Module 2:
Exorcizing seismic ghosts

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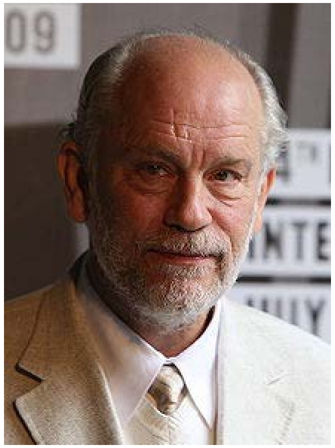
April 24, 2013

Geophysicicists do not lack self confidence, but will we ever solve the deghosting problem?

“I wouldn’t describe myself as lacking in confidence, but I would just say that – the ghosts you chase you never catch“

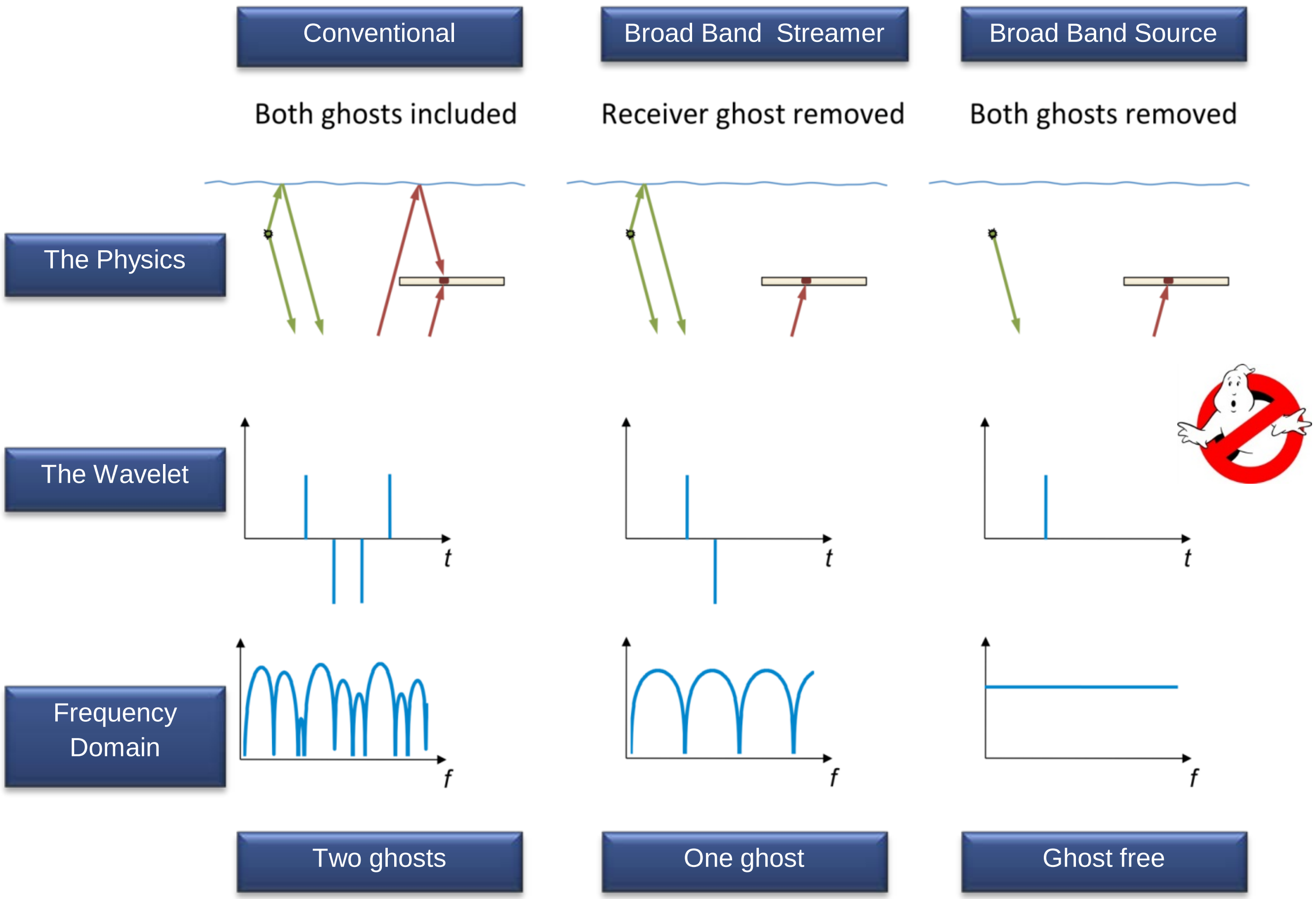
John Malkovich (1953–)



To understand how geophysical priests exorcize seismic ghosts, either through developing sensor technologies or new data processing methods, you need to understand what ghosts are and how they bandlimit seismic data

Long-standing, and today “hot” research area.

Ghosts and de-ghosting



A source ghost is an event starting its propagation upward from the source, and a receiver ghost ends its propagation moving downward at the receiver. They both have a reflection at the sea surface, which leads to a reduction of the useful frequency bandwidth and therefore damages seismic resolution.

Some history

- The effect of the sea or land surface is a well-known obstacle in seismic exploration and has been addressed since the outset of reflection seismology (Leet, 1937)
- Van Melle and Weatherburn (1953) dubbed the reflections from energy initially reflected above the level of the source, by optical analogy, ‘ghosts’
- Lindsey (1960) the first (?) to present a ghost removal or deghosting solution by observing that a downgoing source signal followed by a ghost with specified time lag can be eliminated by designing a positive feedback loop
- Ghost function G can be eliminated theoretically by applying the inverse filter $D = 1/G$ to the data: $D\,G = 1$
- *More history in BroadBand Seismic Module*

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- Pressure and vertical component of particle velocity
- Ghosts on hydrophones and on vertical geophones
- Deghosting methods, incl GeoStreamer and IsoMetrix

Statoil

Wave equations 3D, 1D, 2D

$$\left(\frac{d^2}{dx^2}+\frac{d^2}{dy^2}+\frac{d^2}{dz^2}+k^2\right)P(x,y,z)=-a(\omega)\delta(x-x_s)\delta(y-y_s)\delta(z-z_s)$$

$$\left(\frac{d^2}{dx^2}+\frac{d^2}{dz^2}+k^2\right)P(x,z)=-a(\omega)\delta(x-x_s)\delta(z-z_s)$$

$$k=\frac{\omega}{c}=\frac{2\pi f}{c}$$

$$\left(\frac{d^2}{dz^2}+k^2\right)P(z)=-a(\omega)\delta(z-z_s)$$

Fourier transform over x and y to kx and ky reduces WEs to 1D Helmholtz equation

$$\left(\frac{d^2}{dz^2}+k_z^2\right)p(z)=-a(\omega)\delta(z-z_s)$$

Plane wave decomposition of spherical/cylindrical wavefield

Dispersion relation

$$k_z=\sqrt{k^2-k_x^2-k_y^2} \quad \text{3D}$$

$$k_z=\sqrt{k^2-k_x^2} \quad \text{2D}$$

$$k_z=k \quad \text{1D}$$

Acoustic 4C

$$\rho \partial_t v_x = -\partial_x p$$
$$\rho \partial_t v_y = -\partial_y p$$
$$\rho \partial_t v_z = -\partial_z p$$

In seismic (acoustic) multicomponent acquisition, the spatial derivatives of the pressure may be directly calculated from the measured particle velocities or particle accelerations (Robertsson et al Geophysics 2008)

p and vz measured on GeoStreamer
p, vz and vy measured on Nessie-6

Hydrophones measure pressure (p) changes, while geophones and accelerometers are sensitive to particle motion. The hydrophone is omni-directional and measures the sum of upgoing and downgoing pressure waves. The vertically oriented geophone (vz) has directional sensitivity and measures their difference.

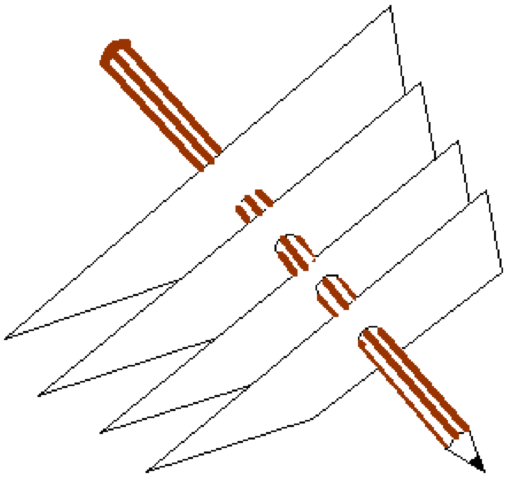
Wavenumbers, slowness, angles …

$k = \frac{\omega}{c} = \frac{2\pi f}{c}$ wavenumber, ω circular frequency, f frequency, c velocity

k_x, k_y horizontal wavenumbers, $k_y = 0$ in 2D

$k_z = \sqrt{k^2 - k_x^2 - k_y^2}$ vertical wavenumber

In a lossless isotropic medium the direction of the wavevector ***k***=(k_x, k_y ,k_z) is the same as the direction of wave propagation



The pencil represents the direction of the **k**-vector. The pencil can point in any direction in space, defined by its Cartesian components: k_x, k_y *and* k_z.

Wavenumbers, slowness, angles …

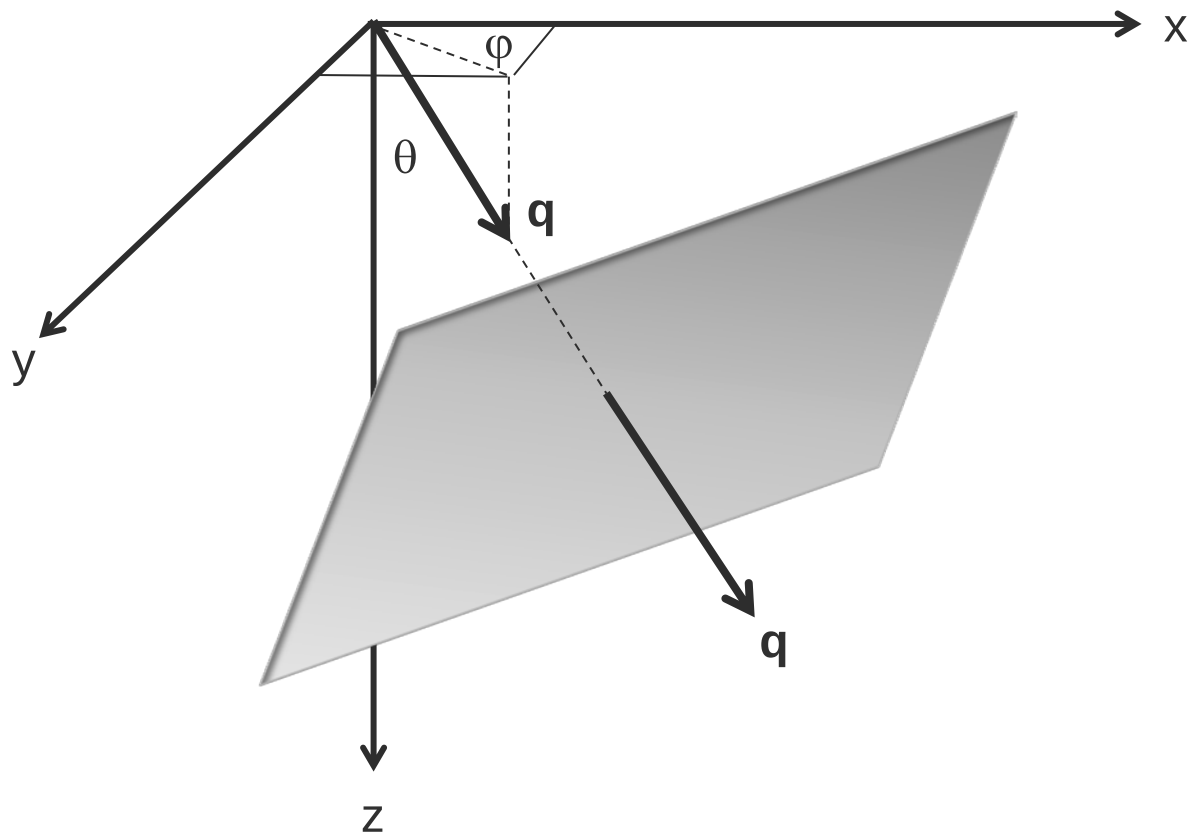
The slowness of a single plane wave is related to incidence (dip) angle θ to the vertical and azimuth φ

$$q_x = \frac{k_x}{\omega} = \frac{dt}{dx} = \frac{1}{c}\cos\varphi\sin\theta \quad \text{horizontal slowness,} \; q_x = \frac{1}{c}\sin\theta \text{ in 2D}$$

$$q_y = \frac{k_y}{\omega} = \frac{dt}{dy} = \frac{1}{c}\sin\varphi\sin\theta \quad \text{horizontal slowness,} \; q_y = 0 \quad \text{in 2D}$$

$$q_z = \frac{k_z}{\omega} = \frac{dt}{dz} = \sqrt{c^{-2} - q_x^2 - q_y^2} = \frac{1}{c}\cos\theta \quad \text{vertical slowness}$$

Plane wave front, perpendicular to slowness vector **q**



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Fourier transform sign convention

The sign is up to you to choose, but the inverse transform has to have the opposite sign. For the inverse Fourier transform, our choice is

$$f(t,x,y) = \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} d\omega dk_x dk_y \exp\left(-i\left(\omega t - k_x x - k_y y\right)\right) F(\omega, k_x, k_y)$$

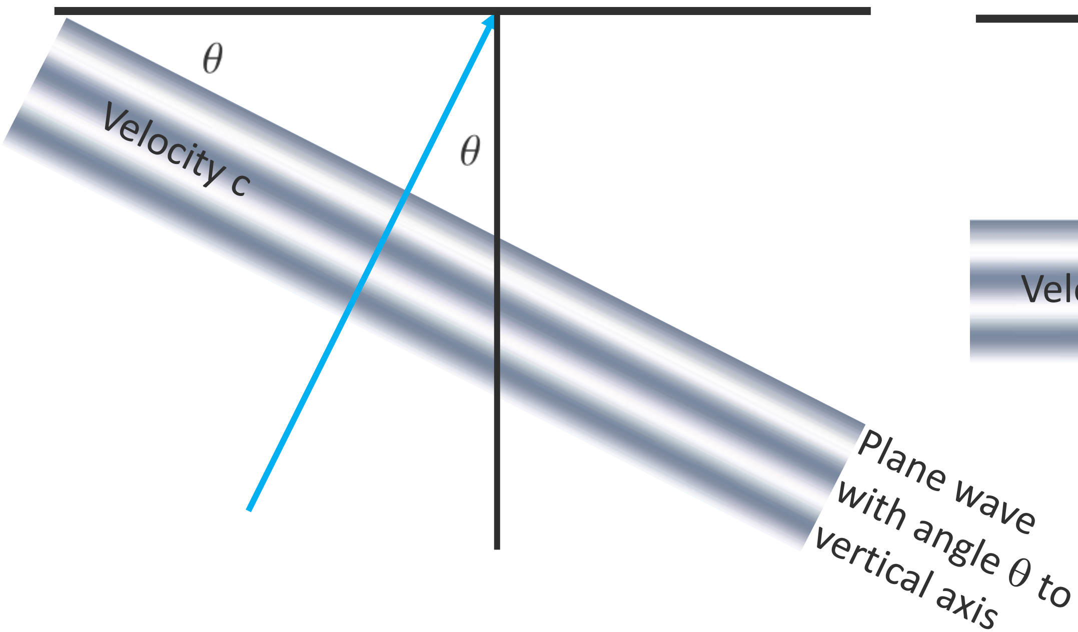
This integrand can be interpreted, in the case of positive *kx, ky*, and ω, as a wave propagating in the positive directions of x and y.

Let z be positive downwards. In a homogeneous medium, the solutions to the Helmholtz equation are

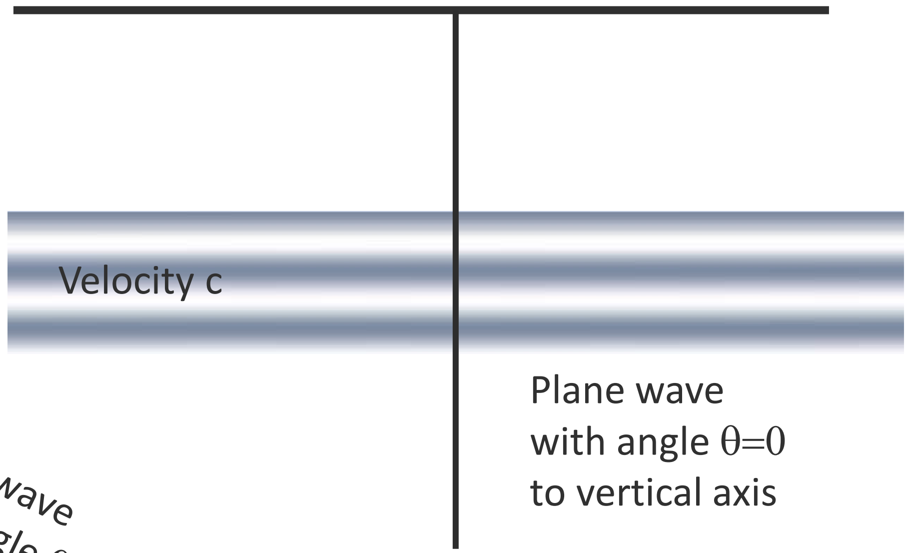
$$P\left(k_x, k_y, z, \omega\right) \propto \exp(\pm i k_z z)$$

representing downgoing (+) and upgoing (-) waves.

Plane waves (2D) traveling along the vertical axis



$$k_x = \frac{\omega}{c}\sin\theta$$
$$k_z = \frac{\omega}{c}\cos\theta$$



$$k_x = 0$$
$$k_z = \frac{\omega}{c}$$

1D wave equation (Helmholtz equation) Solution homogeneous medium with source

1D WE

$$\left(\frac{d^2}{dz^2} + k_z^2\right) P(z) = -a(\omega)\delta(z - z_s)$$

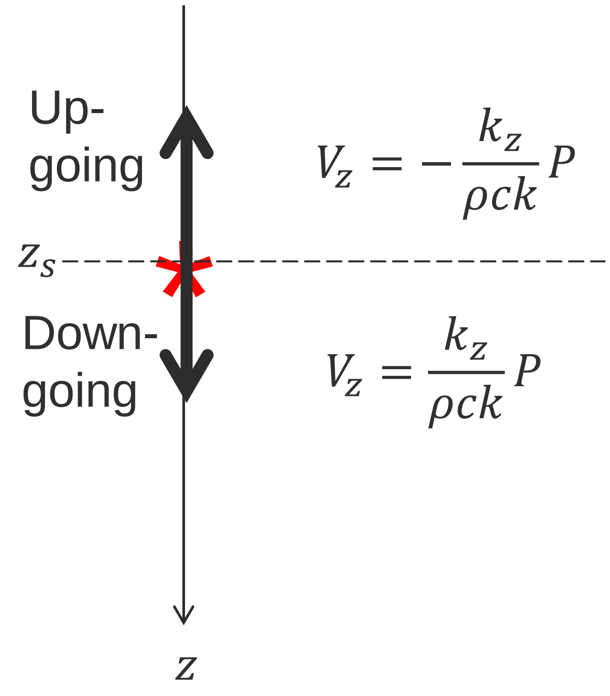
Pressure field for monopole point source with wavelet a(w) is omni-directional.

$$P(z) = \exp(ik_z|z - z_s|)\, s$$

$$s = -\frac{a(\omega)}{2ik_z} \quad \text{source radiation pattern}$$

Associated particle velocity

$$V_z(z) = (i\omega\rho)^{-1}\frac{dP(z)}{dz} = \frac{k_z}{\rho c k}\text{sign}(z - z_s)P(z)$$



Solution homogeneous medium bounded by free surface P(z=0)=0

$$P(z) = [\exp(ik_z|z - z_s|) - [\exp(ik_z(z + z_s))] \ s$$

Incident wave　　　Ghost

$$\mathbf{0 < z < z_s}: \ P(z) = \exp(ik_z(z_s - z)) \ G\_(z) \ s$$

$G\_(z) = 1 - \exp(2ik_z z)$ receiver ghost function for pressure

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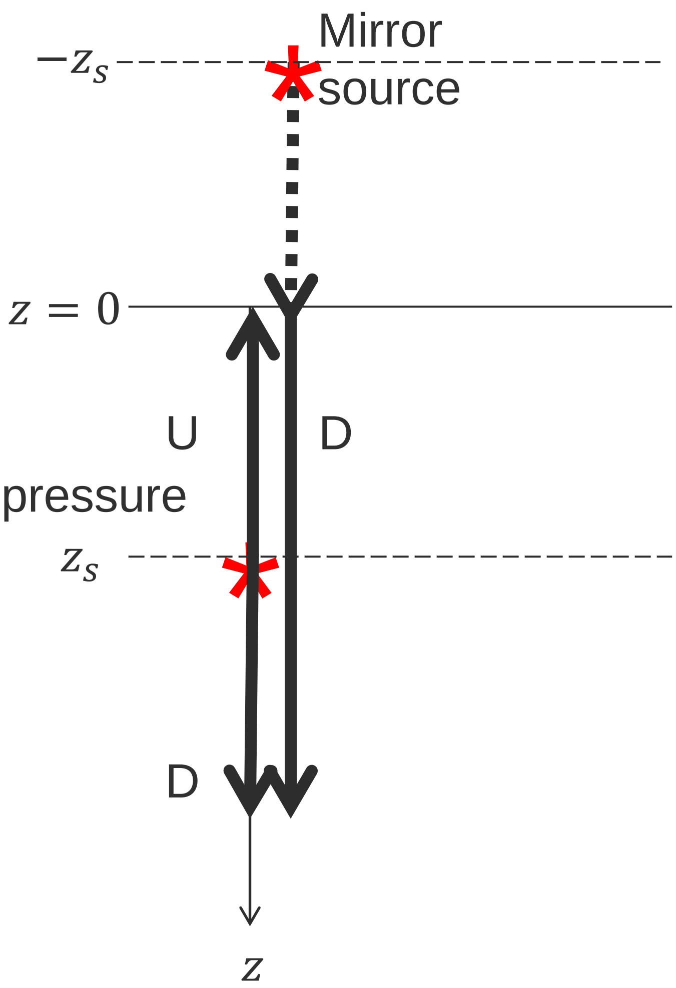
$$\mathbf{z > z_s}: \ P(z) = \exp(ik_z(z - z_s) \ G\_(z_s) \ s$$

$G\_(z_s) = 1 - \exp(2ik_z z_s)$ source ghost function



Solution homogeneous medium bounded by free surface P(z=0)=0

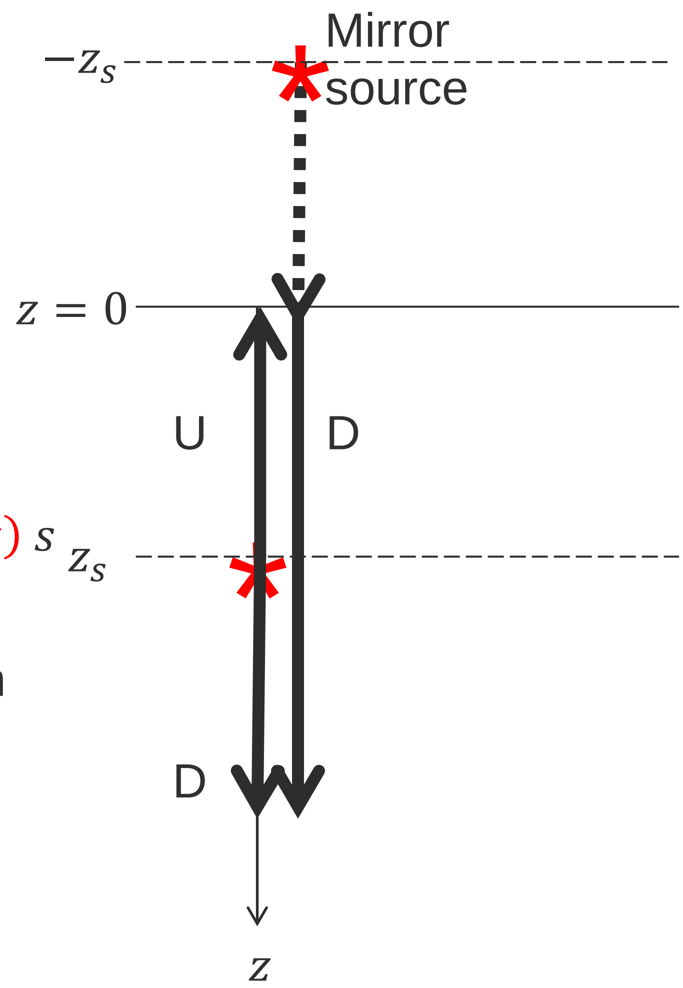
Consider $\mathbf{0 < z < z_s}$

$$P(z) = \exp(ik_z(z_s - z))\ G_-(z)\ s$$

$G_-(z) = 1 - \exp(2ik_z z)$ receiver ghost function for pressure

$$V_z(z) = (i\omega\rho)^{-1}\frac{dP(z)}{dz} = -\frac{k_z}{\rho c k}\exp(ik_z(z_s - z))\ G_+(z)\ s$$

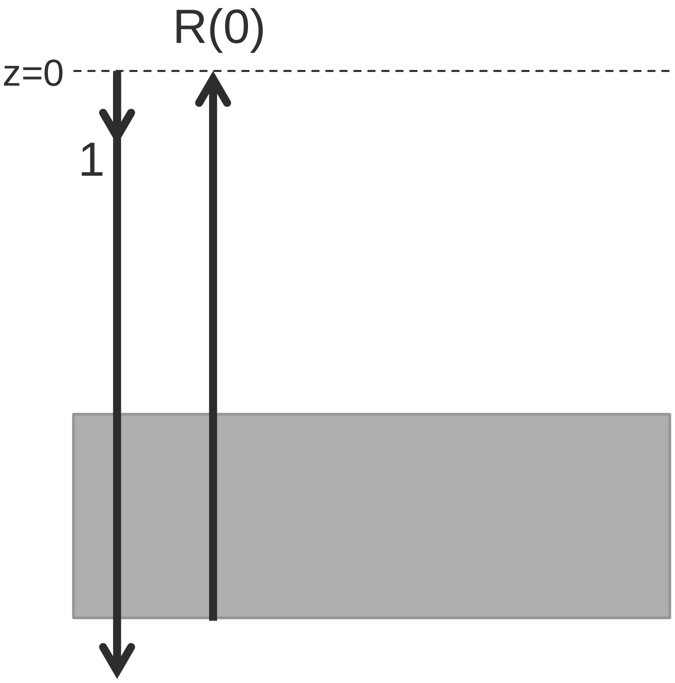
$G_+(z) = 1 + \exp(2ik_z z)$ receiver ghost function for particle velocity

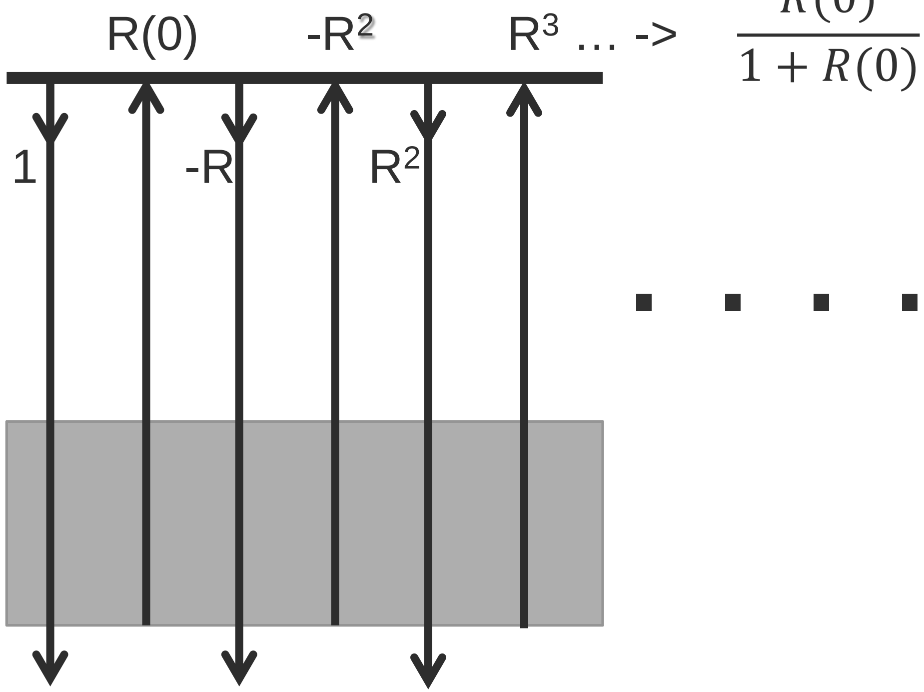


Note + sign in ghost function , since:

$$\frac{d}{dz}[\exp(-ik_z z) - \exp(ik_z z)] = -ik_z[\exp(-ik_z z) + \exp(ik_z z)] = -ik_z\exp(-ik_z z)\ \ G_+(z)$$

Layered medium: reflection response at z=0



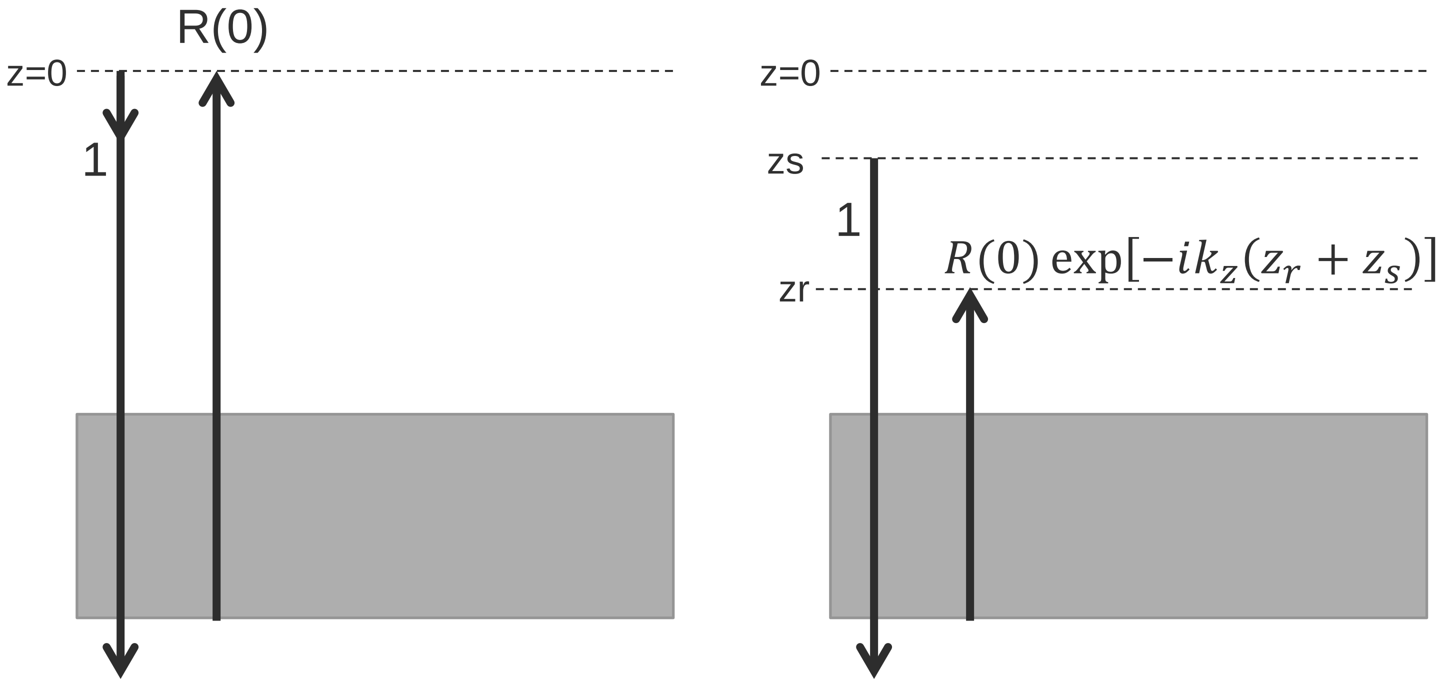


$$\frac{R(0)}{1 + R(0)}$$

Free surface absent

Free surface present

Layered medium: reflection response at receiver depth zr from source at depth zs



Layered medidum: pressure response

Pressure = Direct wave (incident + ghost)
+ Reflected wavefield (due to layering)

Reflected pressure wavefield = Reflection response R(0) adusted to zs and zr
x free-surface related multiples
x receiver ghost function
x source ghost function
x source radiation pattern

Layered medium, zr>zs

Pressure

$$P(z_r) = P_d(z_r) + P_r(z_r)$$

$$P_d(z_r) = \exp[ik_z(z_r - z_s)]\, G_-(z_s)\, s \qquad s = -\frac{a(\omega)}{2ik_z}$$

$$P_r(z_r) = \frac{R(0)}{1+R(0)} \exp[-ik_z(z_r + z_s)]\; G_-(z_r)\, G_-(z_s)\, s$$

Particle velocity (neglect direct part)

$$V_z(z_r) = -\frac{k_z}{\rho\omega}\frac{R(0)}{1+R(0)} \exp[-ik_z(z_r + z_s)]\; G_+(z_r)\, G_-(z_s)\, s$$

$$V_z(z_r) = -\frac{k_z}{\rho\omega}\frac{G_+(z_r)}{G_-(z_r)} P(z_r)$$

$V_z - P$ relation (Amundsen, 1993, 1995)

(for low frequencies, not very sensitive to errors in receiver depth … reflection coefficient close to -1)

Statoil

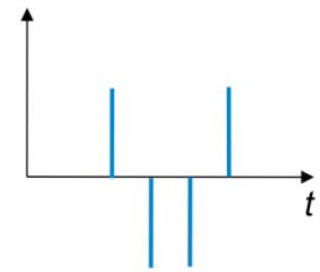
Layered medium, Ghosts 1D

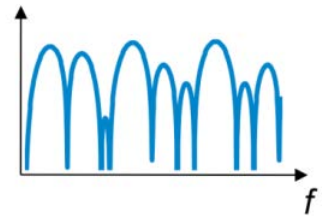
Frequency domain

$$G_{-}(z_r)G_{-}(z_s) = 1 - \exp(2ikz_r) - \exp(2ikz_s) + \exp(2ik(z_r + z_s))$$

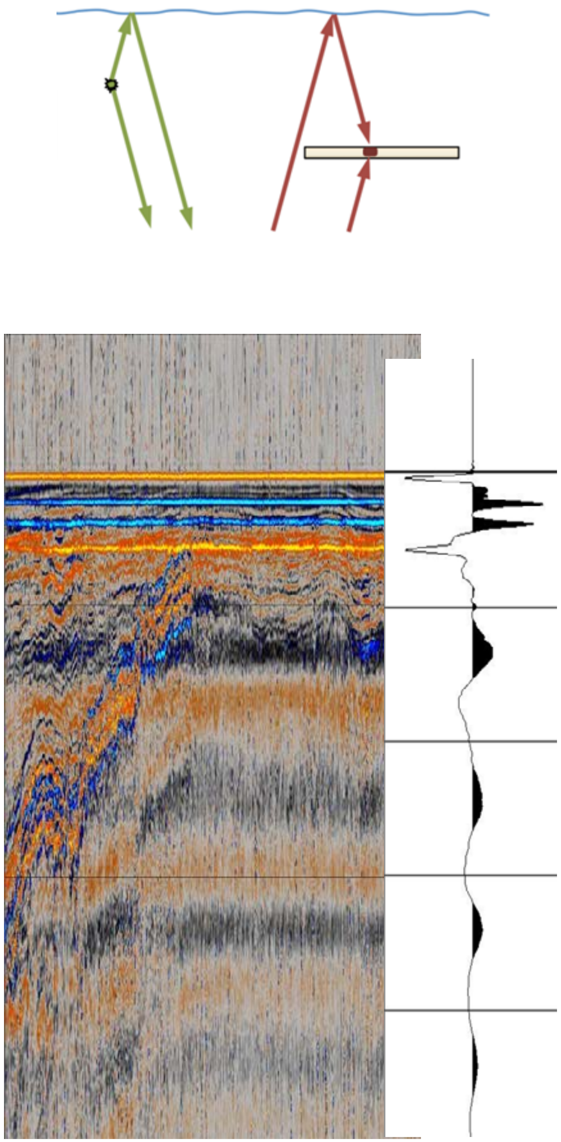
Time domain

$$s(t) - s(t - 2z_r/c) - s(t - 2z_s/c) + s(t - 2(z_r + z_s)/c)$$

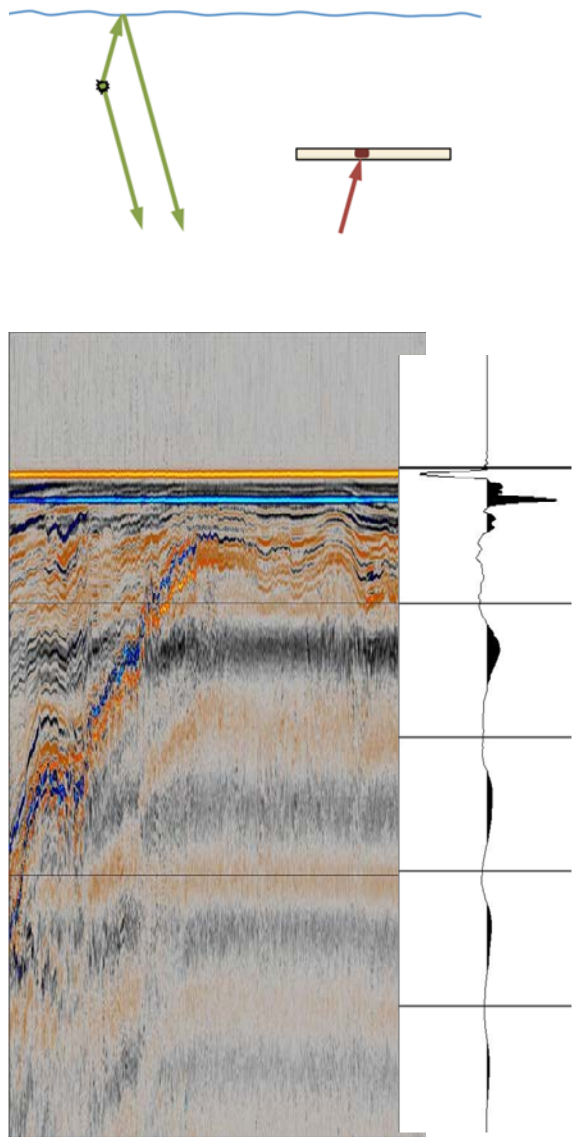




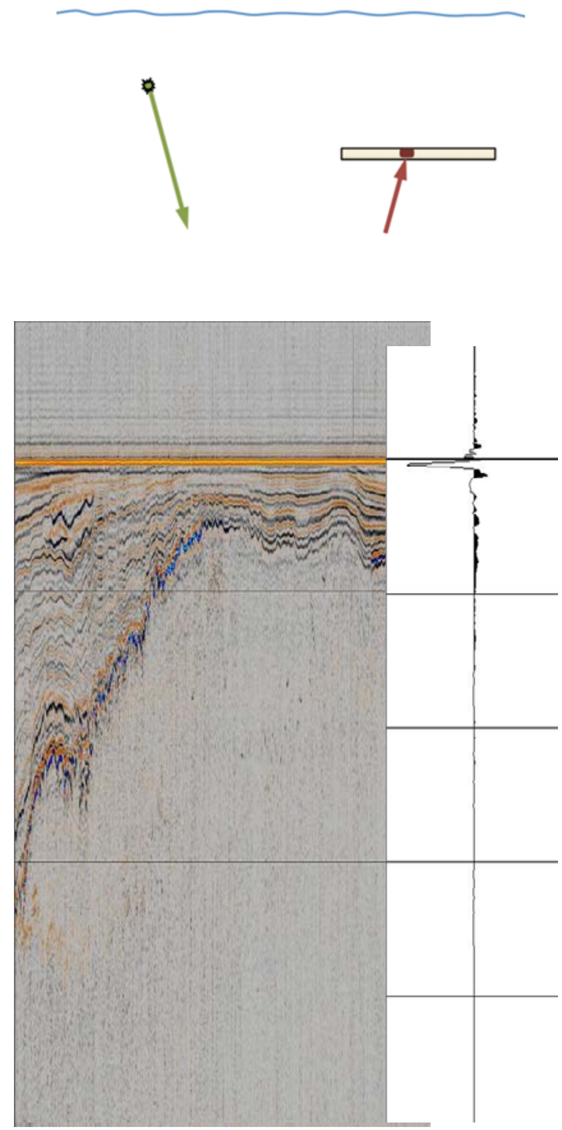
Both ghosts included



Receiver ghost removed



Both ghosts removed and de-signature



Effect of sea surface may modify ghosts (in particular for higher frequencies)

$$G_{\pm}(z_r) = 1 \pm |r(\omega)| \exp(2ik_z z_r)$$

The receiver ghost model depends on

- Receiver depth *zr*
- Velocity *c*
- Sea surface reflection coefficient; for flat sea $r(\omega)=-1$

Ghost effect on hydrophones (1D)

Ghost function

$$G_{-}(z) = 1 - \exp(2ikz)$$

Frequency spectrum has zeroes or ‘notches’ at frequencies

$$|G_{-}(z)| = 2\sin(kz) = \mathbf{2}\sin(2\pi f z/c)$$

$$f_n = \frac{nc}{2z} \quad (n = 0,1,2,3,\dots)$$

Ghosts interfere constructively or destructively with the primary reflections

$$\frac{d|G_{-}(z)|}{df} \propto z \quad \text{for low frequencies}$$

The inverse of the ghost function, 1/G-, has simple poles at *fn*.

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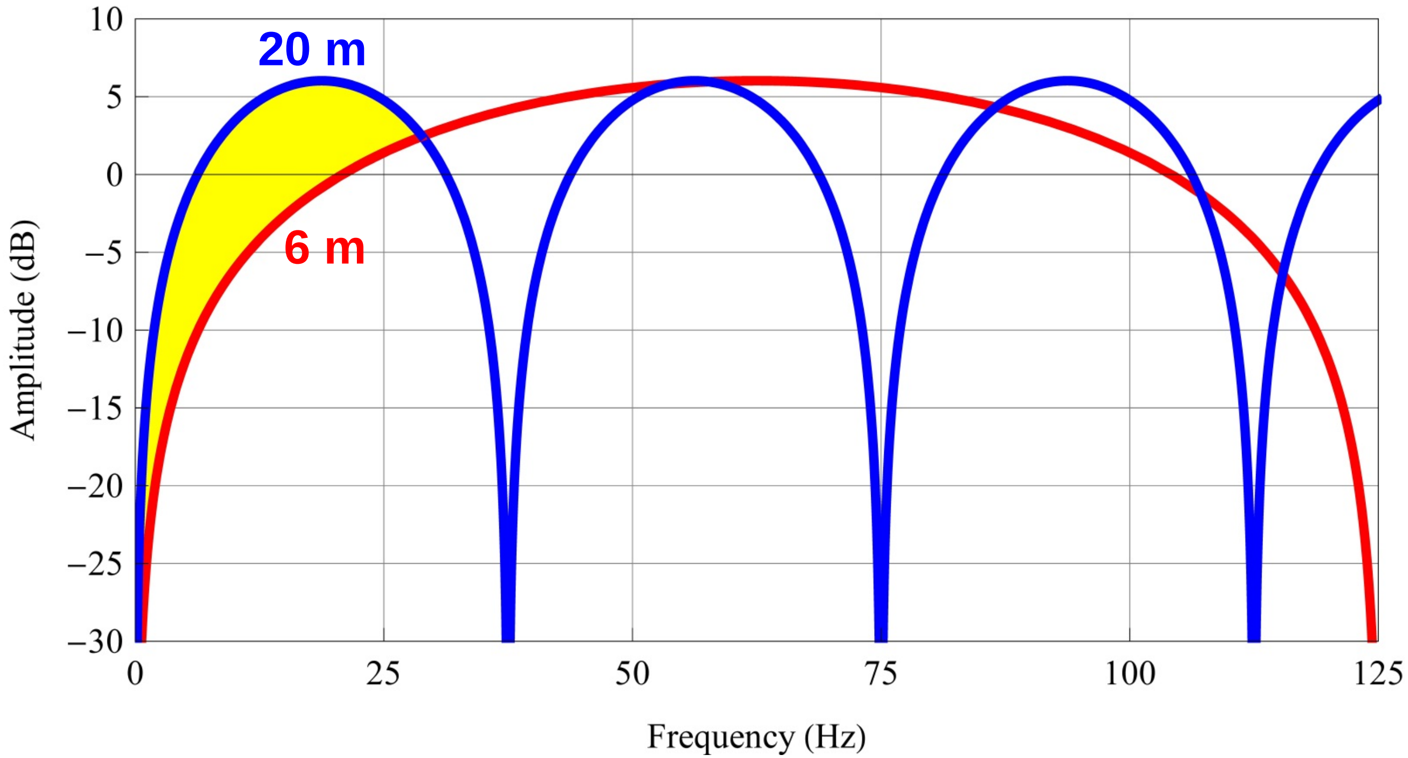
Ghost effect on hydrophones (1D)



The ghost modulates the pressure spectrum, reducing energy at the notch frequencies. The first notch is at f=0, resulting in strong loss of useful low-frequency energy. Similar losses at the second and higher notch frequencies. The useable seismic pressure bandwidth is normally between the first and second notch. Towing streamers shallowly favors the higher frequencies at the expense of attenuating the low frequencies, while towing streamers deeper favors the lower frequencies, at the expense of attenuating frequencies within the seismic bandwidth.

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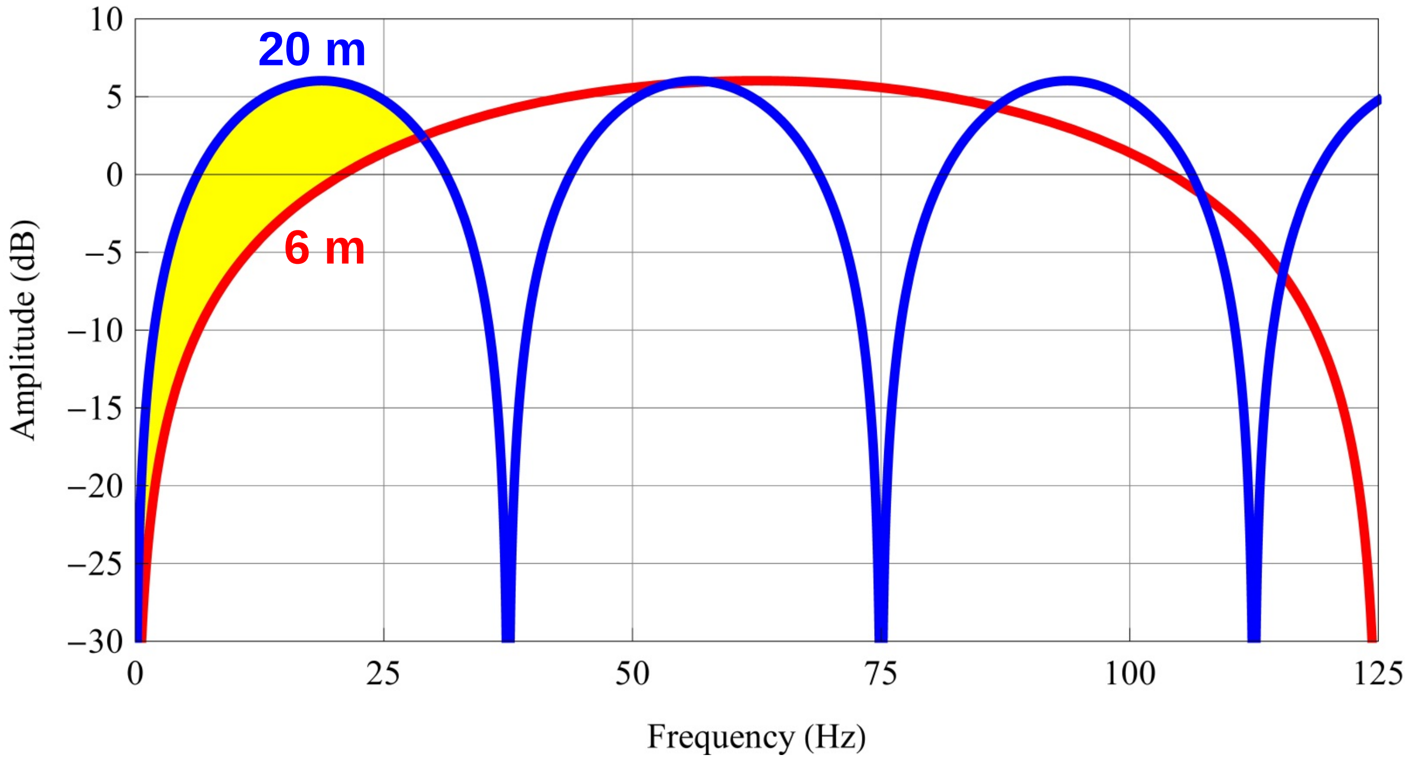
Ghost effect on hydrophones (1D)



Ghost responses for deep-tow at 20m and shallow-tow at 6m. The ghost amplifies some frequencies (amplitude >0 dB) and attenuates other frequencies (amplitude <0 dB). By towing deeper the pressure signal is improved below ~30 Hz. Although deep-tow yields nice low-frequency characteristics, the second notch at 37.5 Hz has a detrimental effect on resolution.

Ghost effect on hydrophones (3D)

Ghost function

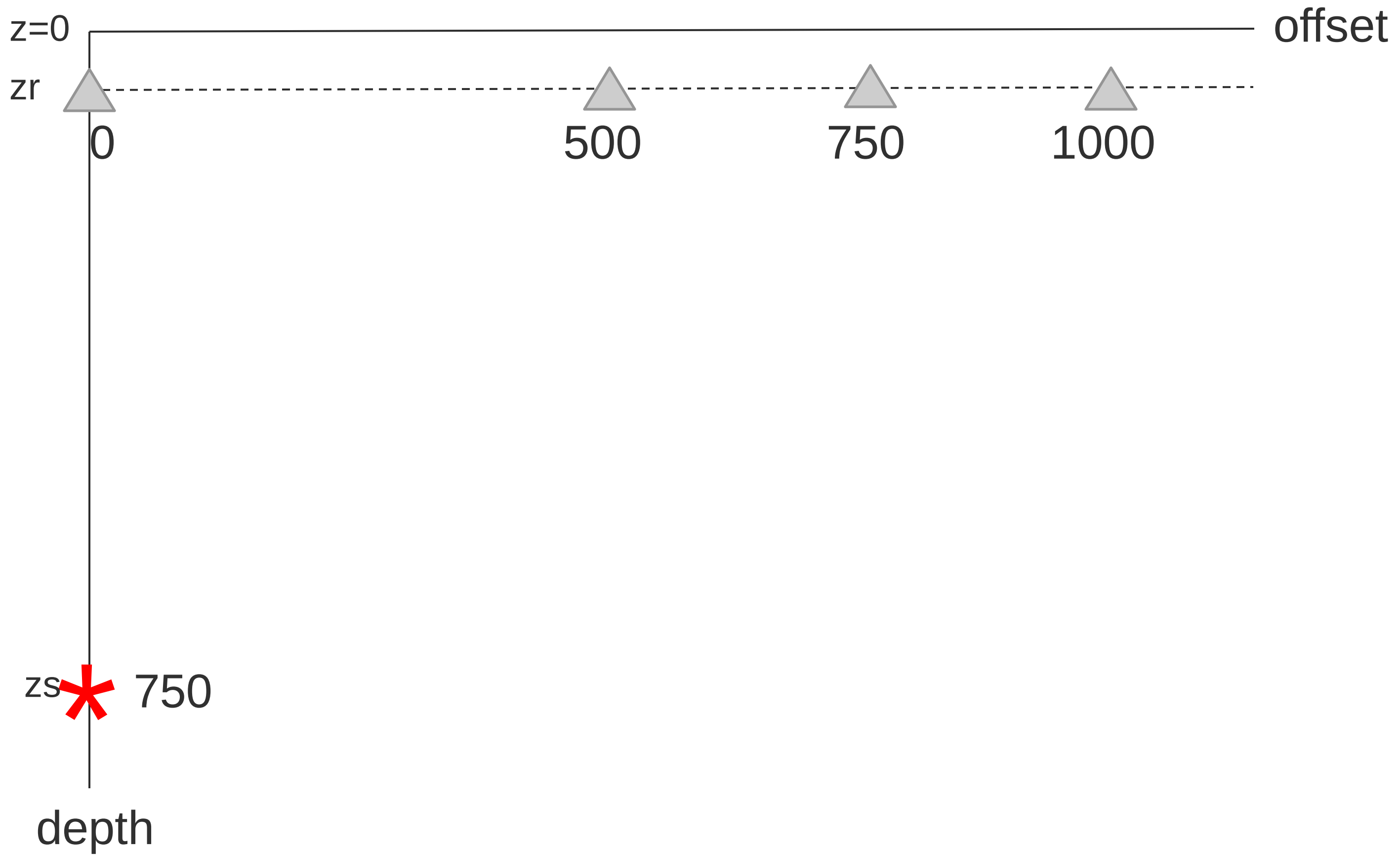
$$G_{-}(z) = 1 - \exp(2ik_z z)$$

$$k_z = k\cos\theta$$

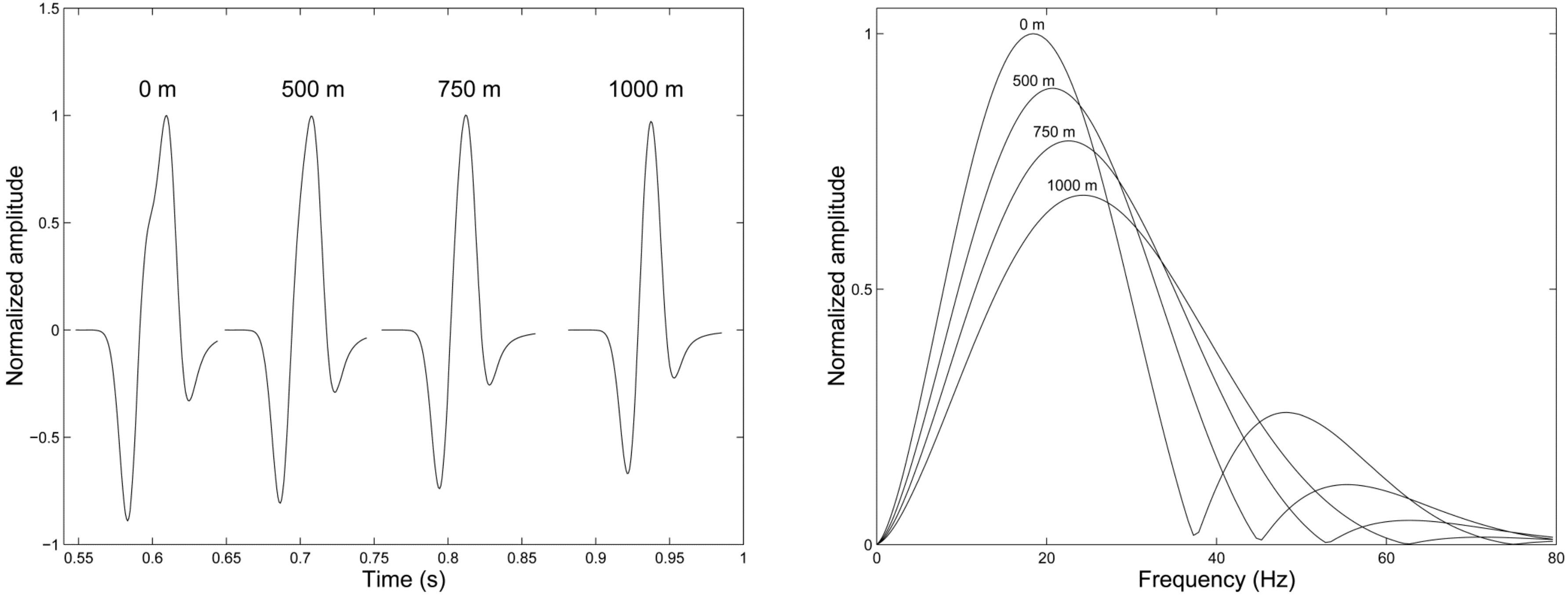
Frequency spectrum has notches at frequencies

$$f_{n,\theta} = \frac{nc}{2z\cos\theta} = \frac{f_n}{\cos\theta} \geq f_n \quad (n = 0,1,2,3,\dots)$$

Simple test model



Ghost effect vs offset/angle (2D)



Pressure data consisting of incident wave and ghost at z=20 m for a few offsets.
For zero offset the (second) notch frequency is at f1 = 37.5 Hz. Observe that this notch frequency increases with increasing offset.

Ghost effect on geophones (3D)

Ghost function

$$G_{+}(z) = 1 + \exp(2ik_z z)$$

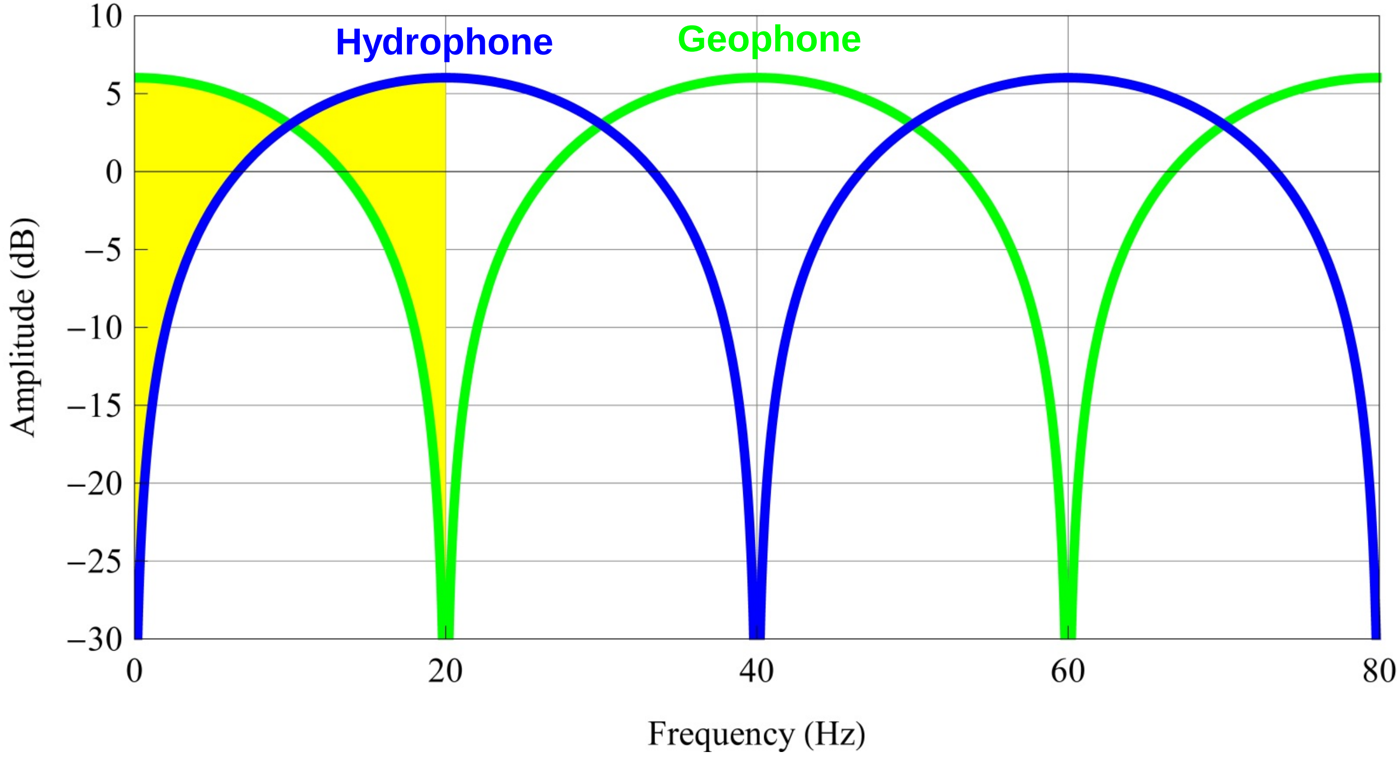
Frequency spectrum has notches at frequencies

$$|G_{+}(z)| = 2\cos(k_z z) = 2\cos(2\pi f z \cos\theta\ /c)$$

$$f_{n,\theta} = \frac{(2n+1)c}{4z\cos\theta} \qquad (n = 0,1,2,3,\dots)$$

These notches are lying mid-between the notches in pressure recordings

Hydrophones and geophones measure complementary information



Receiver ghost responses for hydrophone and geophone at 18.75m depth. The geophone has maximum response (+6 dB) where the hydrophone has notch, and vice versa. For the low frequencies f<20 Hz, the real geophone signal is too noisy, and deghosting is achieved using the geophone-hydrophone model.

Deghosting of pressure by inverse filtering

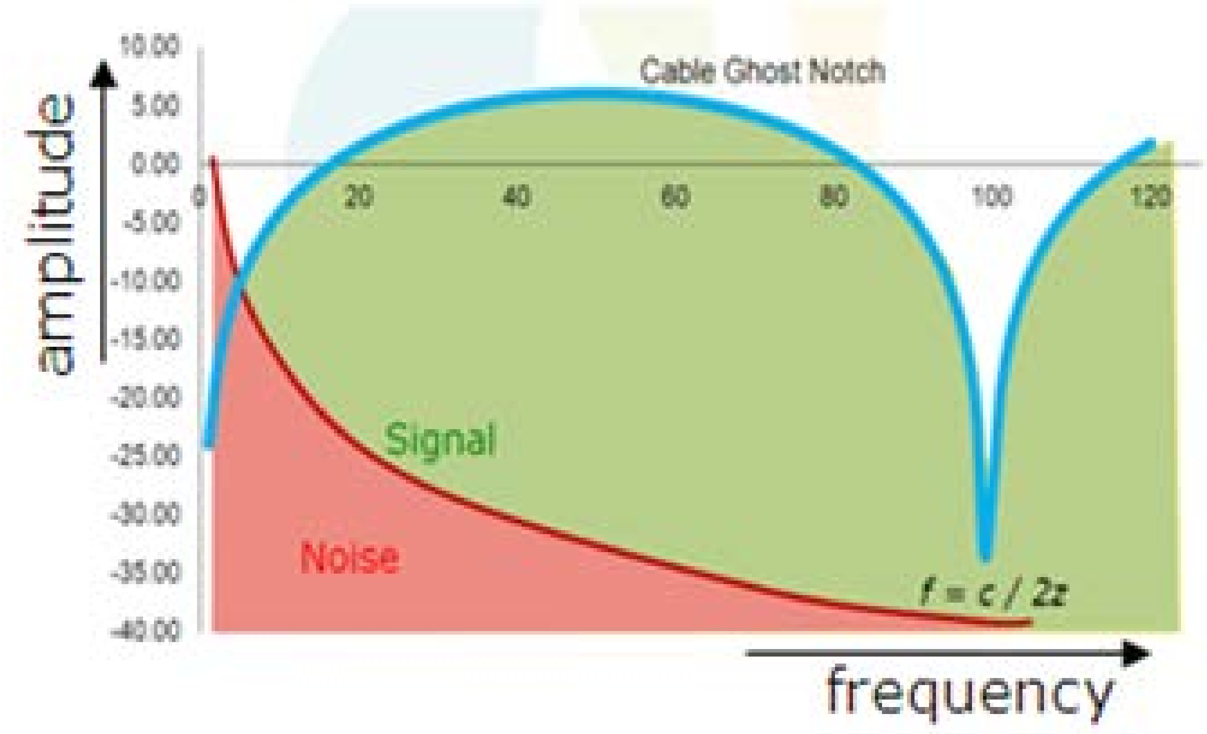
$$P_r(z_r) = \frac{R(0)}{1+R(0)} \exp[-ik_z(z_r + z_s)]\, G_-(z_r)\, G_-(z_s)\, s$$

Inverse filtering

$$P_r^{dg}(z_r) = P_r(z_r)\frac{1}{G_-(z_r)} = \frac{R(0)}{1+R(0)} \exp[-ik_z(z_r + z_s)]\; G_-(z_s)\, s$$

$$\frac{1}{G_-(z_r)} = \frac{1}{1-\exp(2ik_z z_r)}$$

For deghosting by inverse filtering, S/N>1



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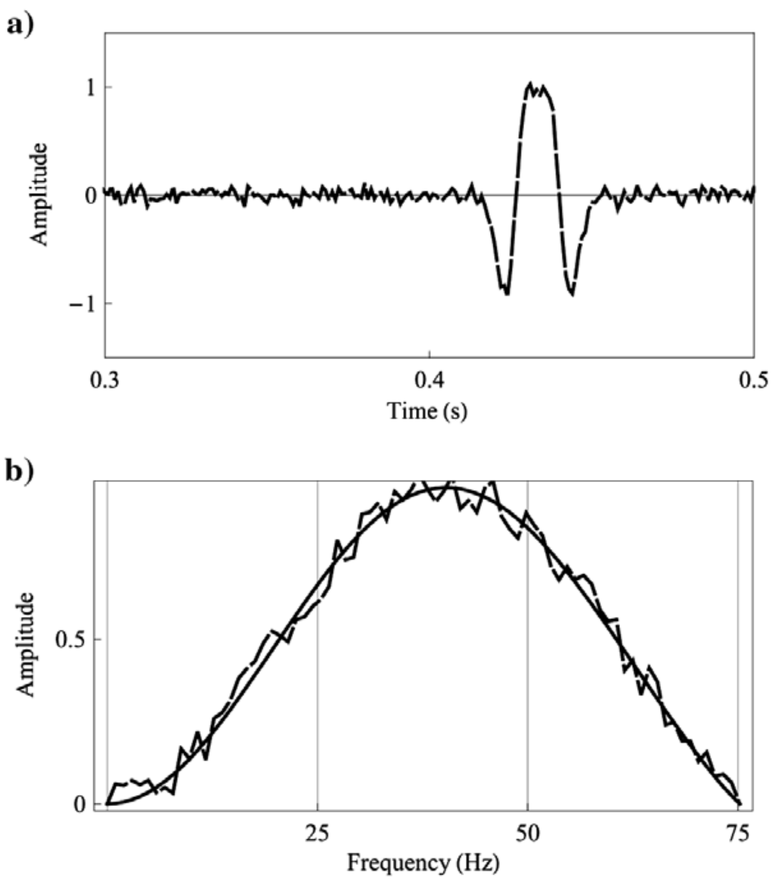
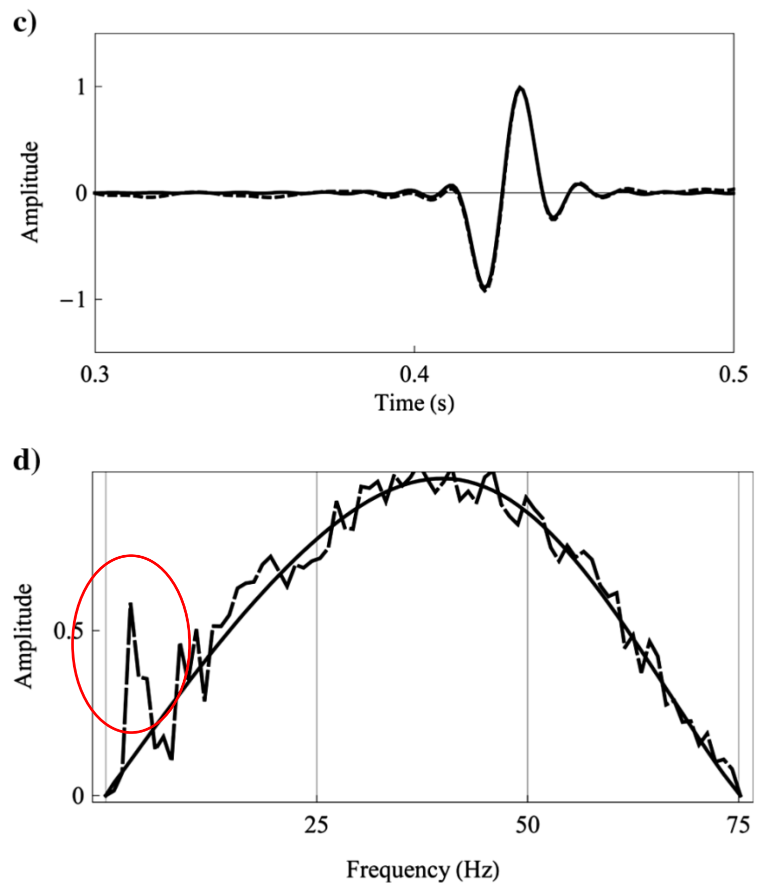
Inverse filtering «impossible» close to notch when S/N<1



Signal and signal + noise (noise to 150 Hz)



Deghosted signal and signal + noise (highcut 75Hz)
Noise down-weighted below 3 Hz

Low-frequency trace deghosting (LFD)

$$G_{-}(z_r) = 1 - \exp(2ikz_r)$$

Significant contribution for low frequencies

$$\frac{1}{G_{-}(z_r)} = \frac{1}{1 - \exp(2ikz_r)} \approx \frac{1}{2}\left[1 - \frac{c}{i\omega z_r} + \frac{i\omega z_r}{3c} + \cdots\right]$$

$$P_r^{dg}(x,t) = \frac{1}{2}\left[P_r(x,t) + \frac{c}{z_r}\int_0^t dt' P_r(x,t') + \frac{z_r}{3c}\partial_t P_r(x,t)\right]$$

For *kz=k*, the low-frequency deghosted pressure field can be computed trace-by-trace as the sum of the pressure field and its scaled temporally integrated and temporally differentiated fields. LFD deghosts data up to a frequency that is typically half of the second notch frequency. LFD can be appropriate to apply to the part of seismic data that have penetrated and reflected beneath complex and attenuating overburdens such as basalt, salt, and chalk.

Amundsen and Zhou Geophysics 2013

Deghosting by P-Vz summation!

- You can exorcize the receiver ghost by "summing" *P* and scaled *Vz.*
- Receiver side deghosting, equivalent to computing the upgoing component of the pressure field, can be done from *P-Vz* measurements.
- This is a fundamental basis of both PGS's GeoStreamer solution and WesternGeco's IsoMetrix solution
- Independent of the sea surface

$$P_r(z_r) = \frac{R(0)}{1+R(0)} \exp[-ik_z(z_r+z_s)]\, G_-(z_r)\, G_-(z_s)\, s$$

$$-\frac{\rho\omega}{k_z} V_z(z_r) = \frac{R(0)}{1+R(0)} \exp[-ik_z(z_r+z_s)]\, G_+(z_r)\, G_-(z_s)\, s$$

$$P_r^{dg}(z_r) = \frac{1}{2}\left[P(z_r) - \frac{\rho\omega}{k_z} V_z(z_r)\right]$$

$$k_z = \frac{\omega}{c}\cos\theta$$

Vz-P relation

$$V_z(z_r) = -\frac{k_z}{\rho\omega}\frac{G_+(z_r)}{G_-(z_r)}P(z_r)$$

- For low frequencies where *Vz* is noisy, *Vz* can be estimated from P by deghosting the pressure (multiplying *P* by *1/G–*) and ghosting the result (multiplying by *G+*)
- This Vz estimation is used at low frequencies whereas the *Vz* measurements are used at higher frequencies in GeoStreamer deghosting (Anthony Day, Tilman Klüver, Walter Söllner, Hocine Tabti, and David Carlson, Geophysics 2013)
- *Vz-P* model is used also in 3D deghosting of IsoMetrix measurements, not necessarily to replace low-frequency Vz data but rather to further constrain the cross-line reconstruction problem

- *P-Vz* sensor streamers have no direct benefits for low frequency recording as they use only the hydrophone at low frequencies; the geophones are used to infill the higher frequency ghost notches
- To get high-quality pressure measurements at low frequencies, the cables must be towed deep where the pressure ghost notch has minimum effect and the *S/N* ratio is good

Simultaneous deghosting and crossline wavefield reconstruction (GMP)

$$P(k_y) = G_-(k_y)P^{dg}(k_y) = H_1(k_y)P^{dg}(k_y) = S_1(k_y)$$
$$\partial_y P(k_y) = ik_y G_-(k_y)P^{dg}(k_y) = H_2(k_y)P^{dg}(k_y) = S_2(k_y)$$
$$\partial_z P(k_y) = ik_z G_+(k_y)P^{dg}(k_y) = H_3(k_y)P^{dg}(k_y) = S_3(k_y)$$

Note: Ghost models are used to relate each input to the unknown deghosted pressure

Model the deghosted pressure at position *yn* as a sum of complex exponential basis functions, each described by amplitude, wavenumber and phase

$$P^{dg}(y_n) = \sum_m A_m \exp\left(ik_{y,m}y_n + \varphi_m\right)$$

Expand multi-measurement input signals (i=1,2,3) at position *yn* as a combination of the same basis functions

$$S_i(y_n) = \sum_m H_i(k_{y,m})A_m \exp\left(ik_{y,m}y_n + \varphi_m\right)$$

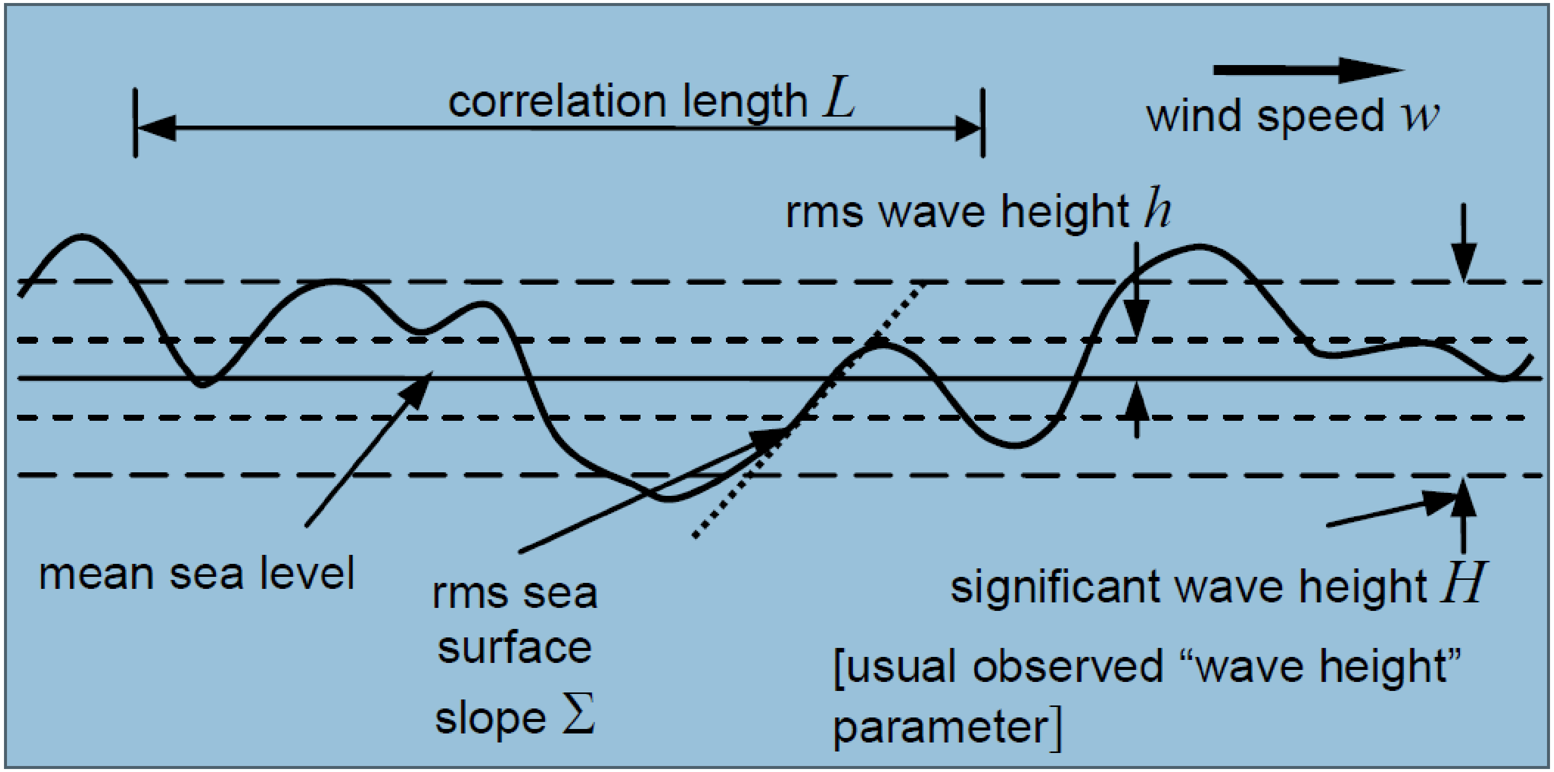
GMP (Generalized Matching Pursuit) iteratively looks for a set of basis functions that match all input signals at the measured positions (Özbek et al Geophysics 2010).

Statoil

Effect of sea surface may modify ghosts (for higher frequencies)

$$G_{-}(z_r) = 1 - |r(\omega)| \exp(2ik_z z_r)$$

Parameters for a rough surface



Jones et al 2009

Kirchhoff model

For randomly rough surface, the Kirchhoff approximation to scattering and the assumption of a Gaussian probability of surface elevations of standard deviation *h* (rms roughness) gives

$$r(\omega) = -\exp(-2(kh\cos\theta)^2)$$

Relation to wind velocity v_{20} measured 20 m above the sea surface

$$h^2 = Dv_{20}^4$$

The ghost notch effect is less in rough weather … good or bad?

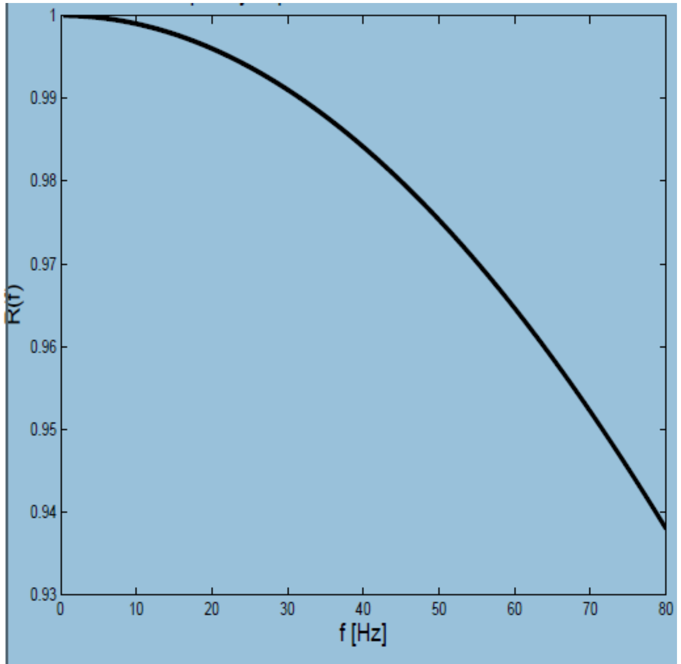


Figure 1: Reflection coefficient at zero offset $c = 1500$ m/s, $v_{20} = 10$ m/s

Reitan, Landrø, Amundsen 2013 (work in progress)

Statoil

The kz problem

We don't have properly sampled data in cross-line (y) direction: streamer separation is 75-100 m. Processing data on each streamer Individually, we implicitly assume that ky=0, and as a result, kz is systematically overestimated.

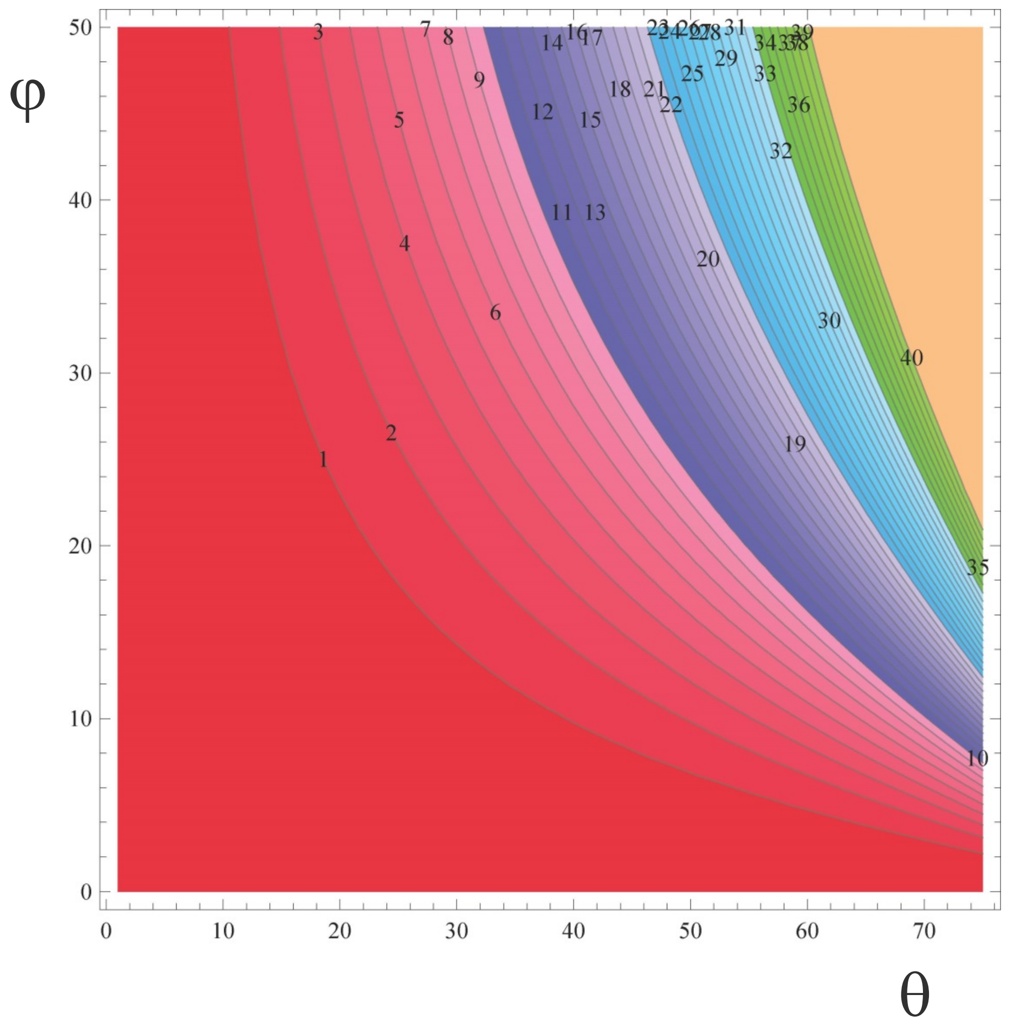
The 3D vertical wavenumber can be expanded in the ky direction. The 2D inline direction part can be implemented accurately in the in-line direction. If the second spatial derivative of data in the cross-line direction is measured, one may achieve better 3D deghosting (Robertsson 2005)

$$k_z^{2D} = \sqrt{k^2 - k_x^2}$$

$$k_z^{3D} = \sqrt{k^2 - k_x^2 - k_y^2} = k_z^{2D}\left(1 - \frac{k_y^2}{2(k_z^{2D})^2} + O(k_y^4)\right)$$

One may derive slownesses (and 3D vertical wavenumber) from 4C acoustic measurements (Vassallo 2009; Amundsen, Ikelle, Robertsson, Westerdahl SEG 2013), using an assumption of non-overlapping primaries/multiples

$$(1/q_z^{3D} - 1/q_z^{2D})/(1/q_z^{3D})$$



$$q_x = \frac{1}{c}\cos\varphi\sin\theta$$

$$q_y = \frac{1}{c}\sin\varphi\sin\theta$$

$$q_z^{3D} = \sqrt{c^{-2} - q_x^2 - q_y^2} = \frac{1}{c}\cos\theta$$

$$q_z^{2D} = \frac{1}{c}\sqrt{1 - (\cos\varphi\sin\theta)^2}$$

Statoil

Deghosting by spatial convolution

Wavenumber domain multiplication

$$P\,\frac{1}{G_{-}(z_r)} = P\,\frac{1}{1 - r(\omega)\exp(2ik_z z_r)}$$

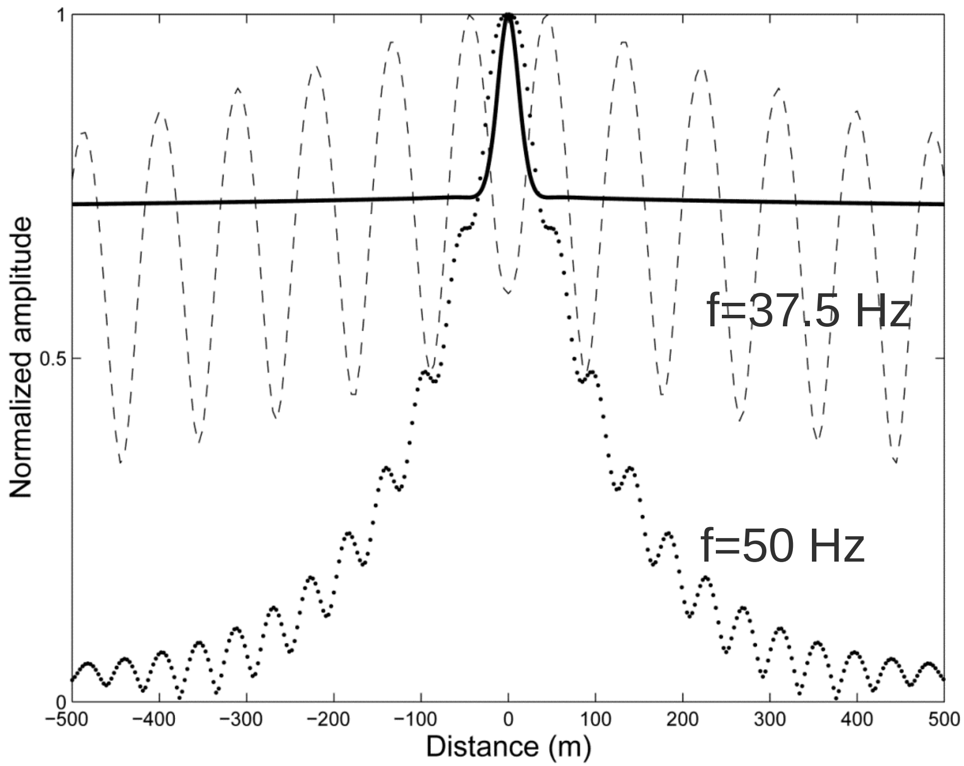
Space domain (2D) convolution

$$P * \frac{i(1+|r|)}{2}\sum_{n=0}^{\infty}|r|^n H(kR_n) \qquad R_n = \sqrt{(x - x_r)^2 + (1+2n)^2 z_r^2}$$

H Hankel function

Amundsen,
Weglein and
Reitan,
Geophysics 2013

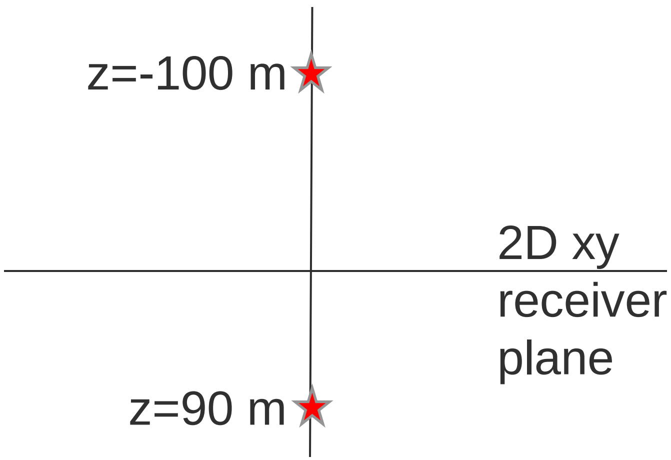
Deghosting filters in space domain are very long



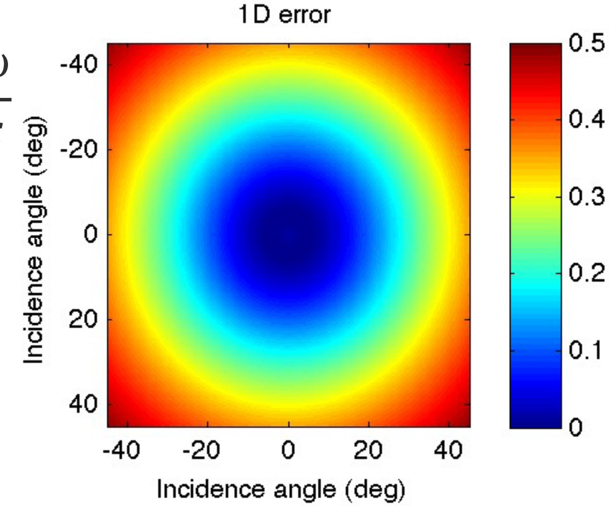
Amundsen, Weglein and Reitan, Geophysics 2013

Amplitude spectra of deghosting filters with offset (in the range ±500 m) for z=20 m. For fundamental frequencies fn = nc/(2 z) the deghosting filter has logarithmic singularities or "peaks". These fundamental frequencies correspond to the notch frequencies in the ghost function for plane waves that travel vertically ($\theta = 0$).

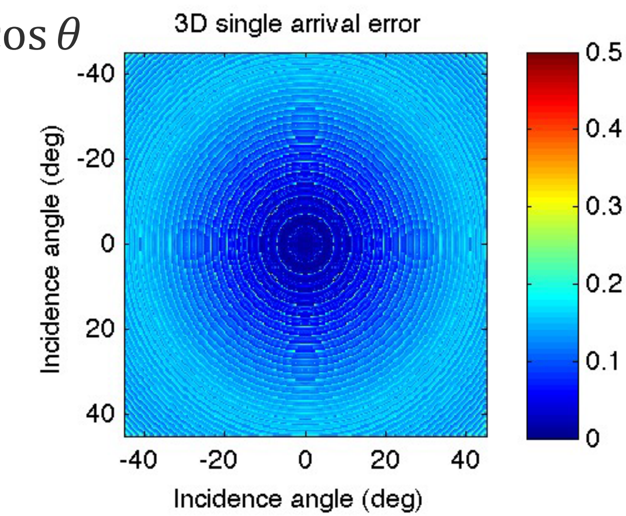
Test of deghosting error 1D vs 3D (OBS seismic)



$$k_z = \frac{\omega}{c}$$



$$k_z = \frac{\omega}{c}\cos\theta$$



Modeling in homogeneous medium upgoing and downgoing waves with opposite wavelet polarities gives traces with ghost on xy receiver plane.
Error plots showing the relative error (rms of the difference over traces normalized by the rms of reference solution trace).

Amundsen, Ikelle, Robertsson, Westerdahl, SEG 2013