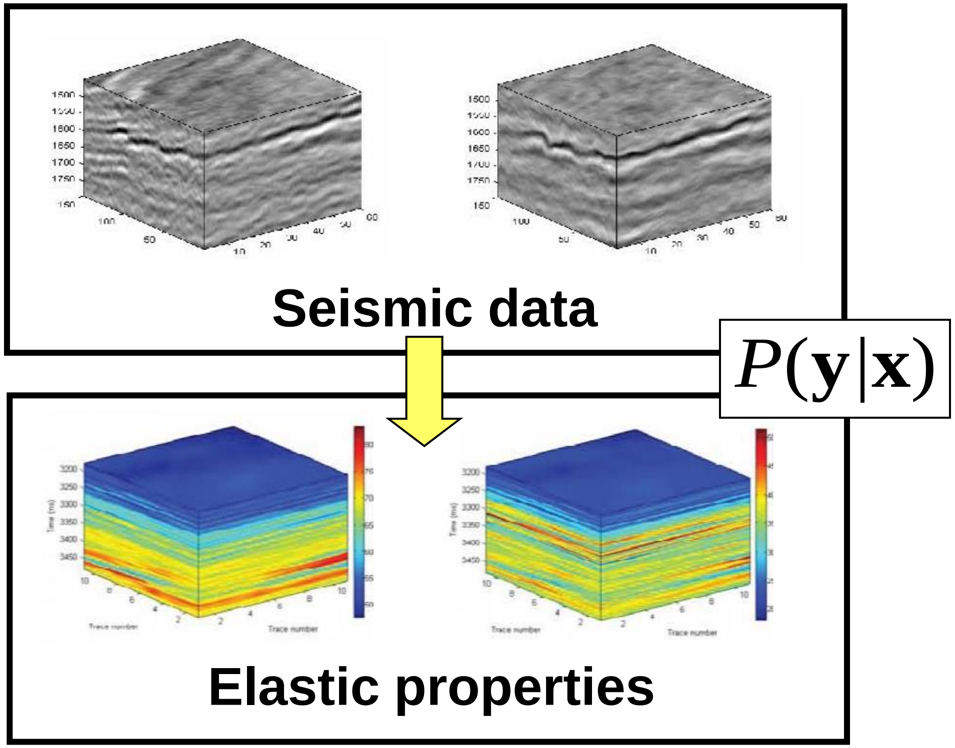
$$P\left(\begin{bmatrix}\mathbf{R}\\ \Delta\mathbf{D}\end{bmatrix} \middle| \begin{bmatrix}\mathbf{m}\\ \Delta\mathbf{m}\end{bmatrix}\right) \propto P\left(\begin{bmatrix}\mathbf{m}\\ \Delta\mathbf{m}\end{bmatrix} \middle| \begin{bmatrix}\mathbf{R}\\ \Delta\mathbf{D}\end{bmatrix}\right) P\left(\begin{bmatrix}\mathbf{R}\\ \Delta\mathbf{D}\end{bmatrix}\right)$$

3. We combine $P(\mathbf{y}|\mathbf{x})$ [1] and $P(\mathbf{w}|\mathbf{y})$ [2] using Chapman-Kolmogorov equation

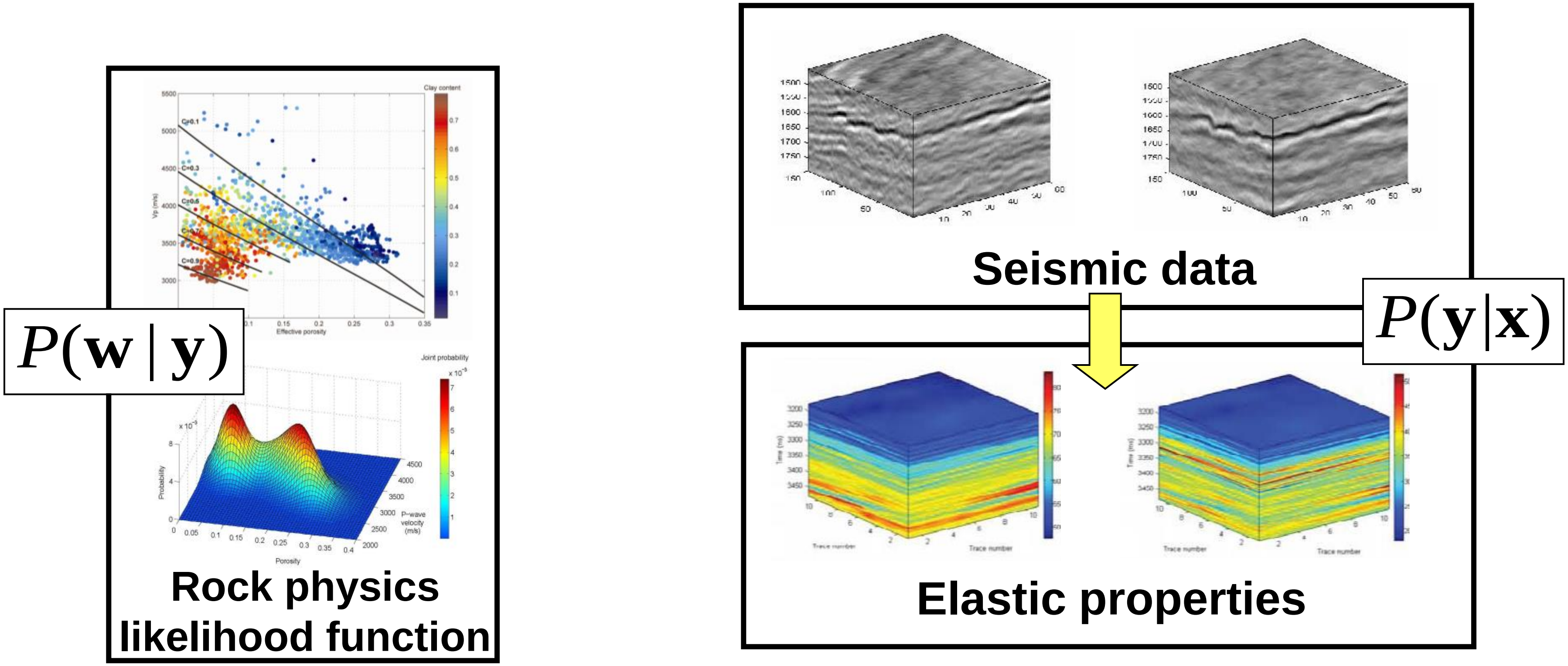
$$P(\mathbf{w} \mid \mathbf{x}) = \int_{\Re^m} P(\mathbf{w} \mid \mathbf{y}) P(\mathbf{y} \mid \mathbf{x}) dy$$

Grana and Della Rossa , 2010

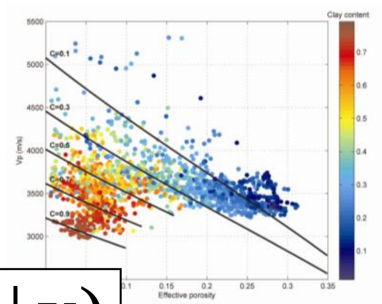
Inversion workflow

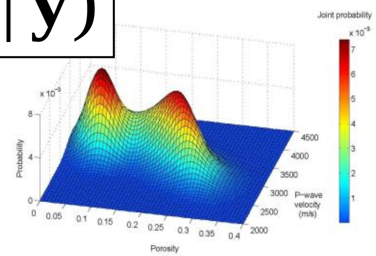


Inversion workflow



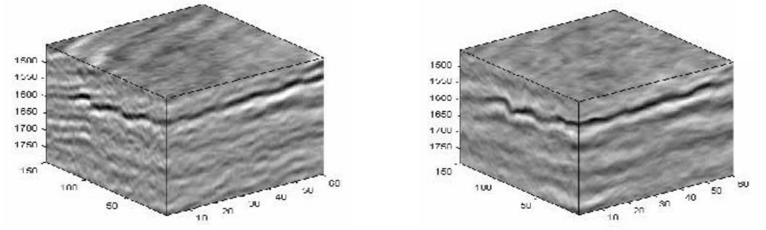
Inversion workflow





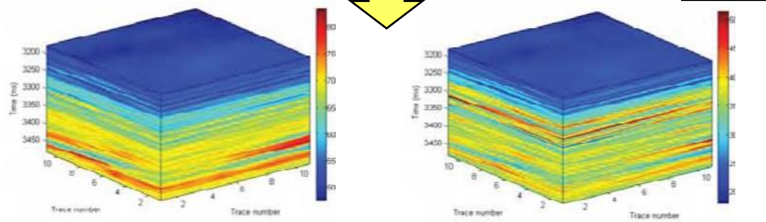
Rock physics likelihood function

$$P(\mathbf{w}\,|\,\mathbf{y})$$

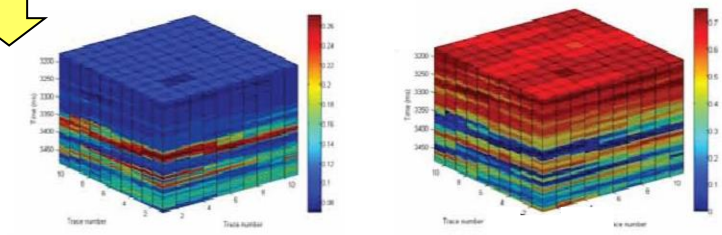


Seismic data

$$P(\mathbf{y}|\mathbf{x})$$



Elastic properties



Reservoir properties

$$P(\mathbf{w}|\mathbf{x}) = P(\begin{bmatrix}\mathbf{R}\\ \Delta\mathbf{D}\end{bmatrix} | \begin{bmatrix}\mathbf{S}\\ \Delta\mathbf{S}\end{bmatrix})$$

(Chapman-Kolmogorov)

Simultaneous Bayesian 4D inversion

Combined Inverse problem

$$\begin{bmatrix} \mathbf{S}^{base} \\ \Delta\mathbf{S} \end{bmatrix} = \begin{bmatrix} \mathbf{G} & \mathbf{0} \\ \mathbf{0} & \mathbf{G} \end{bmatrix} \begin{bmatrix} \mathbf{m} \\ \Delta\mathbf{m} \end{bmatrix} + \begin{bmatrix} \boldsymbol{\varepsilon} \\ \mathbf{e} \end{bmatrix}$$

We estimate the posterior distribution

$$P(\begin{bmatrix} \mathbf{m} \\ \Delta\mathbf{m} \end{bmatrix} | \begin{bmatrix} \mathbf{S} \\ \Delta\mathbf{S} \end{bmatrix}) \propto P(\begin{bmatrix} \mathbf{S} \\ \Delta\mathbf{S} \end{bmatrix} | \begin{bmatrix} \mathbf{m} \\ \Delta\mathbf{m} \end{bmatrix}) P(\begin{bmatrix} \mathbf{m} \\ \Delta\mathbf{m} \end{bmatrix})$$

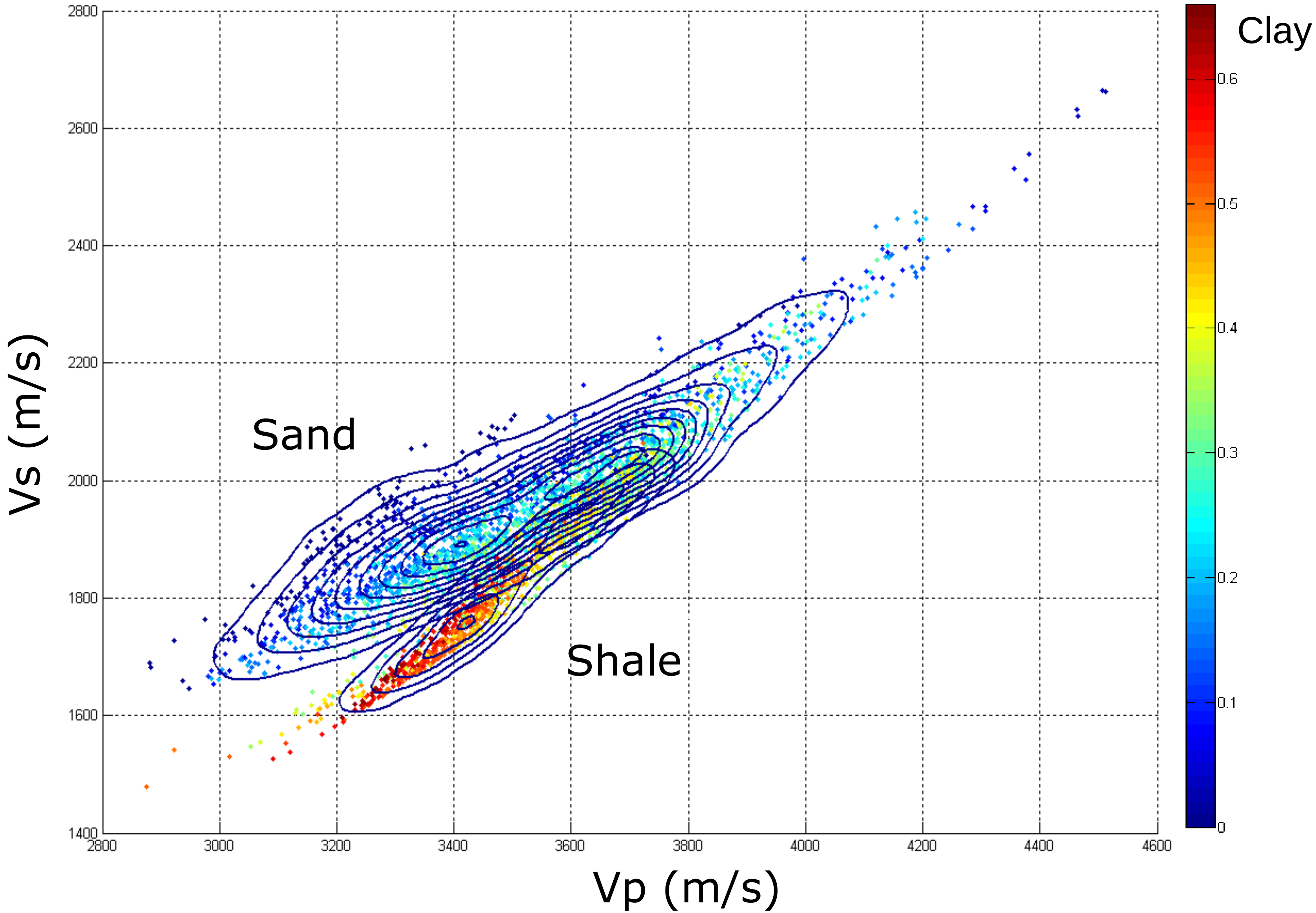
Reservoir property estimation

Using statistical rock physics we estimate the likelihood

$$P\left(\begin{bmatrix} \mathbf{R} \\ \Delta\mathbf{D} \end{bmatrix} \middle| \begin{bmatrix} \mathbf{m} \\ \Delta\mathbf{m} \end{bmatrix}\right) \propto P\left(\begin{bmatrix} \mathbf{m} \\ \Delta\mathbf{m} \end{bmatrix} \middle| \begin{bmatrix} \mathbf{R} \\ \Delta\mathbf{D} \end{bmatrix}\right) P\left(\begin{bmatrix} \mathbf{R} \\ \Delta\mathbf{D} \end{bmatrix}\right)$$

We use non-parametric pdfs and we estimate them using Kernel Density Estimation (KDE)

Non-parametric pdfs: KDE



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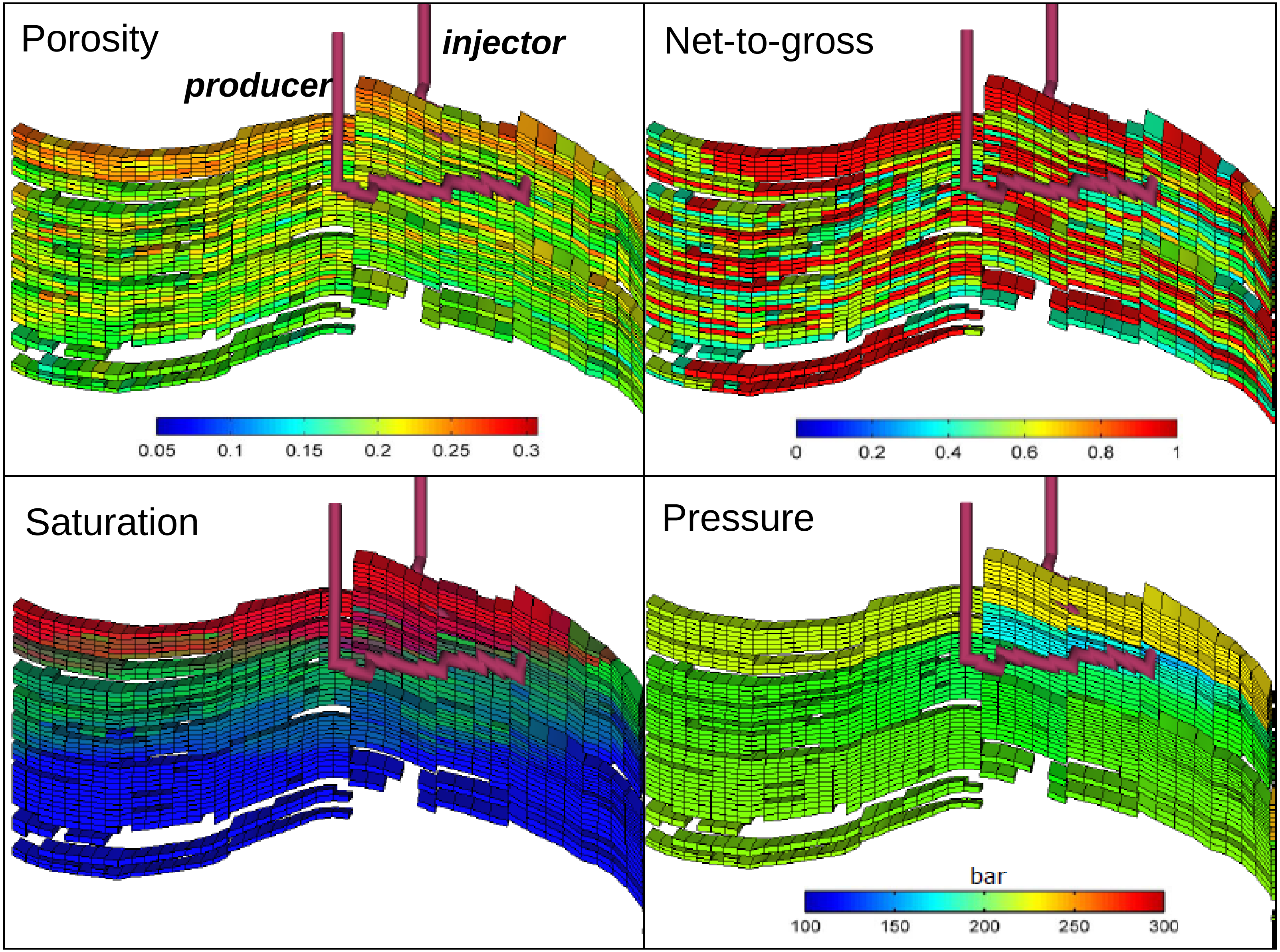
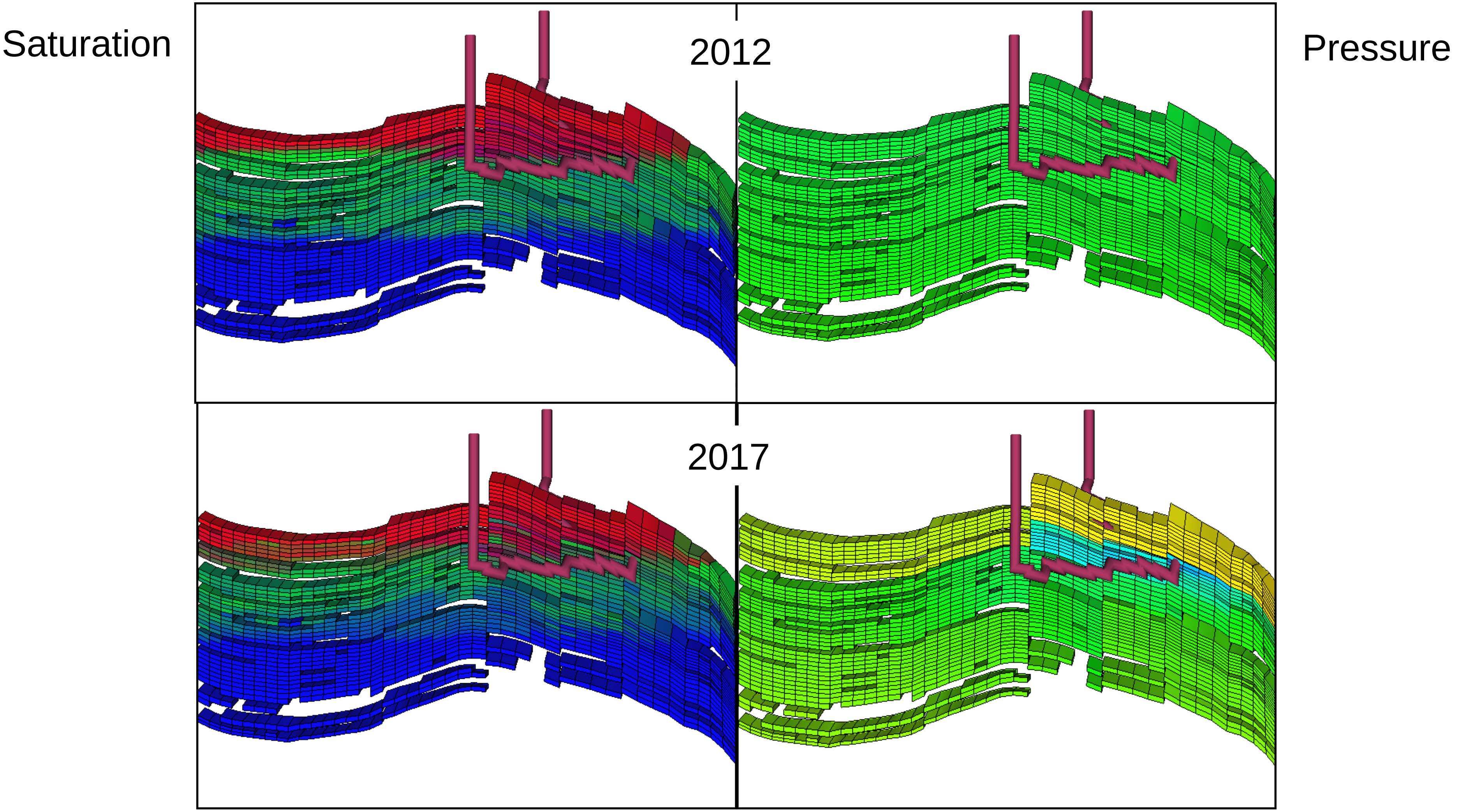
2D application: synthetic model (from Eclipse)



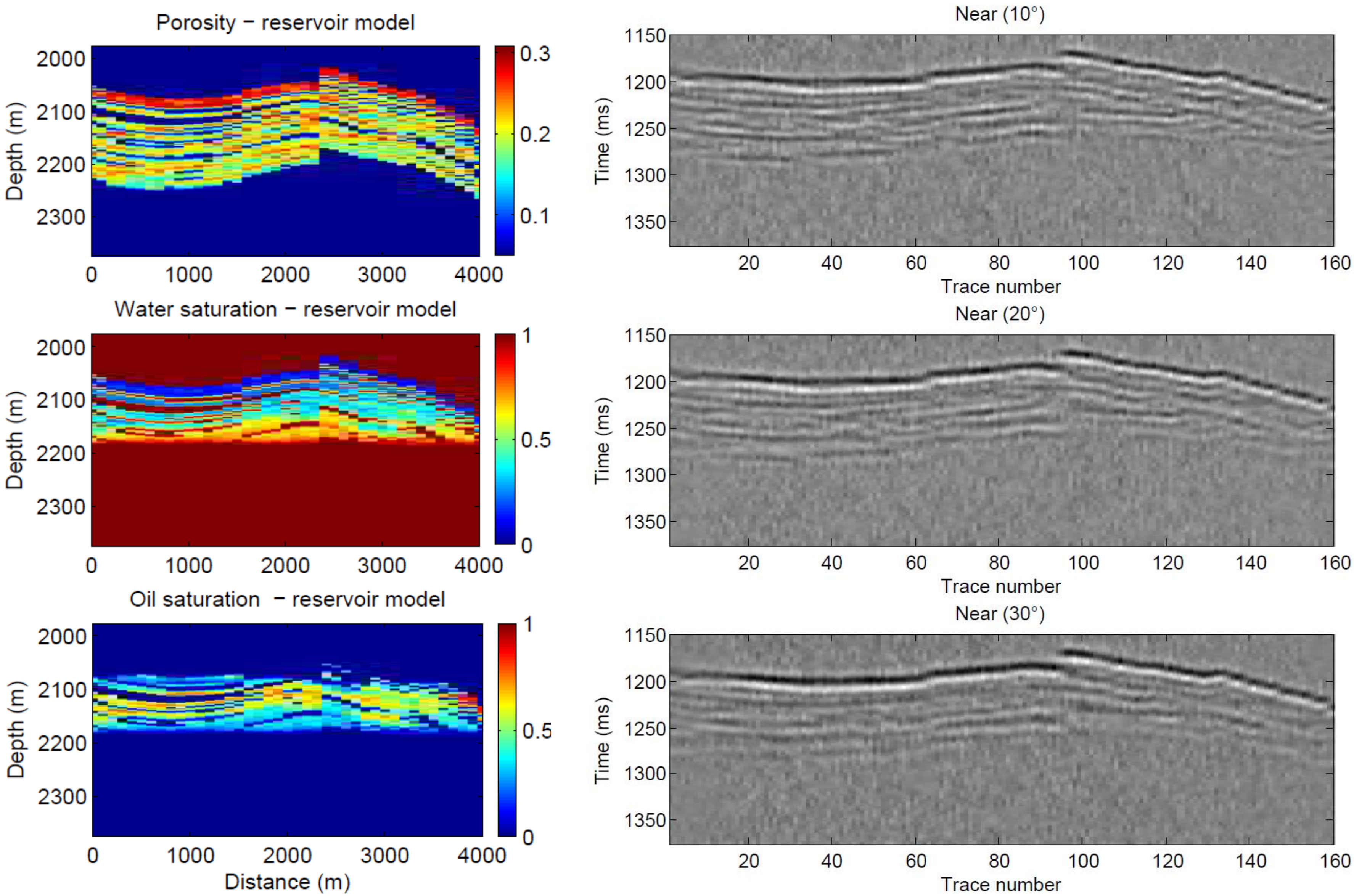
2D application: synthetic model (from Eclipse)



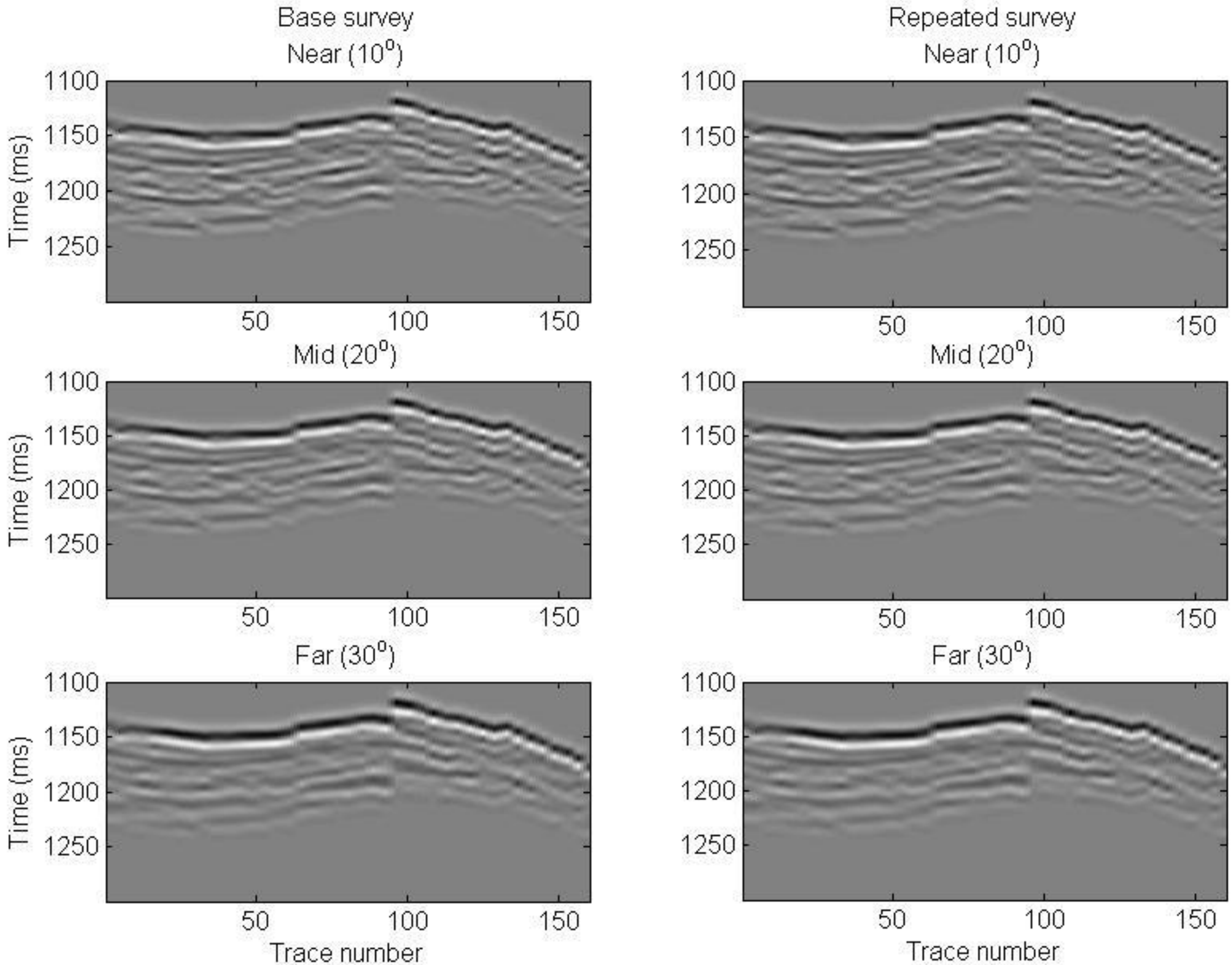
Assumptions

- Porosity does not change in time (no compaction effect).
- Initial pressure and saturation (pre-production) are known.

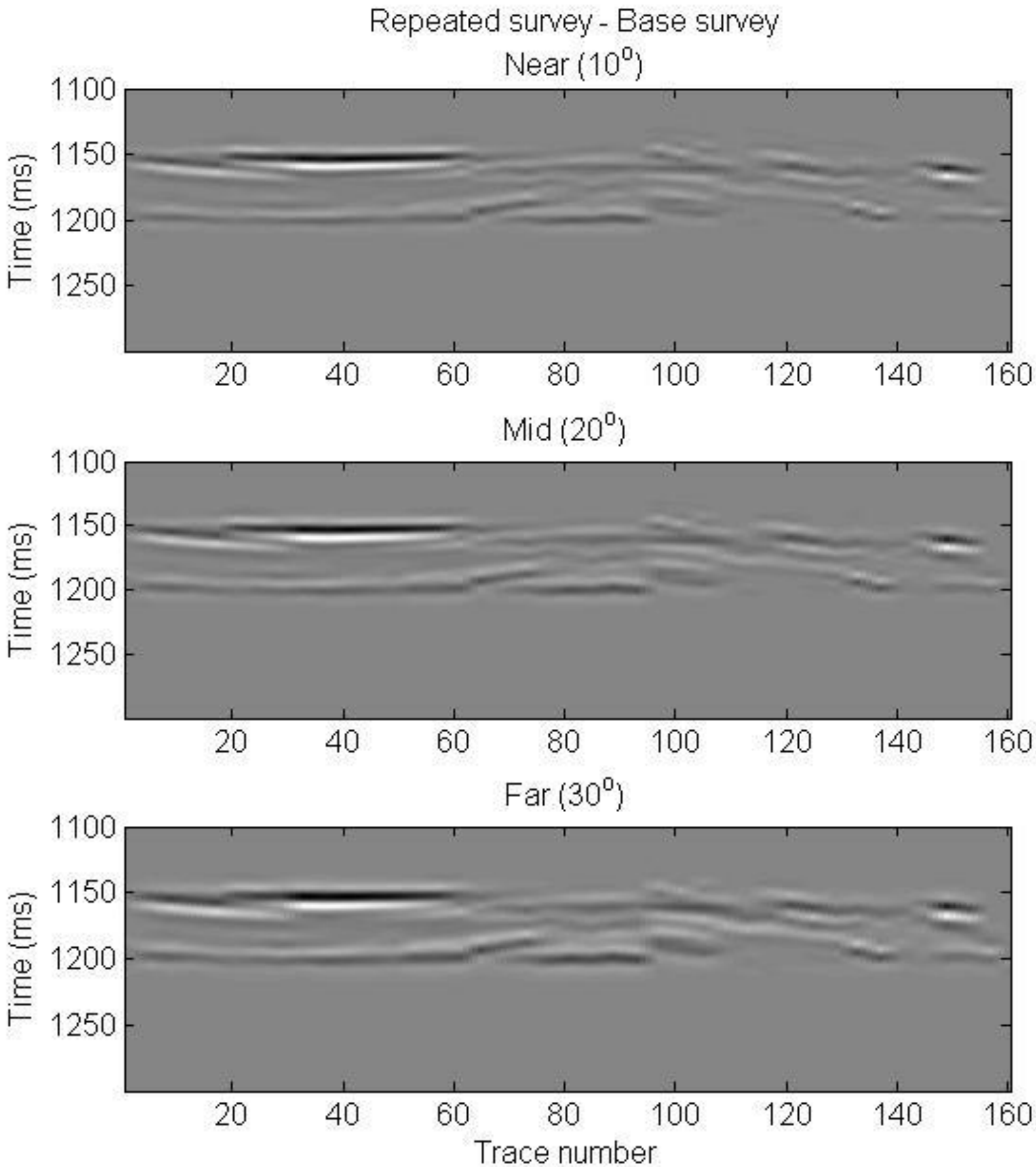
2D application: synthetic seismic model



Synthetic seismic surveys: base and repeated



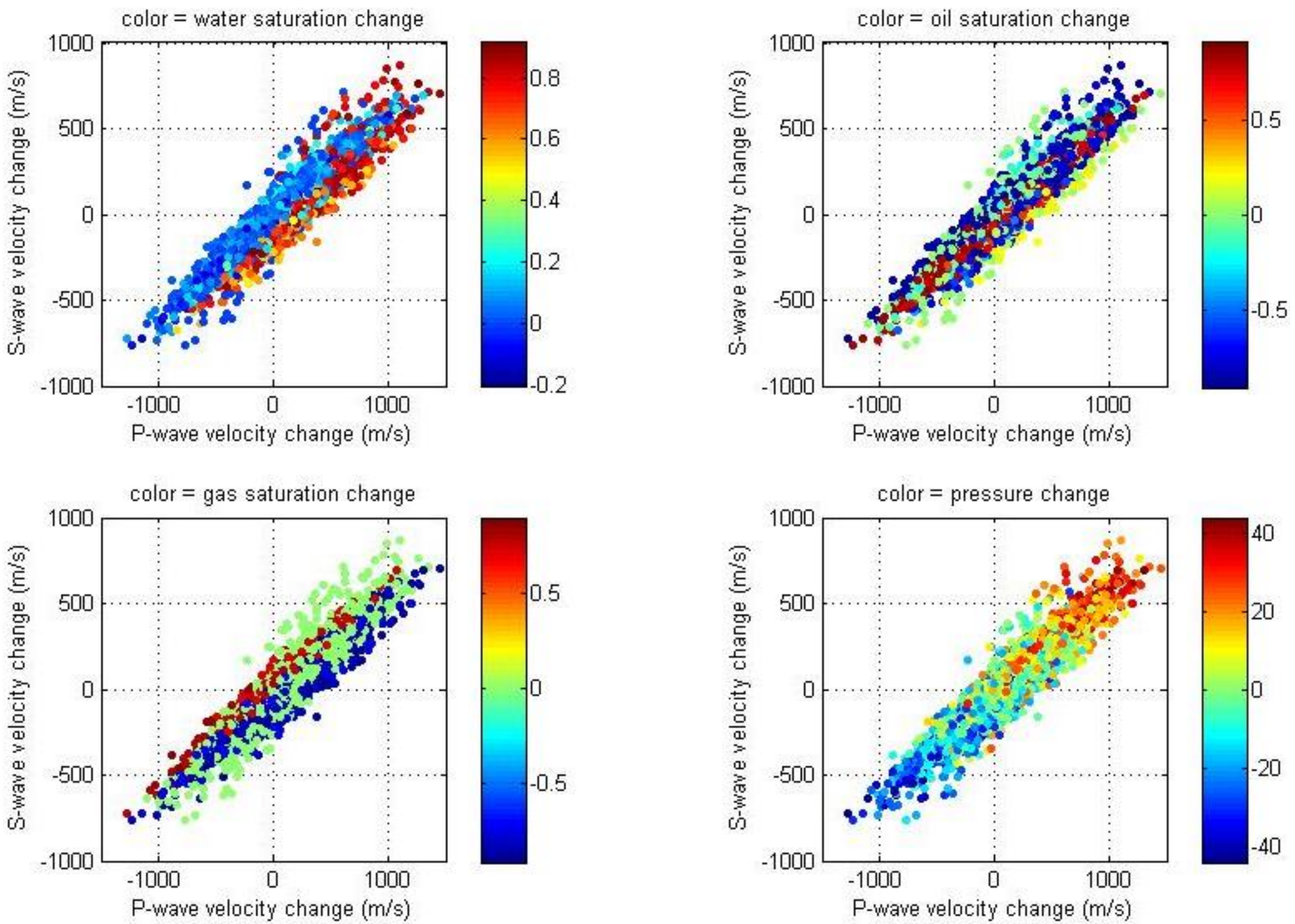
Time-lapse seismic differences



Rock physics likelihood

- Different scenarios:
  - Pressure decreases – Saturation insitu
  - Pressure increases – Saturation insitu
  - Pressure insitu – Water replaced oil
  - Pressure insitu – Gas replaced oil
  - Pressure decreases – Water replaced oil
  - Pressure decreases – Gas replaced oil
  - Pressure increases – Water replaced oil
  - Pressure increases – Gas replaced oil

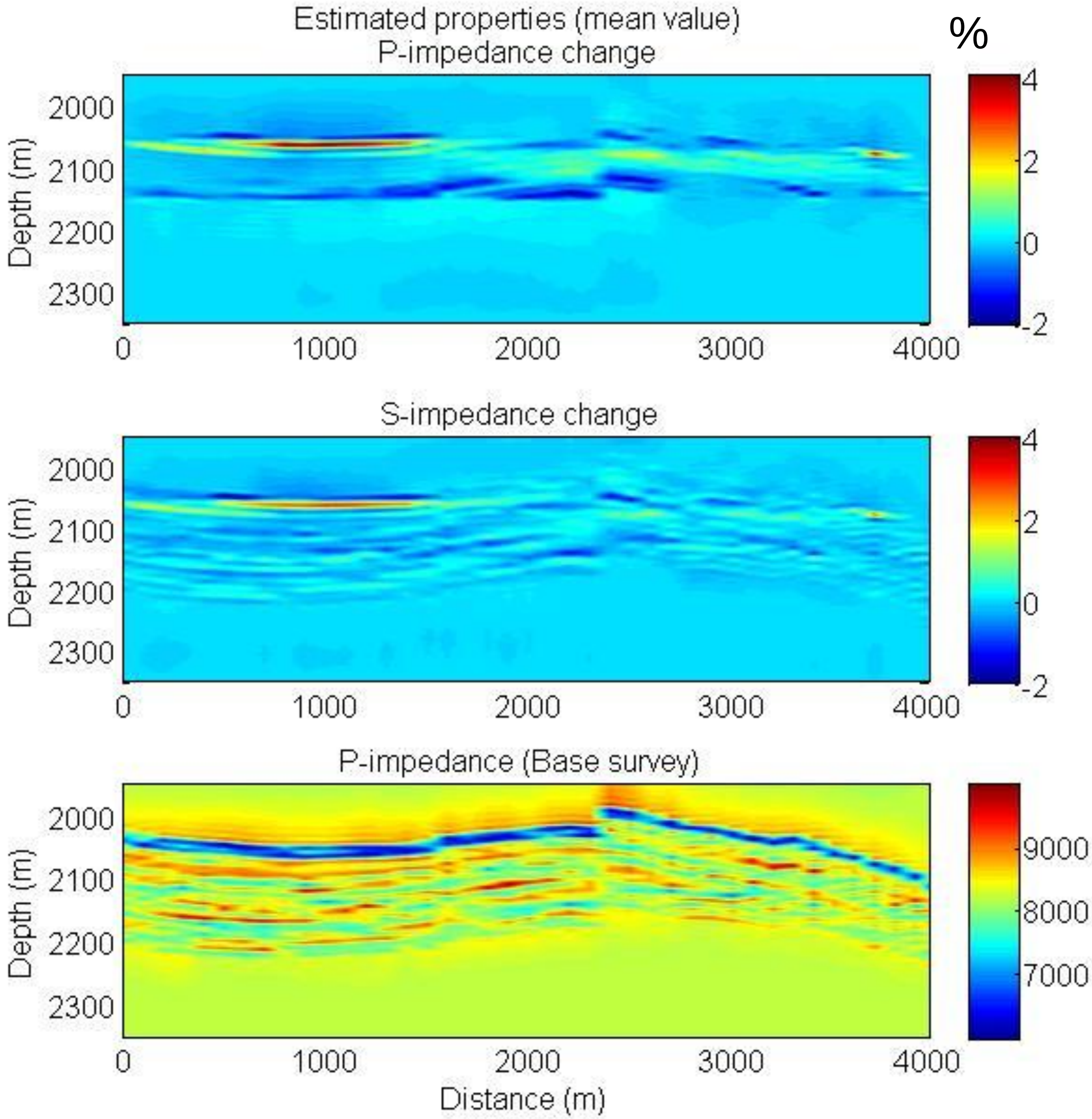
Rock physics likelihood



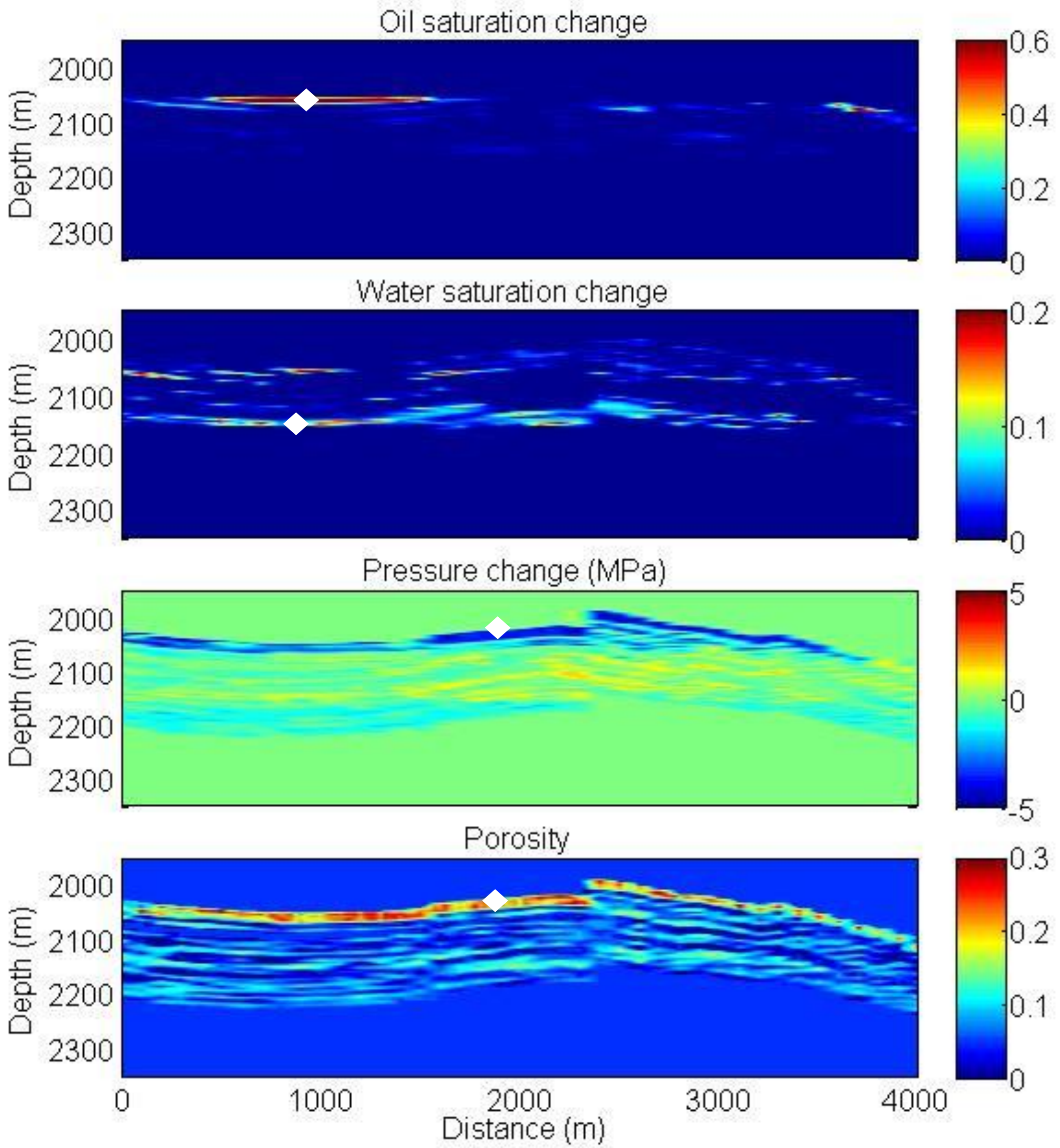
Elastic property changes (relative)

$$\text{relative change} = 1 - \frac{I_P^{rep}}{I_P^{base}}$$

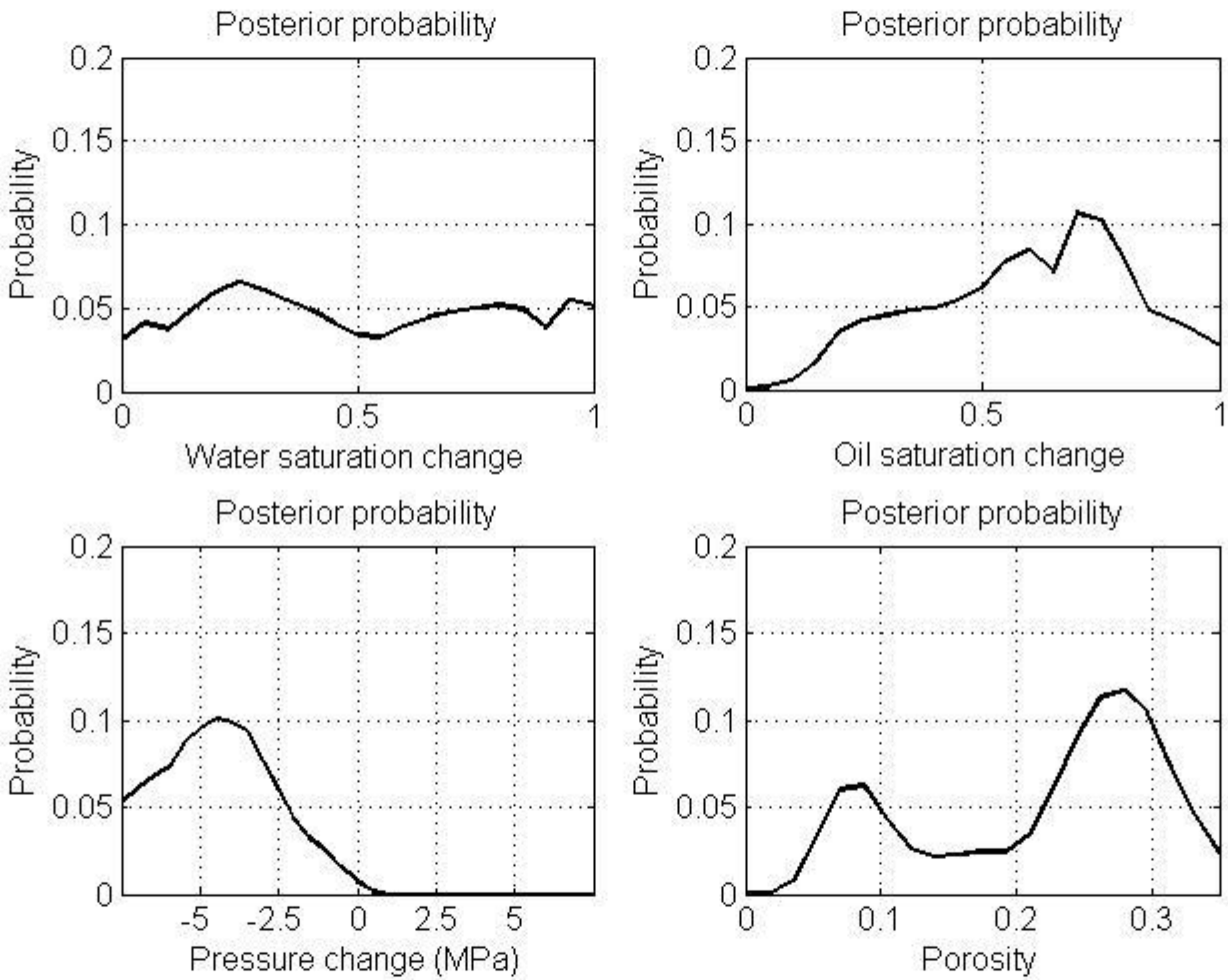
$$\text{relative change} = 1 - \frac{I_S^{rep}}{I_S^{base}}$$



Reservoir property changes



Point-wise posterior probabilities (examples)



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Conclusions

- We presented a full Bayesian methodology to estimate reservoir properties and their changes from seismic data
- The method allows to assess the uncertainty in the estimation of reservoir properties
- Inverted data can be used in seismic history matching to improve the reservoir description

Acknowledgements

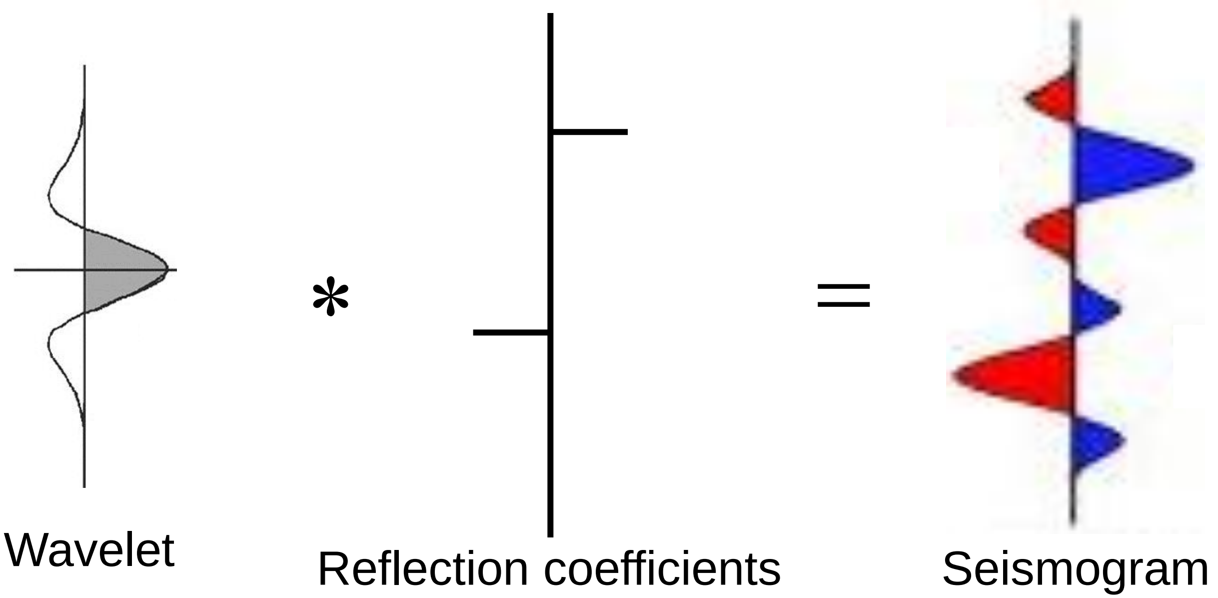
- Prof. Martin Landrø and NTNU for the invitation
- Stanford University, SRB and SCRF for supporting my research
- Thank you for the attention

Backup

Physical model

Seismic forward model:

- Wavelet convolution
- Linearized Aki-Richards approximation of Zoeppritz equations



$$r_{PP}(\theta) = h(\mathbf{m}, \theta)$$

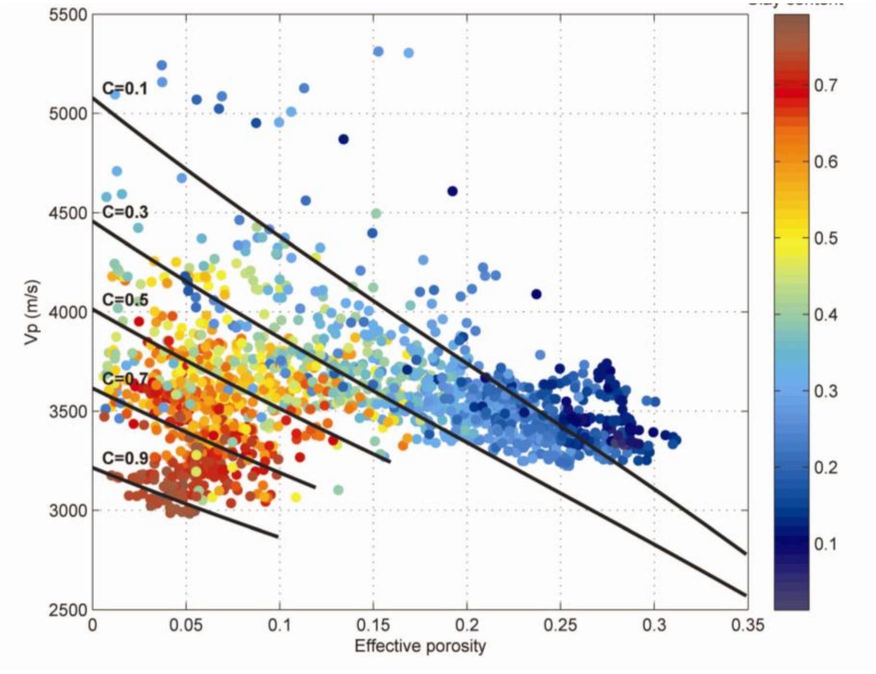
Physical model

Rock physics forward model:

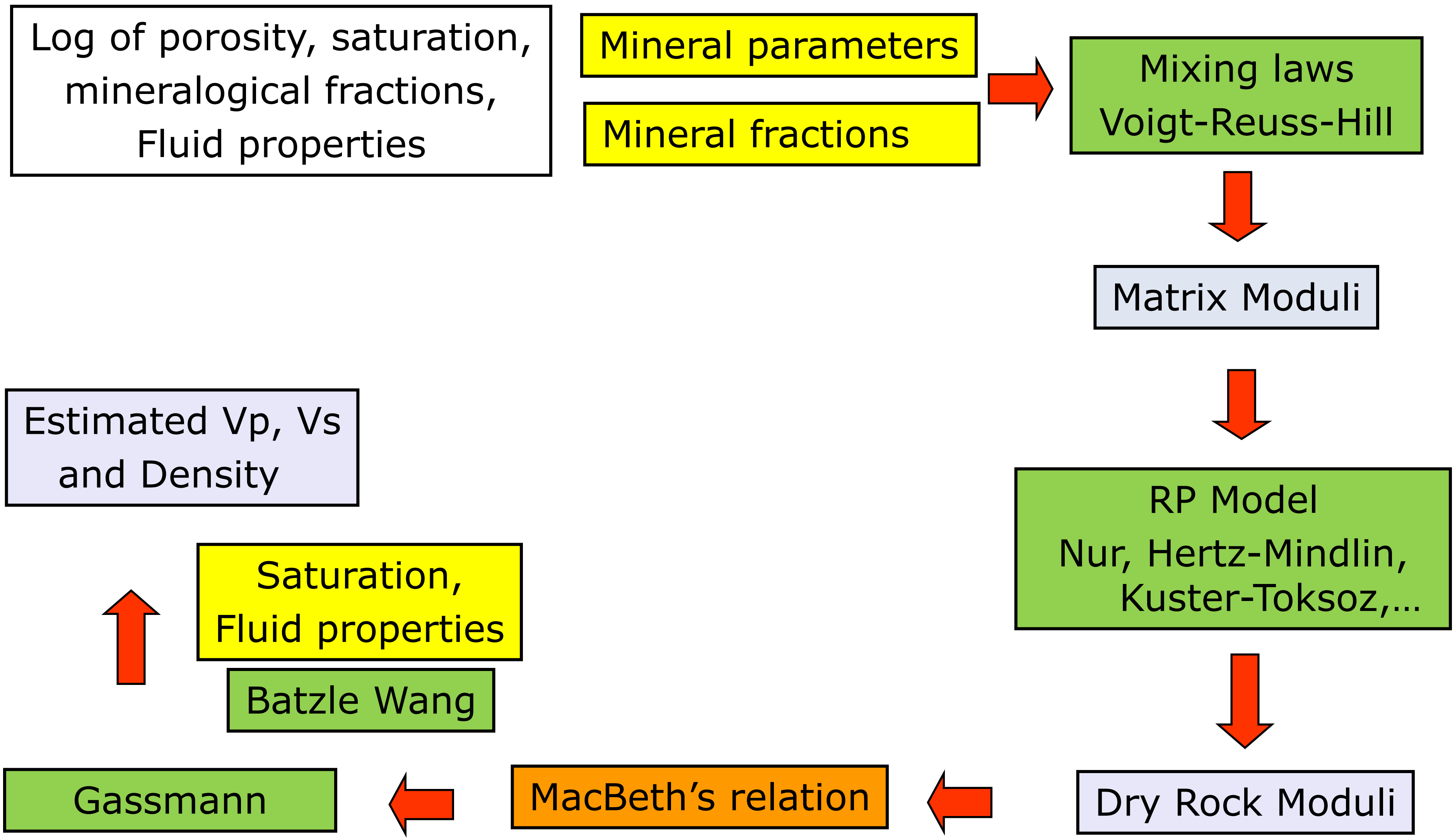
- Granular media models (Hertz-Mindlin contact theory)
- Gassmann's equations
- Velocity-pressure relations

$$\begin{bmatrix} V_P \\ V_S \\ \rho \end{bmatrix} = \mathbf{f}_{RPM}\left(\begin{bmatrix} \phi \\ sw \\ p \end{bmatrix}\right)$$

P-wave velocity versus effective porosity



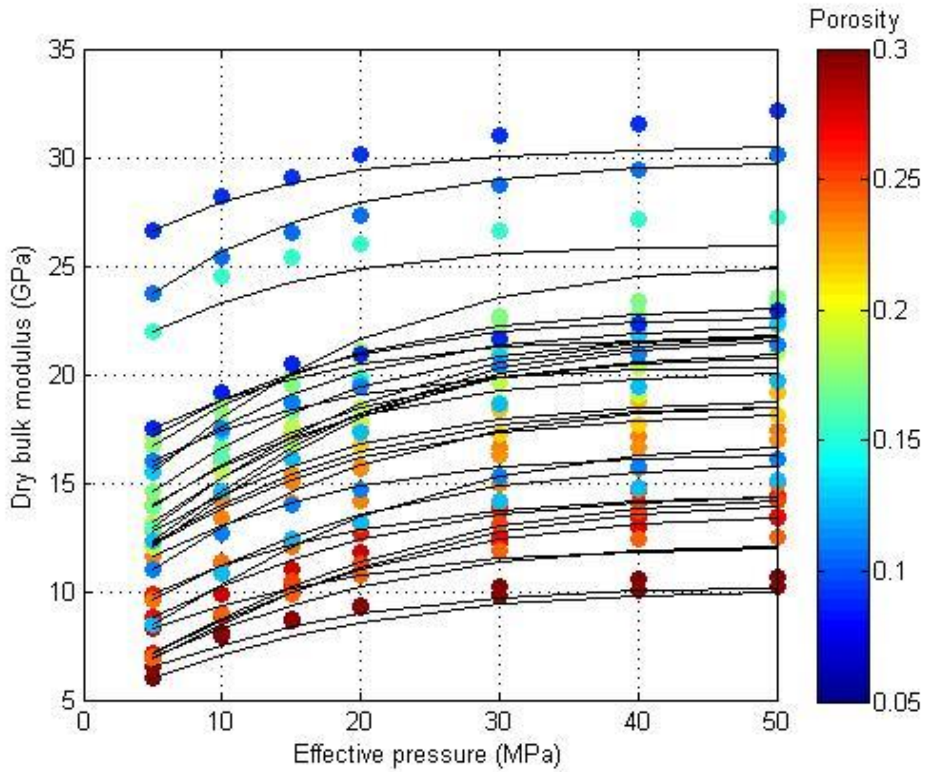
Rock physics model



MacBeth modified (bulk modulus)

$$K_{dry}(p) = \frac{K^{\infty}}{1 + \frac{K^{\infty} - K_0}{K_0} e^{-\frac{(p-p_0)}{p_K}}}, \qquad K_0 = K_{dry}(p = p_0)$$

We assume that $K^{\infty} = \lambda_1(\phi + 0.3C) + \lambda_2$



Bayesian 3D inversion

Seismic Inverse problem

$$\mathbf{S}^{base} = \mathbf{G}\mathbf{m} + \boldsymbol{\varepsilon}$$

$$\mathbf{m} = \begin{bmatrix} \ln\left(I_P^{base}\right) \\ \ln\left(I_S^{base}\right) \end{bmatrix}$$

Bayesian 3D inversion

Seismic Inverse problem

$$\mathbf{S}^{base} = \mathbf{G}\mathbf{m} + \boldsymbol{\varepsilon}$$

$$\mathbf{m} = \begin{bmatrix} \ln\left(I_P^{base}\right) \\ \ln\left(I_S^{base}\right) \end{bmatrix}$$

If the prior distribution of $\mathbf{m}$ is Gaussian

If the model $\mathbf{G}$ is linear

Then the posterior distribution $\mathbf{m} \,|\, \mathbf{S}^{base}$ is Gaussian

Buland and Omre (2003)

Bayesian 4D inversion

Time-lapse Inverse problem

$$\Delta\mathbf{S} = \mathbf{G}\,\Delta\mathbf{m} + \mathbf{e}$$

$$\Delta\mathbf{S} = \mathbf{S}^{rep} - \mathbf{S}^{base}$$

$$\Delta\mathbf{m} = \begin{bmatrix} \ln\left(\dfrac{I_P^{rep}}{I_P^{base}}\right) \\ \ln\left(\dfrac{I_S^{rep}}{I_S^{base}}\right) \end{bmatrix}$$

Bayesian 4D inversion

Time-lapse Inverse problem

$$\Delta \mathbf{S} = \mathbf{G}\,\Delta \mathbf{m} + \mathbf{e}$$

$$\Delta \mathbf{S} = \mathbf{S}^{rep} - \mathbf{S}^{base}$$

$$\Delta \mathbf{m} = \begin{bmatrix} \ln\left(\dfrac{I_P^{rep}}{I_P^{base}}\right) \\ \ln\left(\dfrac{I_S^{rep}}{I_S^{base}}\right) \end{bmatrix}$$

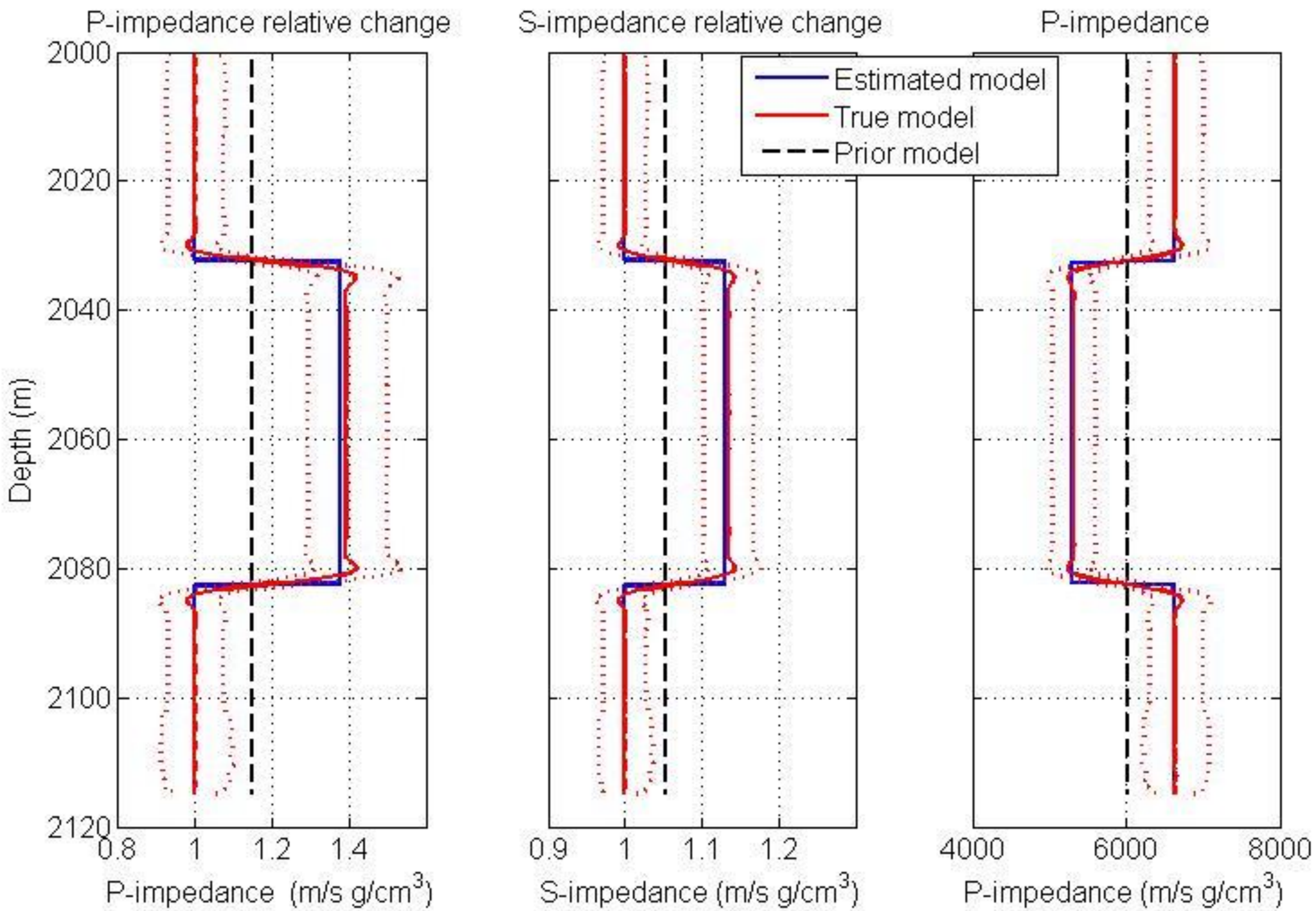
If the prior distribution of $\Delta \mathbf{m}$ is Gaussian

If the model $\mathbf{G}$ is linear

Then the posterior distribution $\Delta \mathbf{m} \mid \Delta \mathbf{S}$ is Gaussian

Buland and El Ouair (2006)

Base seismic and time-lapse inversion



We then compute the relative impedance change as $\Delta I_P = 1 - I_P^{rep}/I_P^{base}$

Base seismic and time-lapse inversion (ex. 2)