Normal modes in anisotropic VTI media

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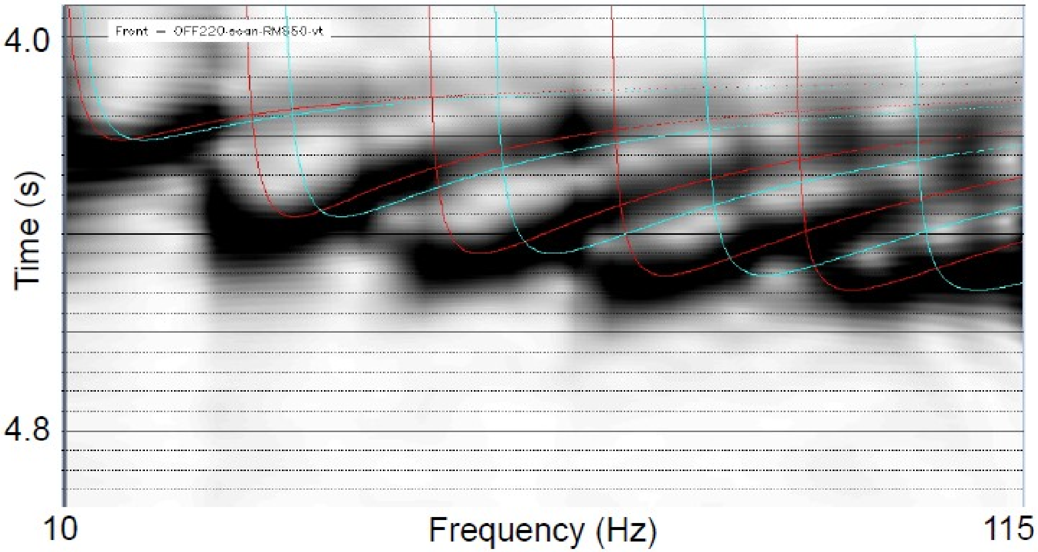
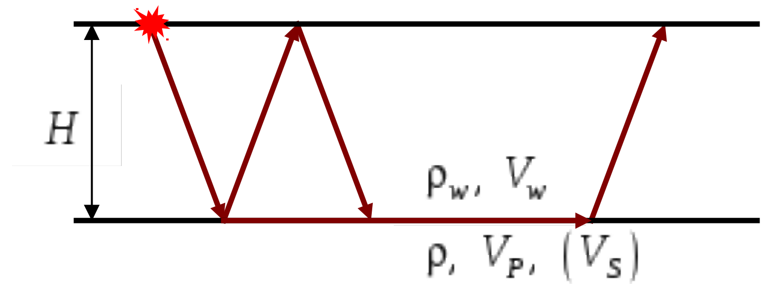
Normal Modes



Landrø and Hatchell, 2012



- Acoustic Isotropy : Pekeris, 1948

Wavenumber

Phase velocity

$$\tan\left(kH\sqrt{\frac{c^2}{V_w^2}-1}\right) = -\frac{\rho\sqrt{\frac{c^2}{V_w^2}-1}}{\rho_w\sqrt{1-\frac{c^2}{V_P^2}}}$$

- Elastic Isotropy: Ewing, 1950
- Elastic Anisotropy: Anderson, 1961

- Geomechanical purposes
- Platform installation planning
- Input for FWI

Normal modes vs anisotropy parameters

Outline

- Dispersion equation
- Sensitivity analysis
- Conclusions

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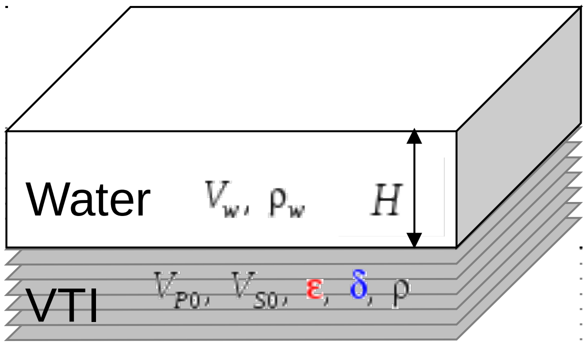
Dispersion Equation



$$\tan\left(kH\sqrt{\frac{c^2}{V_w^2}-1}\right)=\frac{\rho\sqrt{\frac{c^2}{V_w^2}-1}}{\rho_w c^2\left(L_1+L_2\right)\sqrt{V_{PH}^2-c^2}}\times$$

$$\times\left\{\left[\left(GV_{P0}^2-V_{S0}^2\right)^2+V_{P0}^{\ 2}\left(c^2-V_{PH}^2\right)\right]\sqrt{V_{S0}^2-c^2}+c^2V_{P0}V_{S0}\sqrt{V_{PH}^2-c^2}\right\}$$

Epsilon-related term

$$V_{PH}=V_{P0}\sqrt{2\varepsilon+1}$$

Delta-related term

$$G=\sqrt{2\left(1-\frac{V_{S0}^2}{V_{P0}^2}\right)\delta+\left(1-\frac{V_{S0}^2}{V_{P0}^2}\right)^2}$$

$$L_j^2=\left[\frac{1}{2}M_1\pm\frac{1}{2}\sqrt{M_1^2-M_2}\right],\quad j=1,2$$

$$M_1=-G^2V_{P0}^4-V_{P0}^2\left(c^2-V_{PH}^2\right)-V_{S0}^2\left(c^2-V_{S0}^2\right)$$

$$M_2=4V_{P0}^2V_{S0}^2\left(c^2-V_{PH}^2\right)\left(c^2-V_{S0}^2\right)$$

Dispersion Equation

Elastic Anisotropy

$$\tan\left(kH\sqrt{\frac{c^2}{V_w^2}-1}\right)=\frac{\rho\sqrt{\frac{c^2}{V_w^2}-1}}{\rho_w c^2\left(L_1+L_2\right)\sqrt{V_{PH}^2-c^2}}\times$$

$$\times\left\{\left[\left(GV_{P0}^2-V_{S0}^2\right)^2+V_{P0}{}^2\left(c^2-V_{PH}^2\right)\right]\sqrt{V_{S0}^2-c^2}+c^2V_{P0}V_{S0}\sqrt{V_{PH}^2-c^2}\right\}$$

$V_{S0}=0$

$\varepsilon=\delta=0$

$\varepsilon=\delta=0$

$V_{S0}=0$

Acoustic Isotropy

$$\tan\left(kH\sqrt{\frac{c^2}{V_w^2}-1}\right)=-\frac{\rho\sqrt{\frac{c^2}{V_w^2}-1}}{\rho_w\sqrt{1-\frac{c^2}{V_{P0}^2}}}$$

Pekeris, 1948

Elastic Isotropy

$$\tan\left(kH\sqrt{\frac{c^2}{V_w^2}-1}\right)=\frac{\rho V_{S0}^4\sqrt{\frac{c^2}{V_w^2}-1}}{c^4\rho_w\sqrt{1-\frac{c^2}{V_{P0}^2}}}\times$$

$$\times\left\{4\sqrt{1-\frac{c^2}{V_{P0}^2}}\sqrt{1-\frac{c^2}{V_{S0}^2}}-\left[2-\frac{c^2}{V_{S0}^2}\right]^2\right\}$$

Ewing, 1950

Acoustic Anisotropy

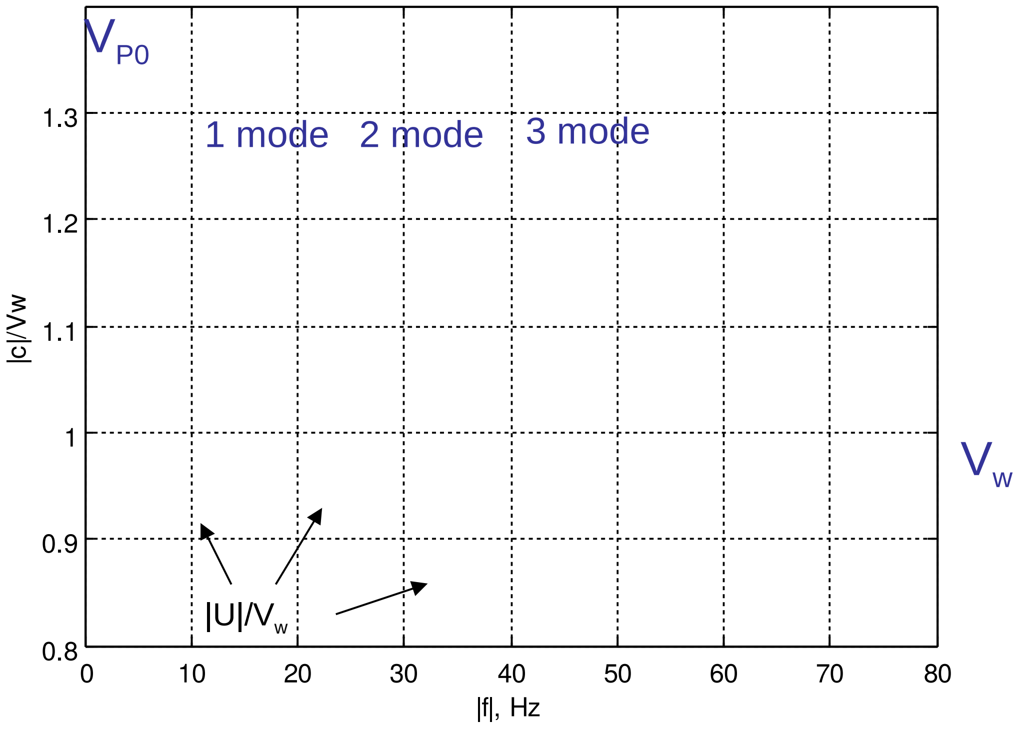
$$\tan\left(kH\sqrt{\frac{c^2}{V_w^2}-1}\right)=-\frac{\rho V_{P0}\sqrt{\frac{c^2}{V_w^2}-1}}{\rho_w c\sqrt{\left(2\varepsilon+1\right)V_{P0}^2-c^2}}\times$$

$$\times\sqrt{c^2-2\left(\varepsilon-\delta\right)V_{P0}^2}$$

Dispersion Equation

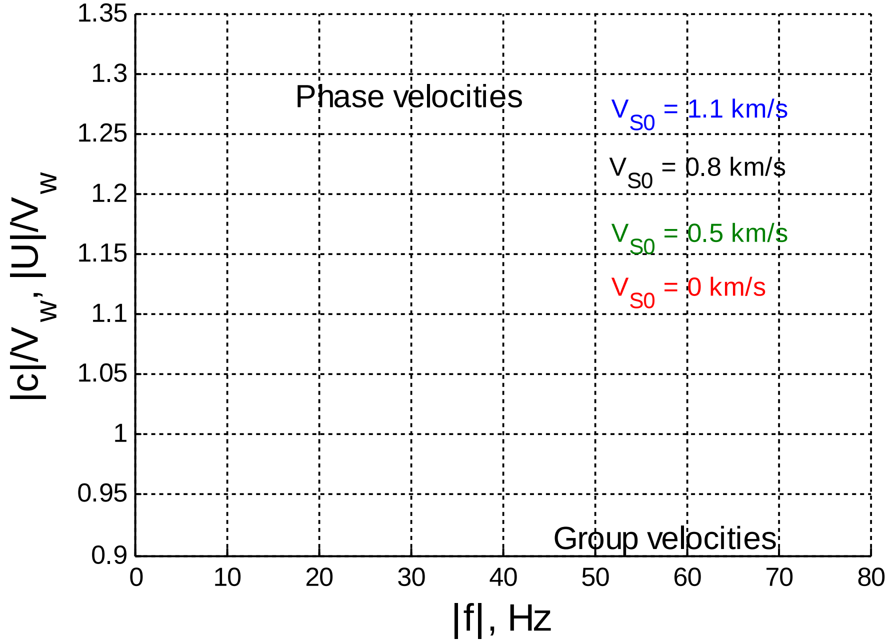
$$\tan\left(kH\sqrt{\frac{c^2}{V_w^2}-1}\right)=F(c,\mathbf{m})$$

Phase velocity $c(k)$ → Group velocity $U=c+k\frac{dc}{dk}$

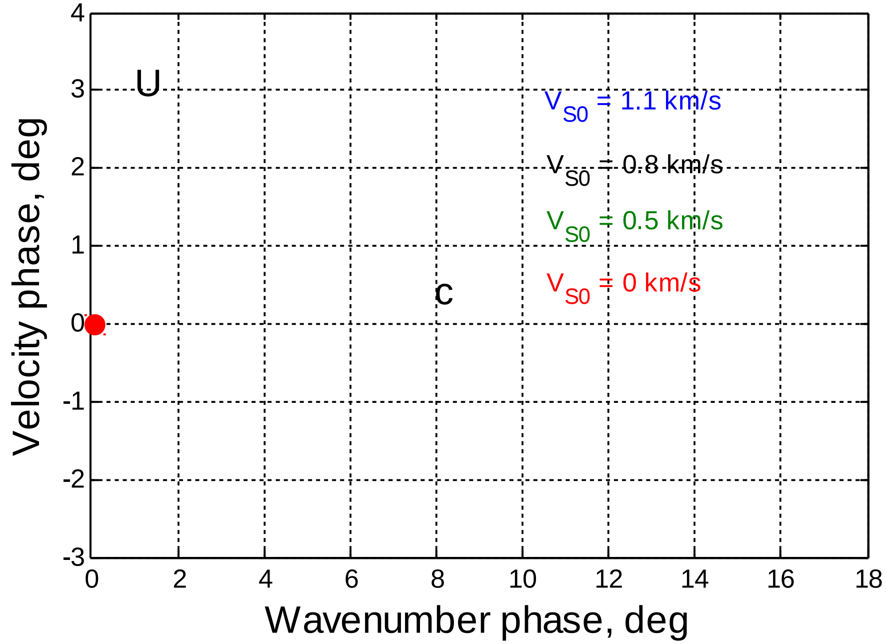


Elastic vs Acoustic

Amplitude domain



Phase domain



- Acoustic equation can be used for unconsolidated sediments
- Elastic equation should be used for consolidated sediments

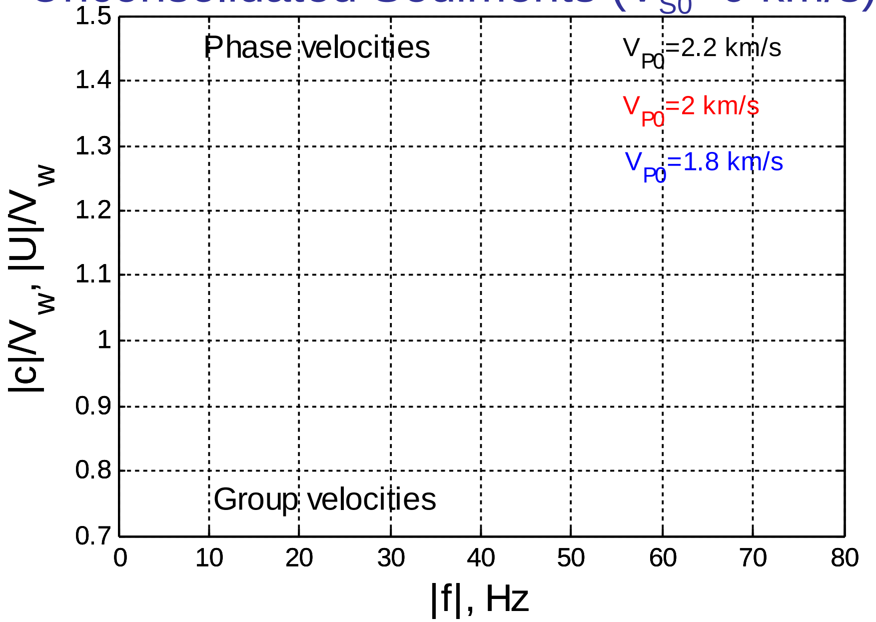
Outline

- Dispersion equation
- Sensitivity analysis
- Conclusions

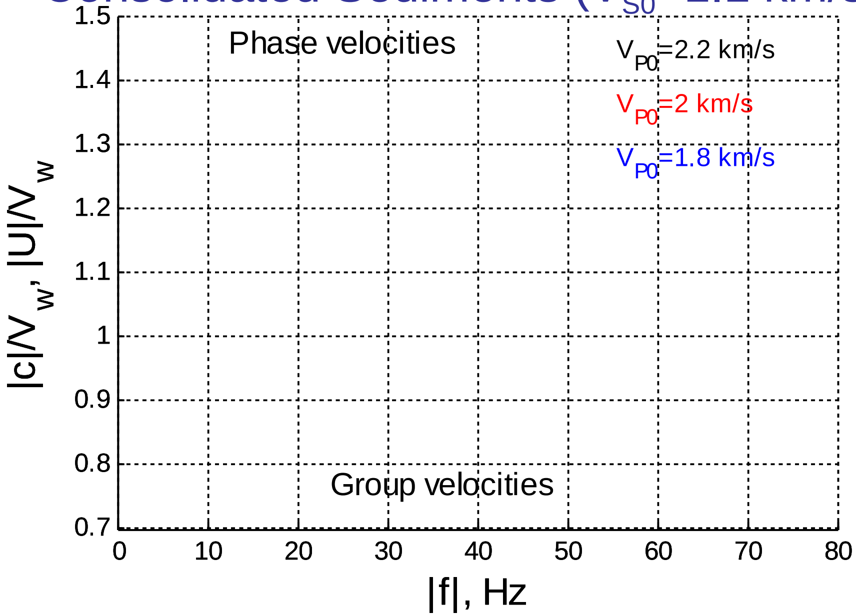
Sensitivity analysis

P-wave velocity effect

Unconsolidated Sediments (V_{S0}=0 km/s)



Consolidated Sediments (V_{S0}=1.1 km/s)



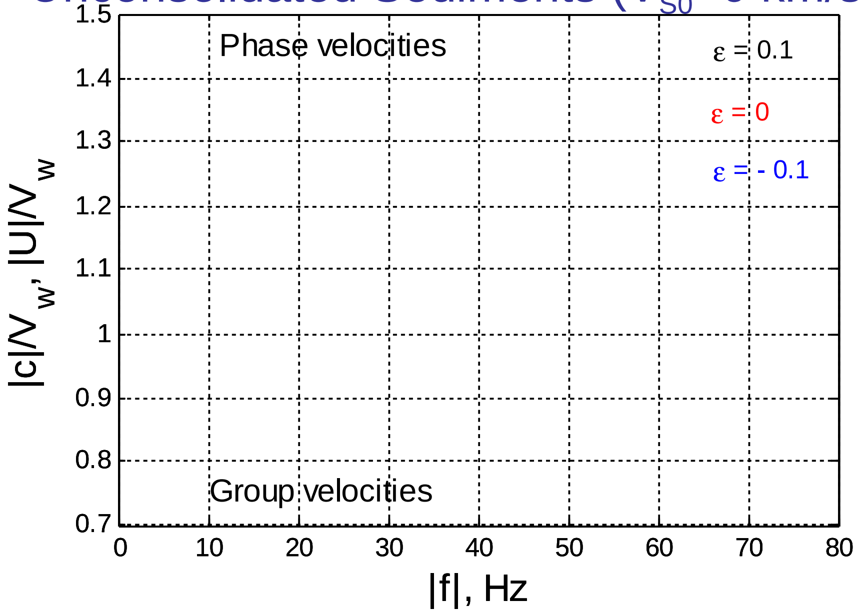
- Changes in maximum group and phase velocity
- Asymmetric shift of cut off frequency
- Changes in minimum group velocity
- Increased sensitivity of higher modes

Sensitivity analysis

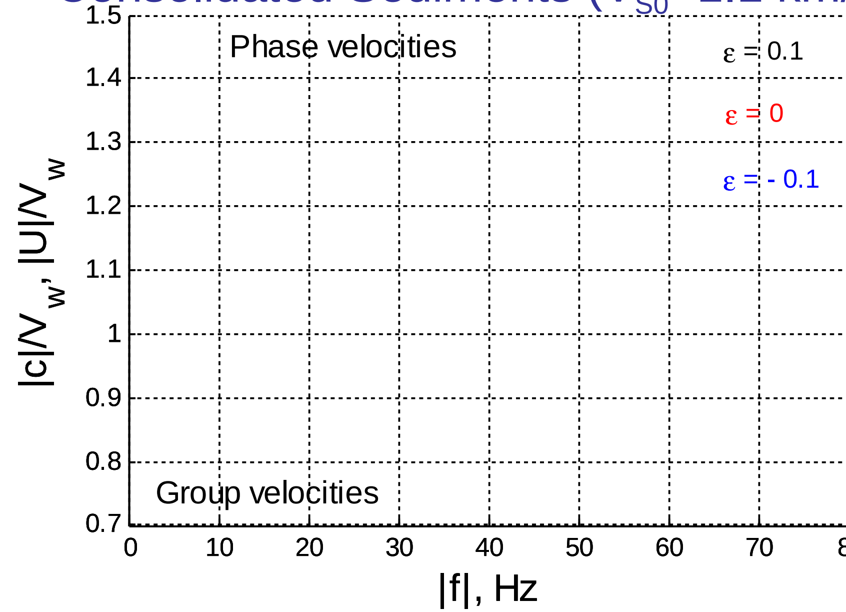
Epsilon effect

V_{P0}=2.0 km/s

Unconsolidated Sediments (V_{S0}=0 km/s)



Consolidated Sediments (V_{S0}=1.1 km/s)



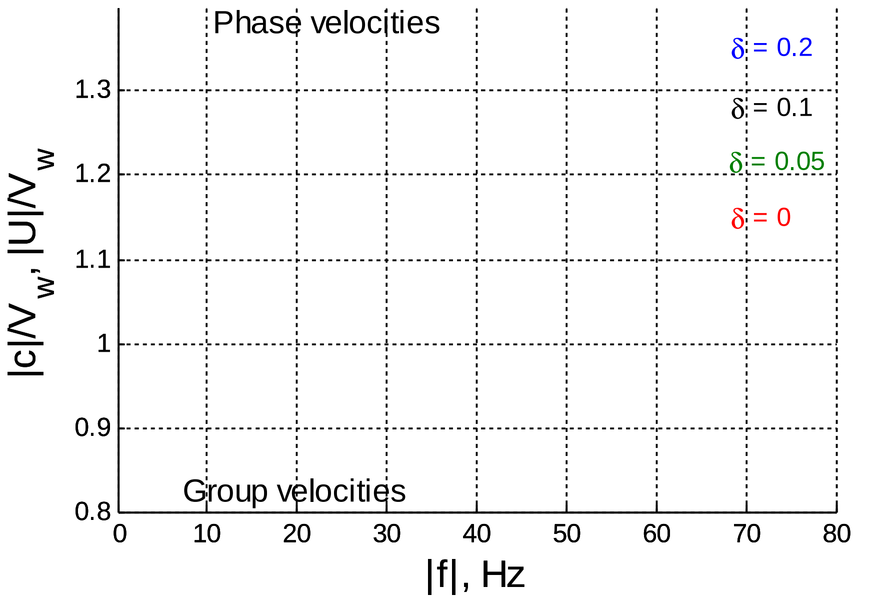
- Changes in maximum group and phase velocity
- Asymmetric shift in cut off frequency
- Changes in minimum group velocity
- Epsilon effect increases with mode increase

Sensitivity analysis

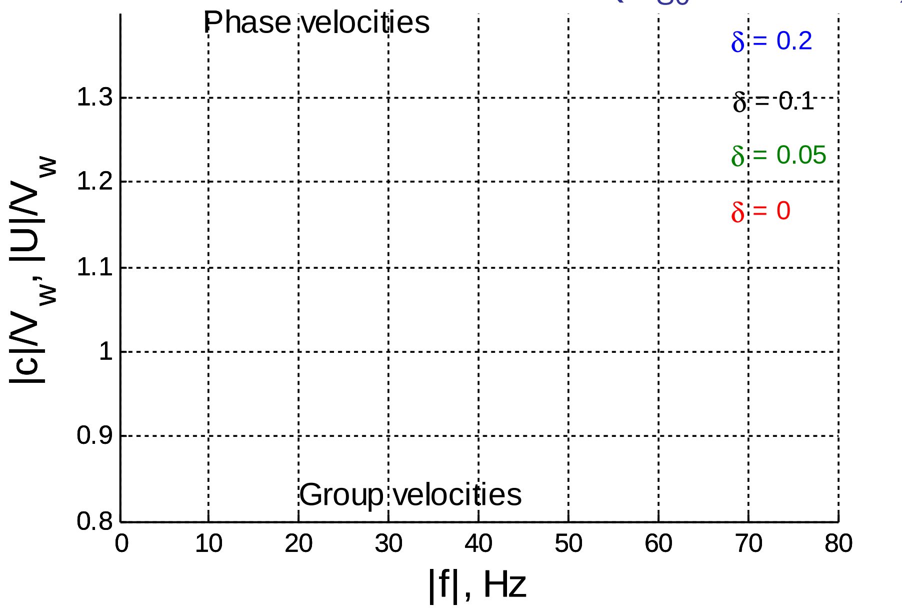
Delta effect

V_{P0}=2.0 km/s

Unconsolidated Sediments (V_{S0}=0 km/s)



Consolidated Sediments (V_{S0}=1.1 km/s)



- Delta effect is minor and observed mostly on group velocity minima
- Consolidated sediments are more sensitive to delta changes
- Delta effect does not increase with mode increase

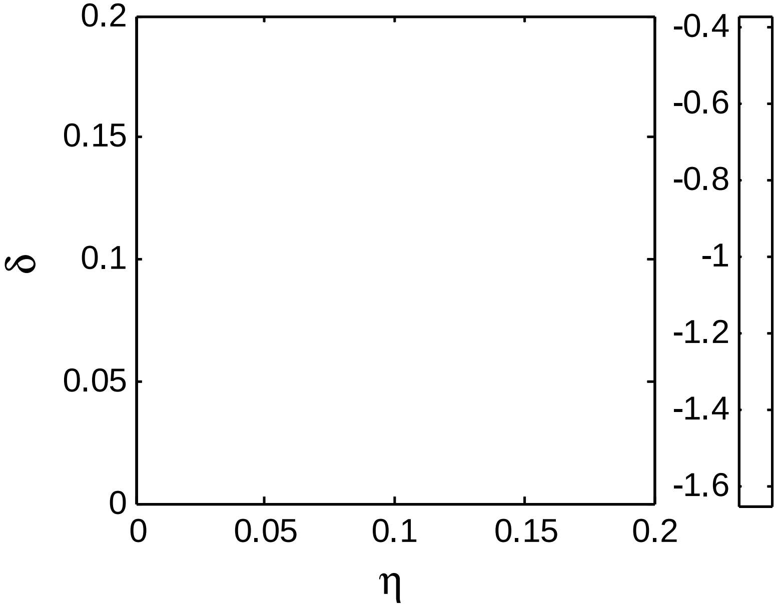
Sensitivity analysis

Eta effect

$$\tan\left(kH\sqrt{\frac{c^2}{V_w^2}-1}\right) = -\frac{\rho V_{P0}\sqrt{\frac{c^2}{V_w^2}-1}\sqrt{c^2-2\eta V_{nmo}^2}}{\rho_w c\sqrt{(2\eta+1)V_{nmo}^2-c^2}}$$

$$V_{nmo} = V_{P0}\sqrt{1+2\delta}$$

$$\eta = \frac{\varepsilon-\delta}{1+2\delta}$$



- Linear trade off between delta and eta
- Delta is less resolved

Conclusions

- Slope of disperion curves is sensitive to the S-wave velocity changes
- Asymmetric effect on the dispersion curves is due to changes in P-wave velocity and epsilon
- Delta changes cause minor effect on normal modes while epsilon changes can be easily observed

Acknowledgments

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