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Prestack traveltimes for DTI media

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Pavel Golikov[1], Tariq Alkhalifah[2] and Alexey Stovas[1]

[1]NTNU, Trondheim, Norway
[2]KAUST, Thuwal, Saudi Arabia

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Motivation

- DSR - the double-square-root operator being used for migration before stacking
- DTI - the special (but most practical) case for TTI media with symmetry axis being orthogonal to the dip

Outline

Double-square-root (DSR) equation

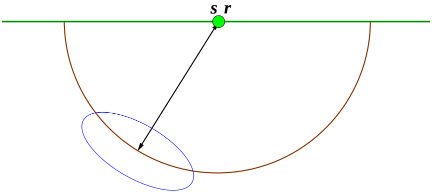
Anisotropic media with tilted symmetry axis

DSR equation for DTI

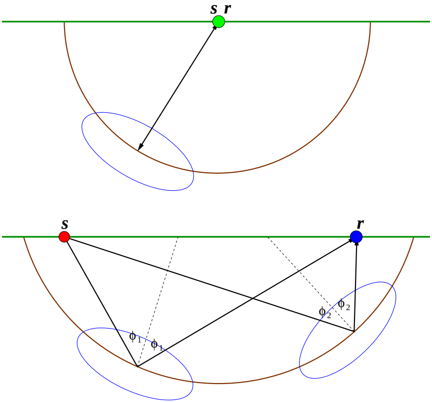
DTI traveltime parameters

Conclusions

DSR Equation

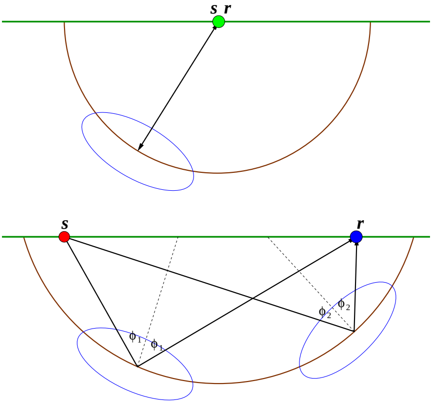


DSR Equation



Alkhalifah & Sava, 2010

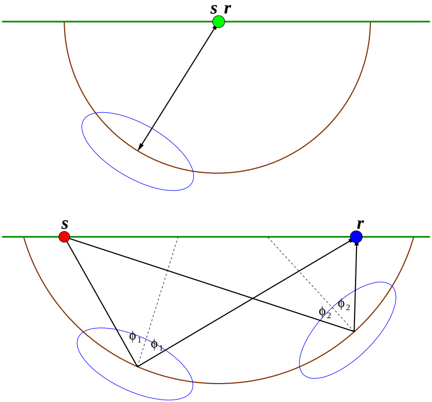
DSR Equation



$$t = \sqrt{\frac{(s_x - x)^2 + (s_y - y)^2 + z^2}{v_g^2(\phi)}} + \sqrt{\frac{(r_x - x)^2 + (r_y - y)^2 + z^2}{v_g^2(\phi)}}$$

Alkhalifah & Sava, 2010

DSR Equation



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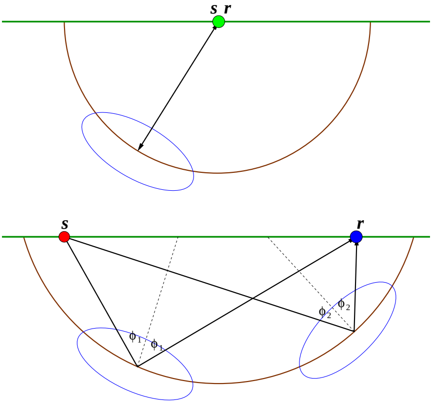
$$\frac{\partial t}{\partial z} = \sqrt{\frac{1}{v^2(\theta)} - \left(\frac{\partial t}{\partial s}\right)^2} + \sqrt{\frac{1}{v^2(\theta)} - \left(\frac{\partial t}{\partial r}\right)^2}$$

Alkhalifah & Sava, 2010

DSR Equation



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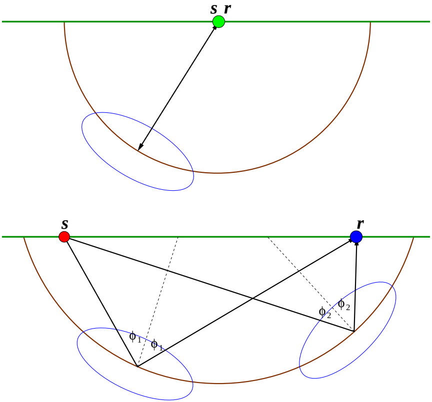
$$q = \sqrt{1 - p_s^2 v^2(\theta)} + \sqrt{1 - p_r^2 v^2(\theta)}$$

Alkhalifah & Sava, 2010

DSR Equation



Alkhalifah & Sava, 2010

$$t = \sqrt{\frac{(s_x - x)^2 + (s_y - y)^2 + z^2}{v_g^2(\phi)}} + \sqrt{\frac{(r_x - x)^2 + (r_y - y)^2 + z^2}{v_g^2(\phi)}}$$

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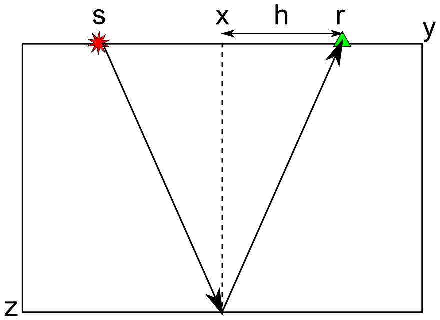
$$q = \sqrt{1 - p_s^2 v^2(\theta)} + \sqrt{1 - p_r^2 v^2(\theta)}$$

$$\frac{\partial U}{\partial z} = \frac{-i\omega q}{v} U$$

Jon Claerbout, 1985, Imaging of the Earth Interior

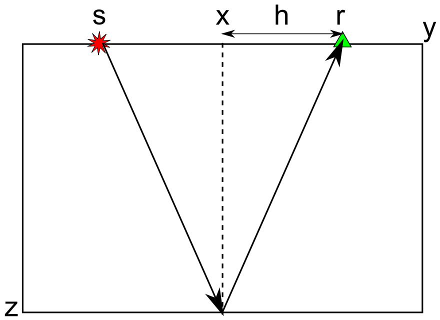
s-r and x-h coordinate systems



$$x = \frac{s + r}{2}$$

$$h = \frac{s - r}{2}$$

s-r and x-h coordinate systems



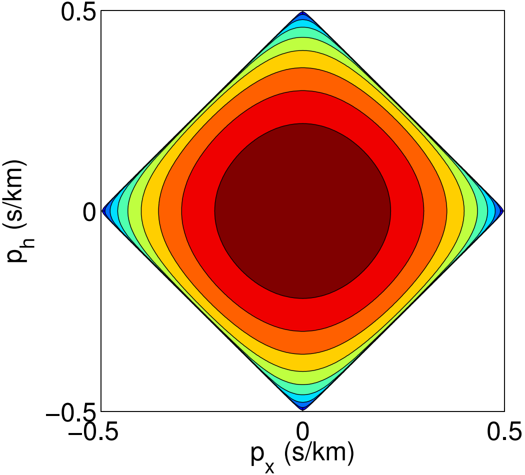
$$x = \frac{s + r}{2}$$

$$h = \frac{s - r}{2}$$

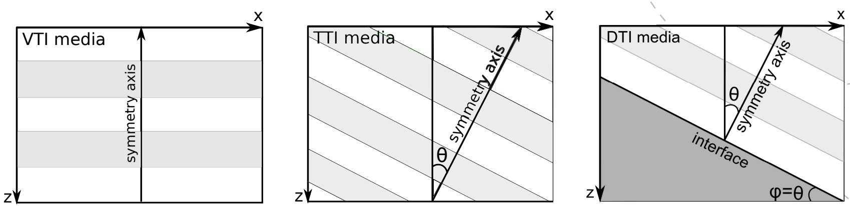
$$p_x = p_s + p_r$$

$$p_h = p_s - p_r$$

Isotropic DSR

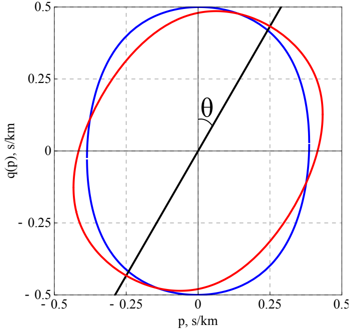


Tilted TI media



- Symmetry axis for transversely-isotropic (TI) media can be not only vertical (VTI) or horizontal (HTI) but also tilted (TTI).
- One of the most reasonable approximations for TTI is the model with the symmetry axis being normal to the reflector plane - the DTI model.

Slowness surface rotation



Slowness surfaces for VTI and TTI (tilt is 30°) media.

VTI slowness surface in acoustic approximation

$$q'^2_P - \frac{1}{v_0^2}\frac{1-(1+2\eta)(1+2\delta)p'^2 v_0^2}{1-2\eta(1+2\delta)p'^2 v_0^2} = 0$$

The rotation operator is defined as

$$\begin{pmatrix} p' \\ q' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} p \\ q \end{pmatrix}$$

where
v_0 is the symmetry-direction velocity,
$\eta = (\varepsilon - \delta)/(1+2\delta)$
q and p are vertical and horizontal slowness, respectively

TTI slowness surface

$$v_0^2(q\cos\theta + p\sin\theta)^2(1 - 2\eta(p\cos\theta - q\sin\theta)^2 v_n^2) +$$
$$(1 + 2\eta)(p\cos\theta - q\sin\theta)^2 v_n^2 - 1 = 0.$$

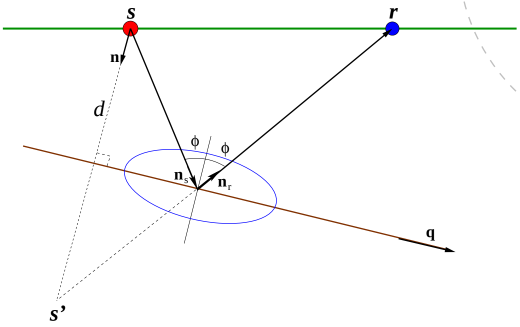
TTI slowness surface

$$v_0^2(q\cos\theta + p\sin\theta)^2(1 - 2\eta(p\cos\theta - q\sin\theta)^2 v_n^2) + (1+2\eta)(p\cos\theta - q\sin\theta)^2 v_n^2 - 1 = 0.$$

$$\Downarrow$$

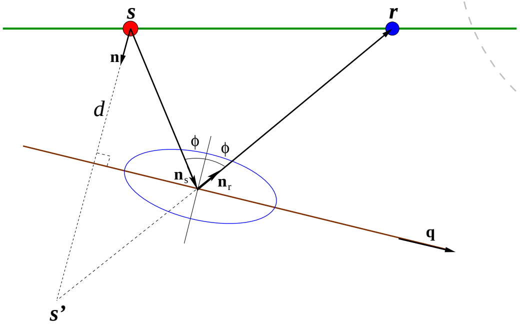
$$\begin{cases} v_0^2(q_s\cos\theta + p_s\sin\theta)^2(1 - 2\eta(p_s\cos\theta - q_s\sin\theta)^2 v_n^2) + (1+2\eta)(p_s\cos\theta - q_s\sin\theta)^2 v_n^2 - 1 = 0 \\ v_0^2(q_r\cos\theta + p_r\sin\theta)^2(1 - 2\eta(p_r\cos\theta - q_r\sin\theta)^2 v_n^2) + (1+2\eta)(p_r\cos\theta - q_r\sin\theta)^2 v_n^2 - 1 = 0 \end{cases}$$

DTI formulation



Alkhalifah & Sava, 2010

DTI formulation



Alkhalifah & Sava, 2010

$$\sin\theta = \frac{p_x}{\sqrt{p_x^2 + q^2}} = \frac{p_s + p_r}{\sqrt{(p_s + p_r)^2 + (q_s + q_r)^2}},$$

$$\cos\theta = \frac{q}{\sqrt{p_x^2 + q^2}} = \frac{q_s + q_r}{\sqrt{(p_s + p_r)^2 + (q_s + q_r)^2}},$$

DTI slowness surface

$$\begin{cases} f_1(q_s, q_r, p_s, p_r, v_0, \delta, \eta) = 0 \\ f_2(q_s, q_r, p_s, p_r, v_0, \delta, \eta) = 0 \end{cases}$$

where f_1 and f_2 are 8^{th}-order polynomial.

DTI slowness surface

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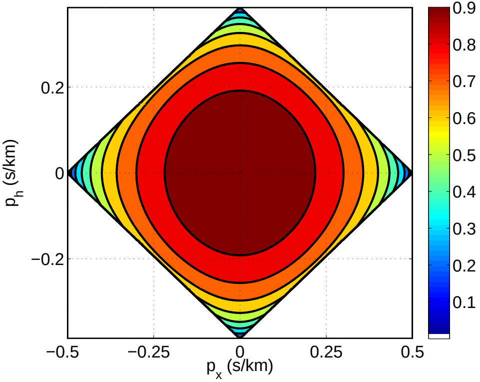
where f_1 and f_2 are 8^{th}-order polynomial.

Solution can be found:

- Numerically using least-squares minimization
- Analytically using perturbation theory

DTI slowness surface



Exact DSR surface for DTI as a numerical solution for the model with $v_0 = 2\,km/s$, $\eta = 0.2$, $\delta = 0.1$.

Perturbation in anelliptical parameter η

Assuming η is a small parameter

$$\begin{cases} q_s = q_{s0} + q_{s1}\eta + q_{s2}\eta^2 \\ q_r = q_{r0} + q_{r1}\eta + q_{r2}\eta^2 \end{cases}$$

Perturbation in anelliptical parameter η

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$$\begin{cases} q_s = q_{s0} + q_{s1}\eta + q_{s2}\eta^2 \\ q_r = q_{r0} + q_{r1}\eta + q_{r2}\eta^2 \end{cases}$$

$$q_{s0} = \frac{1}{v_0}\sqrt{\frac{1-(1+2\delta)(p_h-p_x)^2v_0^2+2\delta A^2}{(1+2\delta)}},$$

$$q_{r0} = \frac{1}{v_0}\sqrt{\frac{1-(1+2\delta)(p_h+p_x)^2v_0^2+2\delta A^2}{(1+2\delta)}},$$

$$A = \frac{1}{2}\left[\sqrt{(1-p_xv_0)^2-(1+2\delta)p_h^2v_0^2}+\sqrt{(1+p_xv_0)^2-(1+2\delta)p_h^2v_0^2}\right]$$

Shanks transformation

$$A_0 = q_{s0} + q_{r0}$$

$$A_1 = A_0 + (q_{s1} + q_{r1})\eta$$

$$A_2 = A_1 + (q_{s2} + q_{r2})\eta^2$$

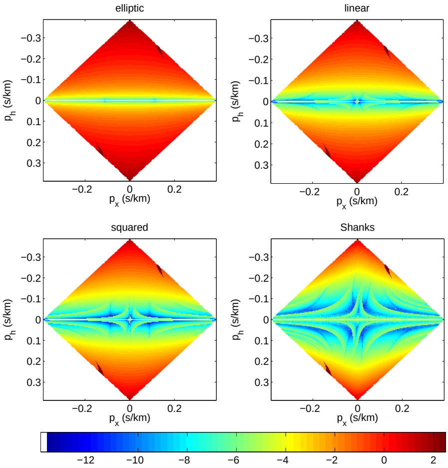
Shanks transformation

$$A_0 = q_{s0} + q_{r0}$$
$$A_1 = A_0 + (q_{s1} + q_{r1})\eta$$
$$A_2 = A_1 + (q_{s2} + q_{r2})\eta^2$$

$$q_{Sh} = \frac{A_0 A_2 - A_1^2}{A_0 + A_2 - 2A_1}$$

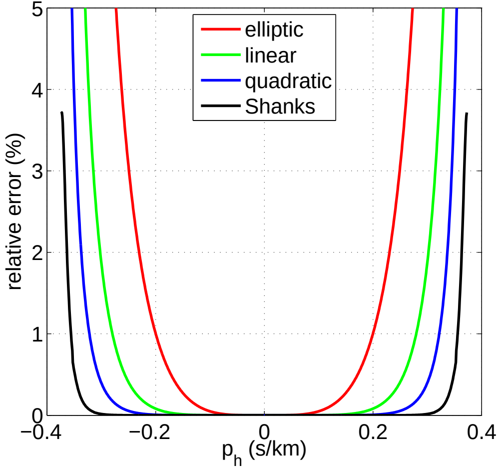
Bender and Orszag, 1978

Accuracy test

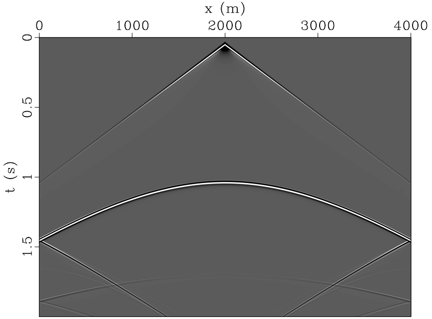


$v_0 = 2\,km/s$, $\delta = 0.1$ and $\eta = 0.2$

Accuracy test

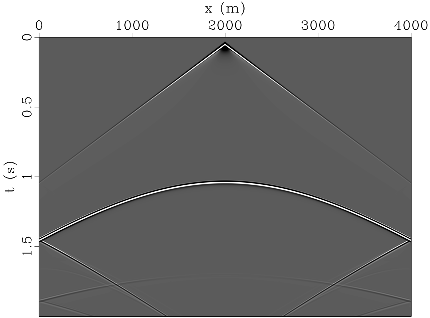


Traveltime parameters



Recorded data

Traveltime parameters



Recorded data

Taylor expansion for traveltime:

$$t^2(x) \approx t_0^2 + \frac{x^2}{v_n^2} + \frac{(1 - S_2)x^4}{4v_n^4 t_0^2}$$

DTI traveltime parameters

$$q = q_0 + q_1 p_h + q_2 p_h^2 + q_3 p_h^3 + q_4 p_h^4$$

DTI traveltime parameters

$$q = q_0 + q_1 p_h + q_2 p_h^2 + q_3 p_h^3 + q_4 p_h^4$$

$$t_0 = z q_0$$
$$v_n^2 = -2 q_2 / q_0$$
$$S_2 = -2 q_0 q_4 / q_2^2$$

DTI traveltime parameters

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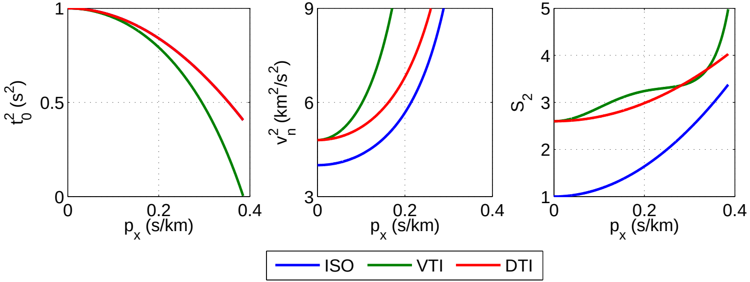
$$t_0 = z q_0$$
$$v_n^2 = -2 q_2 / q_0$$
$$S_2 = -2 q_0 q_4 / q_2^2$$

$$\longrightarrow$$

$$t_0 = \frac{2z}{v_0} \sqrt{1 - p_x^2 v_0^2}$$
$$v_n^2 = \frac{v_0^2 (1 + 2\delta)}{(1 - p_x^2 v_0^2)^2}$$
$$S_2 = 1 + 4 p_x^2 v_0^2 + 8\eta (1 - p_x^2 v_0^2)$$

DTI traveltime parameters



Traveltime parameters for isotropic (blue), VTI (green) and DTI (red) models with parameters $v_0 = 2\,km/s$, $\eta = 0.2$, $\delta = 0.1$ and $z = 1\,km$.

Inversion of traveltime parameters

Assuming z and p_x are known

$$t_0 = \frac{2z}{v_0}\sqrt{1 - p_x^2 v_0^2}$$

$$v_n^2 = \frac{v_0^2(1+2\delta)}{(1-p_x^2 v_0^2)^2}$$

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$$v_n^2 = \frac{v_0^2(1+2\delta)}{(1 - p_x^2 v_0^2)^2} \quad \rightarrow$$

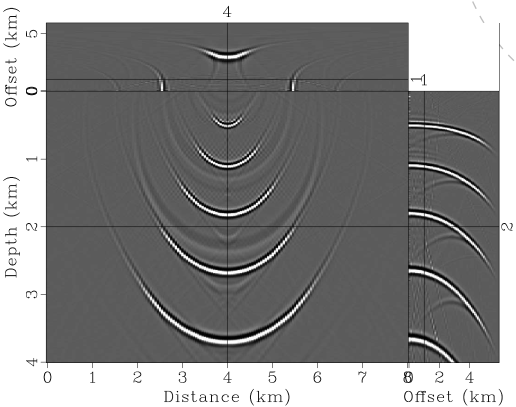
$$S_2 = 1 + 4p_x^2 v_0^2 + 8\eta(1 - p_x^2 v_0^2)$$

$$v_0 = \frac{2z}{\sqrt{t_0^2 + 4z^2 p_x^2}}$$

$$\delta = \frac{v_n^2(1 - p_x^2 v_0^2)^2}{2v_0^2} - 1$$

$$\eta = \frac{S_2 - 1 - 4p_x^2 v_0^2}{8(1 - p_x^2 v_0^2)}$$

Impulse response



- 5 pulses at zero-offset and vertical time 0.6, 1.2, 1.8, 2.4, 3 s.
- Model: $v(z) = 1.5 + 0.6z$, $\delta = 0$ and $\eta = 0.2$

Conclusions

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- DSR equation for DTI media derived by perturbation in anelliptic parameter η
- Proposed approximation gives high accuracy for wide range of offsets and dips
- Traveltime parameters for DTI media are given by simple formulas that can be used for inversion

Acknowledgments

We would like to acknowledge the ROSE project and KAUST.

Thank you!