Double-diffraction approximation of the feasible Green's function in geometrical shadow zones

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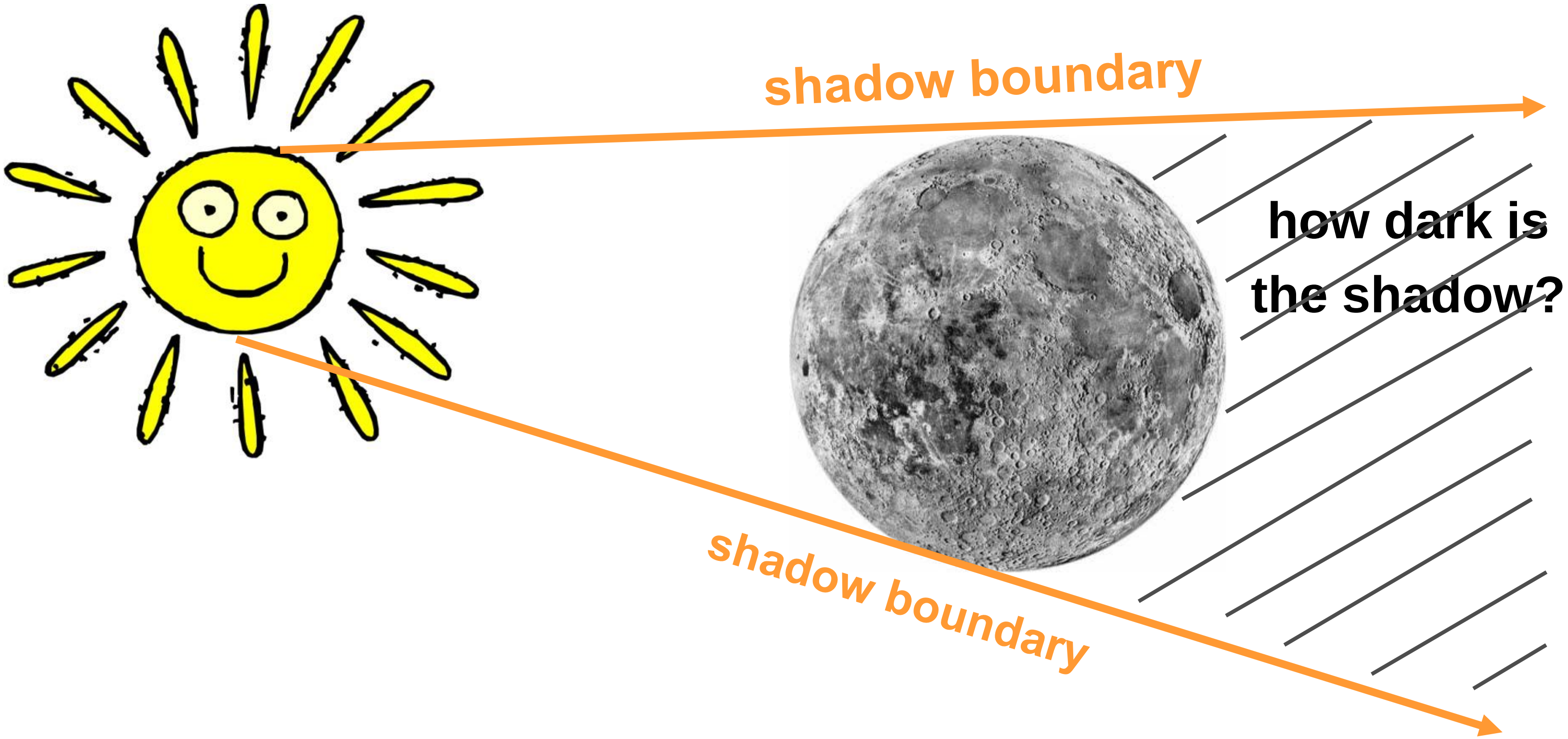

Outline

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Introduction

Objectives

Understanding of the wave structure of the feasible Green’s function (A. Aizenberg and A. Ayzenberg, 2009)

Comparison of the feasible Green’s function with theory of Jones (1973)

Testing on 3 models

Theory

feasible Green's function

$$\mathbf{F} = \sum_{n=0}^{\infty} (\mathbf{PA})^n \mathbf{G}$$

conventional Green's function

1-order diffraction correction of Green's function

$$\mathbf{F} = \mathbf{G} + \mathbf{P}\,\mathbf{A}\,\mathbf{G}$$

Kirchhoff propagation operator

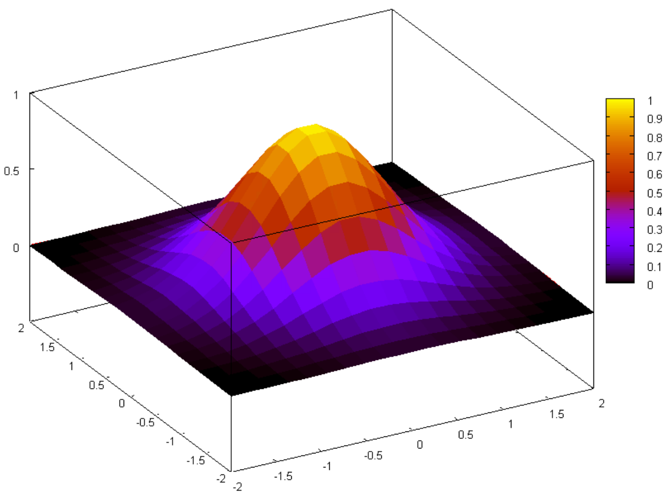
$$\mathbf{P}(\mathrm{s},\mathrm{s}')\langle\ldots\rangle = \iint_S \mathbf{G}(\mathrm{s},\mathrm{s}')\,\mathbf{N}_{\mathrm{s}'}\langle\ldots\rangle\,dS(\mathrm{s}')$$

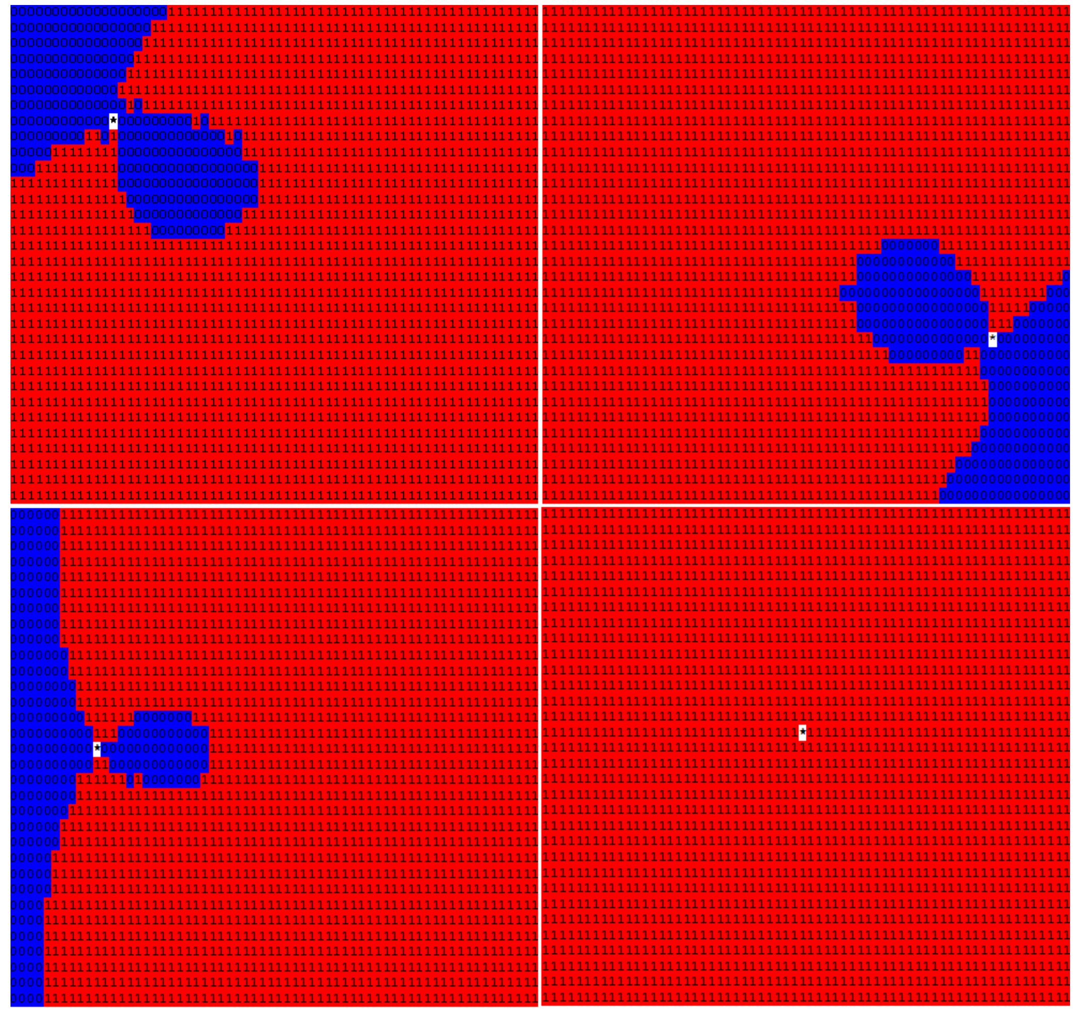
absorption operator

$$\mathbf{A}(\mathrm{s},\mathrm{s}')\langle\ldots\rangle = \iint_S h(\mathrm{s},\mathrm{s}')\,\mathbf{G}(\mathrm{s},\mathrm{s}')\,\mathbf{N}_{\mathrm{s}'}\langle\ldots\rangle\,dS(\mathrm{s}')$$

shadow function

Shadow function





Model

$$v = 2.0\, km/s$$

$$\rho = 2.1\, g/cm^3$$

wavelet $$e^{-(2\tau)^2} \cos\left(2\pi\tau\right)$$

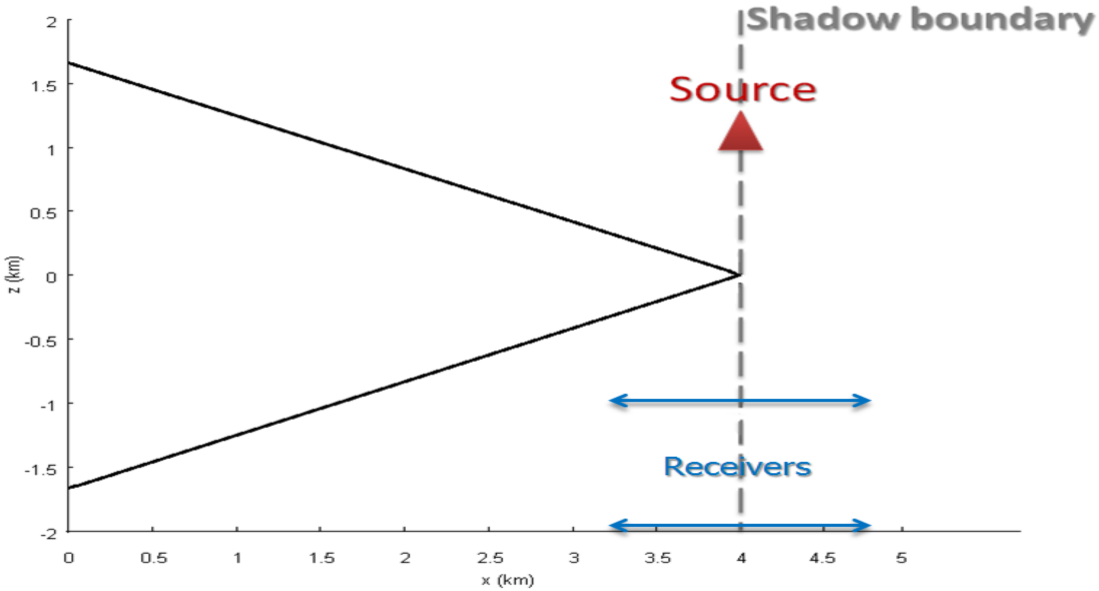
$$\tau = t/T - 2, \quad T = 0.032\, s$$

$$\lambda = 0.064\, km$$

$$f = 31.25\, Hz$$

$$define \quad DAC = \frac{\mathbf{F}}{\mathbf{G}}$$

Test 1: V-shaped boundary



$$upper \quad z = 0.41(x_0 - x)$$

$$lower \quad z = -0.41(x_0 - x)$$

$$x_0 = 4$$

$$\mathbf{F}_V^{(1)} \cong \mathbf{G} + \mathbf{D}_V^{(1)}, \quad \mathbf{D}_V^{(1)} = \mathbf{P}_{x2}\mathbf{A}_{21}\mathbf{G}_1$$

$$relative \quad error \quad in \quad amplitudes \quad \leq 4\%$$

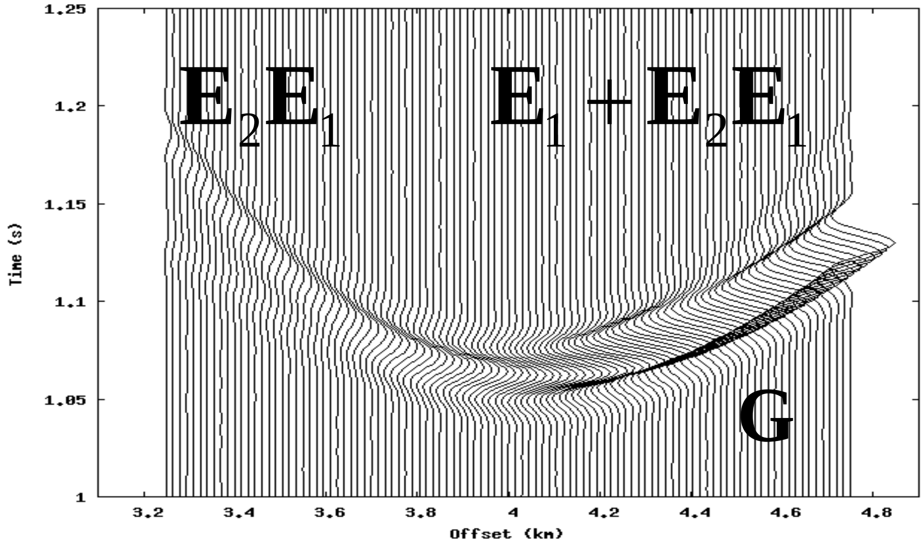
$$absolute \quad error \quad in \quad traveltime \quad \approx 0.002\ s$$

Test 1: V-shaped boundary

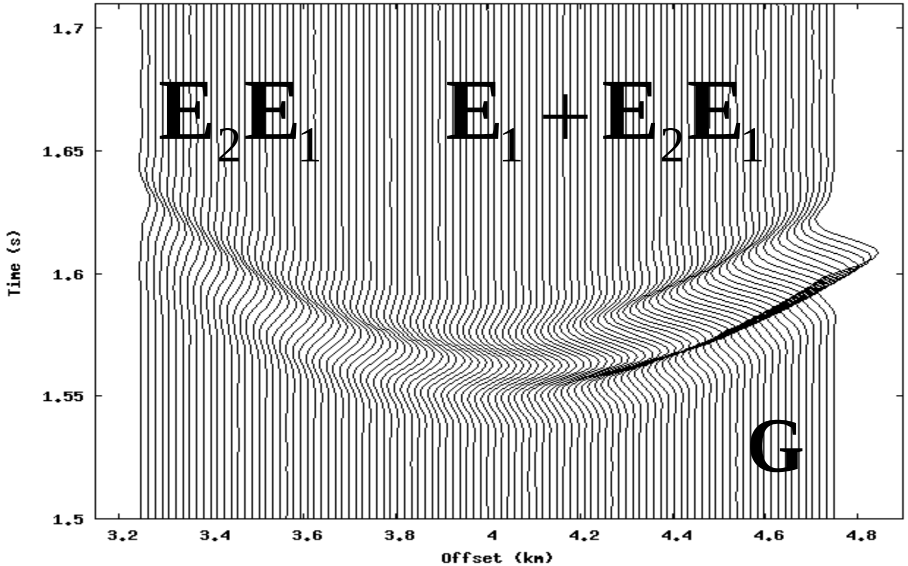
$$\mathbf{A}(s, s') = \begin{bmatrix} \mathbf{A}(s_1, s_1') & \mathbf{A}(s_1, s_2') \\ \mathbf{A}(s_2, s_1') & \mathbf{A}(s_2, s_2') \end{bmatrix}$$

$$\mathbf{A}(s, s') \cong \begin{bmatrix} \mathbf{O} & \mathbf{O} \\ \mathbf{A}(s_2, s_1') & \mathbf{O} \end{bmatrix}$$

Test 1: V-shaped boundary

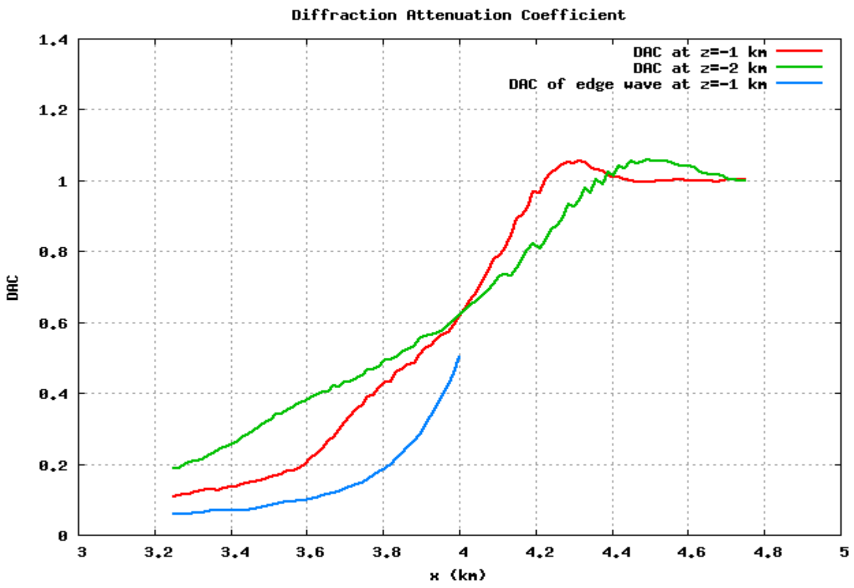


$line\,1 \quad z = -1$



$line\,2 \quad z = -2$

Test 1: V-shaped boundary



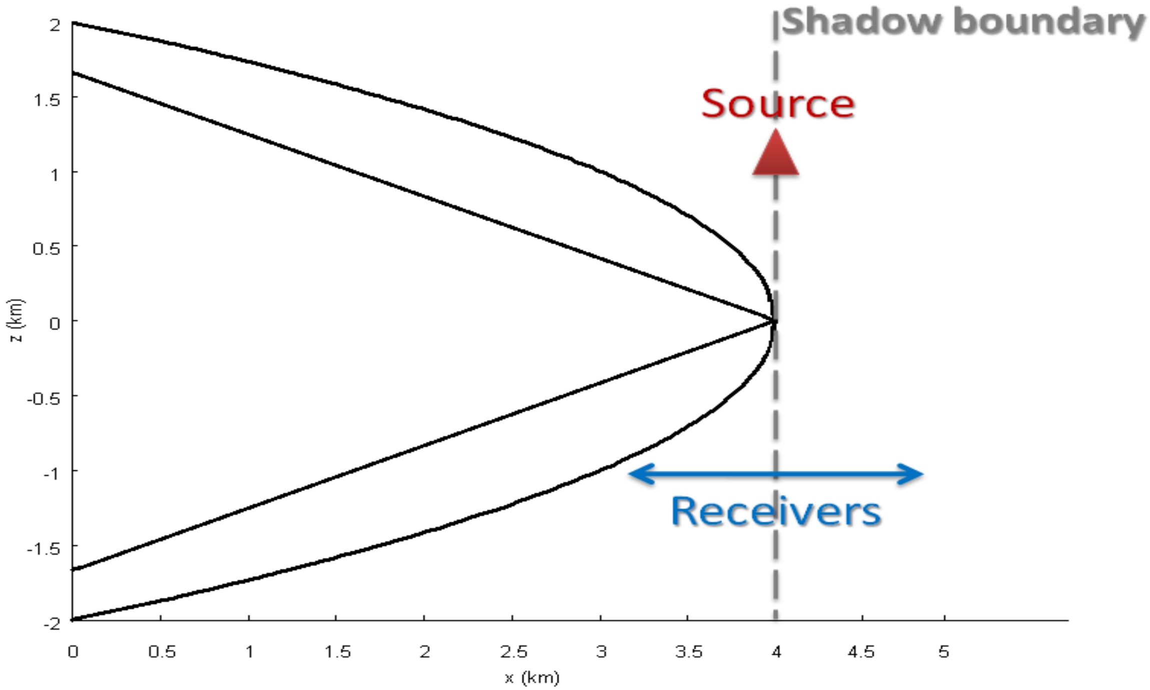
$$line\,1 \quad DAC(4) = 0.617$$
$$line\,2 \quad DAC(4) = 0.622$$

$$edge \quad wave \quad DAC^{(1)}(4) = 0.5$$

(Modified Fresnel integral)

$$double \quad edge \quad diffraction \quad wave \quad DAC^{(2)}(4) = 0.125$$
$$DAC(4) = DAC^{(1)}(4) + DAC^{(2)}(4) = 0.625$$

Test 2: U-shaped boundary



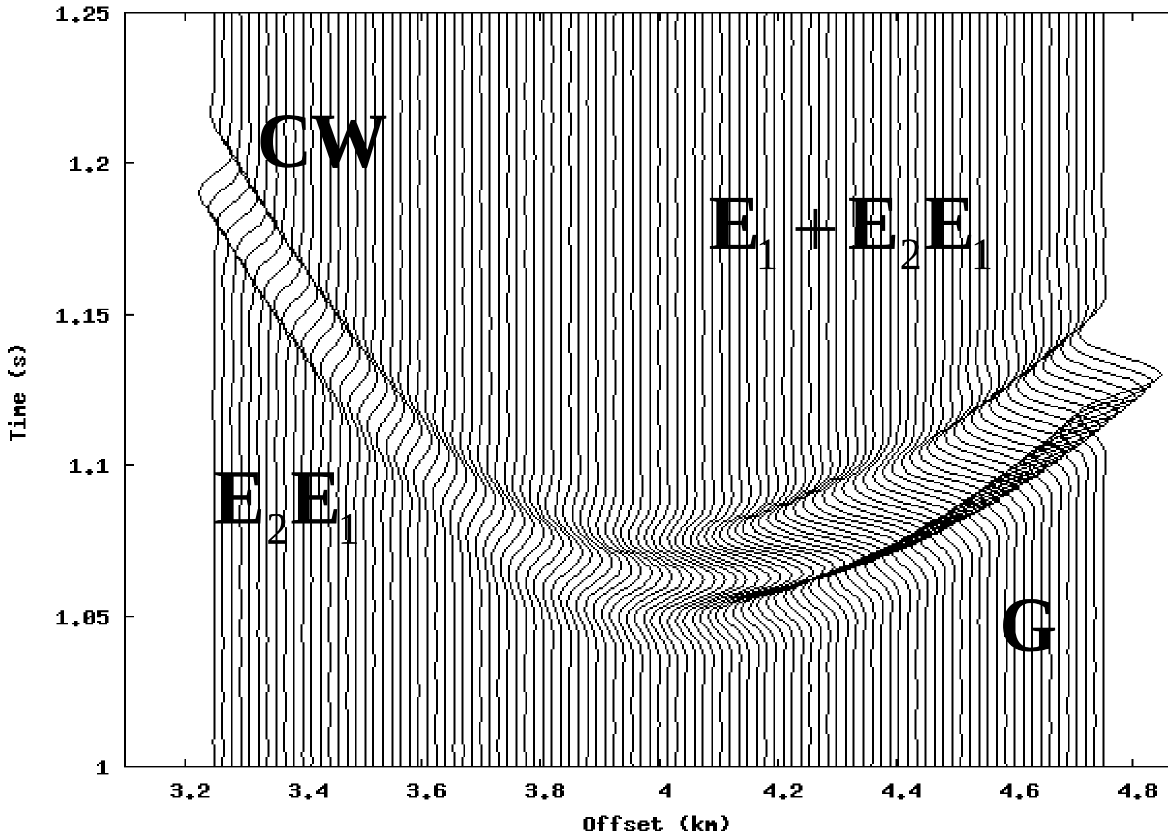
$$upper \quad z = \sqrt{x_0 - x}$$

$$lower \quad z = -\sqrt{x_0 - x}$$

$$x_0 = 4$$

$$radius \quad of \quad curvature \quad 5\,km \quad at \quad 4.0$$

Test 2: U-shaped boundary

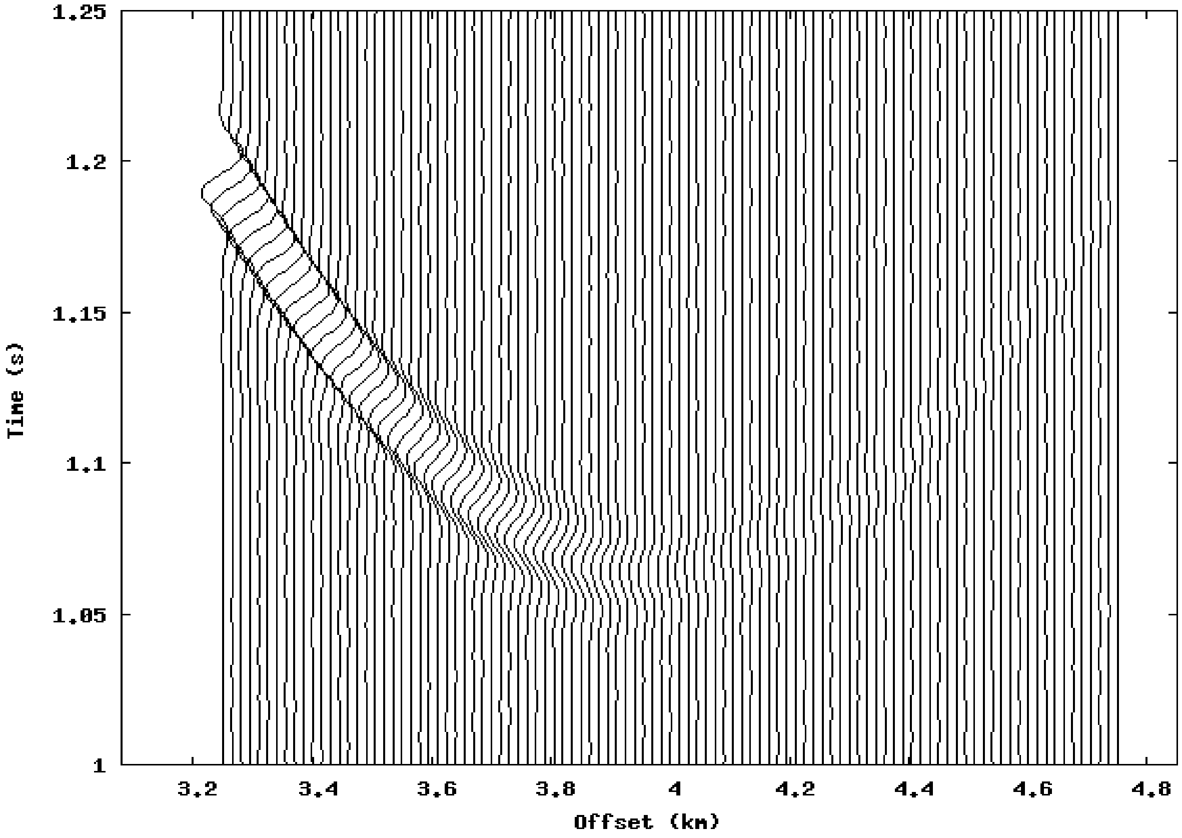


$$\mathbf{F}^{(1)} \cong \mathbf{G} + \mathbf{D}^{(1)} \quad \mathbf{D}^{(1)} = \mathbf{D}^{(1)}_{S_2S_1} + \mathbf{D}^{(1)}_{S_2S_2}$$

$$\mathbf{D}^{(1)}_{S_2S_1} = \mathbf{P}_{x2}\mathbf{A}_{21}\mathbf{G}_1 \quad \mathbf{D}^{(1)}_{S_2S_2} = \mathbf{P}_{x2}\mathbf{A}_{22}\mathbf{G}_2$$

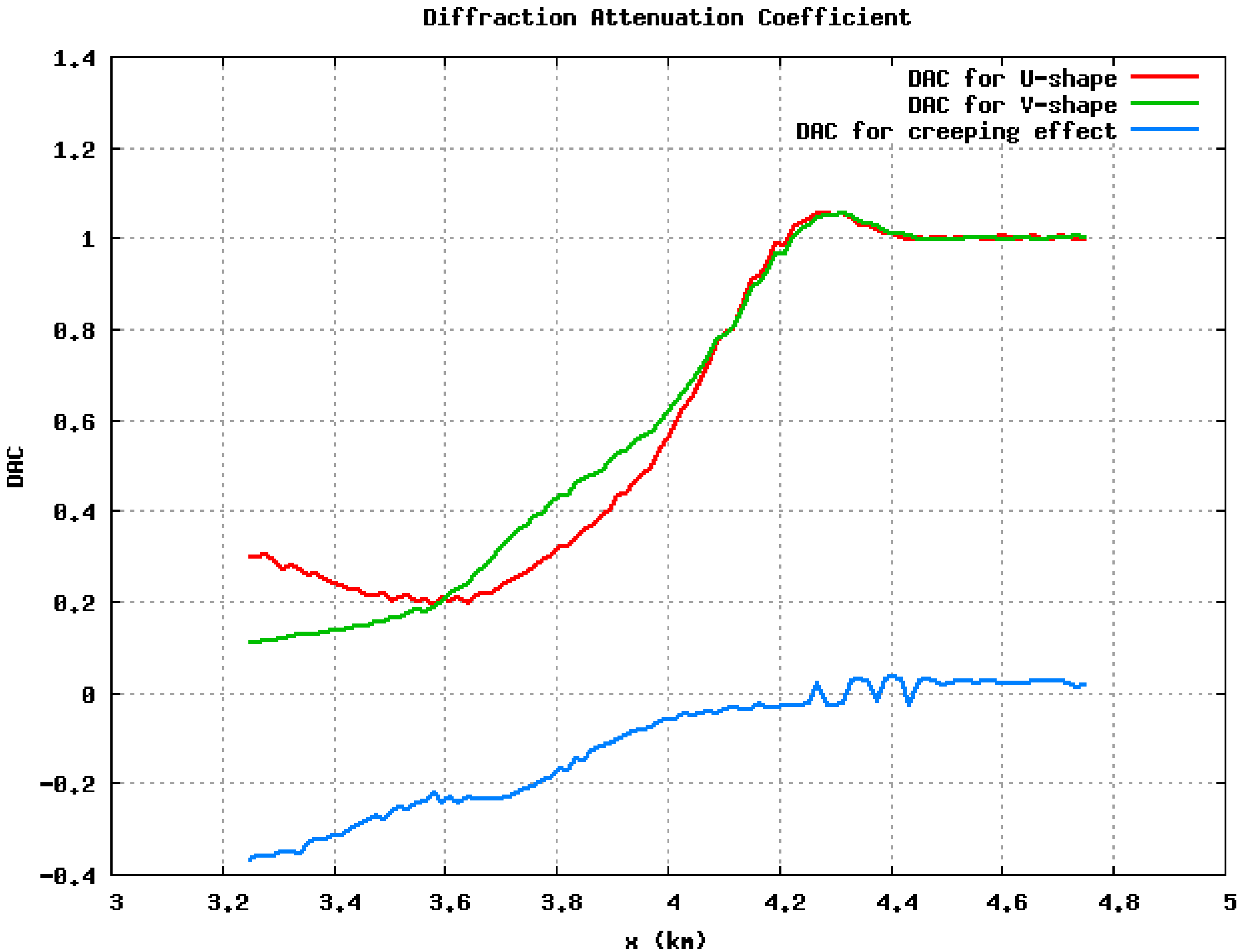
V-shaped diffraction — creeping wave

Test 2: U-shaped boundary. Creeping wave

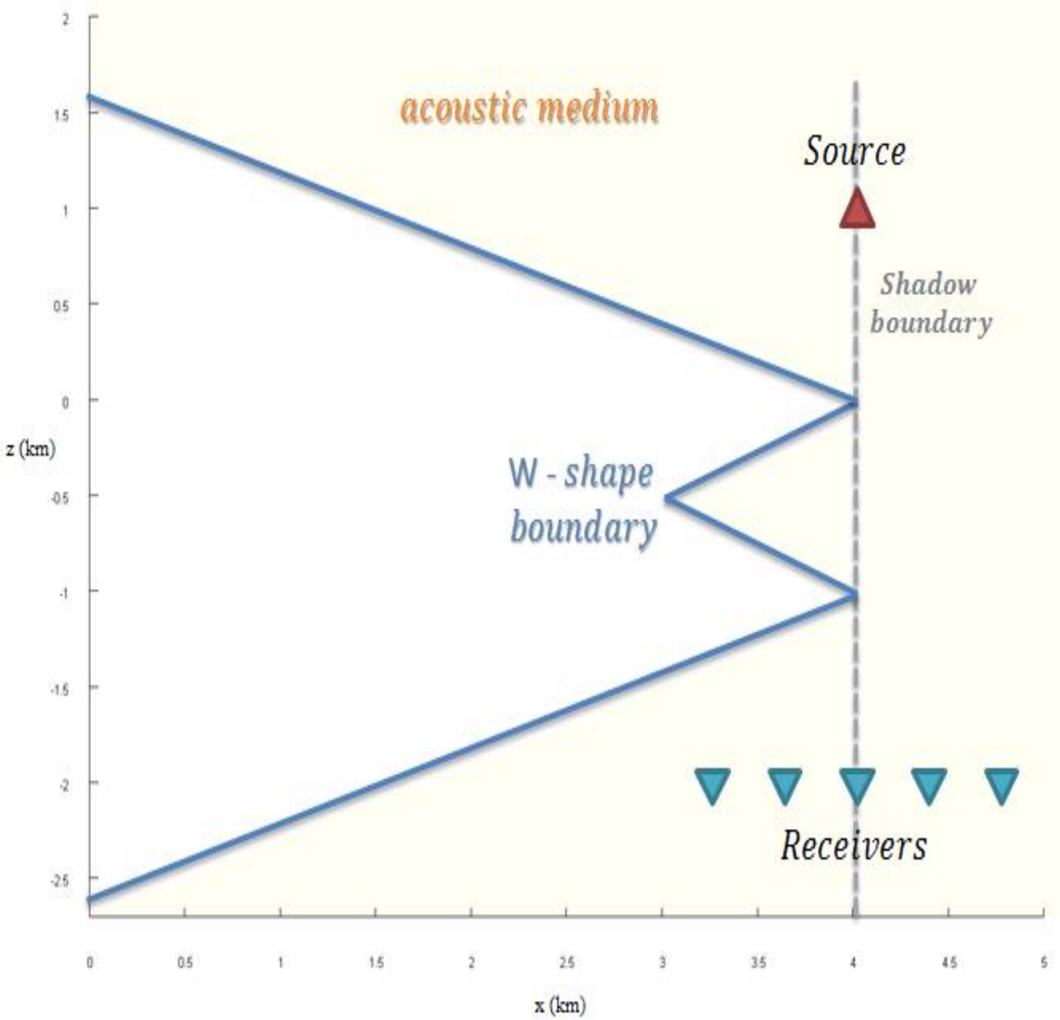


$$\mathbf{P}_{x2}\mathbf{A}_{22}\mathbf{G}_{2}$$

Test 2: U-shaped boundary



Test 3: W-shaped boundary



Test 3: W-shaped boundary

$$\mathbf{F}^{(1)} = \mathbf{G} + \mathbf{D}^{(1)}$$

$$\mathbf{D}^{(1)} = \mathbf{D}^{(1)}_{S_2 S_1} + \mathbf{D}^{(1)}_{S_4 S_3} + \mathbf{D}^{(1)}_{S_4 S_2}$$

$$\mathbf{D}^{(1)}_{S_2 S_1} = \mathbf{P}_{x2}\ \mathbf{A}_{21}\ \mathbf{G}_{1}$$

$$\mathbf{D}^{(1)}_{S_4 S_3} = \mathbf{P}_{x4}\ \mathbf{A}_{43}\ \mathbf{G}_{3}$$

$$\mathbf{D}^{(1)}_{S_4 S_2} = \mathbf{P}_{x4}\ \mathbf{A}_{42}\ \mathbf{G}_{2}$$

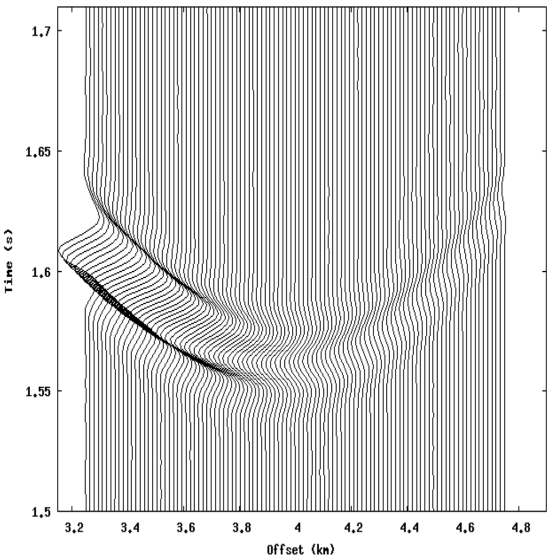
Test 3: W-shaped boundary

$$\mathbf{A}(s, s') = \begin{bmatrix} \mathbf{A}(s_1, s_1') & \mathbf{A}(s_1, s_2') & \mathbf{A}(s_1, s_3') & \mathbf{A}(s_1, s_4') \\ \mathbf{A}(s_2, s_1') & \mathbf{A}(s_2, s_2') & \mathbf{A}(s_2, s_3') & \mathbf{A}(s_2, s_4') \\ \mathbf{A}(s_3, s_1') & \mathbf{A}(s_3, s_2') & \mathbf{A}(s_3, s_3') & \mathbf{A}(s_3, s_4') \\ \mathbf{A}(s_4, s_1') & \mathbf{A}(s_4, s_2') & \mathbf{A}(s_4, s_3') & \mathbf{A}(s_4, s_4') \end{bmatrix}$$

$$\mathbf{A}(s, s') = \begin{bmatrix} \mathbf{O} & \mathbf{O} & \mathbf{O} & \mathbf{O} \\ \mathbf{A}(s_2, s_1') & \mathbf{O} & \mathbf{O} & \mathbf{O} \\ \mathbf{O} & \mathbf{O} & \mathbf{O} & \mathbf{O} \\ \mathbf{O} & \mathbf{A}(s_4, s_2') & \mathbf{A}(s_4, s_3') & \mathbf{O} \end{bmatrix}$$

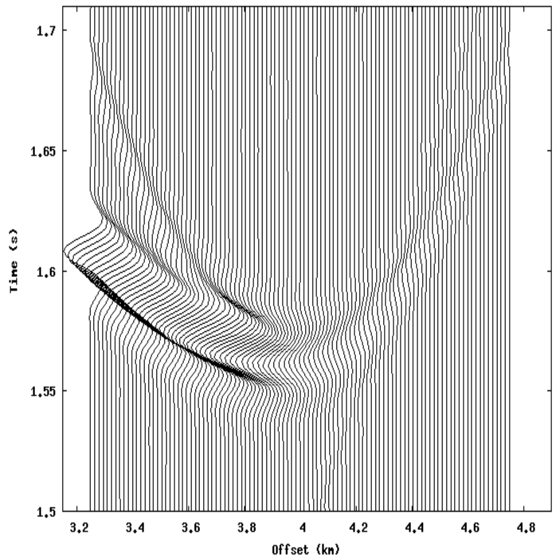
Test 3: W-shaped boundary

(1,2)



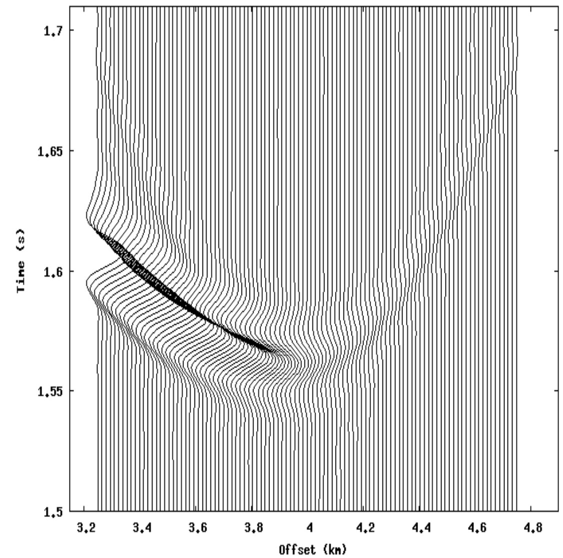
$$\mathbf{D}^{(1)}_{S_2S_1} = \mathbf{P}_{x2}\ \mathbf{A}_{21}\ \mathbf{G}_1$$

(3,4)



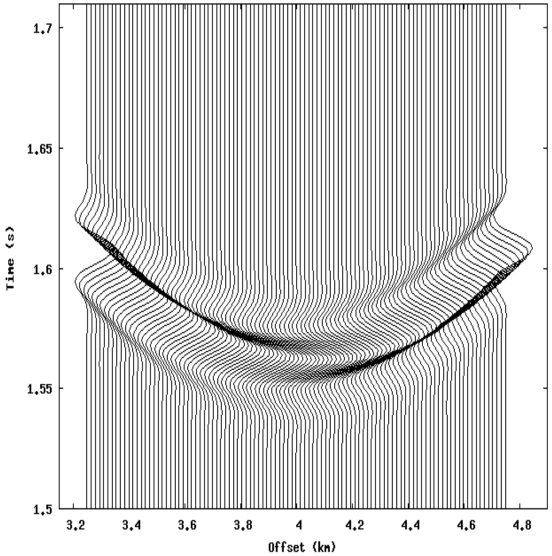
$$\mathbf{D}^{(1)}_{S_4S_3} = \mathbf{P}_{x4}\ \mathbf{A}_{43}\ \mathbf{G}_3$$

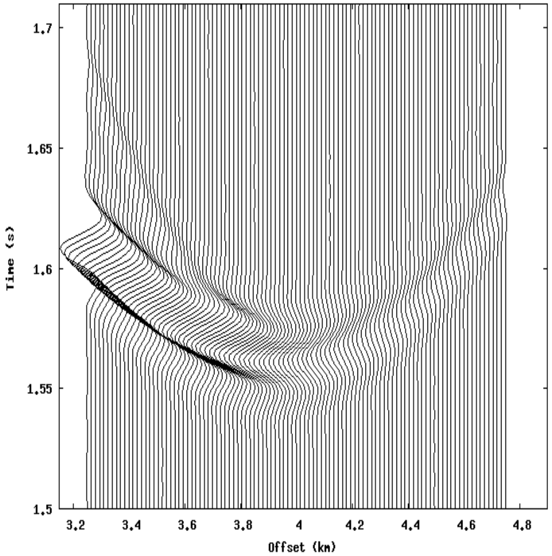
(2,4)

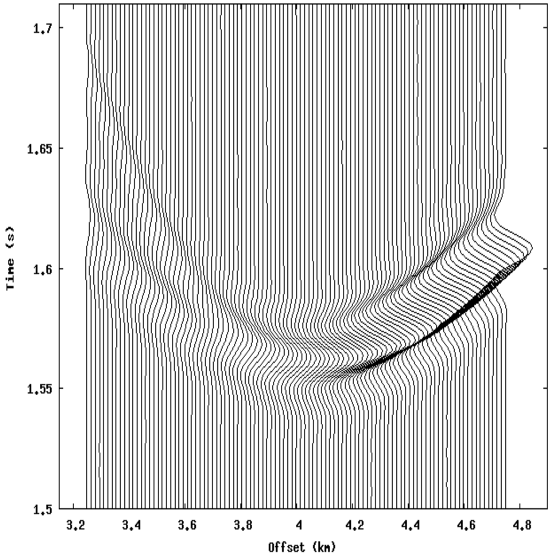


$$\mathbf{D}^{(1)}_{S_4S_2} = \mathbf{P}_{x4}\ \mathbf{A}_{42}\ \mathbf{G}_2$$

Test 3: W-shaped boundary







$$\mathbf{G}$$

$$\mathbf{D}^{(1)} = \mathbf{D}^{(1)}_{S_2 S_1} + \mathbf{D}^{(1)}_{S_4 S_3} + \mathbf{D}^{(1)}_{S_4 S_2}$$

$$\mathbf{F}^{(1)} = \mathbf{G} + \mathbf{D}^{(1)}$$

Conclusions

- 3 numerical models are used to test the representation of the feasible Green's function
- The 1-order diffraction approximation of the feasible Green's function corresponds to the double-diffraction theory of Jones (1973)
- The 1-order term of the feasible Green's function expresses single and double edge waves and creeping wave
- It is shown that 1-order diffraction at V-shaped boundary is equivalent to double knife-edge diffraction at two half-screens with the near edges

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