

# **Curved edge diffraction modeling**

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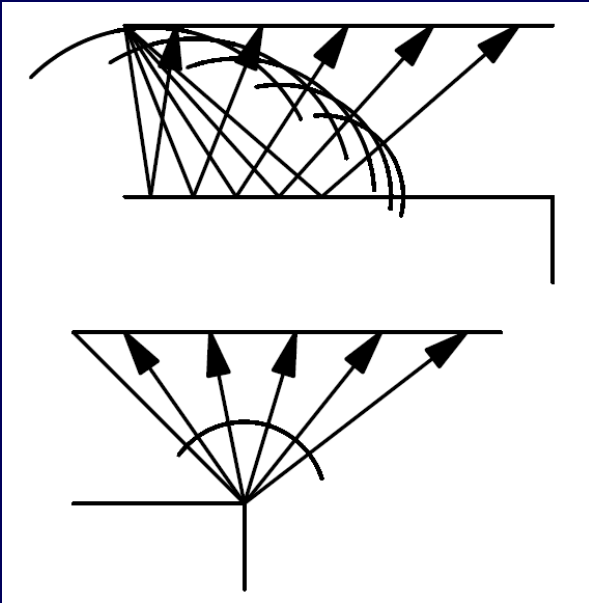
**ROSE Meeting**

**Trondheim April 2012**

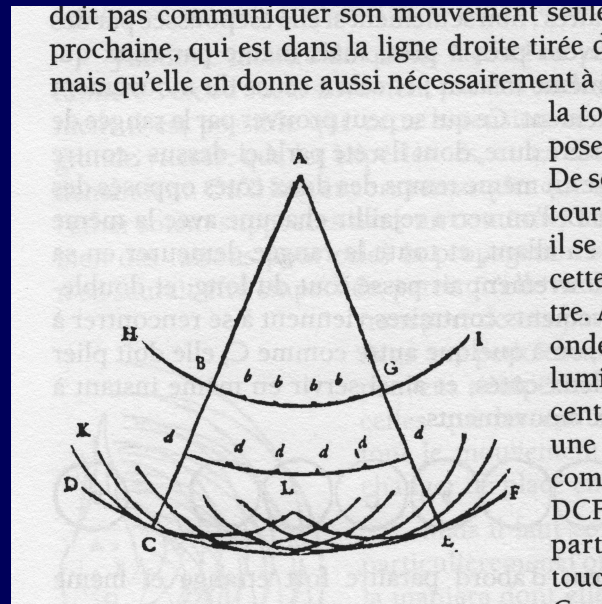
# Outline

- **Diffractions - introduction**
- **Diffraction - analysis, imaging, modeling, inversion**
- **Diffraction modeling - ray-Born**
- **Edge and tip diffractions**
- **Curved edge diffraction modeling**
- **Examples – synthetic, GOM-B**

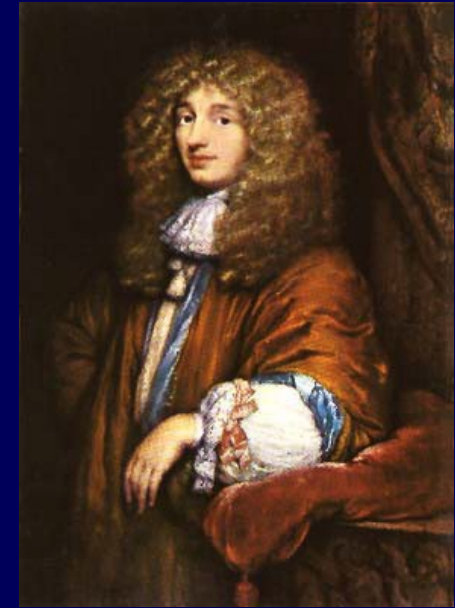
# Diffractions – introduction



**Reflections vs.  
Diffractions**

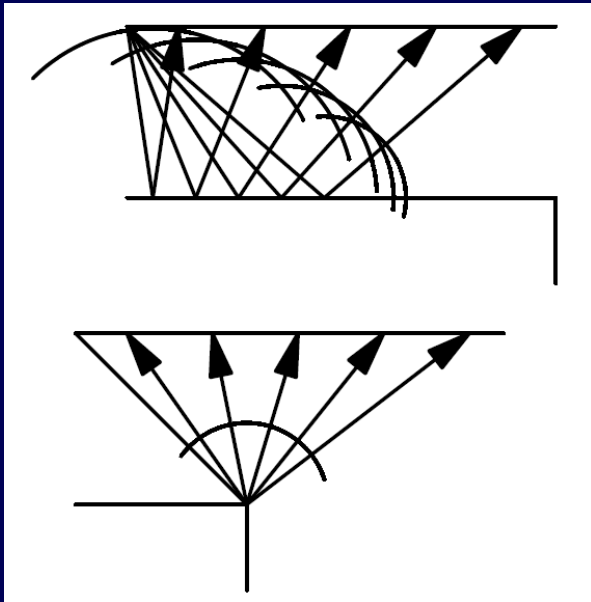


**Traité de la Lumière, Christiaan Huygens  
(’s-Gravenhage, 1629-95)**



# Diffractions – introduction

## Why diffractions?

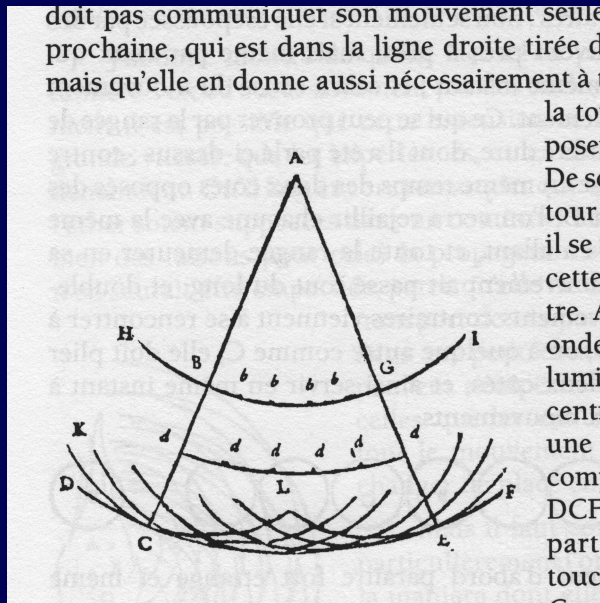


**Geology – structural interpretation, super-resolution: faults, pinch-outs, small-size scattering objects**

**Physics – conventional processing/imaging flow uses specular reflections, diffractions are not well preserved**

***“Diffractions are the abandoned stepchildren of traditional seismic processing and imaging!”  
(Khaidukov, Landa & Moser, Geophysics, 2004)***

# Diffractions – analysis, imaging, modeling, inversion



**What can we do with  
diffractions?**

**Diffraction ...**

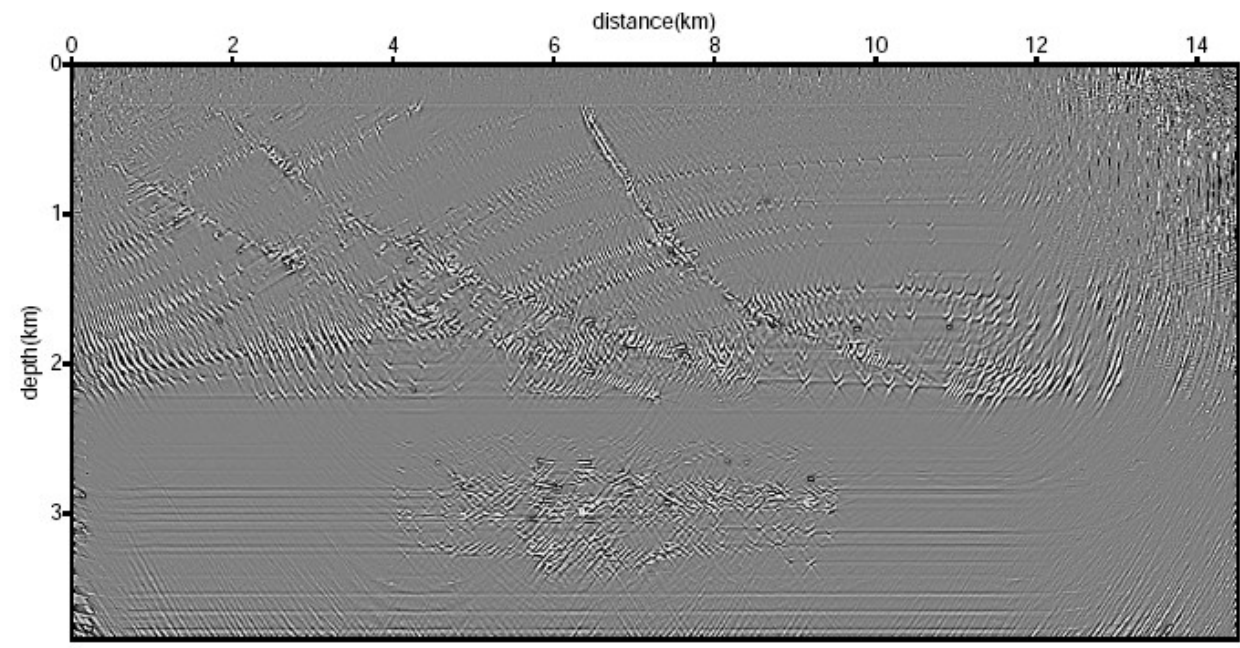
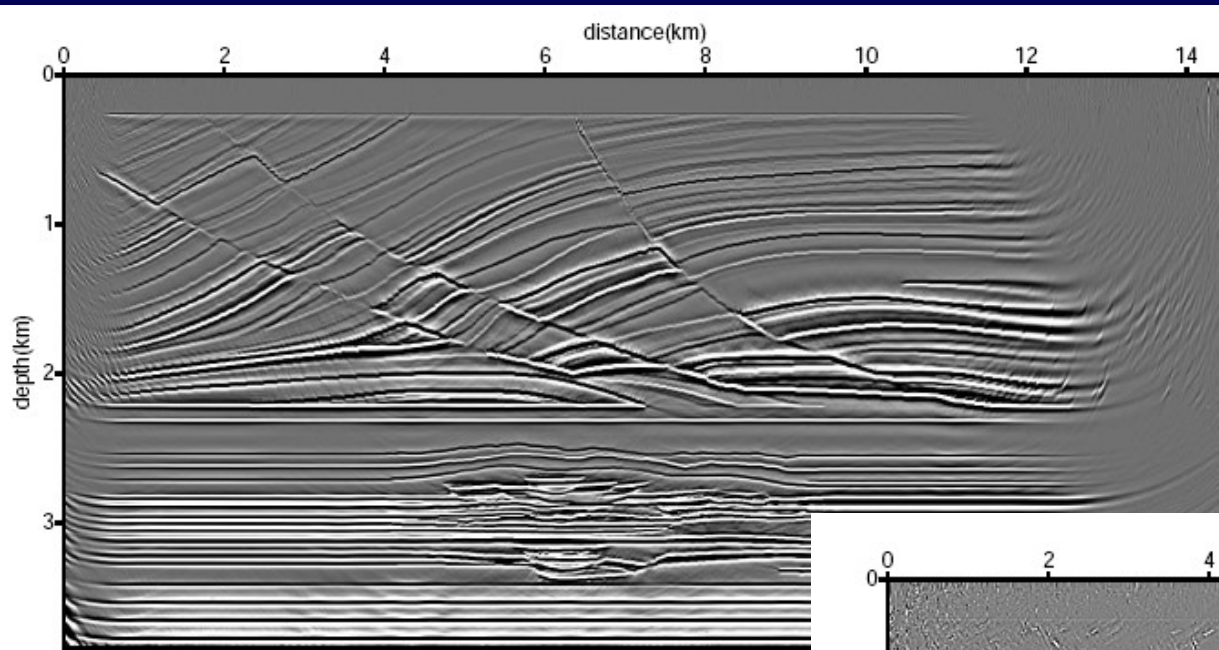
**... Analysis**

**... Imaging**

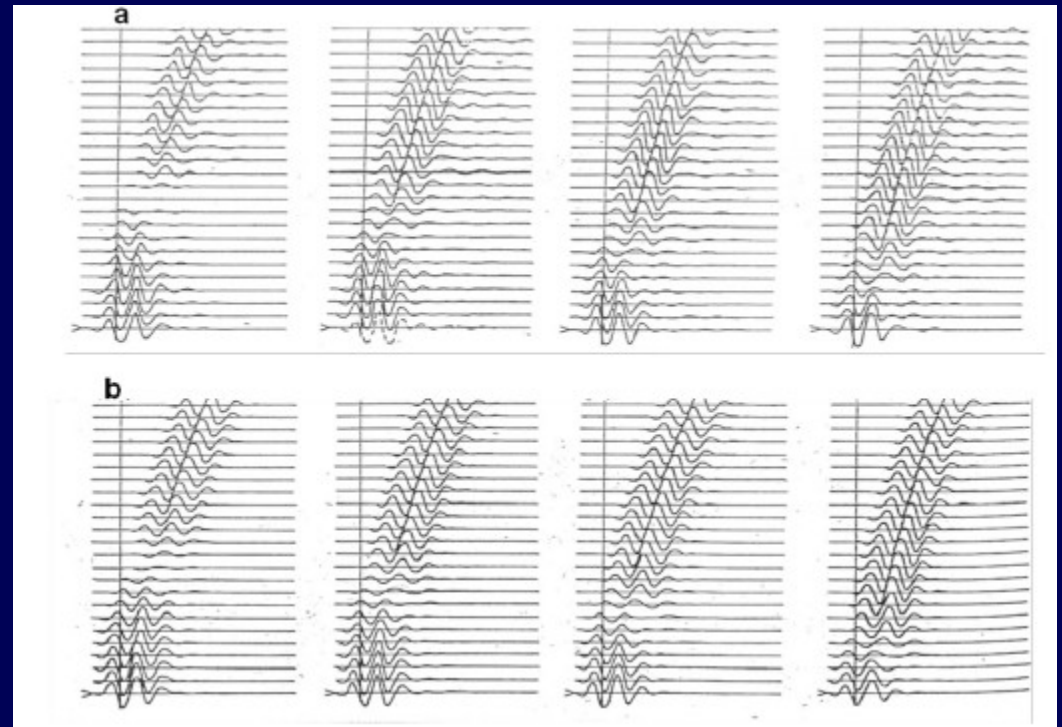
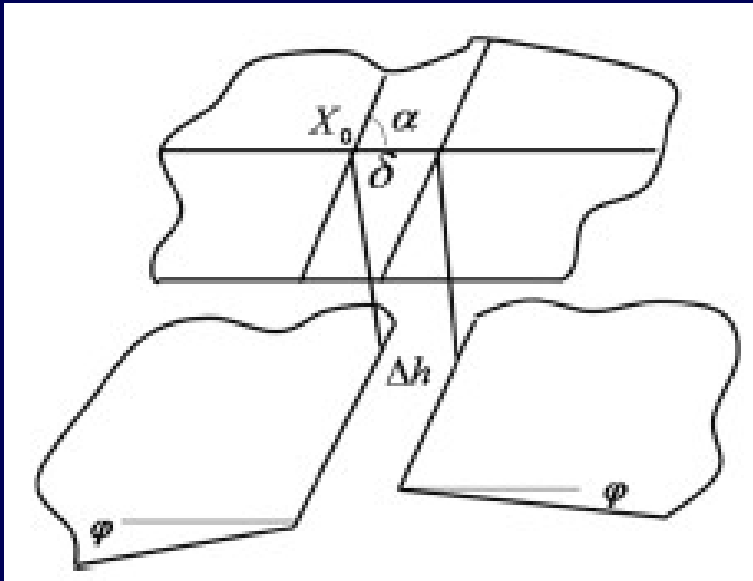
**... Modeling**

**... Inversion**

# Diffractions – analysis, imaging, modeling, inversion



# Diffractions – analysis, imaging, modeling, inversion



$X_0$  hor. position,  $\alpha$  fault azimuth,  $\delta$  hor. shift,  $\Delta h$  fault  
amplitude,  $\varphi$  reflector dip  
a) modeled b) observed seismicograms

# Diffraction modeling

## modeling - Ray-Born integral

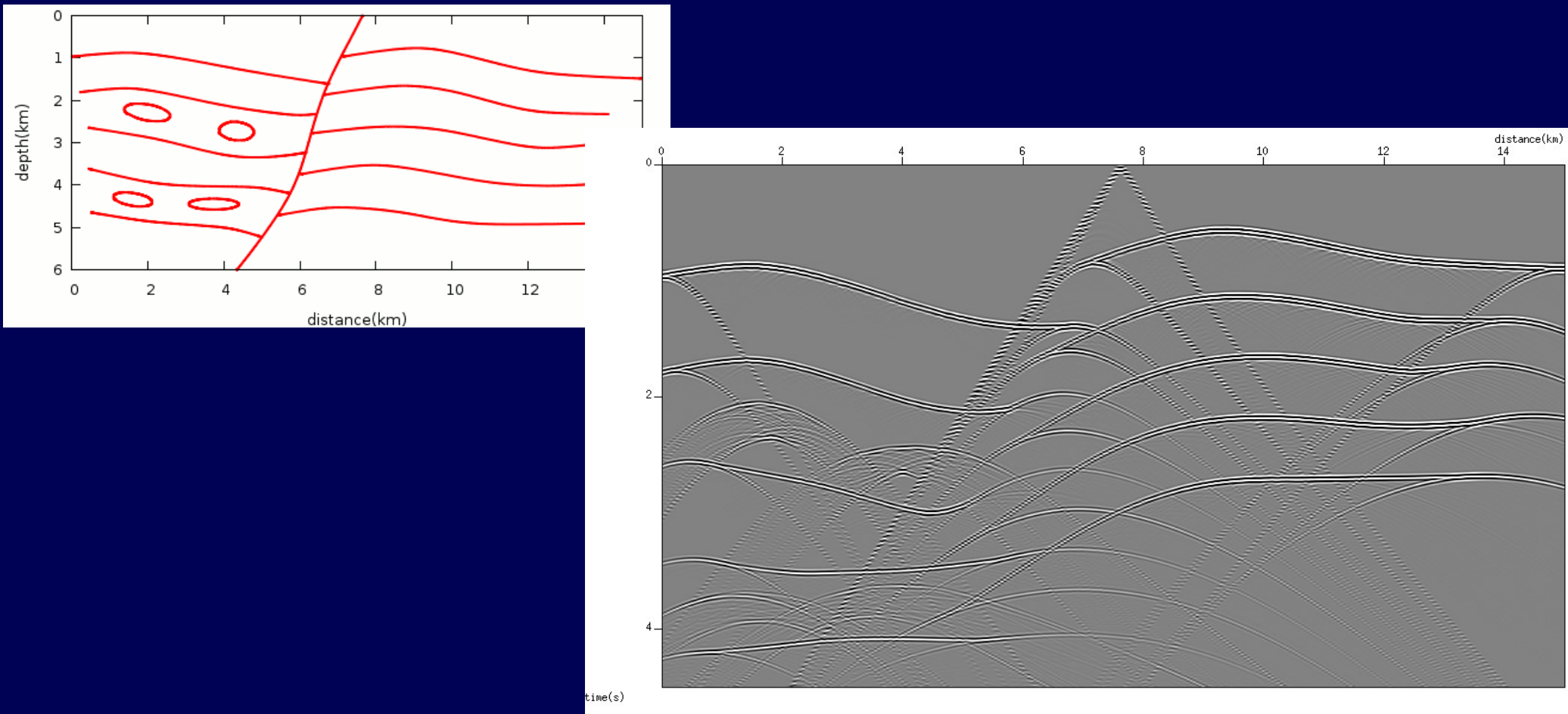
$$U(\mathbf{s}, \mathbf{r}, t) = \int A(\mathbf{s}, \mathbf{x}, \mathbf{r}) V(\mathbf{x}) w(t - T(\mathbf{s}, \mathbf{x}, \mathbf{r})) d\mathbf{x}$$

- **Linear — Inversion, migration + primaries only**
- **Ray-theory travel time  $T(\mathbf{s}, \mathbf{x}, \mathbf{r})$  and amplitude  $A(\mathbf{s}, \mathbf{x}, \mathbf{r})$  in smooth background**
- **Volume integral over scattering potential  $V(\mathbf{x})$**
- **No smoothness constraints for  $V(\mathbf{x})$ , only integrability**  
→ *Diffractions*



# Diffraction modeling

## Ray-Born modeling - example

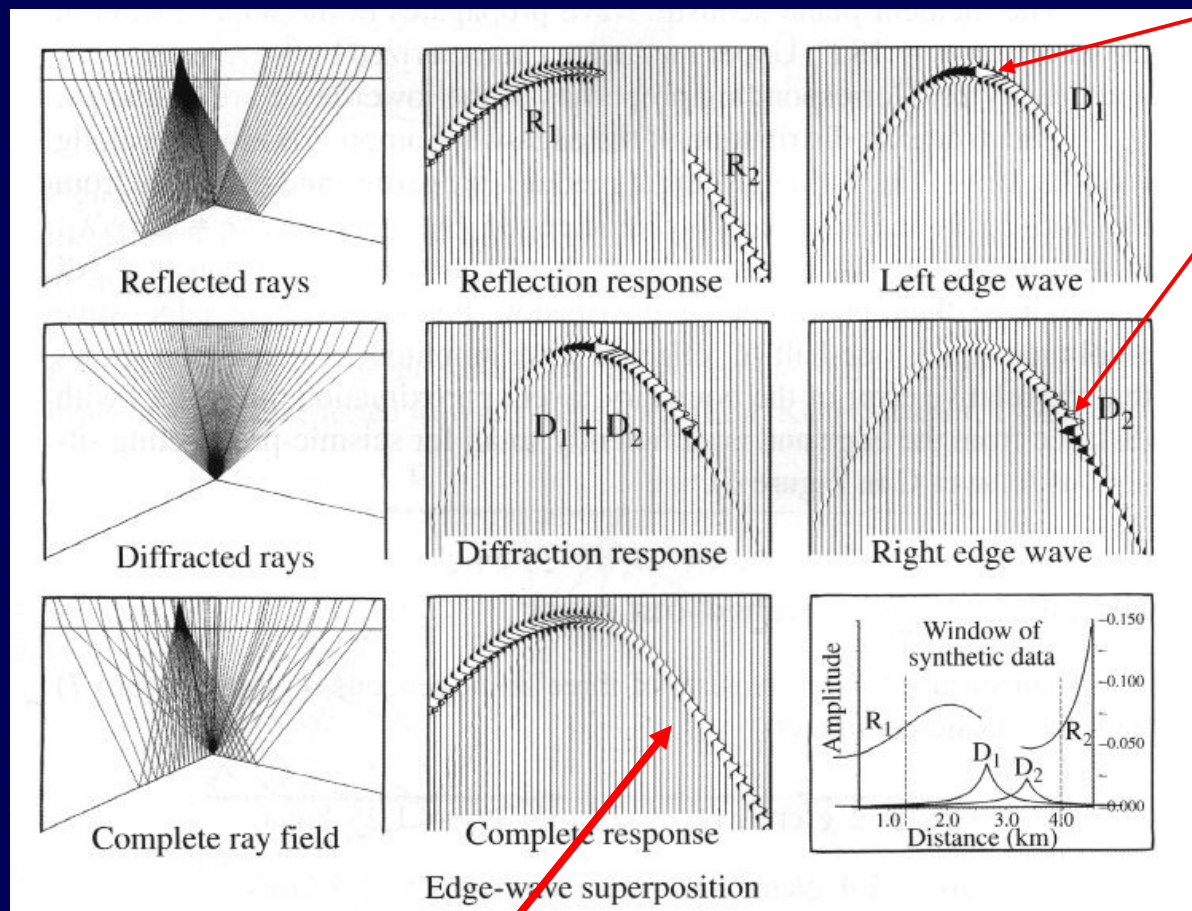


**Complicated fault model**

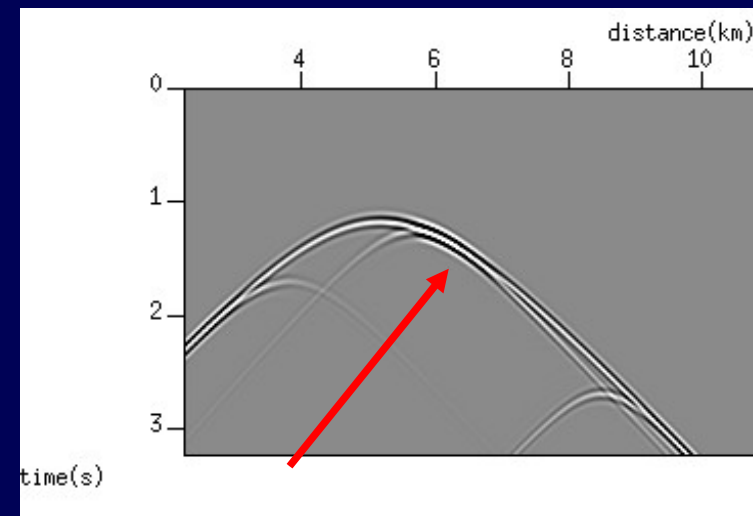
**→ *Edge diffractions + Caustic diffractions***

# Edge and tip diffractions

## Edge wave theory

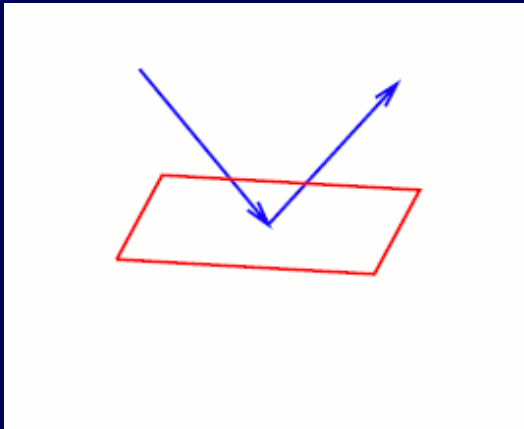


**Polarity change across reflection tangent point**

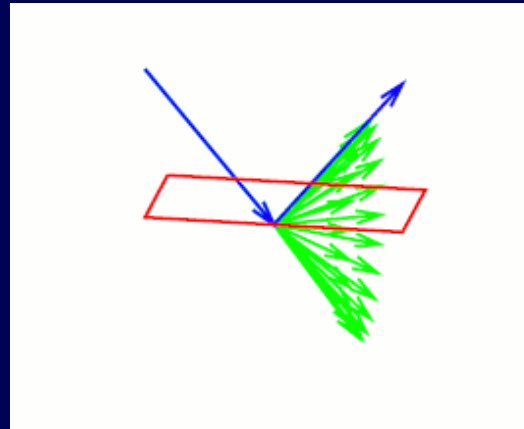


**ray-Born (shot at 4km)**

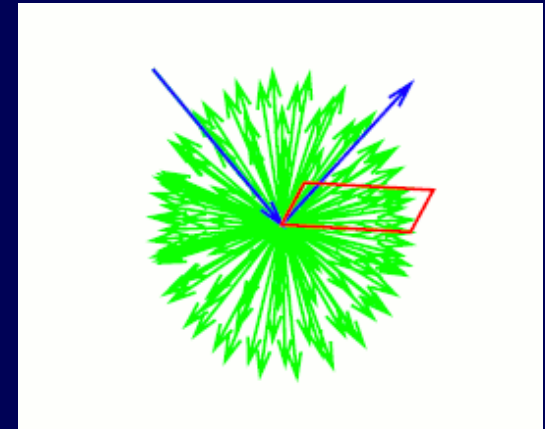
# Edge and tip diffractions



**Plane reflection**



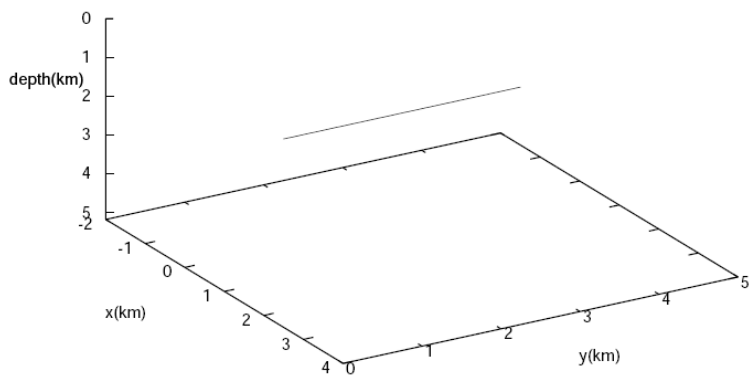
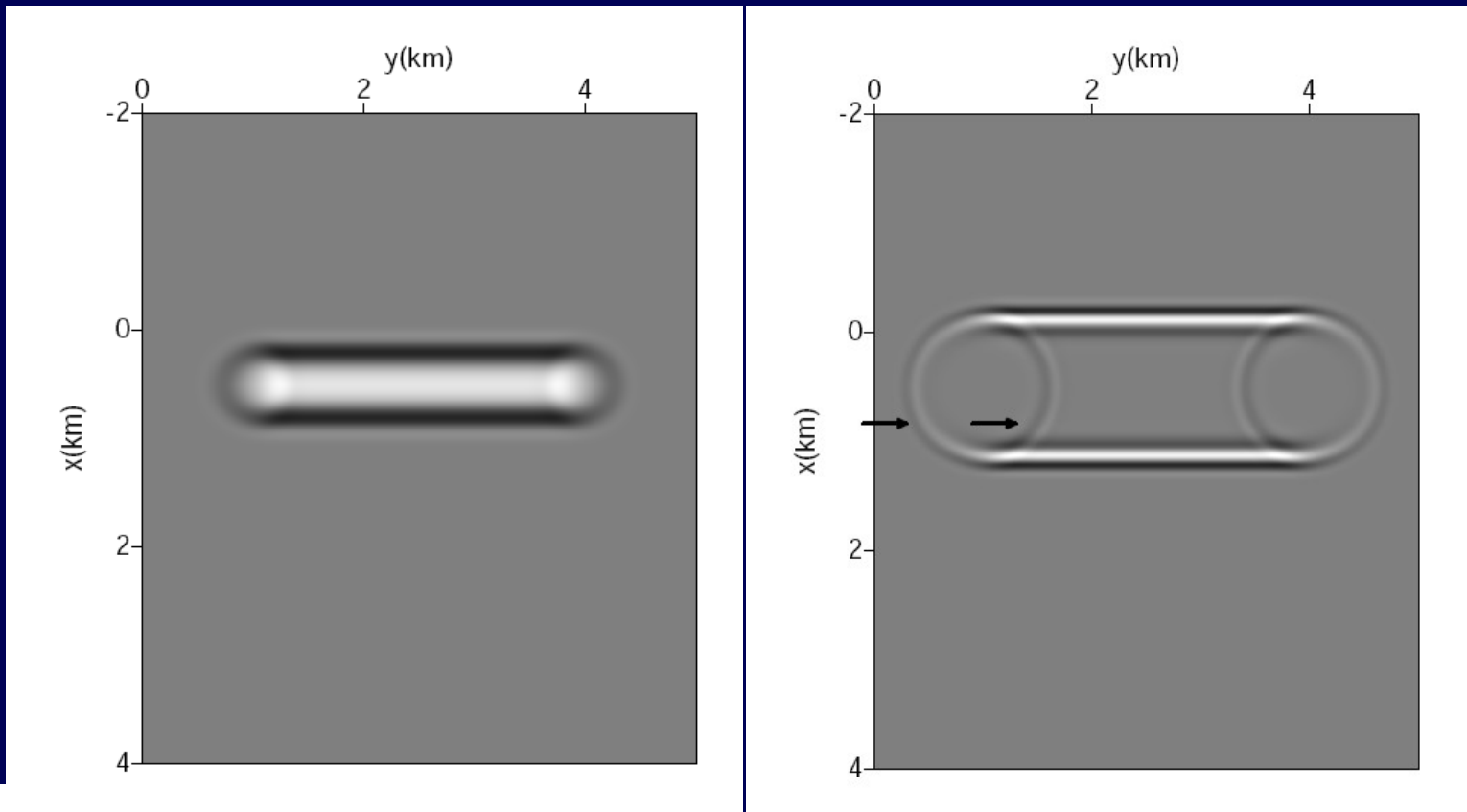
**Edge diffraction**



**Tip diffraction**

# Edge and tip diffractions

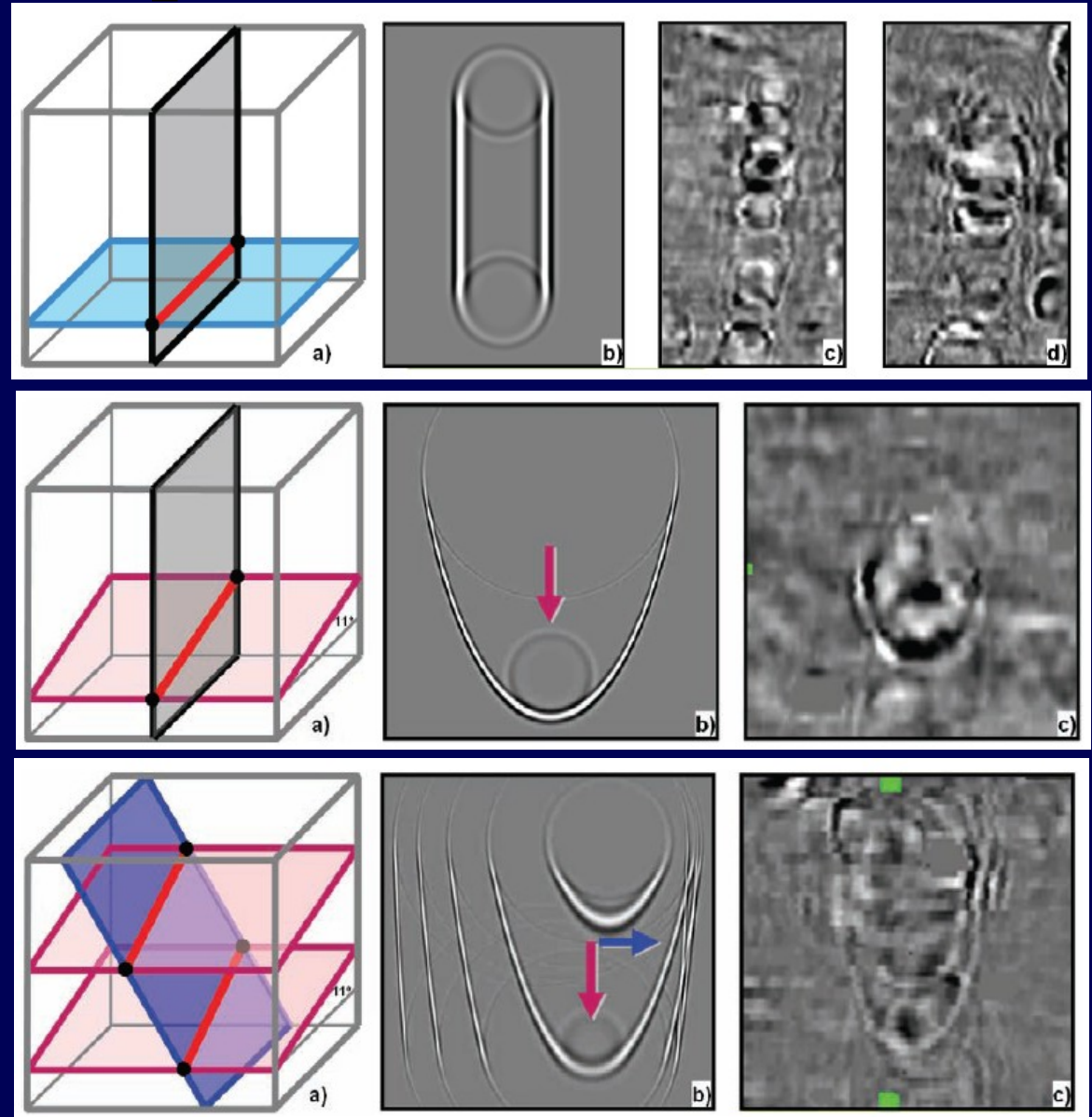
## 3D line/tip diffraction



Time slices  $t=2.0, 2.2\text{s}$

Line diffractor

# Edge and tip diffractions



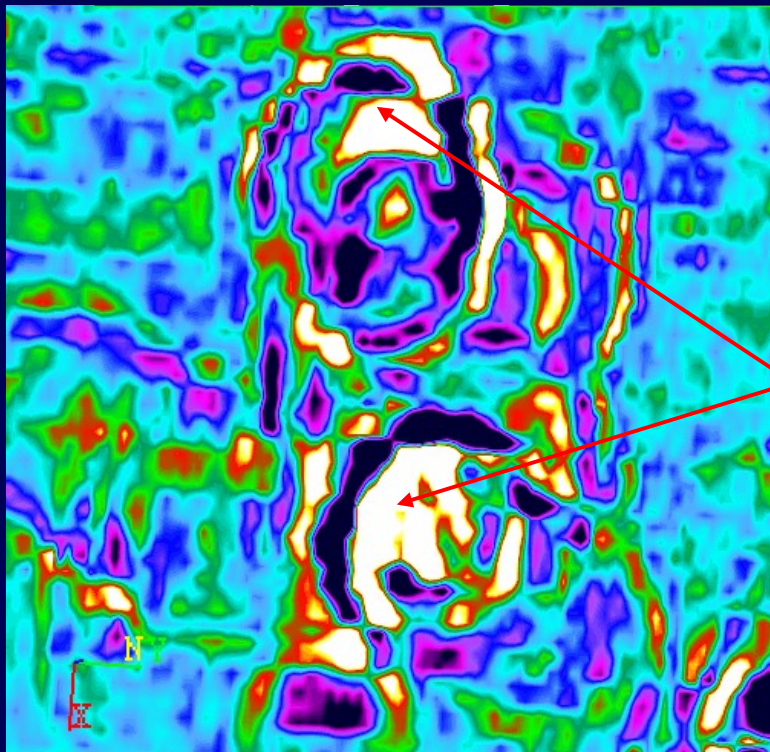
(Grasmueck, Moser and Pelissier  
2011, CSL report)

**GPR data over Cassis quarry  
(fracture and karst network)**

# Edge and tip diffractions

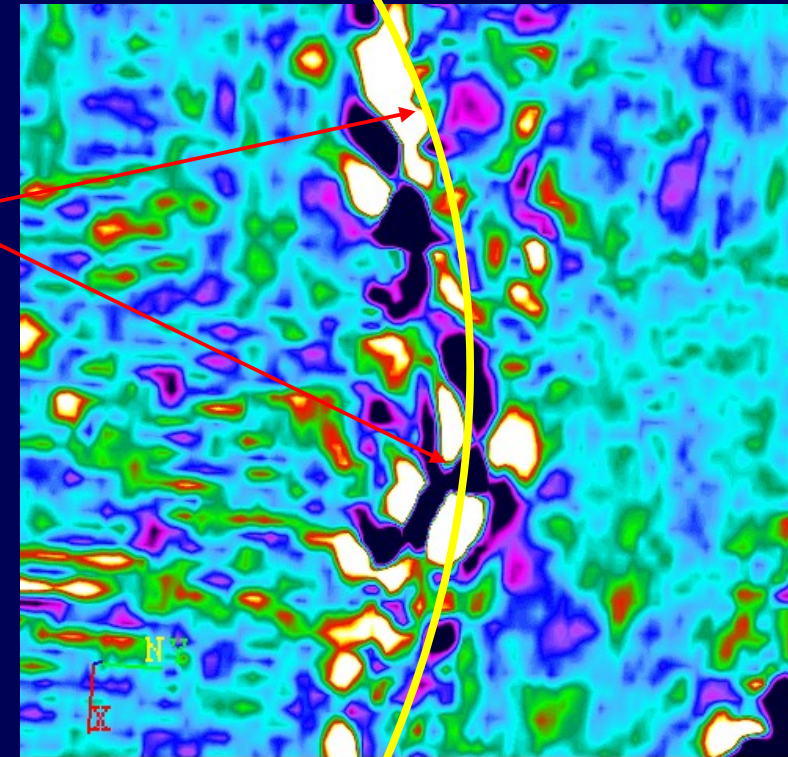
GPR data time slice (zero offset)

... migrated



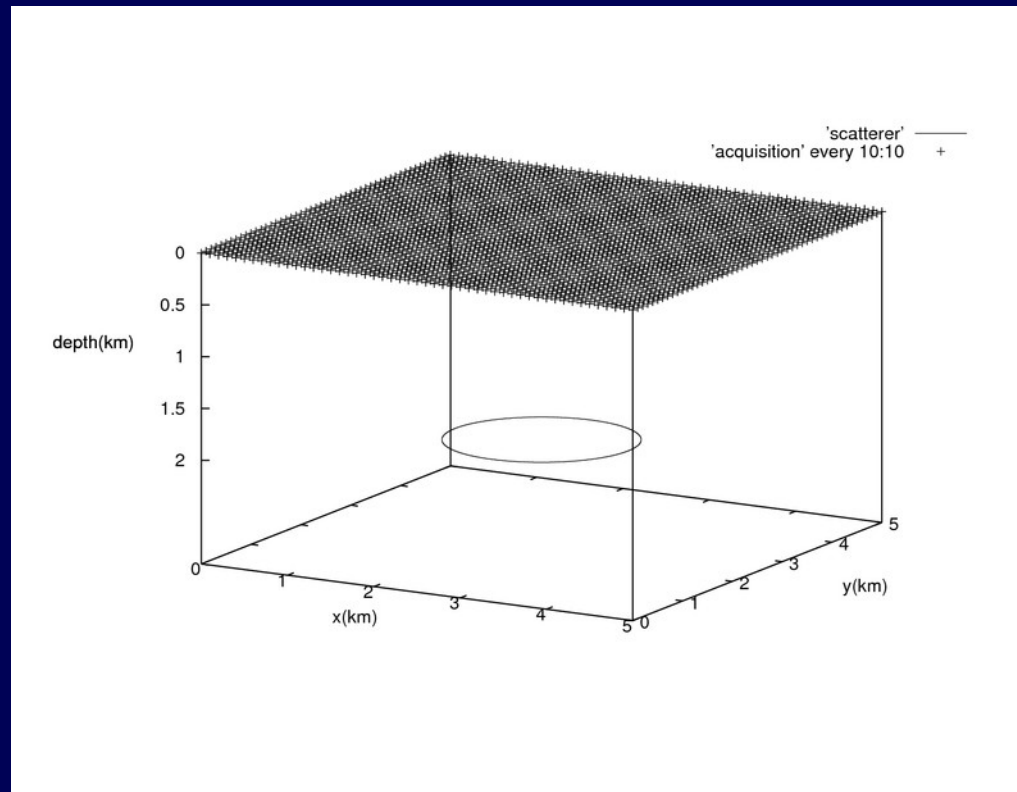
Fractures

Semi-circles



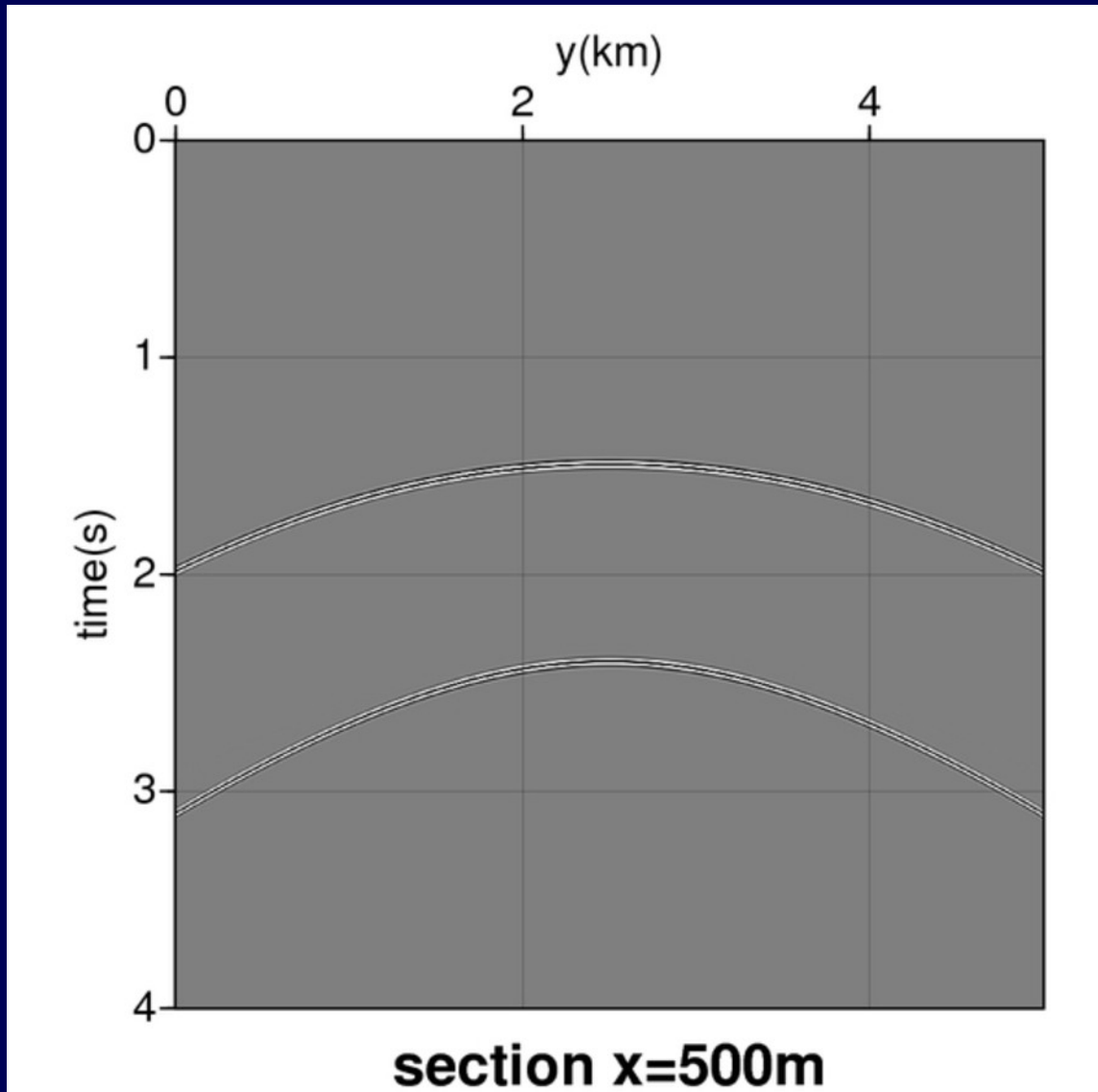
Curved edge

# Curved edge diffractions – synthetic – plane circle



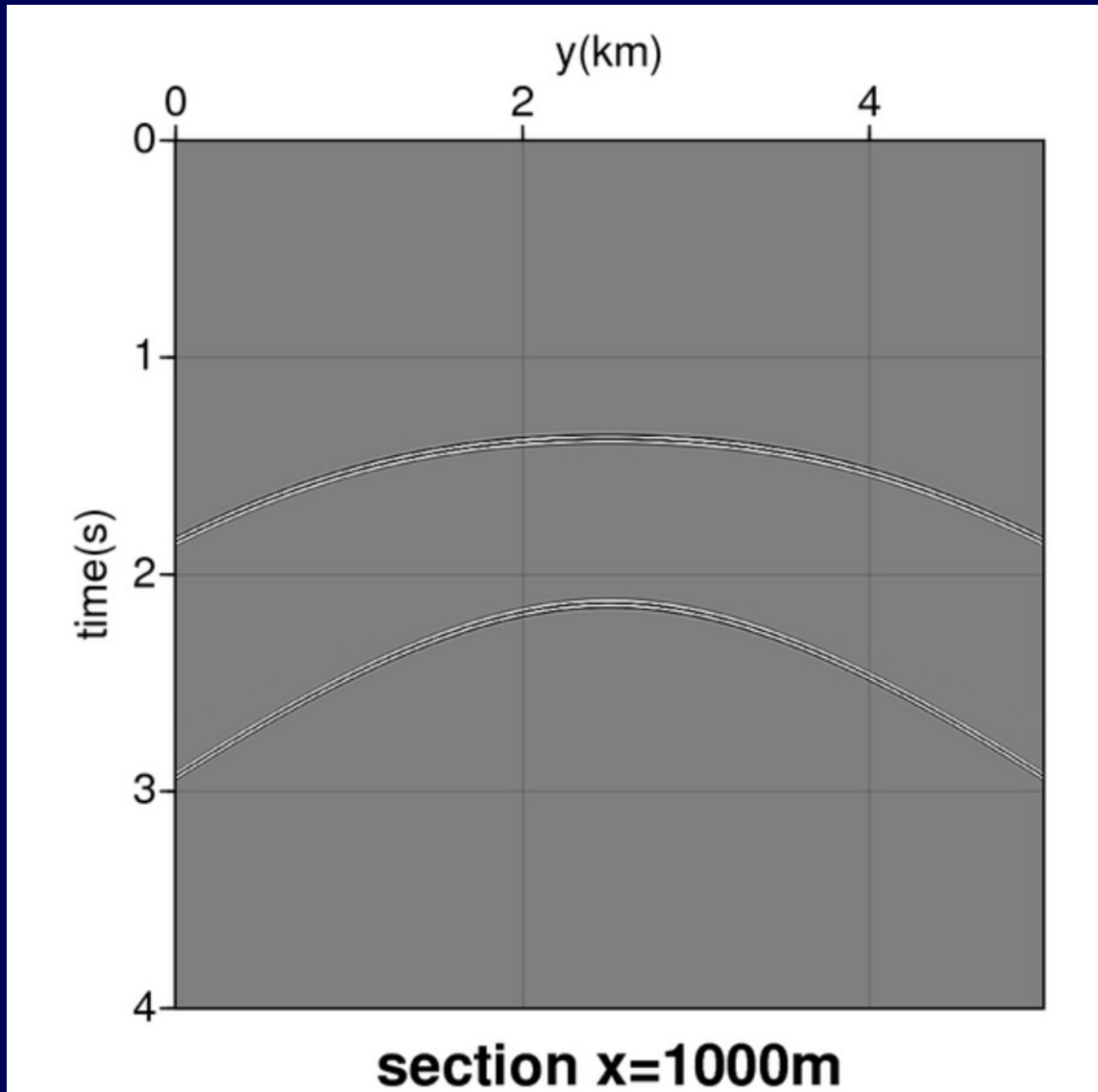
**Zero offset → Stacked section**

# Curved edge diffractions – synthetic – plane circle

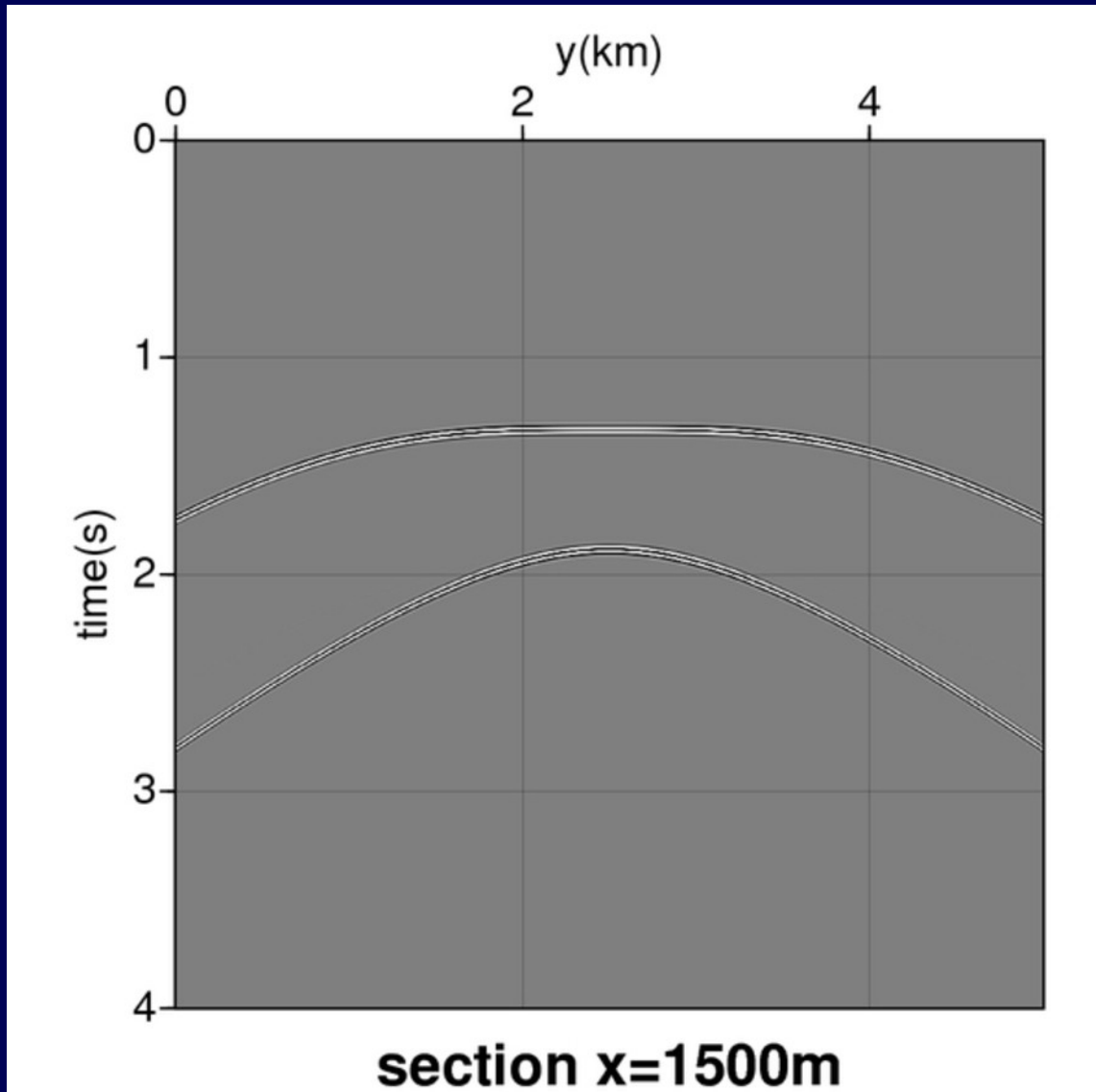




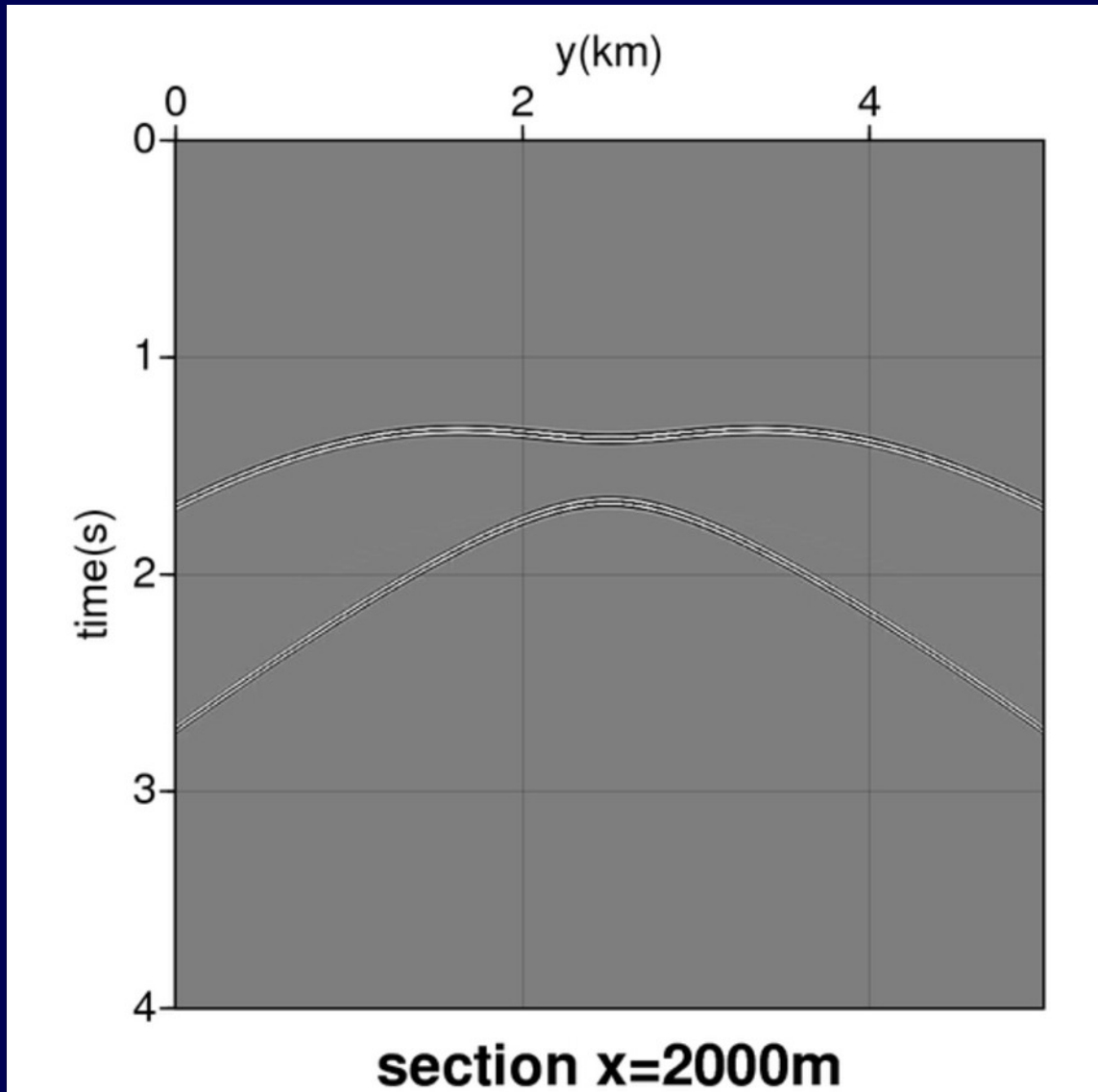
# Curved edge diffractions – synthetic – plane circle



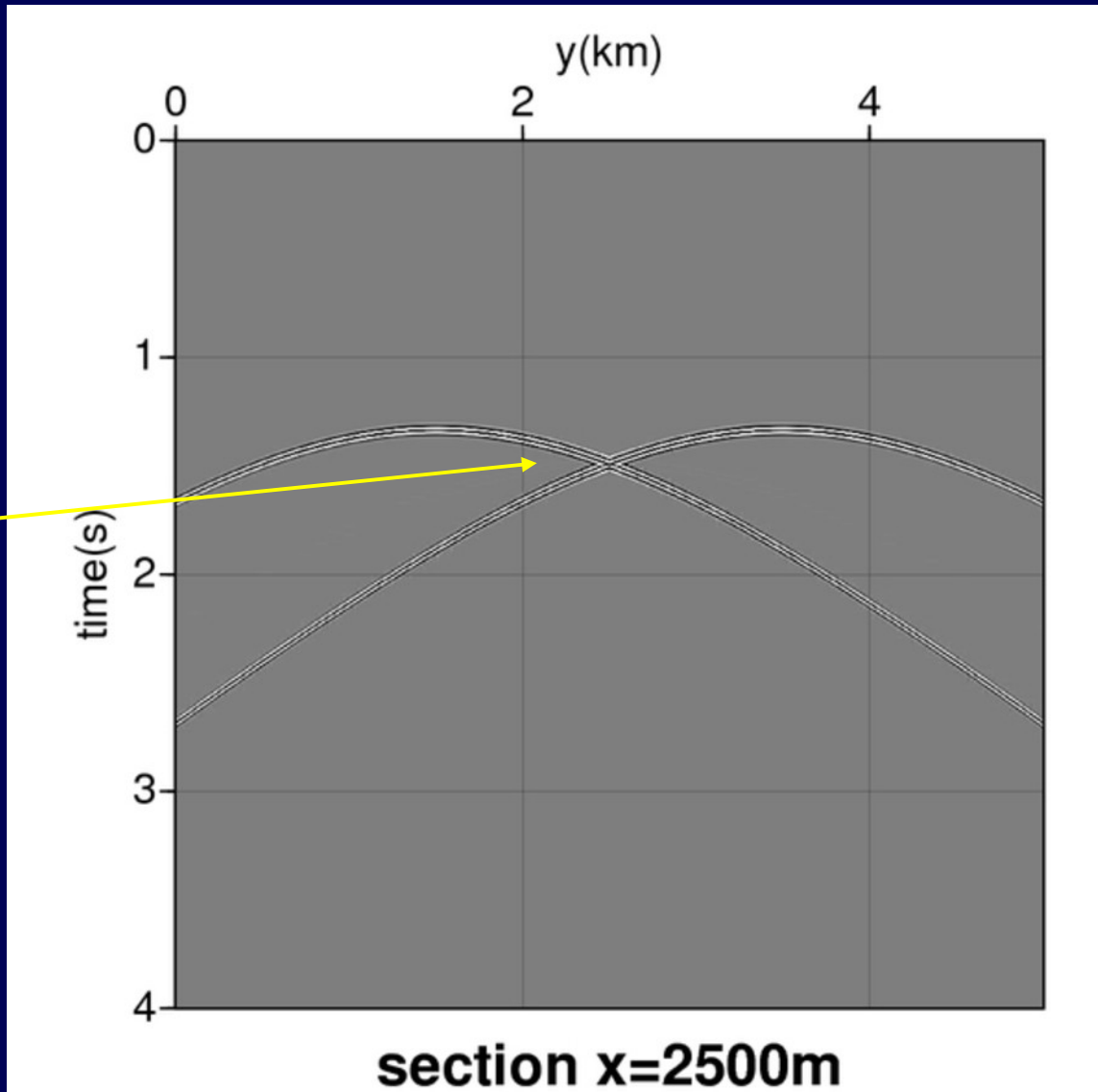
# Curved edge diffractions – synthetic – plane circle



# Curved edge diffractions – synthetic – plane circle

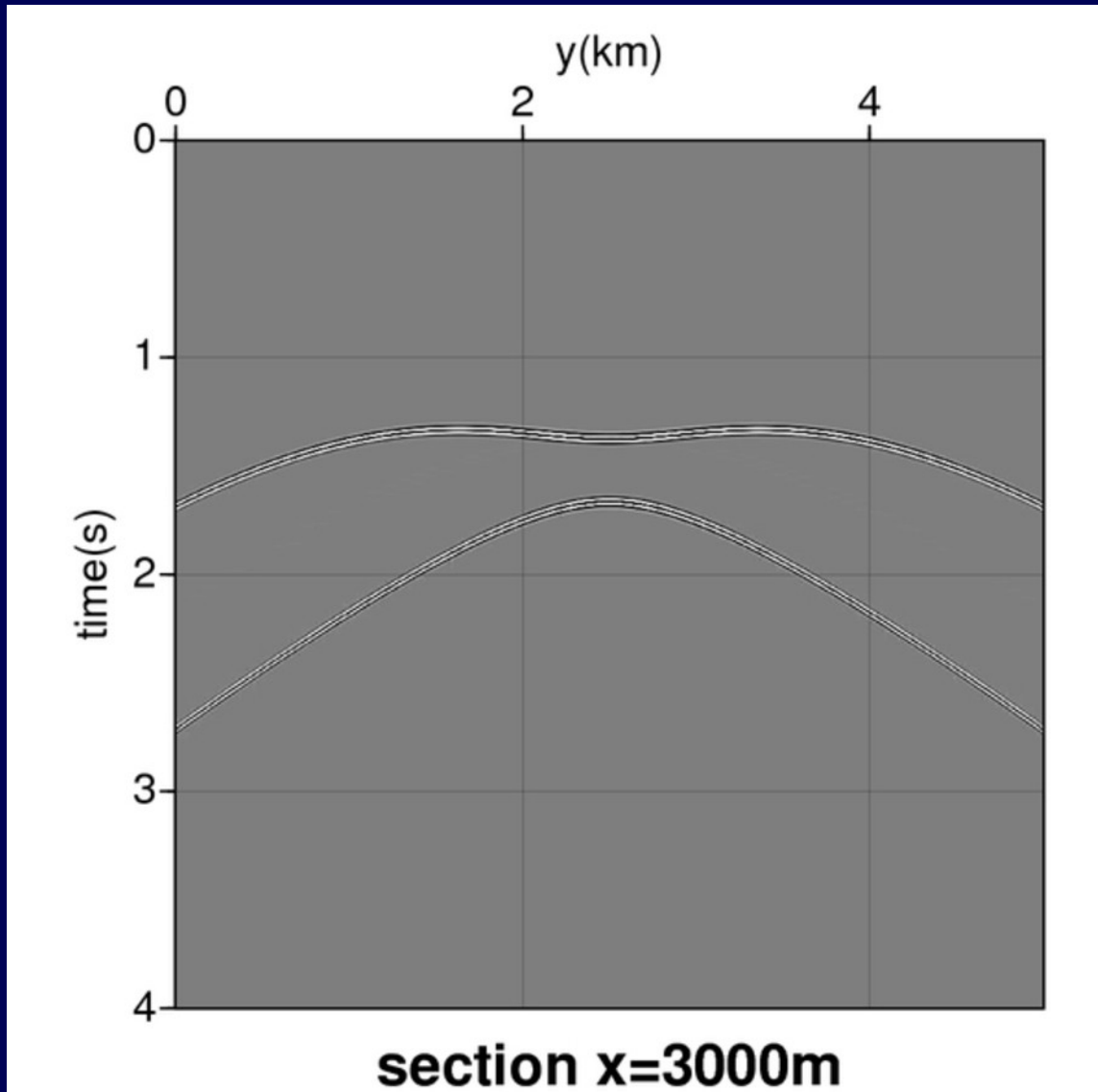


# Curved edge diffractions – synthetic – plane circle

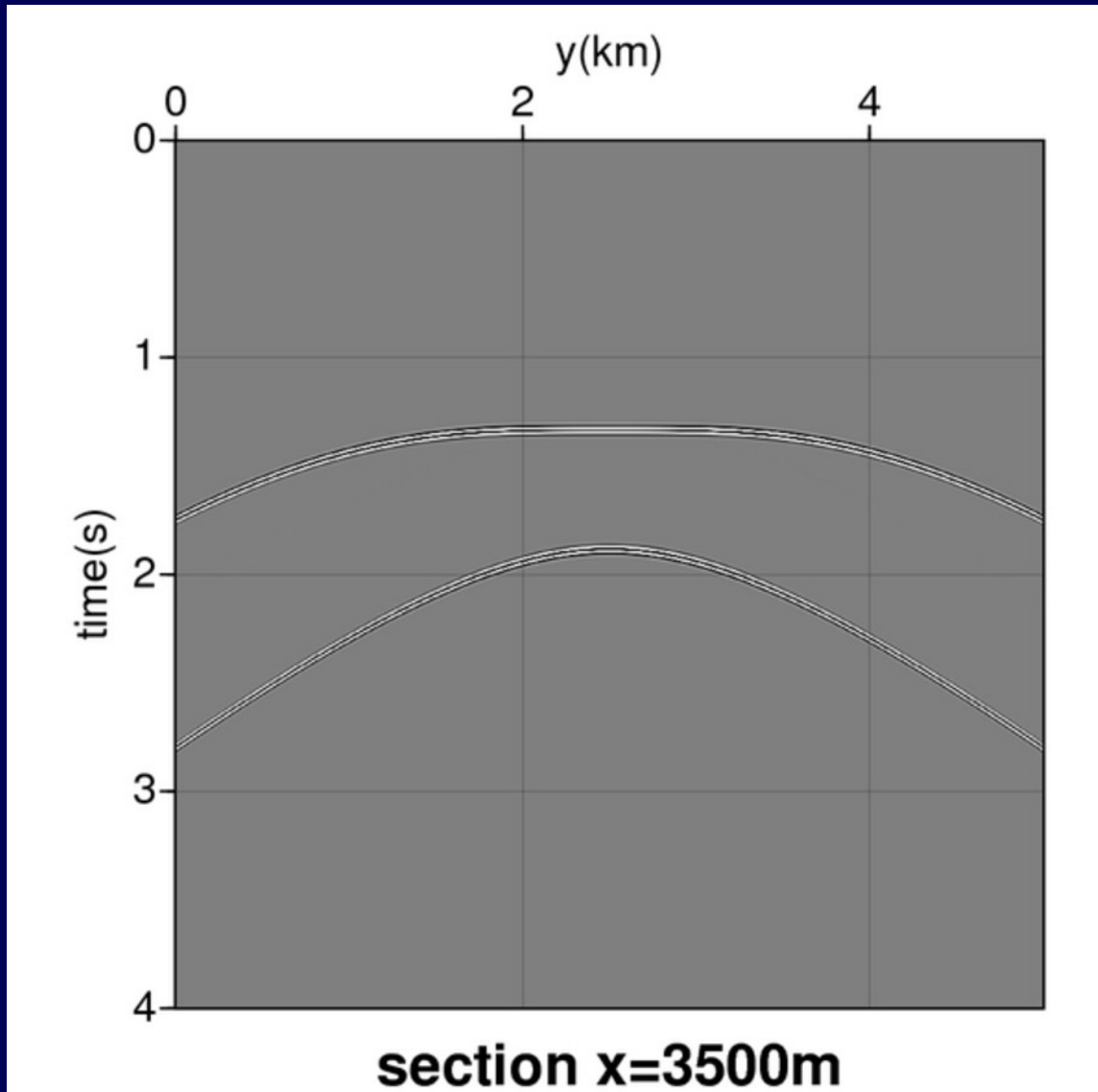


**Two  
branches  
crossing  
over**

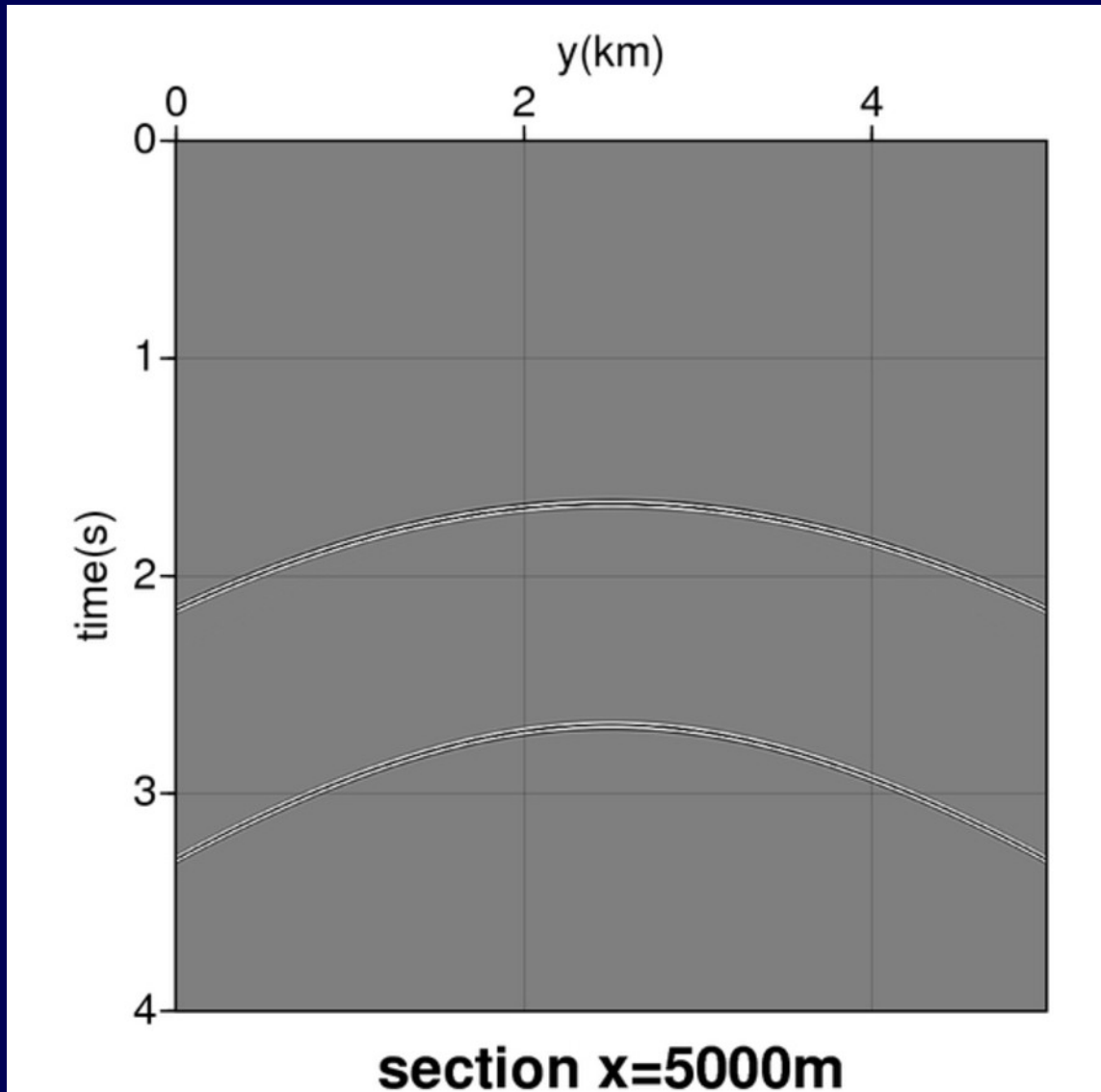
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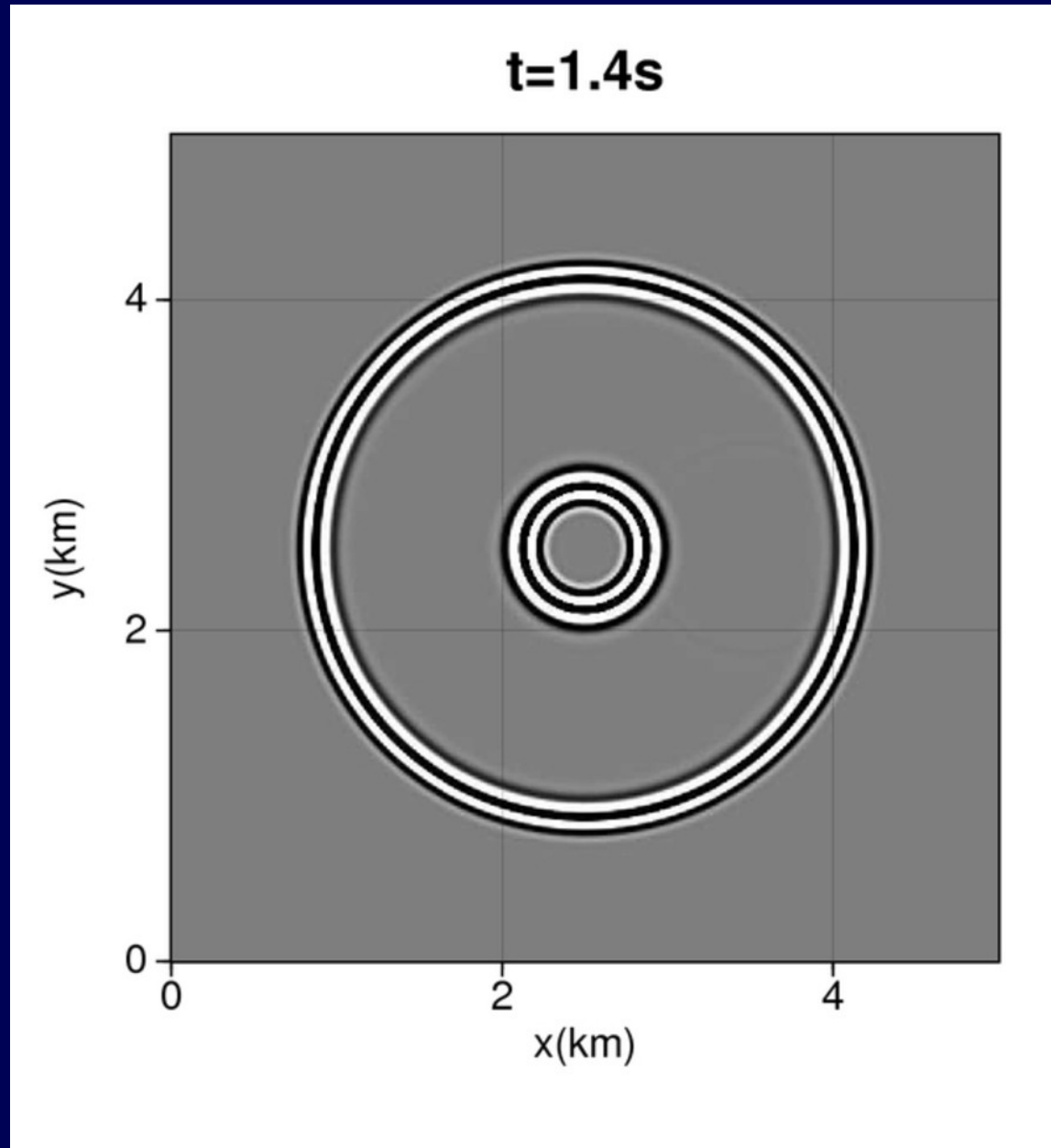
# Curved edge diffractions – synthetic – plane circle



# Curved edge diffractions – synthetic – plane circle

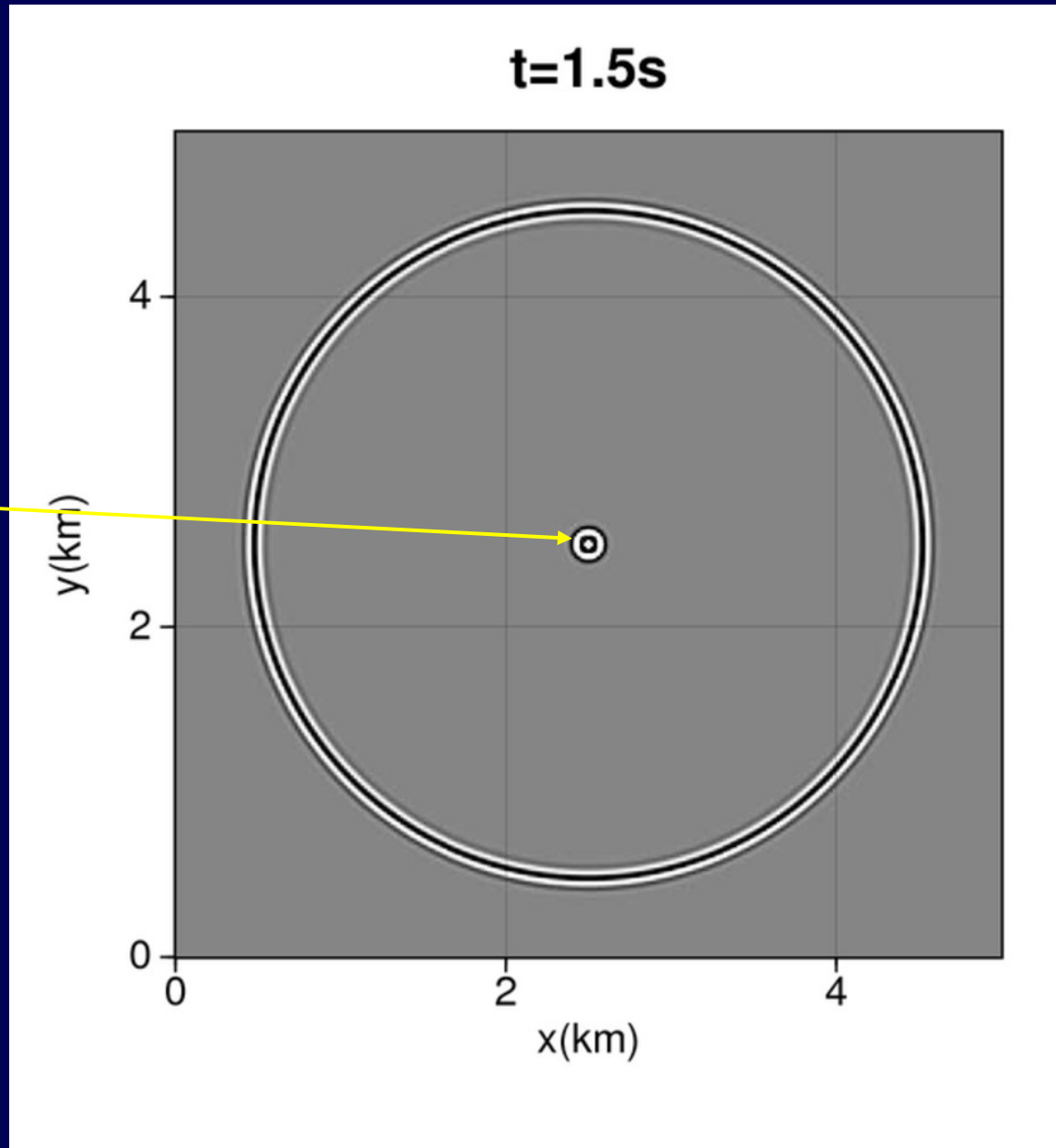


# Curved edge diffractions – synthetic – plane circle



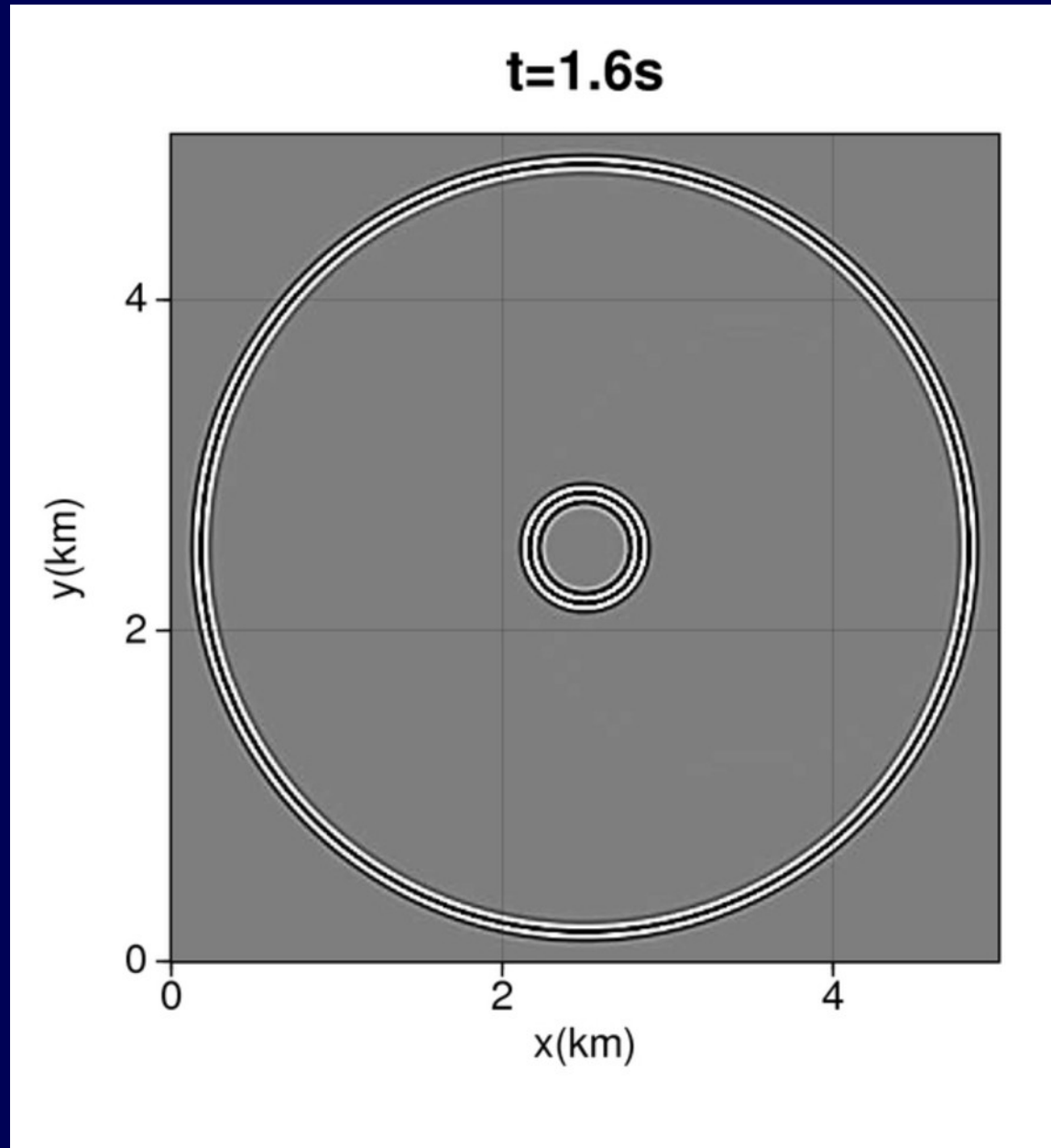


# Curved edge diffractions – synthetic – plane circle

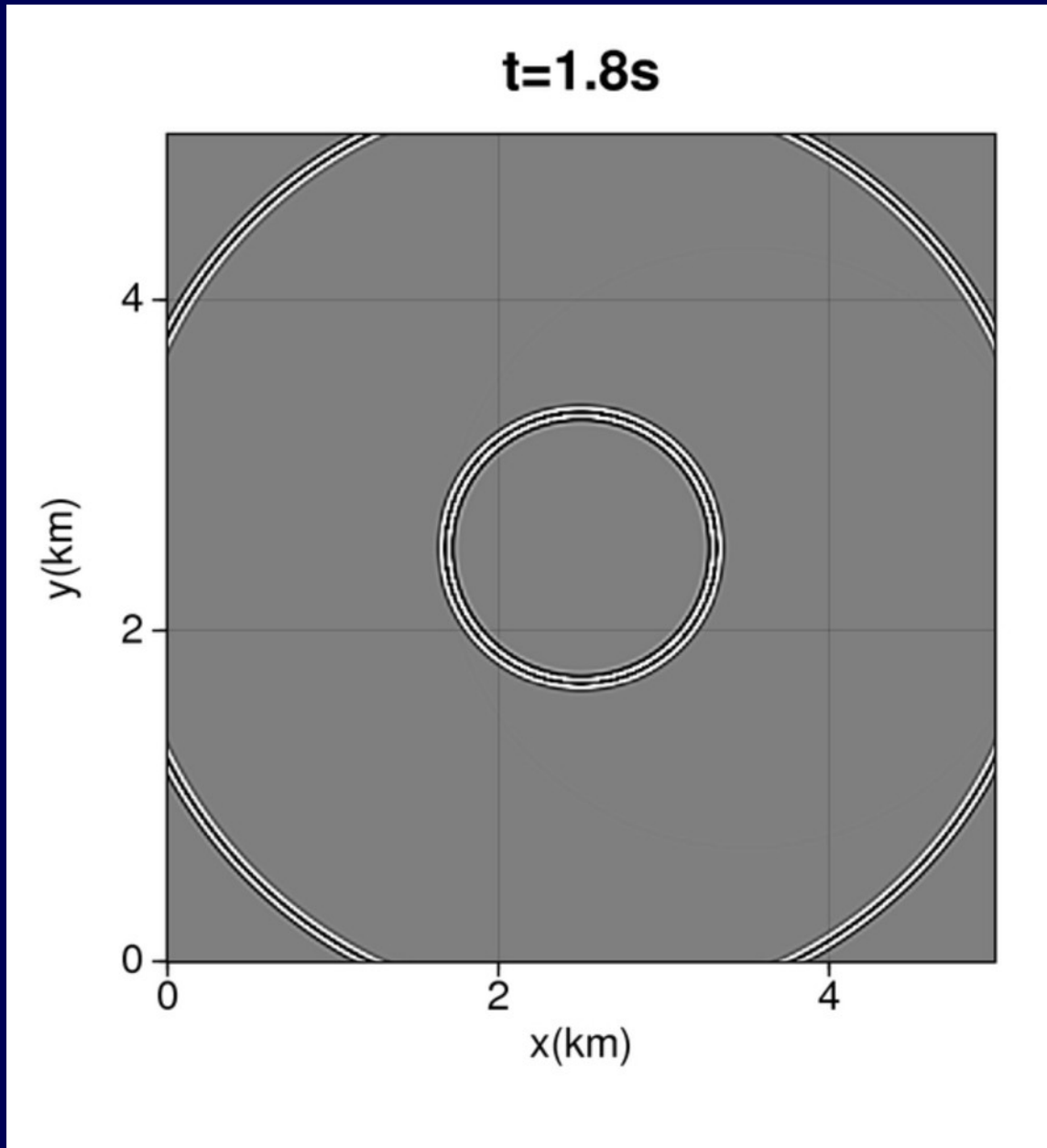


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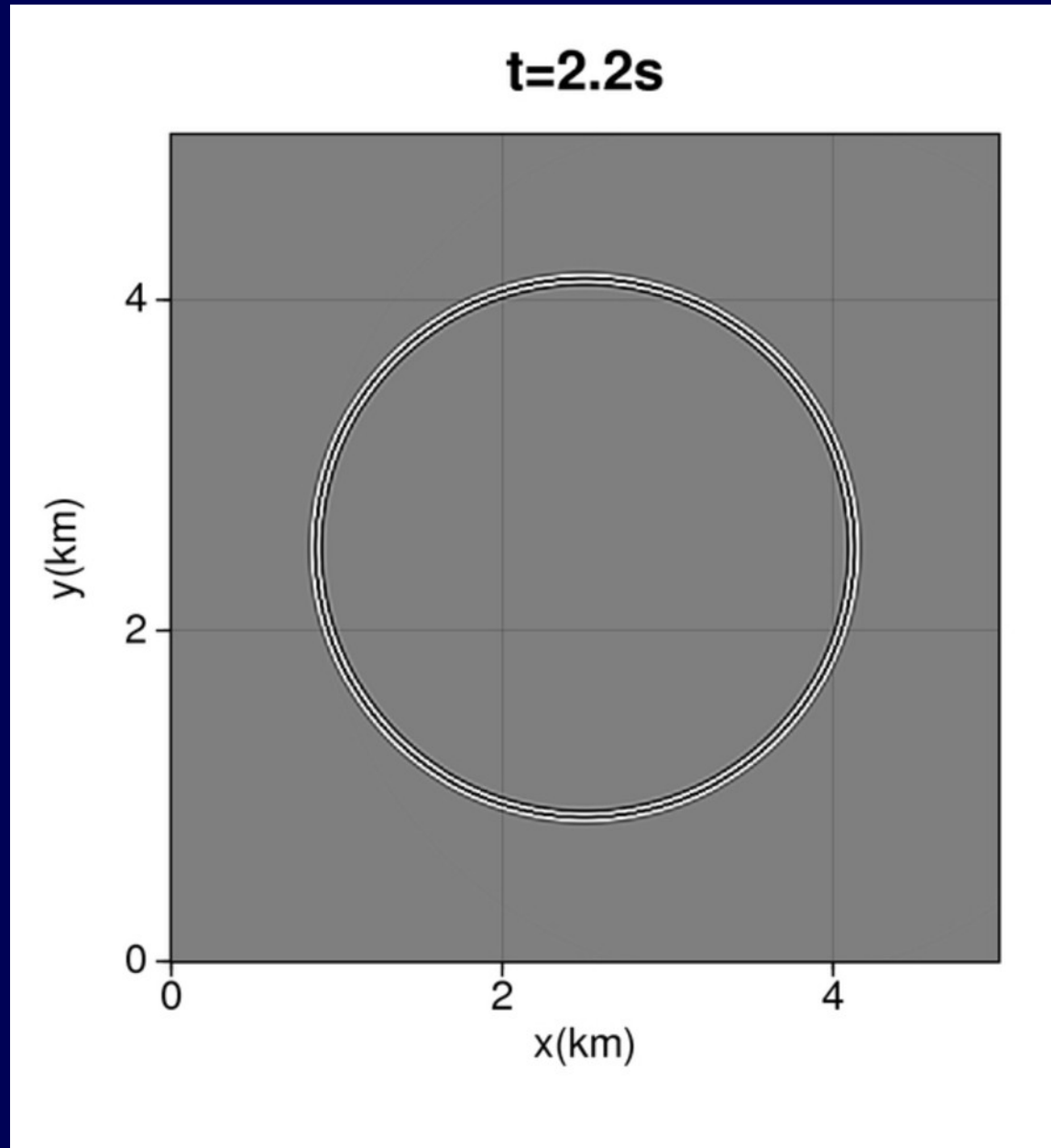
# Curved edge diffractions – synthetic – plane circle



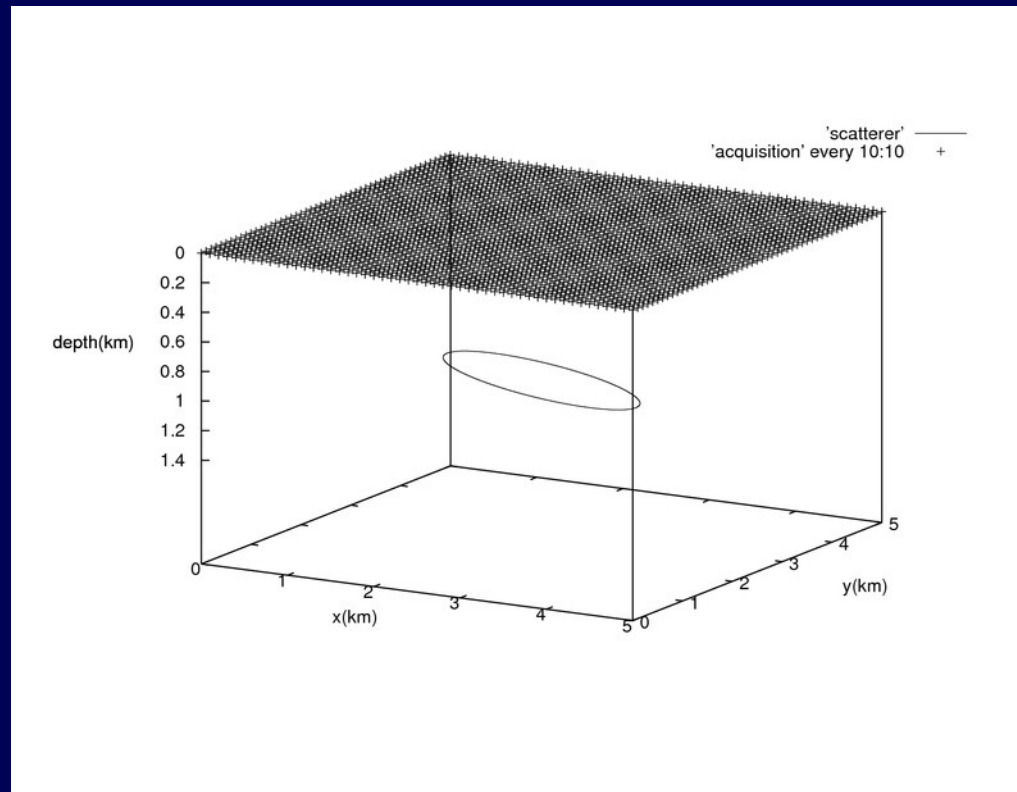
# Curved edge diffractions – synthetic – plane circle



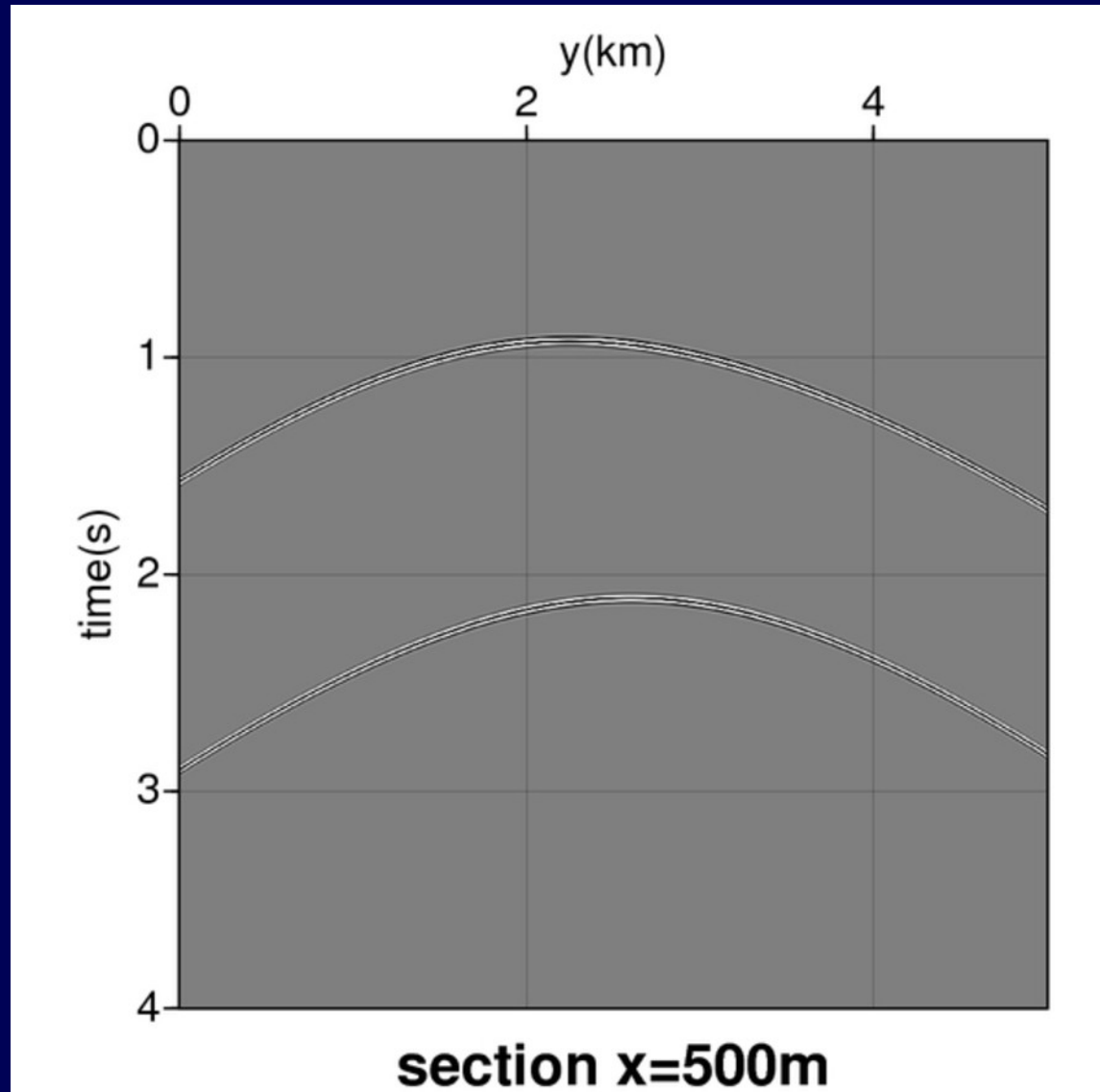
# Curved edge diffractions – synthetic – plane circle



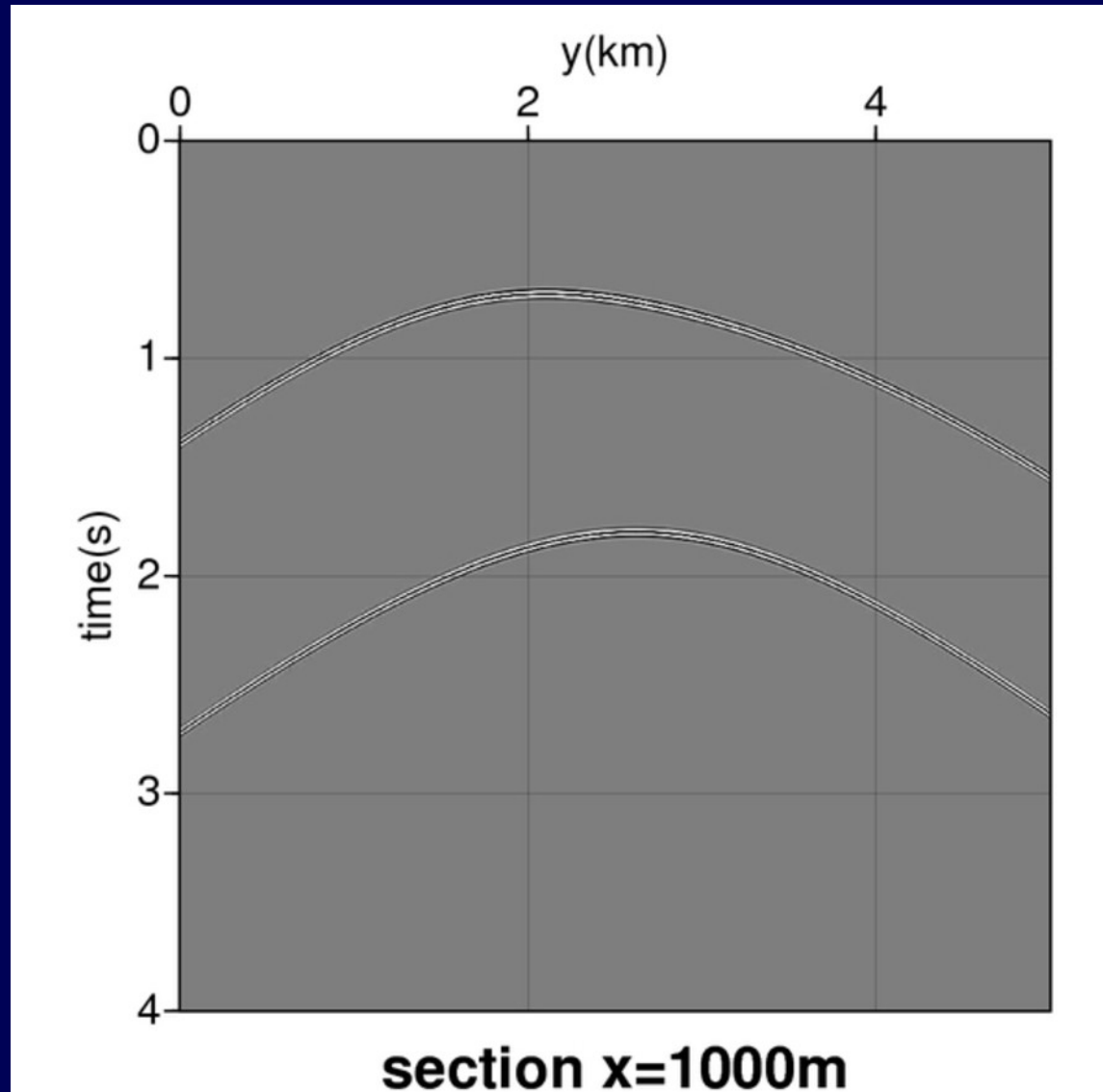
# Curved edge diffractions – synthetic – dipping circle



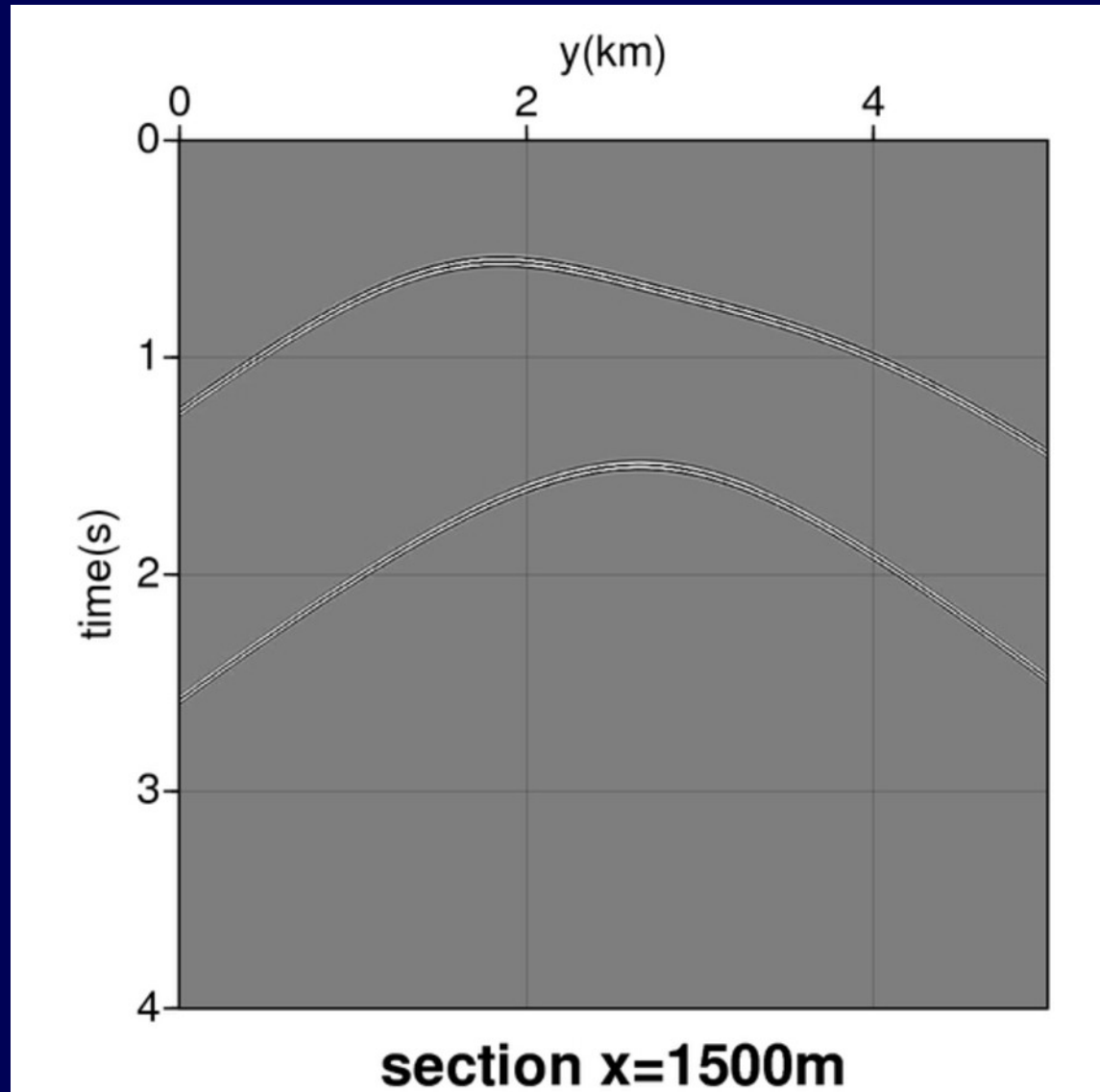
# Curved edge diffractions – synthetic – dipping circle



# Curved edge diffractions – synthetic – dipping circle

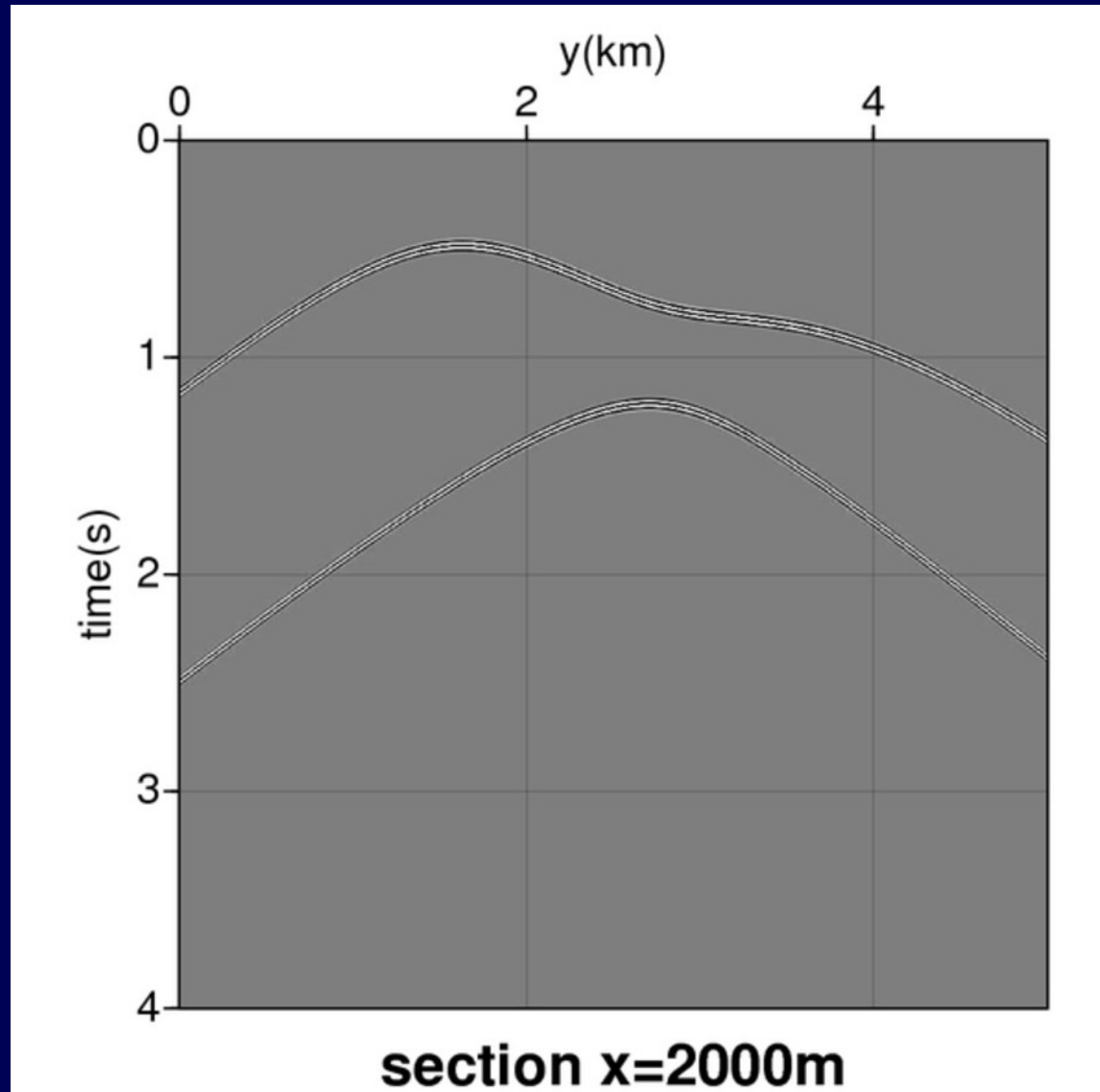


# Curved edge diffractions – synthetic – dipping circle

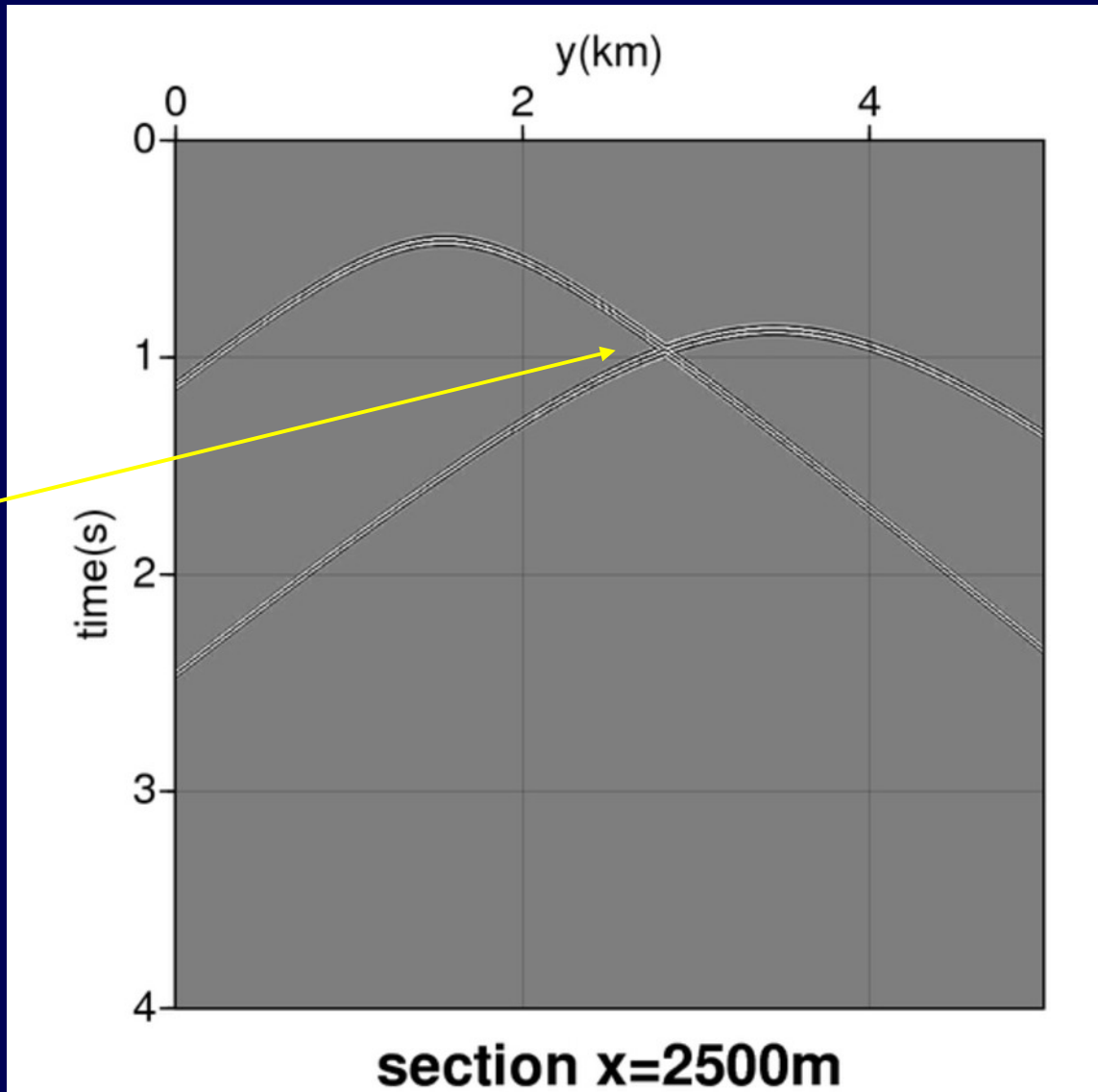




# Curved edge diffractions – synthetic – dipping circle

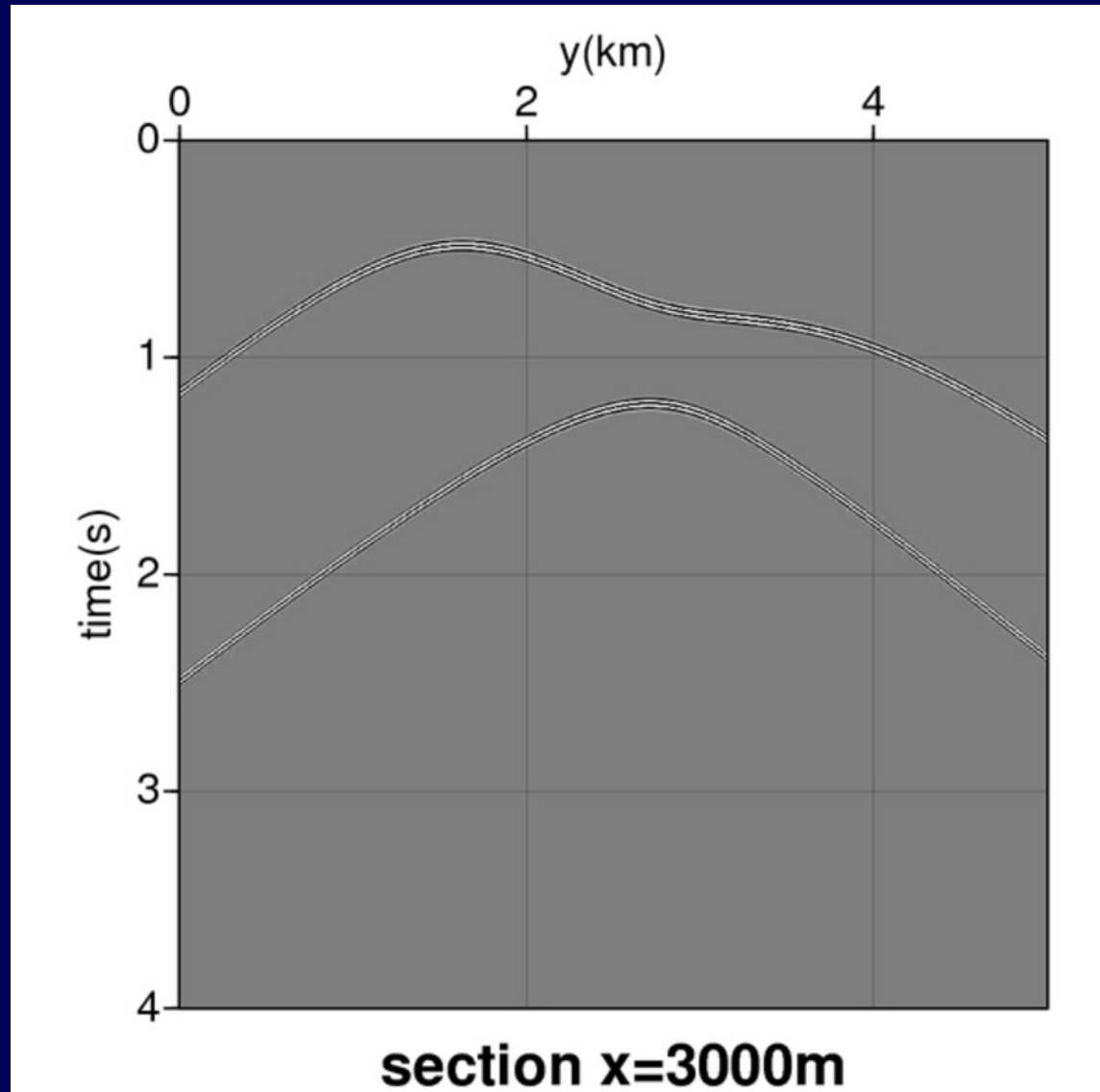


# Curved edge diffractions – synthetic – dipping circle

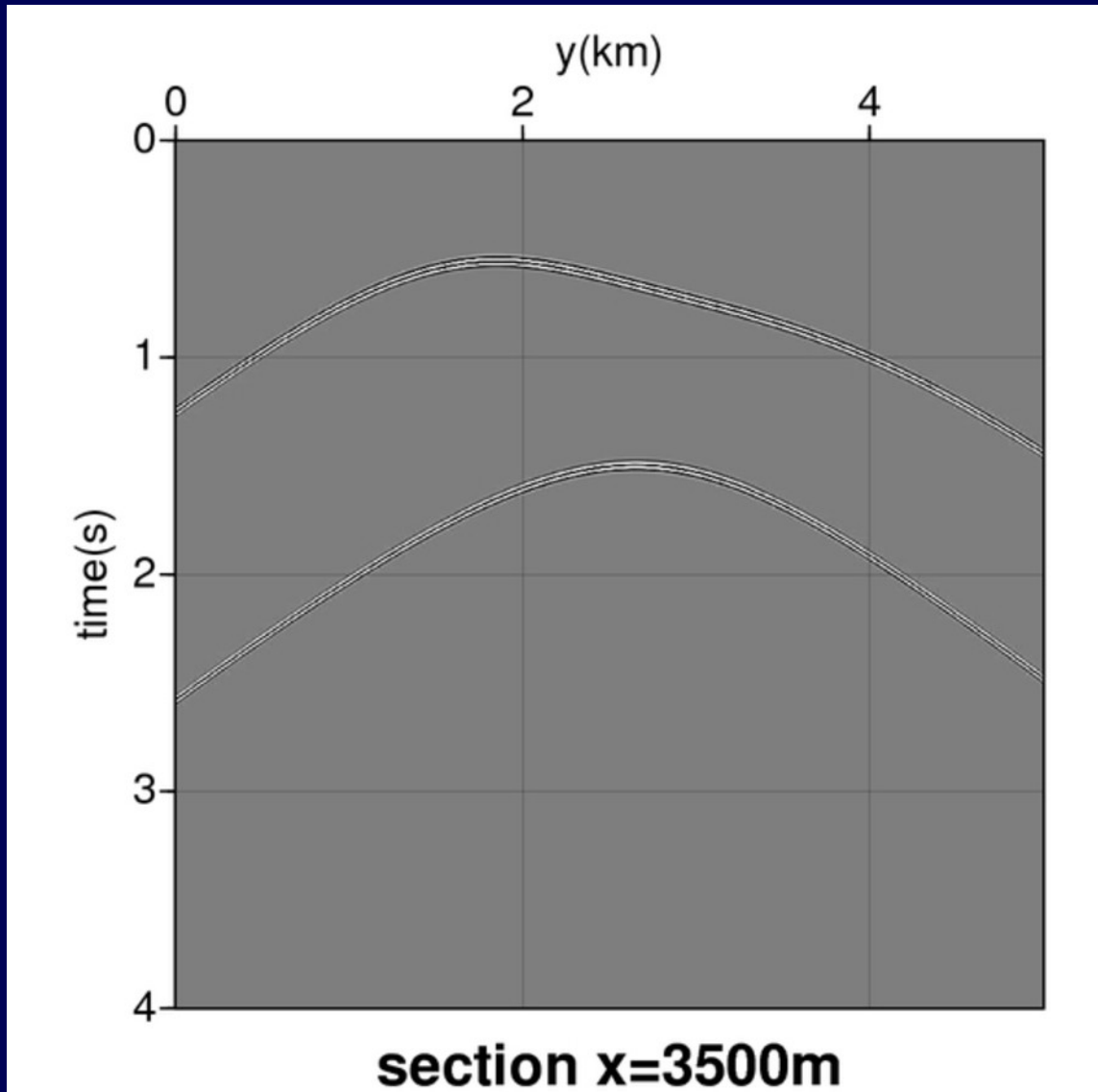


**Two  
branches  
crossing  
over**

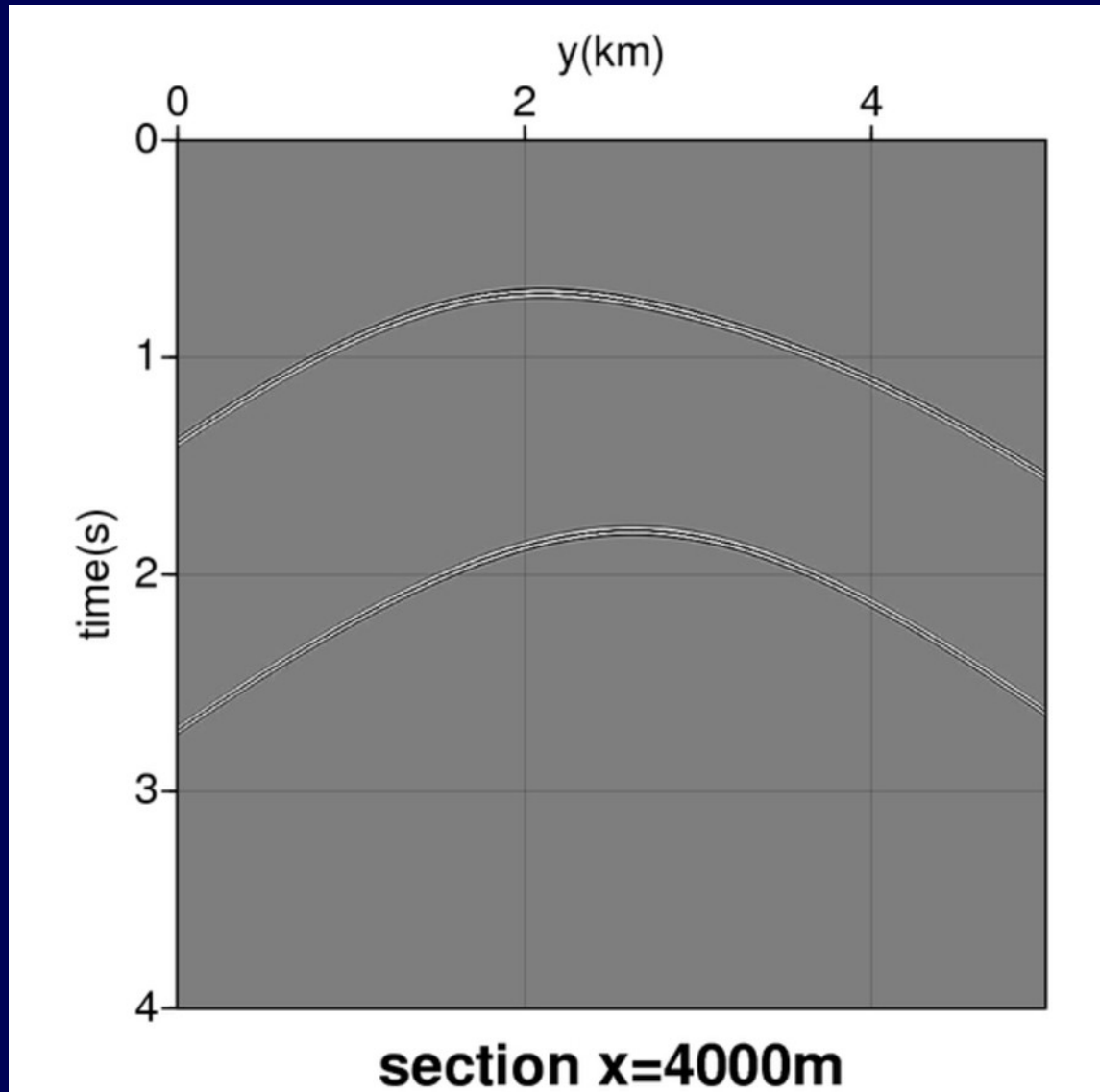
# Curved edge diffractions – synthetic – dipping circle



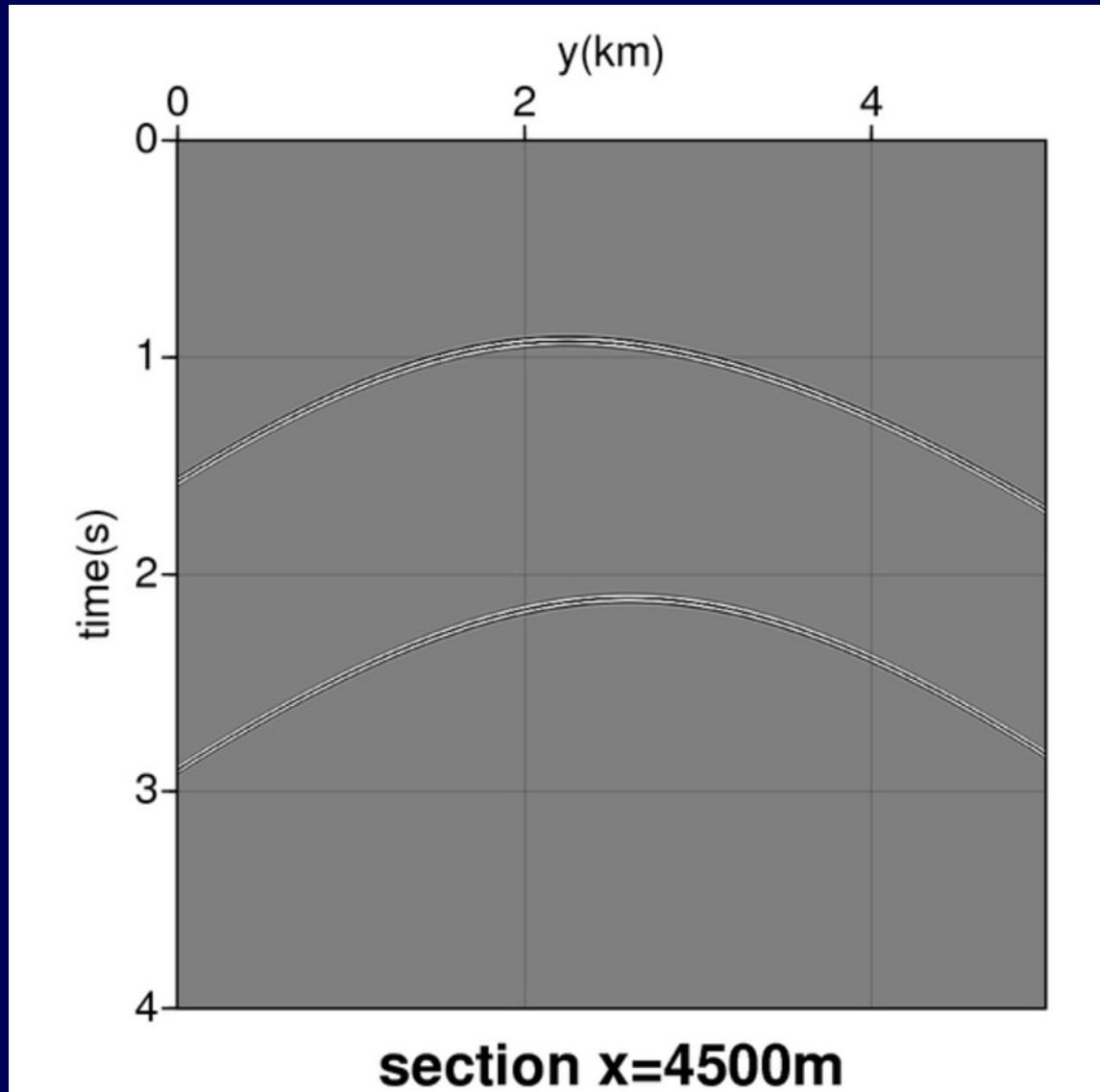
# Curved edge diffractions – synthetic – dipping circle



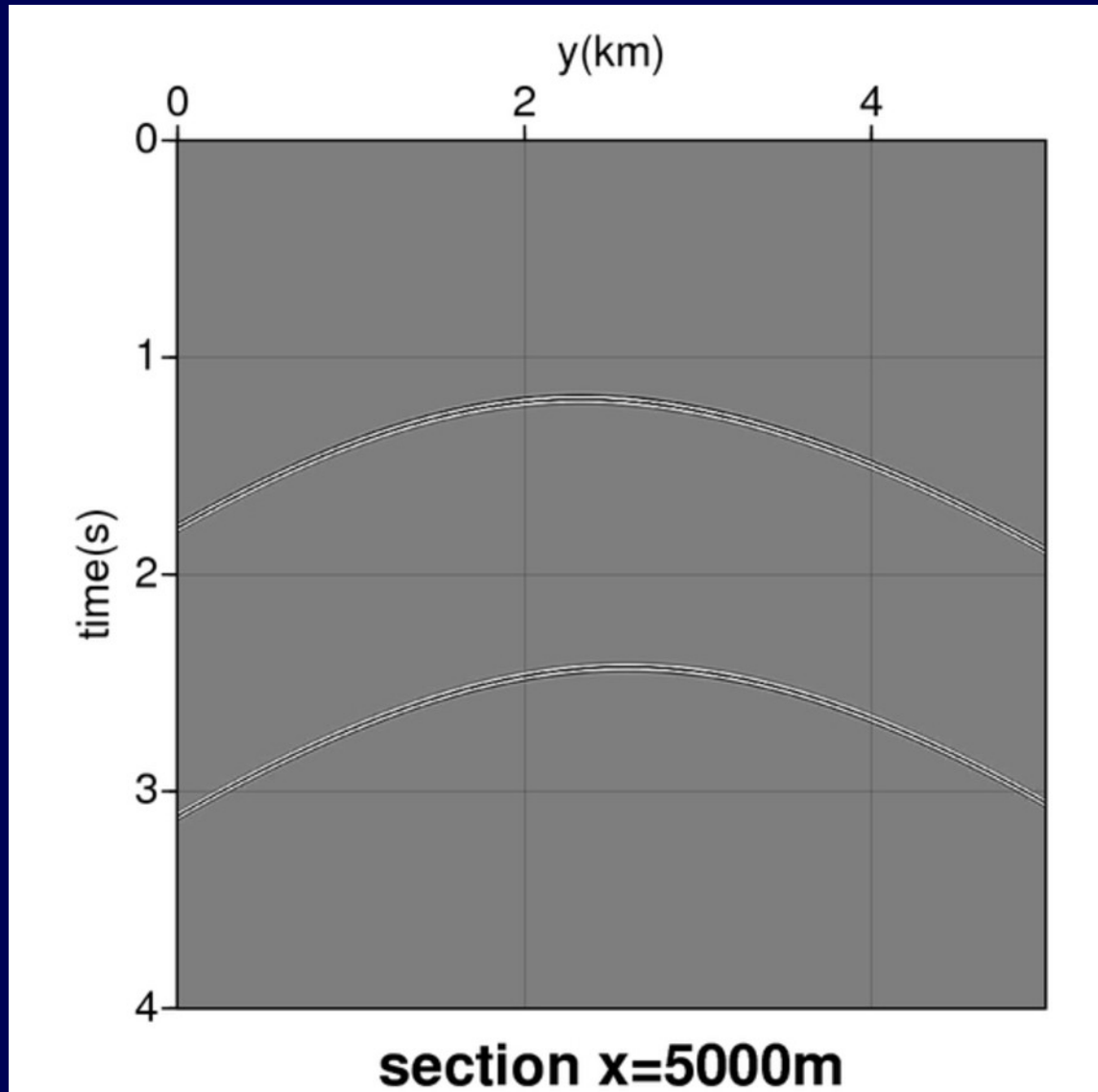
# Curved edge diffractions – synthetic – dipping circle



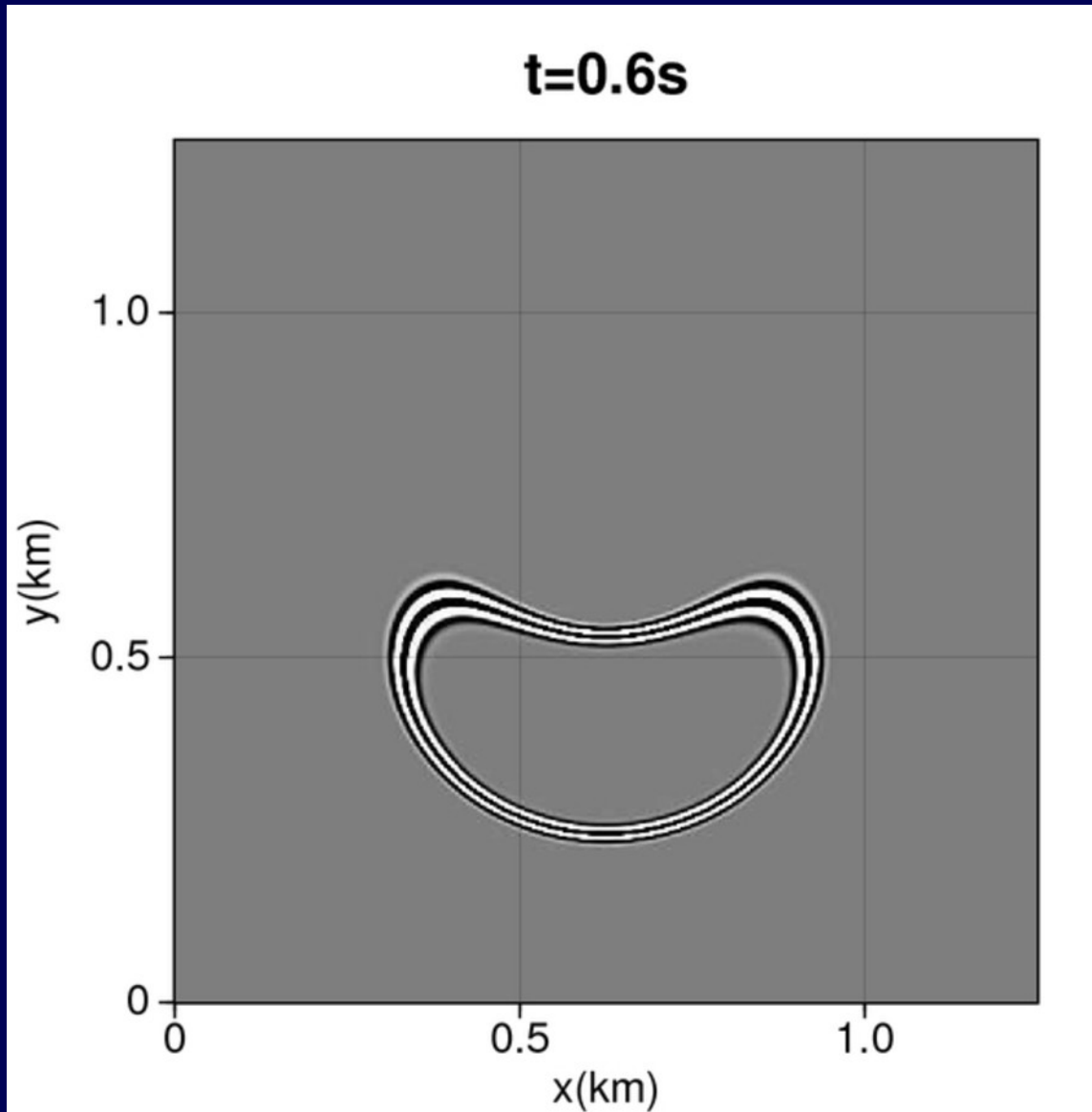
# Curved edge diffractions – synthetic – dipping circle



# Curved edge diffractions – synthetic – dipping circle

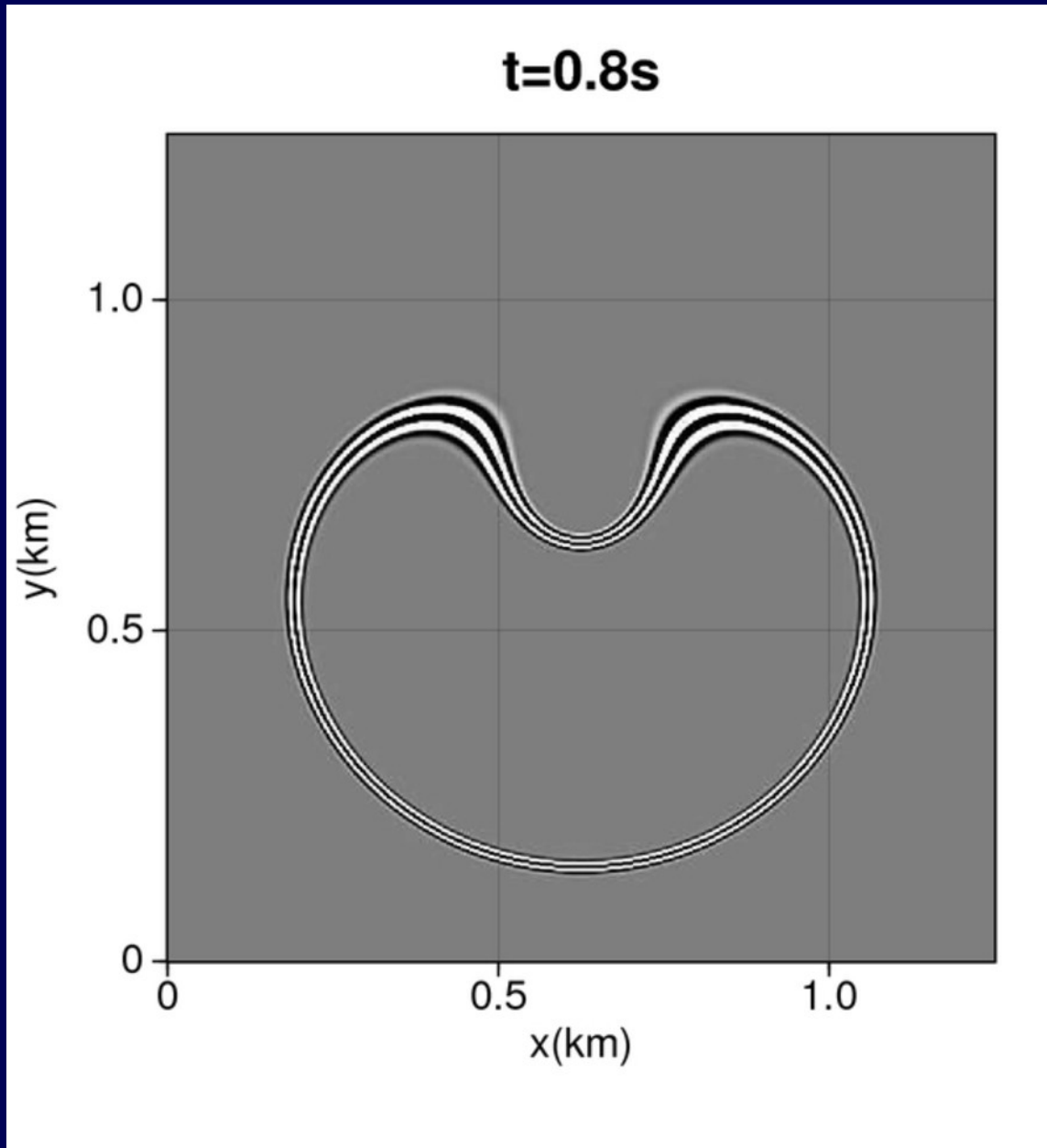


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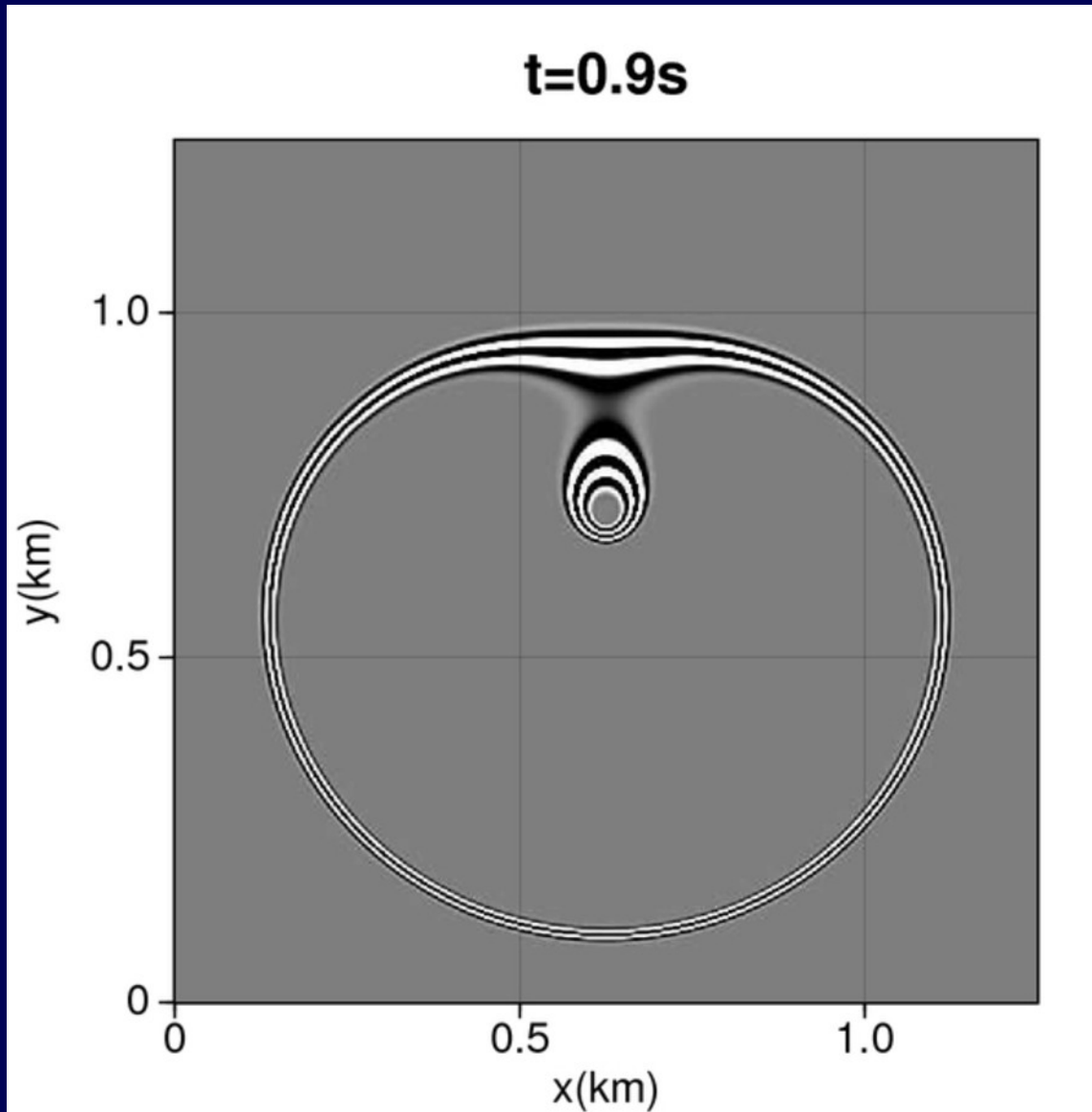




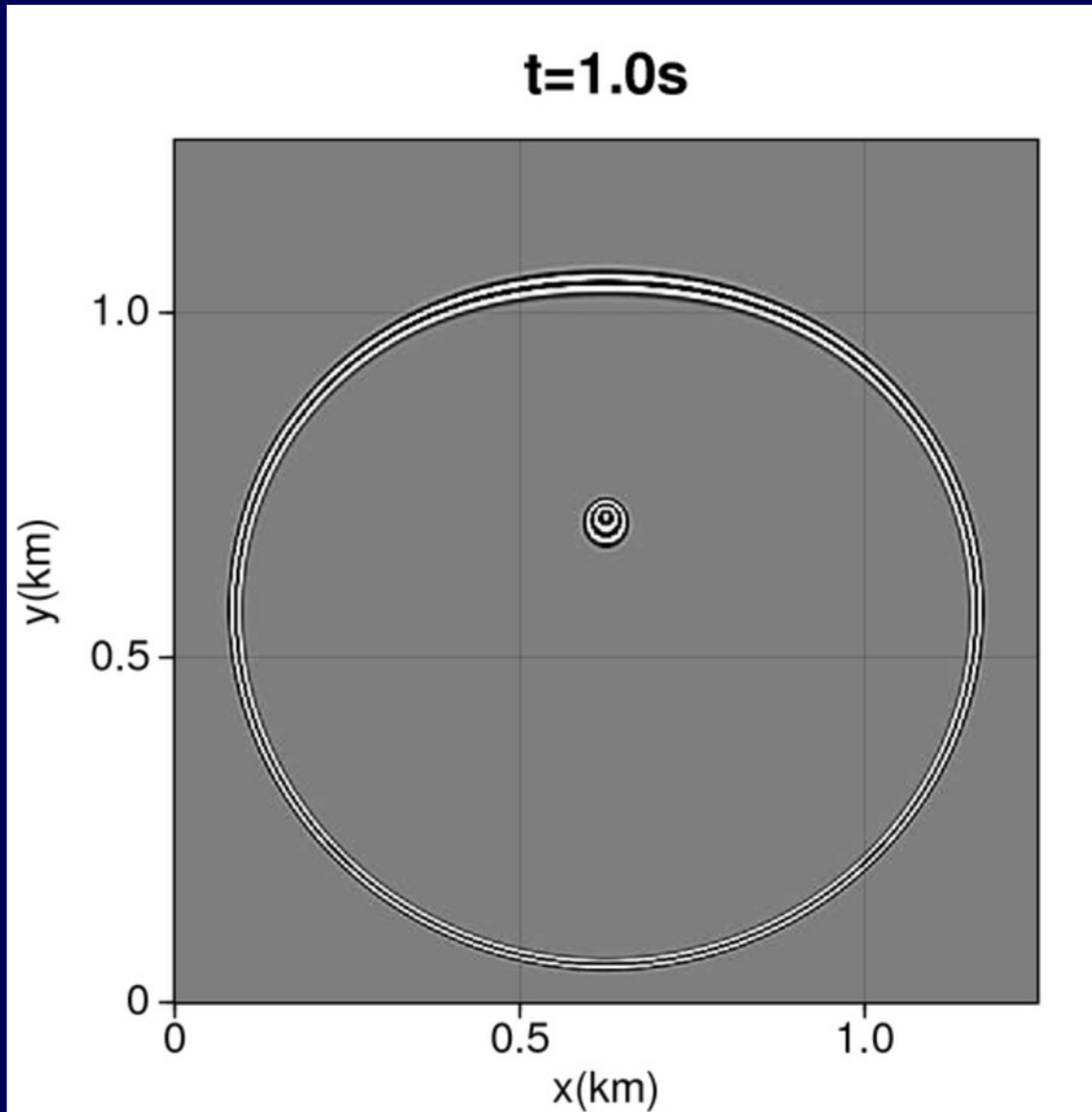
# Curved edge diffractions – synthetic – dipping circle



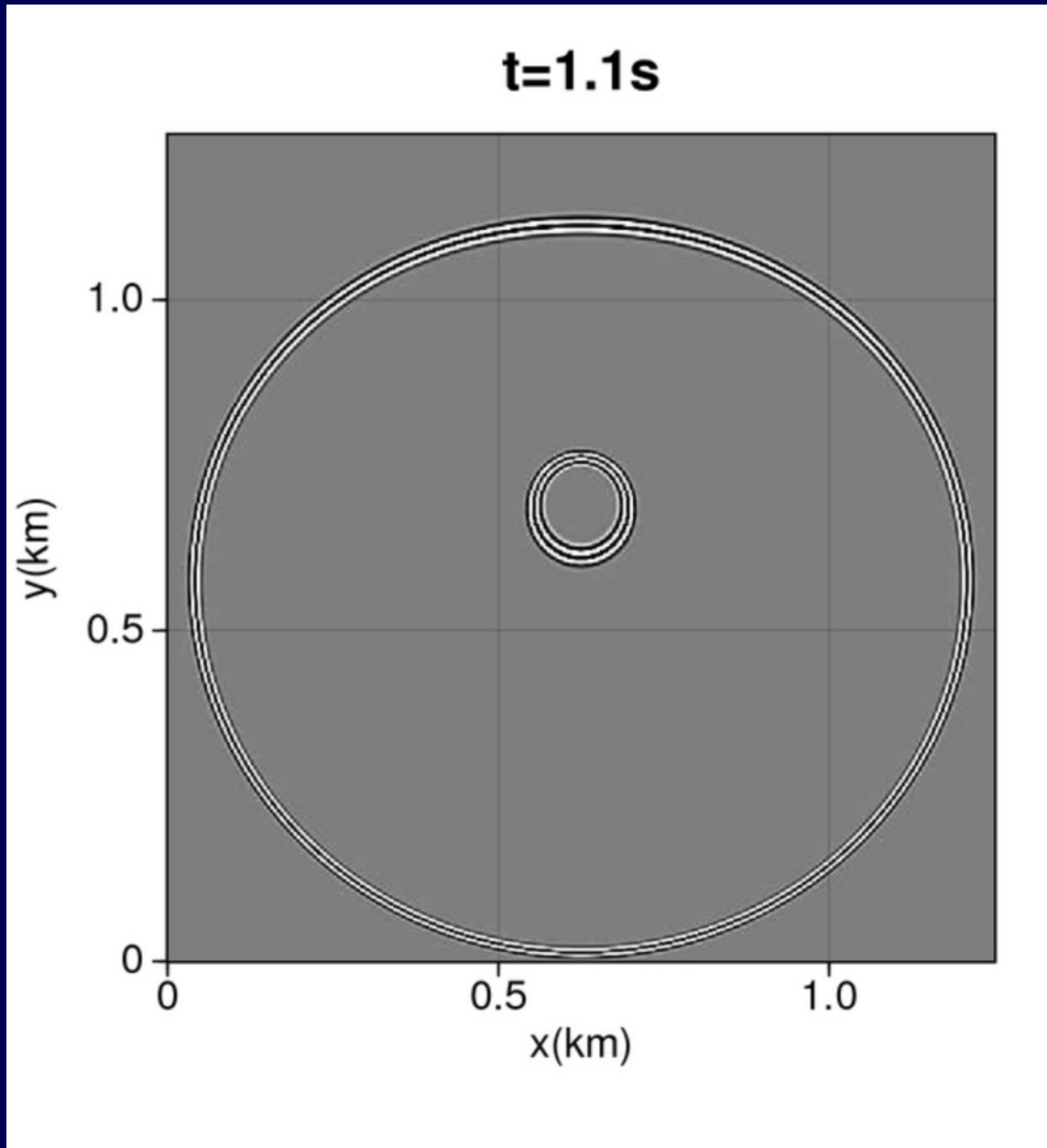
# Curved edge diffractions – synthetic – dipping circle



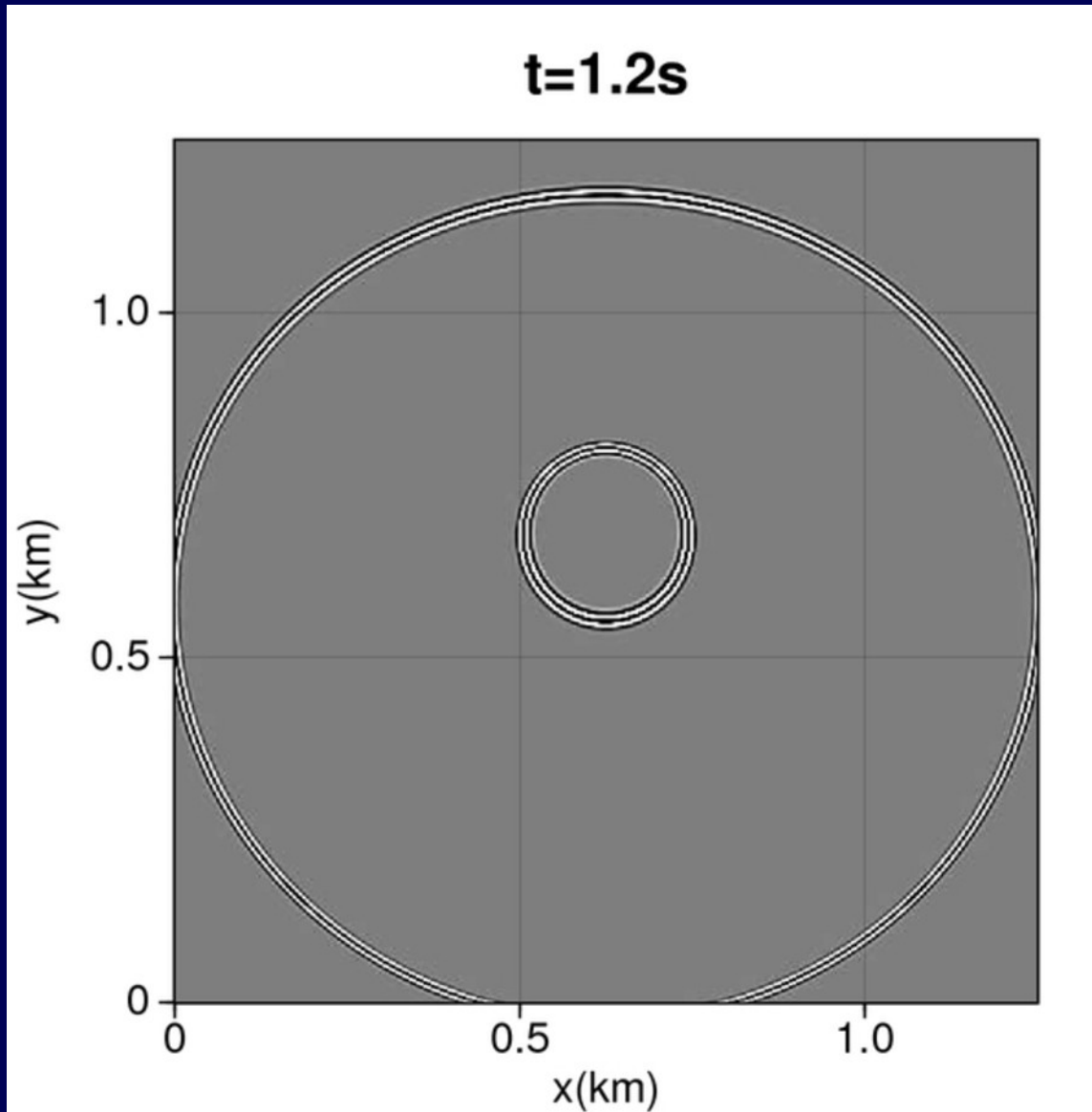
# Curved edge diffractions – synthetic – dipping circle



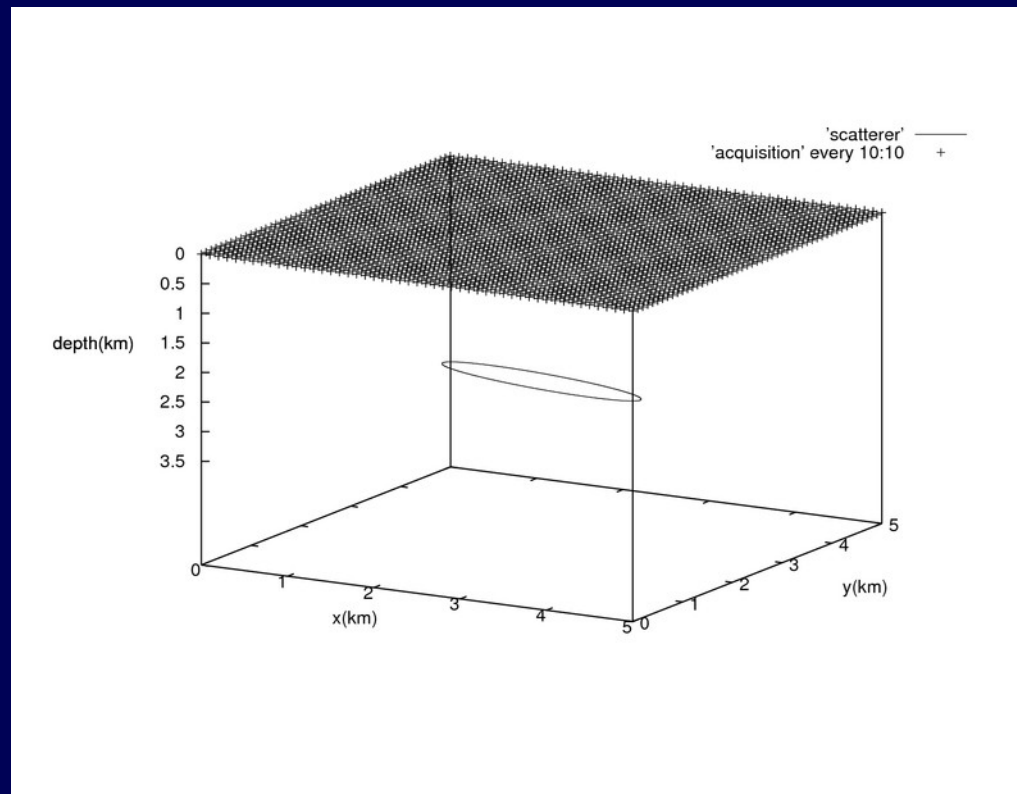
# Curved edge diffractions – synthetic – dipping circle



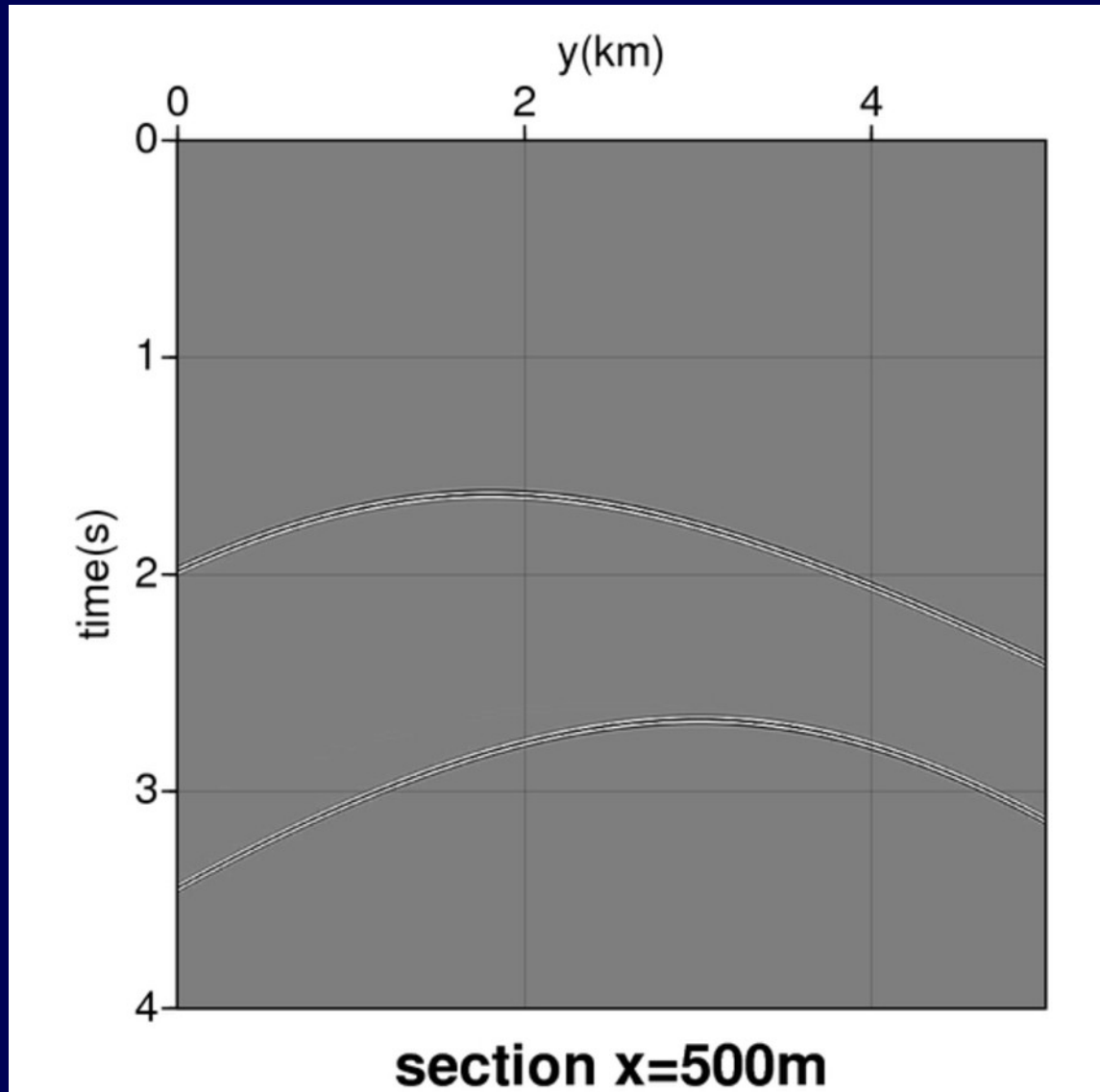
# Curved edge diffractions – synthetic – dipping circle



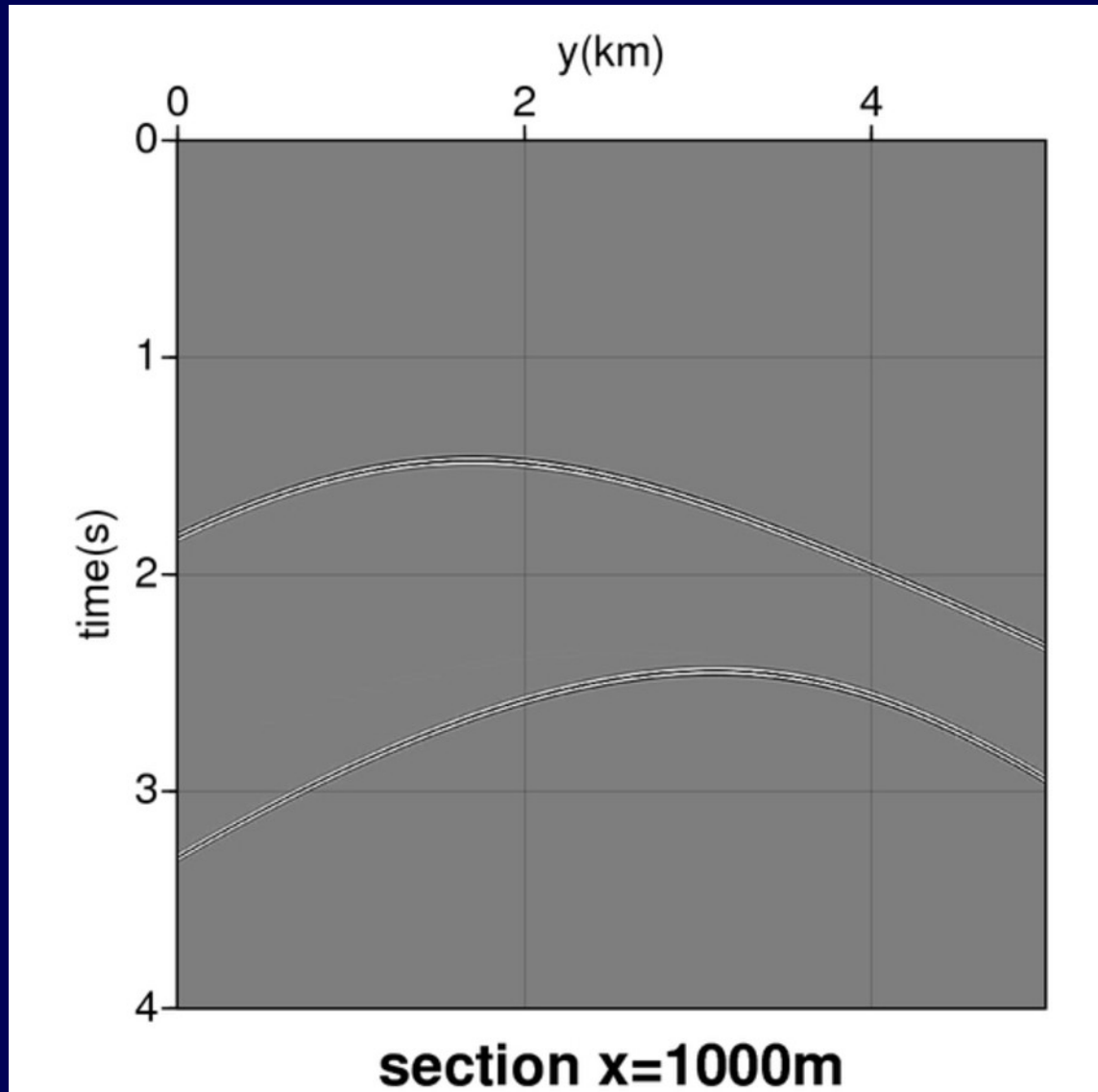
# Curved edge diffractions – synthetic – dipping ellipse



# Curved edge diffractions – synthetic – dipping ellipse

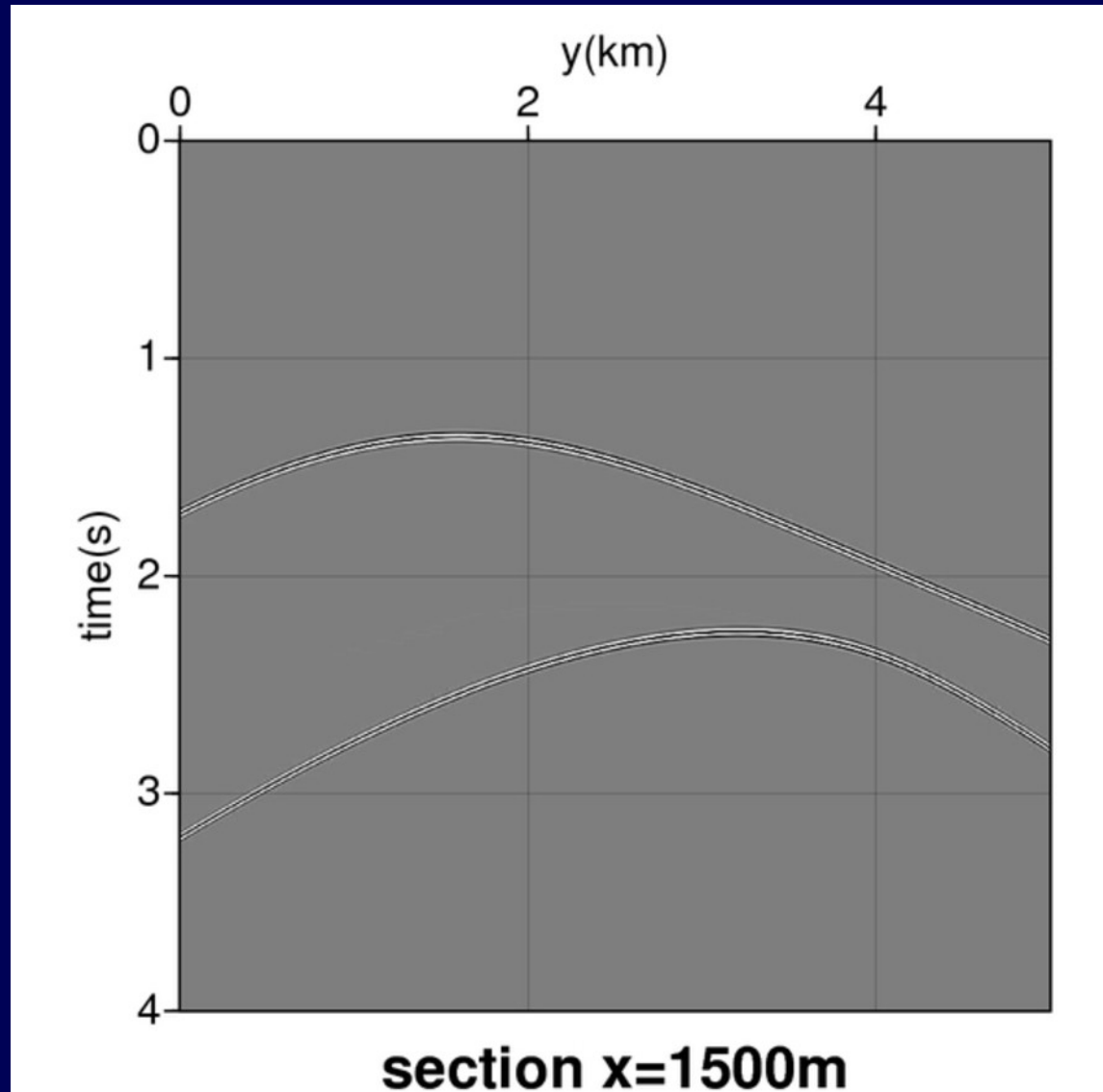


# Curved edge diffractions – synthetic – dipping ellipse

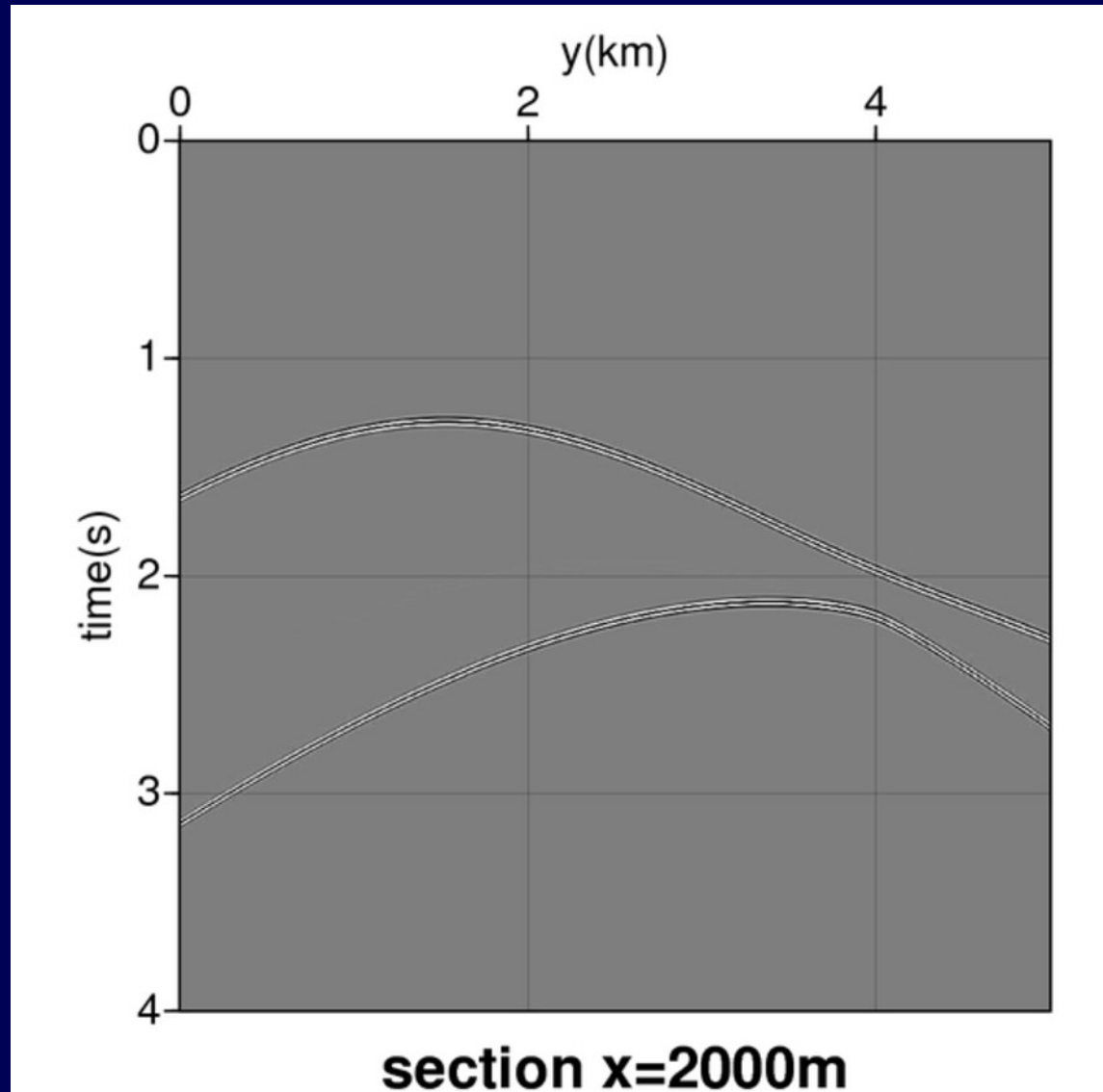




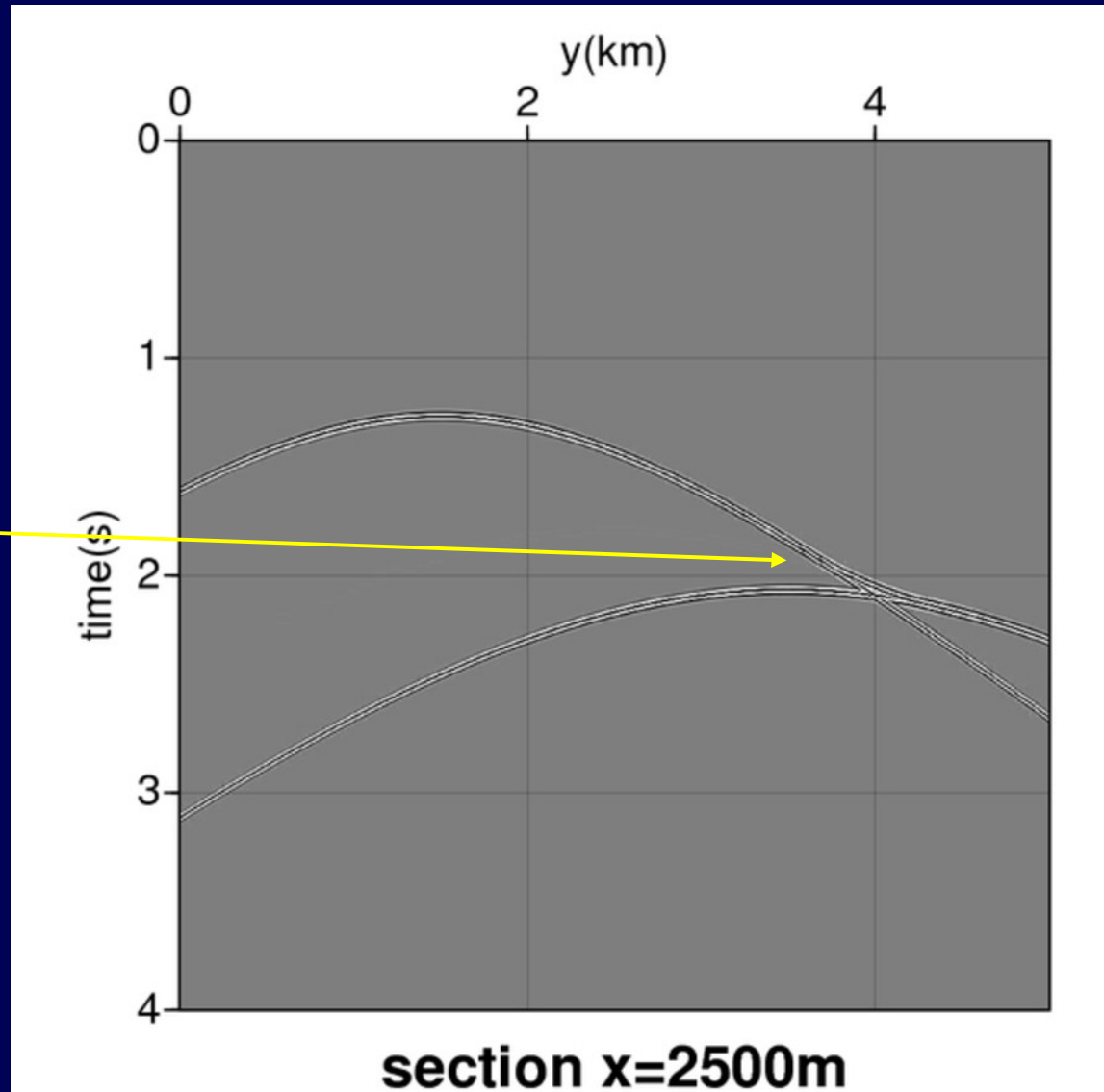
# Curved edge diffractions – synthetic – dipping ellipse



# Curved edge diffractions – synthetic – dipping ellipse

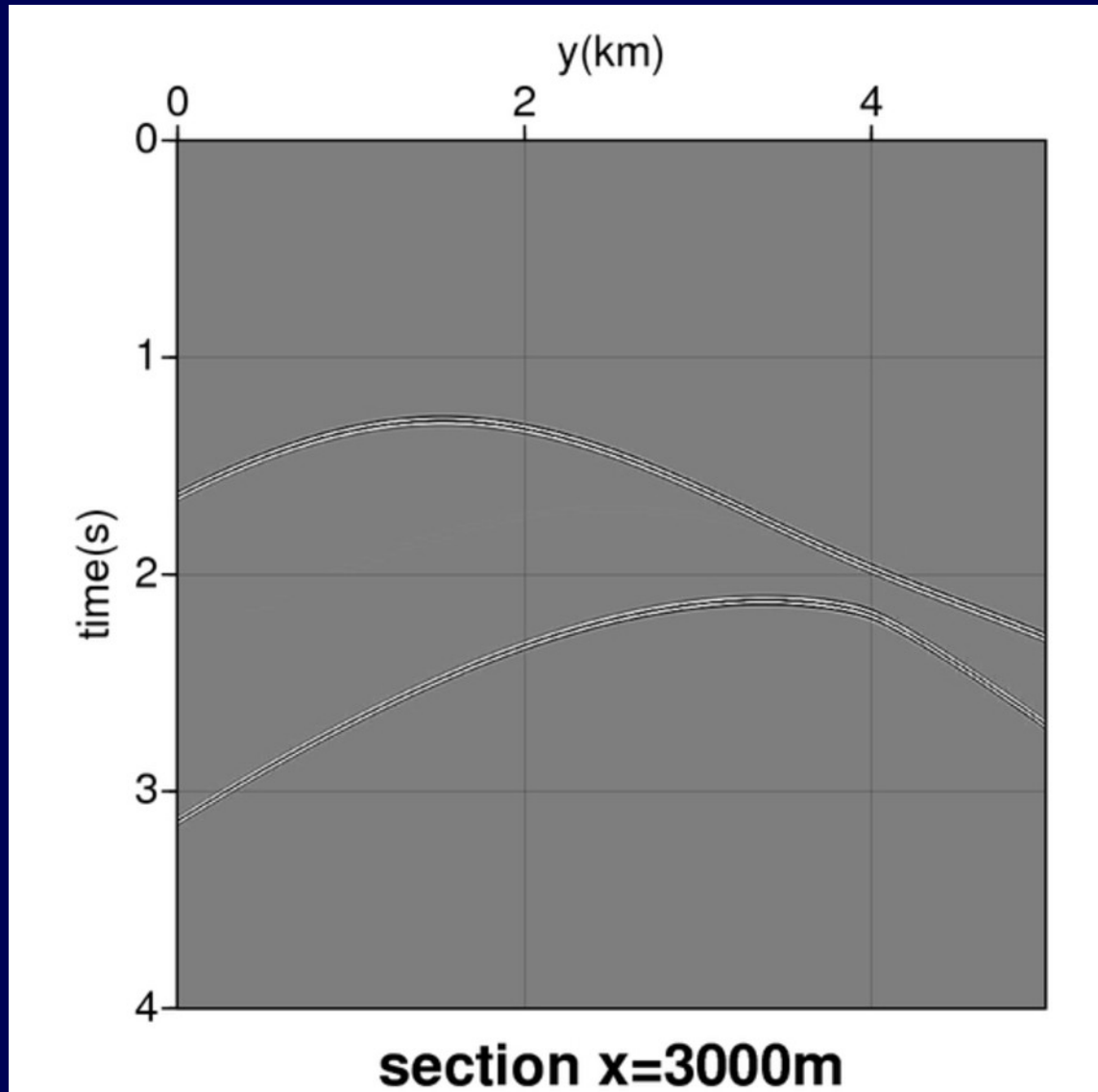


# Curved edge diffractions – synthetic – dipping ellipse

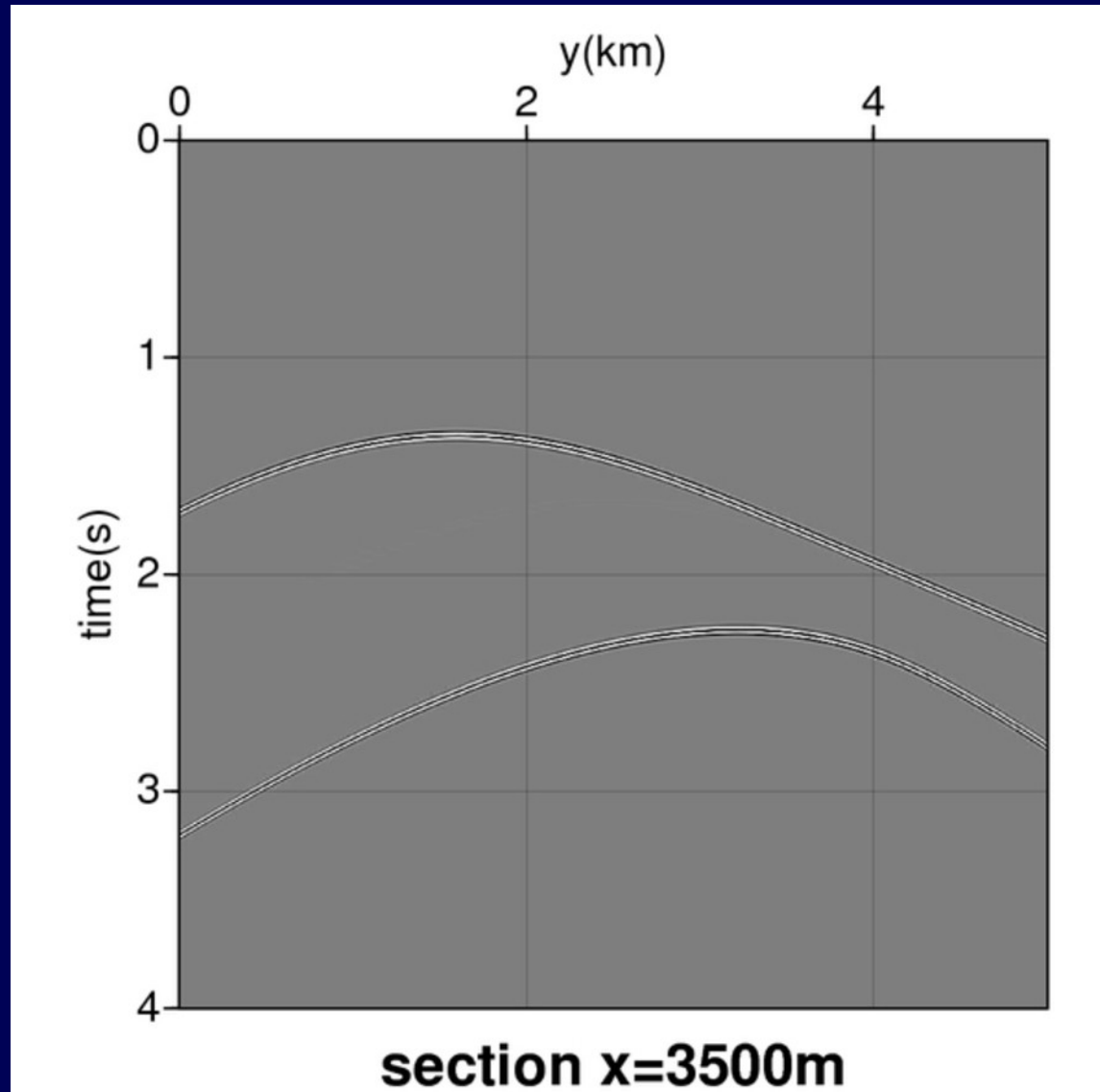


**Triplicated  
branch +  
Caustic**

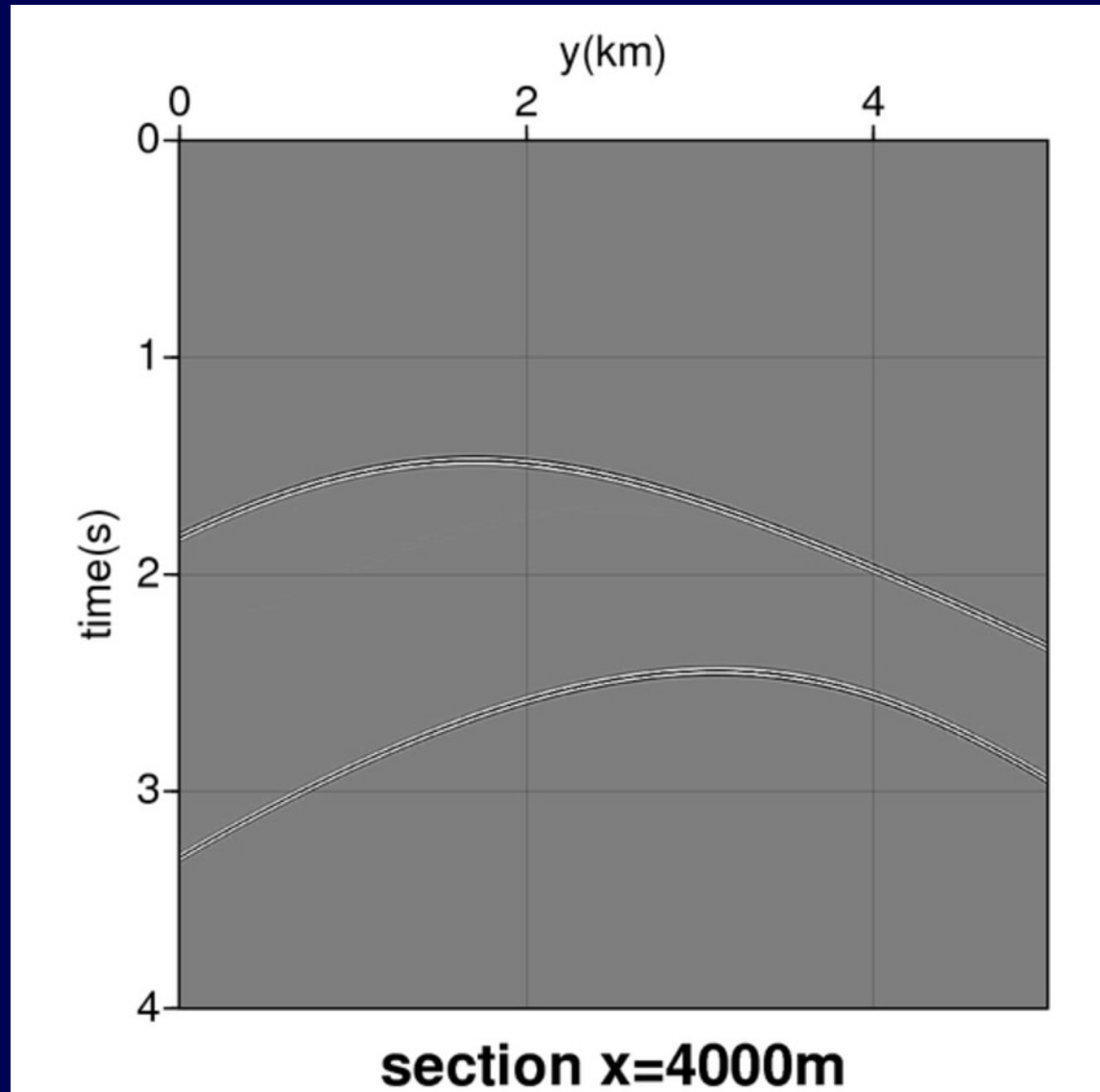
# Curved edge diffractions – synthetic – dipping ellipse



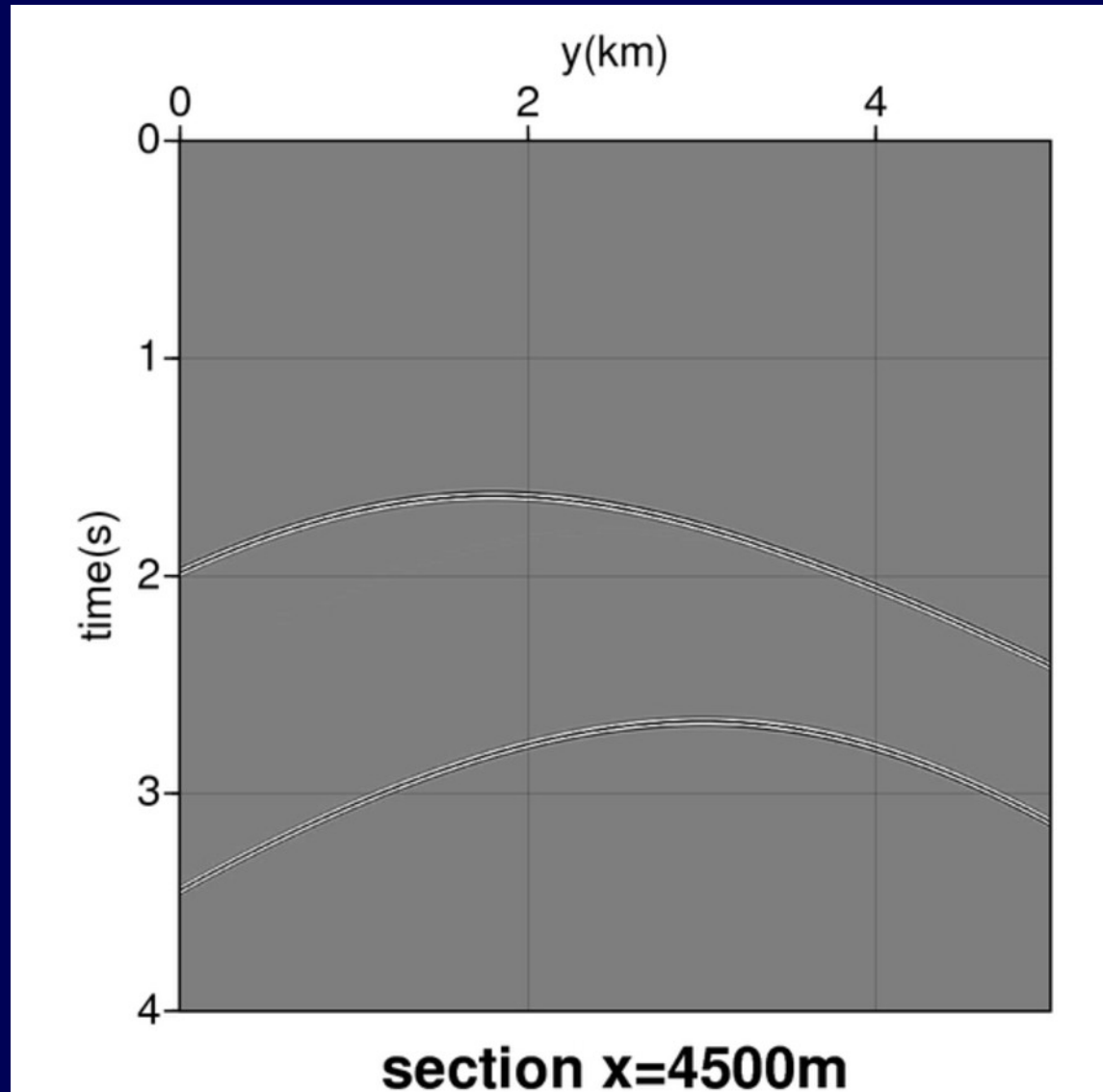
# Curved edge diffractions – synthetic – dipping ellipse



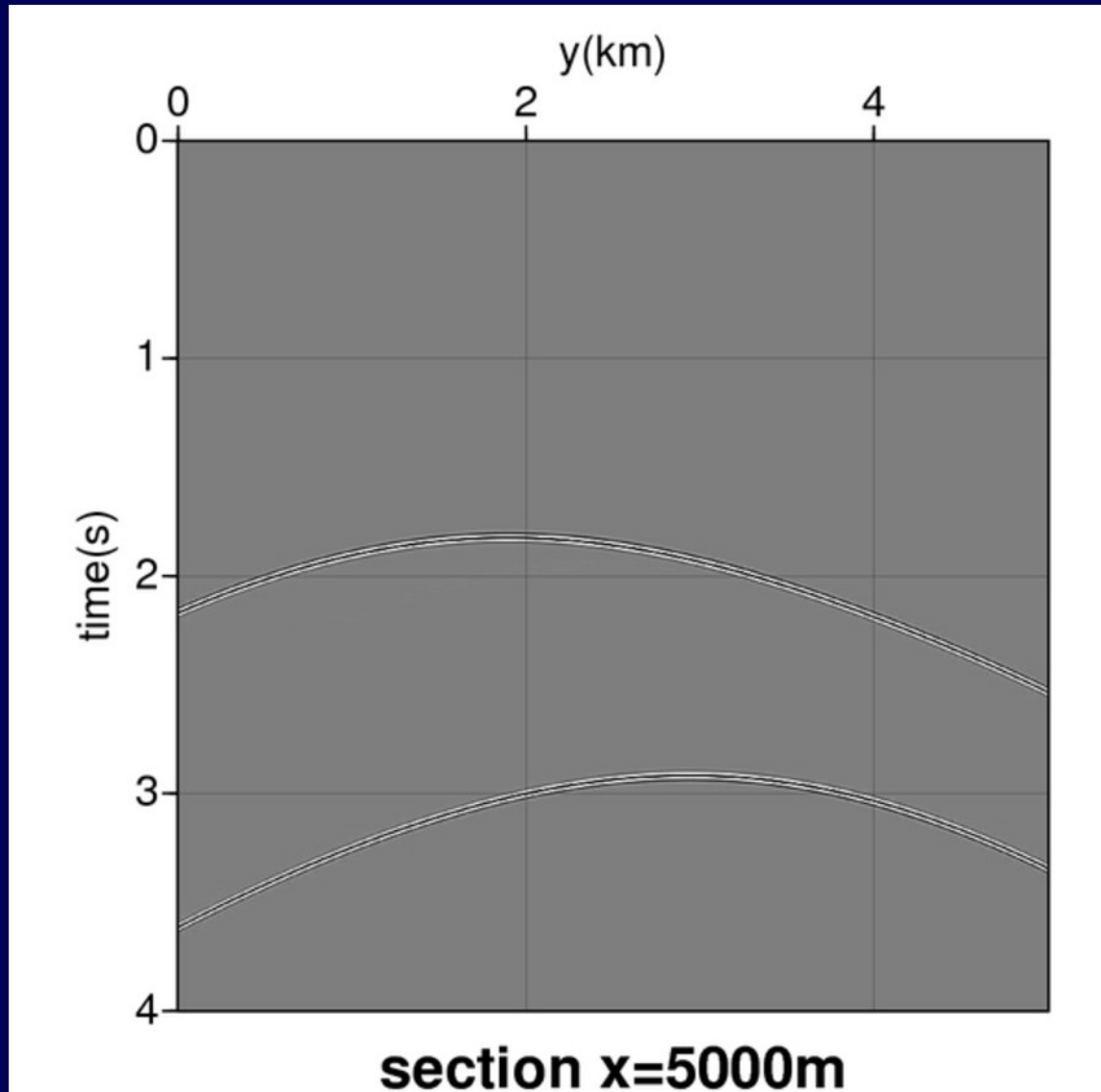
# Curved edge diffractions – synthetic – dipping ellipse



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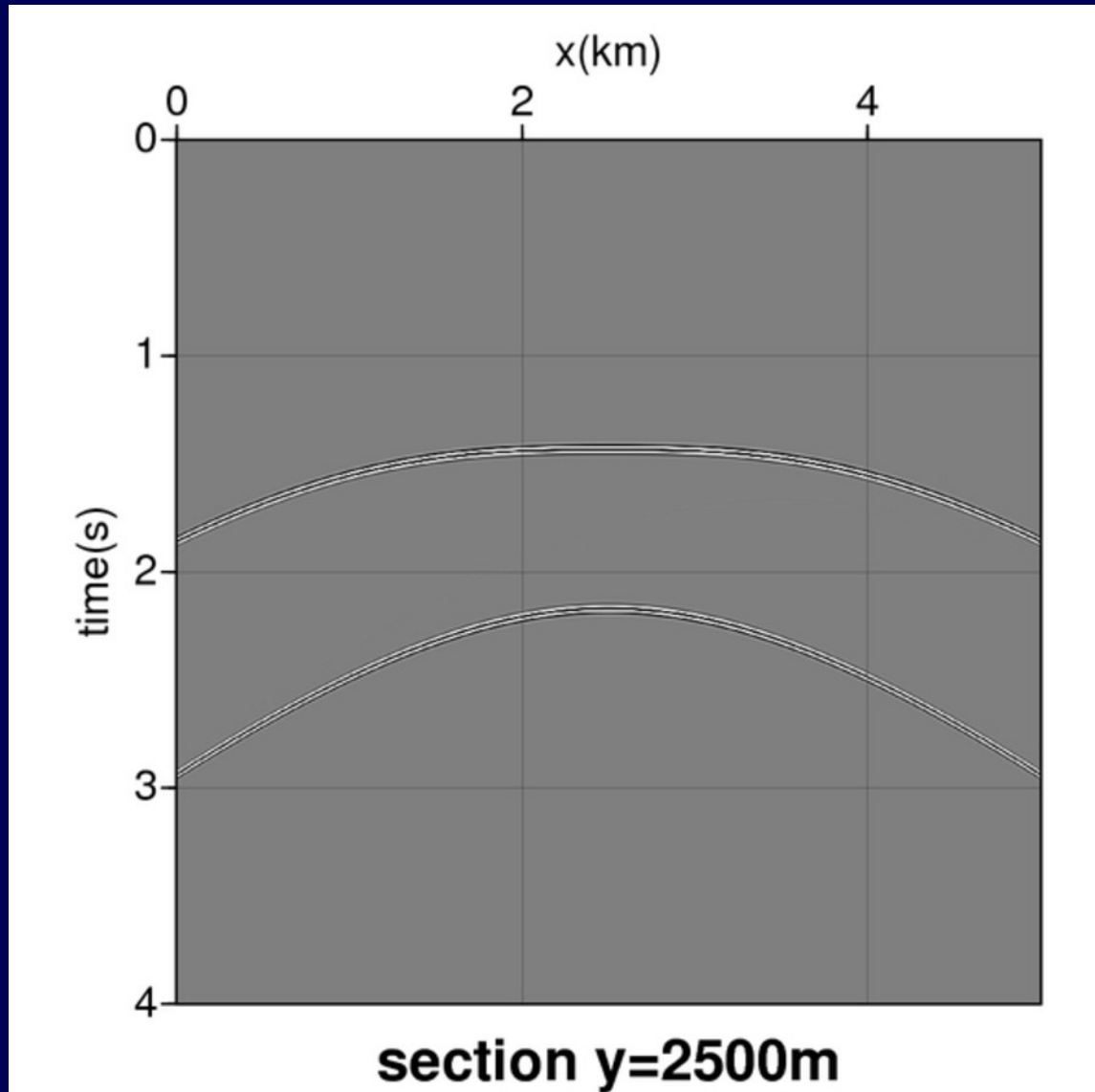


# Curved edge diffractions – synthetic – dipping ellipse

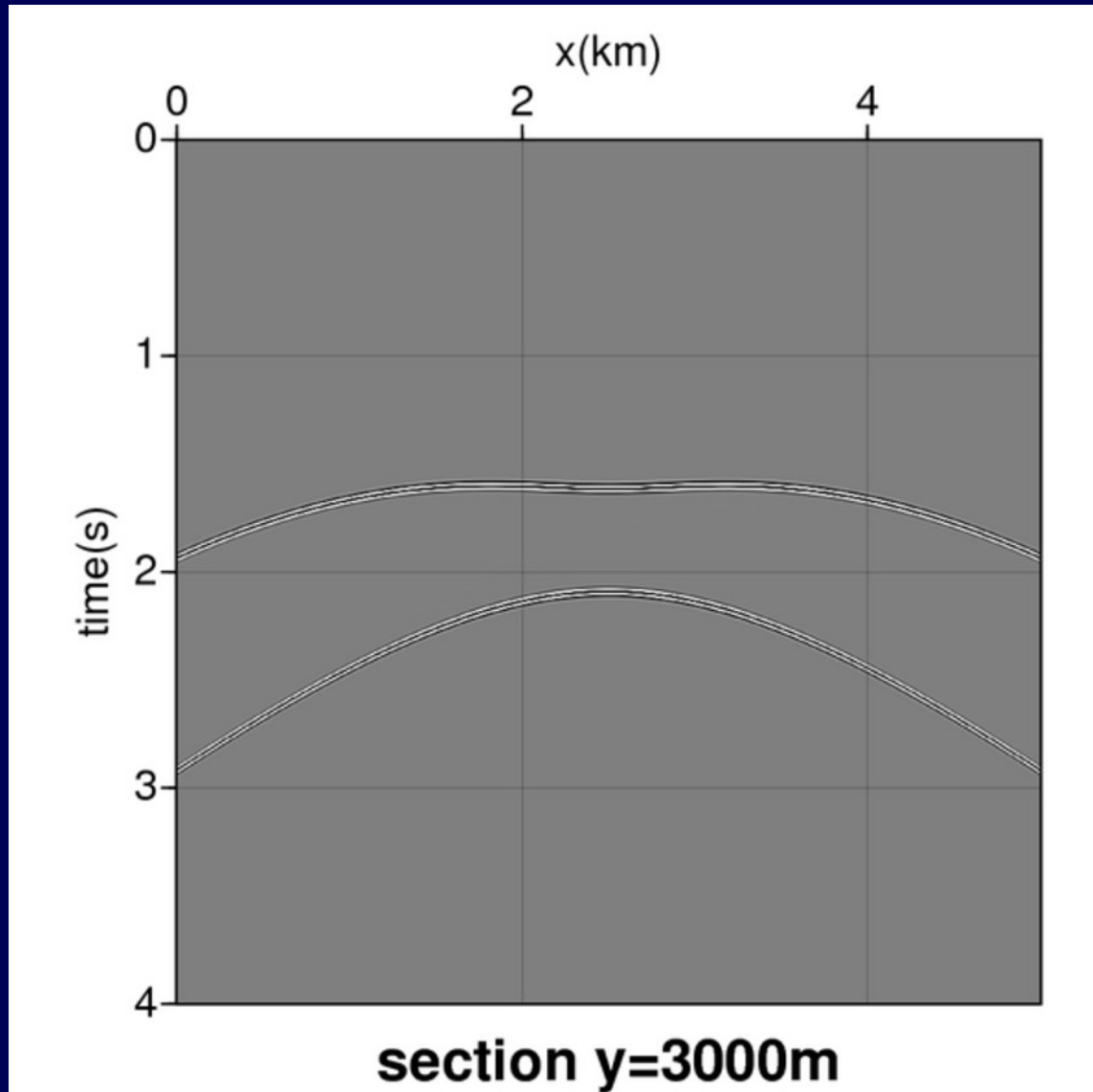




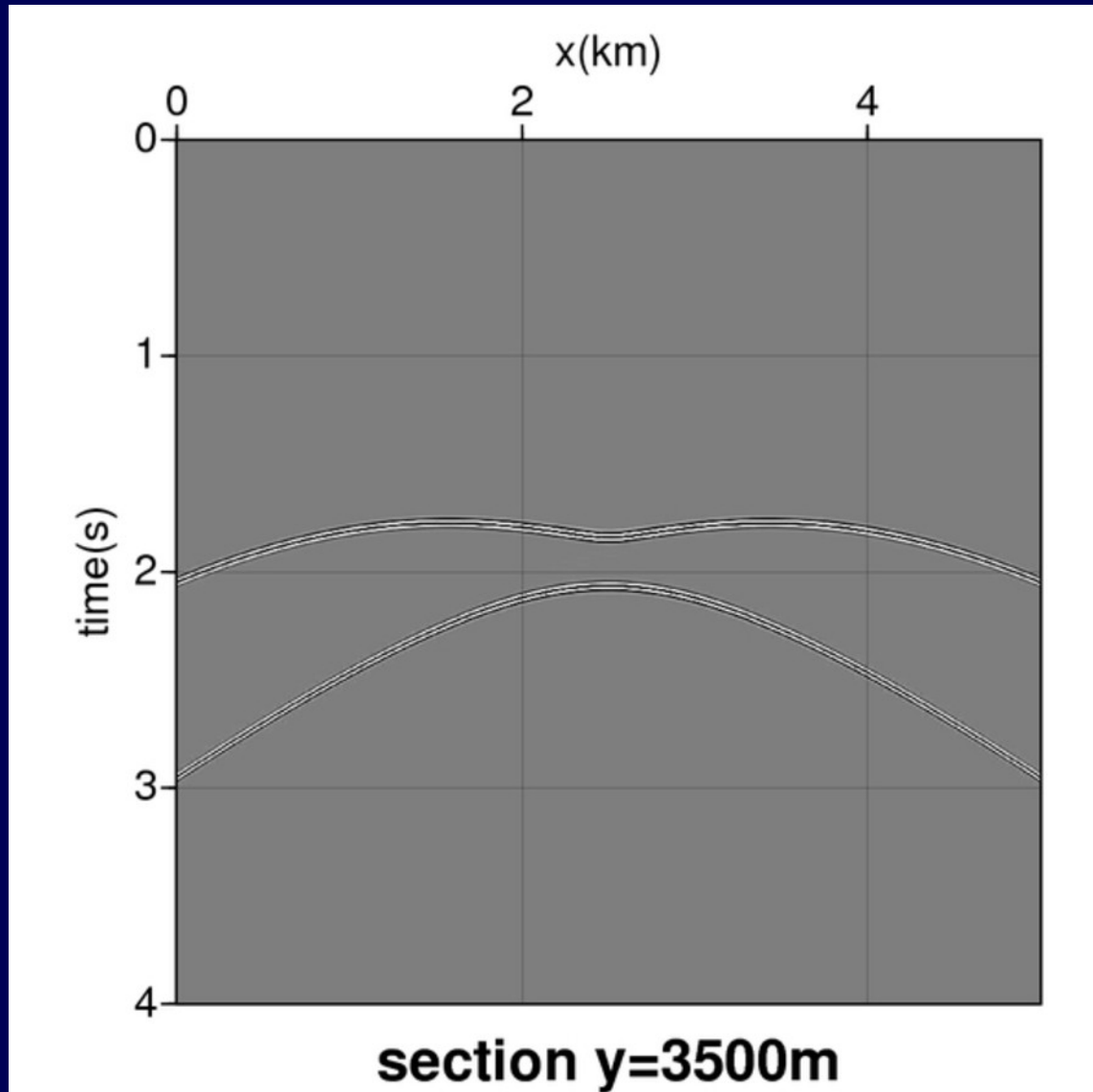
# Curved edge diffractions – synthetic – dipping ellipse



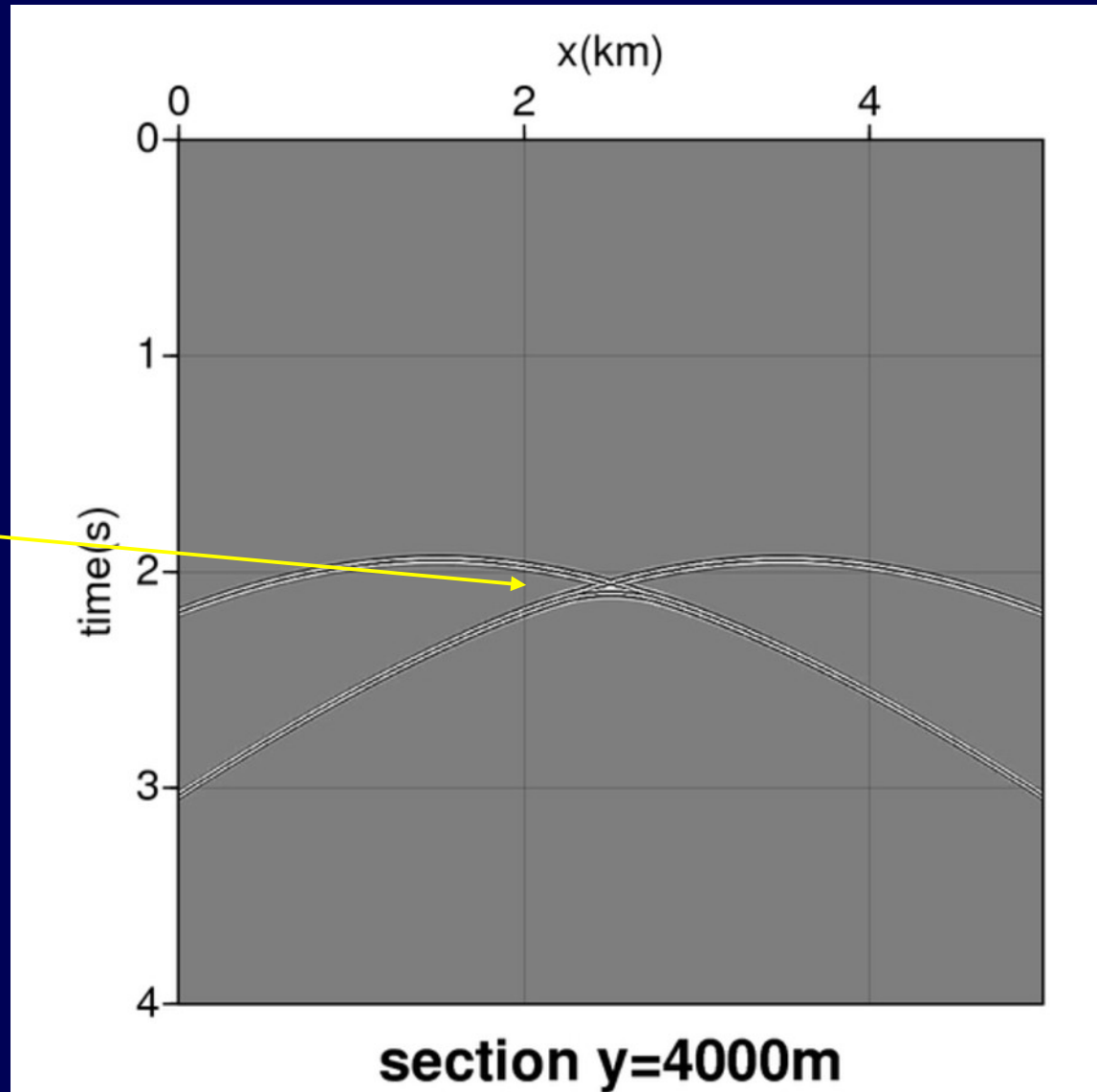
# Curved edge diffractions – synthetic – dipping ellipse



# Curved edge diffractions – synthetic – dipping ellipse

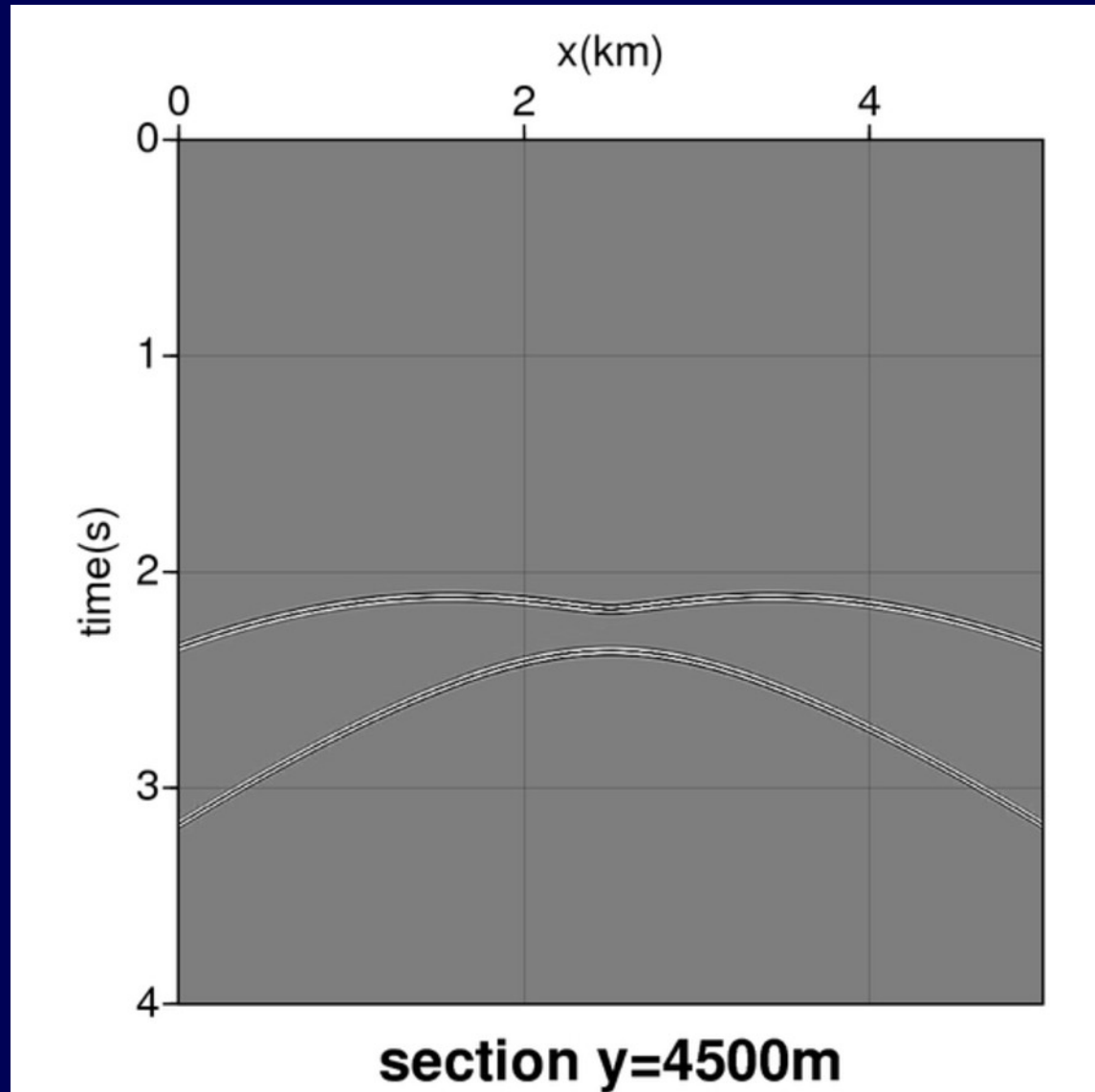


# Curved edge diffractions – synthetic – dipping ellipse

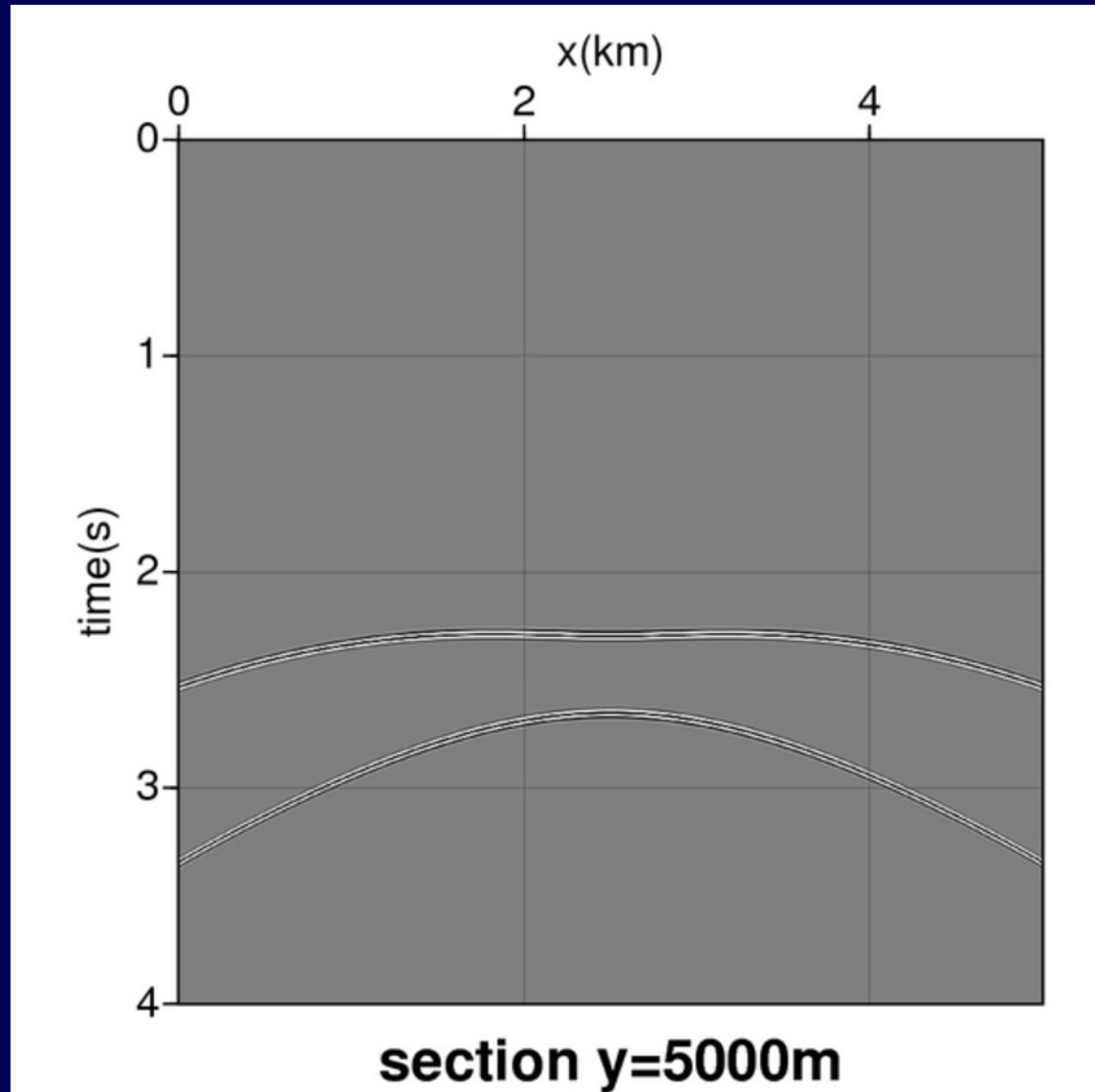


**Triplicated  
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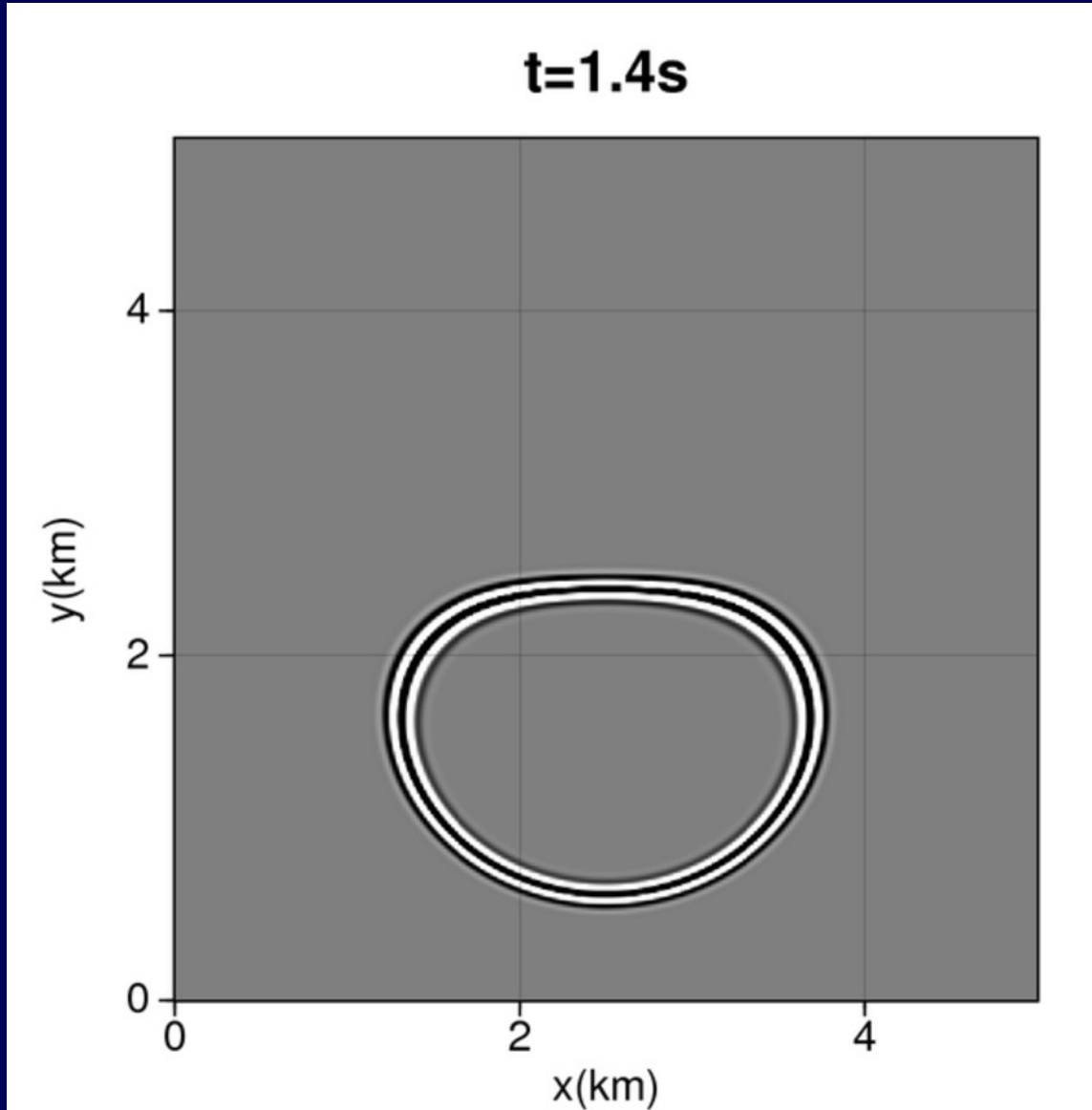
# Curved edge diffractions – synthetic – dipping ellipse



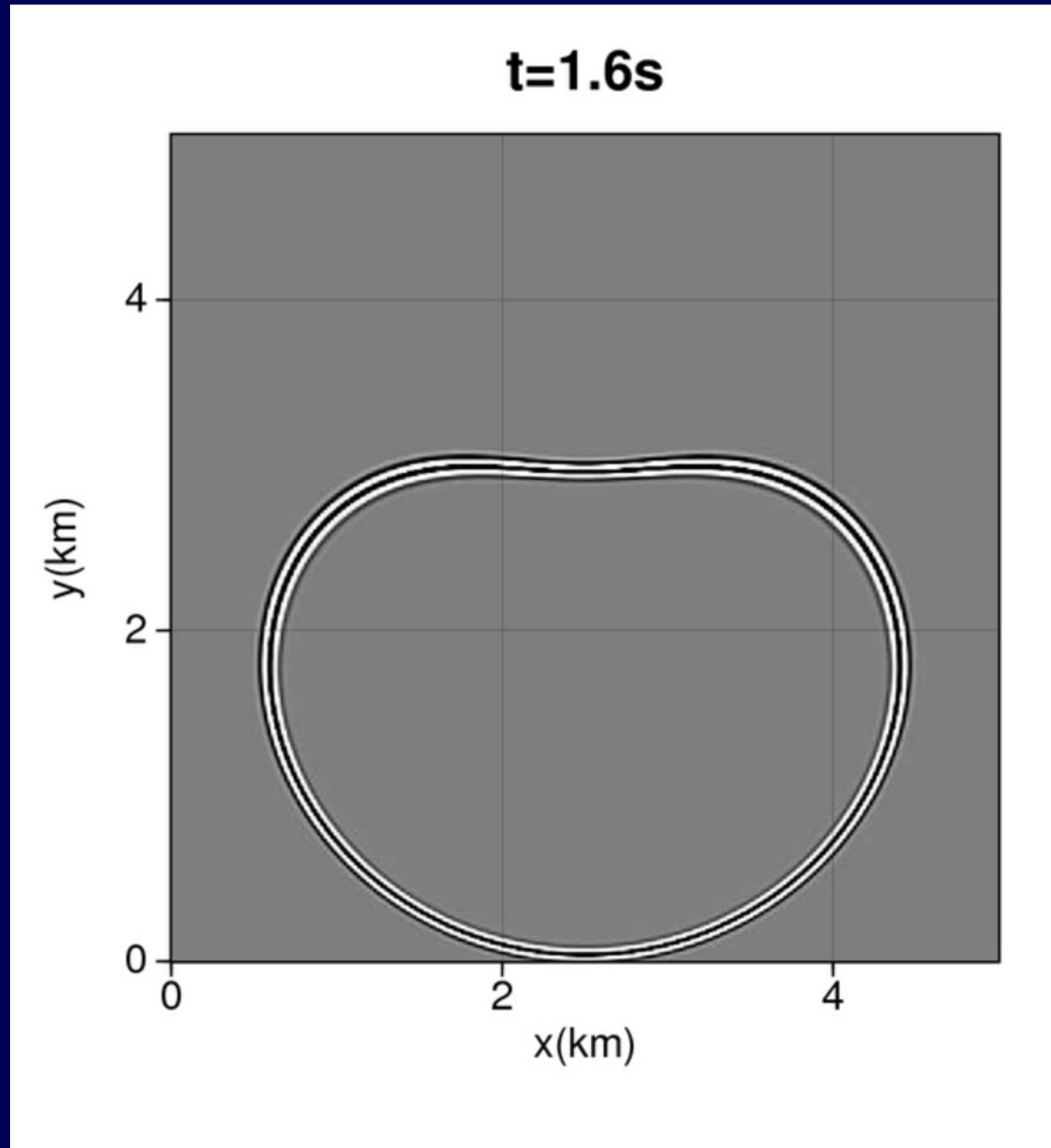
# Curved edge diffractions – synthetic – dipping ellipse



# Curved edge diffractions – synthetic – dipping ellipse

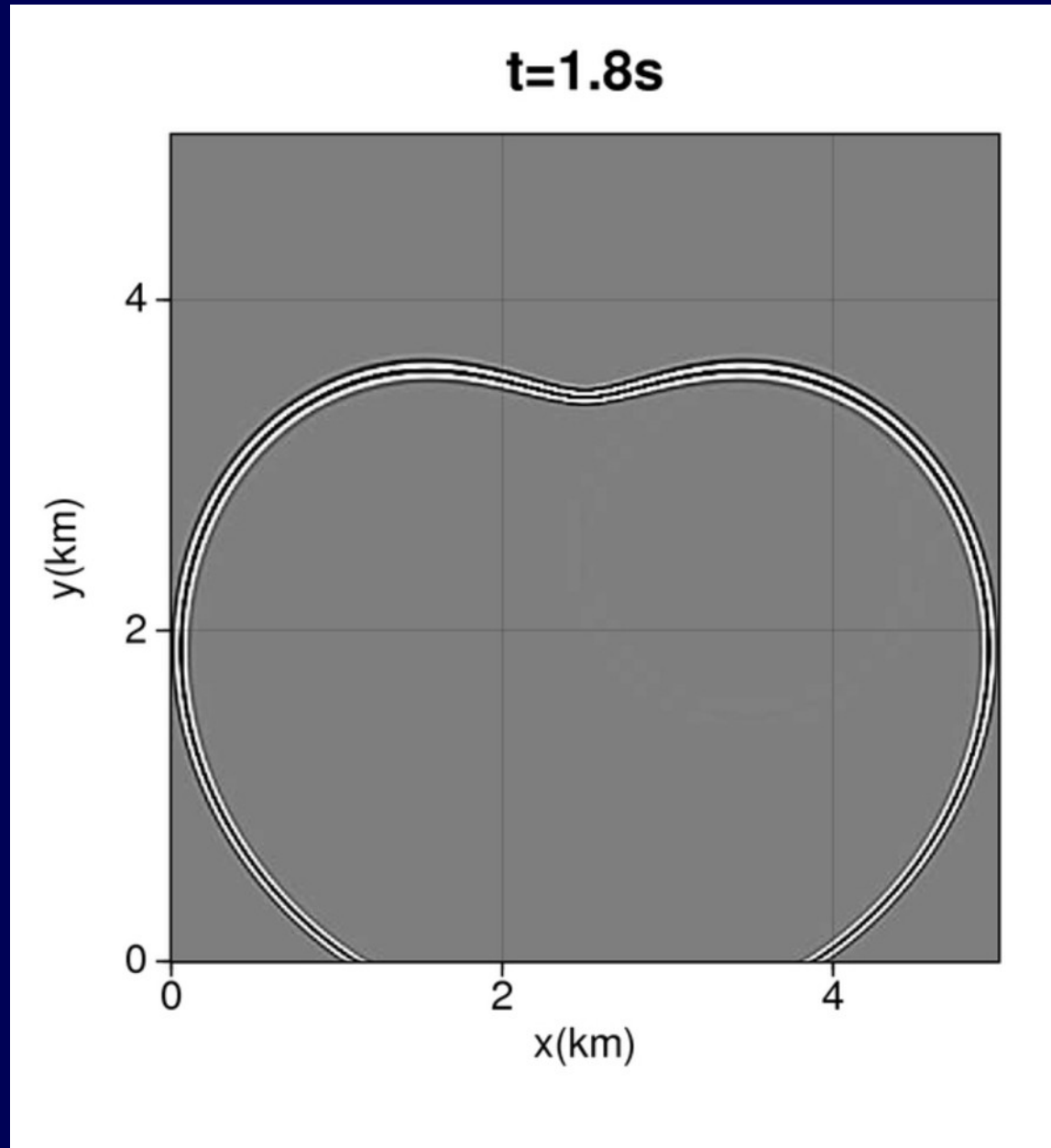


# Curved edge diffractions – synthetic – dipping ellipse

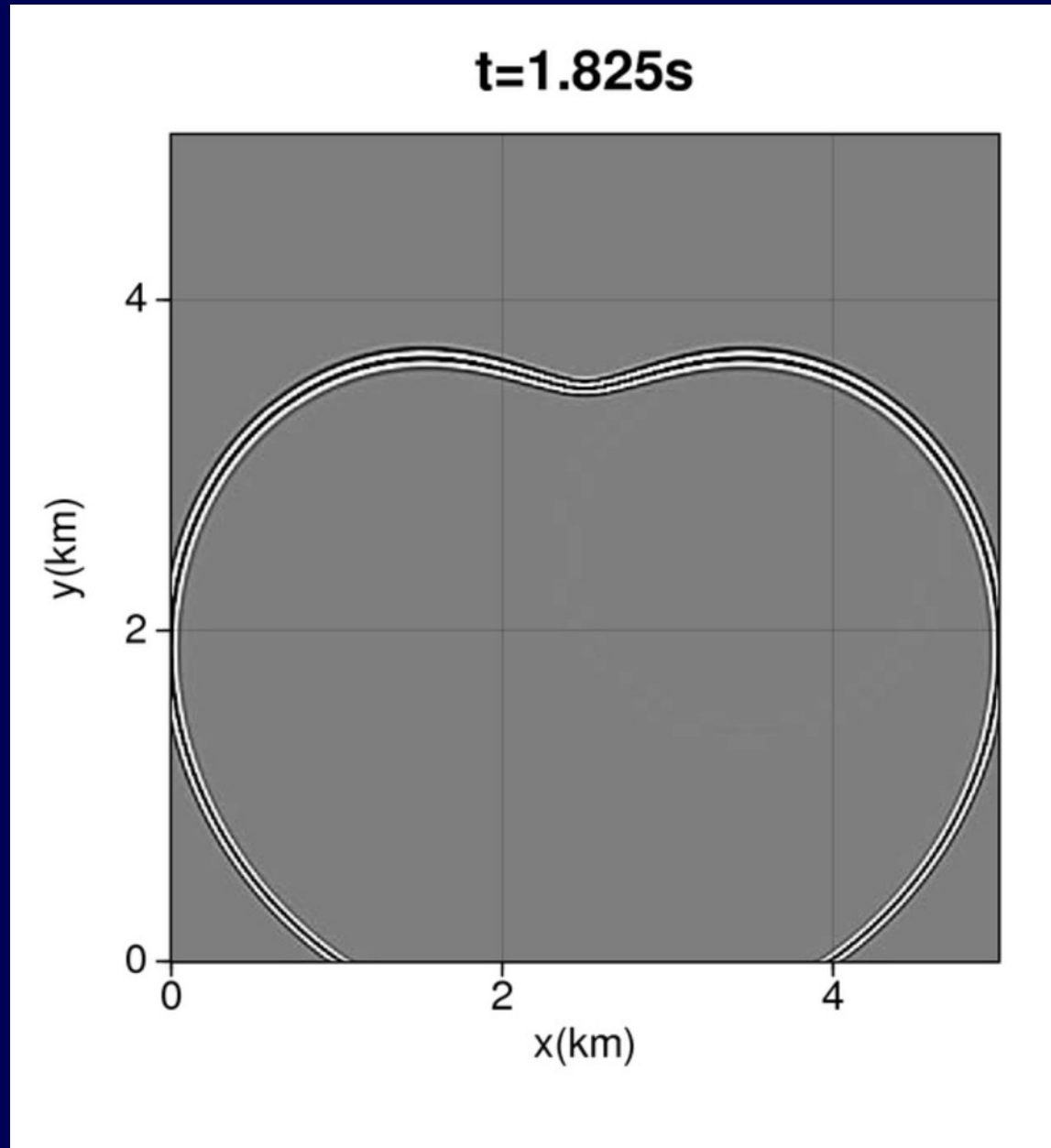




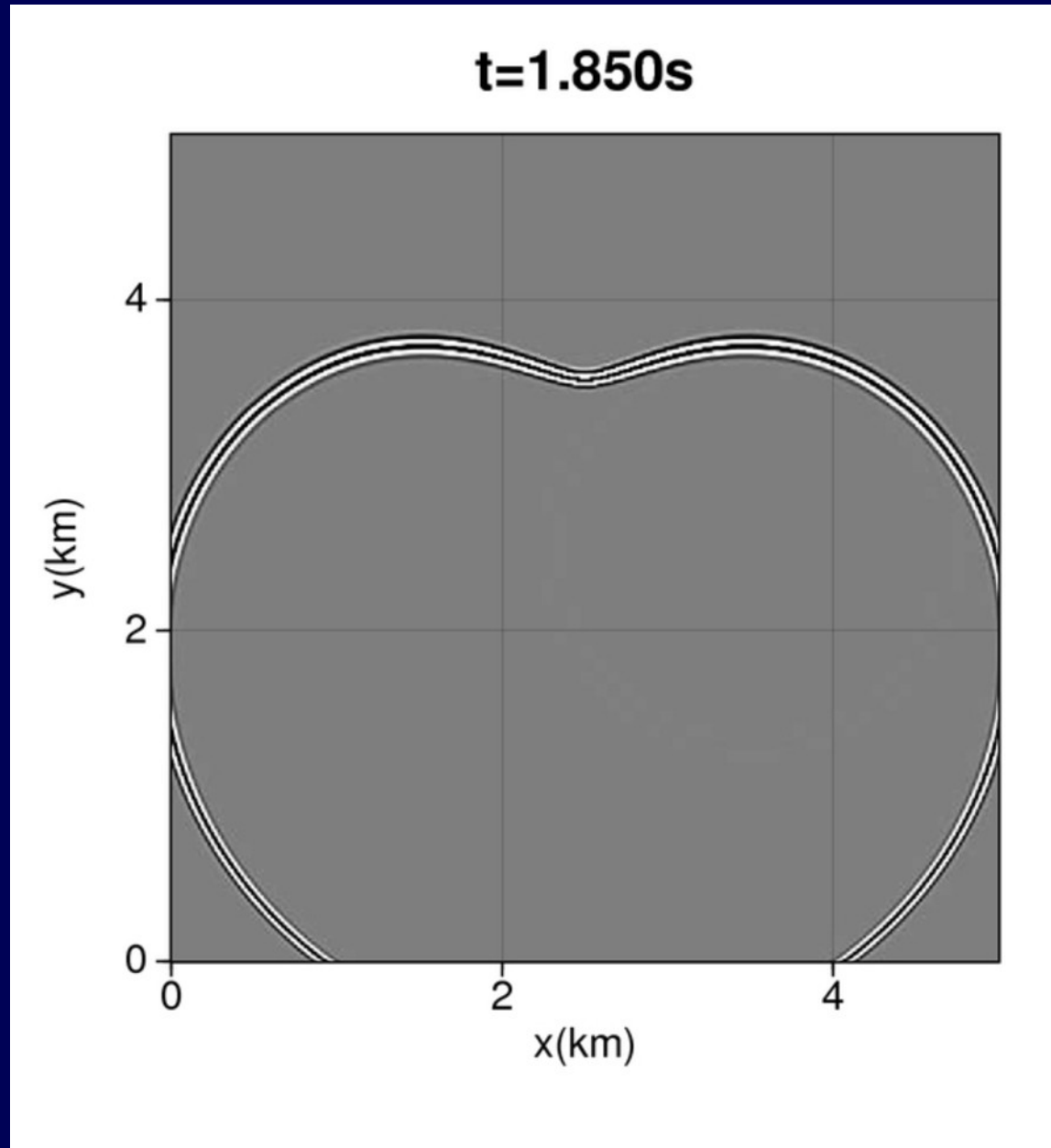
# Curved edge diffractions – synthetic – dipping ellipse



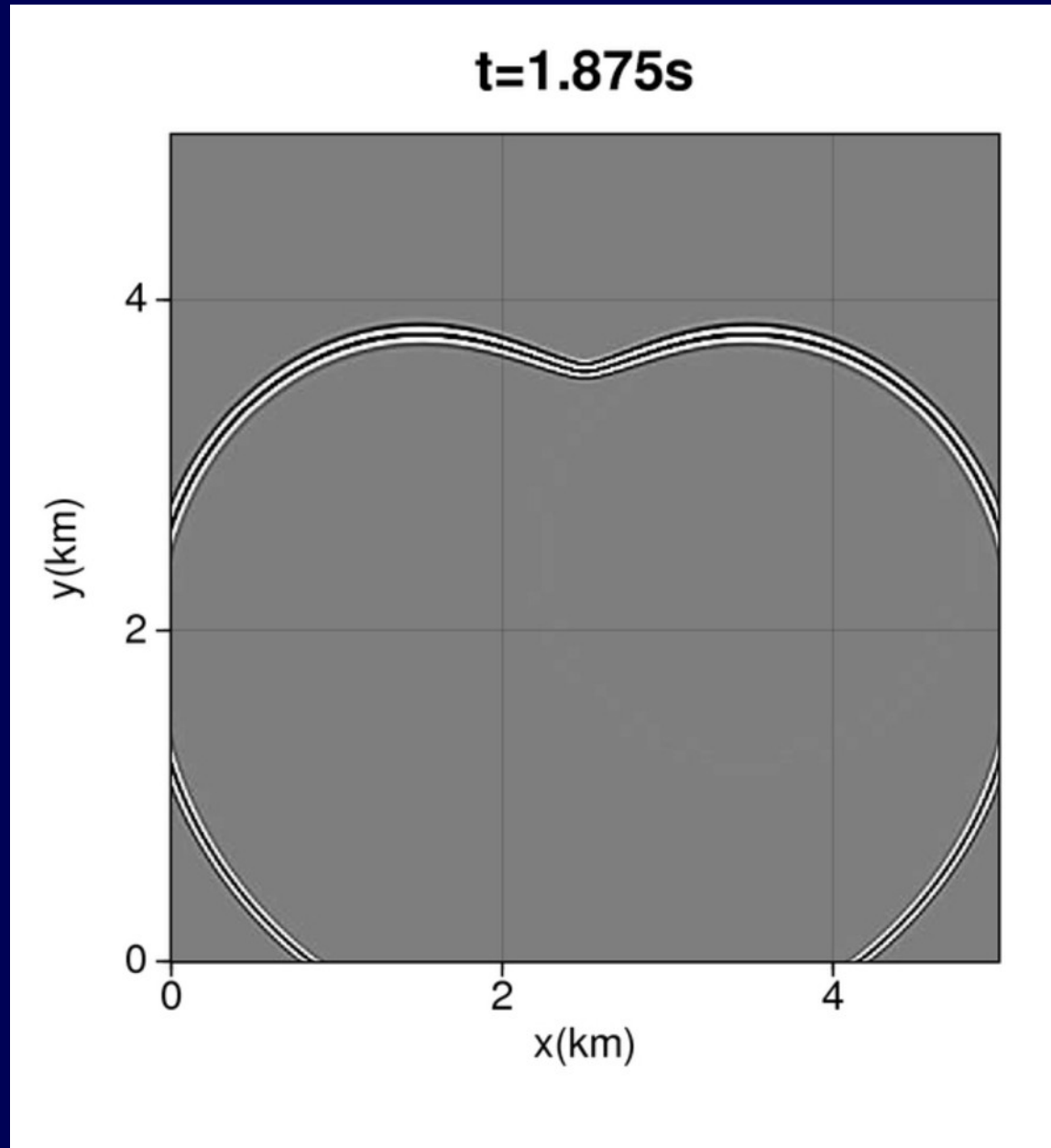
# Curved edge diffractions – synthetic – dipping ellipse



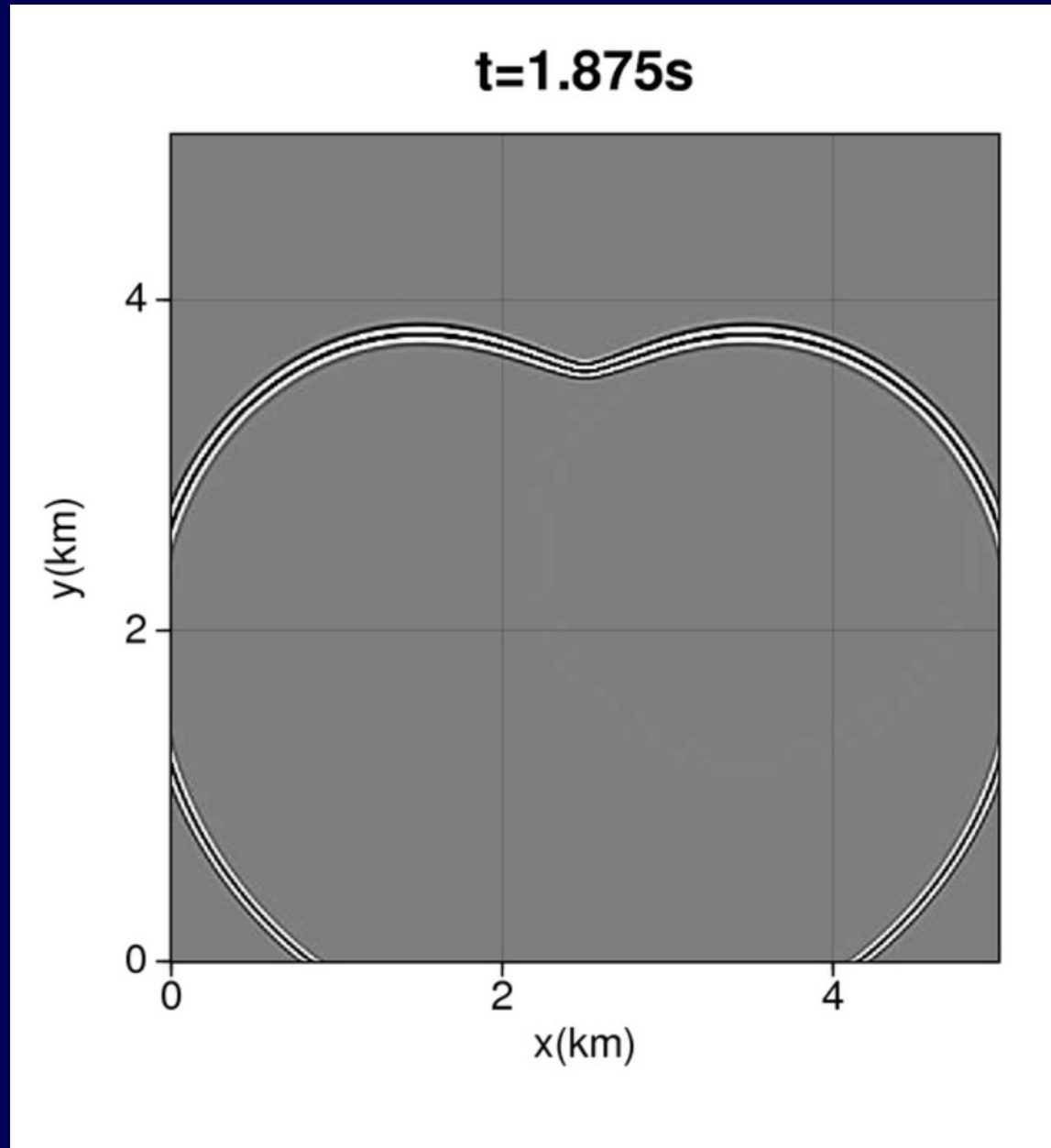
# Curved edge diffractions – synthetic – dipping ellipse



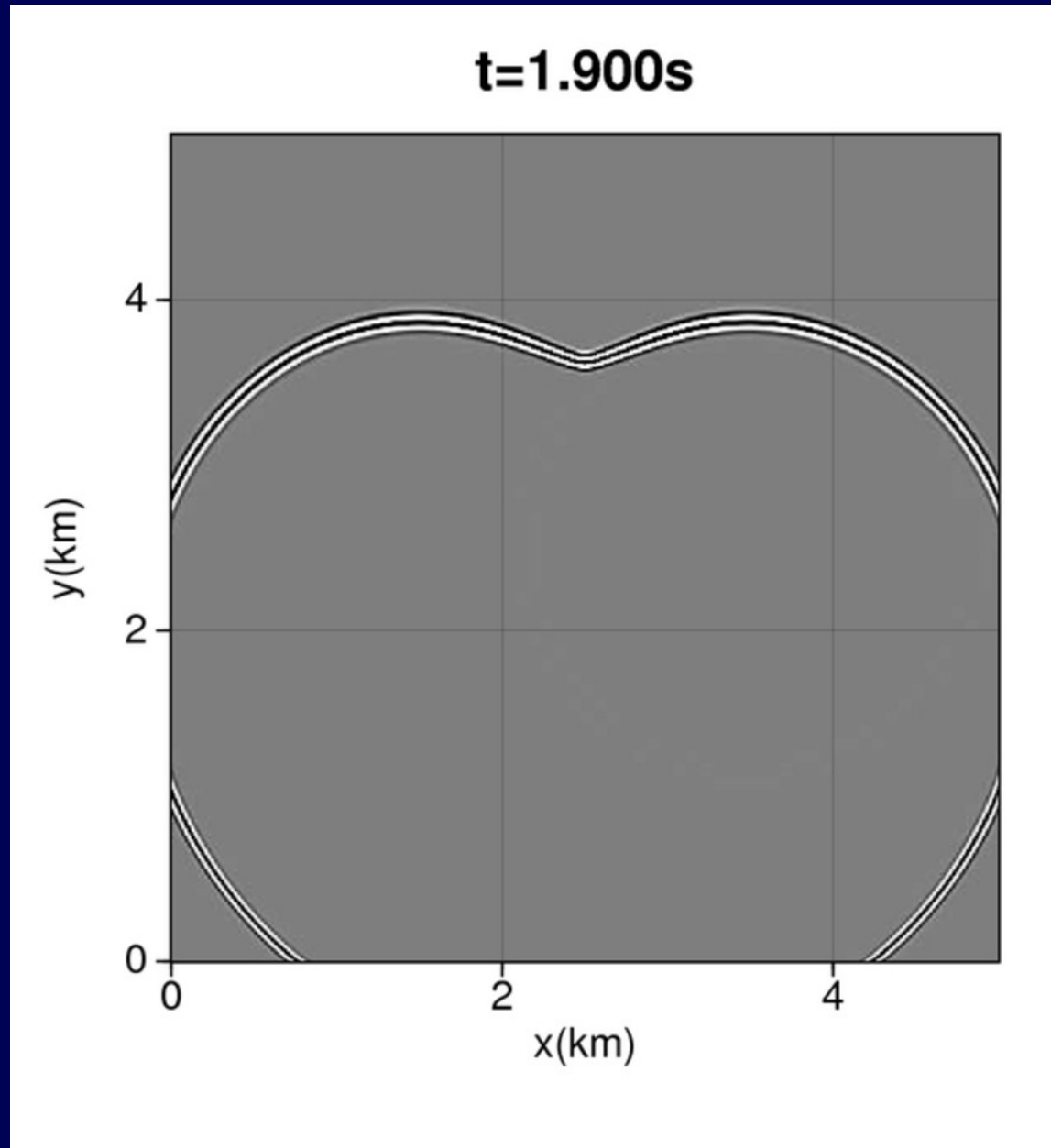
# Curved edge diffractions – synthetic – dipping ellipse



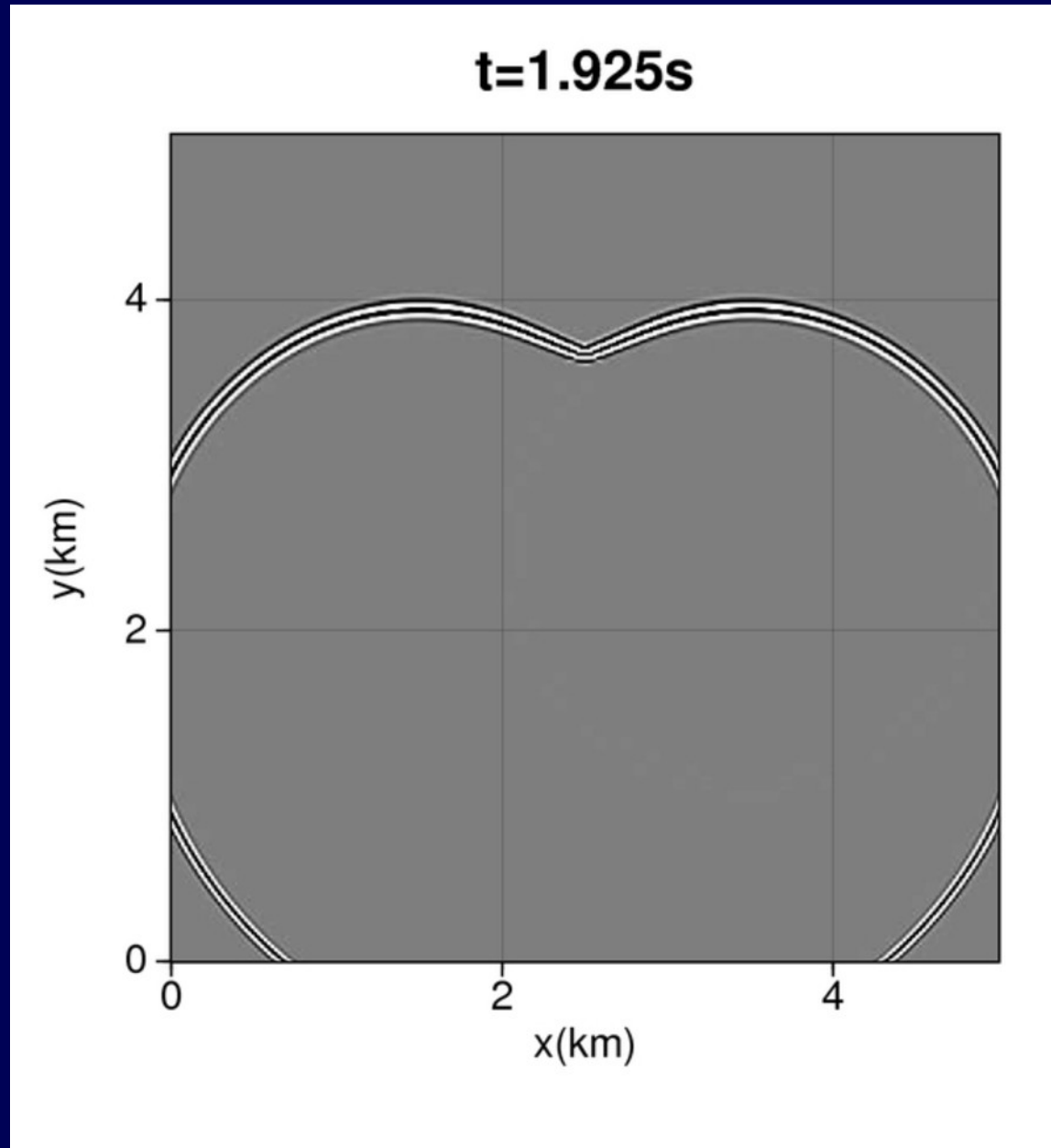
# Curved edge diffractions – synthetic – dipping ellipse



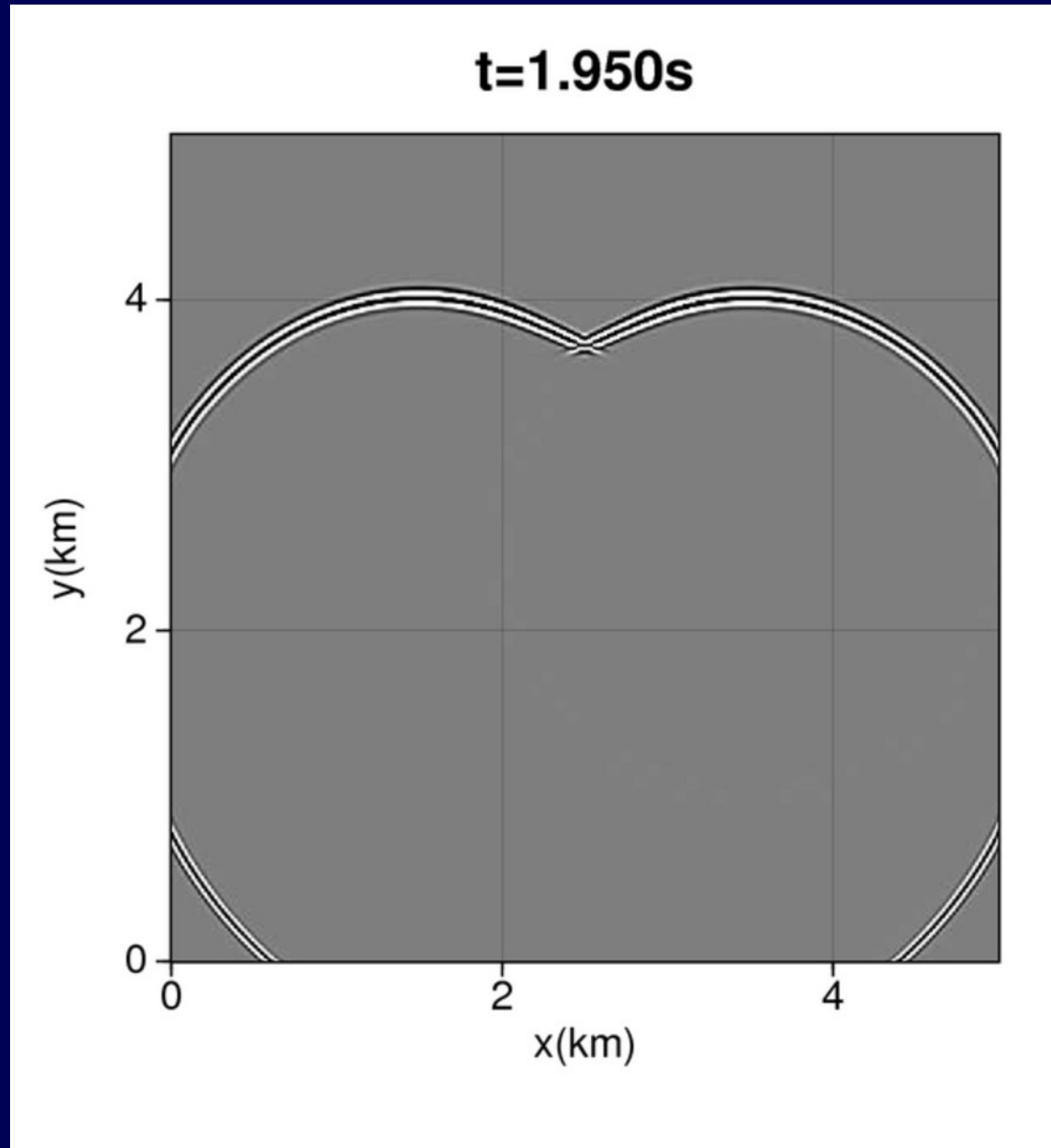
# Curved edge diffractions – synthetic – dipping ellipse



# Curved edge diffractions – synthetic – dipping ellipse

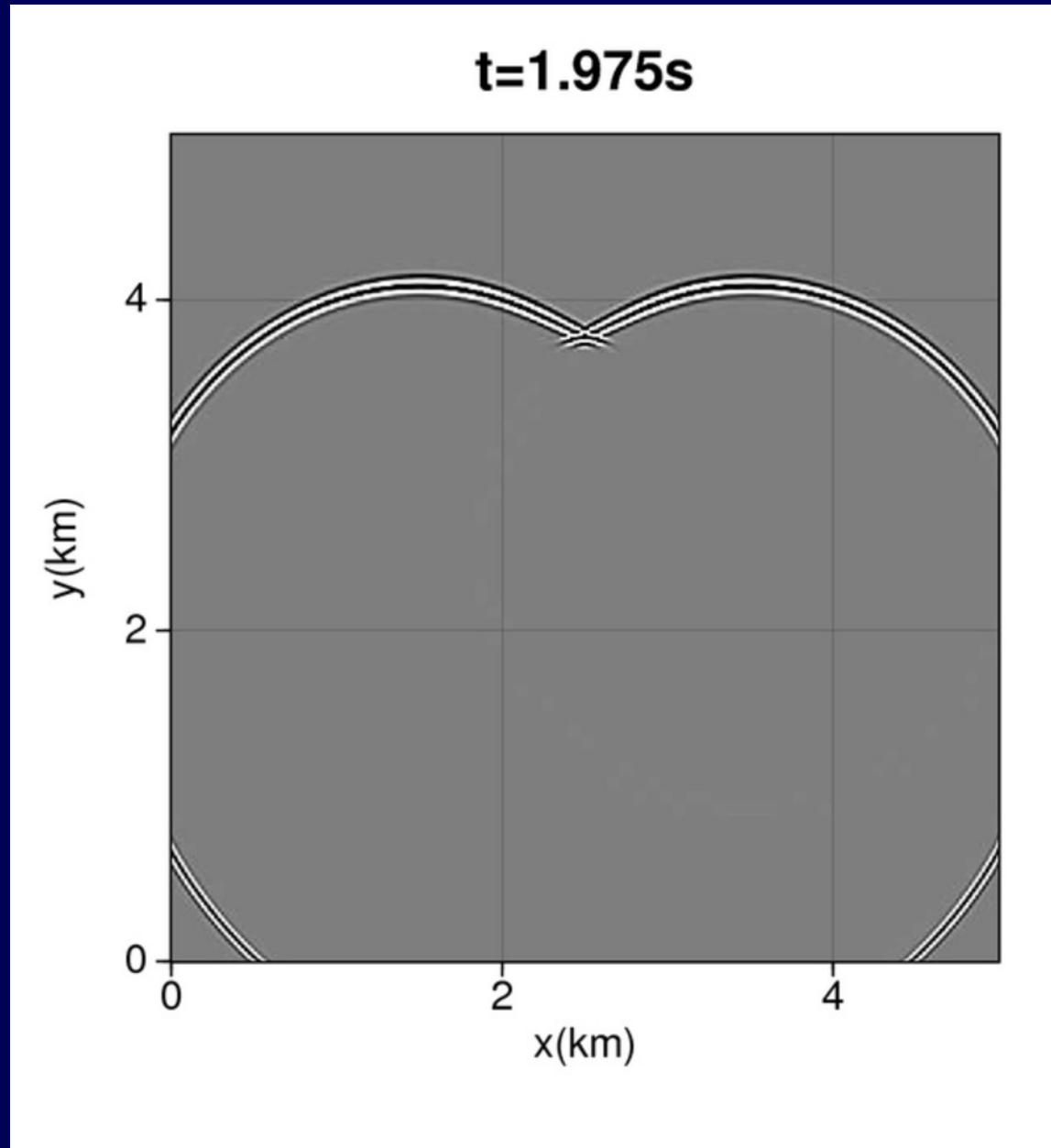


# Curved edge diffractions – synthetic – dipping ellipse

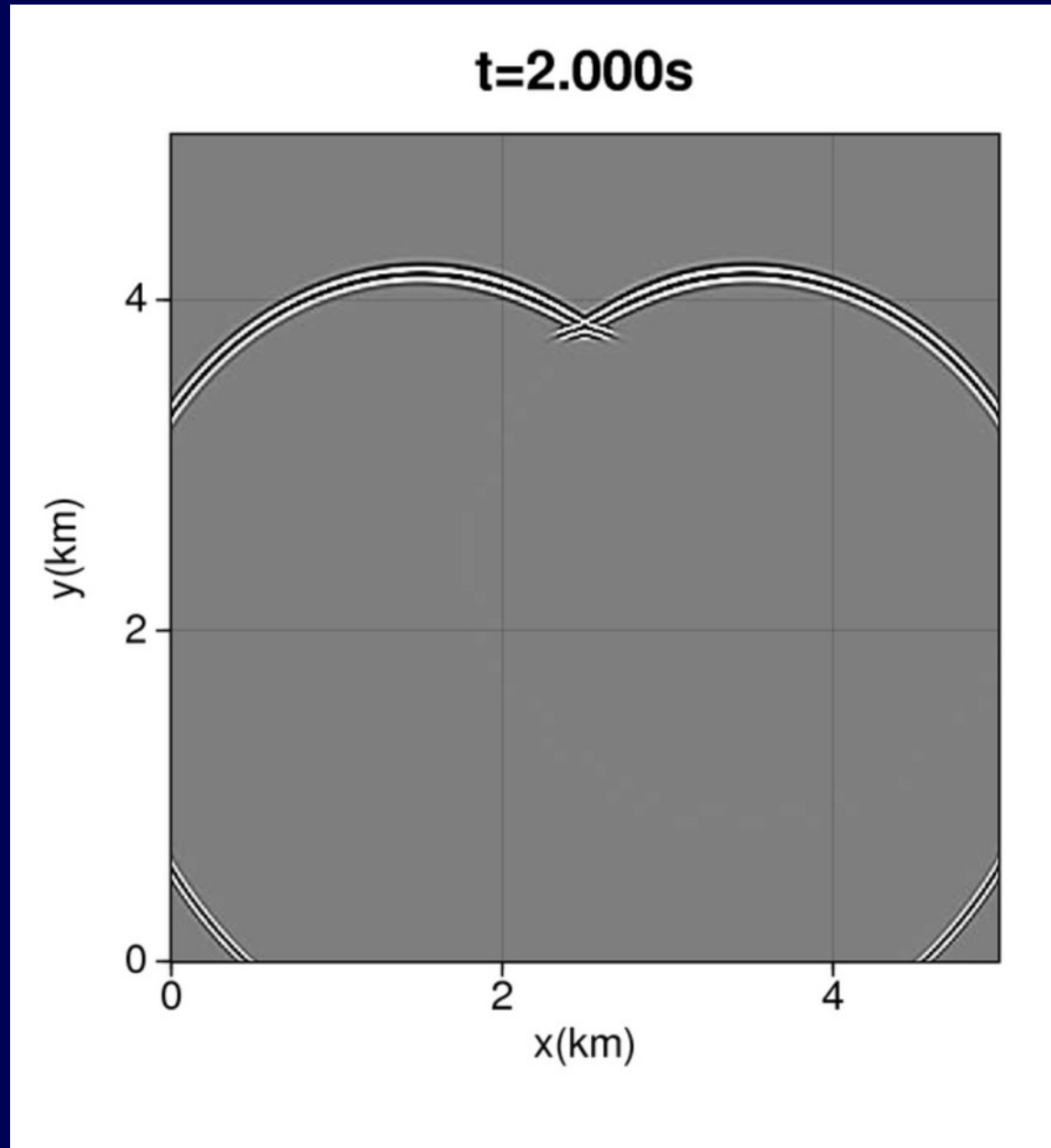




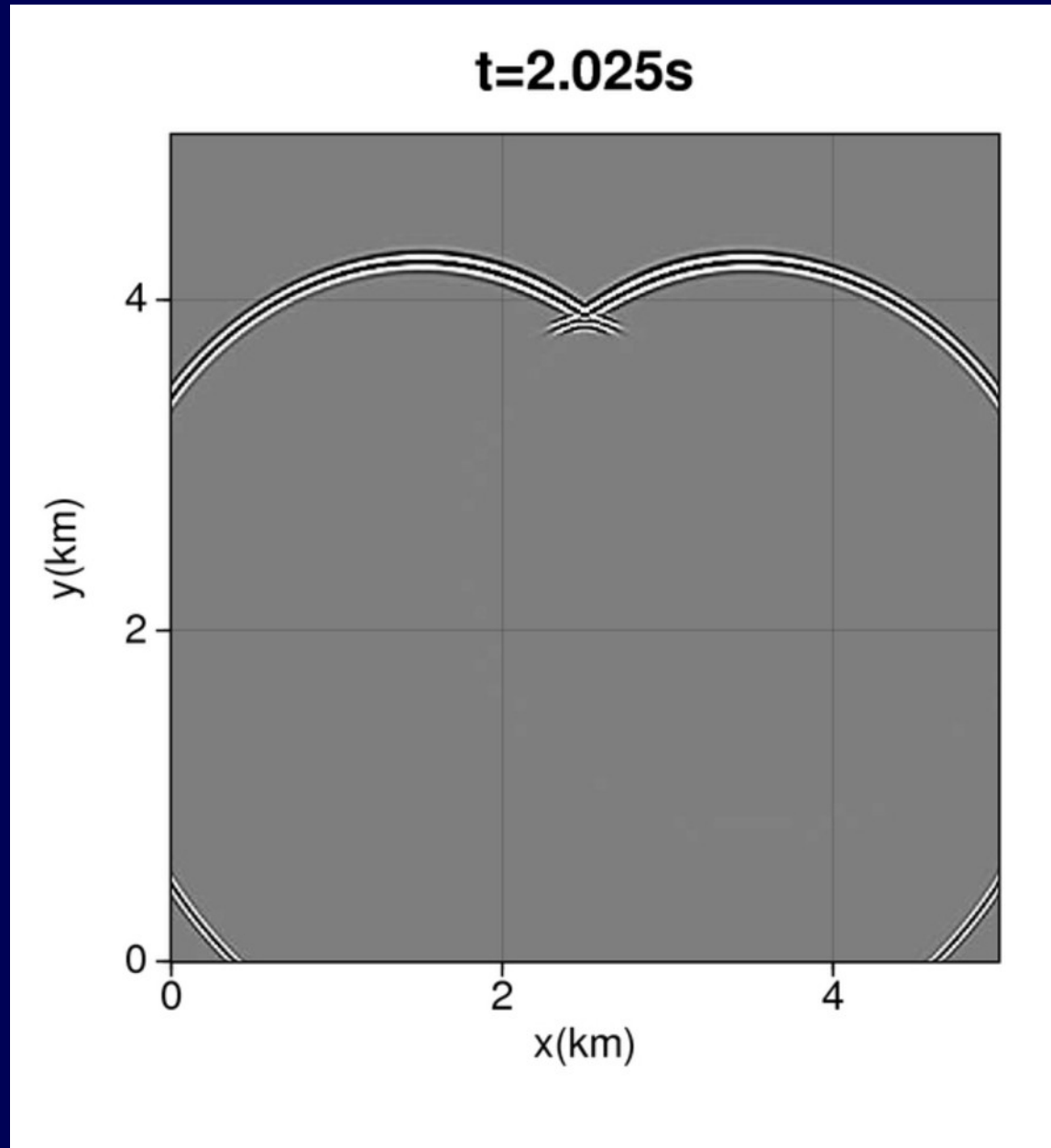
# Curved edge diffractions – synthetic – dipping ellipse



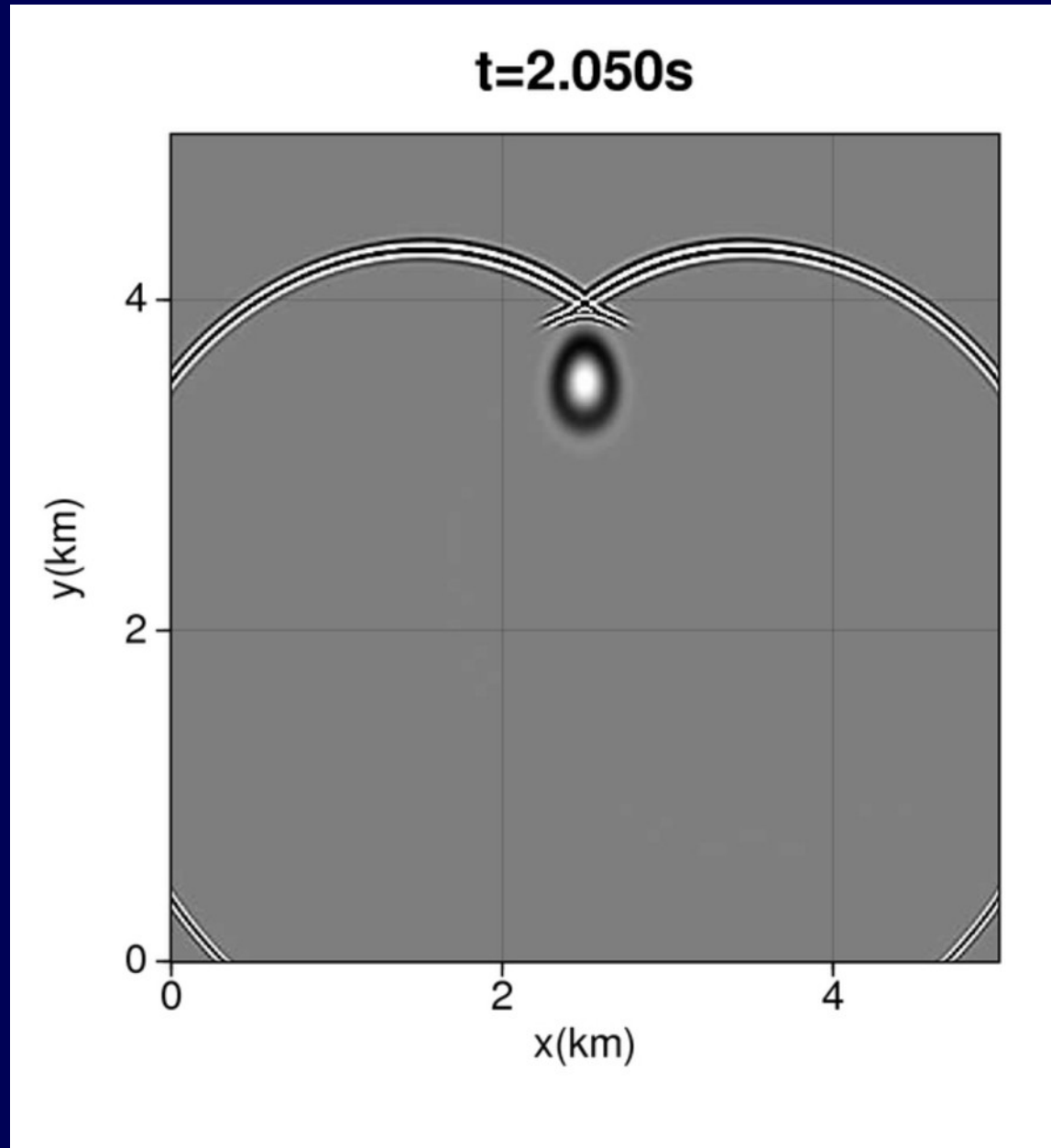
# Curved edge diffractions – synthetic – dipping ellipse



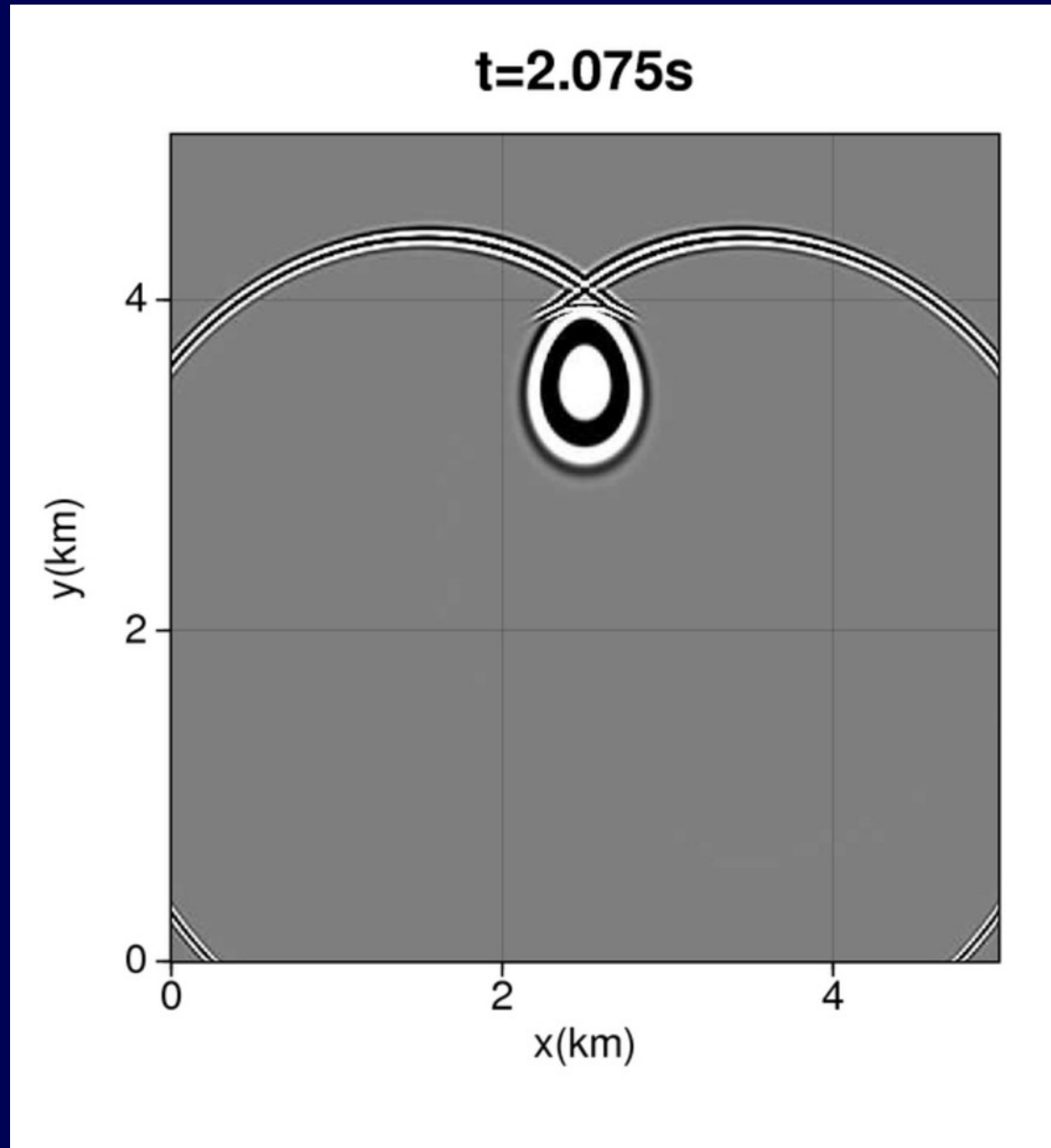
# Curved edge diffractions – synthetic – dipping ellipse



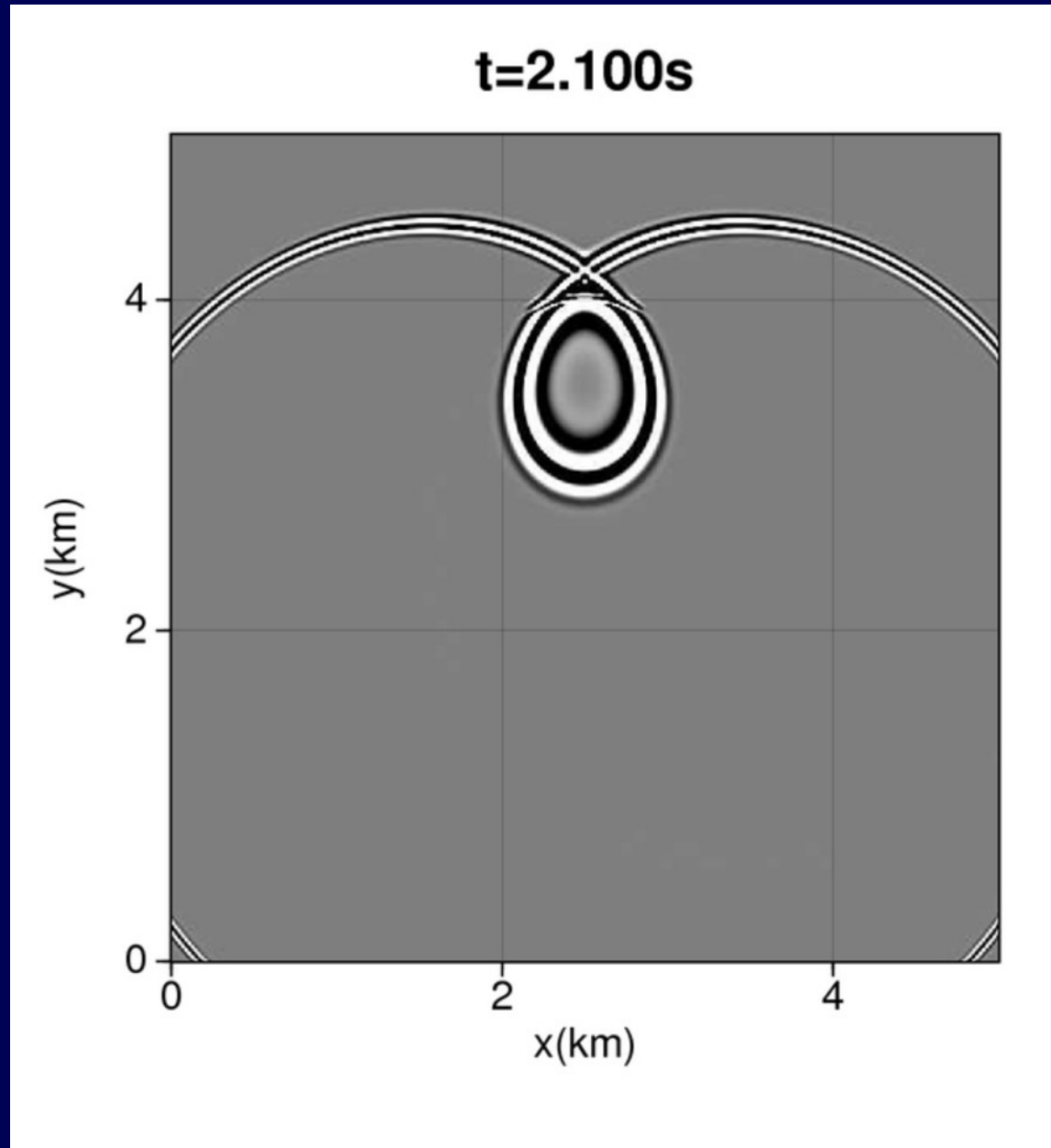
# Curved edge diffractions – synthetic – dipping ellipse



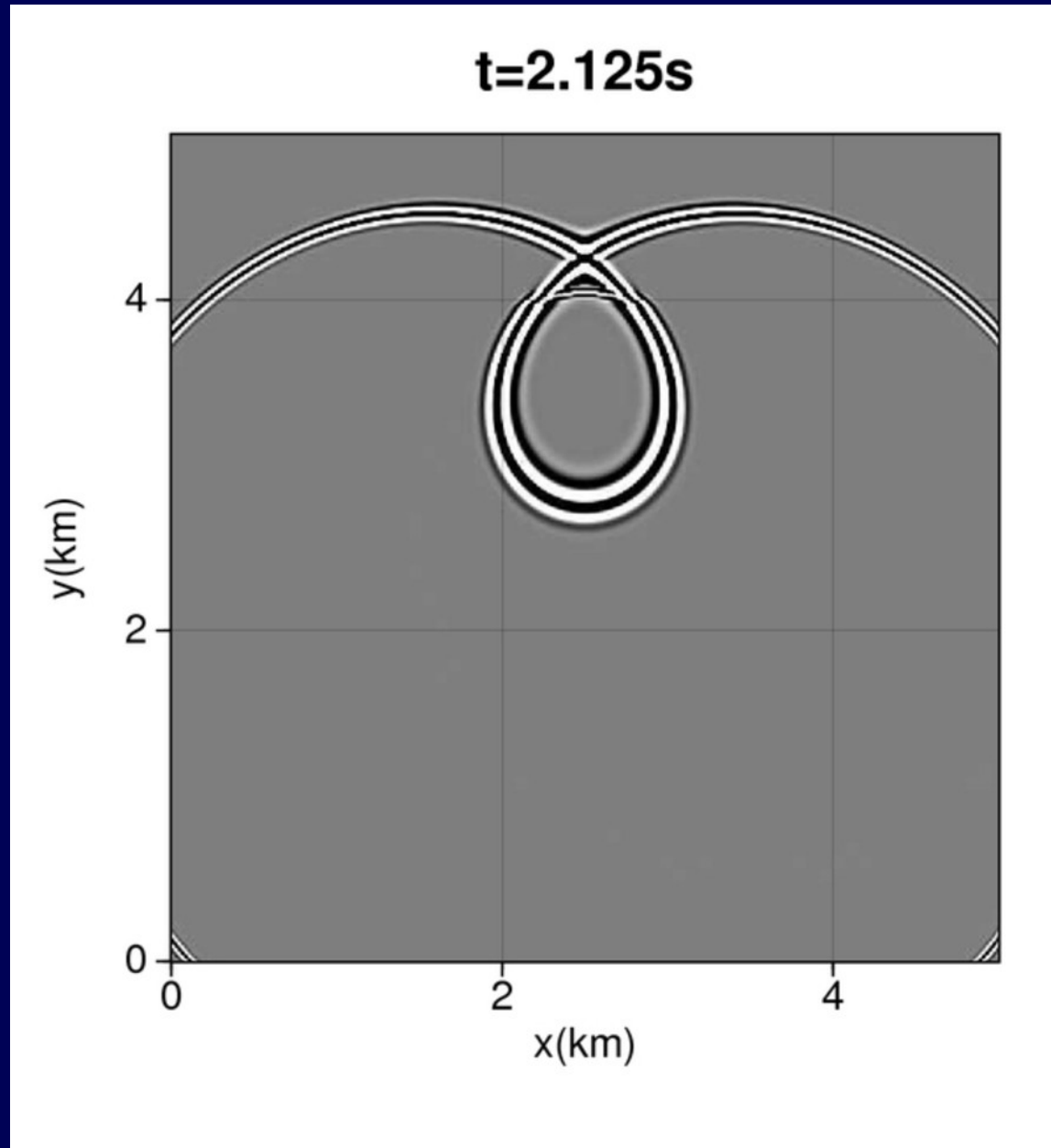
# Curved edge diffractions – synthetic – dipping ellipse



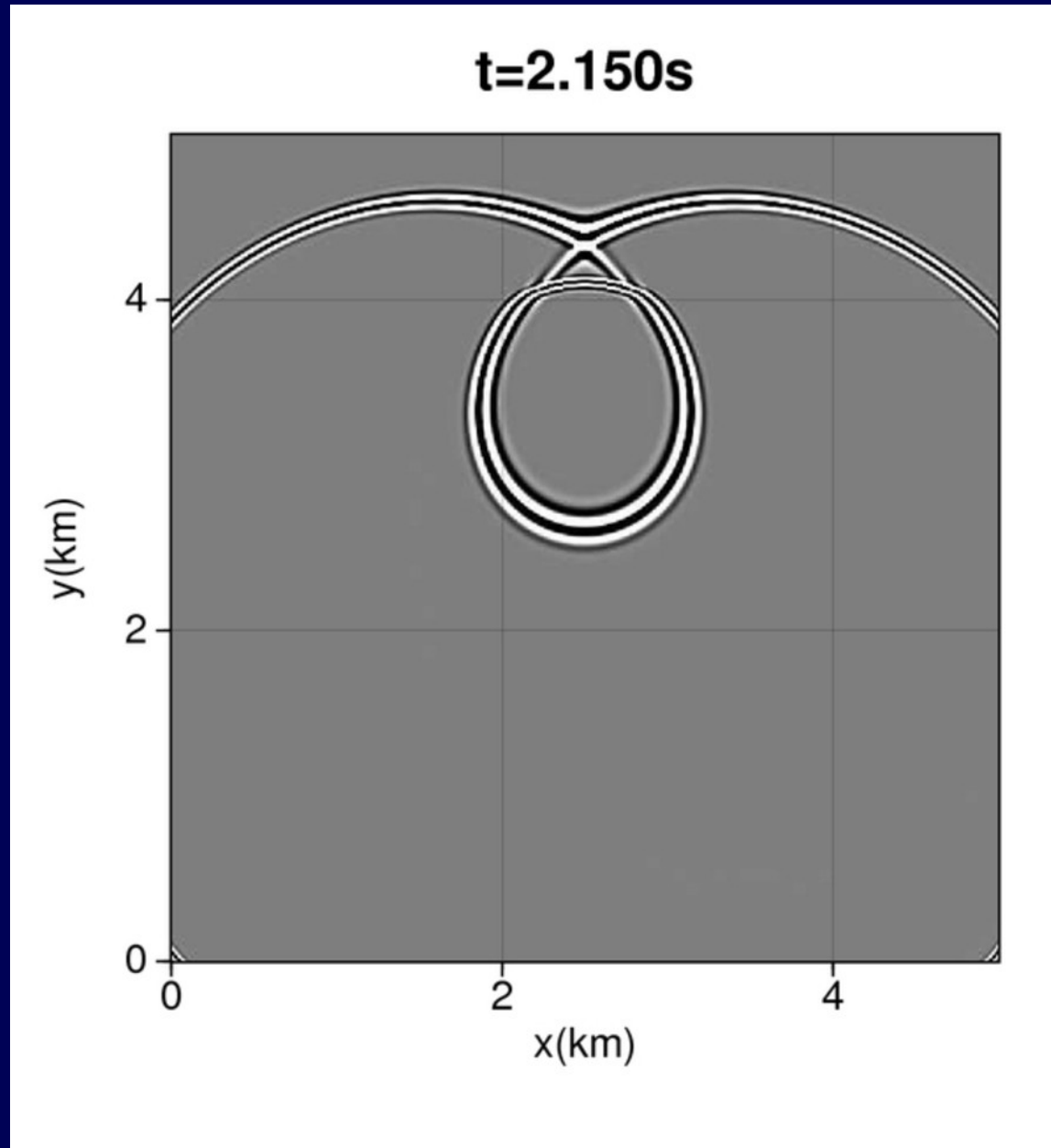
# Curved edge diffractions – synthetic – dipping ellipse



# Curved edge diffractions – synthetic – dipping ellipse

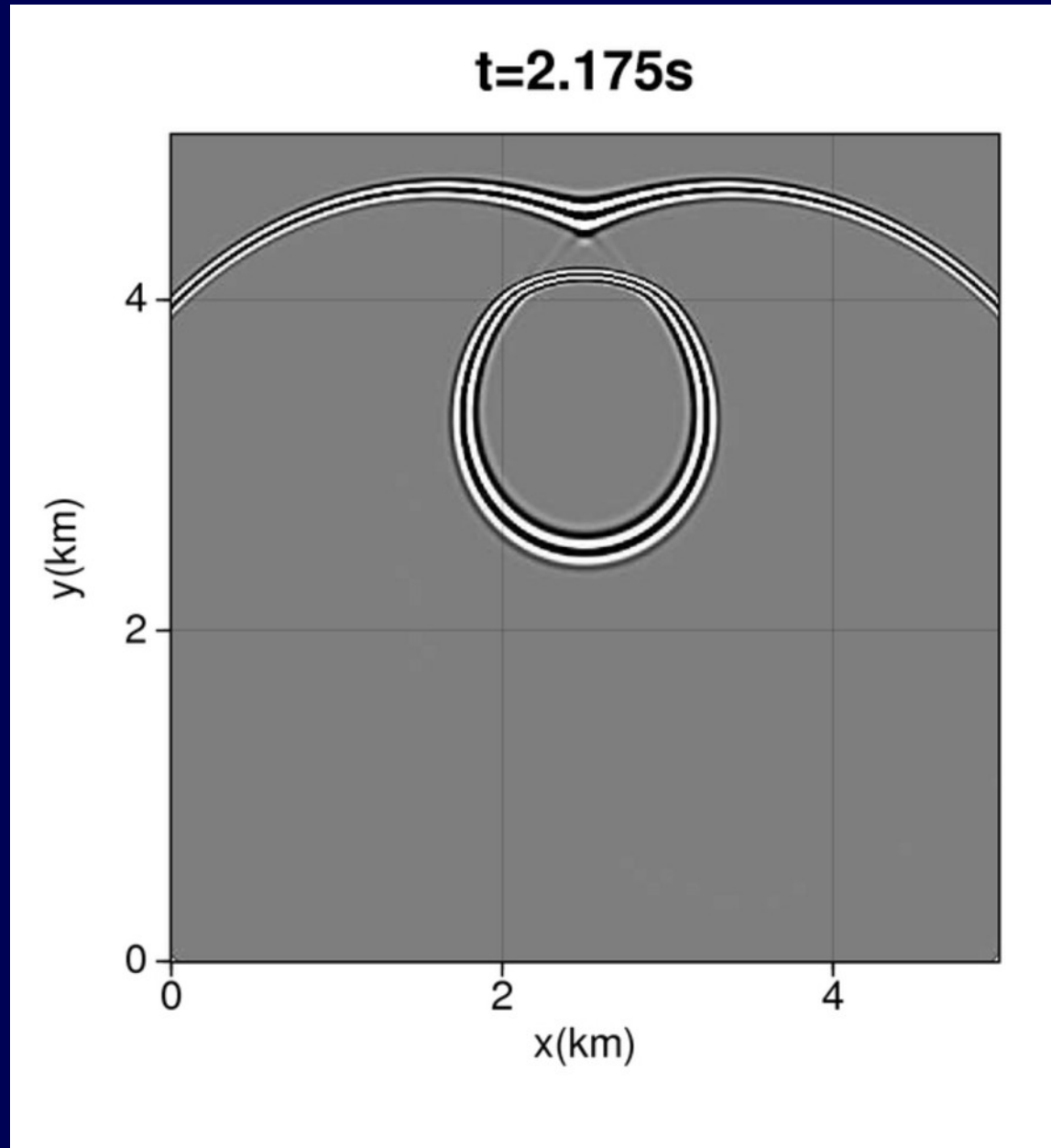


# Curved edge diffractions – synthetic – dipping ellipse

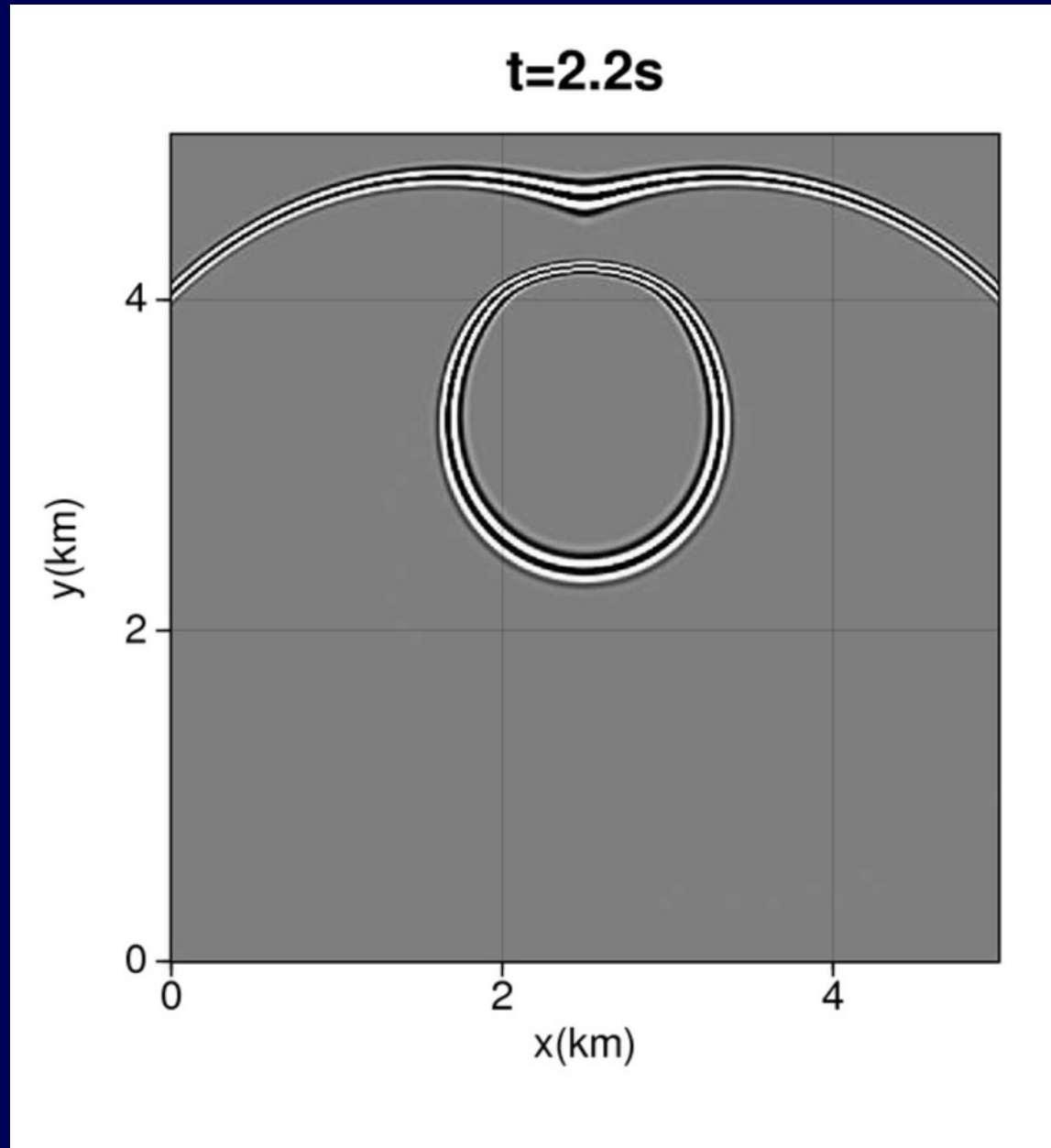




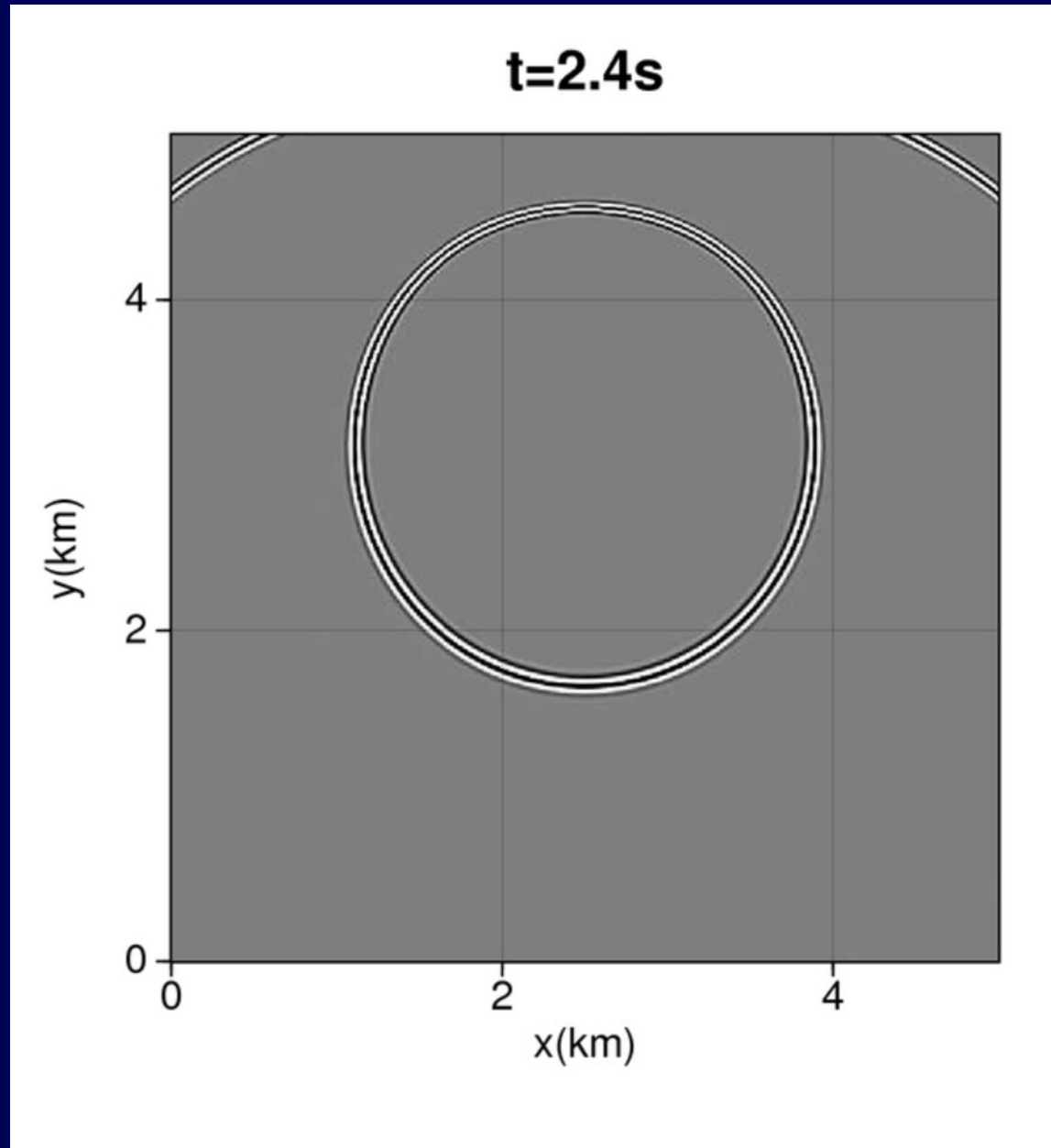
# Curved edge diffractions – synthetic – dipping ellipse



# Curved edge diffractions – synthetic – dipping ellipse



# Curved edge diffractions – synthetic – dipping ellipse



# Curved edge diffractions – analytic

## Envelope of diffraction hyperboloids

Consider a 3D line diffractor given by the curve

$$\mathbf{x}(q) = (x_1(q), x_2(q), x_3(q))^T, \quad (1)$$

where  $q$  is a parameter along the curve and  $\mathbf{x}$  the spatial location. We assume a constant model with velocity equal to one. Then, point diffraction surfaces at each point  $q$  are given by the hyperboloids

$$\mathbf{h}(u, v, q) = \begin{bmatrix} h_1(u, v, q) \\ h_2(u, v, q) \\ h_3(u, v, q) \end{bmatrix} = \begin{bmatrix} x_1(q) + u \\ x_2(q) + v \\ \sqrt{u^2 + v^2 + x_3(q)^2} \end{bmatrix}. \quad (2)$$

In (2),  $u, v$  are parameters pointing in  $x_1, x_2$ -direction. The apices of the hyperboloids are located at the line diffractor:  $\mathbf{h}(0, 0, q) = \mathbf{x}(q)$ .

$\mathbf{h}(u, v, q)$  is a one-parameter family of 3D surfaces for parameter  $q$ . Its envelope is given by combinations of  $u, v, q$  satisfying

$$\det[\mathbf{h}_u \ \mathbf{h}_v \ \mathbf{h}_q] = 0. \quad (3)$$

# Curved edge diffractions – analytic

## Envelope of diffraction hyperboloids

Expanding (3) gives

$$\begin{aligned} 0 = \det[\mathbf{h}_u \ \mathbf{h}_v \ \mathbf{h}_q] &= \det \begin{vmatrix} h_{1u} & h_{1v} & h_{1q} \\ h_{2u} & h_{2v} & h_{2q} \\ h_{3u} & h_{3v} & h_{3q} \end{vmatrix} = \det \begin{vmatrix} 1 & 0 & x'_1 \\ 0 & 1 & x'_2 \\ u/h_3 & v/h_3 & x_3 x'_3/h_3 \end{vmatrix} \\ &= (x_3 x'_3 - v x'_2 - u x'_1)/h_3, \end{aligned} \quad (4)$$

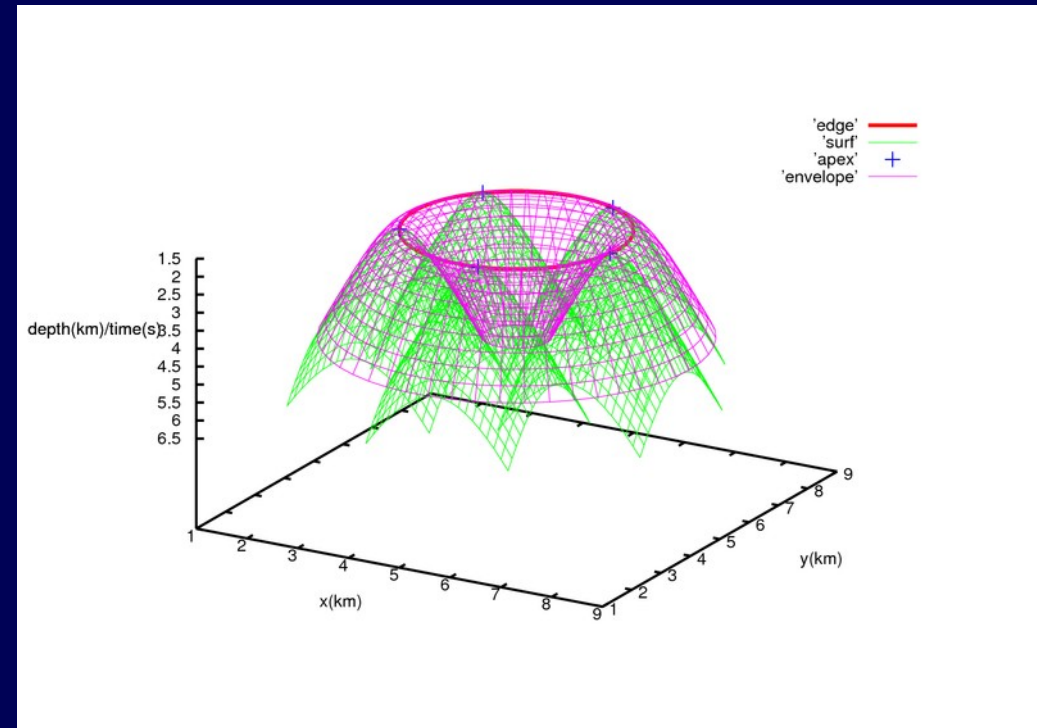
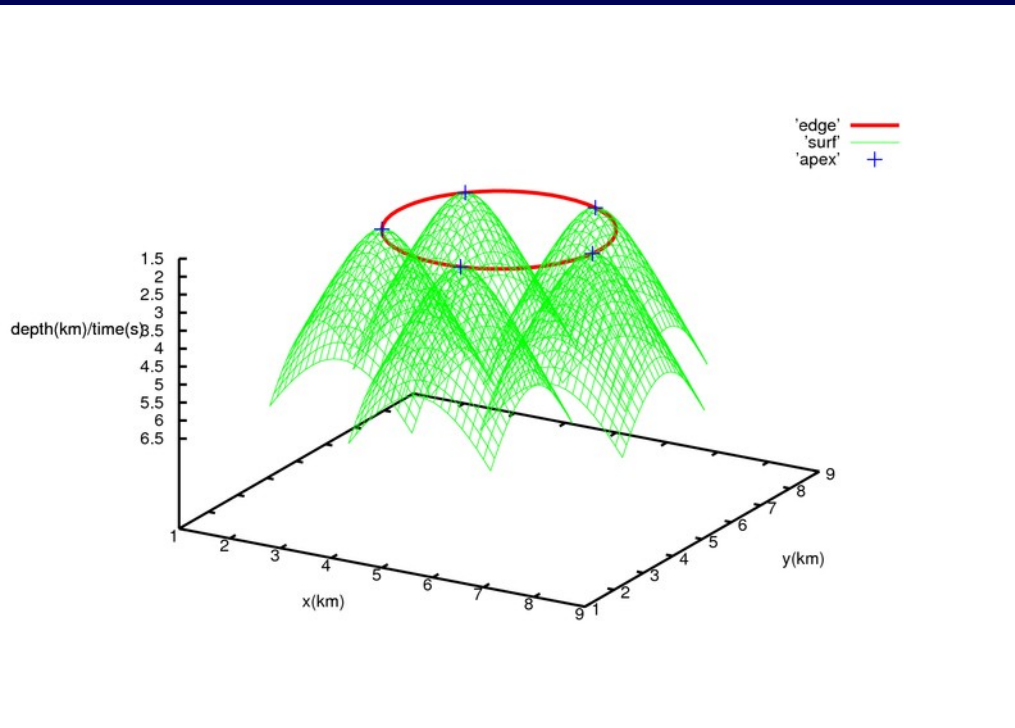
where  $x'_i = dx_i/dq$  ( $i = 1, 2, 3$ ). Therefore for fixed  $q$ , (3) is a line in the  $u, v$ -plane. By parametrizing it as

$$u(w) = x'_1 x_3 x'_3 / x_n + w x'_2, \quad v(w) = x'_2 x_3 x'_3 / x_n - w x'_1, \quad x_n = x_1'^2 + x_2'^2, \quad (5)$$

the line diffraction surface is given by  $\mathbf{h}(u(w), v(w), q)$  and (5).

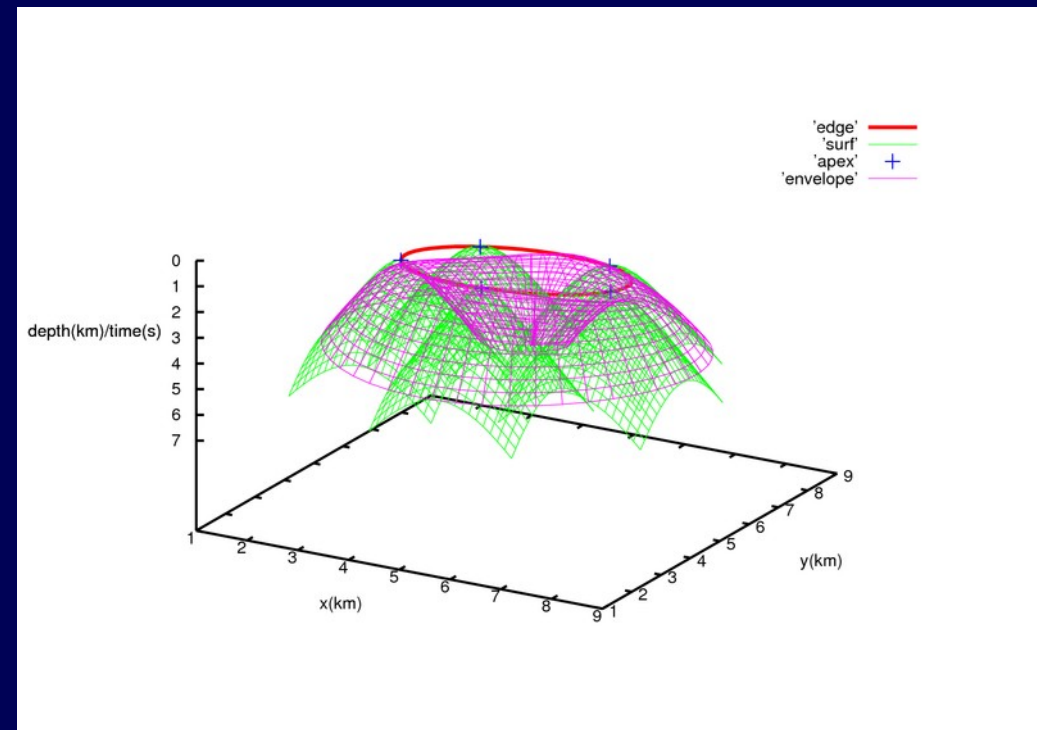
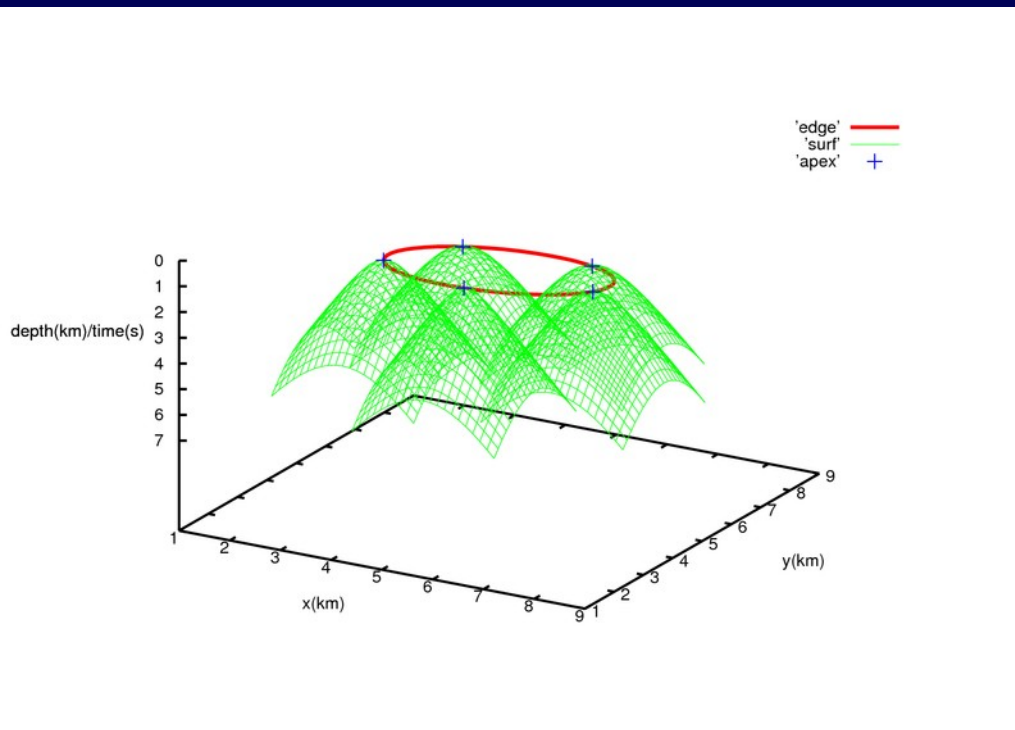
For a fixed  $q$ ,  $\mathbf{h}(u(w), v(w), q)$  is a hyperbola as function of  $w$ . For a horizontal line diffractor,  $x'_3 = 0$ , and the hyperbola has its apex  $w = 0$  on the diffraction line. For a dipping line diffractor this is not the case.

# Curved edge diffractions – analytic



**Flat circular edge**

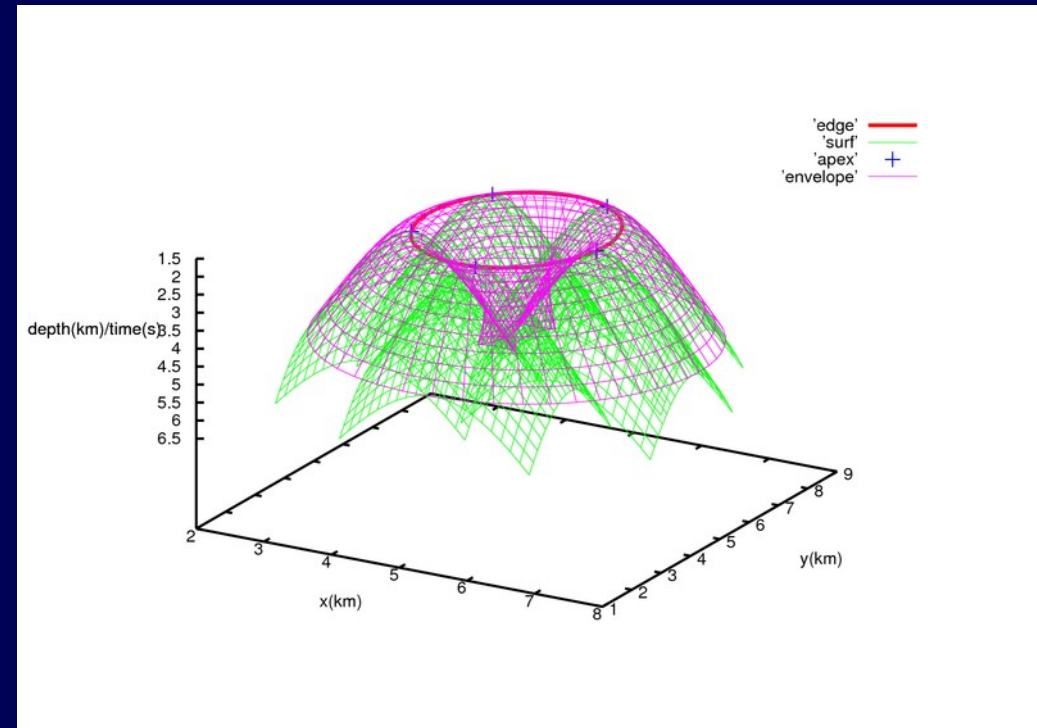
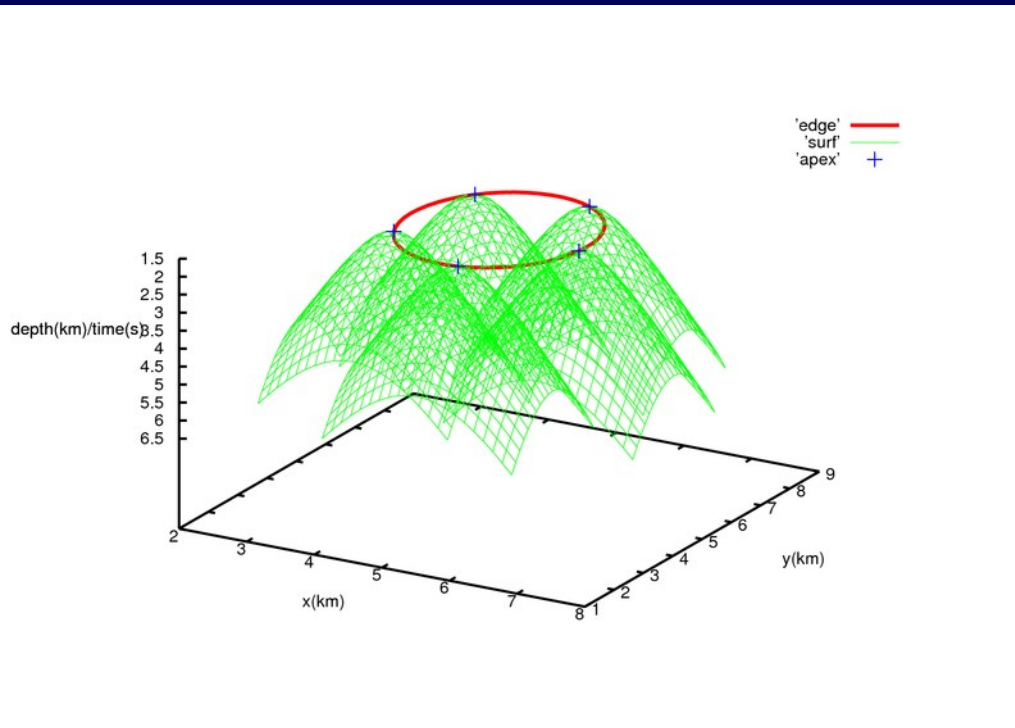
# Curved edge diffractions – analytic



*Note: envelope does not contain diffractor*

**Dipping circular edge**

# Curved edge diffractions – analytic

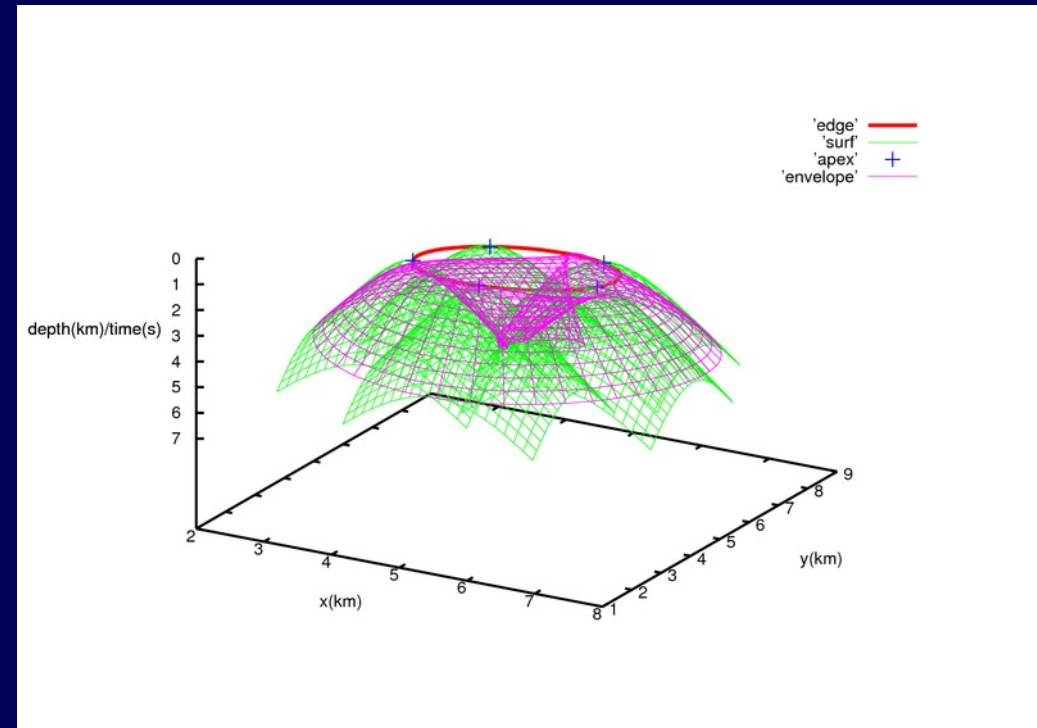
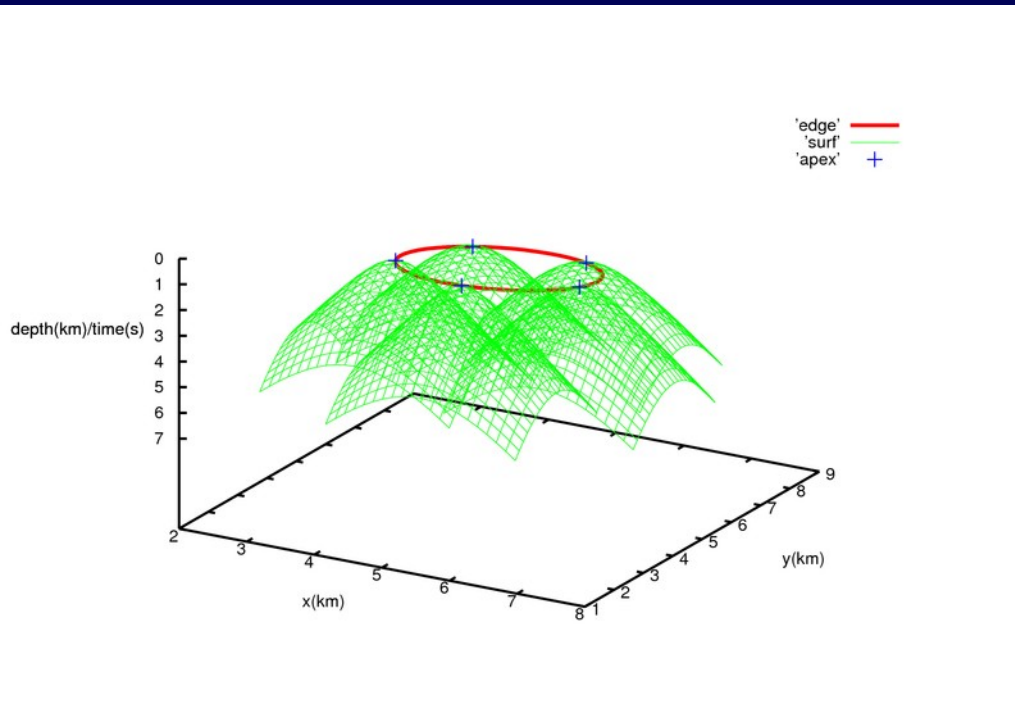


*Note: caustics*

**Flat elliptical edge**

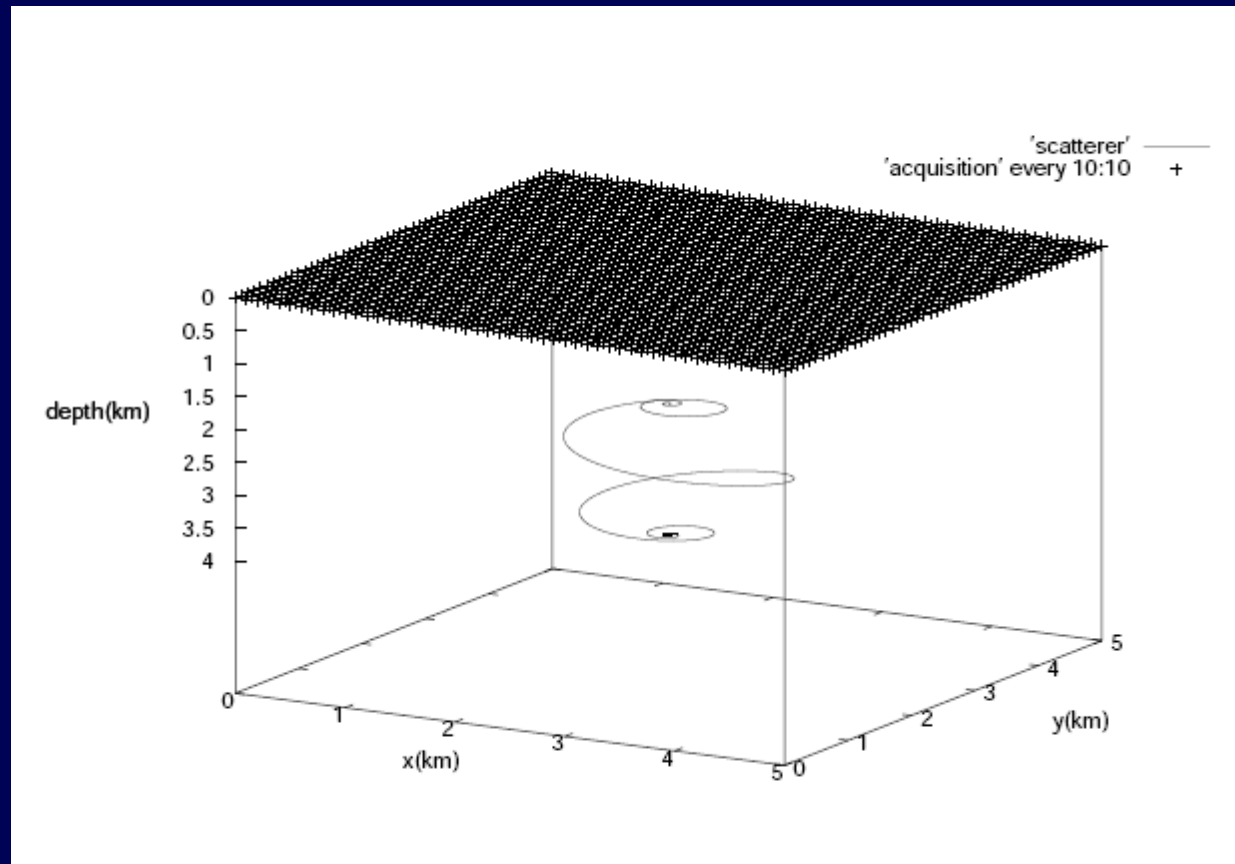


# Curved edge diffractions – analytic



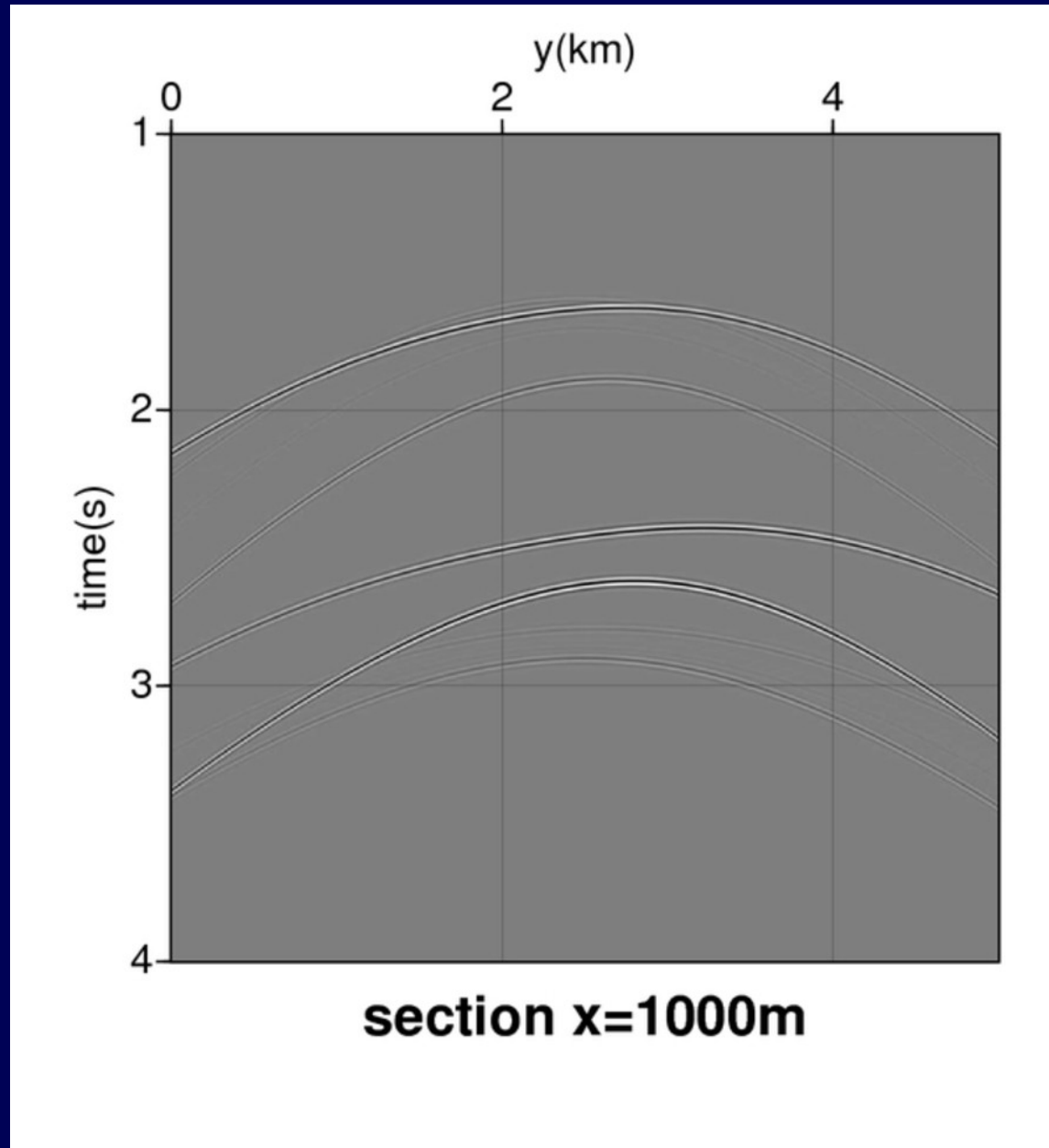
**Dipping elliptical edge**

# Curved edge diffractions – synthetic – spherical spiral

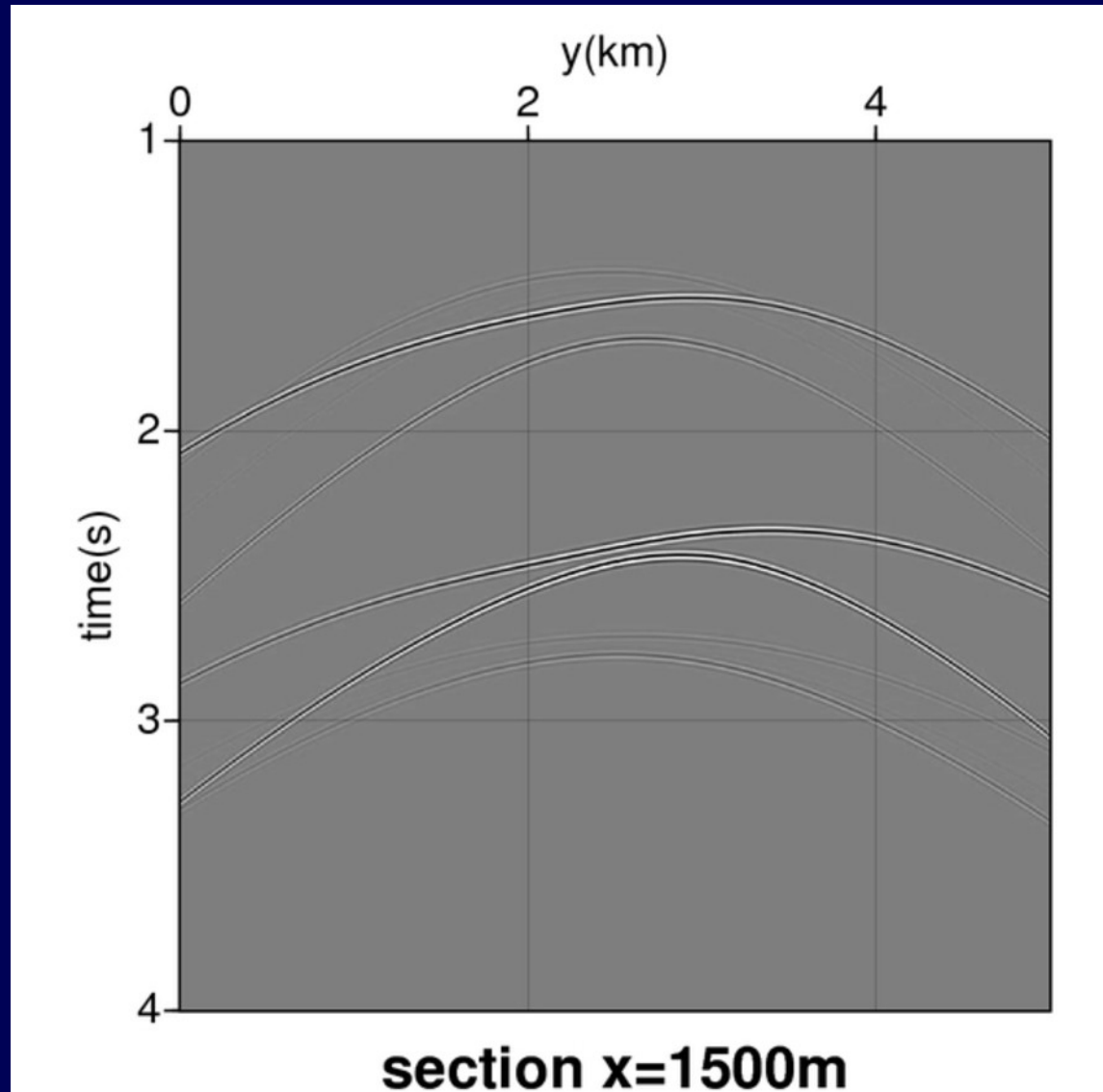


*Curvature going to infinity +  
introducing torsion*

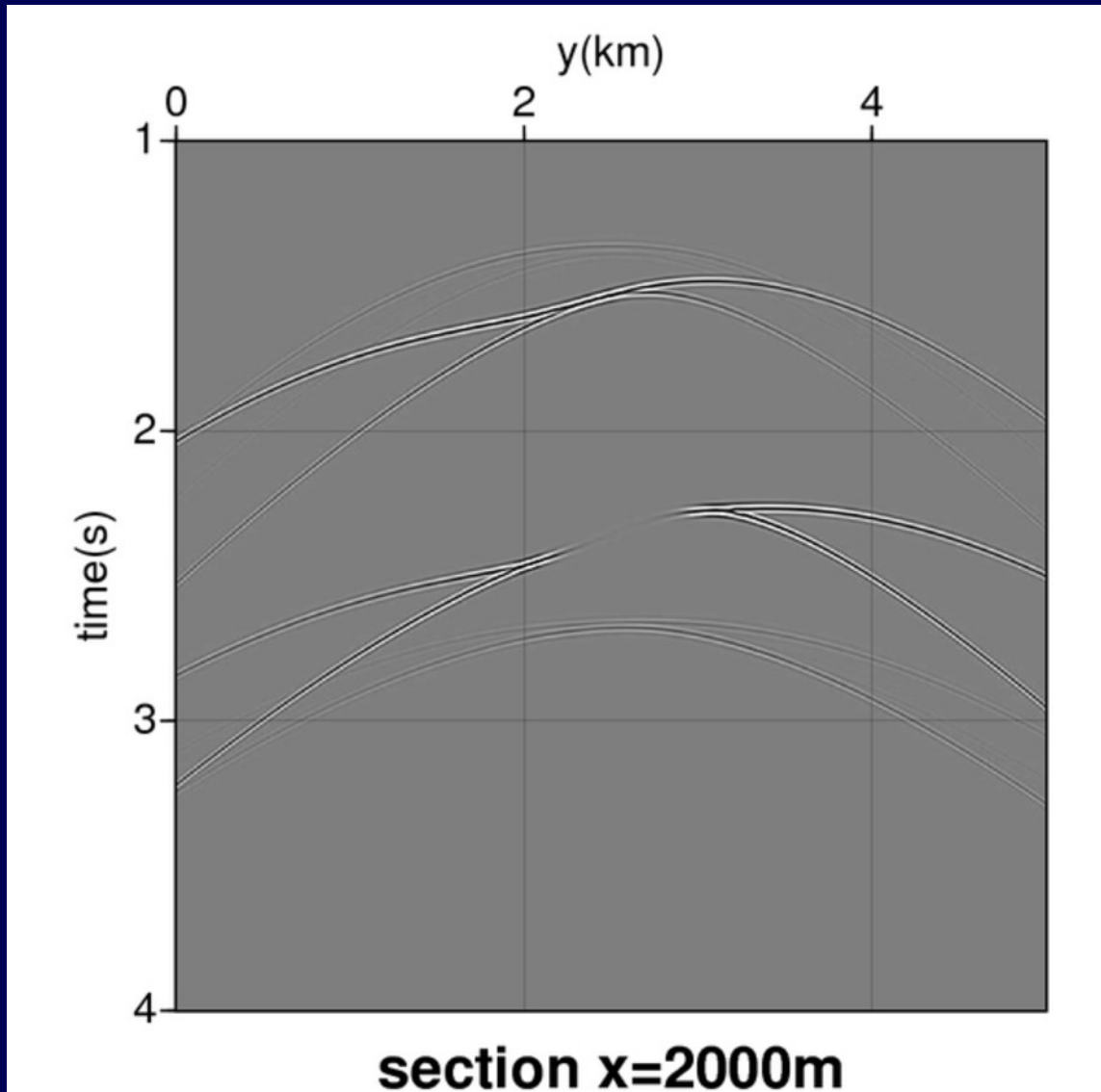
# Curved edge diffractions – synthetic – spherical spiral



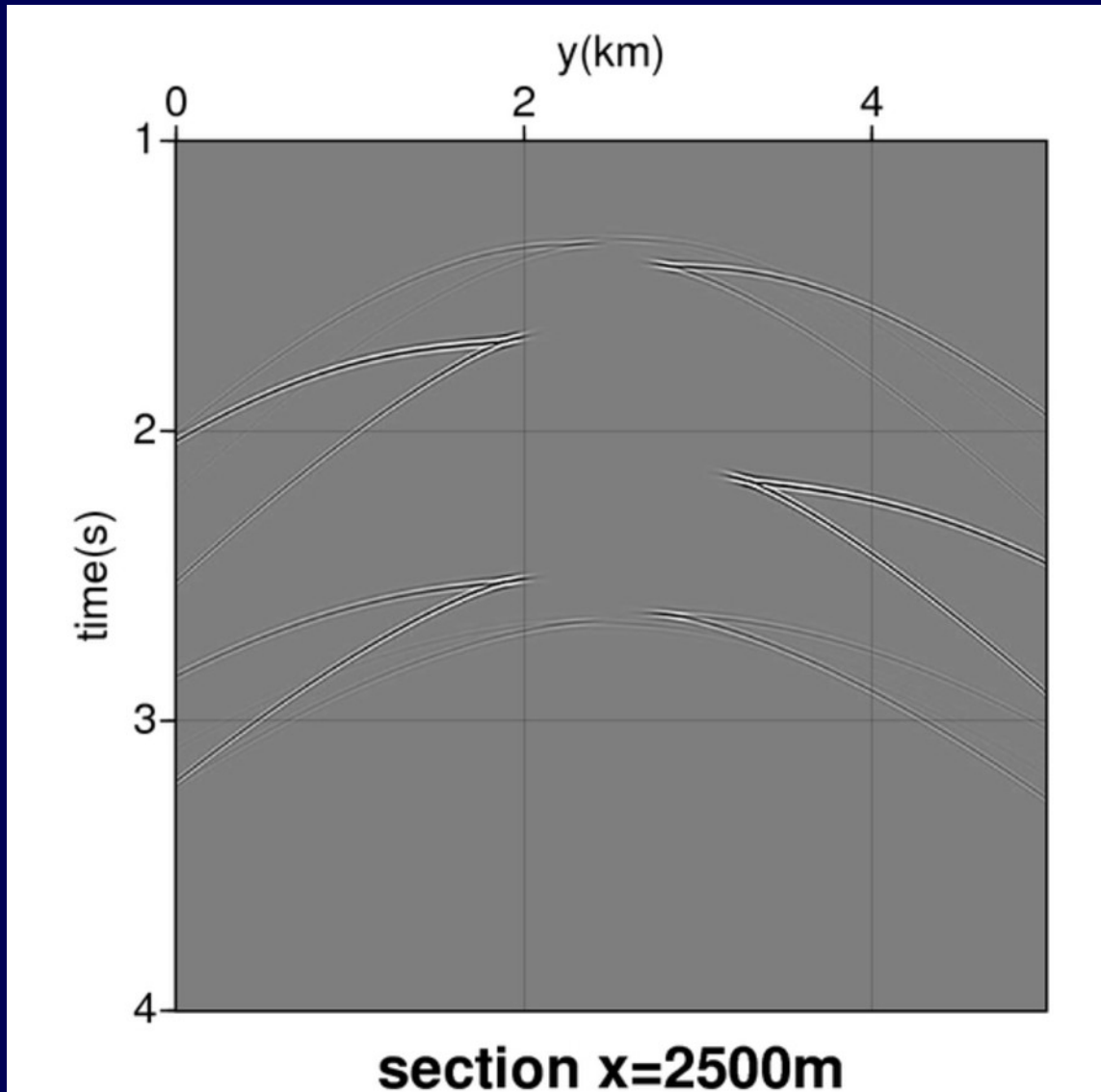
# Curved edge diffractions – synthetic – spherical spiral



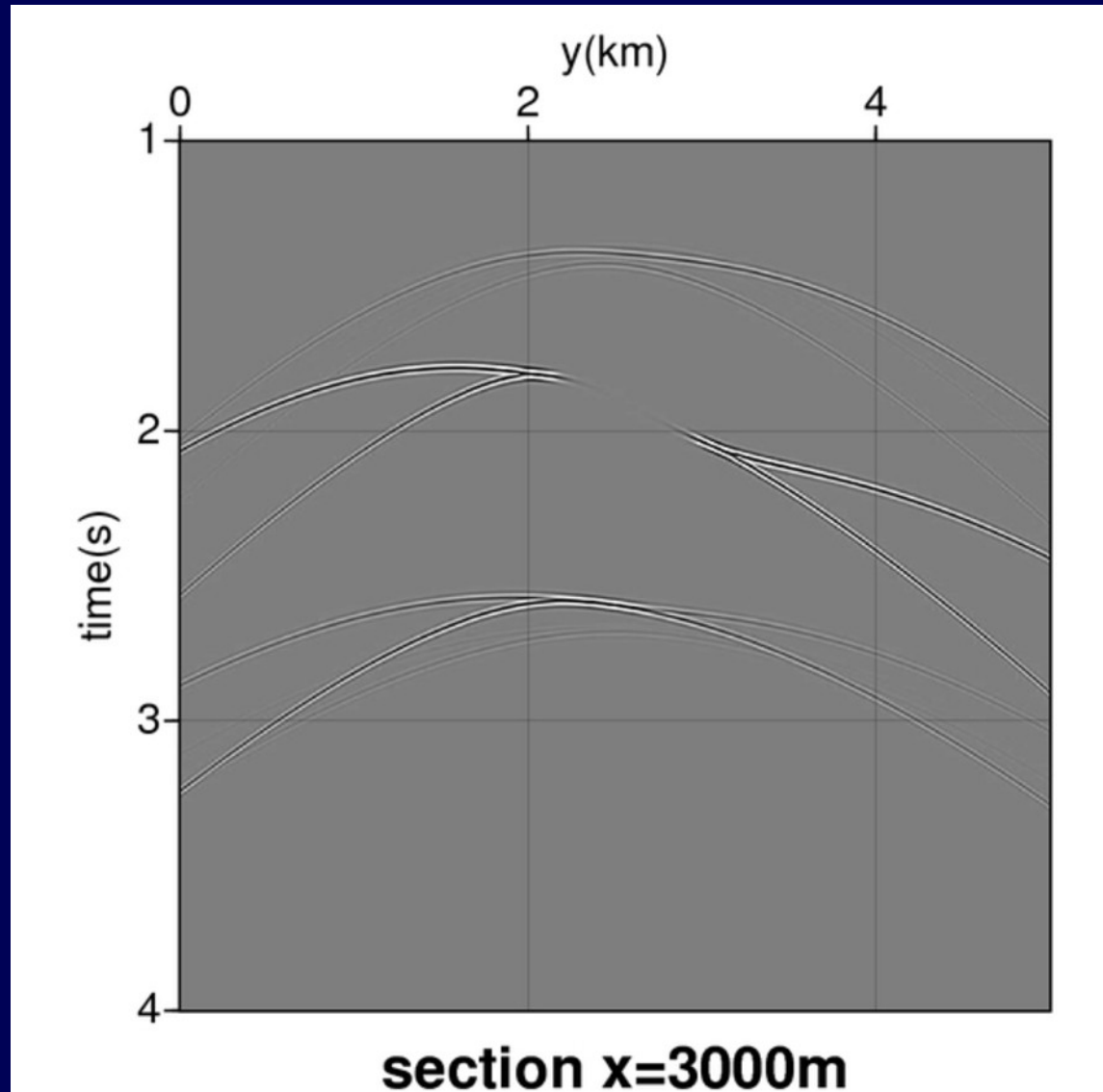
# Curved edge diffractions – synthetic – spherical spiral



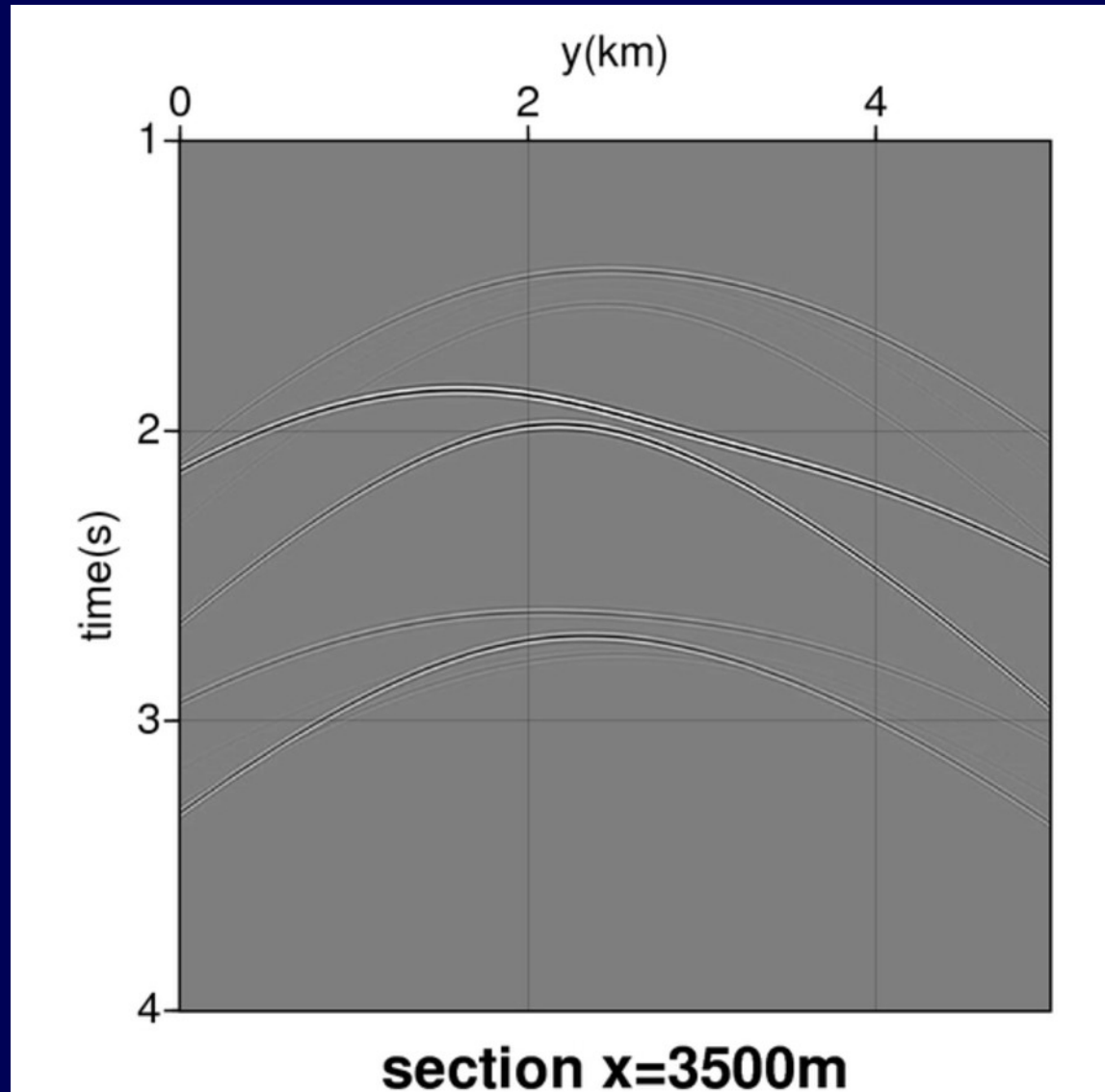
# Curved edge diffractions – synthetic – spherical spiral



# Curved edge diffractions – synthetic – spherical spiral

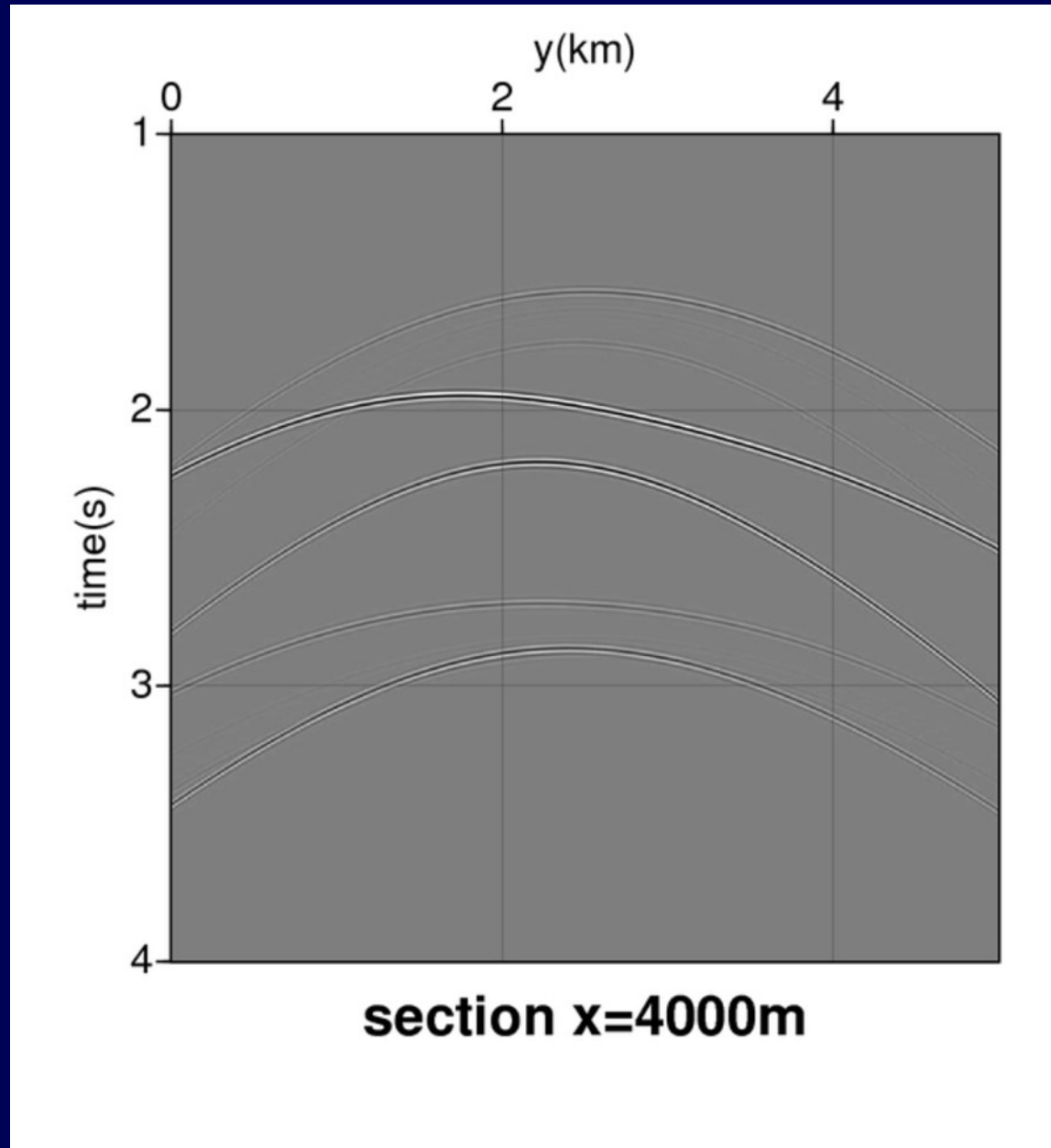


# Curved edge diffractions – synthetic – spherical spiral

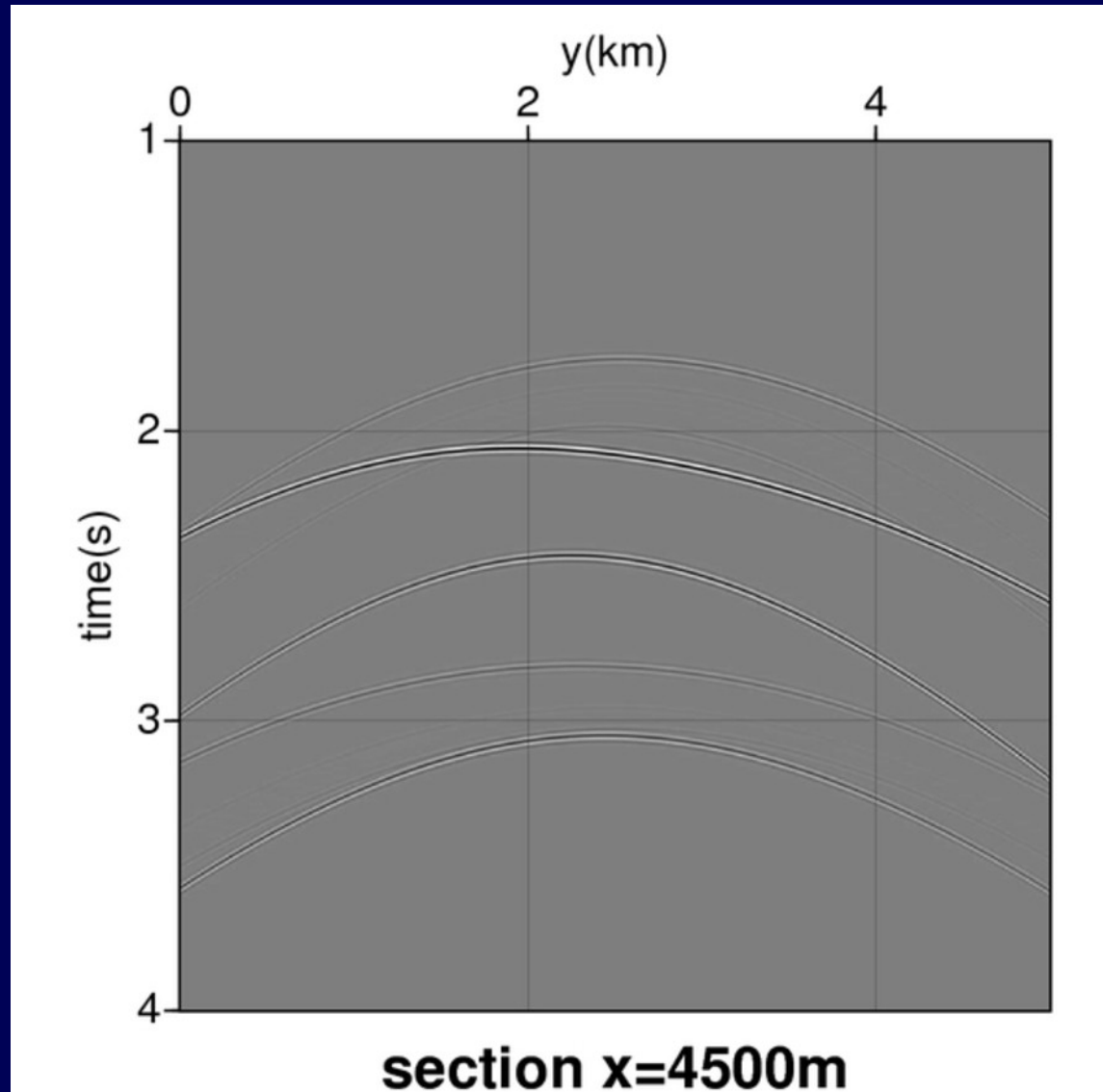




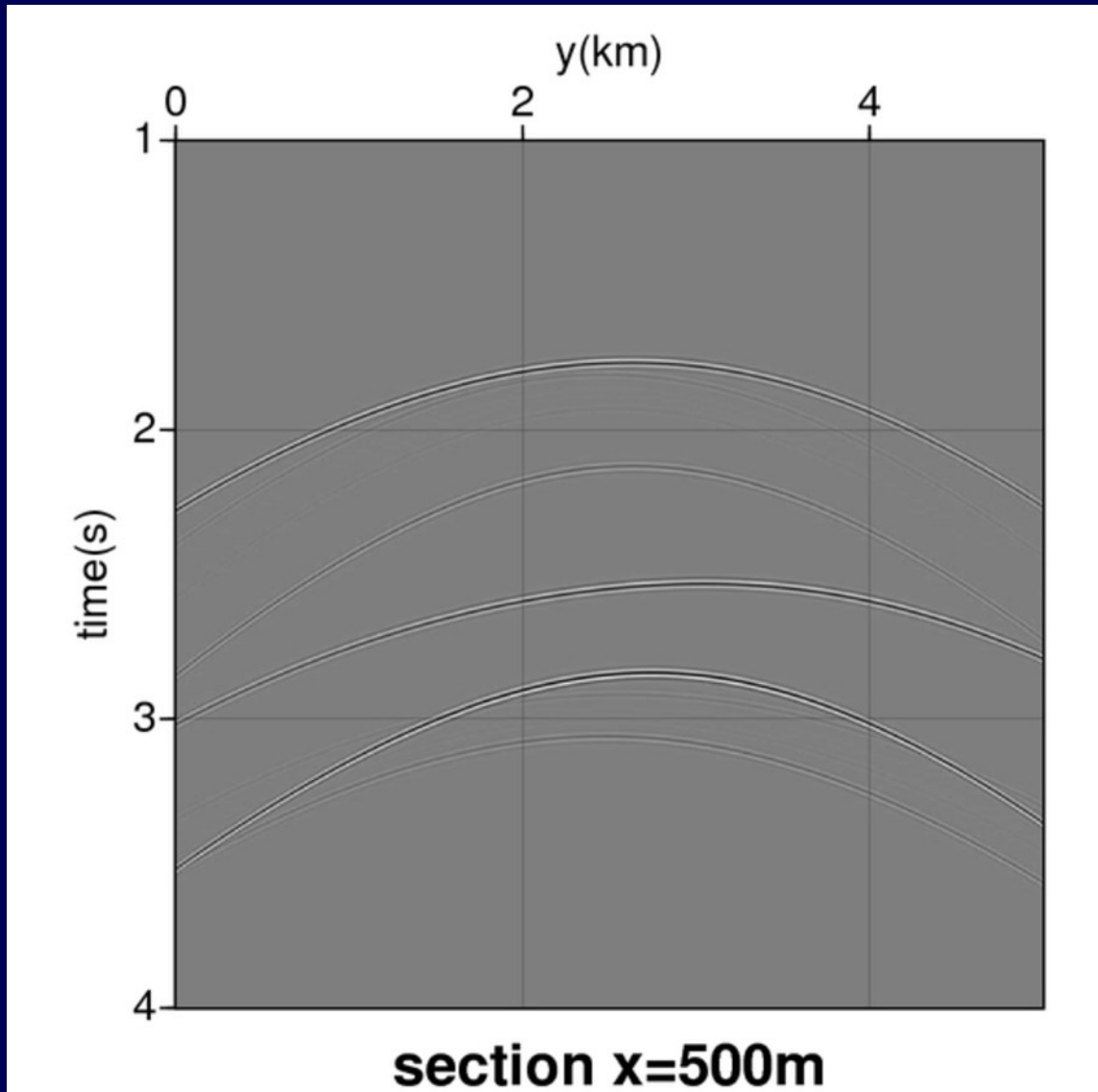
# Curved edge diffractions – synthetic – spherical spiral



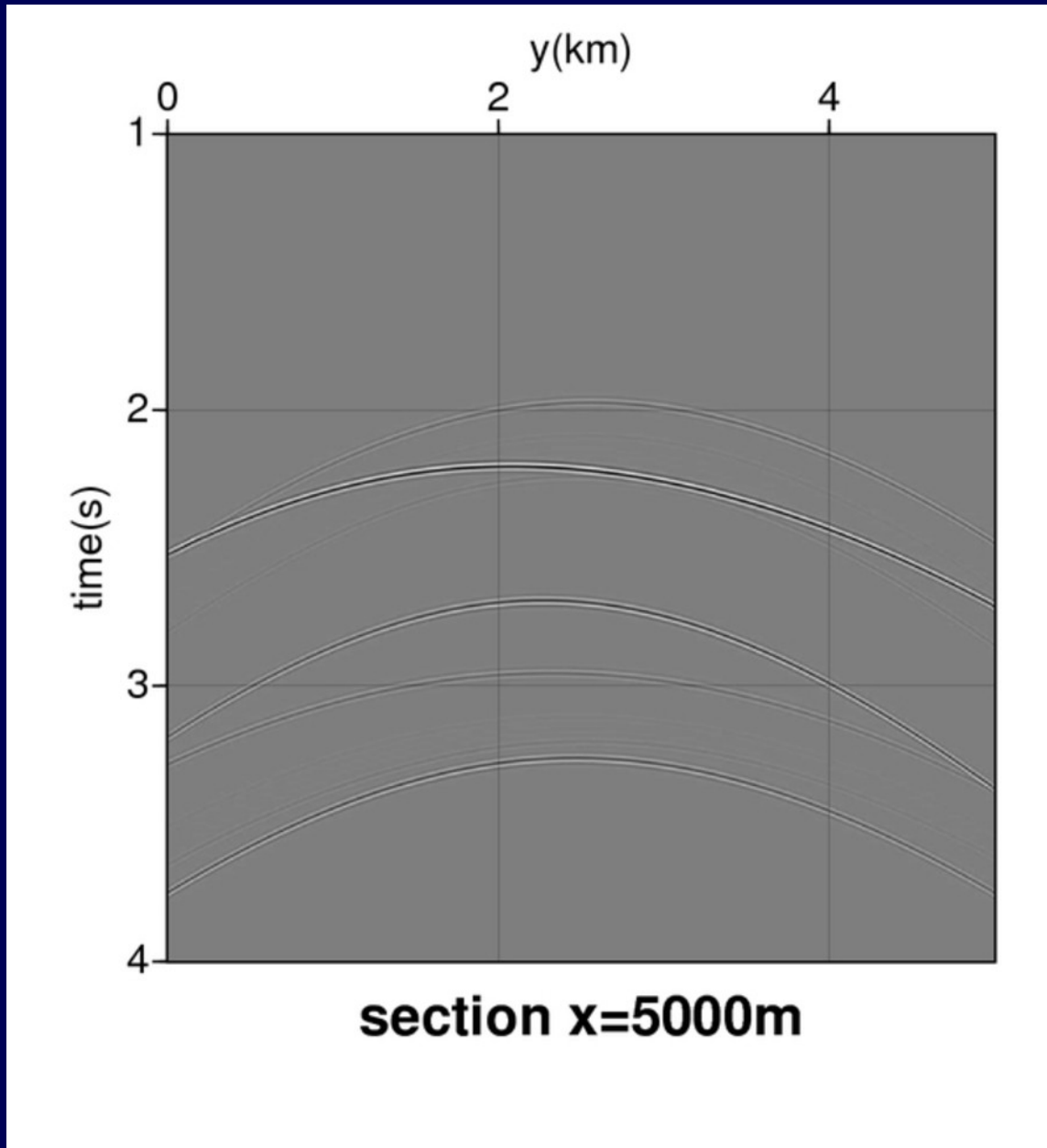
# Curved edge diffractions – synthetic – spherical spiral



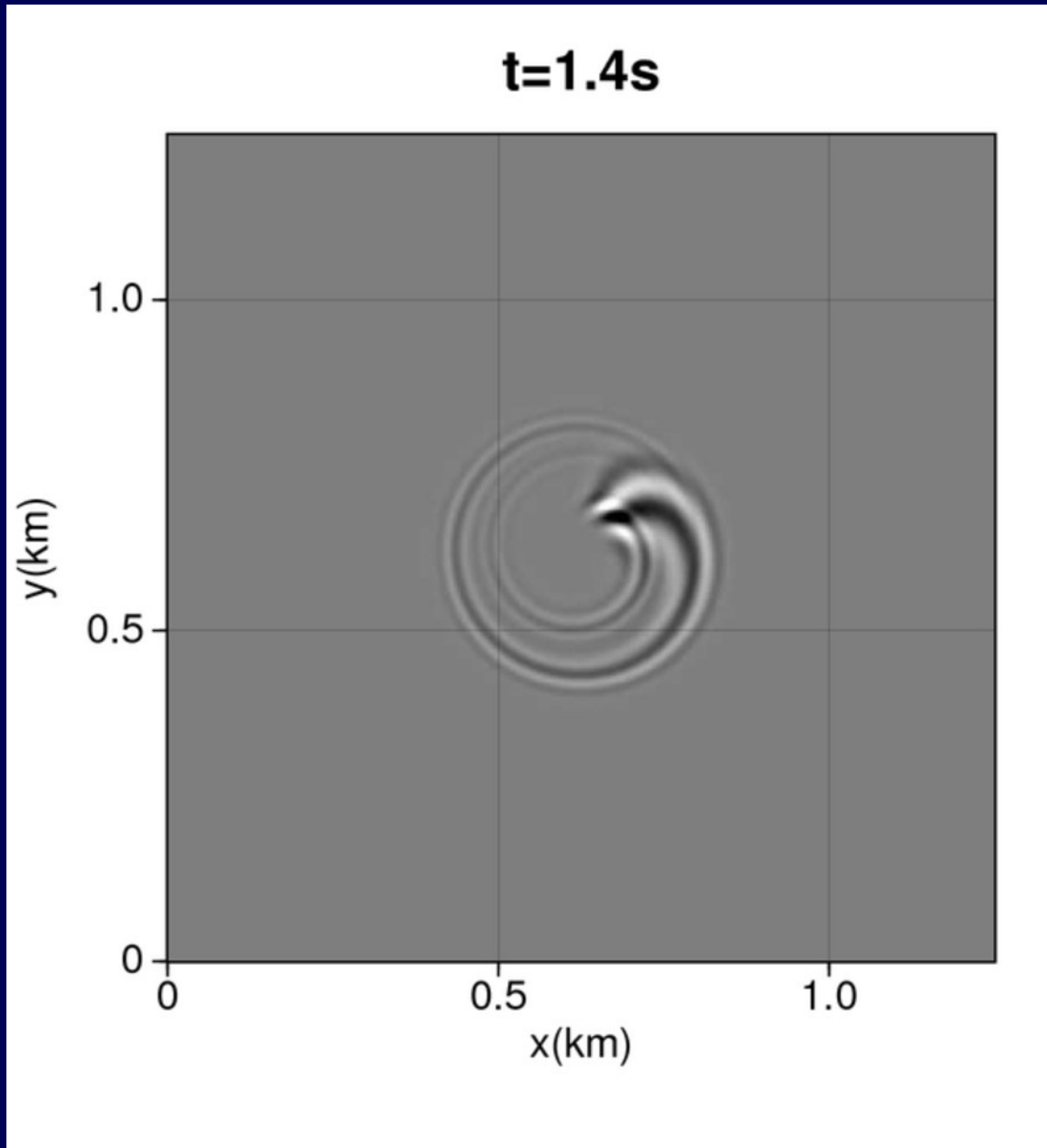
# Curved edge diffractions – synthetic – spherical spiral



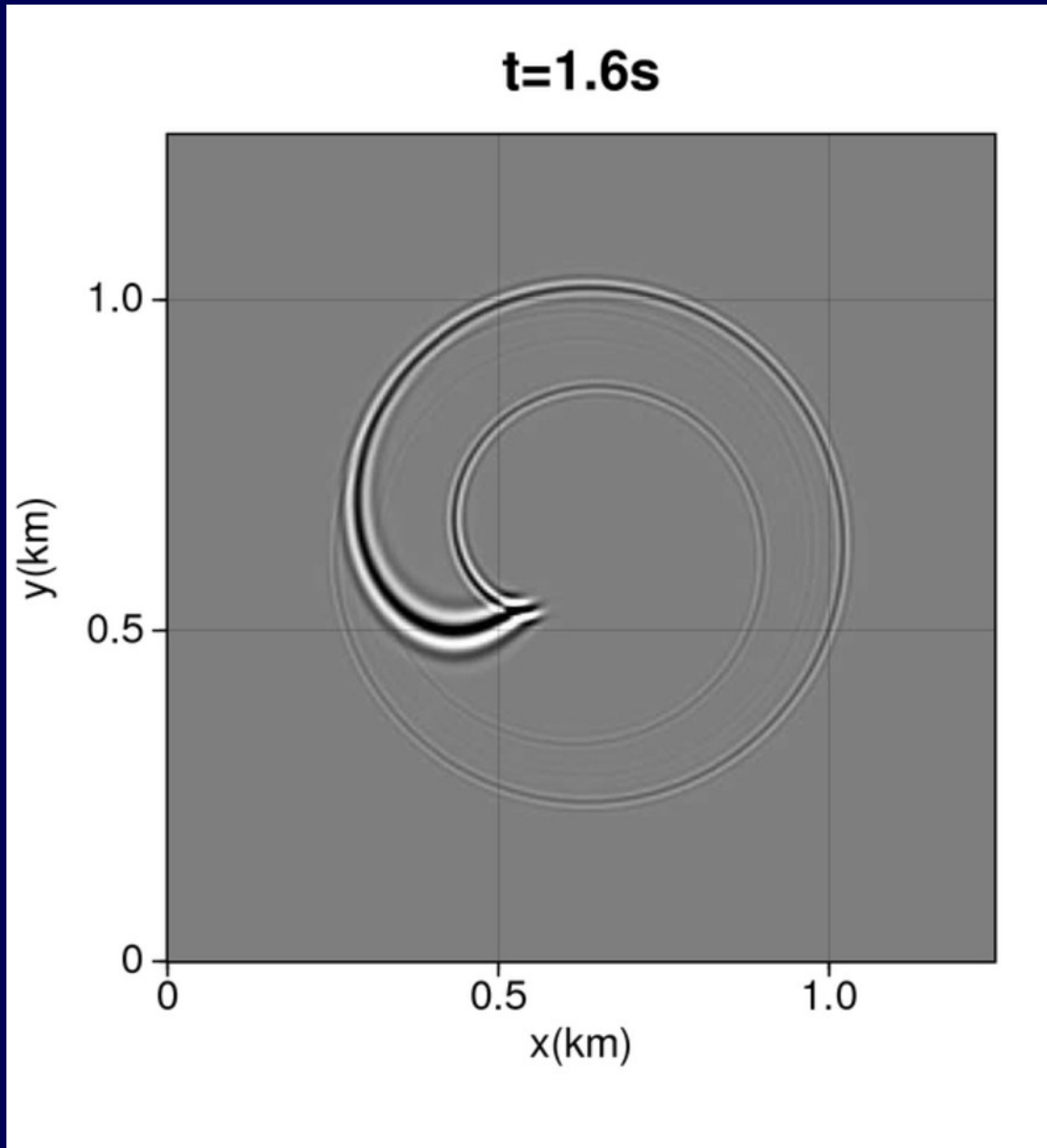
# Curved edge diffractions – synthetic – spherical spiral



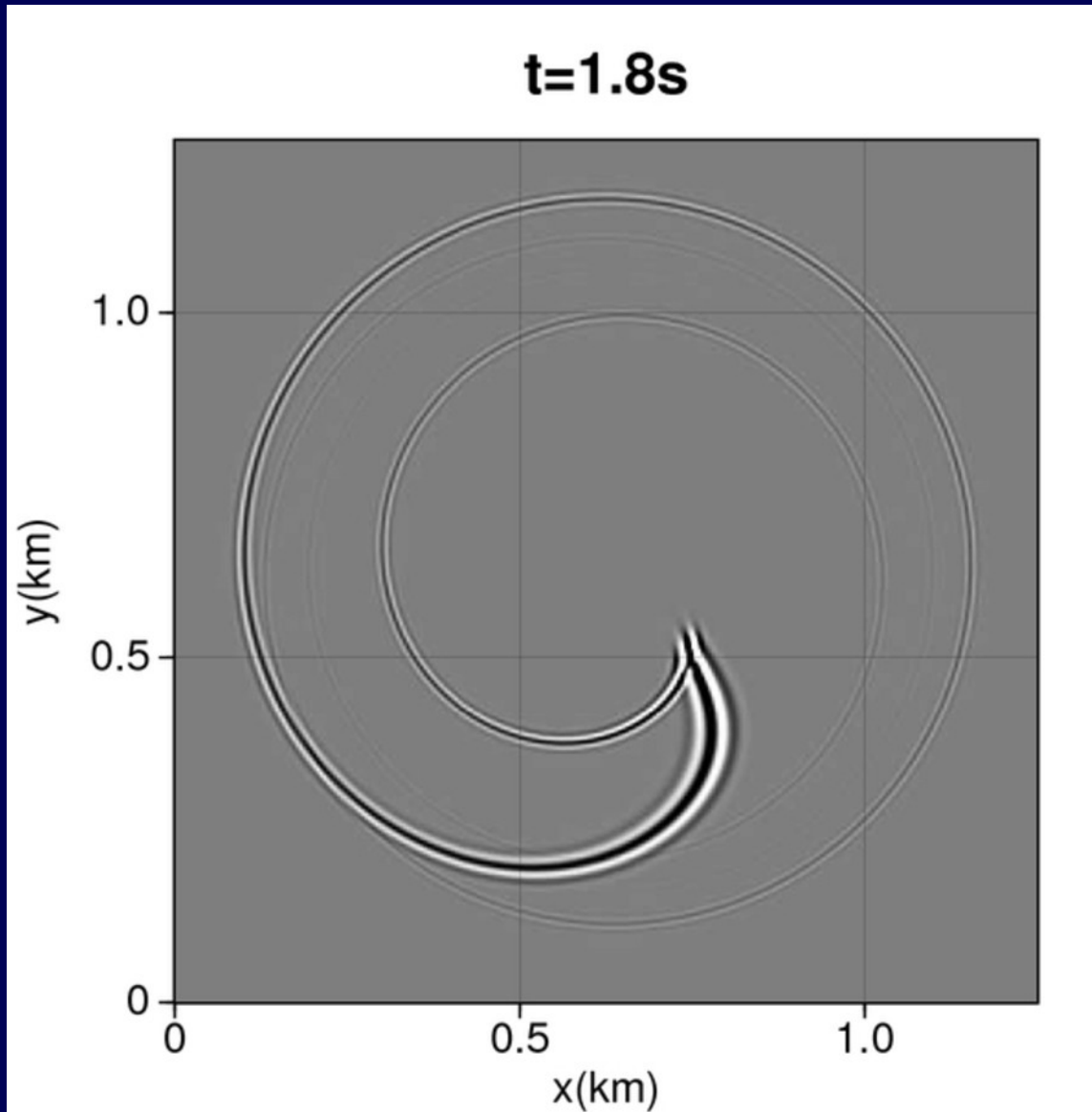
# Curved edge diffractions – synthetic – spherical spiral



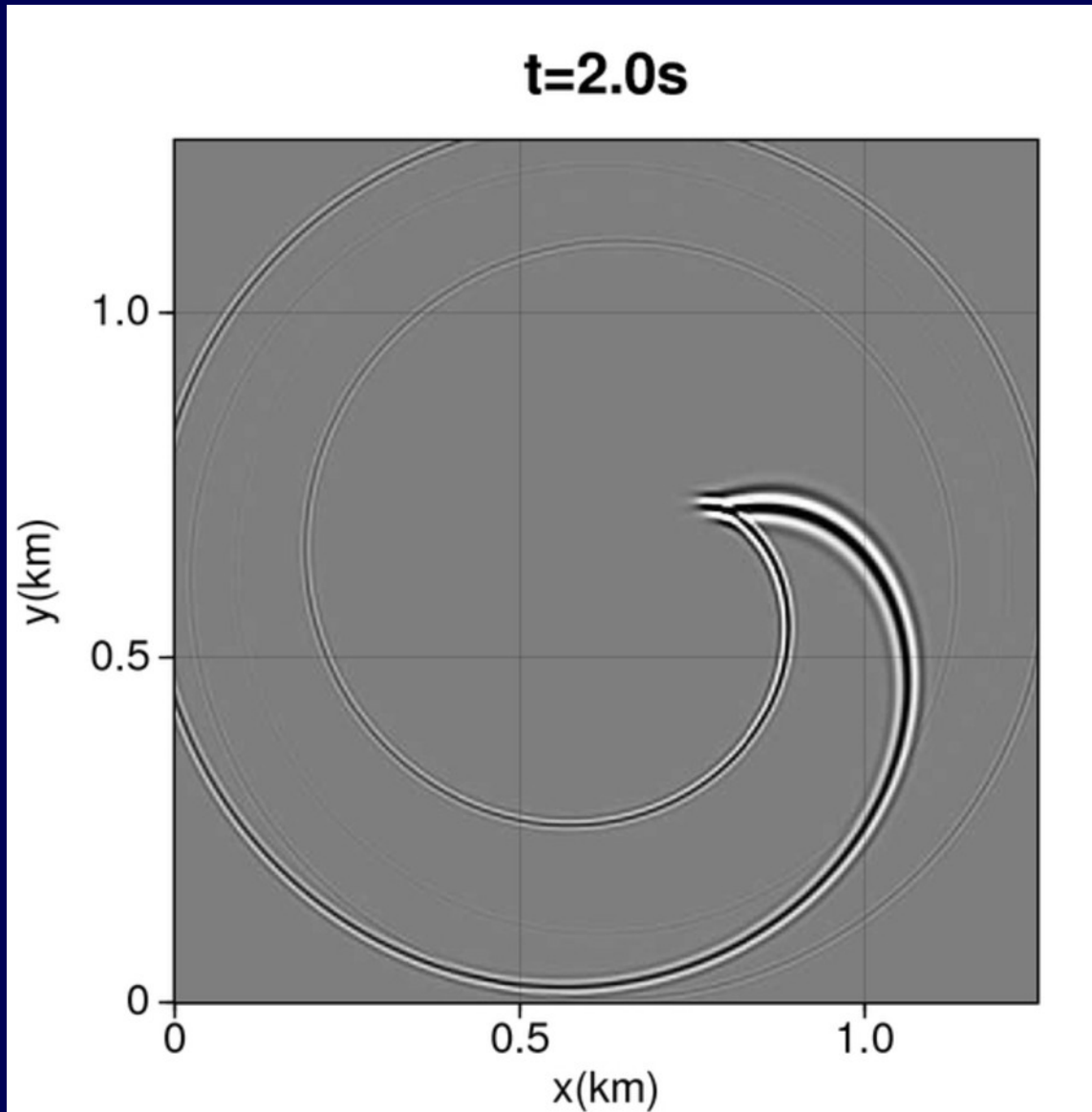
# Curved edge diffractions – synthetic – spherical spiral



# Curved edge diffractions – synthetic – spherical spiral

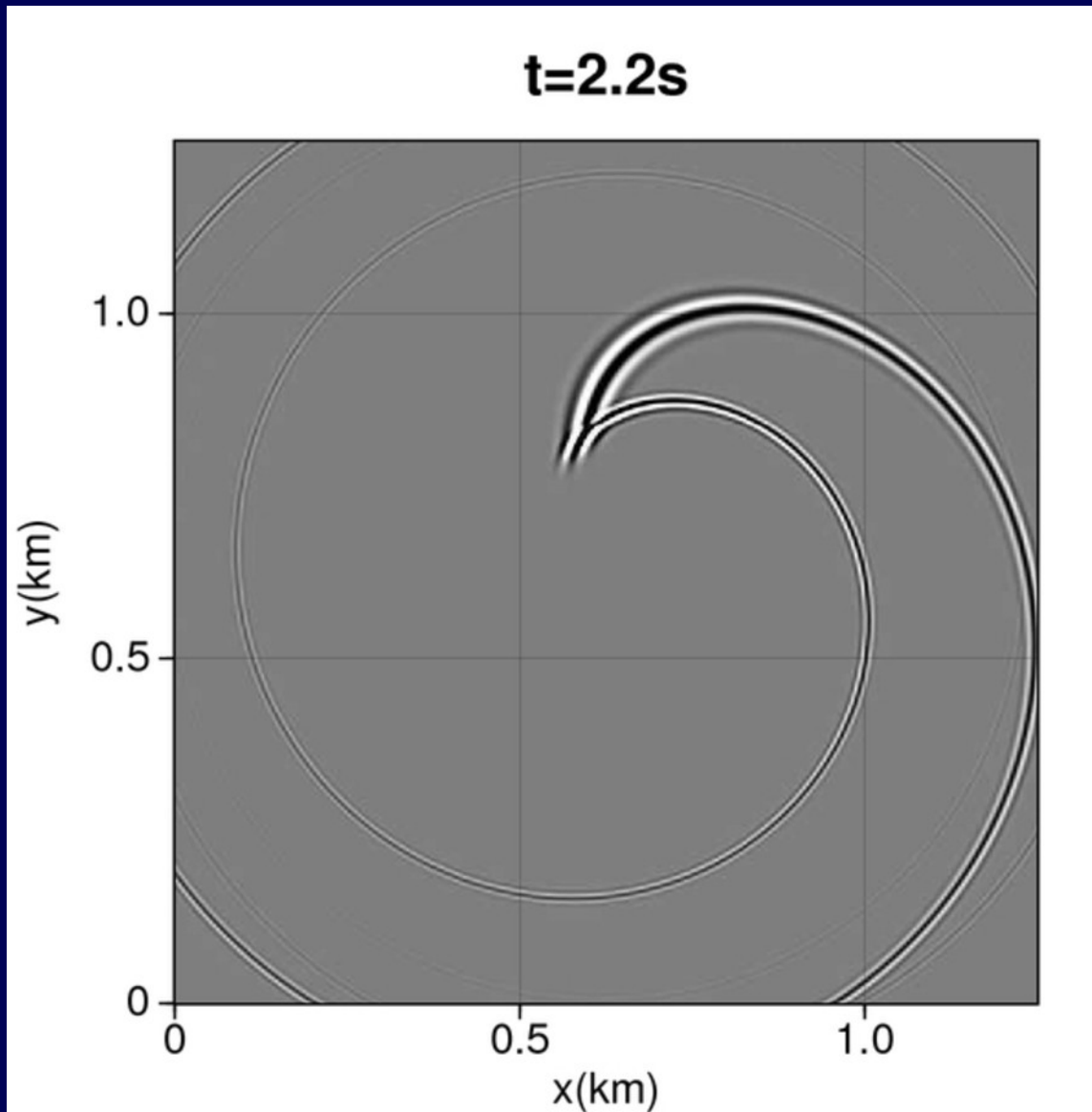


# Curved edge diffractions – synthetic – spherical spiral

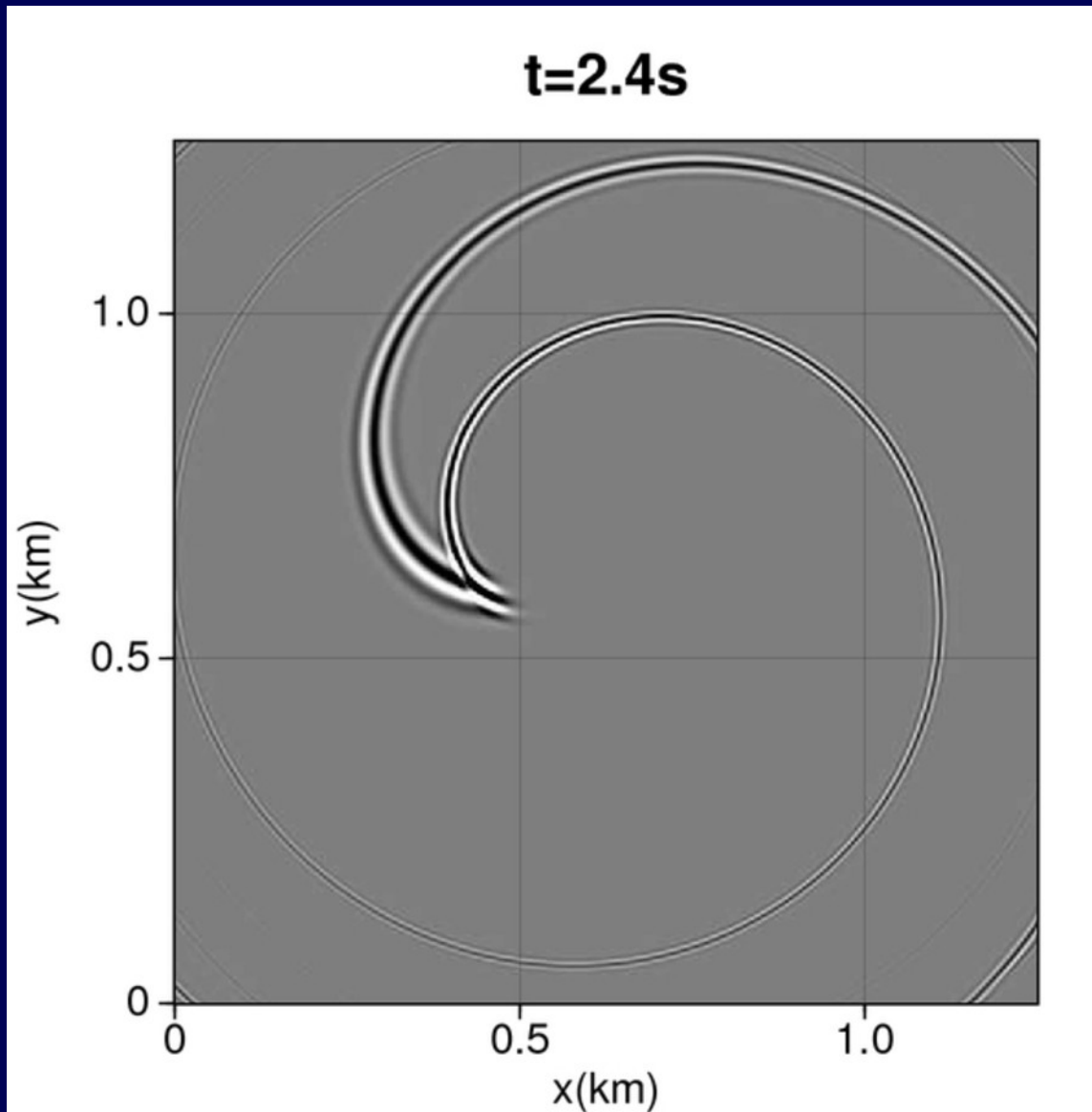




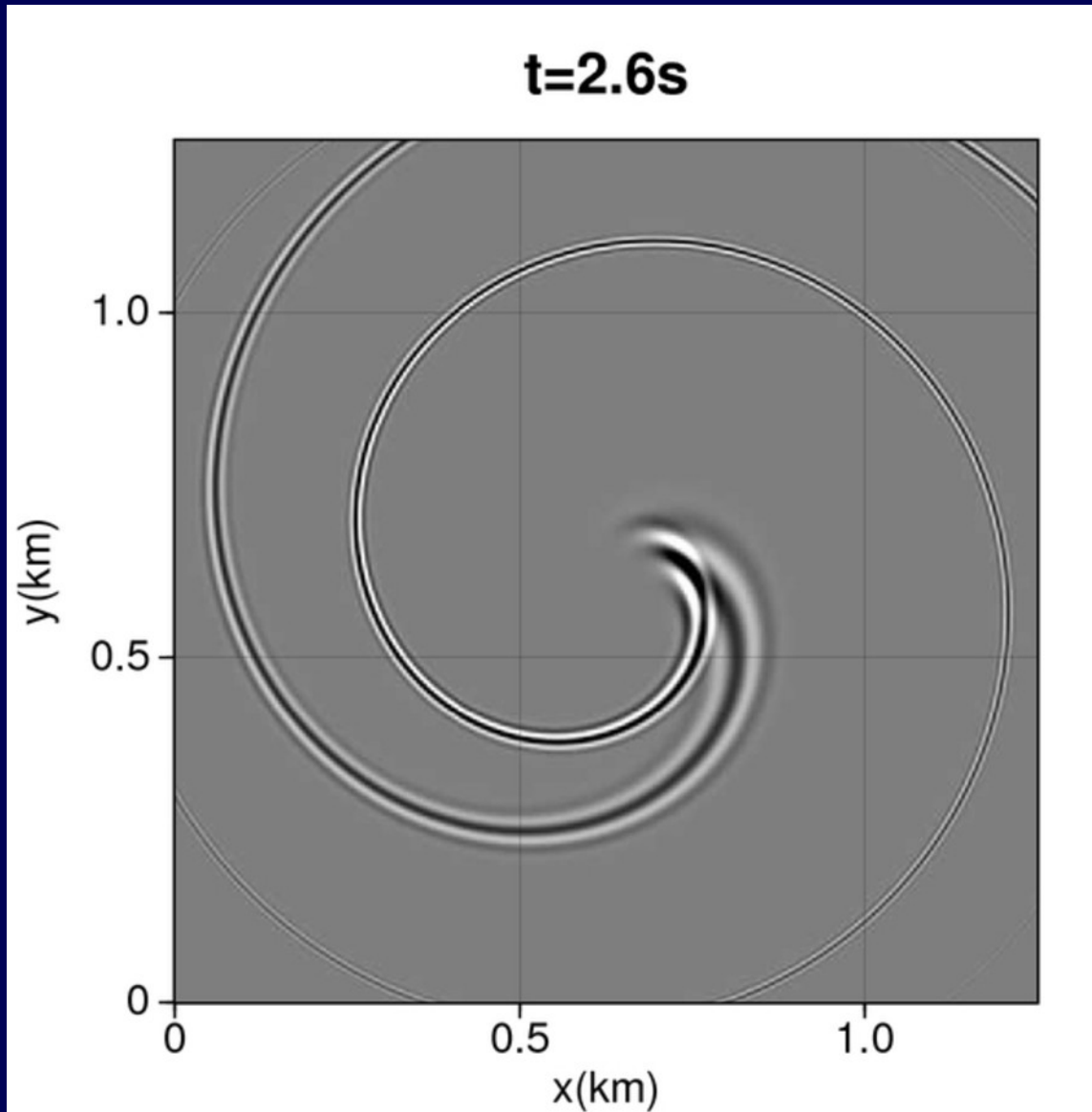
# Curved edge diffractions – synthetic – spherical spiral



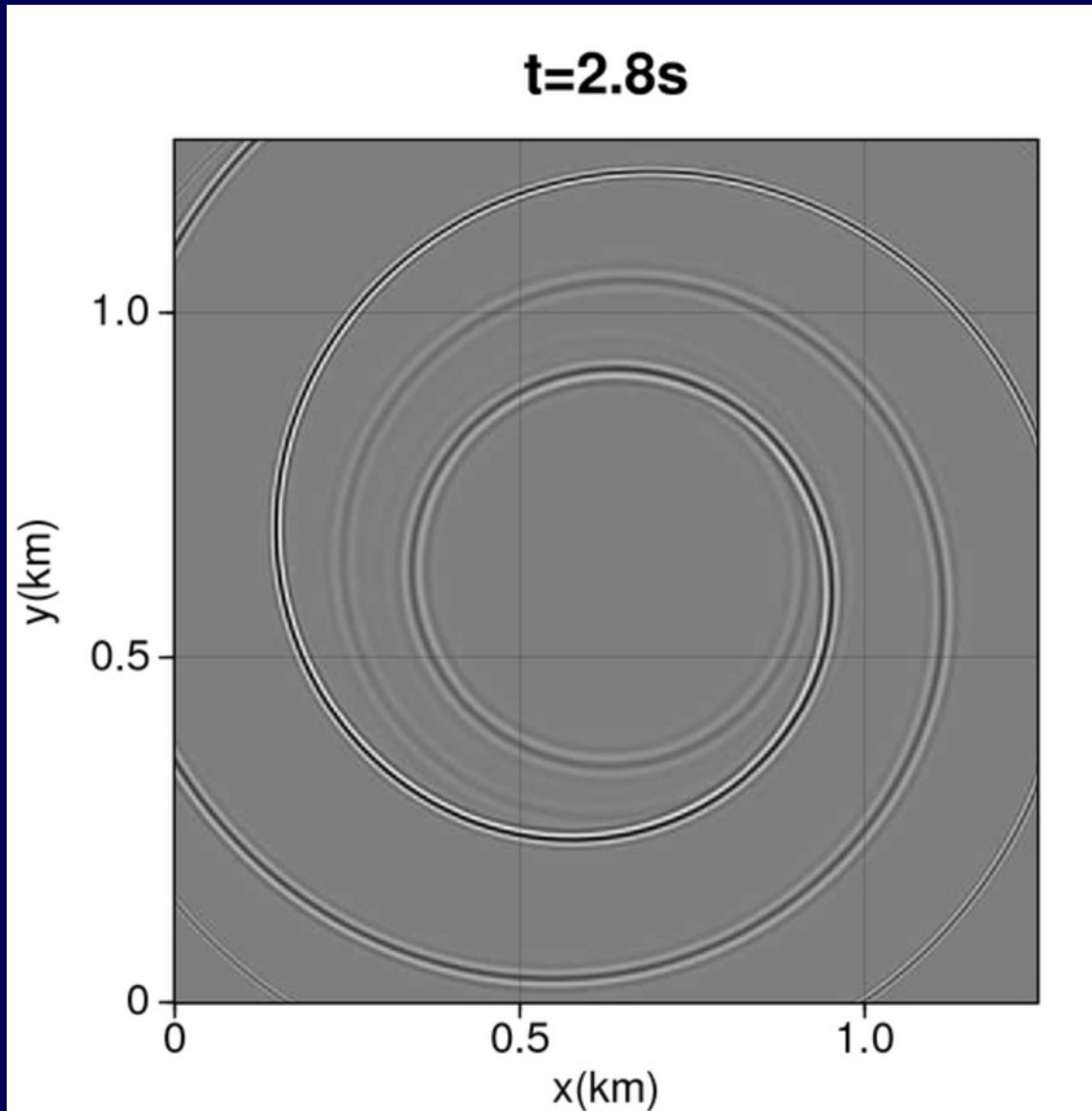
# Curved edge diffractions – synthetic – spherical spiral



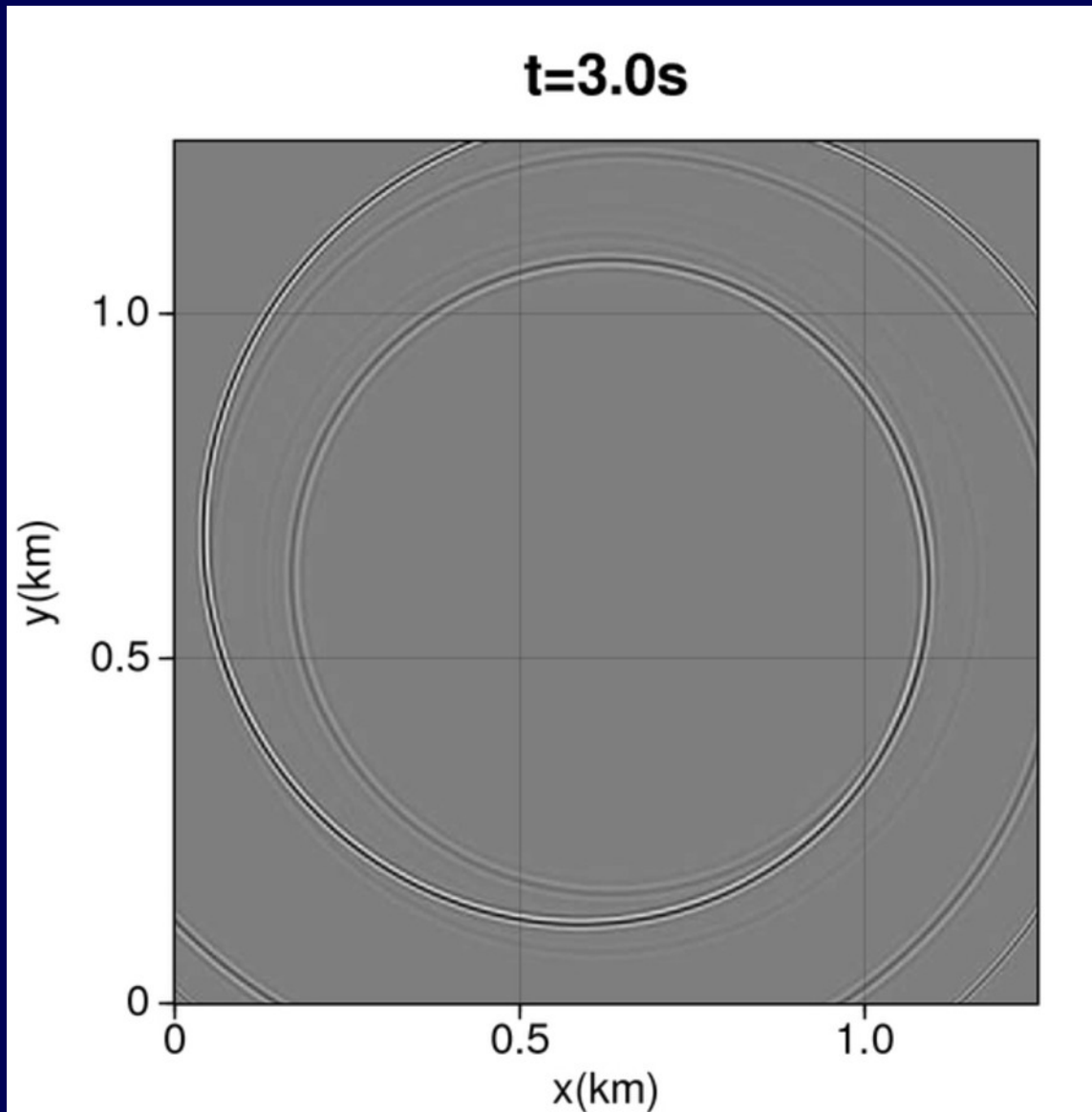
# Curved edge diffractions – synthetic – spherical spiral



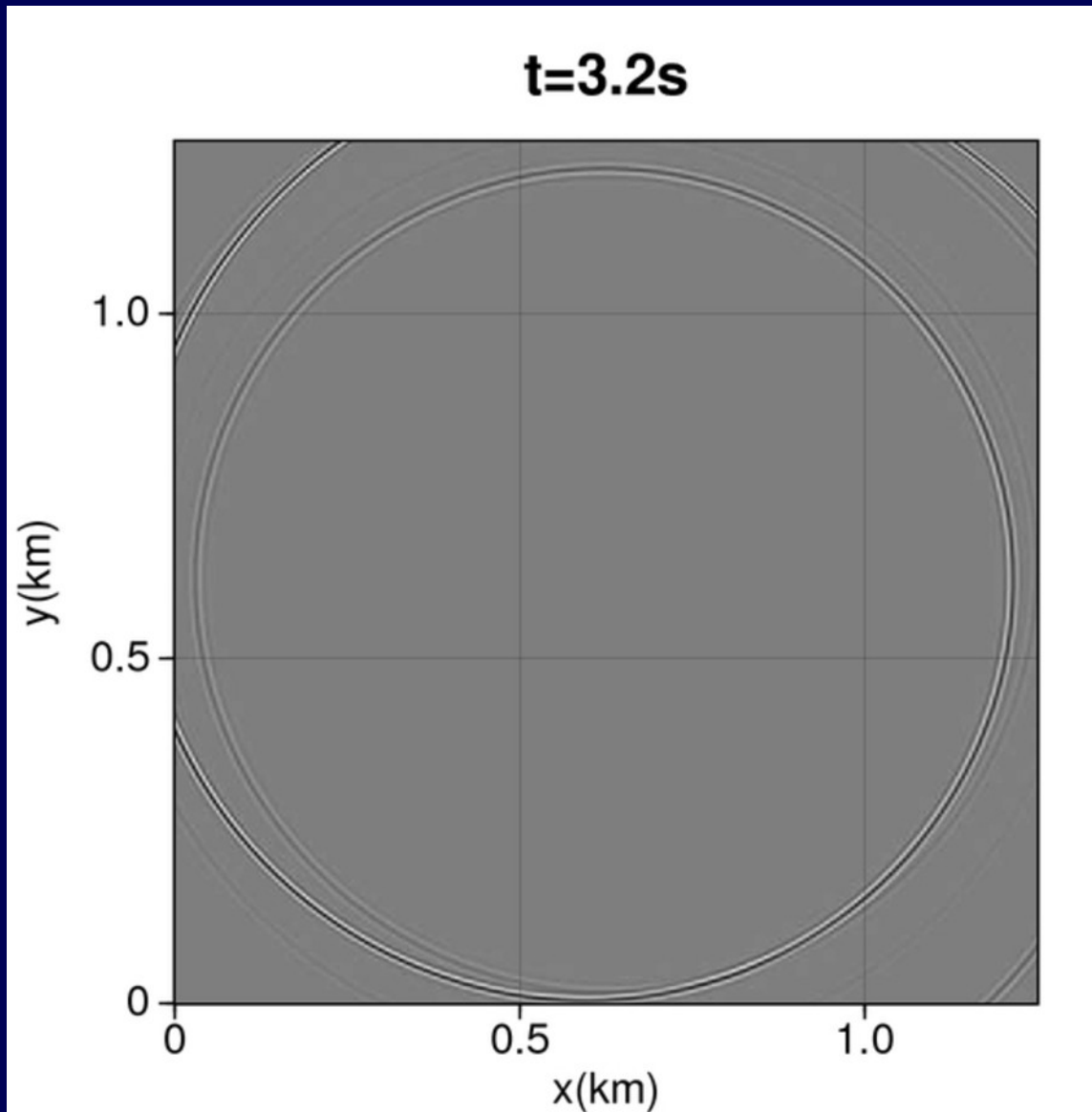
# Curved edge diffractions – synthetic – spherical spiral



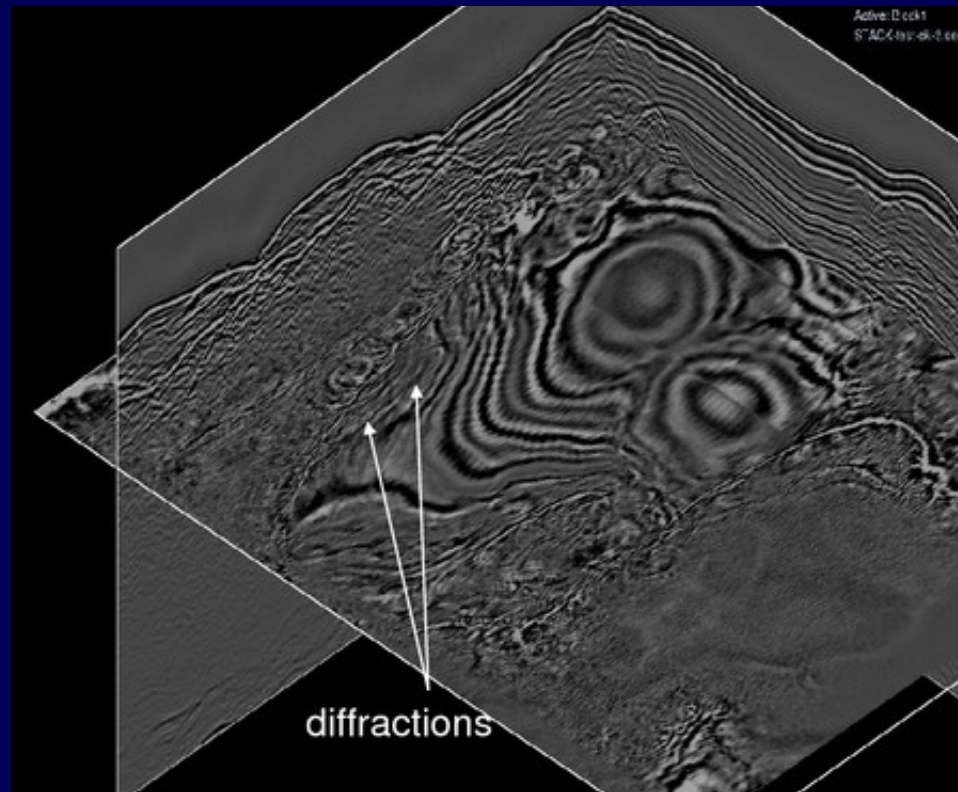
# Curved edge diffractions – synthetic – spherical spiral



# Curved edge diffractions – synthetic – spherical spiral



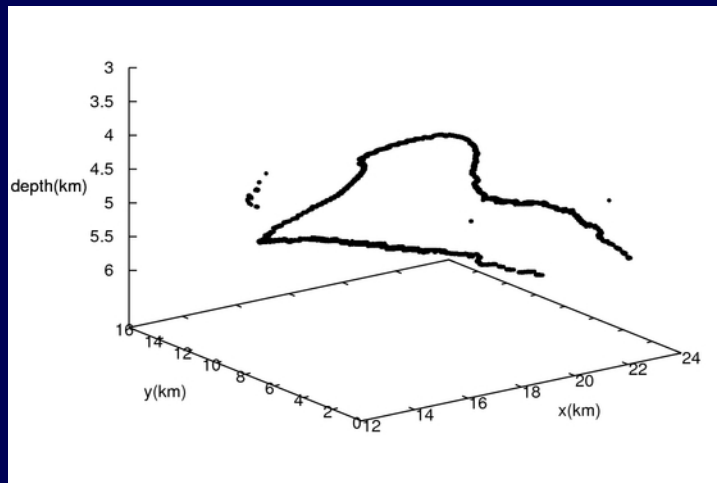
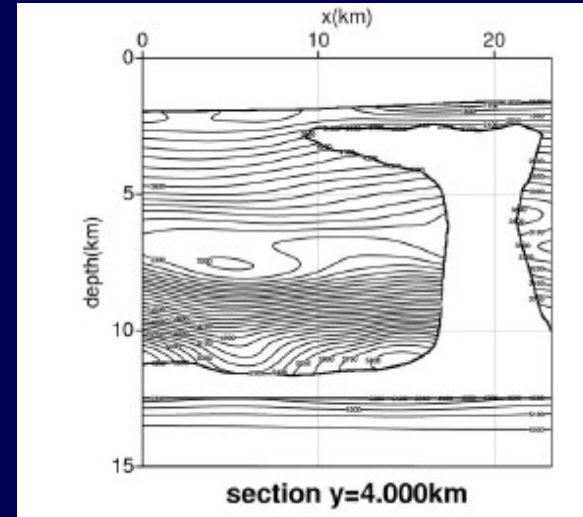
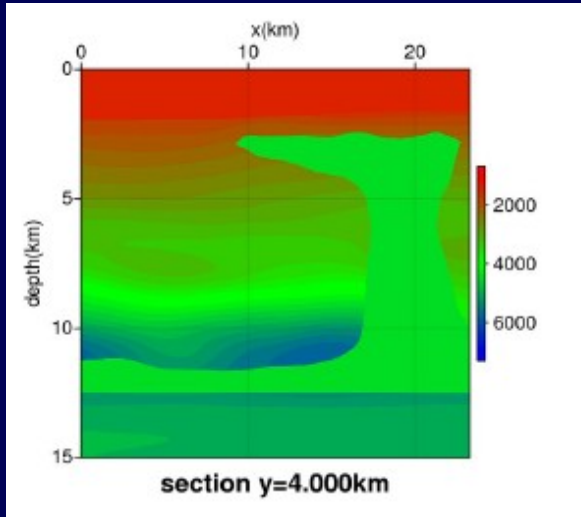
# Curved edge diffractions – Salt diapirs



## Diffractions from salt diapirs:

- How to model them?
- Can they support interpretation ?
- Can they distinguish between:
  - Edge/tip diffractions - fault tectonic
  - Convex bodies - erosional tectonic

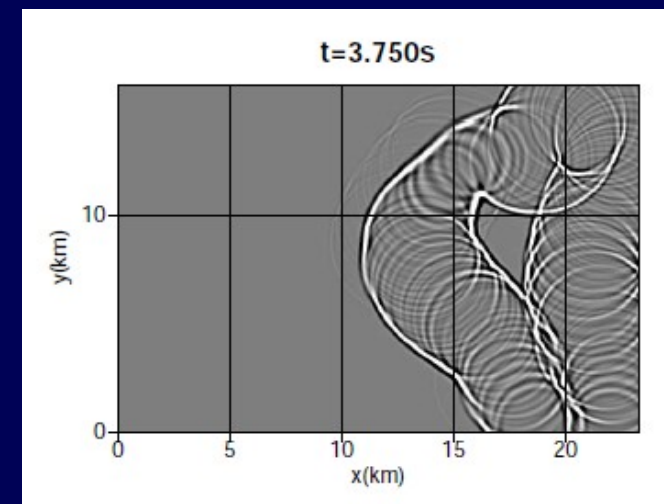
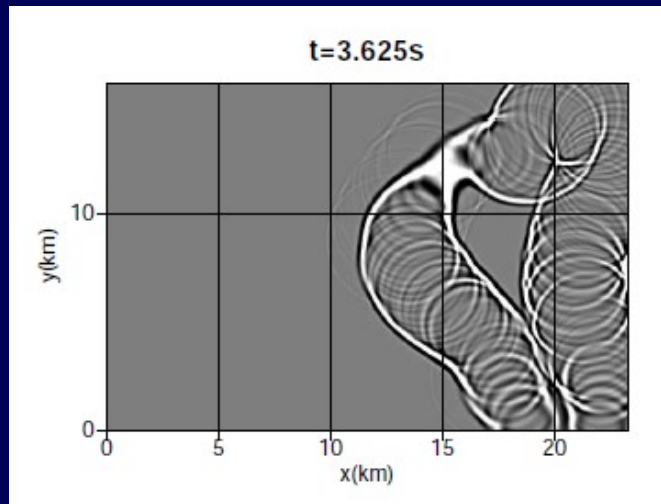
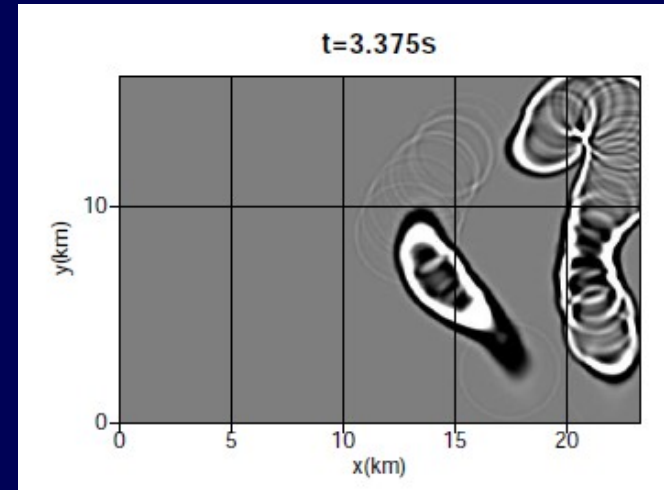
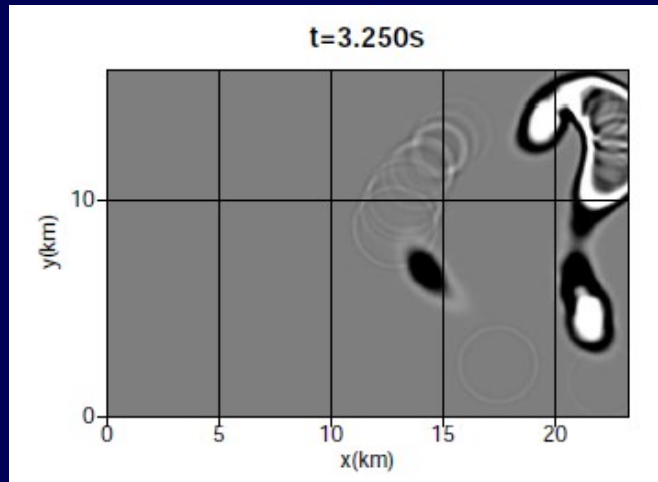
# Curved edge diffractions – GOM-B Salt diapir model



*Extraction of intersection of salt dome with 3km/s iso-velocity surface*



# Curved edge diffractions – GOM-B Salt diapir model



**Time slices**

*Note caustics resembling elliptical edge modeling*

# **Curved edge diffraction modeling – Conclusions**

- **Ray-Born versatile modeling tool for diffractions**
- **Diffraction-based workflow as a complement to reflection-based workflow**
- **Edge and tip diffractions elementary building elements for composite diffractions**
- **Importance of curved edge modeling (GPR, Salt diapir, ...)**
- **Impact on interpretation**

# **Acknowledgments**

**We thank Statoil for permission to show the GOM-B data and model.**

**We thank Mark Grasmueck (Miami Univ. Comparative Sedimentology Lab) for making available the GPR data.**

## **Events**

**Dedicated session 'Case Studies in Diffraction Imaging and Interpretation', EAGE Copenhagen 2012**

**Workshop 'Diffraction Methods for Fault and Fracture Detection', SEG Las Vegas 2012**