LOW-FREQUENCY LAYER-INDUCED ANISOTROPY

A.Stovas (NTNU, Norway)
Y.Roganov (USGPI, Ukraine)

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Outline

- **Upscaling in seismic**
- **Methods**

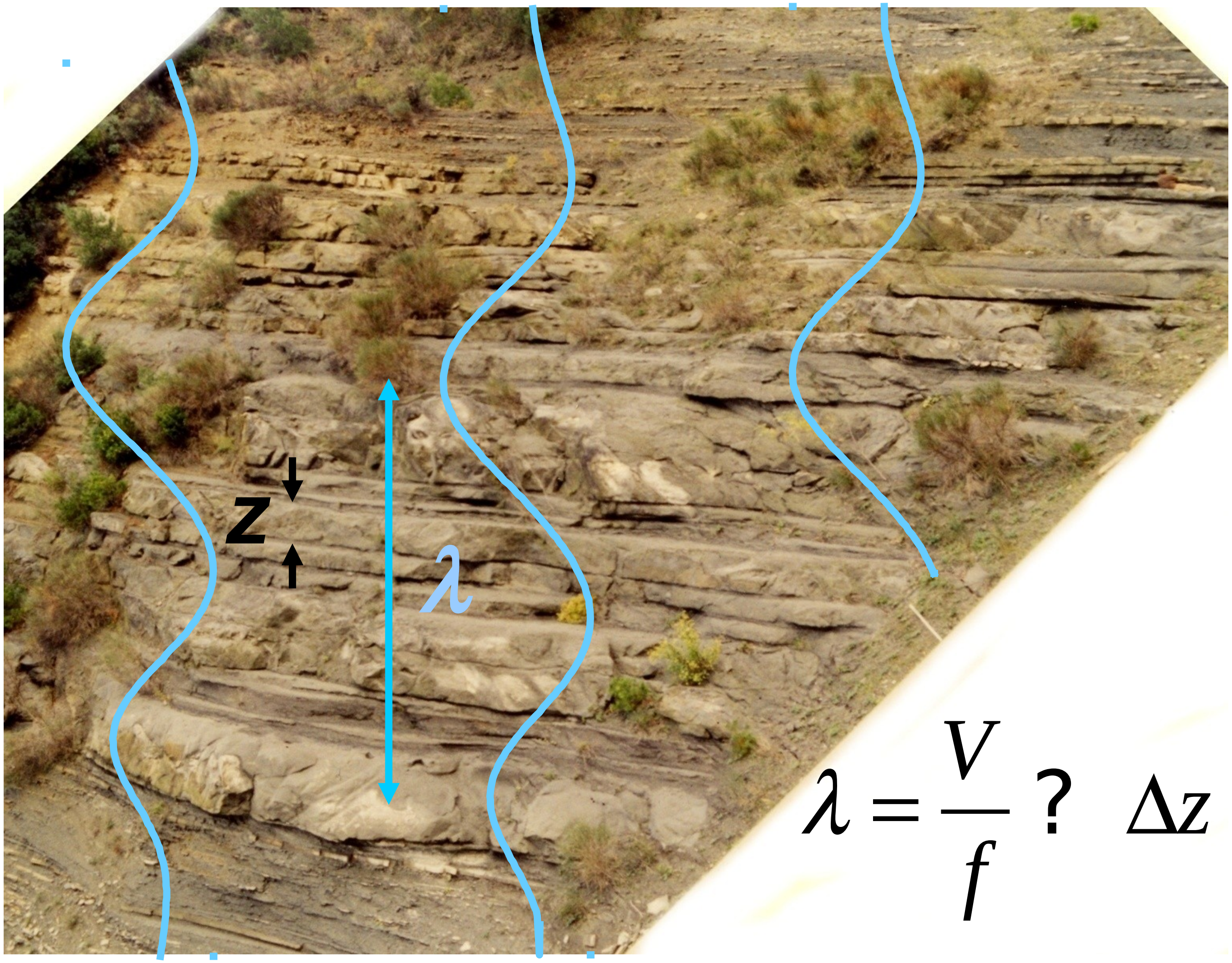
  - Static (Backus)

  - Dynamic (low-frequency)
- **Numerics**

  - Periodically layered medium

  - Real well log data example
- **Conclusions**

Upscaling in seismic



$$\lambda = \frac{V}{f} ? \ \Delta z$$

Backus averaging

Rytov, 1956; Backus, 1962; Schoenberg and Muir, 1989

$$\frac{d\mathbf{b}}{dz} = i\omega \mathbf{A}_j \mathbf{b}$$

$$\tilde{\mathbf{A}} = \left\langle \mathbf{A}_j \right\rangle = \frac{1}{H}\sum_{j=1}^{N} h_j \mathbf{A}_j$$

$$\mathbf{A}_j = \begin{pmatrix} 0 & \mathbf{M}_j \\ \mathbf{N}_j & 0 \end{pmatrix}$$

$$\tilde{c}_{11} = \left\langle c_{11} - \frac{c_{13}^2}{c_{33}} \right\rangle + \left\langle \frac{c_{13}}{c_{33}} \right\rangle^2 \left\langle c_{33}^{-1} \right\rangle^{-1}$$

$$\tilde{c}_{13} = \left\langle \frac{c_{13}}{c_{33}} \right\rangle \left\langle c_{33}^{-1} \right\rangle^{-1}$$

$$\tilde{c}_{33} = \left\langle c_{33}^{-1} \right\rangle^{-1}$$

$$\tilde{c}_{44} = \left\langle c_{44}^{-1} \right\rangle^{-1}$$

$$\tilde{c}_{66} = \left\langle c_{66} \right\rangle$$

$$\tilde{\rho} = \left\langle \rho \right\rangle$$

$$\mathbf{M}_j = \begin{pmatrix} c_{33j}^{-1} & p c_{13j} c_{33j}^{-1} \\ p c_{13j} c_{33j}^{-1} & \rho_j - p^2 \left(c_{11j} - c_{13j}^2 c_{33j}^{-1} \right) \end{pmatrix}$$

$$\mathbf{N}_j = \begin{pmatrix} \rho_j & p \\ p & c_{44j}^{-1} \end{pmatrix}$$

Backus from the stack of isotropic/VTI layers always gives a VTI medium

Low frequency upscaling

Roganov and Stovas, 2011

$$\mathbf{P}(\omega) = \exp\left(i\omega H \tilde{\mathbf{A}}(\omega)\right)$$

$$\tilde{\mathbf{A}}(\omega) = \frac{1}{i\omega H}\log \mathbf{P}(\omega) = \frac{1}{i\omega H}\log\left(\exp\left(i\omega \Delta z_N \mathbf{A}_N\right)\ldots\exp\left(i\omega \Delta z_1 \mathbf{A}_1\right)\right)$$

$$\tilde{\mathbf{A}}(\omega) = \tilde{\mathbf{A}}_0 + i\omega \tilde{\mathbf{A}}_1 + (i\omega)^2 \tilde{\mathbf{A}}_2 + \ldots$$

We expand the logarithm of propagator matrix.
Using the BCH series we derive the low frequency
*expansion of the fundamental matrix **A**.*

Baker–Campbell–Hausdorff formula

is the solution $\mathbf{Z} = \log\left[\exp(\mathbf{X})\exp(\mathbf{Y})\right]$

for noncommuting matrices **X** and **Y**

(Campbell, 1897; Poincare, 1899; Baker, 1902; Hausdorff,1906)

This formula links Lie groups to Lie algebras

$$\mathbf{Z} = \log\left(\exp(\mathbf{X})\exp(\mathbf{Y})\right) = \mathbf{X} + \mathbf{Y} + \frac{1}{2}[\mathbf{X},\mathbf{Y}] + \frac{1}{12}\left([\mathbf{X},[\mathbf{X},\mathbf{Y}]] - [\mathbf{Y},[\mathbf{X},\mathbf{Y}]]\right)$$

$$-\frac{1}{24}[\mathbf{Y},[\mathbf{X},[\mathbf{X},\mathbf{Y}]]]$$

$$-\frac{1}{720}\left([[[[\mathbf{X},\mathbf{Y}],\mathbf{Y}],\mathbf{Y}],\mathbf{Y}] + [[[[\mathbf{Y},\mathbf{X}],\mathbf{X}],\mathbf{X}],\mathbf{X}]\right)$$

$$+\frac{1}{360}\left([[[[\mathbf{X},\mathbf{Y}],\mathbf{Y}],\mathbf{Y}],\mathbf{X}] + [[[[\mathbf{Y},\mathbf{X}],\mathbf{X}],\mathbf{X}],\mathbf{Y}]\right)$$

$$+\frac{1}{120}\left([[[[\mathbf{Y},\mathbf{X}],\mathbf{Y}],\mathbf{X}],\mathbf{Y}] + [[[[\mathbf{X},\mathbf{Y}],\mathbf{X}],\mathbf{Y}],\mathbf{X}]\right) + \ldots$$

Matrix coefficients (two layers)

$$\tilde{\mathbf{A}}_{\mathbf{0}} = \alpha_1 \mathbf{A}_{\mathbf{1}} + \alpha_2 \mathbf{A}_{\mathbf{2}} \qquad (Backus)$$

$$\tilde{\mathbf{A}}_{\mathbf{1}} = \frac{1}{2}\alpha_1 \alpha_2 \left[\mathbf{A}_{\mathbf{2}}, \mathbf{A}_{\mathbf{1}} \right]$$

$$\tilde{\mathbf{A}}_{\mathbf{2}} = \frac{1}{12}\alpha_1 \alpha_2 \left\{ \alpha_2 \left[\mathbf{A}_{\mathbf{2}}, \left[\mathbf{A}_{\mathbf{2}}, \mathbf{A}_{\mathbf{1}} \right] \right] + \alpha_1 \left[\mathbf{A}_{\mathbf{1}}, \left[\mathbf{A}_{\mathbf{1}}, \mathbf{A}_{\mathbf{2}} \right] \right] \right\}$$

$$\tilde{\mathbf{A}}_{\mathbf{3}} = -\frac{1}{24}\alpha_1^2 \alpha_2^2 \left[\mathbf{A}_{\mathbf{2}} \left[\mathbf{A}_{\mathbf{1}}, \left[\mathbf{A}_{\mathbf{2}}, \mathbf{A}_{\mathbf{1}} \right] \right] \right]$$

...

α is a volume fraction of each layer

$$\left[\mathbf{x}, \mathbf{y} \right] = \mathbf{xy} - \mathbf{yx} \qquad (the\ Lie\ bracket)$$

The case with vertical symmetry

Backus

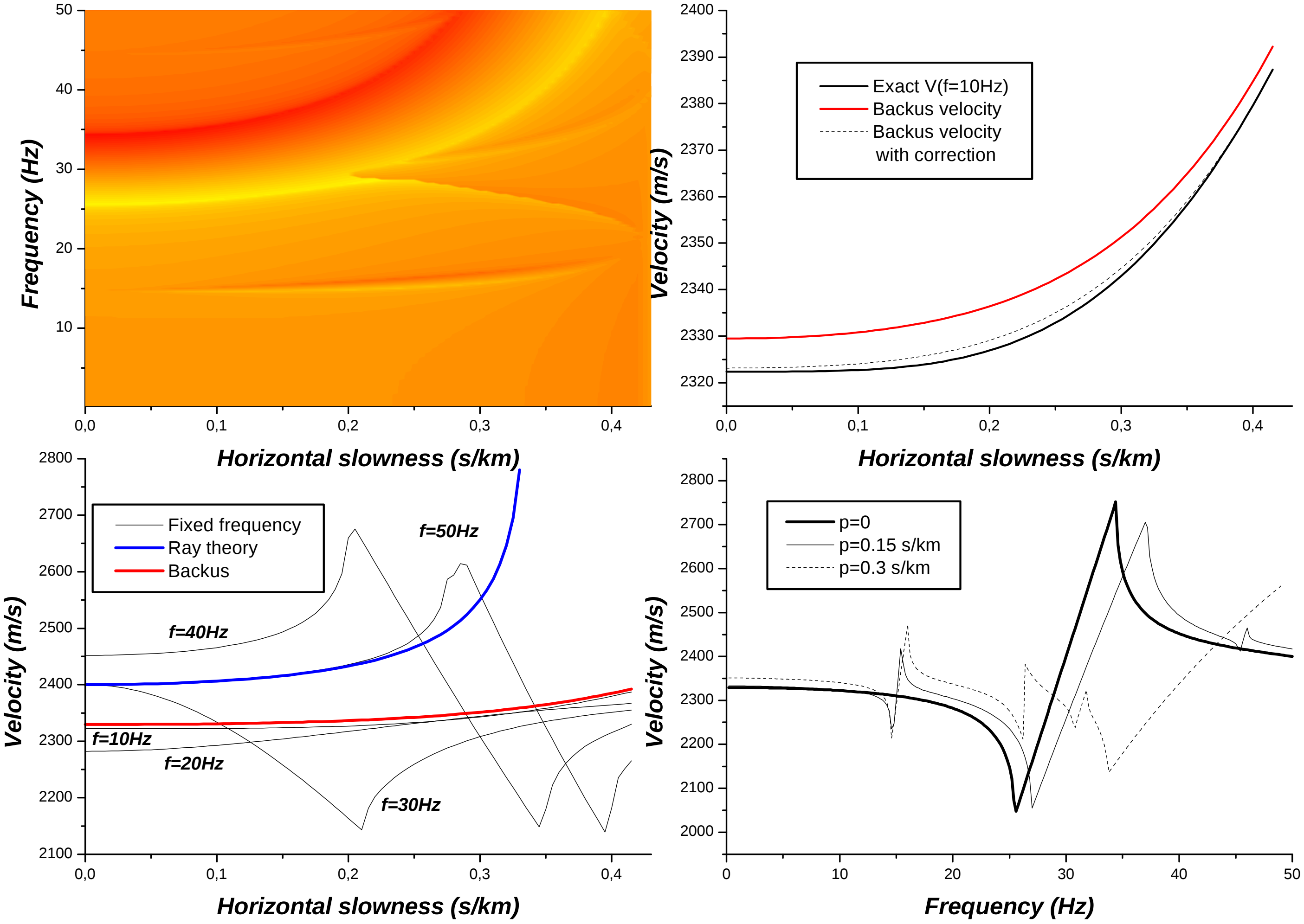
$$\mathbf{A} = \begin{pmatrix} 0 & \mathbf{M} \\ \mathbf{N} & 0 \end{pmatrix} \qquad \tilde{\mathbf{A}}_0 = \begin{pmatrix} 0 & \tilde{\mathbf{M}}_0 \\ \tilde{\mathbf{N}}_0 & 0 \end{pmatrix}$$

Low-frequency expansion

$$\tilde{\mathbf{A}}(\omega) = \begin{pmatrix} \tilde{\mathbf{Q}}(\omega) & \tilde{\mathbf{M}}(\omega) \\ \tilde{\mathbf{N}}(\omega) & \tilde{\mathbf{R}}(\omega) \end{pmatrix} = \begin{pmatrix} (i\omega)\,\tilde{\mathbf{Q}}_1 + \ldots & \tilde{\mathbf{M}}_0 + (i\omega)^2\,\tilde{\mathbf{M}}_2 + \ldots \\ \tilde{\mathbf{N}}_0 + (i\omega)^2\,\tilde{\mathbf{N}}_2 + \ldots & (i\omega)\,\tilde{\mathbf{R}}_1 + \ldots \end{pmatrix}$$

*Note, that the traces of complex matrices **R**(w) and **Q**(w) are zero.*
*Matrix series for **M**(w) and **N**(w) contain the even order terms in frequency, while matrix series for **R**(w) and **Q**(w) – odd order terms.*

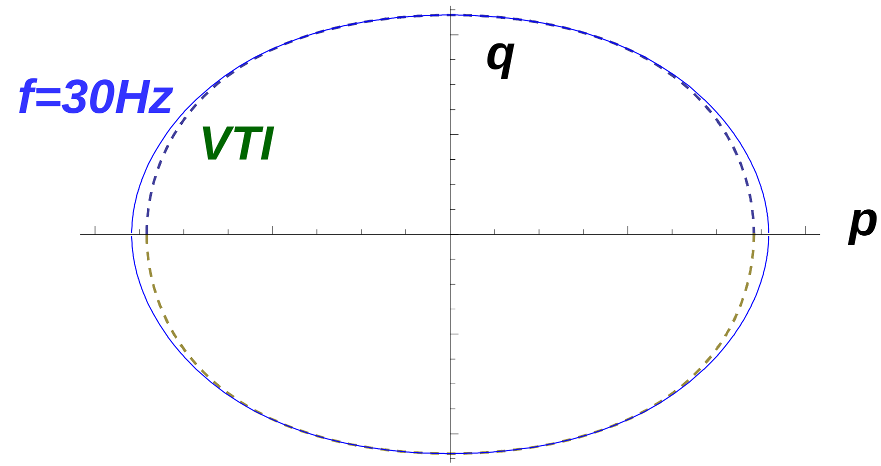
P-wave velocity dispersion



Low frequency upscaling

Compute the eigenvalues
*of the fundamental matrix **M***

$$\tilde{\mathbf{A}}(\omega) = \frac{1}{i\omega H}\log \mathbf{P}(\omega) = \mathbf{E}^{-1}\,\mathrm{diag}(q_m)\,\mathbf{E}$$



Expand the eigenvalues
in series for horizontal slowness

$$q^2(\omega) = q_0^2(\omega) + q_2^2(\omega)\,p^2 + q_4^2(\omega)\,p^4$$

Fit the series coefficients with VTI

$$q^2 = \frac{1}{V_P^2} - (1+2\delta)\,p^2 - \frac{2(\varepsilon-\delta)(1+2\delta-\gamma_0^2)}{(1-\gamma_0^2)}\,p^4 V_P^2 \qquad (qP-wave)$$

$$q^2 = \frac{1}{V_S^2} - (1+2\sigma)\,p^2 + \frac{2\sigma(1+2\delta-\gamma_0^2)}{(1-\gamma_0^2)}\,p^4 V_S^2 \qquad (qSV-wave)$$

$$q^2 = \frac{1}{V_S^2} - (1+2\gamma)\,p^2 \qquad (qSH-wave)$$

The low frequency effective medium is not a VTI medium
but can be approximated as VTI

Weak-contrast

$$\frac{V_B^2}{V_P(\omega)^2} = 1 + R\left((\Delta\rho)^2 + 2\Delta\rho\Delta v_P + (\Delta v_P)^2 \right)$$

$$\delta(\omega) = \delta_B + R\left(-\left(5-\gamma_0^2\right)\Delta\rho\Delta v_S + \left(1+4\gamma_0^2\right)\Delta\rho\Delta v_P + 8\gamma_0^2\Delta v_P\Delta v_S - \frac{(\Delta\delta)^2}{8\gamma_0^2} \right)$$

$$\varepsilon(\omega) = \varepsilon_B + R\left(3\left(1+\gamma_0^2+2\gamma_0^4\right)\Delta\rho\Delta v_S + \frac{\left(1+\gamma_0^2-2\gamma_0^4-4\gamma_0^6\right)}{\gamma_0^2}\Delta\rho\Delta v_P \right.$$

$$\left. +4\left(2-3\gamma_0^2-2\gamma_0^4\right)\Delta v_S\Delta v_P - \frac{1}{2\gamma_0^2}\Delta\delta\Delta\varepsilon + \frac{3\left(1-\gamma_0^2\right)}{8\gamma_0^2}(\Delta\delta)^2 \right)$$

$$\boxed{R = \omega^2 H^2 \alpha^2 (1-\alpha)^2 / 3V_B^2}$$

Vertical P-wave velocity

α *α*

f *f*

$V_P(\alpha,f)$ $V_P(\alpha,f)- VP(\alpha,0)$

Anisotropy parameter Delta

α

f

α

f

$\delta(\alpha,f)$

$\delta(\alpha,f)$- $\delta(\alpha,0)$

Anisotropy parameter Epsilon

α

f

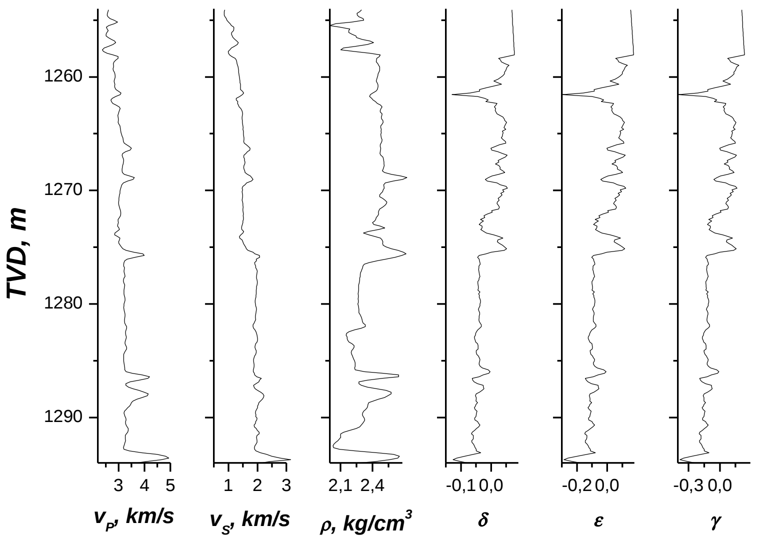
α

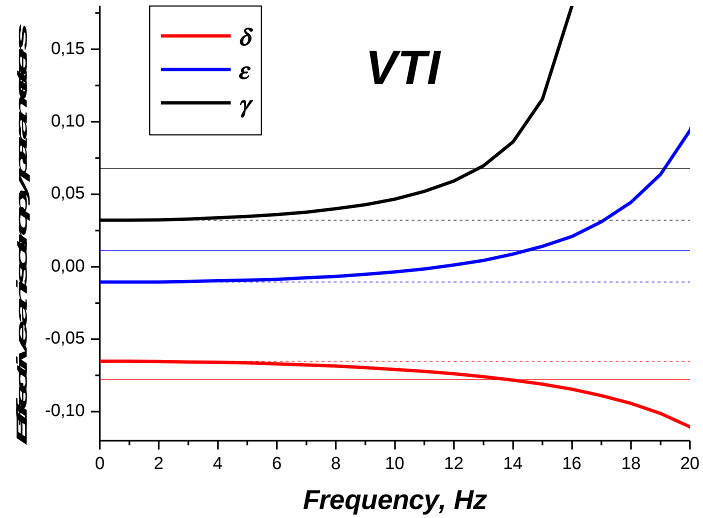
f

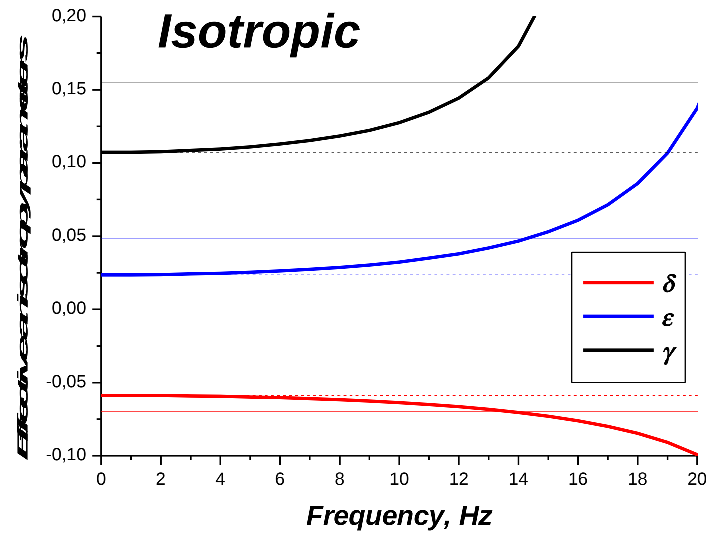
$\varepsilon(\alpha,f)$

$\varepsilon(\alpha,f)- \varepsilon(\alpha,0)$

Real well log data example







$$R(f) = \frac{2 f^2 \exp\left(-\frac{f^2}{f_0^2}\right)}{\sqrt{\pi}\, f_0^3}$$

$$R(f_0, f_m) = \frac{1}{2} Erf\left(\frac{f_m}{f_0}\right) - \frac{f_m \exp\left(-\frac{f_m^2}{f_0^2}\right)}{f_0 \sqrt{\pi}}$$

Conclusions

- We derive the low frequency extension of the Backus averaging method
- The effective medium can be approximated by a VTI
- The typical behaviour of VTI anisotropy parameters:
  - *Epsilon gradually increases with frequency*
  - *Delta gradually decreases with frequency*
- *Method is illustrated on synthetic and real data examples*

Acknowledgement

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