

Borehole Geomechanics

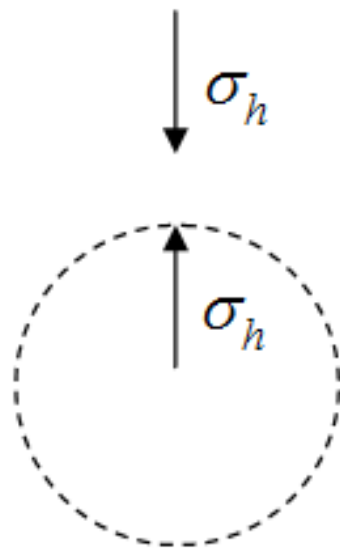
ROSE

Rock Physics and Geomechanics

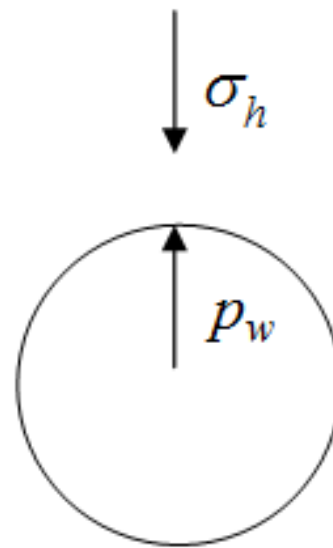
Course 2012

Erling Tjørr

Consider a place in the ground where we are going to drill a hole.



Before
we drill



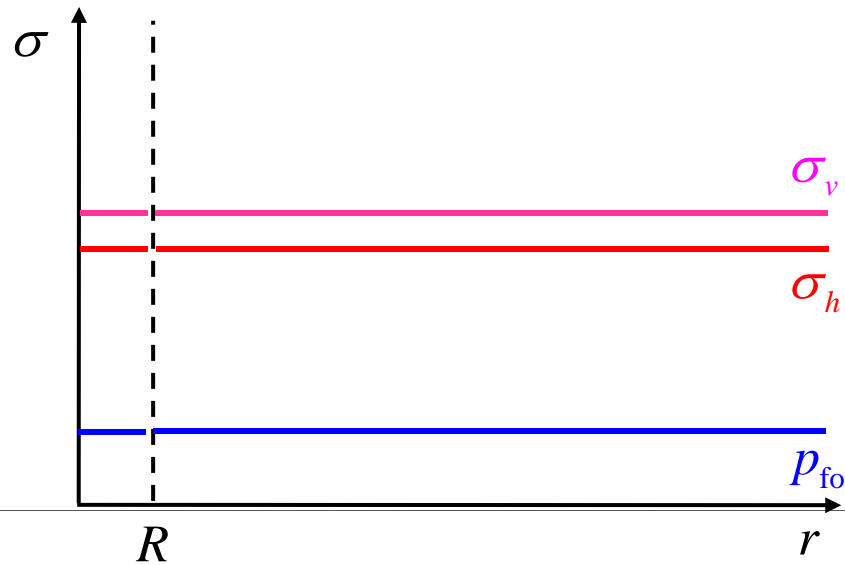
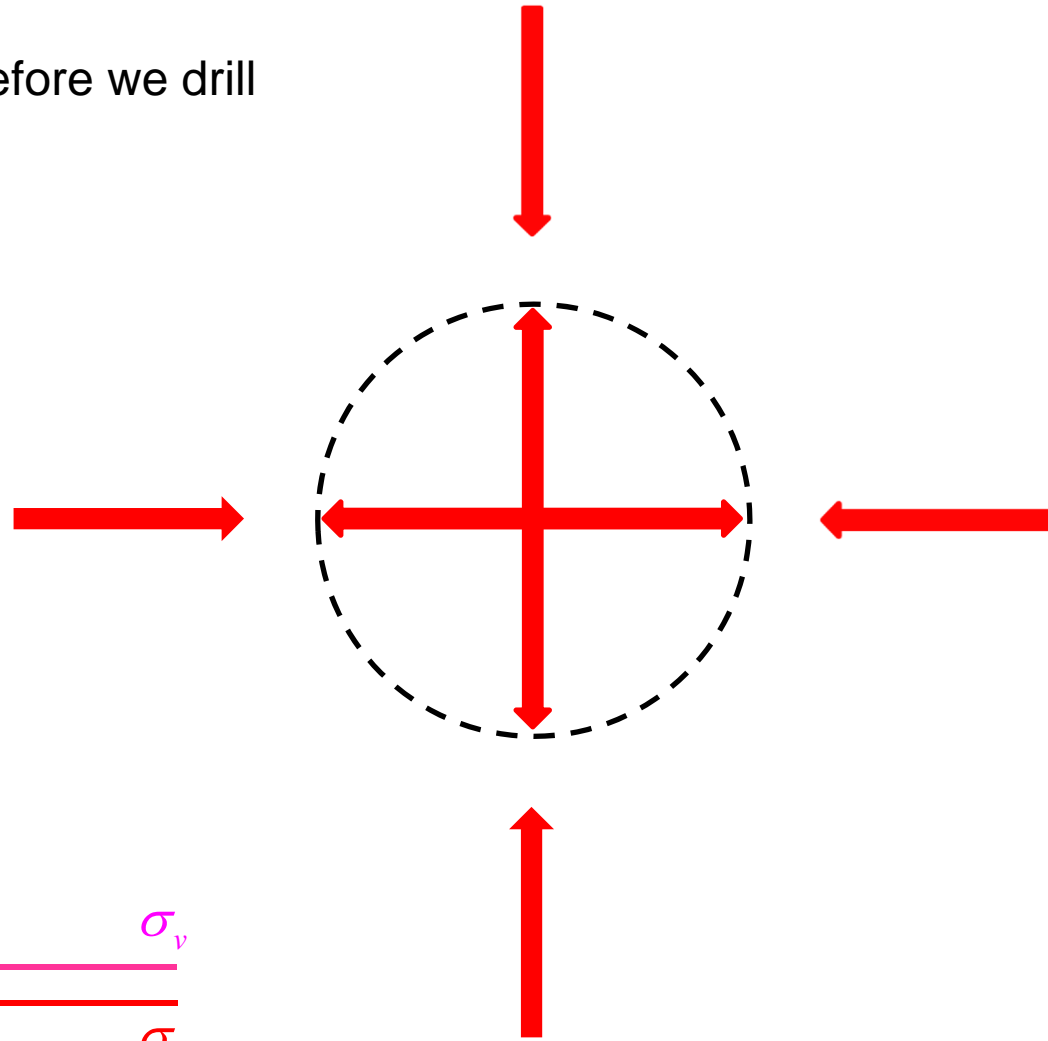
After
we have drilled

p_w = well pressure

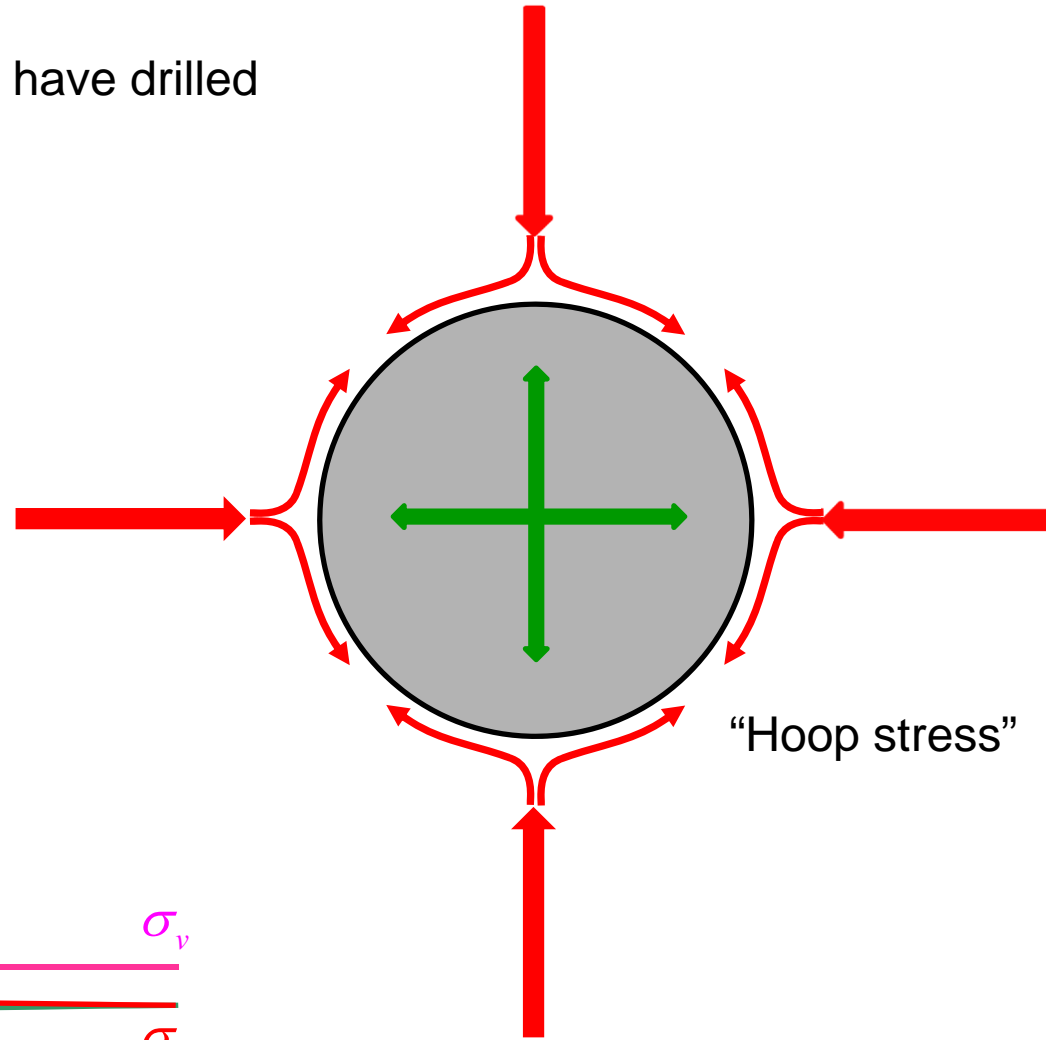
The stress at
the borehole wall
has changed.

What about the
stress around the
hole?

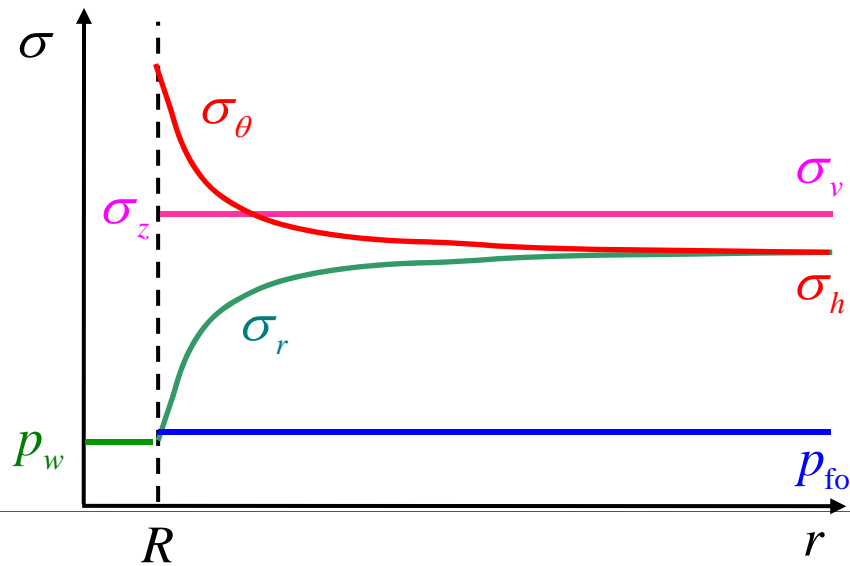
Before we drill



After we have drilled



“Hoop stress”



The hole is cylindrical – we use cylindrical coordinates

Stresses:

$$\sigma_r = \frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2}(\sigma_x - \sigma_y) \cos 2\theta + \tau_{xy} \sin 2\theta$$

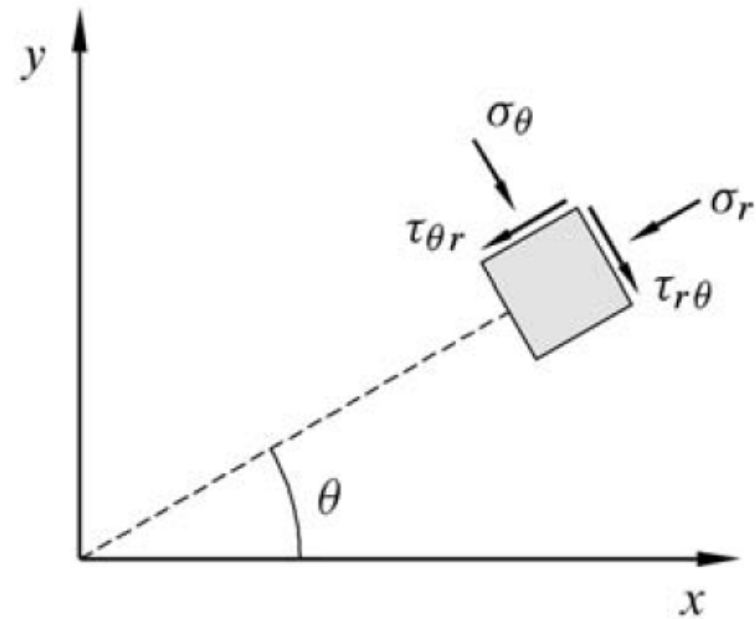
$$\sigma_\theta = \frac{1}{2}(\sigma_x + \sigma_y) - \frac{1}{2}(\sigma_x - \sigma_y) \cos 2\theta - \tau_{xy} \sin 2\theta$$

$$\sigma_z = \sigma_z$$

$$\tau_{r\theta} = \frac{1}{2}(\sigma_y - \sigma_x) \sin 2\theta + \tau_{xy} \cos 2\theta$$

$$\tau_{rz} = \tau_{xz} \cos \theta + \tau_{yz} \sin \theta$$

$$\tau_{\theta z} = \tau_{yz} \cos \theta - \tau_{xz} \sin \theta$$



The hole is cylindrical – we use cylindrical coordinates

Strains:

$$\varepsilon_r = \frac{\partial u}{\partial r}$$

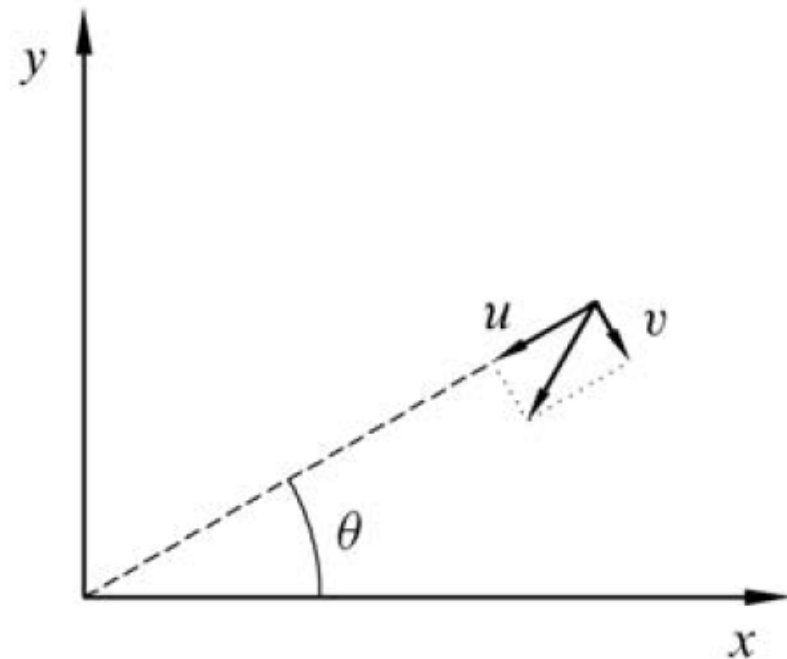
$$\varepsilon_\theta = \frac{u}{r} + \frac{1}{r} \frac{\partial v}{\partial \theta}$$

$$\varepsilon_z = \frac{\partial w}{\partial z}$$

$$\Gamma_{r\theta} = \frac{1}{2r} \left(\frac{\partial u}{\partial \theta} - v \right) + \frac{1}{2} \frac{\partial v}{\partial r}$$

$$\Gamma_{rz} = \frac{1}{2} \left(\frac{\partial w}{\partial r} + \frac{\partial u}{\partial z} \right)$$

$$\Gamma_{\theta z} = \frac{1}{2} \left(\frac{1}{r} \frac{\partial w}{\partial \theta} + \frac{\partial v}{\partial z} \right)$$



The hole is cylindrical – we use cylindrical coordinates

Hooke's law in cylindrical coordinates:

$$\sigma'_r = (\lambda_{\text{fr}} + 2G_{\text{fr}})\varepsilon_r + \lambda_{\text{fr}}\varepsilon_\theta + \lambda_{\text{fr}}\varepsilon_z$$

$$\sigma'_\theta = \lambda_{\text{fr}}\varepsilon_r + (\lambda_{\text{fr}} + 2G_{\text{fr}})\varepsilon_\theta + \lambda_{\text{fr}}\varepsilon_z$$

$$\sigma'_z = \lambda_{\text{fr}}\varepsilon_r + \lambda_{\text{fr}}\varepsilon_\theta + (\lambda_{\text{fr}} + 2G_{\text{fr}})\varepsilon_z$$

$$\tau_{r\theta} = 2G_{\text{fr}}\Gamma_{r\theta}$$

$$\tau_{rz} = 2G_{\text{fr}}\Gamma_{rz}$$

$$\tau_{\theta z} = 2G_{\text{fr}}\Gamma_{\theta z}$$

The hole is cylindrical – we use cylindrical coordinates

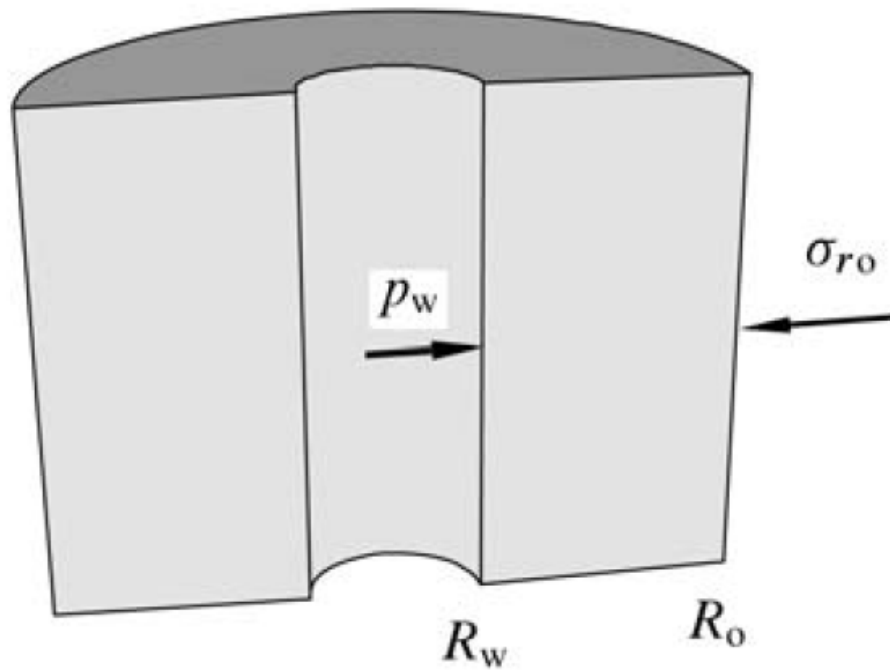
Equations of equilibrium, in cylindrical coordinates:

$$\frac{\partial \sigma_r}{\partial r} + \frac{1}{r} \frac{\partial \tau_{\theta r}}{\partial \theta} + \frac{\partial \tau_{zr}}{\partial z} + \frac{\sigma_r - \sigma_\theta}{r} + \rho f_r = 0$$

$$\frac{1}{r} \frac{\partial \sigma_\theta}{\partial \theta} + \frac{\partial \tau_{r\theta}}{\partial r} + \frac{\partial \tau_{z\theta}}{\partial z} + \frac{2\tau_{r\theta}}{r} + \rho f_\theta = 0$$

$$\frac{\partial \sigma_z}{\partial z} + \frac{\partial \tau_{rz}}{\partial r} + \frac{1}{r} \frac{\partial \tau_{\theta z}}{\partial \theta} + \frac{\tau_{rz}}{r} + \rho f_z = 0$$

The hollow cylinder



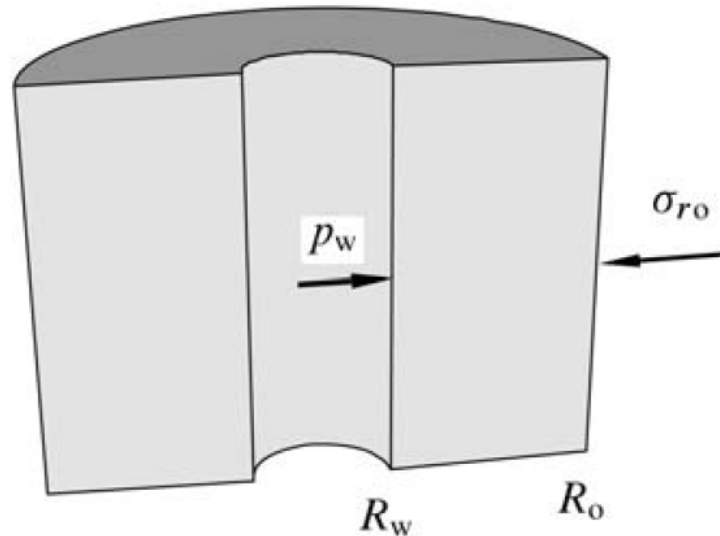
- a simple model of the formation around a well

The hollow cylinder

- a relevant model for laboratory tests



The hollow cylinder



External stresses:

σ_v vertical stress

σ_{ro} lateral stress

p_w pressure in the hole

Assumptions:

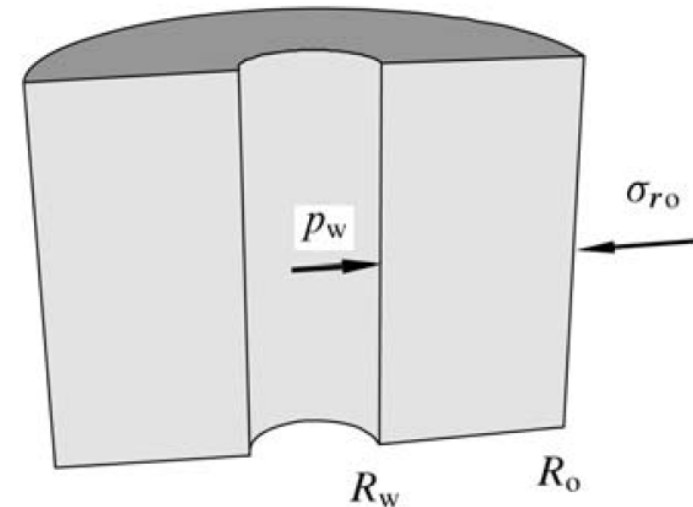
- Full rotational symmetry around the axis
- Full translational symmetry along the axis
- Plane strain

Displacement only in
radial direction

Boundary conditions:

$$\sigma_r = p_w \quad \text{for } r = R_w$$

$$\sigma_r = \sigma_{ro} \quad \text{for } r = R_o$$



⇒ Stresses in the hollow cylinder:

$$\sigma_r = \frac{R_o^2 \sigma_{ro} - R_w^2 p_w}{R_o^2 - R_w^2} - \frac{R_o^2}{R_o^2 - R_w^2} \frac{R_w^2}{r^2} (\sigma_{ro} - p_w)$$

$$\sigma_\theta = \frac{R_o^2 \sigma_{ro} - R_w^2 p_w}{R_o^2 - R_w^2} + \frac{R_o^2}{R_o^2 - R_w^2} \frac{R_w^2}{r^2} (\sigma_{ro} - p_w)$$

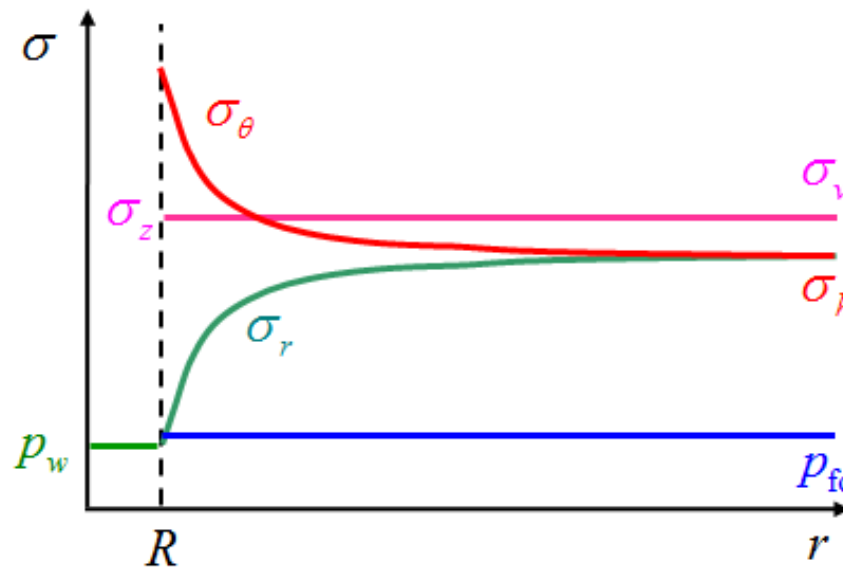
Assuming a vertical well, and isotropic horizontal stresses σ_h
 \Rightarrow

$$\sigma_r = \sigma_h - (\sigma_h - p_w) \frac{R_w^2}{r^2} = \left(1 - \frac{R_w^2}{r^2}\right) \sigma_h + \frac{R_w^2}{r^2} p_w$$

$$\sigma_\theta = \sigma_h + (\sigma_h - p_w) \frac{R_w^2}{r^2} = \left(1 + \frac{R_w^2}{r^2}\right) \sigma_h - \frac{R_w^2}{r^2} p_w$$

$$\sigma_z = \text{const}$$

Assuming constant
pore pressure



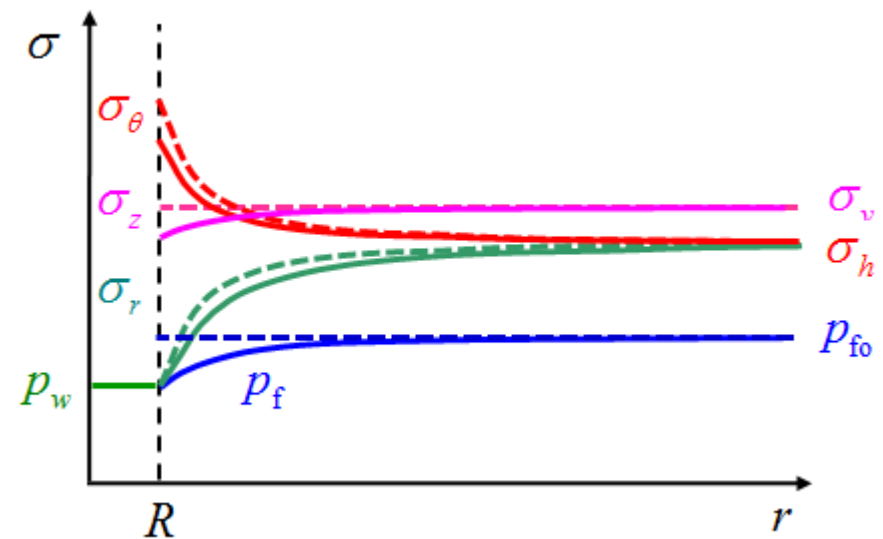
Flow towards a cylindrical hole – impact on stresses

If the drainage radius is much larger than the borehole radius, we have:

$$\sigma_r = \sigma_h - (\sigma_h - p_w) \left(\frac{R_w}{r} \right)^2 + (p_{fo} - p_w) \eta \left[\left(\frac{R_w}{r} \right)^2 - \frac{\ln(R_e/r)}{\ln(R_e/R_w)} \right]$$

$$\sigma_\theta = \sigma_h + (\sigma_h - p_w) \left(\frac{R_w}{r} \right)^2 - (p_{fo} - p_w) \eta \left[\left(\frac{R_w}{r} \right)^2 + \frac{\ln(R_e/r)}{\ln(R_e/R_w)} \right]$$

$$\sigma_z = \sigma_v - (p_{fo} - p_w) \eta \frac{2 \ln(R_e/r) - \nu_{fr}}{\ln(R_e/R_w)}$$



Solid lines: with fluid flow
Dashed lines: without fluid flow

Heat flow around a well – effect on stresses

Note: Hooke's law for

Poroelasticity: $\sigma_{ij} = \lambda_{fr} \varepsilon_{vol} \delta_{ij} + 2G \varepsilon_{ij} + \alpha p_f \delta_{ij}$

Thermoelasticity: $\sigma_{ij} = \lambda \delta_{ij} \varepsilon_{vol} + 2G \varepsilon_{ij} + 3\alpha_T K (T - T_0) \delta_{ij}$

Poroelasticity \leftrightarrow Thermoelasticity

$$p_f \leftrightarrow T - T_0$$

$$\alpha \leftrightarrow 3K\alpha_T = \frac{E}{1-2\nu} \alpha_T$$

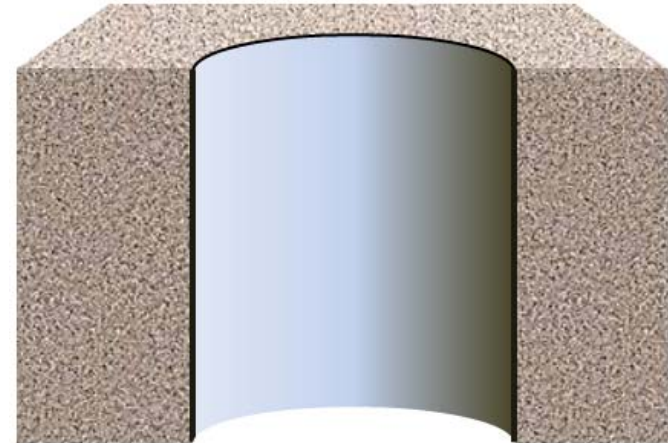
\Rightarrow We may transfer solutions from poroelasticity to thermoelasticity, and vice versa

Cooling of the mud reduces the tangential and axial stresses at the borehole wall

$$\sigma_r = p_w$$

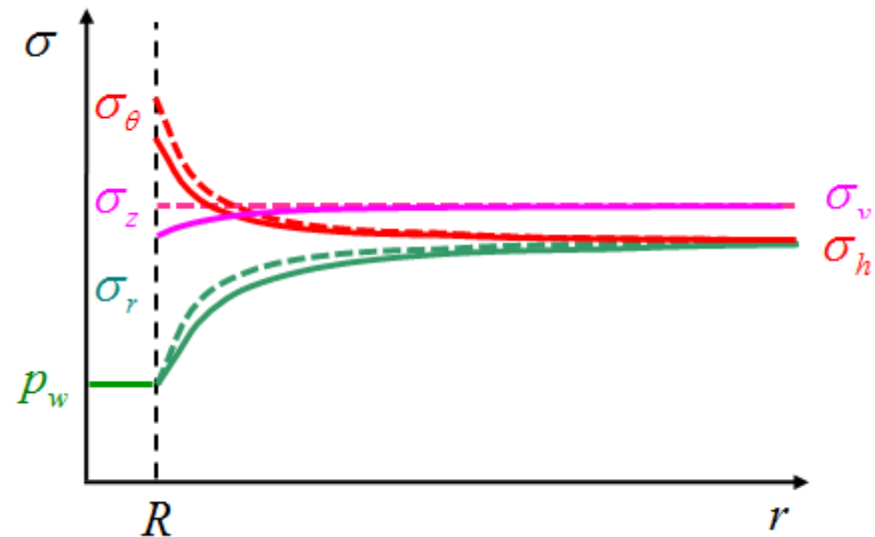
$$\sigma_\theta = 2\sigma_h - p_w + \frac{E_{fr}}{1 - \nu_{fr}} \alpha_T (T_w - T_o)$$

$$\sigma_z = \sigma_v + \frac{E_{fr}}{1 - \nu_{fr}} \alpha_T (T_w - T_o)$$



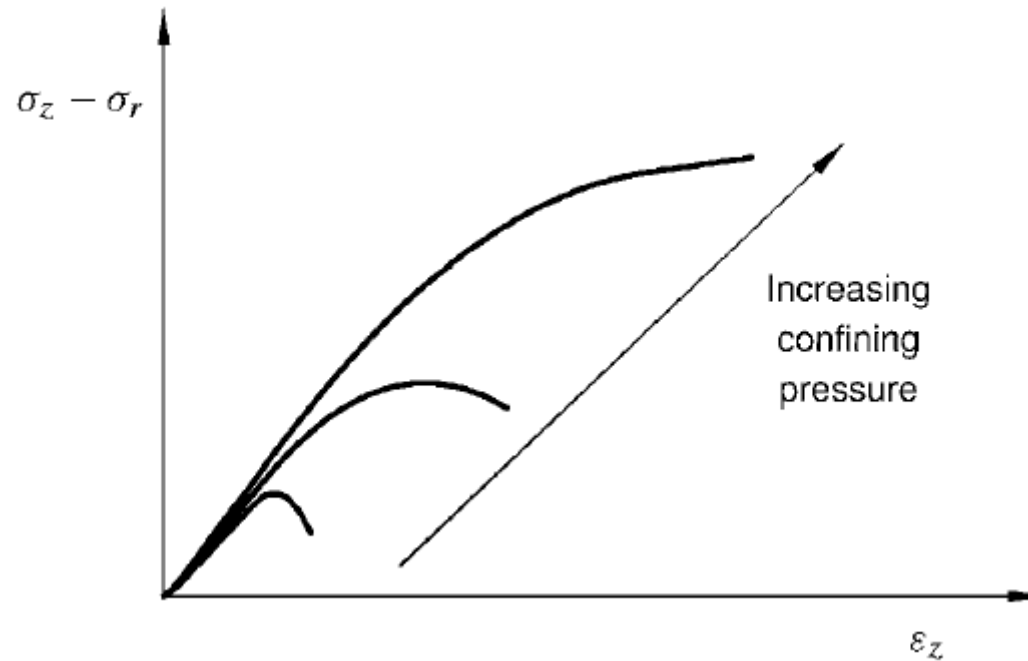
Solid lines: the mud is colder than the formation

Dashed lines: the mud and the formation have the same temperature



Effects of non-linear elasticity

Common observation: Young's modulus increases with increasing confinement.



Effects of non-linear elasticity

Common observation: Young's modulus increases with increasing confinement.

Santarelli et al.: $E(\sigma_r) = E_0 \sigma_r^a$

$$\Rightarrow \sigma_r = \sigma_h \left\{ \left[\left(\frac{p_w}{\sigma_h} \right)^{1-a} - 1 \right] \left(\frac{R_w}{r} \right)^N + 1 \right\}^{\frac{1}{1-a}}$$

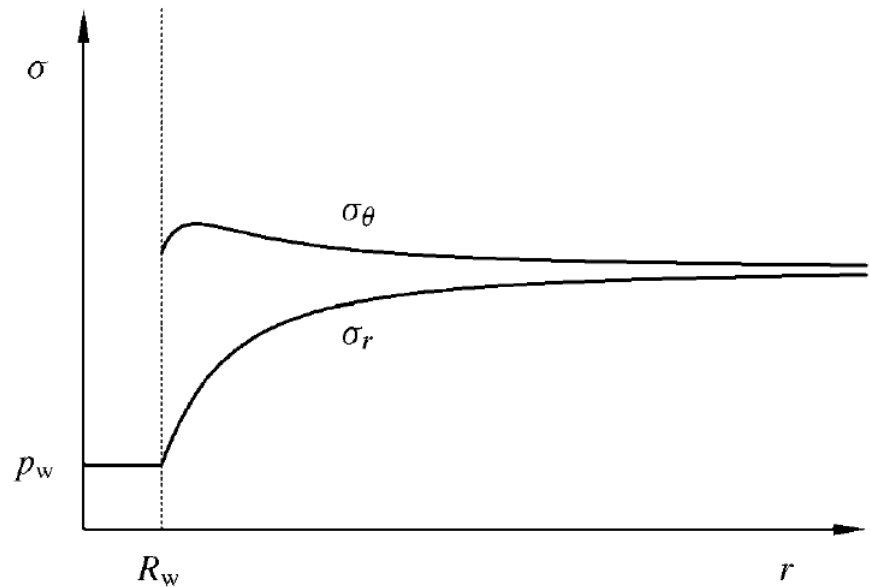
$$\sigma_\theta = \frac{N}{1-a} \sigma_h \left(\frac{\sigma_r}{\sigma_h} \right)^a + M \sigma_r$$

$$N = \frac{1}{1 - \nu_{fr}} [(1 - 2\nu_{fr})(1 - a) + 1]$$

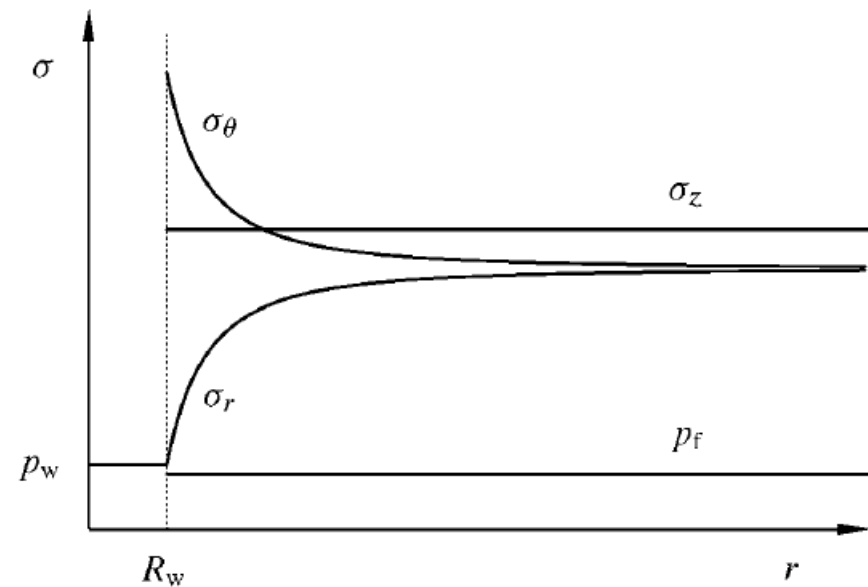
$$M = \frac{\nu_{fr}(1 - a) - 1}{(1 - \nu_{fr})(1 - a)}$$

Effects of non-linear elasticity

$$E(\sigma_r) = E_0 \sigma_r^a$$



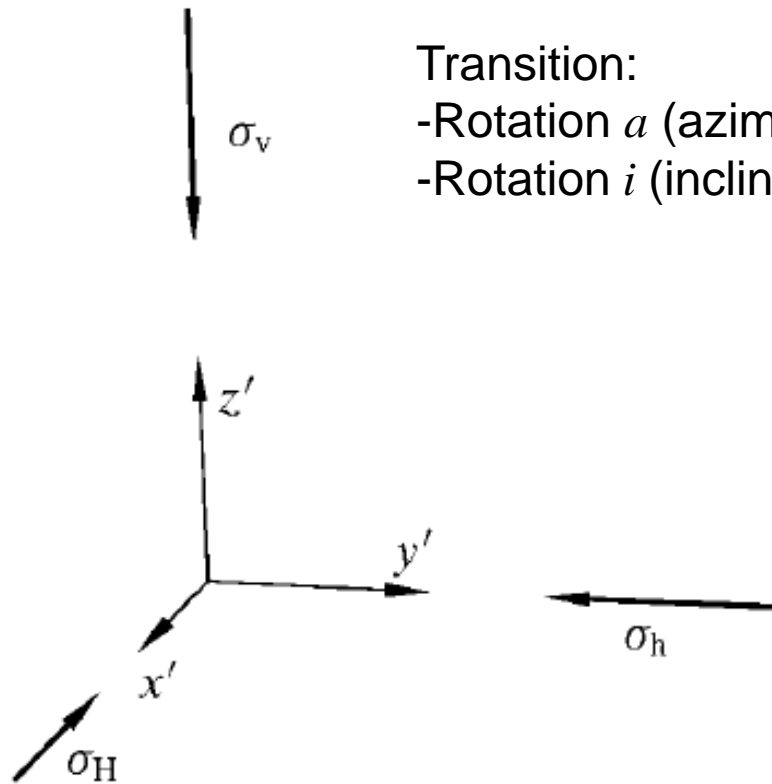
Linear elastic solution



Note: Non-linear elasticity implies a reduction of the tangential (and also the axial) stress in the vicinity of the hole

General solution

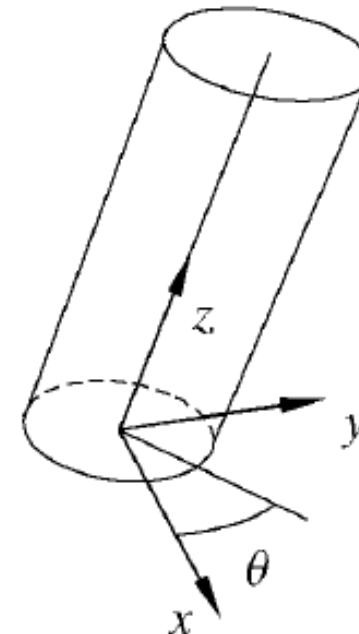
- deviating well, anisotropic stresses



Coordinate system of the stresses

Transition:

- Rotation a (azimuth) around the z' -axis
- Rotation i (inclination) around the y -axis



Coordinate system of the well

Formation stresses expressed in well coordinates:

$$\sigma_x^o = l_{xx'}^2 \sigma_H + l_{xy'}^2 \sigma_h + l_{xz'}^2 \sigma_v$$

$$\sigma_y^o = l_{yx'}^2 \sigma_H + l_{yy'}^2 \sigma_h + l_{yz'}^2 \sigma_v$$

$$\sigma_z^o = l_{zx'}^2 \sigma_H + l_{zy'}^2 \sigma_h + l_{zz'}^2 \sigma_v$$

$$\tau_{xy}^o = l_{xx'} l_{yx'} \sigma_H + l_{xy'} l_{yy'} \sigma_h + l_{xz'} l_{yz'} \sigma_v$$

$$\tau_{yz}^o = l_{yx'} l_{zx'} \sigma_H + l_{yy'} l_{zy'} \sigma_h + l_{yz'} l_{zz'} \sigma_v$$

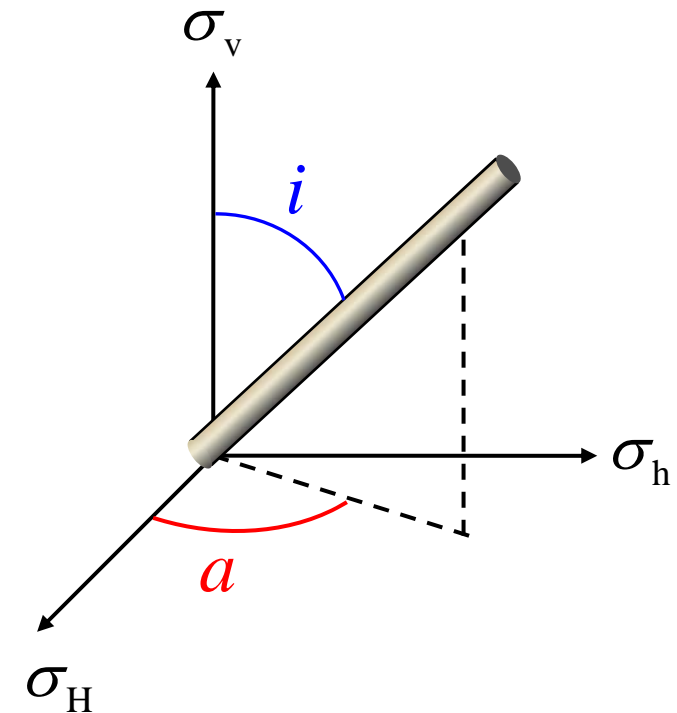
$$\tau_{zx}^o = l_{zx'} l_{xx'} \sigma_H + l_{zy'} l_{xy'} \sigma_h + l_{zz'} l_{xz'} \sigma_v$$

where

$$l_{xx'} = \cos a \cos i, \quad l_{xy'} = \sin a \cos i, \quad l_{xz'} = -\sin i$$

$$l_{yx'} = -\sin a, \quad l_{yy'} = \cos a, \quad l_{yz'} = 0$$

$$l_{zx'} = \cos a \sin i, \quad l_{zy'} = \sin a \sin i, \quad l_{zz'} = \cos i$$



Stresses around the hole:

$$\sigma_r = \frac{\sigma_x^0 + \sigma_y^0}{2} \left(1 - \frac{R_w^2}{r^2}\right) + \frac{\sigma_x^0 - \sigma_y^0}{2} \left(1 + 3\frac{R_w^4}{r^4} - 4\frac{R_w^2}{r^2}\right) \cos 2\theta$$

$$+ \tau_{xy}^0 \left(1 + 3\frac{R_w^4}{r^4} - 4\frac{R_w^2}{r^2}\right) \sin 2\theta + p_w \frac{R_w^2}{r^2}$$

$$\sigma_\theta = \frac{\sigma_x^0 + \sigma_y^0}{2} \left(1 + \frac{R_w^2}{r^2}\right) - \frac{\sigma_x^0 - \sigma_y^0}{2} \left(1 + 3\frac{R_w^4}{r^4}\right) \cos 2\theta$$

$$- \tau_{xy}^0 \left(1 + 3\frac{R_w^4}{r^4}\right) \sin 2\theta - p_w \frac{R_w^2}{r^2}$$

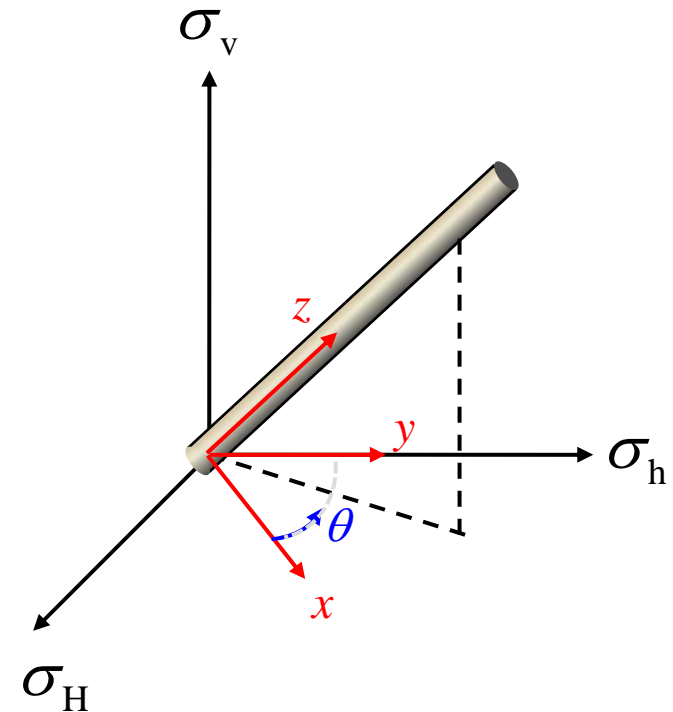
$$\sigma_z = \sigma_z^0 - \nu_{fr} \left[2(\sigma_x^0 - \sigma_y^0) \frac{R_w^2}{r^2} \cos 2\theta + 4\tau_{xy}^0 \frac{R_w^2}{r^2} \sin 2\theta \right]$$

$$\tau_{r\theta} = \frac{\sigma_y^0 - \sigma_x^0}{2} \left(1 - 3\frac{R_w^4}{r^4} + 2\frac{R_w^2}{r^2}\right) \sin 2\theta$$

$$+ \tau_{xy}^0 \left(1 - 3\frac{R_w^4}{r^4} + 2\frac{R_w^2}{r^2}\right) \cos 2\theta$$

$$\tau_{\theta z} = (-\tau_{xz}^0 \sin \theta + \tau_{yz}^0 \cos \theta) \left(1 + \frac{R_w^2}{r^2}\right)$$

$$\tau_{rz} = (\tau_{xz}^0 \cos \theta + \tau_{yz}^0 \sin \theta) \left(1 - \frac{R_w^2}{r^2}\right)$$



The shear stresses do not vanish
 σ_r , σ_θ , σ_z are not principal stresses

Stresses at the borehole wall:

$$\sigma_r = p_w$$

$$\sigma_\theta = \sigma_x^0 + \sigma_y^0 - 2(\sigma_x^0 - \sigma_y^0) \cos 2\theta - 4\tau_{xy}^0 \sin 2\theta - p_w$$

$$\sigma_z = \sigma_z^0 - \nu_{fr} [2(\sigma_x^0 - \sigma_y^0) \cos 2\theta + 4\tau_{xy}^0 \sin 2\theta]$$

$$\tau_{r\theta} = 0$$

$$\tau_{\theta z} = 2(-\tau_{xz}^0 \sin \theta + \tau_{yz}^0 \cos \theta)$$

$$\tau_{rz} = 0$$

At the borehole wall, only the radial stress is a principal stress, in general.

Stresses at the borehole wall:

$$\sigma_r = p_w$$

$$\sigma_\theta = \sigma_x^0 + \sigma_y^0 - 2(\sigma_x^0 - \sigma_y^0) \cos 2\theta - 4\tau_{xy}^0 \sin 2\theta - p_w$$

$$\sigma_z = \sigma_z^0 - \nu_{fr} [2(\sigma_x^0 - \sigma_y^0) \cos 2\theta + 4\tau_{xy}^0 \sin 2\theta]$$

$$\tau_{r\theta} = 0$$

$$\tau_{\theta z} = 2(-\tau_{xz}^0 \sin \theta + \tau_{yz}^0 \cos \theta)$$

$$\tau_{rz} = 0$$

These equations are valid for constant pore pressure.

Due to the superposition principle, the effects of a pore pressure gradient can easily be added.

Special case:
Vertical well,
anisotropic horizontal stresses:

$$\sigma_H > \sigma_h$$

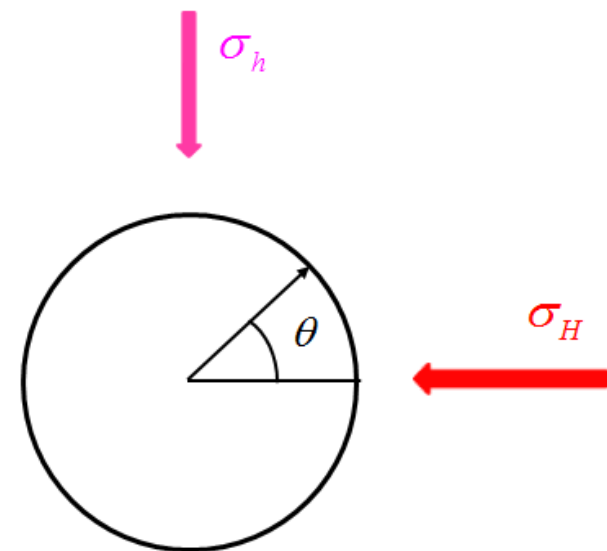
$$\sigma_r = \frac{\sigma_H + \sigma_h}{2} \left(1 - \frac{R_w^2}{r^2}\right) + \frac{\sigma_H - \sigma_h}{2} \left(1 + 3\frac{R_w^4}{r^4} - 4\frac{R_w^2}{r^2}\right) \cos 2\theta + p_w \frac{R_w^2}{r^2}$$

$$\sigma_\theta = \frac{\sigma_H + \sigma_h}{2} \left(1 + \frac{R_w^2}{r^2}\right) - \frac{\sigma_H - \sigma_h}{2} \left(1 + 3\frac{R_w^4}{r^4}\right) \cos 2\theta - p_w \frac{R_w^2}{r^2}$$

$$\sigma_z = \sigma_v - 2\nu_{fr}(\sigma_H - \sigma_h) \frac{R_w^2}{r^2} \cos 2\theta$$

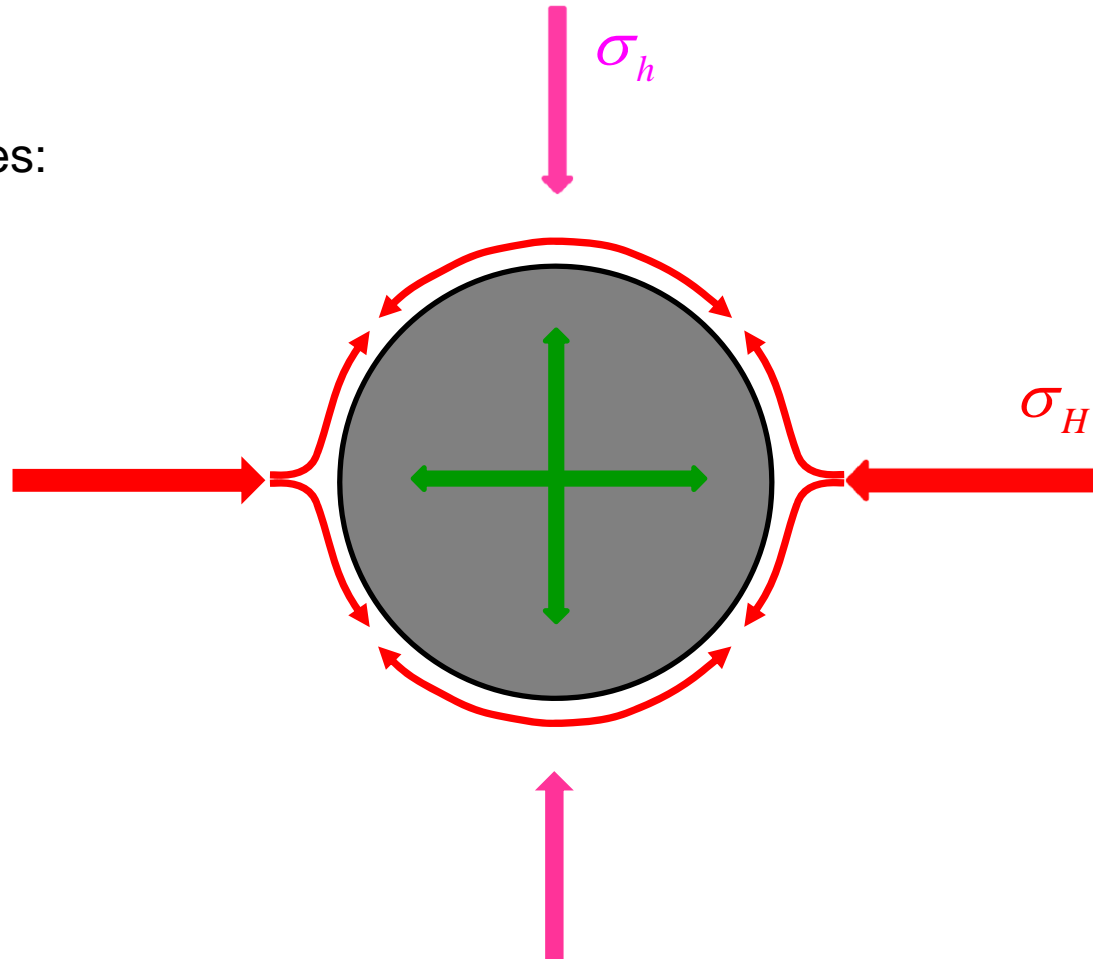
$$\tau_{r\theta} = -\frac{\sigma_H - \sigma_h}{2} \left(1 - 3\frac{R_w^4}{r^4} + 2\frac{R_w^2}{r^2}\right) \sin 2\theta$$

$$\tau_{rz} = \tau_{\theta z} = 0$$



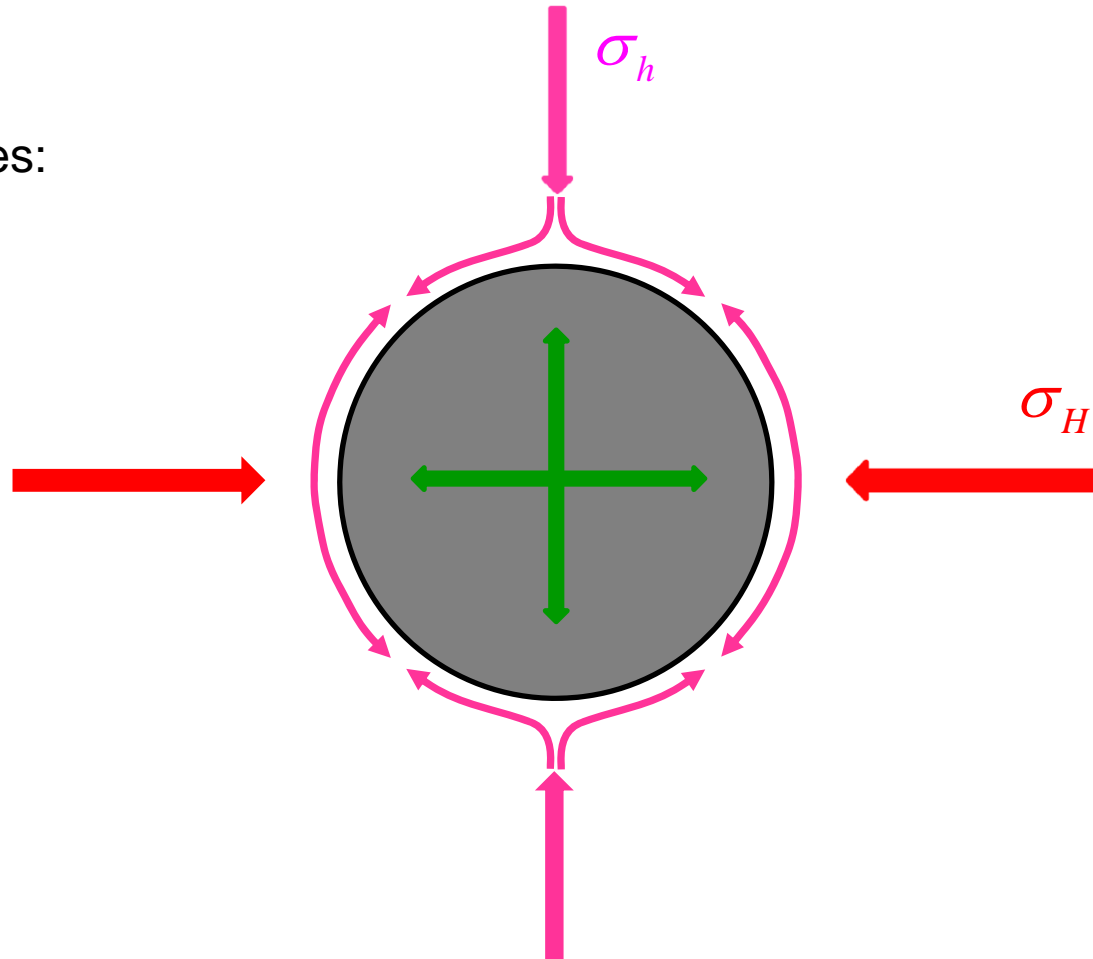
Special case:
Vertical well,
anisotropic horizontal stresses:

$$\sigma_H > \sigma_h$$



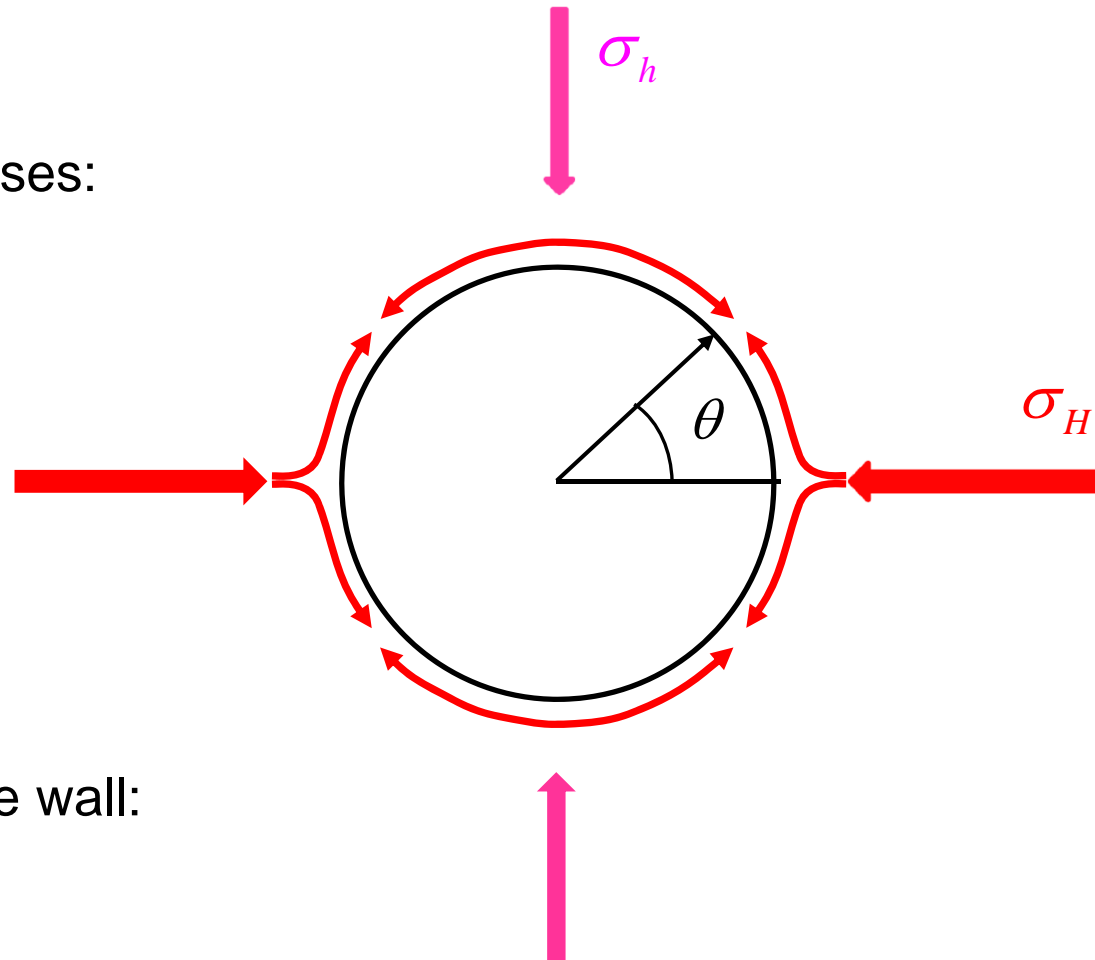
Special case:
Vertical well,
anisotropic horizontal stresses:

$$\sigma_H > \sigma_h$$



Special case:
Vertical well,
anisotropic horizontal stresses:

$$\sigma_H > \sigma_h$$



Stresses at the borehole wall:

$$\sigma_r = p_w$$

$$\sigma_\theta = \sigma_H + \sigma_h - 2(\sigma_H - \sigma_h) \cos 2\theta - p_w$$

$$\sigma_z = \sigma_v - 2\nu_{fr}(\sigma_H - \sigma_h) \cos 2\theta$$

$$\tau_{r\theta} = \tau_{\theta z} = \tau_{rz} = 0$$

The tangential stress varies with
position along the perimeter

What about pore pressure?

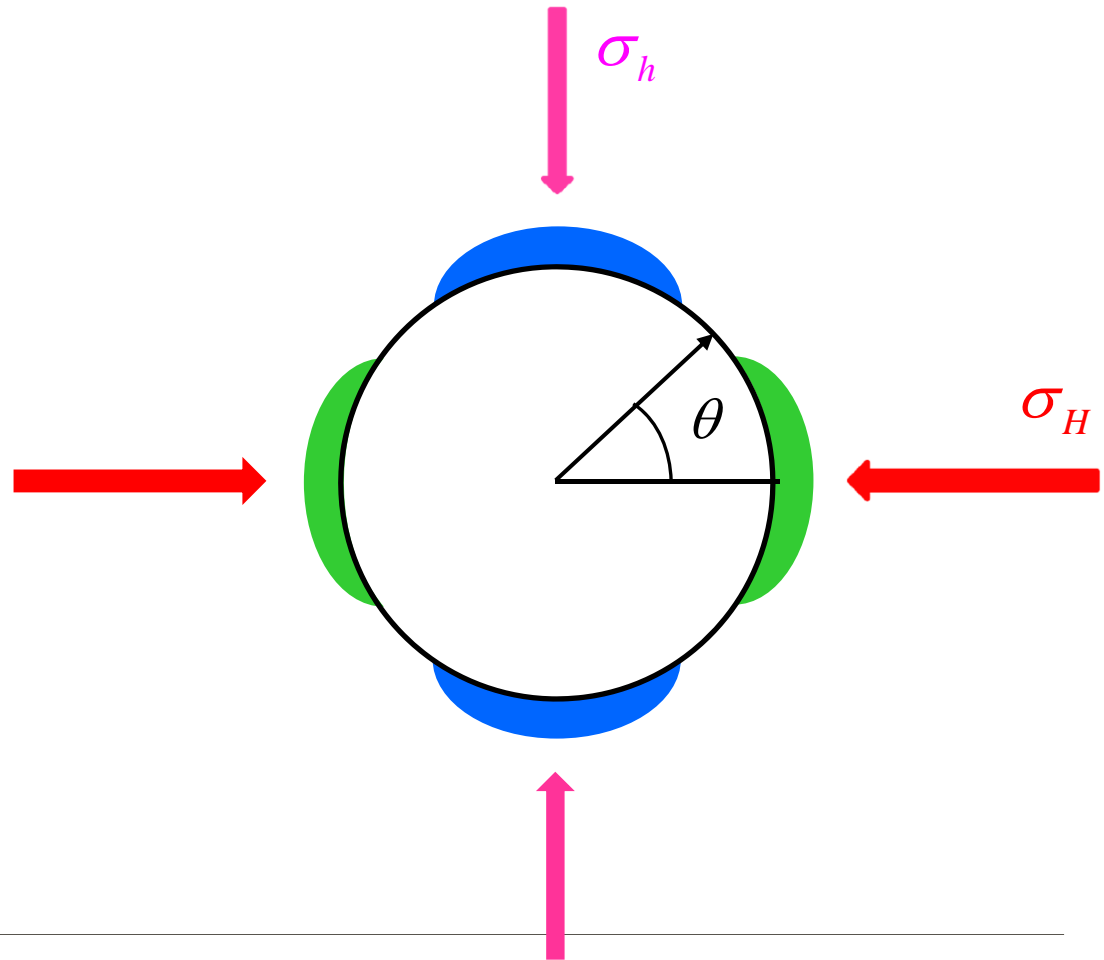
$$\text{Mean stress } 3\bar{\sigma} = \sigma_h + \sigma_H + \sigma_v - 2(1 + \nu_{fr})(\sigma_H - \sigma_h)\cos 2\theta \neq \text{constant}$$

At $\theta = 0^\circ$ and $\theta = 180^\circ$
the mean stress decreases

- ⇒ Volume expansion
- ⇒ **Pore pressure reduction**

At $\theta = 90^\circ$ and $\theta = 270^\circ$
the mean stress increases

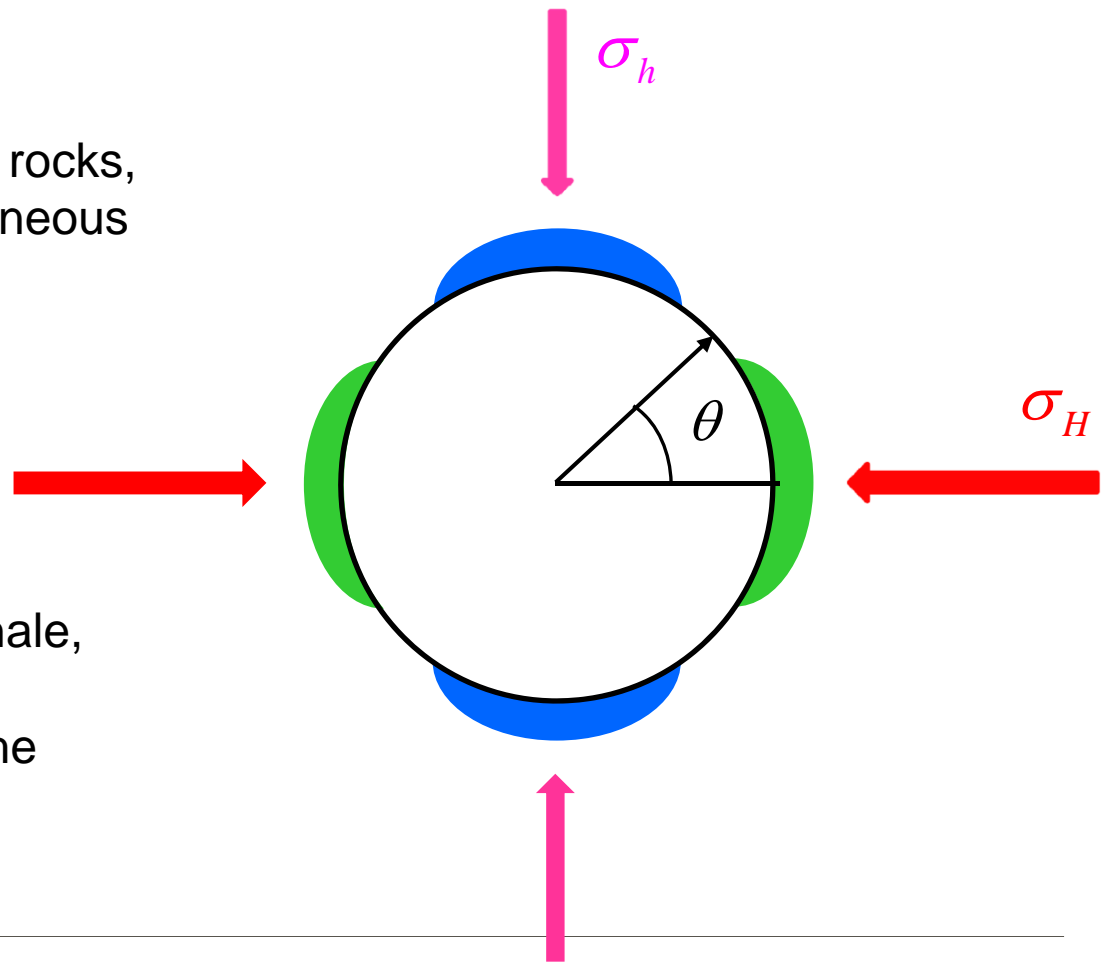
- ⇒ Volume reduction
- ⇒ **Pore pressure increase**



What about pore pressure?

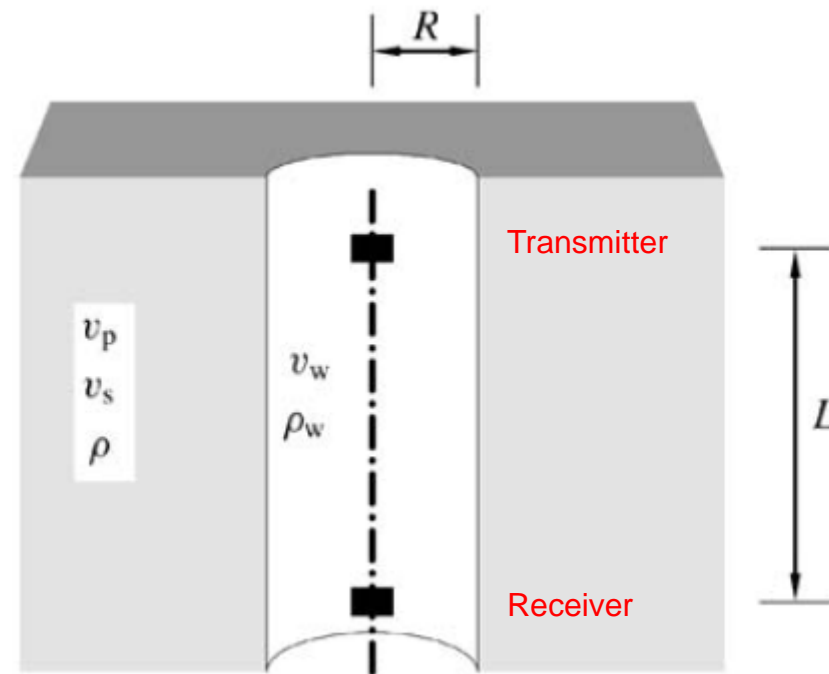
Pore pressure alterations are insignificant in high permeable rocks, like sandstone (nearly instantaneous pore pressure equalization).

In low permeable rocks, like shale, the pore pressure alterations may be significant and affect the stability of the hole.



Borehole acoustics

Principles of a sonic logging tool

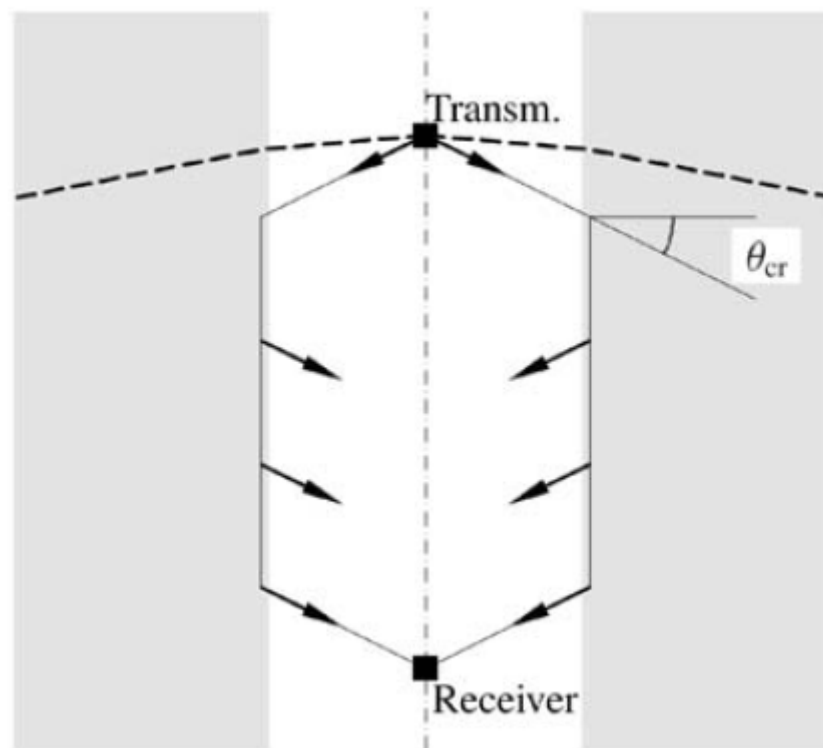


Borehole acoustics

Principles of a sonic logging tool

Traveltime for refracted wave:

$$= \frac{L}{v} + 2R \sqrt{\frac{1}{v_w^2} - \frac{1}{v^2}}$$

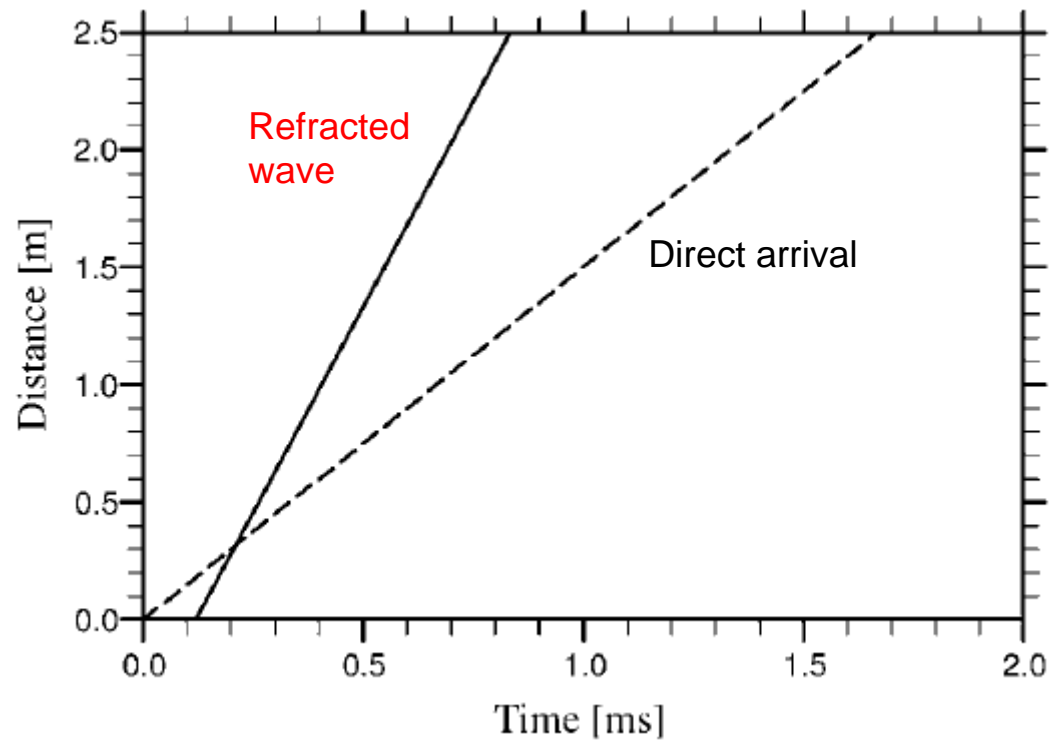


Borehole acoustics

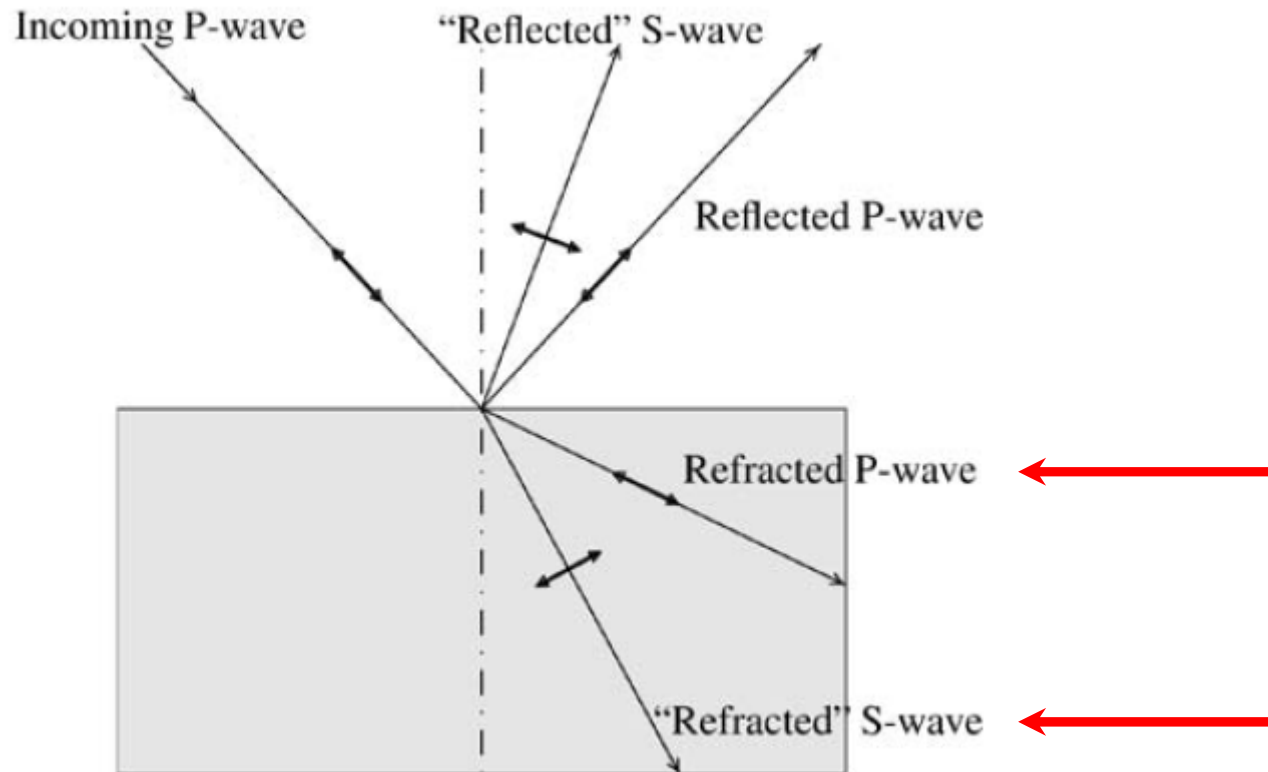
Principles of a sonic logging tool

Traveltime for refracted wave:

$$= \frac{L}{v} + 2R \sqrt{\frac{1}{v_w^2} - \frac{1}{v^2}}$$



Both P- and S-wave generated by refraction



In addition: Borehole modes (exist only at the borehole)

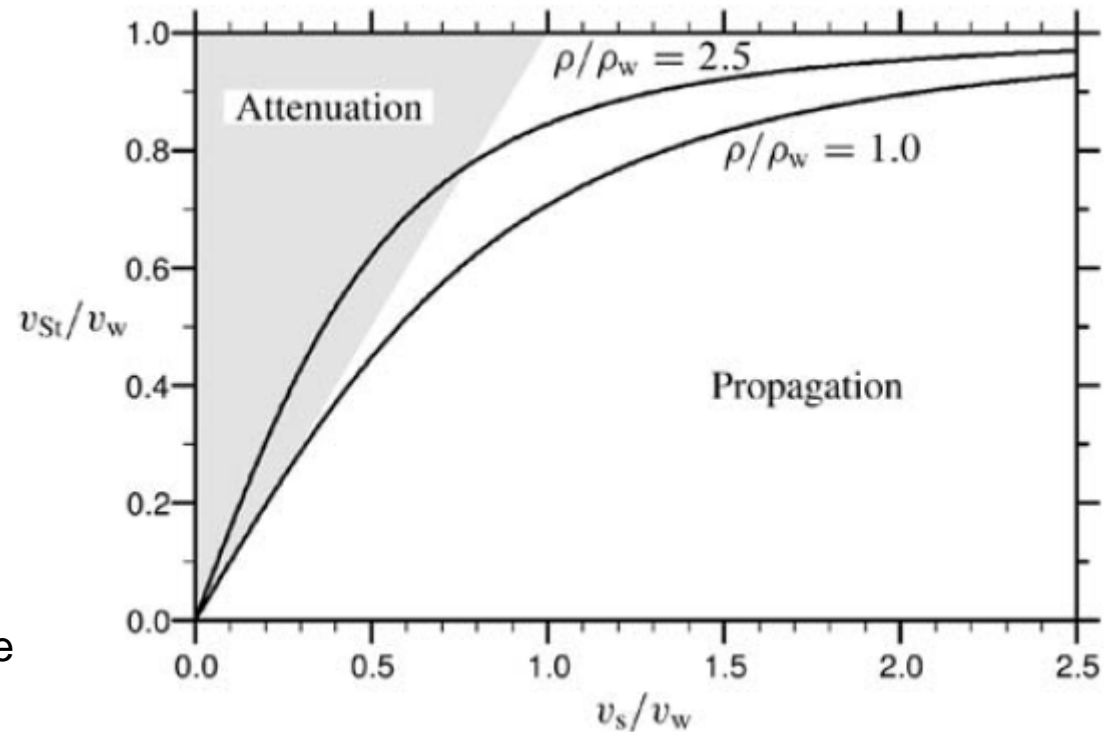
Soneley wave
(actually: Scholte wave)

Velocity:

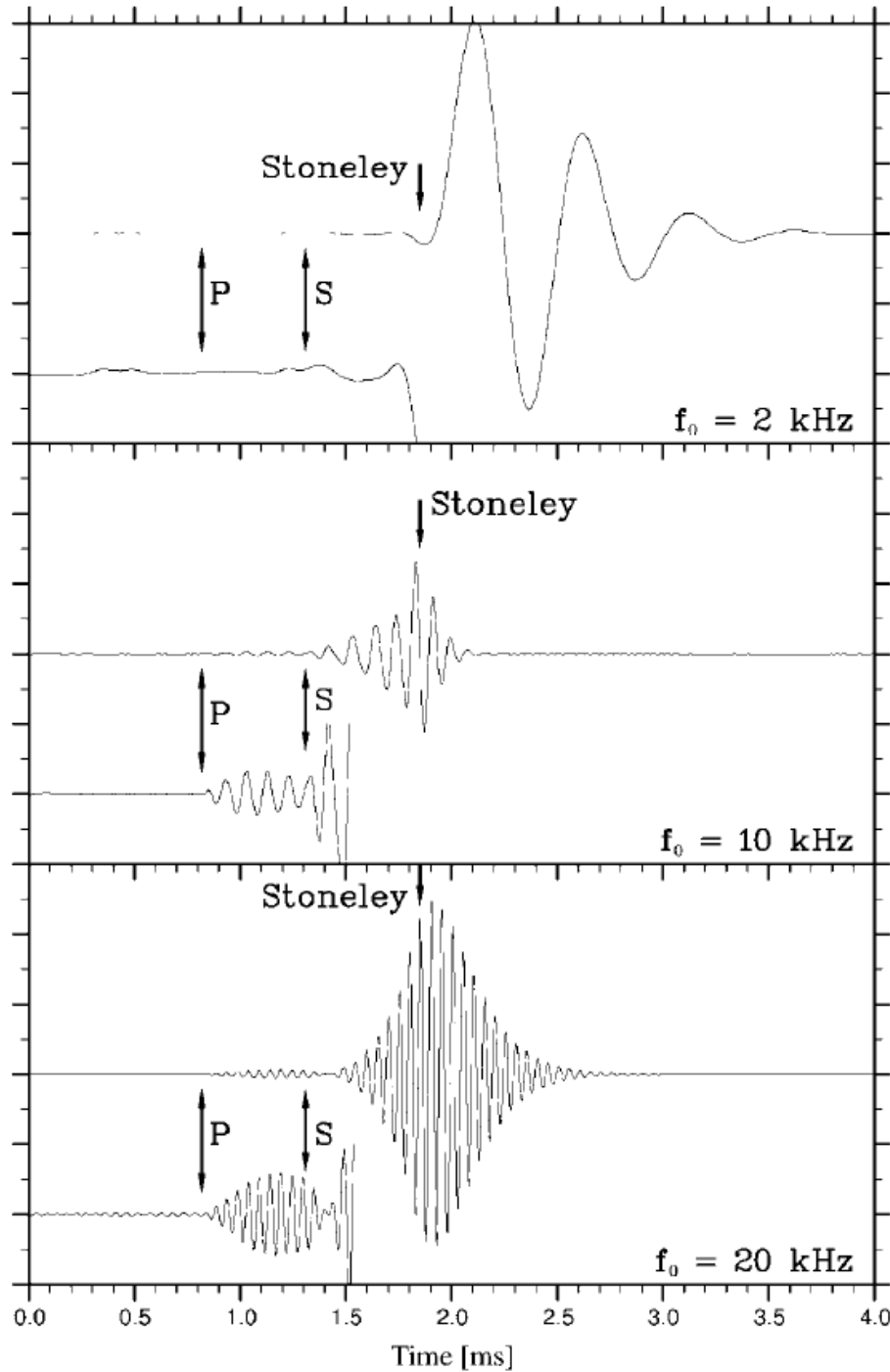
$$v_{St} = \frac{v_w}{\sqrt{1 + \frac{\rho_w v_w^2}{\rho v_s^2}}}$$

May be used to determine the shear wave velocity in slow formations, but -

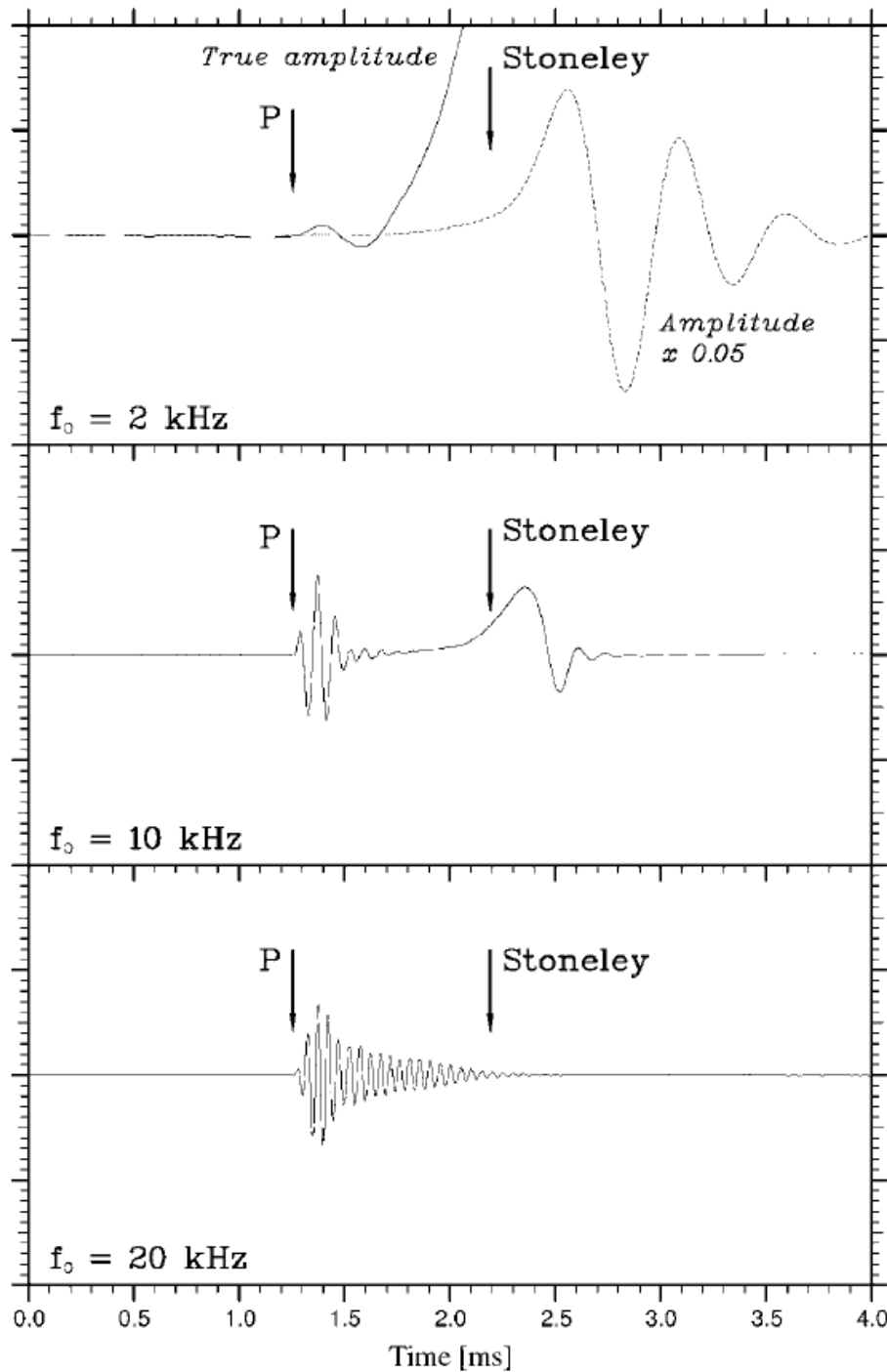
- Attenuation is also high
- Anisotropy complicates
- Permeability complicates



Received signal
in a **fast** formation



Both P- and S-arrivals
can be identified

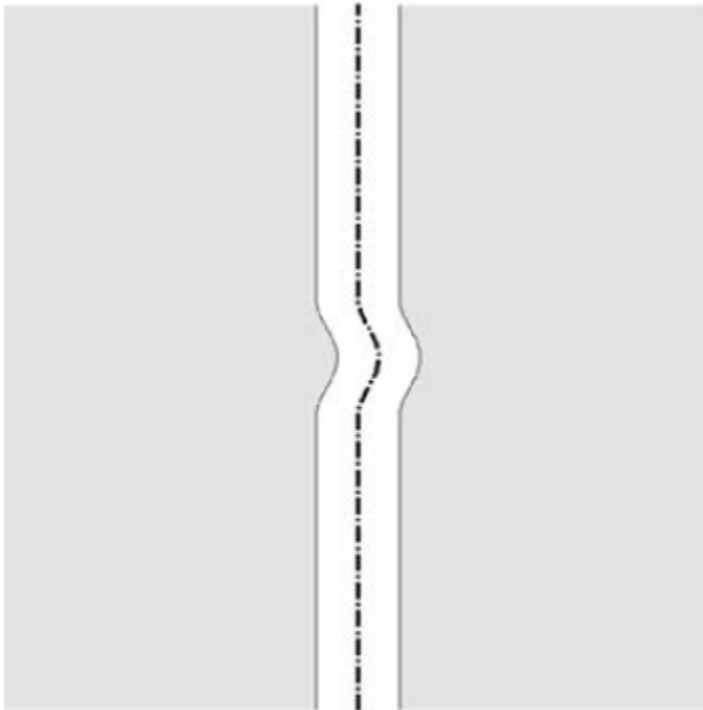


Received signal
in a **slow** formation

Only P-arrival

(+ Stoneley, at low frequencies)

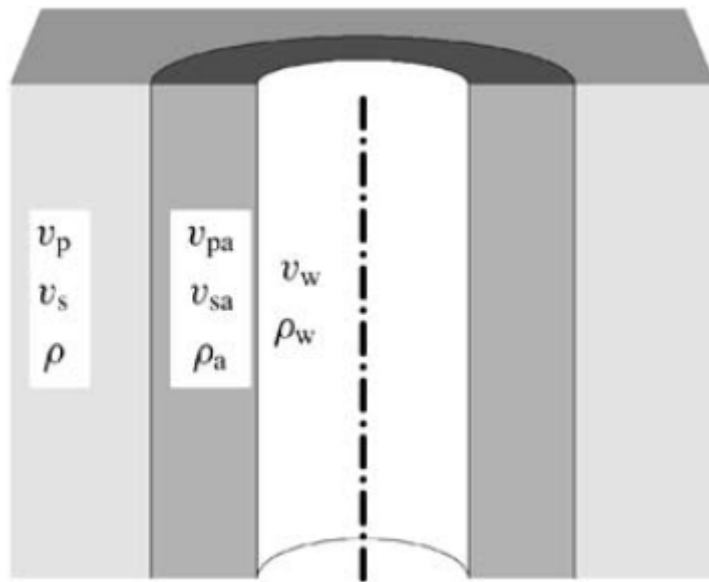
Dipole sources
excite "flexural" mode



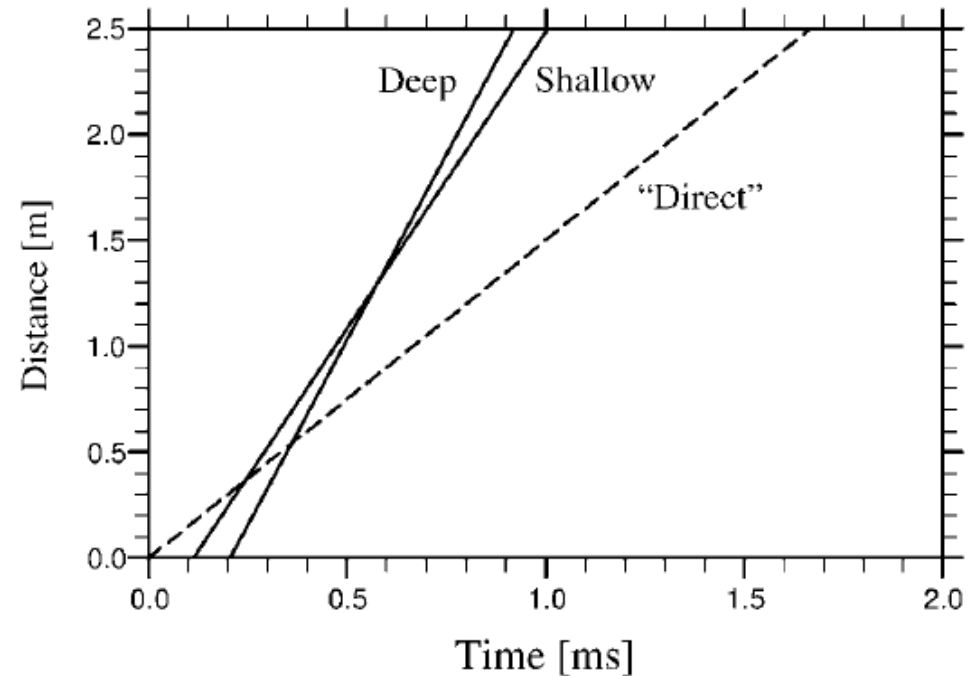
For low frequencies
(wavelength \gg borehole diameter)
the flexural mode travels with the
velocity of the S-wave

Enables measurement of S-wave
velocity also in slow formations

Borehole with an axisymmetric altered zone (due to mud invasion, or stress induced damage)



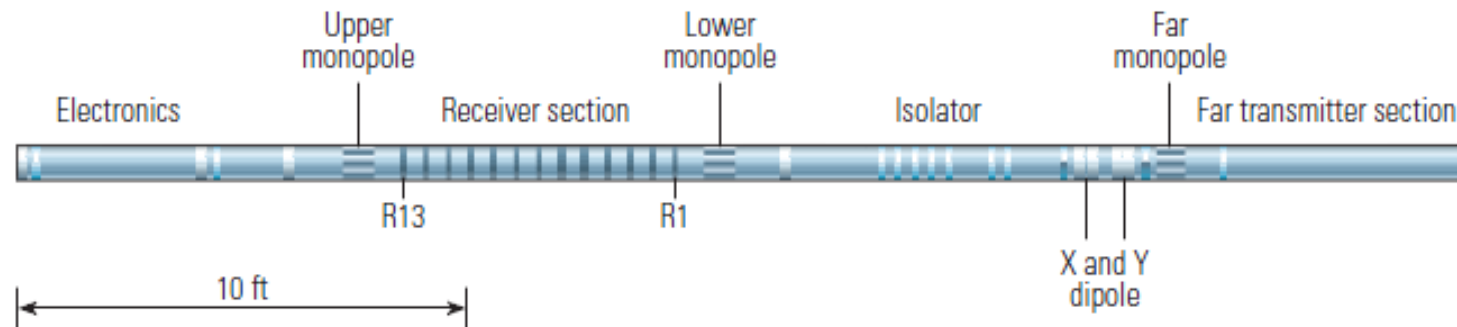
Two refracted waves
– shallow and deep



Standard long-spaced sonic tools
measure the arrivals of the deep refraction
(\leftrightarrow properties of the virgin formation)

New generation of logging tools map the (entire?) velocity field in the vicinity of the hole

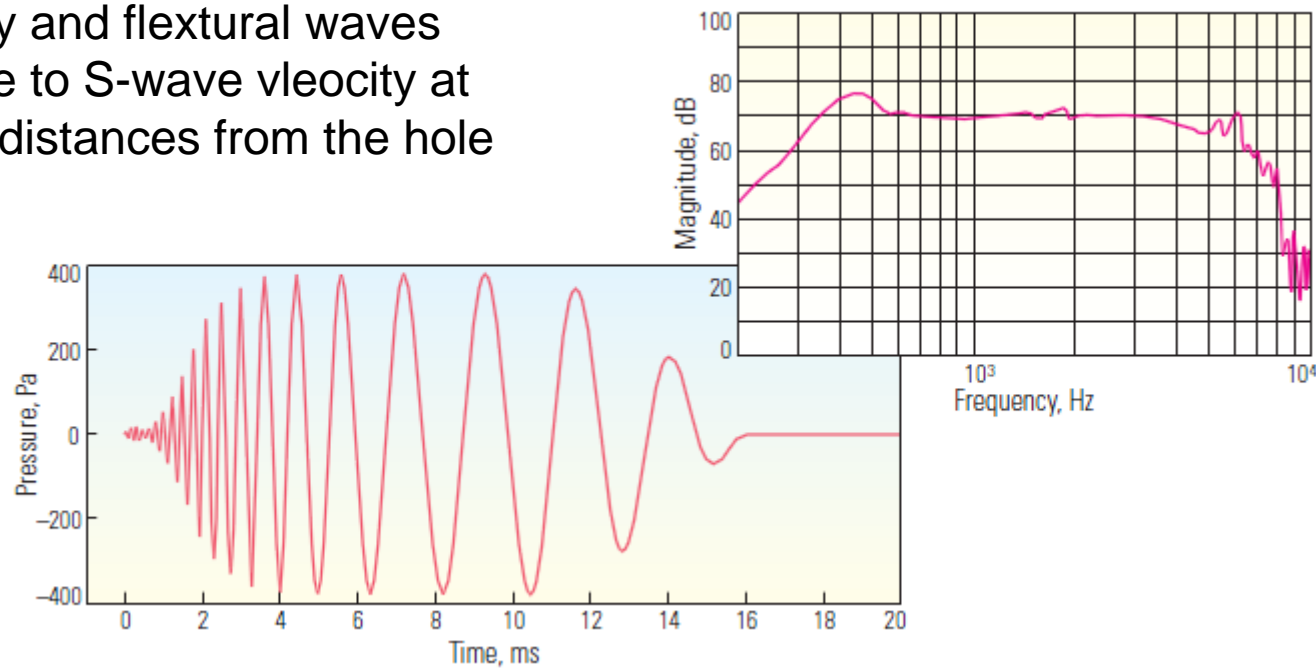
Example: Sonic Scanner (Schlumberger)



New generation of logging tools map the (entire?) velocity field in the vicinity of the hole

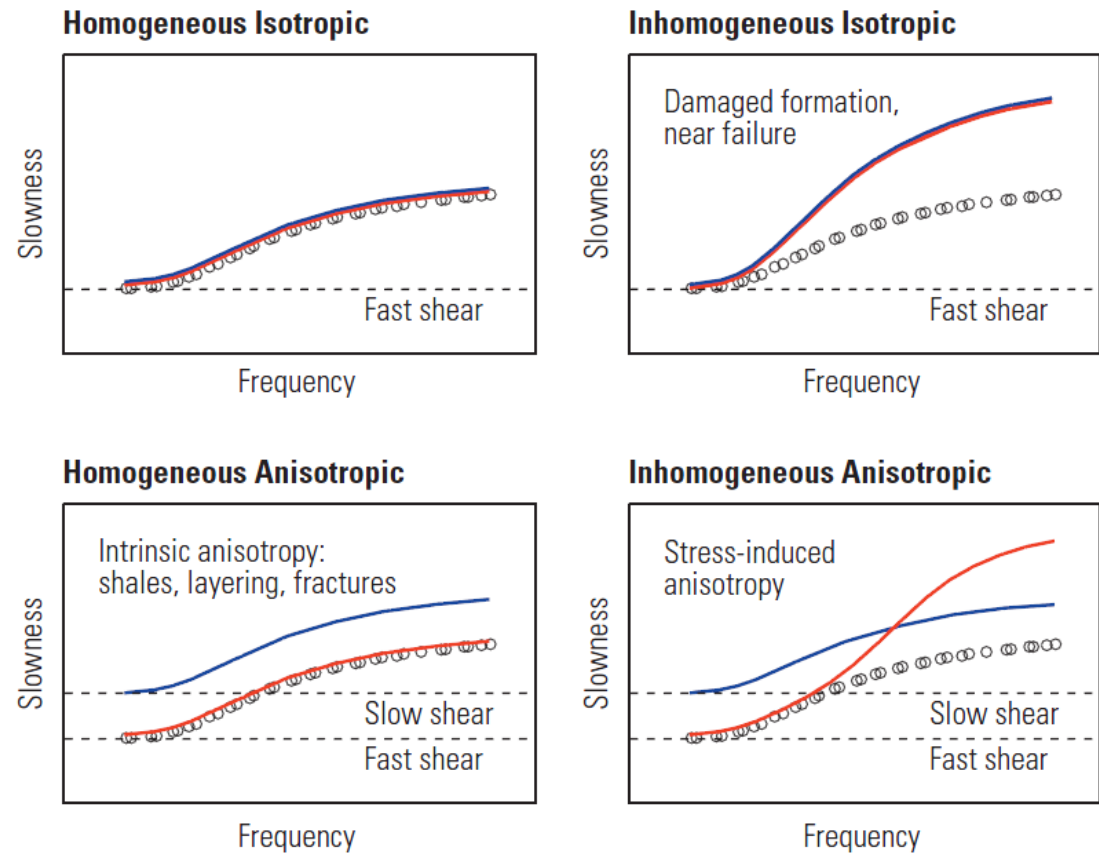
Example: Sonic Scanner (Schlumberger)

Wide-band sources excite Stoneley and flexural waves sensitive to S-wave velocity at various distances from the hole



New generation of logging tools map the (entire?) velocity field in the vicinity of the hole

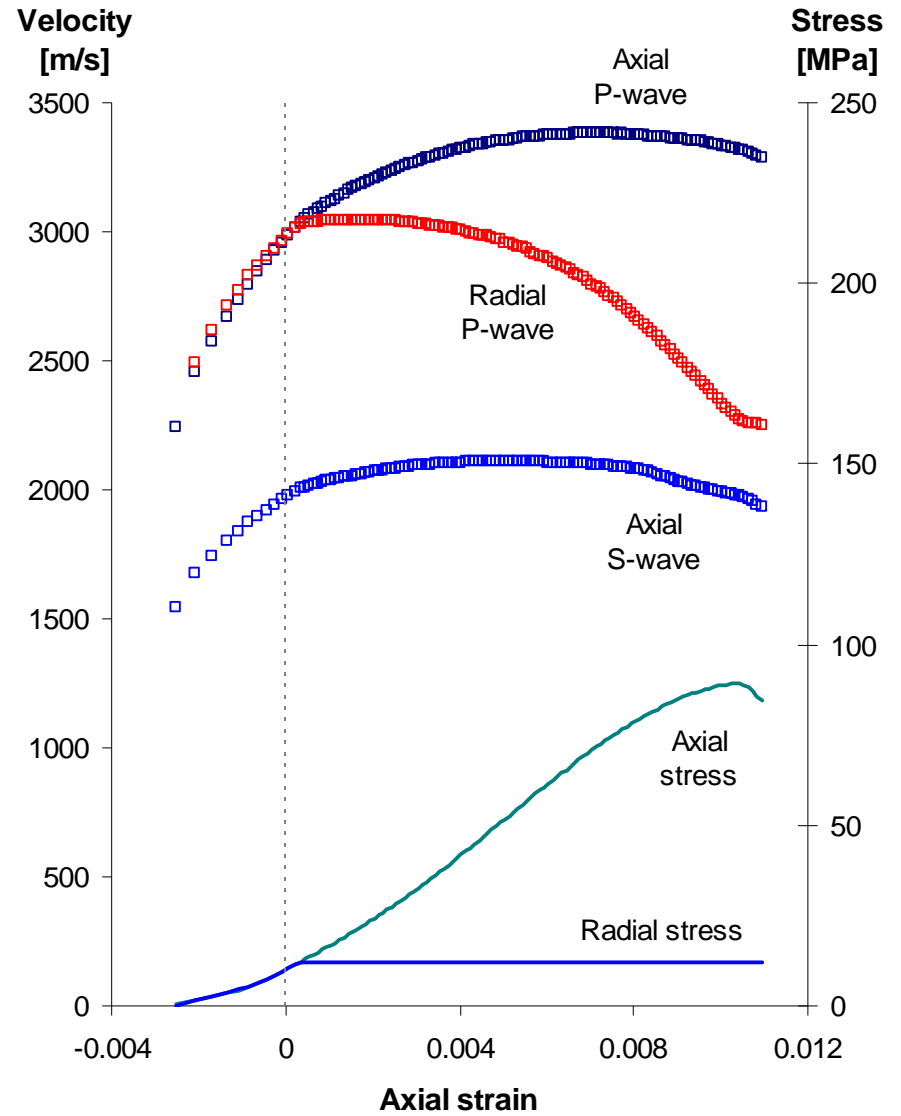
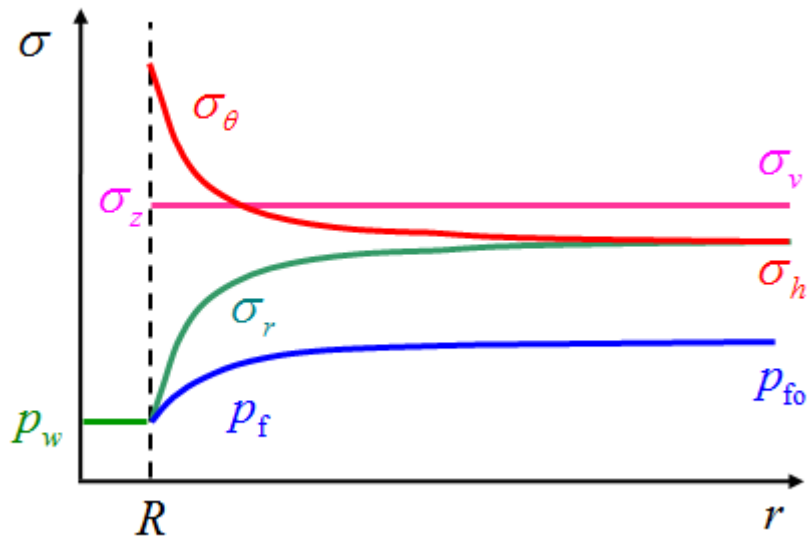
Example: Sonic Scanner (Schlumberger)

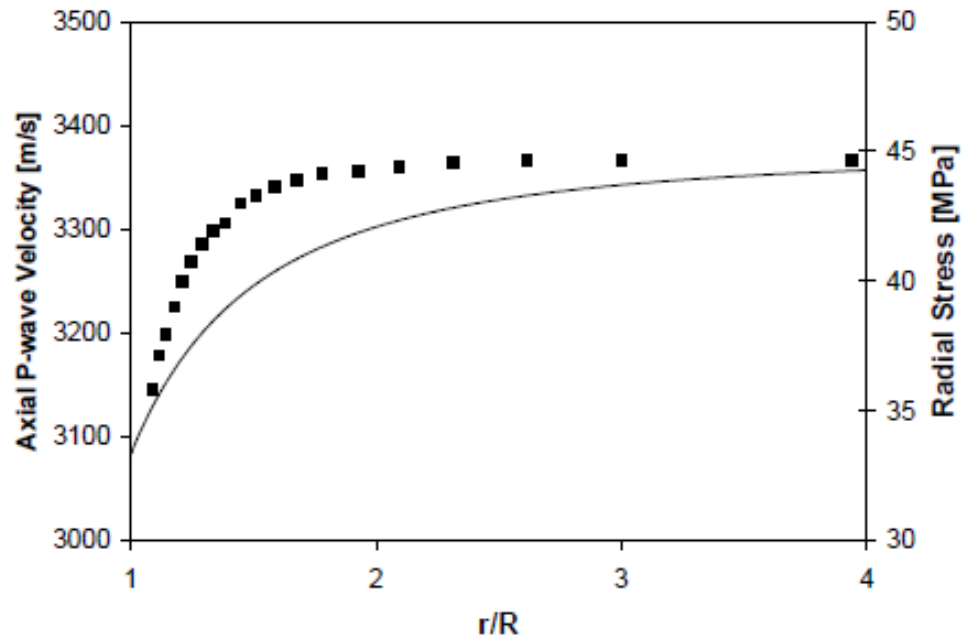


Flexural modes from two orthogonal dipoles

oooo : model for isotropic, homogeneous formation

Velocities are stress dependent
 – can we use the tool to identify
 in situ stresses?

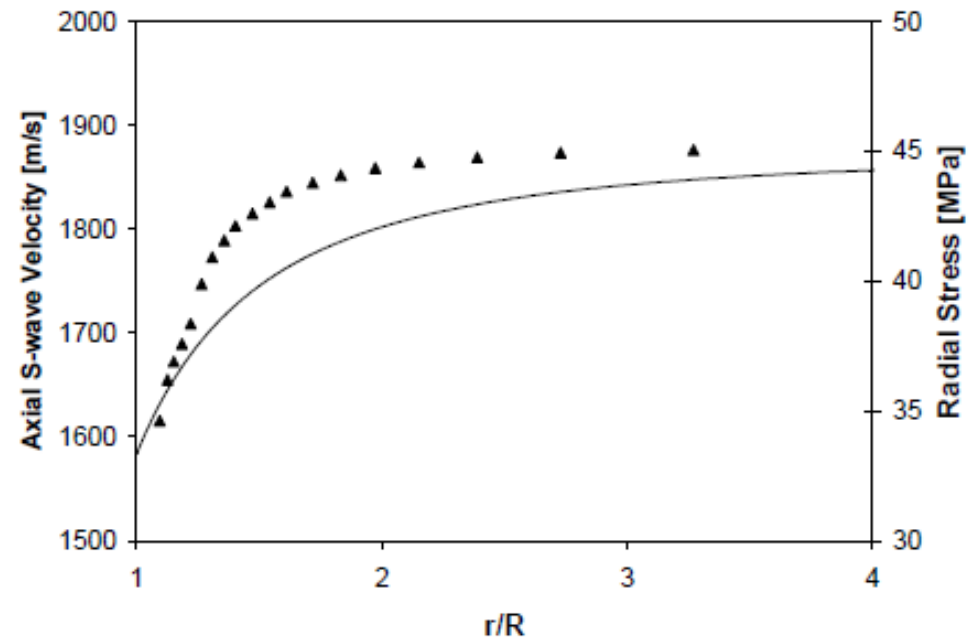




Velocities versus distance from borehole

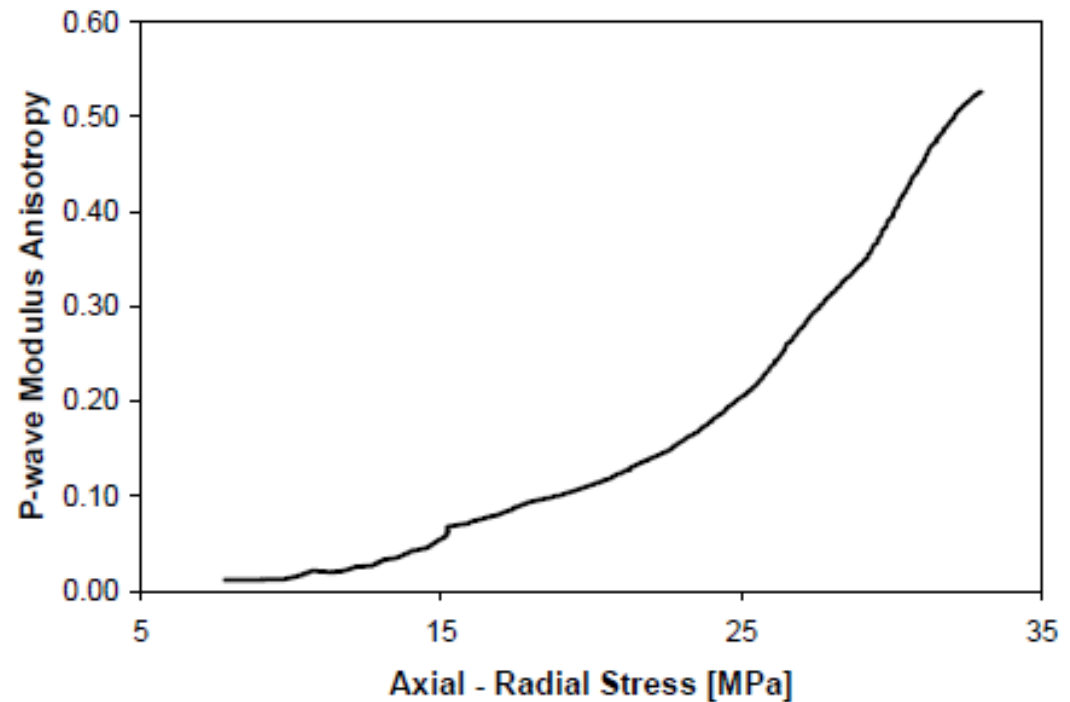
- from tests on rock cemented under stress

Fjær and Holt (1999)



Application for stress determination depends on knowledge about the velocity-stress relations (usually assumed to be linear)

Unique,
but not linear relation



Fjær and Holt (1999)

References:

Fjær, E., Holt, R.M., Horsrud, P., Raaen, A.M. and Risnes, R. (2008) "Petroleum Related Rock Mechanics. 2nd Edition". Elsevier, Amsterdam

Arroyo Franco, J.L., Mercado Ortiz, M.A., De, G.S, Renlie, L. and Williams, S. (2006) "Sonic investigations in and around the borehole". Oilfield Review, **18**, 14-31.

Fjær, E. and Holt, R. M. (1999) "Stress and stress release effects on acoustic velocities from cores, logs and seismics", 40th SPWLA Annual Logging Symposium, Oslo